

**LOAD BEARING CAPACITY OF ALDER,  
SPRUCE AND HEMLOCK  
TAIL TREES**

by

Edwin V. Pugh

A PAPER

submitted to

Oregon State University

in partial fulfillment of  
the requirements for the  
degree of

Master of Forestry

Completed May 1985

Commencement June 1985

APPROVED:

*Marvin R. Pyle*

Assistant Professor of Forest Engineering in charge of major

*Samuel W. Pugh*

Head of the Department of Forest Engineering

Date paper is presented May 6, 1985

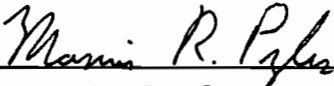
Typed by Judith Sessions for Edwin V. Pugh

AN ABSTRACT OF THE THESIS OF

Edwin V. Pugh for the degree of Master of Forestry in  
Forest Engineering presented on May 6, 1985.

Title: Load Bearing Capacity of Alder, Spruce and Hemlock  
Tail Trees.

Abstract approved: \_\_\_\_\_

  
Dr. Marvin R. Pyles

This paper presents an evaluation of the cable loading support capacity of red alder, Alnus rubra Bong., Sitka spruce, Picea sitchensis (Bong.) Carr, and western hemlock, Tsuga heterophylla (Raf.) Sarg., tail trees. Capacity is measured in terms of combined stress resulting from compression and bending, rather than the traditional methods of buckling or compressive stress alone.

Results from field tests to determine moduli of elasticity, base stiffness values, and functions for moment of inertia are presented to provide strength properties for capacity analysis.

A two dimensional model with one guyline is used to calculate the combined stress at points along the trees. In addition to strength properties of each species, model inputs include front and rear skyline angles, rigging

height, and the following guyline parameters: angle, metallic area, unit weight, modulus of elasticity, and lower end pretension.

The control calculations for each species are made with the guyline angle equal to a rear skyline angle of 45 degrees. A 3/4" guyline with 100 pounds of pretension is used, and the skyline and guyline are placed at a height of 30 feet. Given these conditions, it was found that a skyline angle of about 15 degrees below horizontal maximized combined stress per pound of skyline tension in alder and spruce. An angle of about 10 degrees below horizontal was found to maximize stress in hemlock per pound of skyline tension.

Figures are presented which show that skyline tension to a given level of stress may be a function of tree diameter, if other variables are held constant.

Values for maximum allowable combined stress for each species are set by adjusting published average values downward. Calculations for 16 inch (diameter inside bark) trees indicate that hemlock is able to withstand the greatest skyline tension of the three species before reaching its allowable stress, with alder and spruce following in descending order.

A comparison is made between a 14 inch DIB Douglas-fir, Pseudotsuga menziesii (Mirb.) Franco, and a 16 inch DIB alder, spruce and hemlock. Calculations indicate the

hemlock can withstand about 9% more skyline tension to its allowable stress than the Douglas-fir. An alder slightly over 17 inches DIB would be needed to support the same tension, and a spruce with a DIB over 18 inches, which is outside the range of field data, would be needed.

## TABLE OF CONTENTS

	Page
I. INTRODUCTION .....	1
II. OBJECTIVES AND SCOPE .....	6
III. FIELD STUDY .....	9
A. Study Sites .....	9
B. Study Methods .....	10
IV. RESULTS .....	15
A. Moment of Inertia .....	15
B. Modulus of Elasticity .....	24
C. Base Stiffness .....	33
D. Tail Tree Analysis .....	51
1. Alder Analysis .....	60
2. Spruce Analysis .....	67
3. Hemlock Analysis .....	70
4. Parameter Study .....	72
a. Modulus of Elasticity .....	72
b. Guyline Pretension .....	78
c. Guyline Diameter .....	80
d. Rigging Height .....	82
5. Comparison with Douglas-fir .....	86
V. SUMMARY .....	89
VI. SUGGESTIONS FOR FURTHER RESEARCH.....	91
VII REFERENCES .....	92
APPENDICES .....	93
A. Method for Forcing a Regression Line Through a Point .....	94
B. Variables in Modulus of Elasticity Calculations .....	98

## LIST OF FIGURES

<u>Figure</u>	<u>Page</u>
1. Cable Logging System Support Trees.....	2
2. Simplified Tail Tree with Cable Forces.....	2
(a) Assuming no deflection occurs under loading.	
(b) assuming lateral deflection does occur under loading.	
3. Field Study Rigging Method.....	11
4. Alder Moments of Inertia as a Function of Height..	16
5. Normalized Alder Moments of Inertia and Functions.	18
6. Normalized Hemlock Moments of Inertia and Functions.....	21
7. Normalized Spruce Moments of Inertia and Functions.....	23
8. Relationship Between Rotation and Bending Deflections.....	25
9. Structural Model of a Tree.....	35
10. Relationship of Base Moment to Base Rotation.....	37
11. Alder Base Stiffness as a Function of Tree Diameter.....	42
12. Hemlock Base Stiffness as a Function of Tree Diameter.....	44
13. Spruce Base Stiffness as a Function of Tree Diameter.....	47
14. Model for Combined Stress Calculations.....	54
15. Skyline Tension to Allowable Stress in Alder for Various Skyline Angles.....	59
16. Combined stress vs. Vertical Load for an 18 Inch DIB Alder.....	61
17. Combined stress vs. Skyline tension for an 18 Inch DIB Alder.....	64

## LIST OF FIGURES

(continued)

<u>Figure</u>		<u>Page</u>
18.	Combined stress vs. Skyline Tension for Various Alder Diameters.....	65
19.	Skyline Tension to Allowable Stress in Alder vs. Tree Diameter.....	66
20.	Combined stress vs. Skyline Tension for Various Spruce Diameters.....	68
21.	Skyline Tension to Allowable Stress in Spruce vs. Tree Diameter.....	69
22.	Combined stress vs. Skyline Tension for Various Hemlock Diameters.....	71
23.	Combined stress in a 16 Inch DIB Alder vs. Skyline Tension for Alternative Moduli of Elasticity.....	74
24.	Combined stress in a 16 Inch DIB Spruce vs. Skyline Tension for Alternative Moduli of Elasticity.....	76
25.	Combined stress in a 16 Inch DIB Hemlock vs. Skyline Tension for Alternative Moduli of Elasticity.....	77
26.	Combined stress in a 16 Inch DIB Alder vs. Skyline Tension for Various Guyline Pretensions.....	79
27.	Combined stress in a 16 Inch DIB Alder vs. Skyline Tension for Various Guyline Diameters.....	81
28.	Combined stress in a 16 Inch DIB Hemlock vs. Skyline Tension for Various Rigging Heights.....	83
29.	Combined stress in a 16 Inch DIB Spruce vs. Skyline Tension for Various Rigging Heights.....	84
30.	Combined stress in a 16 Inch DIB Alder vs. Skyline Tension for Various Rigging Heights.....	85
31.	Combined stress vs. Skyline Tension for a 14 Inch DIB Douglas-fir, and a 16 Inch DIB Alder, Spruce and Hemlock.....	87



## LIST OF TABLES

<u>Table</u>		<u>Page</u>
1	Alder Modulus of Elasticity Results .....	28
2	Hemlock Modulus of Elasticity Results .....	32
3	Spruce Modulus of Elasticity Results .....	34
4	Alder Base Stiffness Values .....	39
5.	Hemlock Base Stiffness Values .....	43
6.	Spruce Base Stiffness Values .....	46
7.	Alder Base Stiffness Values for Decreasing Load Heights .....	49
8.	Spruce and Hemlock Base Stiffness Values for Decreasing Load Heights. ....	50
9	Maximum Allowable Compressive Stress Values .....	57
10.	Alternative Values for Modulus of Elasticity .....	73

## I. INTRODUCTION

Concern for logging impacts on soil and water quality, and economics often necessitates the use of tail trees in cable logging to provide sufficient lift to a turn of logs. Intermediate supports are sometimes also needed to meet suspension requirements. Figure 1 illustrates the use of a tail tree and a double tree intermediate support.

The Logging Division of the Oregon Occupational Safety and Health Code, Appendix 80-K (1984) contains a table of recommended minimum diameters for west coast Douglas-fir, Pseudotsuga menziesii (Mirb.) Franco, tail trees. Appendix 80-J contains a similar table for double-tree intermediate support systems. Two blanket recommendations are made for non-Douglas-fir species:

1. add two inches to recommended diameters for Douglas-fir tail trees if using another conifer species, and
2. reduce recommended Douglas-fir intermediate support loads by 25% when using other conifer species.

Both tables address the use of conifer species; however, conifers are not always located where a support is needed. Along the Coast Range of Oregon, red alder, Alnus rubra Bong., Sitka spruce, Picea sitchensis (Bong.) Carr,

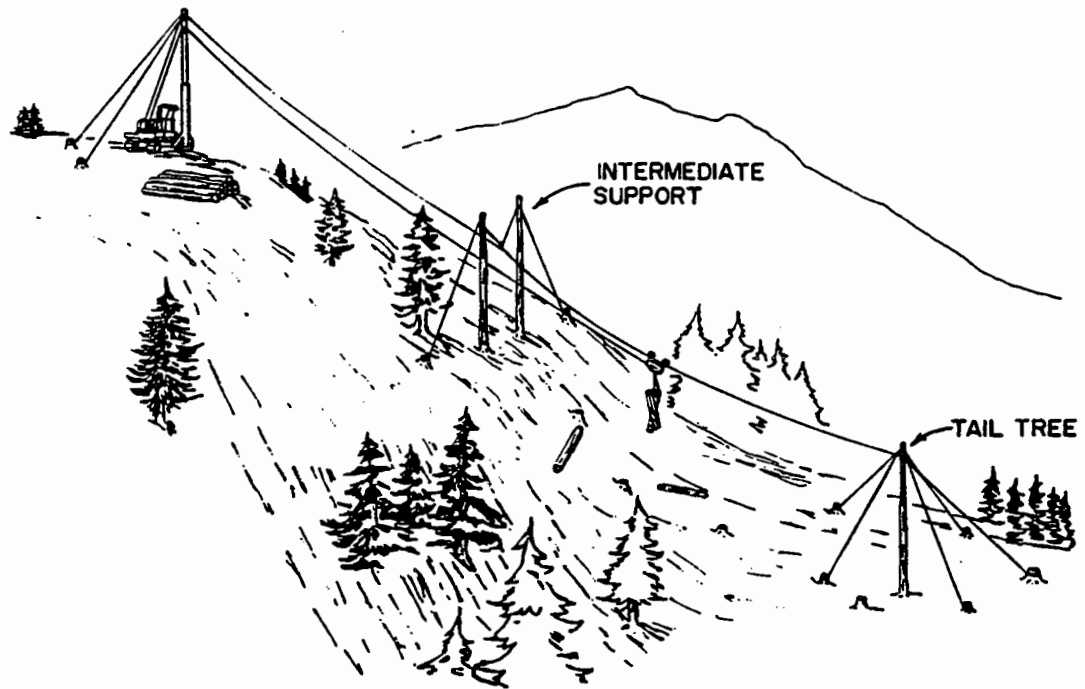


Figure 1. Cable Logging System Support Trees

Pyles, 1984

and western hemlock, Tsuga heterophylla (Raf.) Sarg., are very common species and many times are major components in a harvest unit. Until now, the load bearing capacity of these species has not been quantified.

If we look at a simplified side view of a tail tree rigging configuration (Figure 2a), and assume that the horizontal component of the skyline tension is offset by guylines (which are required in Oregon), then the major force of concern, with respect to failure, is the vertical (axial) component of the load ( $P_v$ ):

As summarized by Pyles (1984), a tree subjected to axial compression (load  $P_v$ ) to the point of failure can behave in one of three ways:

1. It can fail in direct compression if the compressive strength of the fibers is exceeded.
2. It can fail by exceeding the combined bending and compressive strength of the fibers when in a stable, elastic, deflected shape, or
3. It can fail catastrophically by becoming unstable and buckling.

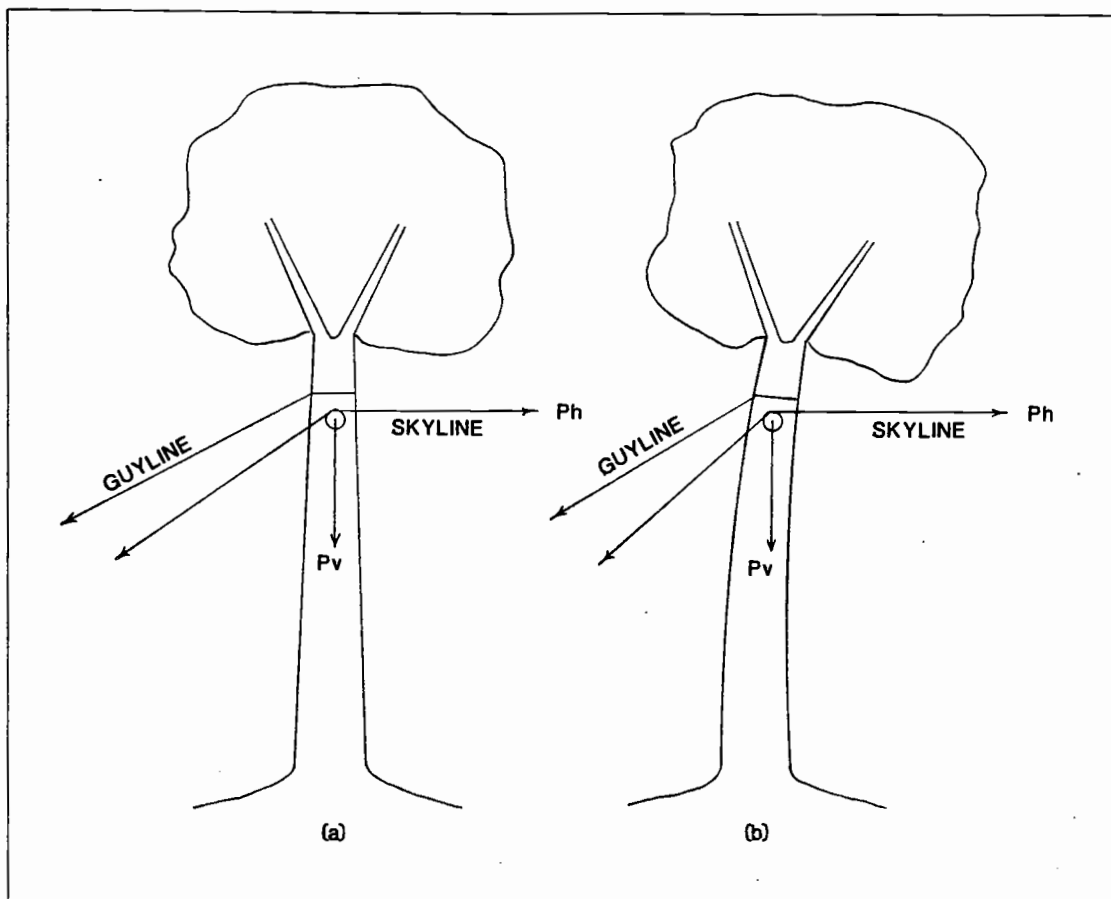


Figure 2. Simplified Tail Tree with Cable Forces  
(a) Assuming no deflection occurs under loading  
(b) assuming lateral deflection does occur  
under loading

If we assume that some lateral deflection of the top of the tree takes place, as shown in Figure 2b, then a combination of axial and flexural stresses will lead to failure. Pyles (1984) states this mode of failure (bending) is more likely to occur in cable support trees than a buckling failure.

This paper will concentrate on bending as the mode of failure. Comparisons with buckling as the mode of failure will also be made.

## II. OBJECTIVES AND SCOPE

To determine the load bearing capacity of tail trees in terms of combined axial and bending stresses, solutions will be needed for the following relationship between bending moment and deflection for an elastic curve:

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \quad (1)$$

where:

- y = lateral deflection of the tree, inches
- x = vertical position along the tree, inches
- M = bending moment, inch - pounds
- E = modulus of elasticity, psi
- I = moment of inertia, inches<sup>4</sup>
- D = column diameter, inches

The Wood Handbook (1974) contains values for modulus of elasticity, E, for many species, including red alder, Sitka spruce and western hemlock. However, the values are determined from tests on small, clear, straight-grained specimens.

The first objective of this study was to determine if the Wood Handbook values for modulus of elasticity could be used in predicting allowable cable loadings for support trees.

The moment of inertia,  $I$ , for a column of constant circular cross-section is easily calculated; however, trees have a varying cross-section due to taper, so the second objective of this study was to determine mathematical relationships for moment of inertia as a function of height for all three species.

One requirement for a solution to EQ (1) is a set of known boundary conditions. The necessary boundary condition at the base of the tree is that the moment at the base of the tree be equal to the rotation of the base of the tree times the base stiffness of the tree. The third objective of this study was to determine values of base stiffness for alder, spruce and hemlock.

Once the values for modulus of elasticity, base stiffness, and moment of inertia were obtained, we would be able to achieve the fourth objective of evaluating the capacity of alder, spruce and hemlock support trees. This would also allow comparison with Douglas-fir.

The scope of the analyses and results presented in this paper is limited. A two dimensional tail tree model with one guyline is used to calculate combined stress. All calculations are assumed to be within the elastic limits of the trees and cables. The effects of load eccentricity are not included, nor are the effects of tree lean.



Summary of Objectives

1. Determine values for modulus of elasticity.
2. Develop relationships for moment of inertia as functions of height.
3. Determine base stiffness values.
4. Evaluate the capacity of alder, spruce and hemlock to support cable loadings.
5. Compare alder, spruce and hemlock support capacities to Douglas-fir.

### III. FIELD STUDY

#### A. Study Sites

Trees were studied in two areas, both on the Hebo Ranger District, Siuslaw National Forest, in the Coast Range of northwest Oregon.

The alder site was at about 1500 feet elevation on the west side of Mt. Hebo. Most of Mt. Hebo burned in the early 1900's, and large tracts of alder are present.

Six alder trees were studied in a small flat about 1/2 acre in size, located at the base of a small ridge. The trees were located on a well-drained area, although a wet depression was located within 70 feet of tested trees.

Diameters at breast height ranged from 14.2" to 26.5", and all trees were about 100 feet tall.

Trees were chosen to obtain a good range in diameters, and have the outward appearance of soundness. However, the trees were not perfect. They all had some lean, ranging from 0.29° to 7.61° (0.5% to 13.4%) from vertical.

Tree 2, which was the largest alder tested, had a fork at the base, with a secondary bole 10" in diameter extending to about 70 feet in height, and Tree 6 had an old wound in its base, in addition to a more recent wound which appeared to have been caused during road construction.

The study was conducted from June 18 to July 5, 1984, and weather conditions ranged from sunny and warm to steady downpour.

The hemlock/spruce study area was located within the Cascade Head Experimental Forest, just north of Lincoln City, Oregon. Four hemlock ranging from 8.9 to 20 inches DBH, and four spruce with diameters from 11.1 to 19.2 inches were studied. Spruce heights ranged from 35 to 119 feet, and hemlock ranged from 60 to 115 feet.

These trees were more spatially separated than the alder, and all were located on well-drained sites. They were tested from July 9 to July 27, 1984. The weather was mostly clear and sunny.

These trees also had some lean. Hemlock lean ranged from 0 to 2.3° (4%), and spruce ranged from 0.7° to 3.8° (1% to 7%) from the vertical.

## B. Study Methods

Figure 3 shows the basic rigging configuration that was used to obtain the needed data.

Survey targets were placed at 5-foot intervals beginning at 5 feet. The highest pull heights were 32.5 feet for Alder #2, 35 feet for Hemlock #4, and 30 feet for Spruce #3. All the alder forked a few feet beyond the heights at which they were rigged. For safety and practical considerations, we did not go beyond the forks.

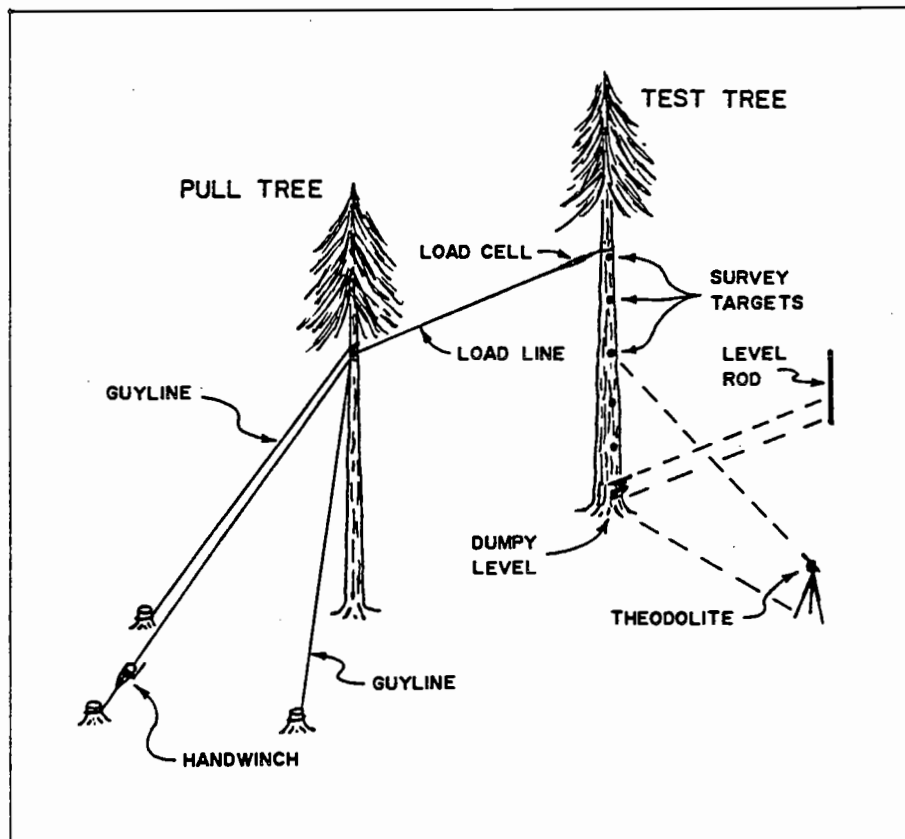


Figure 3. Field Study Rigging Method.

Pyles, 1984

Diameters and bark thickness were measured at each target point on the alder and spruce to determine the functions for moment of inertia. Outside diameter only was measured on the hemlock, and a regression equation (Stuck 1974) was used to determine inside bark diameters.

The load line was placed at a height on the pull tree such that the pull direction was horizontal.

The load was applied to the alder with a Skagit B20F drum set, and was applied to the spruce and hemlock with a hand winch. The load was measured using an electric load cell with a rated capacity of 10,000 pounds. The load sensing element was a four-arm resistive strain gage bridge. Bridge excitation and signal reading was done with a Baldwin-Lima-Hamilton model 120 strain indicator. Using the model 120 strain indicator, the maximum theoretical resolution of the load cell was 10 pounds. Repeated readings at constant load however, show that the measuring system was only accurate to about 30 pounds.

At least three loads were applied to each tree, in each of three directions.

As previously mentioned, virtually all the trees had some lean. To determine the effects of lean, if any, on the structural properties, a set of three loads was applied in the direction of lean, a set against the lean, and a set at a right angle to the lean (not in any particular order).

Lateral deflections of the survey targets were measured with a Wild T-2 theodolite. Unloaded readings were recorded for each target while the load line was slack. A load would then be applied and held constant while the loaded deflection angles were recorded. The difference between the readings was the total deflection at a given height and load, due to both bending and rotation at the base. This data would be used in the modulus of elasticity calculations.

A dumpy level was attached to the base of the test tree as close to the ground as possible, its line of site parallel to the pull direction. It was assumed that no bending occurred at the base of the tree, so that the difference between the loaded and unloaded level rod readings was the amount of base rotation for a given load. The base rotation data would be used both in the modulus and base stiffness calculations.

We were also interested in the effect that load height might have on base stiffness. After a tree had been pulled in all three directions, the direction which had the most rotation per foot-pound of applied moment, i.e. the weakest direction, was determined. The load line was then moved down the tree to a target approximately  $3/4$  of the original load height, and three new loads were applied in the weakest direction.

The largest of the new loads applied in the weakest direction was designed to yield the same moment about the base of the tree as the maximum load at the upper height had. For example, let's assume the maximum load applied at 30 feet had been 1,500 pounds (a base moment of 45,000 foot-pounds). The load point would be moved down to 20 feet, about  $3/4$  of 30 feet, and three new loads would be applied, the largest of which would be about 2,250 pounds, which would yield a base moment of 45,000 pounds. The load point would then be moved down to 10 feet and three additional loads would be applied.

After each set of readings were taken, both at the upper and lower load heights, the load line was slacked and the unloaded theodolite and level rod readings were again recorded.

A major objective in the testing was to apply loads that would create deflections the instruments were capable of measuring, but not to apply loads large enough to displace the roots. In other words, we wanted to determine the structural properties within the elastic range of behavior.

Any significant difference between unloaded readings before testing and unloaded readings after testing would indicate the elastic limit had been exceeded.

## IV. RESULTS

### A. Moment of Inertia

Moment of inertia,  $I$ , as a function of height was needed for the modulus of elasticity, bending and buckling calculations. For solid circular columns,

$$I = \frac{\pi D^4}{64}$$

Although trees are not perfectly circular, they were assumed to be for the purposes of this project. Diameter ( $D$ ) was taken to be the inside bark diameter. It is assumed that the bark does not significantly contribute to the structural properties of a tree.

#### 1. Alder Moment of Inertia

Figure 4 shows the plots of moment of inertia versus height for alder, and it can be seen that alder does not have uniform taper. Diameter measurements below 5 feet were stopped when the diameter tape failed to make continuous contact with the tree because of flutes.

Ideally, we would like to normalize the values for moment of inertia, so that we could get a good idea about its value at any height on a tree by making a single measurement rather than having to make many measurements.



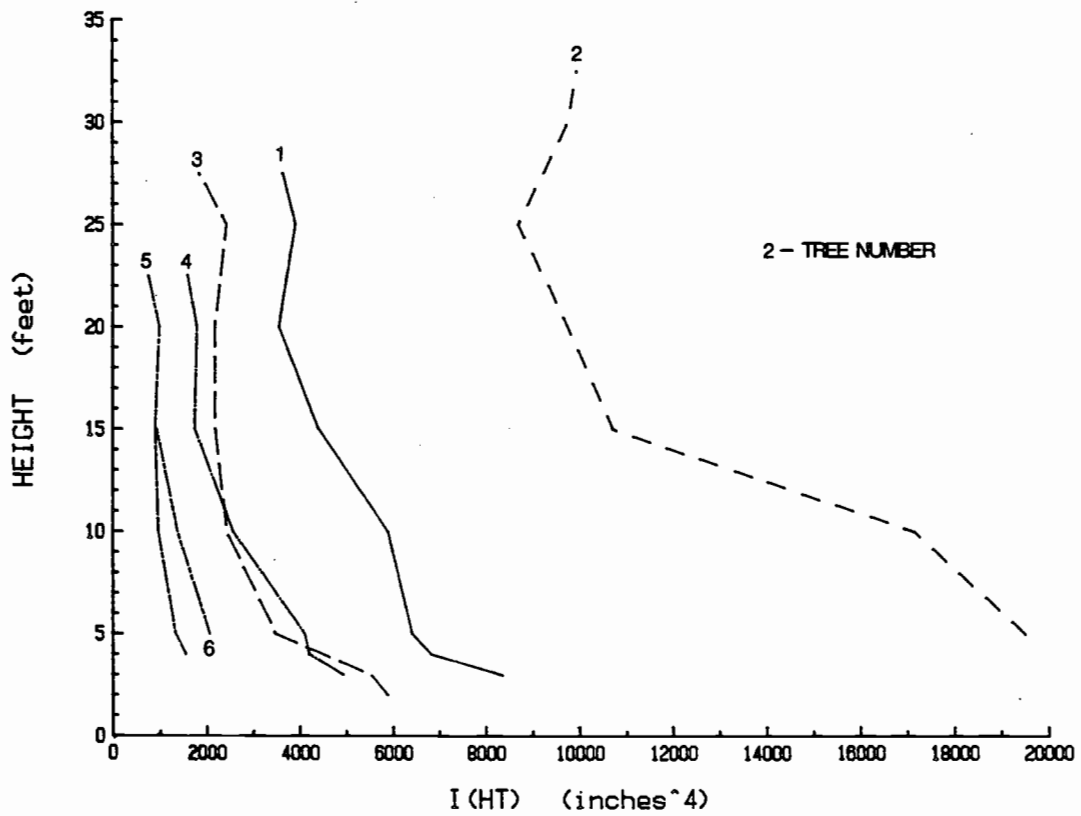


Figure 4. Alder Moments of Inertia as a Function of Height

Pyles (1984) was able to normalize the moments of inertia at given heights ( $I_h$ ) on Douglas-fir by dividing the values calculated from measured diameters for a given tree, by the moment of inertia at 5 feet ( $I_5$ ) for that tree. He fit the values of the ratios to an equation of the form  $I_h/I_5 = ah^b$ . Then, by simply measuring a tree's diameter at 5 feet, calculating the moment of inertia ( $I_5$ ), and multiplying  $I_5$  by  $ah^b$ , a good estimate of the moment of inertia ( $I_h$ ) at any height  $h$  could be obtained.

Several methods were tried to normalize the values of  $I_h$  for alder, but because of the wide variation in values, no method could be found to adequately reduce the scatter. However, for the bending and buckling calculations, a function for moment of inertia had to be developed. This is because most methods used for calculating bending stresses and buckling loads for tapered columns break them into many segments, and values for moment of inertia are needed for each of the segments.

What was needed, from the engineering standpoint, was a conservative function--one that we were reasonably certain would not overpredict resistance to bending.

It was decided that a function of the form  $I_h/I_5 = f(\text{height})$  would be the most practical because moment of inertia at 5 feet ( $I_5$ ) can be readily determined. Figure 5

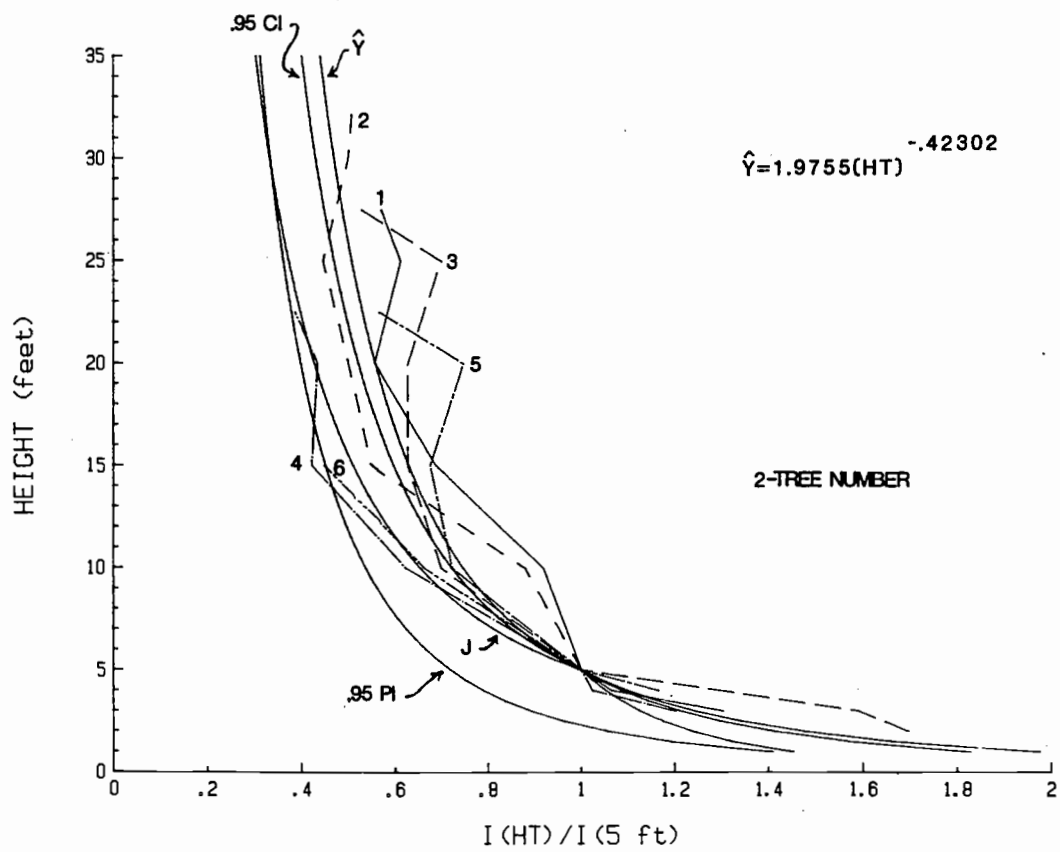


Figure 5. Normalized Alder Moments of Inertia and Functions

displays the plots of  $I_h/I_5$  versus height for the six alder. The data lines for all trees pass through 1 at 5 feet because  $I_5/I_5 = 1$ .

Also plotted in Figure 5 are four curves; the explanation of each follows. To graphically convey a picture of how moment of inertia varies with height, height has been plotted on the ordinate. It is pointed out that height is the independent variable.

- $\hat{Y}$  - this is the least squares regression line (point estimator) of the mean response of  $I_h/I_5$  given a value of height. The  $R^2$  value is 0.82.
- .95 CI - this is the lower 95% confidence interval(CI) for the mean response ( $\hat{Y}$ ) line. A transformation (Appendix A) analogous to forcing a regression line through the origin, was made to force the  $\hat{Y}$  and CI lines through the point (5,1). This was desirable because:
  - 1) physically, the line has to go through the point, ie.  $I_5/I_5$  will always equal 1, and
  - 2) moving the line to the left would cause an unnecessary loss in moment of inertia in the lower portion of the trees, with an accompanying loss of bending resistance.
- .95 PI - this is the lower 95% prediction interval for an individual outcome of  $I_h/I_5$  given a value for height. Note that this line does not go through the point (5,1) as we know it should. A statistically based method for forcing a prediction line through a point was not found.

Since we wanted a function to determine new values for  $I_h/I_5$  given a value of height, a prediction line seemed in

order. However, the method for determining its location did not recognize the physical reality that  $I_h/I_5$  will always equal 1 when  $h = 5$ .

Therefore, we were faced with the choice of using an accepted statistical method, based on probability, that did not represent physical reality, or of using an alternative method based on judgement.

The fourth curve in Figure 5, labeled J is of identical form as the .95 CI, except that the value of 8 was substituted for the 95% t -value of 2.712 (2-tail, 38 degrees of freedom).

From 15 feet up, only 1 data point lies within the .95 PI that is not within J. A total of 5 data points out of 39 (13%) fall outside the J line. It was felt this function represented a conservative estimator of  $I_h/I_5$  and met physical reality, so it was used in the tail tree analysis.

While statistical tools were used to determine the location of the J line, and as such it is a repeatable process, no statistical significance can be attached to its location. Its location is based on judgement.

## 2. Hemlock Moment of Inertia

Hemlock had more uniform taper than alder. For the modulus calculations, power curves were fit to each tree using the least squares method. Figure 6 displays the values of  $I_h/I_5$  for the four trees.

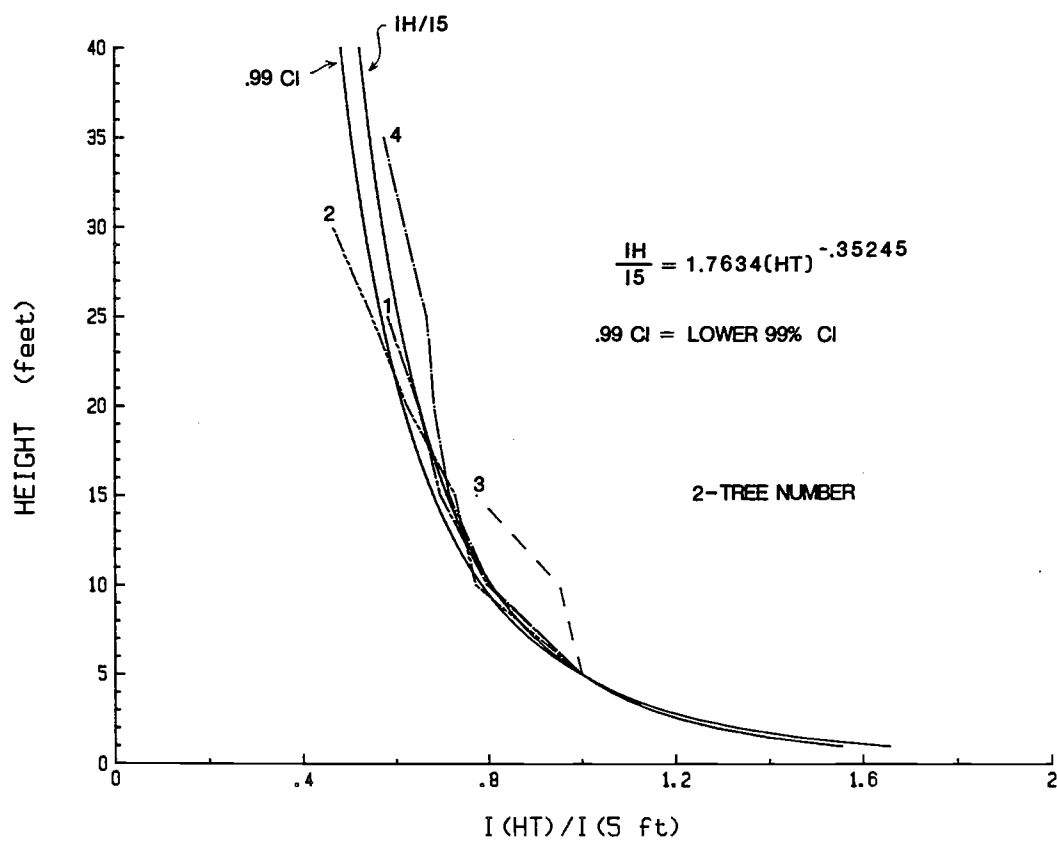


Figure 6. Normalized Hemlock Moments of Inertia and Functions

As was done for alder, a mean regression line was fit to the data through (5,1), with an  $R^2 = .88$ . The lower 99% expected value line (.99 CI) is also shown on Figure 6, and is the function used in the tail tree analysis.

### 3. Spruce Moment of Inertia

The  $I_h/I_5$  lines for spruce are plotted in Figure 7. Tree 1, a 9.9", 15 foot tree, was dropped from the moment of inertia analysis because of highly abnormal form. The mean line ( $\hat{Y}$ ) is shifted to the left side of the plotted data as a result of forcing it through (5,1), and has an  $R^2$  of only 0.66. The  $R^2$  could be increased by not forcing the line through (5,1), but this would yield a function not representative of physical realities. The lower 99% expected value line (.99 CI) was used in the tail tree analysis.

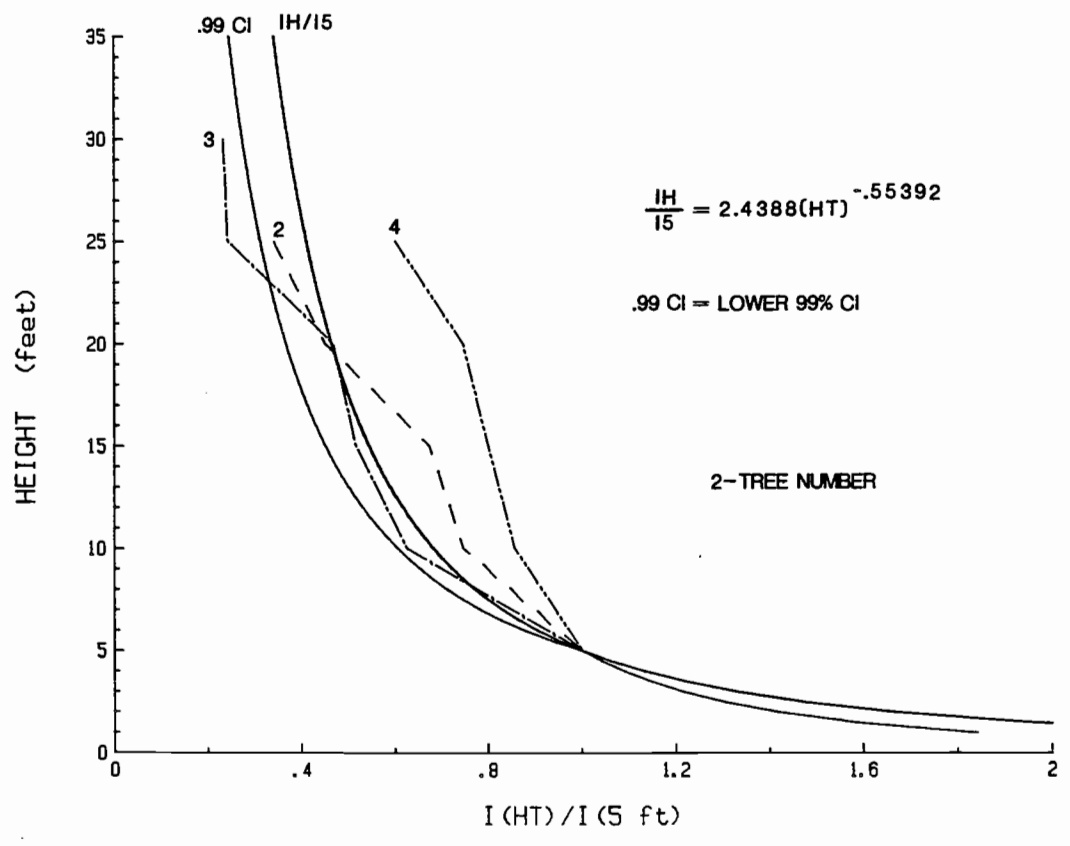


Figure 7. Normalized Spruce Moments of Inertia and Functions



## B. Modulus of Elasticity

The total lateral deflection measured for a given load at a given height is the sum of two components:

1. the component due to rotation of the tree about its base, and
2. the component due to bending of the tree.

To calculate the modulus of elasticity, it was necessary to determine what portion of the lateral deflection measured with the theodolite at a given target height was due to the bending of the tree.

By measuring the base rotation with the dumpy level, the tree's rotation angle,  $\theta_0$ , was known. Multiplying the tangent of  $\theta_0$  by a given target's height ( $h$ ) yielded the amount of deflection at that height due to base rotation,  $Y_b$ . Subtracting this value from the total deflection measured at the height,  $Y$ , yielded the amount of deflection the point due to bending,  $Y_c$ . These relationships are illustrated in Figure 8.

Pyles (1984) developed two methods for calculating modulus of elasticity, both of which were used in this study.

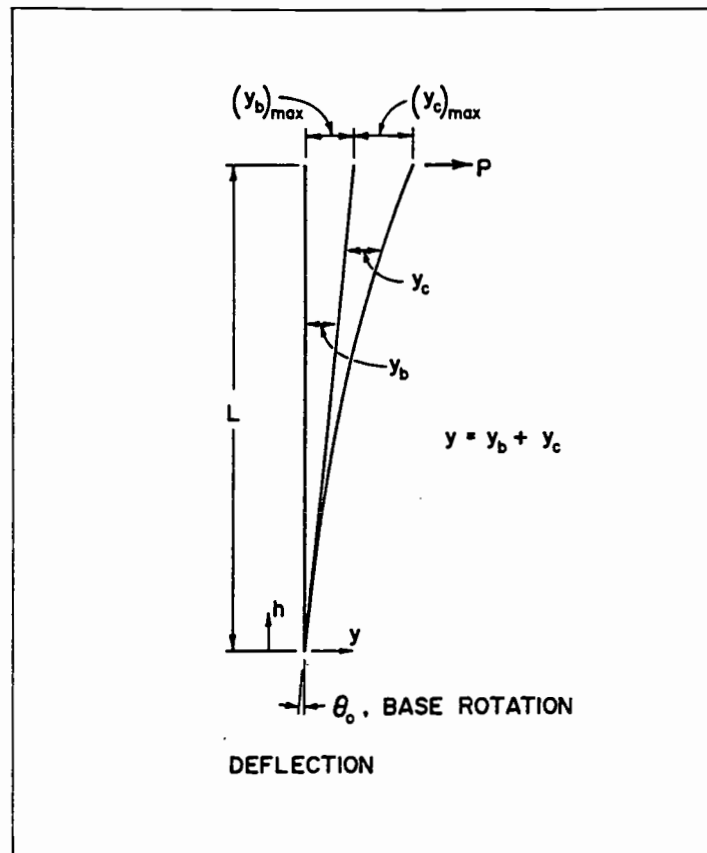


Figure 8. Relationship Between Rotation and Bending Deflections

The first method allows one to calculate the modulus over segments of the test tree. This was the only method used on the alder. The relationship is:

$$E = \frac{P (L-h) \Delta h}{ah^b \Delta \theta} \quad (4.12, \text{Pyles})$$

where: E = modulus of elasticity, psi

P = the applied load, pounds

The divisor portion,  $ah^b$ , represents a power function fit to each tree for moment of inertia at height, h. Because of the nonuniformity of taper, which caused wide variation in moment of inertia, it was decided to obtain the values of moment of inertia for the alder modulus calculations by averaging the measured diameters over each section, and using the relation  $I = \pi D^4/64$ . Appendix B displays how the variables in EQ. 4.12 are calculated for a tree section after the deflection due to base rotation has been subtracted.

The second method for calculating E is based on integration, and would allow calculation of one modulus value for a tree over its entire length. The relation is:

$$E = \frac{P}{\theta a} \left[ \frac{Lh^{(-b+1)}}{-b+1} - \frac{h^{(-b+2)}}{-b+2} \right] \quad (4.15, \text{Pyles})$$

where: a,b = regression coefficients  
from each tree's function for  
moment of inertia at height h,  
 $I_h = ah^b$

Because of the sensitivity of the deflection measurements to small breezes, and because of the limited number of trees sampled, it was decided that values of modulus would be calculated over 10 foot segments with EQ. 4.15, as was done with EQ. 4.12. Therefore, the remaining variables are the same as defined in Appendix B except that  $\theta = (\theta_1 + \theta_2)/2$ .

#### 1. Alder Modulus

The Wood Handbook (1974) lists an average green wood modulus value for red alder of 1,170,000 psi.

Calculated values for modulus varied greatly in this study. One reason was that even slight breezes made it difficult to accurately record deflections with the theodolite.

Extremely large values such as 20,000,000 psi, or negative values for modulus were removed from the set of calculated values as being unrealistic.

The average of all remaining calculated values for alder modulus (114 in all) was 1,308,000 psi, with a standard deviation of 955,000 psi (Table 1). The average is

TABLE 1. Alder Modulus of Elasticity (E) Results

Method	Maximum Variation of E for Interval and Direction	Sample Size	Mean Value(psi)	Standard Deviation(psi)	Coefficient of Variation
EQ 4.12	None	114	1,308,000	955,000	.73
	50%	52	1,051,000	500,000	.48
	30%	34	1,156,000	528,000	.46

Note: Wood Handbook (1974) average value for green alder, E =1,170,000 psi

only 12% larger than the published average value of 1,170,000 psi, but the coefficient of variation (ratio of standard deviation to the mean) is 73%. The Wood Handbook (1974) lists an average coefficient of variation of 22% for green wood from approximately 50 species. The alder's coefficient of variation is so high because the 114 modulus values ranged from 300,000 to 6,000,000 psi.

One could argue that a value for alder modulus of 6,000,000 psi is unreasonable, and throw it out, but this involves judgment, which becomes more difficult to apply as values approach the published value. Therefore, an impartial method for selecting "in" and "out" modulus values to be included in the average was desired.

Recall that three loads were applied to each tree in each of three directions, allowing calculation of nine values of modulus for each 5-foot segment. Because of out-of-roundness of the trees, the moment of inertia, i.e. resistance to bending, was probably different in different directions. Therefore, we could expect differences in the values of modulus calculated for the same tree segment, depending on the direction pulled.

However, we should expect the values of modulus to be approximately the same for three loads on the same segment in the same direction. Large differences would indicate measurement errors.

To eliminate modulus values with measurement errors, the range from lowest value to highest value of modulus was limited to a 50% difference for each segment and pull direction. If the maximum value of modulus was more than 50% greater than the minimum value for a given segment and direction, then all values for that segment were eliminated from the data set.

This reduced the number of "in" modulus values from 114 to 52 (Table 1). The resulting average value for modulus was 1,051,000 psi (10% less than the published value), with a standard deviation of 500,000 psi. This reduced the coefficient of variation from 73% to 48%, which is still quite high.

The maximum range from low to high values was then restricted to 30%. This reduced the number of "in" values to 34, with an average modulus of 1,156,000 psi, a standard deviation of 528,000 psi, and a coefficient of variation of 46%.

Further screening of the calculated values would not decrease the coefficient of variation. Shortly, we'll see that the average values for alder modulus had more variation than hemlock or spruce. This is because alder is generally a rougher tree in terms of cross section, which means more variation in moment of inertia, hence more variation in the calculated modulus values.

However, it appears the Wood Handbook's average modulus value of 1,170,000 psi, which is essentially equal to the final field value calculated, is a good value to use in the bending and buckling calculations.

## 2. Hemlock Modulus

Since good relationships for  $I_h/I_5$  for were developed for hemlock, EQ. 4.15 could be used to calculate the modulus values. To compare the methods, modulus values were also calculated using EQ. 4.12. The results are summarized in Table 2.

The Wood Handbook (1974) lists an average green wood modulus for hemlock of 1,310,000 psi. In this case, EQ. 4.15 yields larger values for modulus than EQ. 4.12, but its values have less variation. The values calculated with EQ. 4.12 fluctuated too much to go below the 20% limit on variation.

An unpaired t-test was done on the mean values of 1,810,000 psi and 1,432,000 psi. The values are significantly different at the .05 level.

We can't conclude that one method is better or yields answers closer to the "true" value for the tree's modulus, but we can say that the published mean value of 1,310,000 psi appears to be conservative, and it will be used for the hemlock modulus in the bending calculations.



TABLE 2. Hemlock Modulus of Elasticity (E) Results

Method	Maximum Variation of E for Interval and Direction	Sample Size	Mean Value(psi)	Standard Deviation(psi)	Coefficient of Variation
EQ 4.15	30%	63	1,810,000	445,000	.25
EQ 4.12	30%	20	1,539,000	450,000	.29
EQ 4.15	20%	63	1,810,000*	445,000	.25
EQ 4.12	20%	12	1,432,000*	443,000	.31
EQ 4.15	10%	48	1,771,000	396,000	.22
EQ 4.15	5%	32	1,738,000	331,000	.19

Note: Wood Handbook (1974) average value for green, western hemlock, E = 1,310,000 psi.

\*These means are significantly different at the .05 level. Significance of the difference between the other two pairs of means was not tested.

### 3. Spruce Modulus

The Wood Handbook (1974) lists an average green Sitka spruce modulus value of 1,230,000 psi. Table 3 summarizes the values obtained by the method of EQ. 4.15.

It appears the published value, which is 1% larger than the final field value of 1,222,000 psi, is a good value to use in the bending and buckling calculations.

### C. **Base Stiffness**

To determine load bearing capacity, base stiffness values are needed. The end condition for the base of a tree is somewhere between pinned (free to rotate) and fixed (infinitely stiff). Figure 9 illustrates how such a system might be modeled.

A pin is shown at the bottom, but a spring of stiffness  $K$  is attached at the rotation point. The stiffness could be variable with rotation, or constant. A constant stiffness is defined in units of end moment per unit of angular rotation, ie.  $K = 10 \text{ ft-kips/degree}$  means it would take 10,000 foot-pounds of moment to rotate the base one degree. The work presented here considers the value of  $K$  to be constant over its elastic range of behavior.

TABLE 3. Spruce Modulus of Elasticity (E) Results

Method	Maximum Variation of E for Interval and Direction	Sample Size	Mean Value(psi)	Standard Deviation(psi)	Coefficient of Variation
EQ 4.15	25%	64	1,239,000	418,000	.34
	15%	61	1,241,000	428,000	.34
	10%	52	1,222,000	412,000	.34

Note: Wood Handbook (1974) average value for green Sitka spruce, E = 1,230,000 psi

---

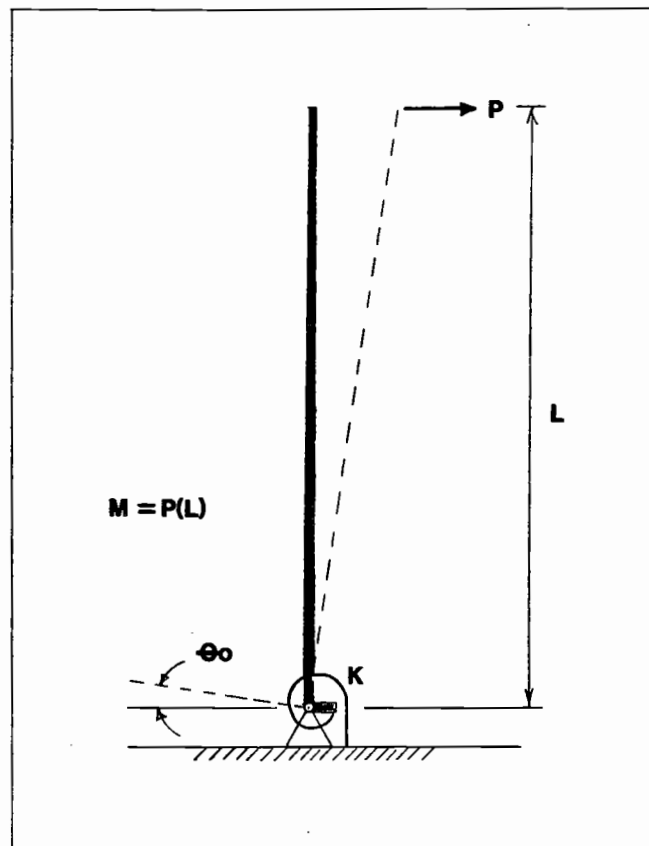


Figure 9. Structural Model of a Tree

Base stiffness values were determined by multiplying loads (P) by the height they were applied (L), and dividing the product by the number of degrees the base rotated ( $\theta_0$ ), which was measured with the leveling rod.

Three loads were applied in each direction at each load height. If the applied loads were within the range of linear, elastic behavior, the three data points would form a line starting at zero, with a constant slope (stiffness) K. The theoretical relationship is:

$$M = K (\theta_0) \quad (4.16, \text{ Pyles})$$

where: M = base moment, ft-kips

K = base stiffness, ft-kips/degree

$\theta_0$  = degrees of base rotation.

The relationship is displayed in Figure 10a. The field data, plus one known value of (0,0) were fit to straight lines using the least squares method. The form of the equations was:

$$M = K (\theta_0) + M_0 \quad (4.17, \text{ Pyles})$$

where the variables are defined for EQ. 4.16, and  $M_0$  is the moment intercept. The relationship is displayed in Figure 10b.

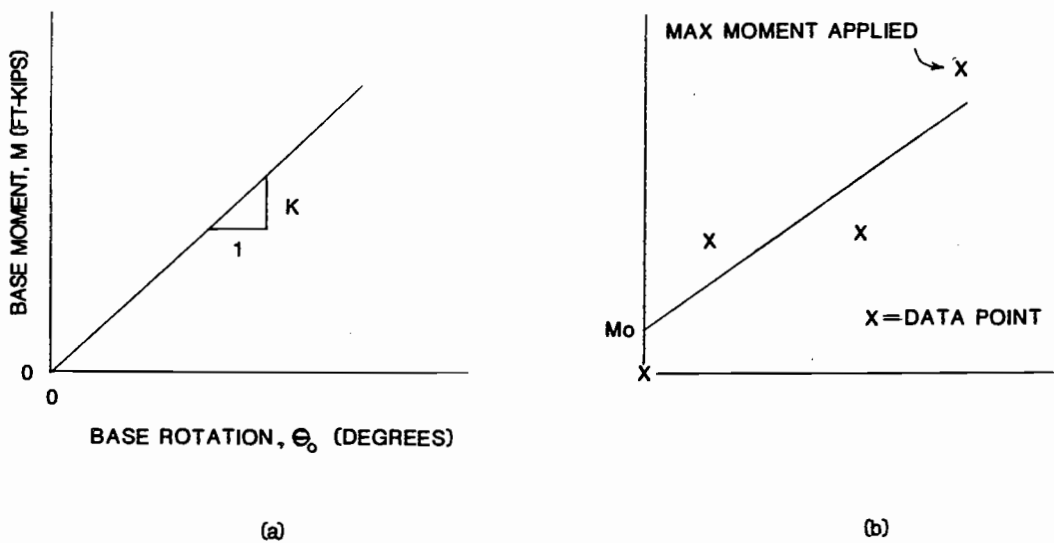


Figure 10. Relationship of Base Moment to Base Rotation

### 1. Alder Base Stiffness

The computed values for alder are displayed in Table 4.

As evidenced by the high values for  $R^2$ , we see the values of K are fairly linear and therefore constant for a given direction.

However, we also see variation in the values of K for each tree. The largest difference from low to high K is found for Tree 3, which has a 225% difference (71.41 to 232.34 ft-kips/degree). The lowest range is found in Tree 5, with a difference of 21% from low to high K.

A reason why base stiffness would vary with pull direction has not been found. The following possible causes were evaluated:

1. The strongest direction could be in the direction of prevailing winds. Azimuths of pull direction were recorded, and highest K values were spread throughout the four quadrants.

2. The highest K for a tree could be related to the direction which had the highest (or lowest) maximum base moment applied. No relationship was apparent.

3. The variation of K could be related to the amount of lean. Tree 5 had the greatest amount of lean from vertical ( $7.61^\circ$ ), and had the least variation in base stiffness. Tree 3 was second in terms of lean ( $6.3^\circ$ ) and had the most variation in K.

TABLE 4. Alder Base Stiffness Values

Tree # (DIB 5 ft)	With Lean			Against Lean			Right Angle to Lean		
	K*	Mo**	R <sup>2</sup>	K*	Mo**	R <sup>2</sup>	K*	Mo**	R <sup>2</sup>
Tree 1 (19")	202.74	2.75	.99	191.05	4.24	.98	291.16	4.0	.987
Tree 2 (25.1")	425.03	6.32	.99	513.57	1.88	.99	279.30	12.32	.97
Tree 3 (16.3")	71.41	9.32	.96	108.97	3.30	.99	232.34	1.44	.99
Tree 4 (17.0")	152.31	-.08	1.00	114.53	.59	1.00	178.21	1.42	.99
Tree 5 (12.8")	65.66	.80	1.00	79.31	-.39	.98	79.35	.53	1.00
Tree 6 (14.3")	47.61	-.22	1.00	49.39	1.06	.99	91.84	-.5	1.00

\* (ft-kips/degree)  
 \*\* (ft-kips)



4. The highest or lowest value of  $K$  could be related to the order in which the direction was pulled. The highest value of  $K$  for a tree was never in the first direction pulled, and the lowest value of  $K$  was never in the direction pulled last.

5. The variation in  $K$  could be related to tree diameter. No relationship was observed between variation and increasing or decreasing tree diameter.

Non-zero values for  $M_0$  are indicative of measurement errors. Some of the values for  $M_0$  appear large, considering the theoretical value is zero. However, all of the values of  $M_0$  are less than 10% of the maximum base moment that was applied in the given direction, and in that context are not considered large (Figure 10b). For example, Tree 3, with lean, has an  $M_0$  of 9.32 ft-Kips, but the maximum base moment that was applied in this direction was 102 ft-kips; hence  $M_0$  is only 9% of the maximum applied moment.

For buckling and bending calculations, a function that relates base stiffness to tree size is needed. Pyles (1984) developed linear, exponential and power functions for Douglas-fir base stiffness as a function of diameter inside bark at five feet height (DIB at 5 ft). He determined that a power function made the most physical sense, because it yields a base stiffness of zero for zero diameter.

The values of base stiffness (K) for alder from Table 4 are plotted versus DIB at 5 ft in Figure 11. Also plotted are power functions of the form  $K = a (DIB)^b$ . The middle curve was fit to all values of K, the upper curve to the highest value of K for each tree, and the bottom curve to the lowest value of K for each tree.

Since there was a great deal of variation in the range of K for each tree, the bottom curve will be used in the tail tree analyses.

#### Hemlock Base Stiffness

As was done for alder, the base moment and base rotation values were fit to lines of the form  $M = K (\theta_0) + M_0$ , and the regression values are displayed in Table 5.

The high  $R^2$  values indicate the applied loads were within a linearly elastic range of behavior. The largest value of  $M_0$  was small compared to the applied moments, being only 7% of the maximum base moment applied in that direction.

Again base stiffness varies for each tree, though not as much as alder on a percentage basis. The reason(s) for the variation are not apparent, although pull azimuths were not recorded for hemlock.

The values of base stiffness versus DIB at 5 ft are plotted in Figure 12. To be conservative, the lowest curve will be used in the tailtree analyses.

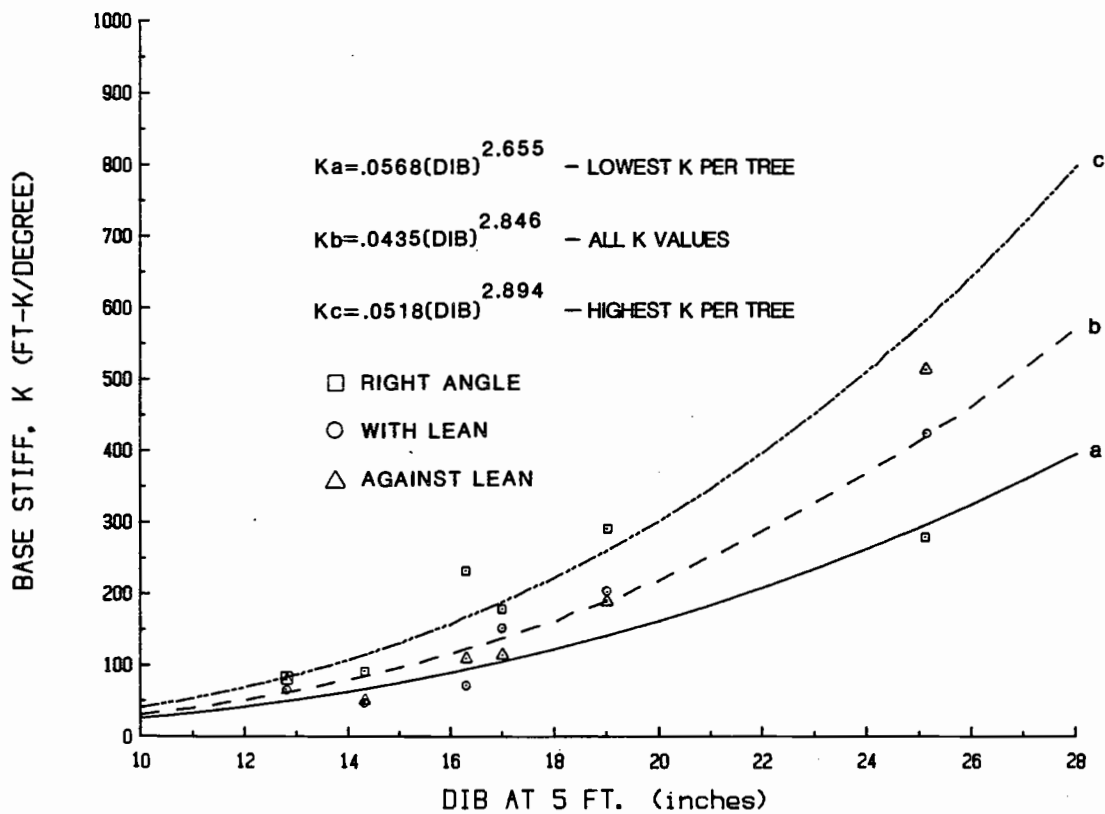


Figure 11. Alder Base Stiffness as a Function of Tree Diameter

TABLE 5. Hemlock Base Stiffness Values

Tree # (DIB 5 ft)	With Lean			Against Lean			Right Angle to Lean		
	K*	Mo**	R <sup>2</sup>	K*	Mo**	R <sup>2</sup>	K*	Mo**	R <sup>2</sup>
Tree 1 (12.6")	53.70	.43	1.00	73.16	0.67	1.00	112.79	.68	1.00
Tree 2 (14.3")	69.56	3.20	.98	83.64	.92	1.00	94.24	2.09	.99
Tree 3 (7.5")	12.78	.51	.99	14.45	.59	.99	17.41	1.04	.98
Tree 4 (16.1")	238.02	1.10	1.00	248.97	2.64	.99	267.75	2.45	.99

\* (ft-kips/degree)

\*\* (ft-kips)

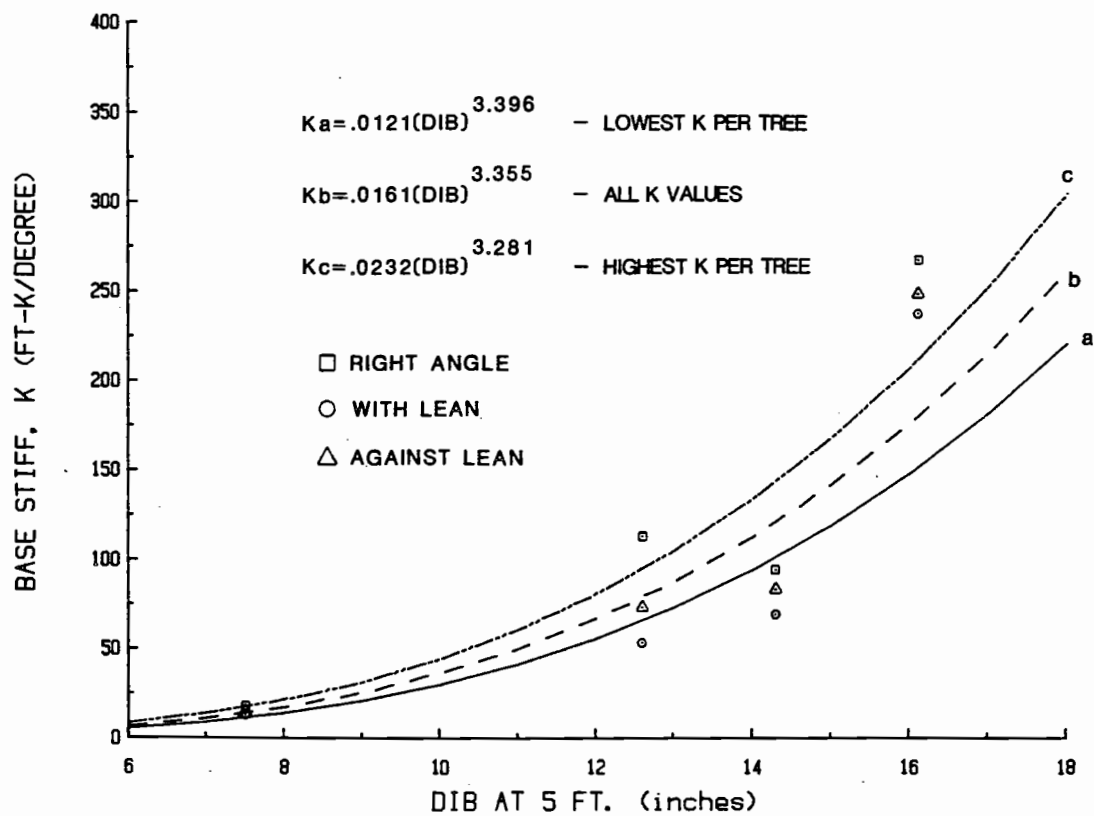


Figure 12. Hemlock Base Stiffness as a Function of Tree Diameter

### Spruce Base Stiffness

Fitted values for  $M = K (\theta_0) + M_0$  are displayed in Table 6. Variation in values of K for each tree is somewhat larger than for hemlock, but less than for alder. For the most part it appears as though a linear model does adequately represent base moment as a function of base rotation.

As was done for hemlock and alder, values of K were fit to curves of the form  $K = a(\text{DIB})^b$ , which are shown in Figure 13. Again the lowest curve will be used in the calculations. It is noted that the location of the curves was heavily influenced by the 18.3 inch tree. When later attempting to pull its stump, it was learned that its root system was heavily entwined with the root system of an adjacent tree. Therefore, its stiffness may not be typical for other trees of its size.

### Base Stiffness Related to Load Height

As stated in Study Methods, we were also interested in the relationship between base stiffness and the height of applied load.

Separate studies of Douglas-fir base stiffness reported by Pyles (1984) and Stoupa (1984) determined different values. Stoupa's work with stumps yielded a higher value for base stiffness than did Pyles' work on support trees, on whose methods most of this paper is based.

TABLE 6. Spruce Base Stiffness Values

Tree # (DIB 5 ft)	Right Angle to Lean			With Lean			Against Lean		
	K*	Mo**	R <sup>2</sup>	K*	Mo**	R <sup>2</sup>	K*	Mo**	R <sup>2</sup>
Tree 1 (9.9")	18.20	.99	.98	7.51	1.84	.93	8.49	.50	.98
Tree 2 (12.7")	25.02	7.16	.89	17.62	7.65	.87	20.68	1.29	.99
Tree 3 (14.4")	22.04	7.74	.92	46.1	.81	1.00	41.68	2.12	.99
Tree 4 (18.3")	347.03	.85	1.00	243.84	-.15	1.00	297.15	.46	1.00

\* (ft-kips/degree)

\*\* (ft-kips)

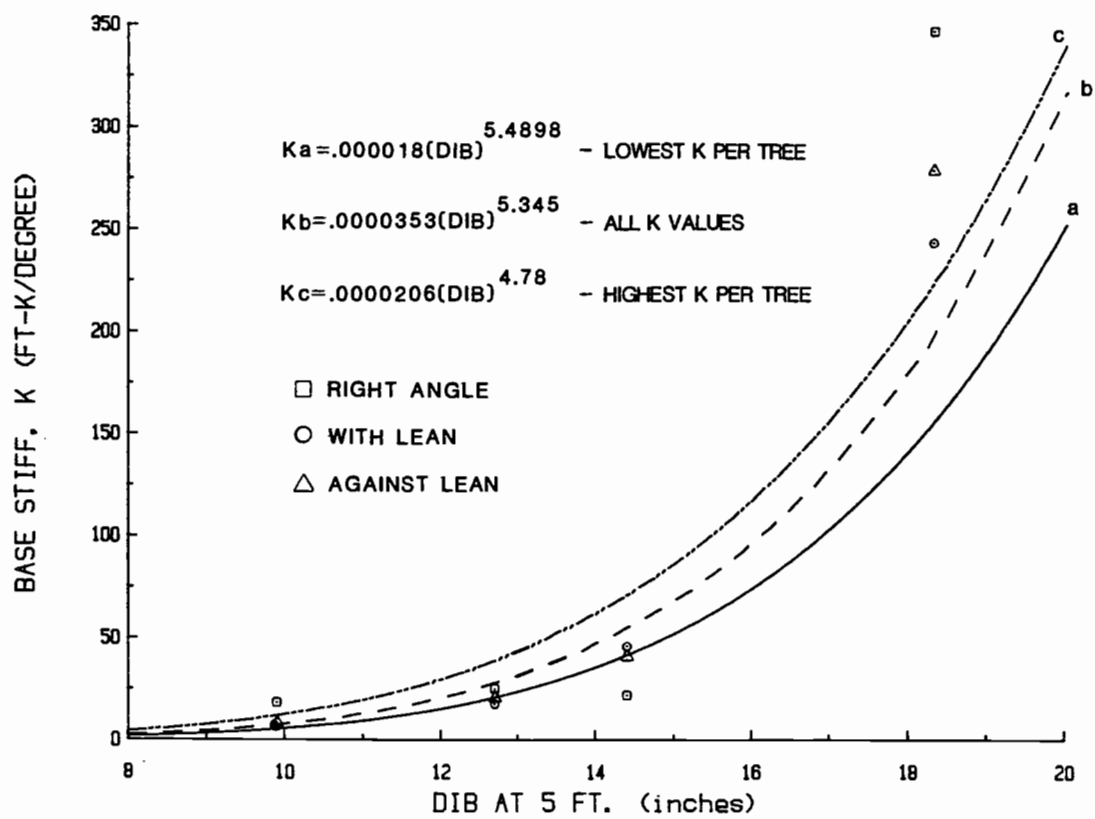


Figure 13. Spruce Base Stiffness as a Function of Tree Diameter



If the model pictured in Figure 9 is a good representation of how the system operates, then moving the load point down the tree should yield the same value of K in the same direction.

The values of base stiffness for alder for differing heights are shown in Table 7. A definite trend of increasing base stiffness with decreasing load height is seen.

All trees had base stiffness values for 15-foot load heights. To put all values on an equal basis, they were normalized by dividing the pull heights by 15 feet, and the base stiffnesses at each height by the base stiffnesses at 15 feet. The normalized values were fit to a straight line of the form:

$$\frac{\text{Pull height}}{15 \text{ ft}} = a + b \left[ \frac{K (\text{pull height})}{K (15 \text{ feet})} \right]$$

where a and b are regression constants,

$$a = 4.57$$

$$b = -3.554$$

$$R^2 = .81$$

The data for alder suggests there may be a relationship between base stiffness and load height.

The effect of load height on spruce and hemlock base stiffness is inconclusive. The values are shown in Table 8. Base stiffness decreases with load height for Spruce 1 and 2, and generally increases for 3 and 4.

TABLE 7. Alder Base Stiffness Values for Decreasing Load Heights<sup>1</sup>

	<u>Load Height (ft)</u>	<u>K(ft-kips/degree)</u>	<u>Mo(ft-kips)</u>
<u>Tree 2, Right Angle Pull</u>	32.5	279.3	12.32
	25	306.48	-5.01
	15	381.28	1.66
<u>Tree3, Pull with Lean</u>	27.5	71.41	9.32
	20	97.61	1.34
	15	101.16	1.58
<u>Tree 4, Pull Against Lean</u>	22.5	114.53	.59
	15	125.88	1.03
	10	125.06	.59
<u>Tree 5, Pull with Lean</u>	22.5	65.66	.80
	15	66.08	.81
	10	71.95	.40
<u>Tree 6, Pull with Lean</u>	20	47.61	-.22
	15	53.81	-.20
	10	56.66	-.14

1/ Insufficient data was taken for Tree 1.

---

TABLE 8. Spruce and Hemlock Base Stiffness Values for Decreasing Load Heights

	<u>Spruce</u>		
	<u>Load Height (ft)</u>	<u>K(ft-kips/degree)</u>	<u>Mo(ft-kips)</u>
<u>Tree 1, Pull With Lean</u>	15	7.51	1.84
	10	6.73	.20
	5	5.43	.98
<u>Tree 2, Pull with Lean</u>	25	17.62	7.65
	20	14.25	1.55
	15	12.62	2.53
<u>Tree 3, Right Angle Pull</u>	30	22.04	7.74
	25	23.73	1.84
	20	26.88	.71
<u>Tree 4, Pull With Lean</u>	25	243.84	-.15
	20	240.8	.53
	15	269.27	1.46
		<u>Hemlock</u>	
<u>Tree 1, Pull with Lean</u>	25	53.7	.43
	20	55.57	.69
	15	57.87	1.18
<u>Tree 2, Pull with Lean</u>	30	69.56	3.2
	20	54.41	2.55
	10	58.55	1.17
<u>Tree 3, Pull Against Lean</u>	15	14.45	.59
	10	14.43	.71
	5	12.42	.55
<u>Tree 4, Pull Against Lean</u>	35	248.97	2.64
	27.5	245.82	1.78
	20	277.03	2.46

Base stiffness increases with decreasing load height in Hemlock 1, generally decreases with load height in Hemlocks 2 and 3, and shows no pattern in Hemlock 4.

#### **D. Tail Tree Analysis**

The Euler equation is commonly used to calculate critical buckling loads, hence maximum cable loadings for tail trees. As pointed out by Pyles (1984) and Sessions, Pyles and Mann (1985), such use of the Euler equation is inappropriate.

Tail tree rigging configurations violate several of the assumptions implicit in use of the Euler equation. Some of the conditions required for its use are:

- 1) That we are dealing with a long, slender column, meaning a slenderness ratio greater than about 150. This translates to a rigging height about 38 times the column's diameter. For example, a 20-inch diameter column would have to be rigged to a height of about 63 feet for buckling to be the expected failure mode.
- 2) The column is initially straight and remains so with increasing axial load. This condition is most likely never met. Even 4 guylines cannot prevent deflection of the tree toward the yarder as skyline tension is applied.

3) The line of action of the load is colinear with the vertical axis of the column. This condition also is probably never met, as the skyline, when run through a block hung on the side of the tree, transmits a load that is several inches from the center of the tree's cross section.

The method of analysis to be used in this paper will be that proposed by Sessions, Pyles and Mann (1985). This method considers tail tree failure to be a result of combined axial and bending stresses rather than due to buckling. The effects of eccentricity of the load (item 3 above) are not included.

To calculate the combined axial and bending stresses in a tail tree, it is first necessary to solve the general relationship between bending moment and deflection for an elastic curve:

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \quad (2, \text{Sessions, et al.})$$

where:

Y = transverse deflection of the member, inches

X = longitudinal position along member, inches

M = modulus of elasticity, psi

I = moment of inertia, inches<sup>4</sup>

Figure 14a illustrates a two-dimensional case with a single guyline. Equation (2) can be rewritten as follows:

$$\frac{d^2y}{dx^2} = \frac{H(1-x) + P(y)}{EI} \quad (7, \text{Sessions, et al.})$$

where:

H = horizontal force on the tail tree, pounds

P = vertical load on the tail tree, pounds

l = length of member between load and base, inches

x, y as defined above.

Because the moment of inertia, I, varies over the length of the tree, a numerical technique is the most direct method for solution of EQ. 7.

A solution to EQ. 7 must be compatible with the following:

1) the reaction at the base of the tree,

$$K_{\text{base}} = \frac{M_{\text{base}}}{\theta_{\text{base}}} \quad (8, \text{Sessions, et al.})$$

where:  $K_{\text{base}}$  = base stiffness value, ft-kips/degree

$M_{\text{base}}$  = moment at base of tree, ft-kips

$\theta_{\text{base}}$  = rotation at base of tree, degrees

2) the reaction at the top of the tree, as defined by the stiffness of the guyline.

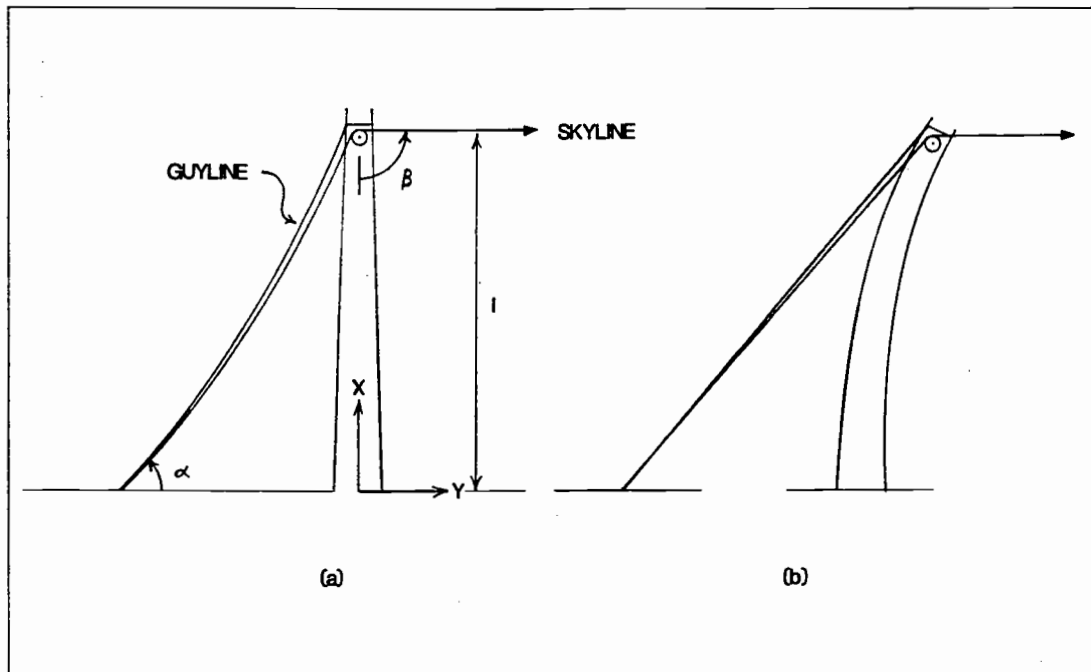


Figure 14. Model for Combined Stress Calculations

Once a solution to EQ. 7 has been found, the combined (total) stress at any point along the tree,  $\sigma_x$ , can be computed with the following equation:

$$\sigma_x = \frac{P}{A_x} + \frac{M_x C_x}{I_x} \quad (9, \text{Sessions, et al.})$$

where:

$\sigma_x$  = combined stress at point x, psi

P = vertical load on the tree, pounds

$A_x$  = cross sectional area at point x, inches<sup>2</sup>

$M_x$  = moment at point x, inch-pounds

$C_x$  = radius inside bark at point x, inches

$I_x$  = moment of inertia at point x, inches<sup>4</sup>

As Sessions et al., point out, the largest value of combined stress will result from summing the axial and bending stress components, ie. using the plus sign in EQ. 9. It's also noted that the point of maximum stress will not necessarily be at the base of the tree, because of taper.

All the functions and values needed to calculate combined stress,  $\sigma_x$ , for alder, Sitka spruce and western hemlock were determined in the previous sections.

Before beginning the calculations, some parameters had to be established. The first of these to consider was what upper value of combined stress would be used for analysis.



The Wood Handbook (1974) lists average values for maximum compression (crushing strength) parallel to grain. It also reports an average coefficient of variation for these values of 18% for 50 species. Since coefficient of variation is a ratio of the standard deviation to the mean, an estimate the standard deviation for a given species can be made by multiplying its mean crushing strength by .18.

In keeping with the desire to make conservative estimates of load bearing capacities, it was decided to subtract 3 standard deviations from the mean values for crushing strength. If the populations were normally distributed, less than 1% of the samples tested would have failed at that stress. The published maximum compressive stresses and estimated lower limit values are displayed in Table 9.

Referring to Figure 14a, the following variable values were set for calculations for varying diameters:

$\alpha$ -guyline,  $\alpha$ -skyline =  $45^\circ$

guyline diameter =  $3/4$ "

guyline metallic area =  $.262 \text{ in}^2$

guyline unit weight =  $1.04 \text{ lb/ft}$

initial lower end guyline tension =  $100 \text{ lb}$ .

guyline modulus of elasticity,  $E = 14,000,000 \text{ psi}$

load height,  $l = 30 \text{ ft}$ .

TABLE 9. Maximum Allowable Compressive Stress,  $\sigma_x$ , Values

Species	Mean Maximum <sup>1</sup> Compressive Stress $\bar{\sigma}_x$ (psi)	Estimated Standard <sup>2</sup> Deviation, s (psi)	Mean Minus 3 Standard Deviations (psi)
Red alder	2960	532.8	1360
Sitka spruce	2670	480.6	1228
Western hemlock	3360	604.8	1546

<sup>1</sup>Wood Handbook (1974). Values are for green wood, maximum crushing strength parallel to grain.

<sup>2</sup>Average coefficient of variation, cv, for 50 species = 0.18

$$cv = s/\bar{\sigma}_x = 0.18, \quad s = \bar{\sigma}_x (.18)$$

A value for skyline angle,  $\beta$ , which would yield the maximum combined stress per pound of skyline tension was desired. Sample runs were made for an 18-inch (inside bark) alder, and the results are displayed in Figure 15. At about  $15^\circ$  below horizontal ( $\beta=75^\circ$ ) a given stress condition is reached with the lowest amount of skyline tension.

A horizontal angle of (-)15 degrees was also found to maximize stress per pound of skyline tension for spruce, but (-)10 degrees ( $\beta = 80^\circ$ ) was found to be "critical" for hemlock.

Skyline angles below (-)30 degrees were not evaluated. As the steepness of the skyline angle increases, the resultant of the forces at the top of the tree rotates in a clockwise direction, and would eventually point toward the rear of the tree. To simulate this condition, a front guyline would be needed, and this has not yet been incorporated in the model used in this study.

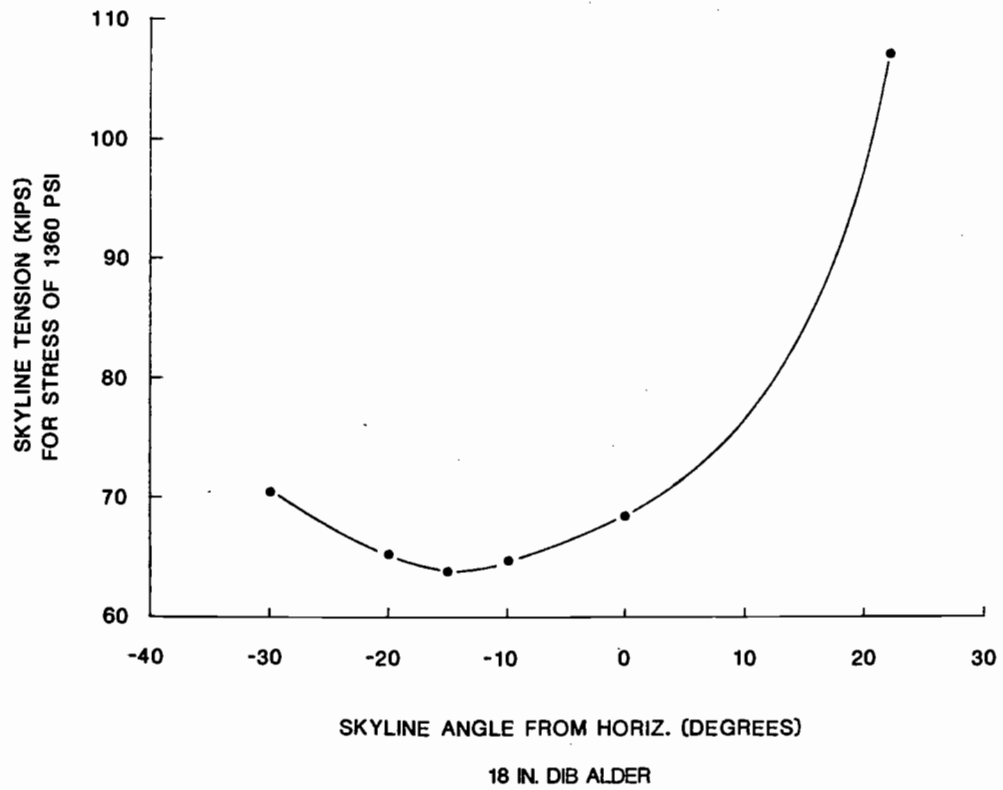


Figure 15. Skyline Tension to Allowable Stress in Alder for Various Skyline Angles

## 1. Alder Analysis

A plot of combined stress versus vertical load in an 18" DIB alder is shown in Figure 16. The 100 pounds pretension causes an initial stress of 47 psi. Stress increases rapidly until about 2500 pounds of vertical load. In this range, the "belly" of the guyline catenary is being taken up as the top of the tree is deflected toward the skyline, Figure 14a,b. After the guyline has tightened, stress increases at a lower rate.

If we define the maximum allowable stress at 1360 psi (Table 9), we see the vertical component of the forces at the top of the tree ( $P$  in equations 7 and 9) is about 82,500 pounds. For comparison, a critical buckling load,  $P_{cr}$ , for the tree was computed using the method presented by Pyles (1984). The method allows calculation of critical buckling loads for columns with varying moment of inertia, and given base stiffness. The same moment of inertia and base stiffness functions were used as in the bending calculations, and the end conditions were pinned at the top, restrained at the bottom. The value of  $P_{cr}$  was 398,700 pounds.

Assuming we would not want to exceed a normal stress of 1360 psi at the rigging height, where the diameter would probably be the smallest, the maximum allowable vertical load would have to be decreased from 398,700 to 199,600 pounds.

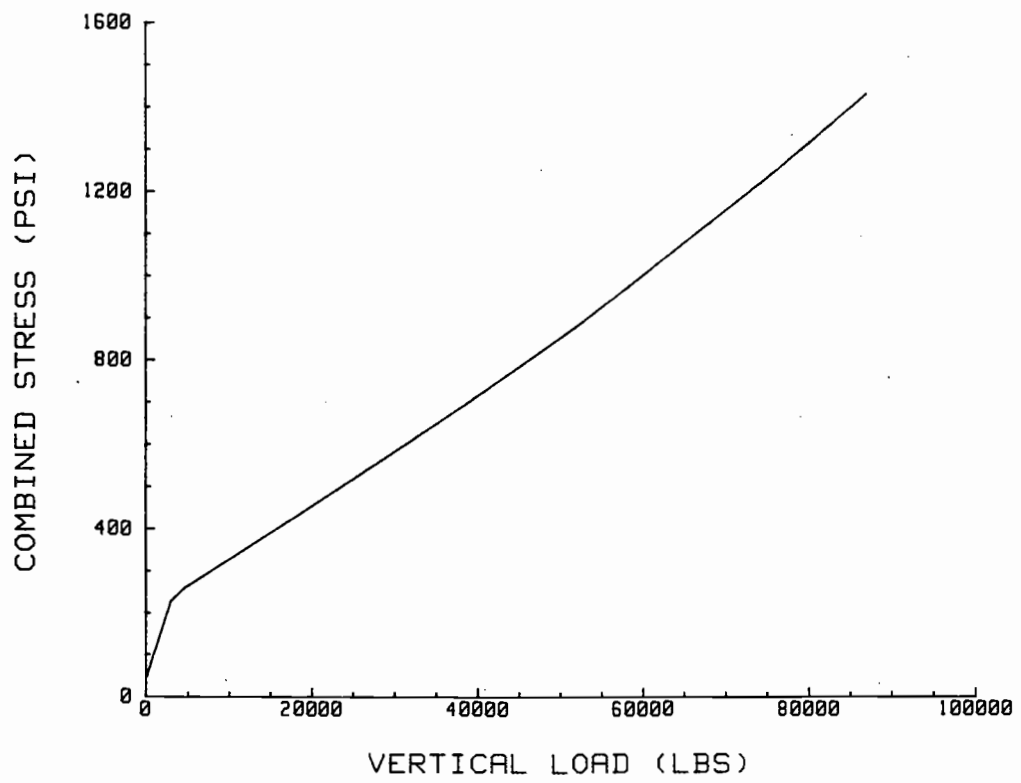


Figure 16. Combined stress vs Vertical Load for an 18 Inch DIB Alder

In this case, when analyzing the tail tree in terms of buckling, maximum axial stress limits the vertical load, but yields a value 142% larger than the bending analysis (199,600 pounds versus 82,500 pounds).

A buckling calculation was also done for a pinned-pinned case, which yielded a critical buckling load of 233,700 pounds. Again, the maximum axial stress of 1360 psi would be limiting, and we would still predict a maximum vertical load of 199,600 pounds.

In fairness to the buckling approach, it should be noted that the criterion of a slenderness ratio greater than 150 is not met by the above example. If we assume an average column diameter of 14.5", which is equal to the diameter at two-thirds the rigging height, the tree would have to be rigged at least 46 feet up. It's doubtful any alder could be rigged to a height that would meet the slenderness ratio criterion, without going past a fork in the tree.

Sessions, Pyles and Mann (1985) compared the combined stress approach to the Euler equation for a Douglas-fir which did meet the slenderness ratio criterion. For an assumed constant cross section, and varying end conditions, they found that critical buckling loads bracket maximum allowable vertical load, as determined from combined stress.

Figure 17 displays combined stress versus skyline tension for the case we've been examining. It would take about 57,500 pounds of skyline tension to create 1,360 psi of combined stress. The point of maximum combined stress would be 15 feet from the base of the tree.

Calculations of combined stress were also made for other diameter classes within the size range that field data was obtained, and the results are displayed in Figure 18. As we would expect, the skyline tension required to reach a given combined stress increases with diameter. To reach a stress of 1,360 psi, about 51,000 pounds of skyline tension would be required for a 14" DIB alder, whereas about 90,000 pounds would be required to cause the same stress condition in a 26" DIB alder.

The values of skyline tension required to produce a combined stress of 1360 psi are shown versus tree diameter in Figure 19. The plot suggests that for a given geometry, pretension, etc., the relationship between skyline tension and a given allowable stress condition may be a well defined function of tree diameter.



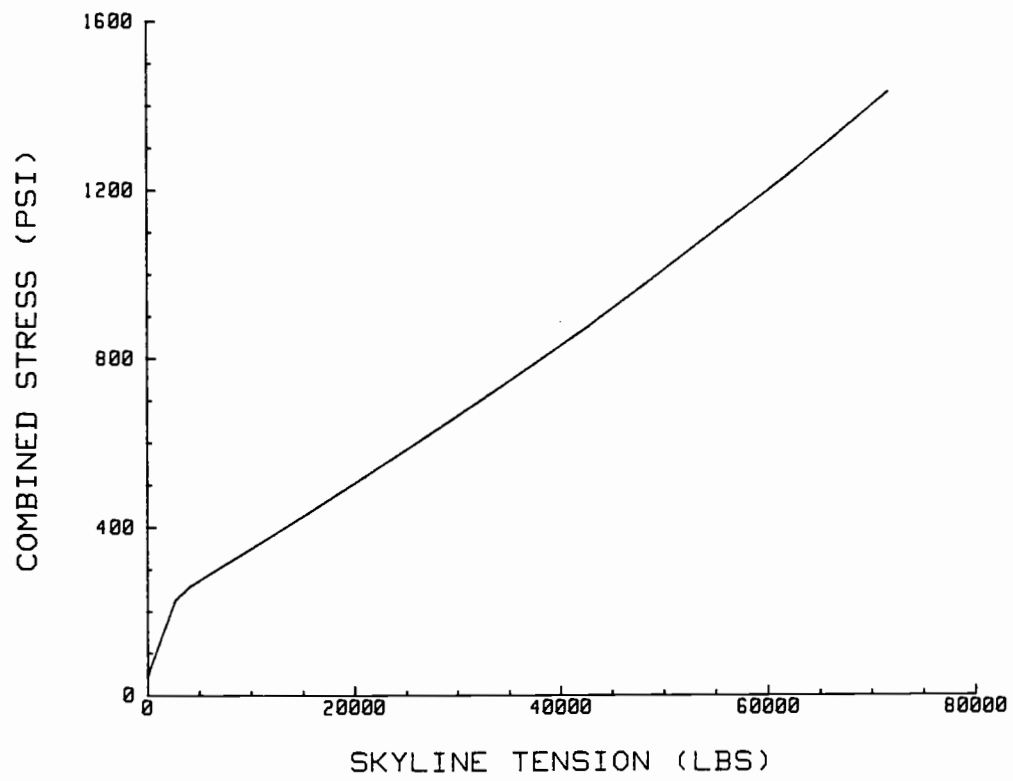


Figure 17. Combined stress vs Skyline tension for an 18 Inch DIB Alder

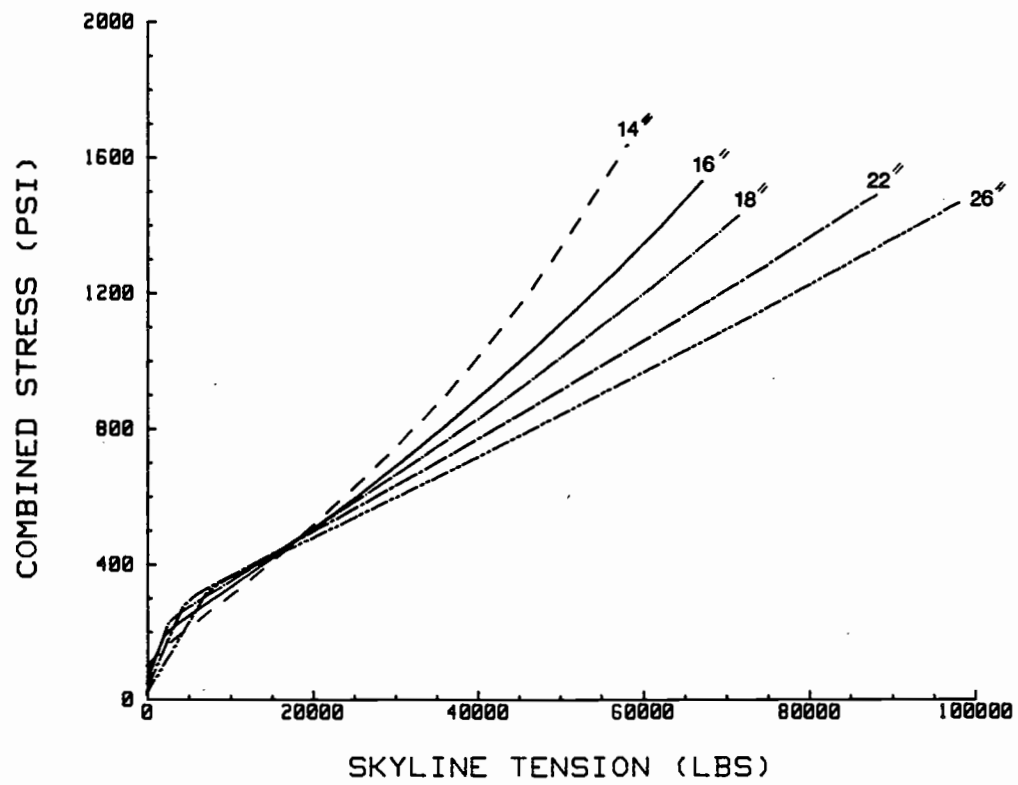


Figure 18. Combined stress vs Skyline Tension for Various Alder Diameters

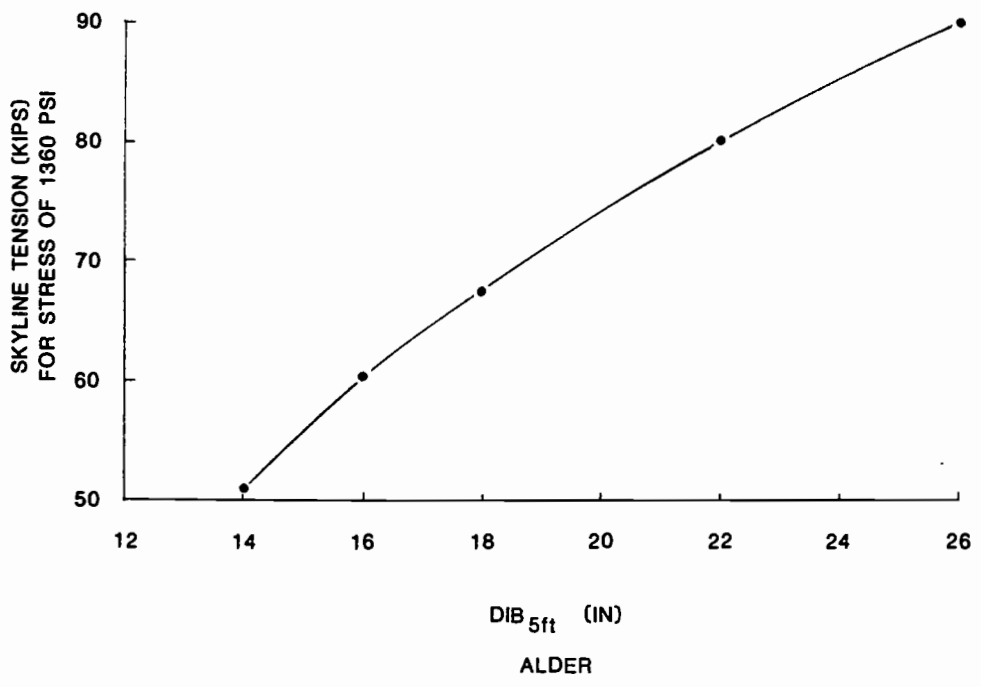


Figure 19. Skyline Tension to Allowable Stress in Alder vs Tree Diameter

## 2. Spruce Analysis

Stress calculations were done for four diameter classes. Input geometry, etc. was identical to alder, except, of course, for the base stiffness, modulus of elasticity, and moment of inertia function. The plots of total stress versus skyline tension are shown in Figure 20.

The 100 pounds of guyline pretension in the 12-inch spruce deflects the tree toward the guyline enough to cause a combined stress of 170 psi. Once skyline tightening begins, it takes about 8,000 pounds of tension to create the same stress condition in the direction of the skyline.

Skyline tensions to the assumed maximum stress of 1228 psi versus diameter are shown in Figure 21. No inference should be drawn about strength characteristics beyond the range displayed.

It is interesting to note that a 16-inch alder can resist about 15% more skyline tension than a 16-inch spruce (60.3 kips vs. 52.3 kips), before reaching its allowable stress.

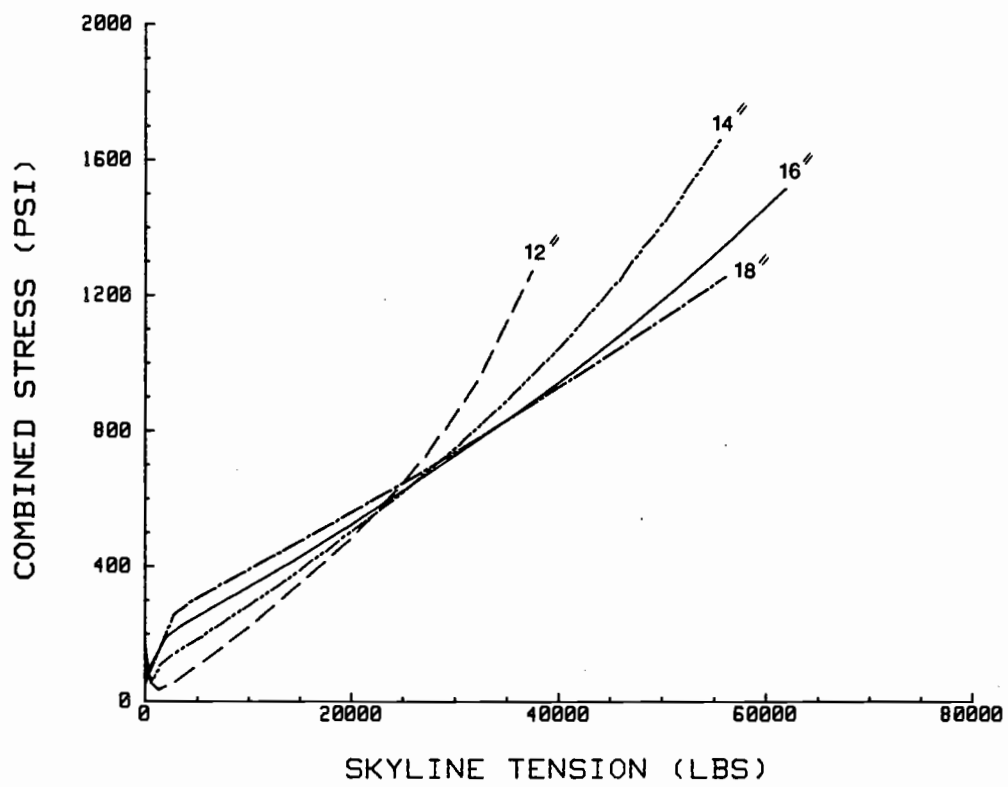


Figure 20. Combined stress vs Skyline Tension for Various Spruce Diameters

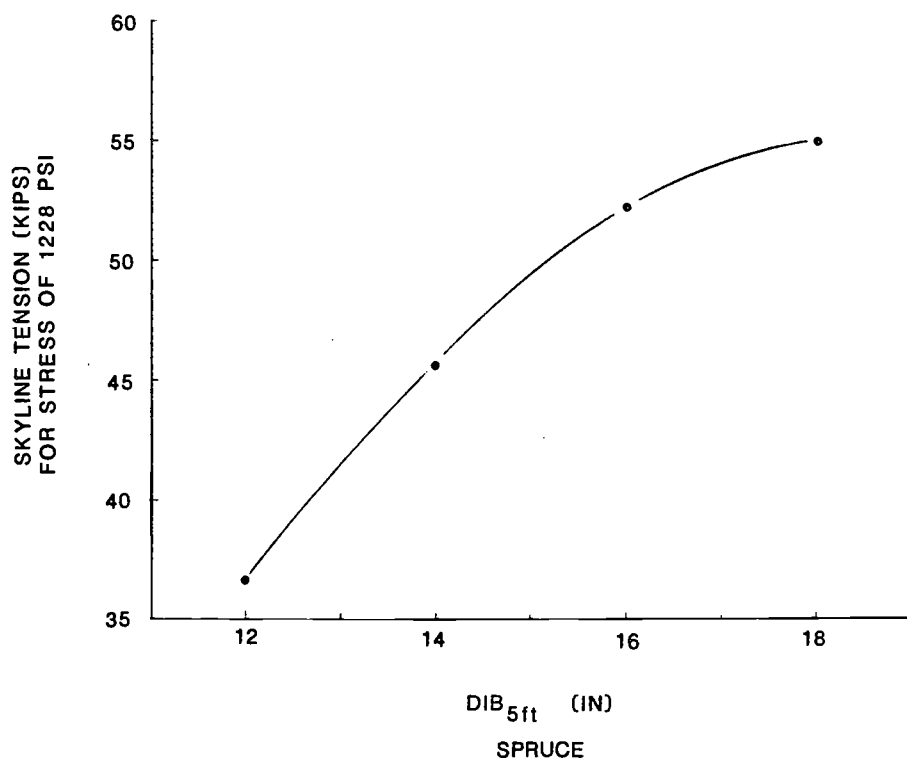


Figure 21. Skyline Tension to Allowable Stress in Spruce vs Tree Diameter

### 3. Hemlock Analysis

As done for alder and spruce, stress calculations were made for the same geometry, etc., except that a skyline angle of  $10^\circ$  below horizontal was used, as it maximized stress per pound of skyline tension. Figure 22 displays total stress versus skyline tension.

No plot of skyline tension to combined stress of 1546 psi is displayed. As there was only a 4-inch range in field tested diameter classes, it may seem trivial. However, looking at the 16-inch class, with a skyline tension of about 70,000 pounds to its maximum stress, hemlock appears to be the strongest of the three species tested. The 16-inch alder had a maximum tension of 60,300 pounds, and the 16-inch spruce a value of 52,300 pounds.

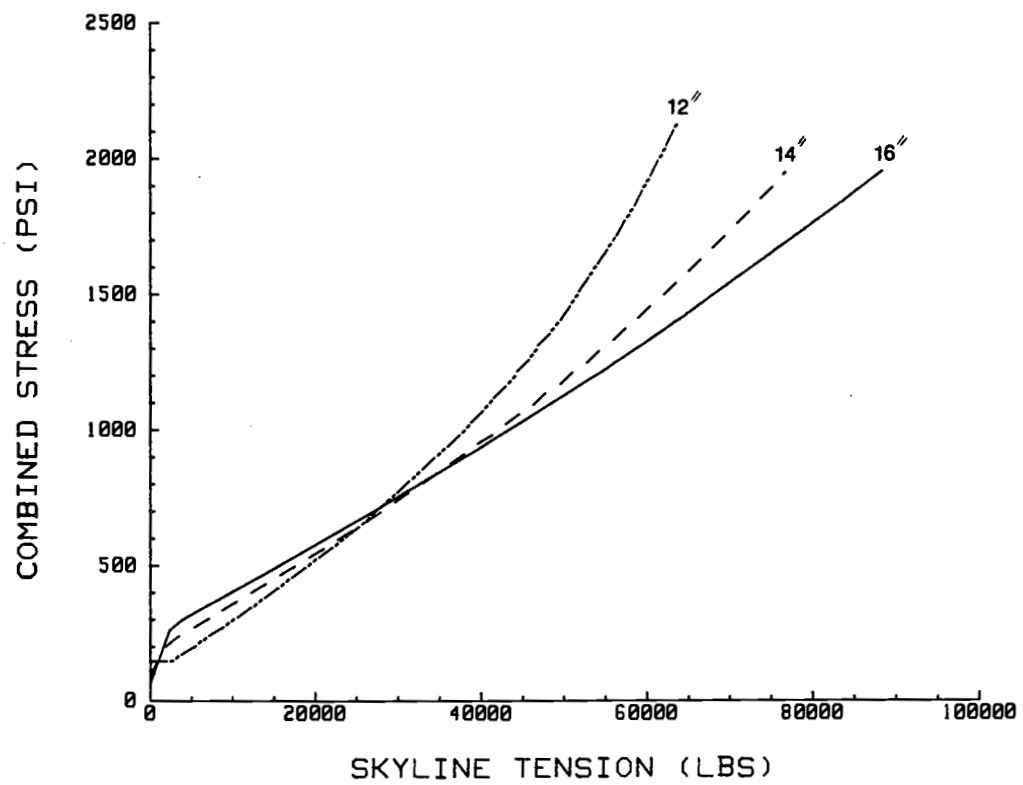


Figure 22. Combined stress vs Skyline Tension for Various Hemlock Diameters



#### 4. Parameter Study

##### a. Modulus of Elasticity

Rather than using the published average Wood Handbook (1974) values for moduli of elasticity, more conservative estimates could have been made using the same method that was used to establish maximum allowable compressive stress values.

The Wood Handbook (1974) reported an average coefficient of variation for modulus of elasticity of 22%, for approximately 50 species tested. Using the coefficient of variation to estimate the standard deviation for each species, the assumption was made that 95% of the modulus test values were within plus or minus 2 standard deviations of the mean values. Combined stress calculations were made for a 16 inch tree of each species, using the upper and lower values for modulus of elasticity.

The upper, lower and mean values for modulus, and the estimated standard deviations are shown in Table 10. The upper and lower values are all about 44% larger or smaller, respectively, than the mean values.

Combined stress versus skyline tension curves for the upper and lower alder modulus values, and for the mean modulus are shown in Figure 23. As noted earlier, the

TABLE 10. Alternative Values for Modulus of Elasticity

<u>Species</u>	<u>Mean<sup>1</sup> Modulus (psi)</u>	<u>Estimated Standard<sup>2</sup> Deviation (psi)</u>	<u>Lower Value (Mean Minus 2) (Standard Deviations)</u>	<u>Upper Value (Mean Plus 2) (Standard Deviations)</u>
Alder	1,170,000	257,400	665,200	1,684,800
Spruce	1,230,000	270,600	688,800	1,771,200
Hemlock	1,310,000	288,200	733,600	1,886,400

<sup>1</sup>Wood Handbook (1974)

<sup>2</sup>Estimated from reported average coefficient of variation of 22%.

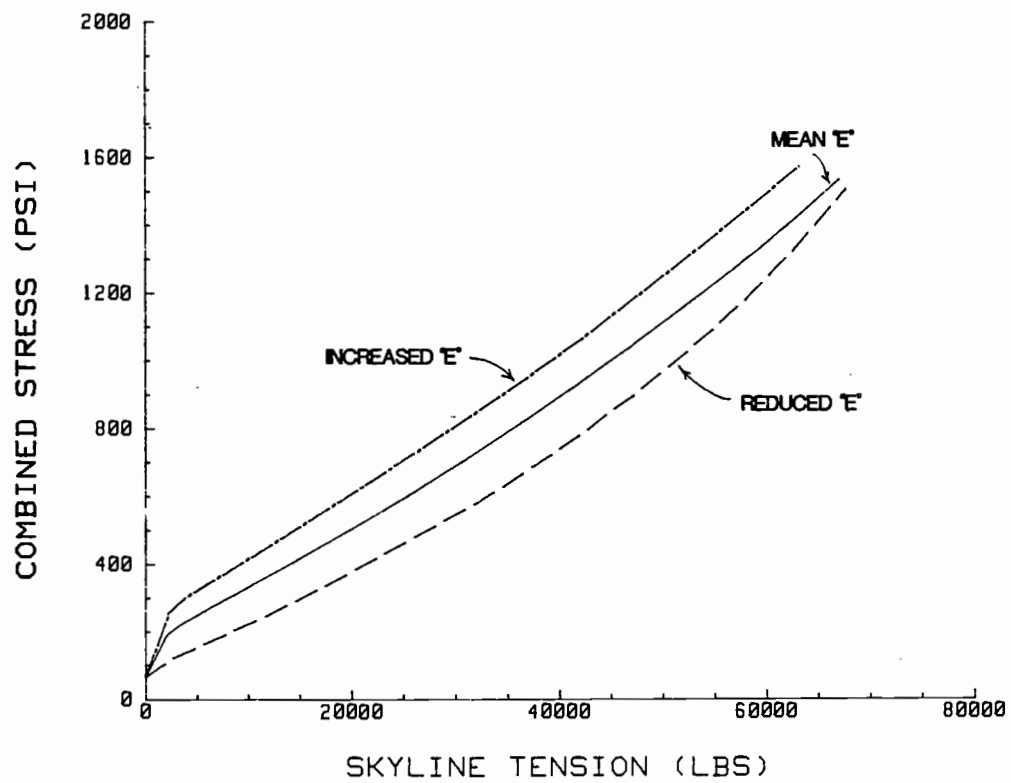


Figure 23. Combined stress in a 16 Inch D18 Alder vs Skyline Tension for Alternative Moduli of Elasticity

calculations for mean modulus reached 1360 psi at about 60.3 kips of skyline tension.

The curve with the upper modulus value reached 1360 psi of stress at about 54.4 kips of skyline tension, and the curve with the lower modulus reached the allowable stress at about 63.5 kips.

Increasing the modulus value by 44% resulted in a 10% decrease in skyline tension required to reach allowable stress. Decreasing the modulus resulted in a 5% increase in allowable skyline tension.

Figure 24 displays stress versus skyline tension curves for varying spruce moduli. The skyline tensions to the allowable stress of 1228 psi are 47.5 kips, 52.3 kips and 54.7 kips for the upper, mean and lower modulus values, respectively. The higher modulus reduced the allowable skyline tension by 9%, and the lower modulus added 5% to allowable skyline tension.

Combined stress versus skyline tension for varying hemlock moduli are shown in Figure 25. The loads to the allowable stress of 1546 psi are 58.7 kips, 70.7 kips and 82.6 kips, respectively. The higher modulus reduced allowable skyline tension by 17%, and the lower modulus increased allowable tension by 17%. It is noted that the guyline stress for the reduced hemlock modulus calculations would probably have exceeded the proportional limit before the tree had reached its allowable stress.

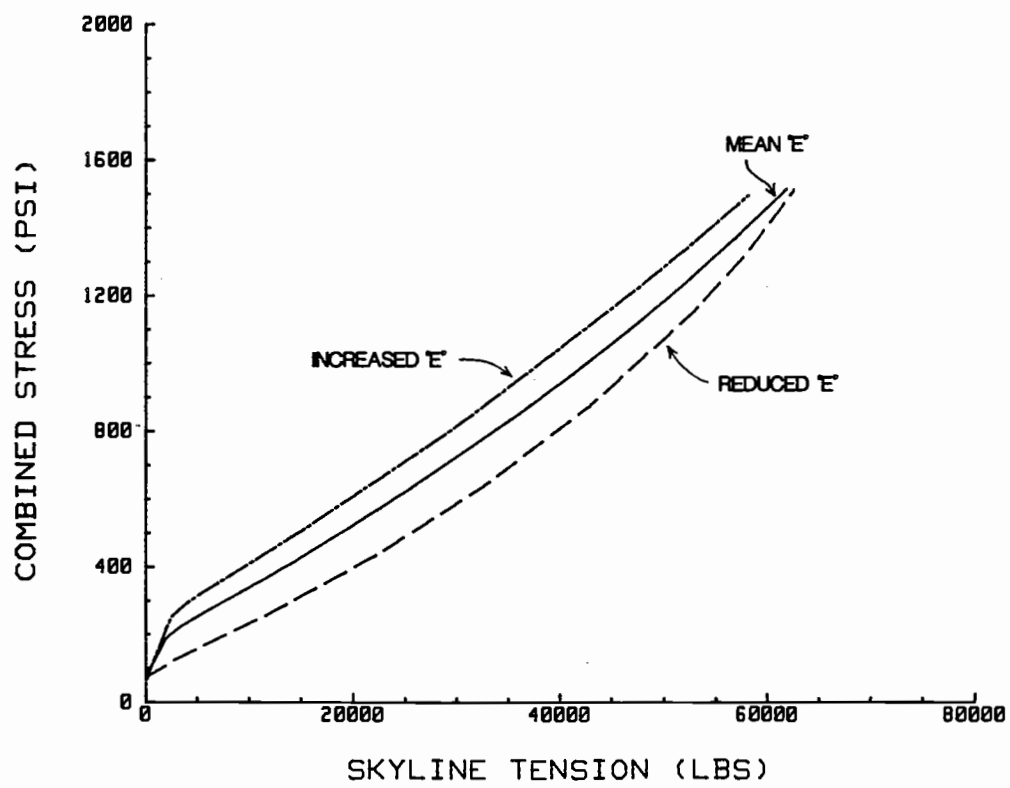


Figure 24. Combined stress in a 16 Inch D1B Spruce vs Skyline Tension for Alternative Moduli of Elasticity

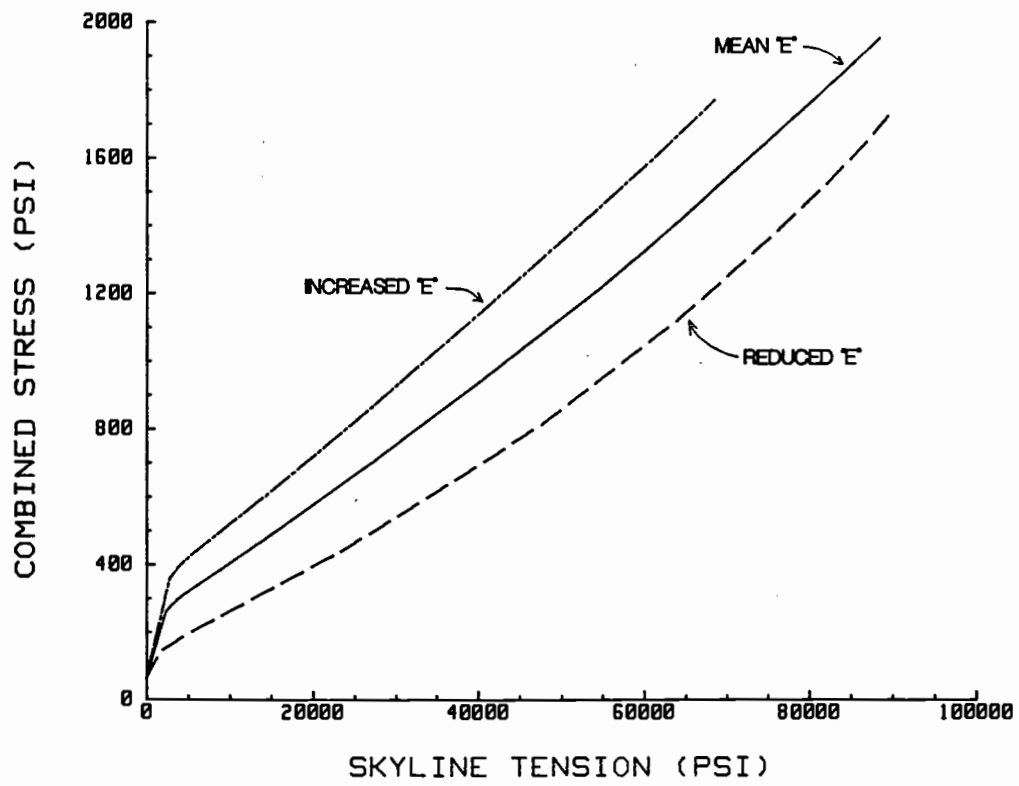


Figure 25. Combined stress in a 16 Inch DIB Hemlock vs Skyline Tension for Alternative Moduli of Elasticity

In calculations with the mean modulus values, and compressive stress values reduced by 3 standard deviations, it has been assumed we have been operating within the proportional limits of the wood. The values of strain would be .0012, .001 and .0012 for alder, spruce and hemlock, respectively. Using the reduced moduli, the values of strain would be .002 for all three. It is assumed we would still be within the proportional limits.

For the geometry, etc. analyzed, hemlock seemed to be the most sensitive to changes in modulus of elasticity, followed by alder and spruce. Considering that changes in modulus on the order of 44% result in a maximum change of 17% in skyline tension to allowable stress, it appears that combined stress calculations are not very sensitive to modulus of elasticity.

b. Guyline Pretension

The effects of guyline pretension are of interest. All previous calculations have been made with an initial 100 pounds of lower end tension in the guyline. Two additional sets of calculations were made for a 16 inch DIB alder, one with 50 pounds of pretension, and one with 150 pounds. The resulting stress versus skyline tension curves, along with the curve for 100 pounds of pretension are shown in Figure 26.

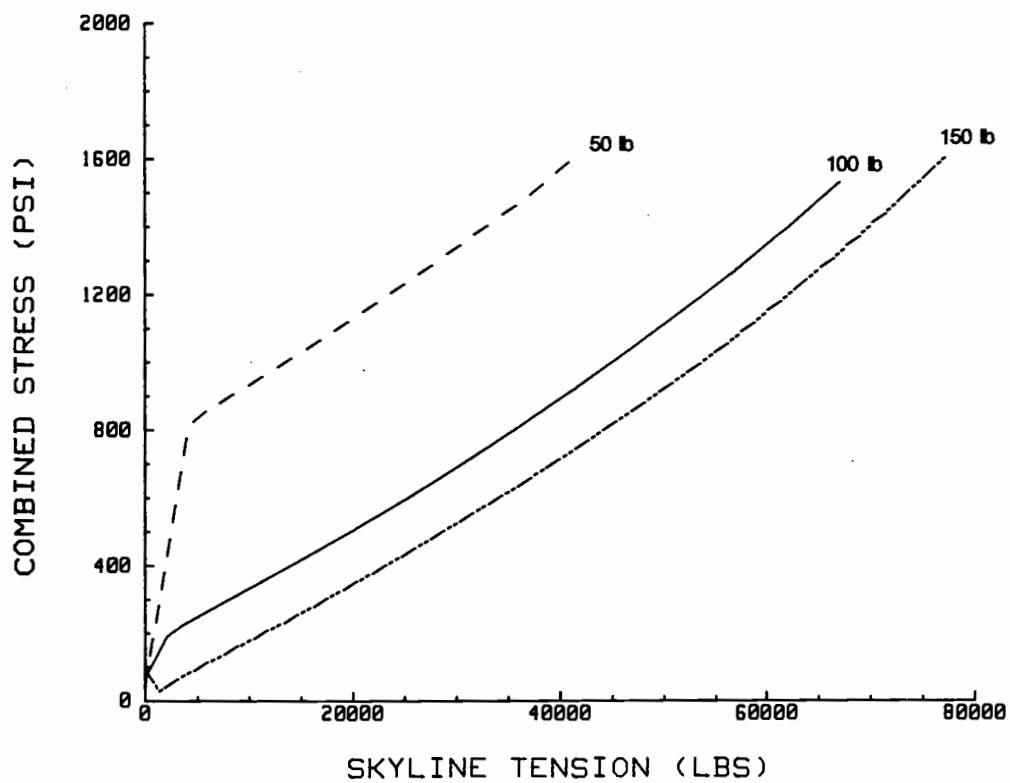


Figure 26. Combined stress in a 16 Inch DIB Alder vs Skyline Tension for Various Guyline Pretensions



Immediately we see that with only 50 pounds of pretension, stress increases rapidly in the tree while slack in the guyline is being taken up. The skyline tensions required to yield a stress of 1360 psi for the given pretensions are: 50 lb - 31 kips, 100 lb - 60.5 kips, 150 lb - 67.5 kips. A 95% increase in maximum skyline tension is obtained in going from 50 to 100 pounds of pretension.

c. Guyline Diameter

Previous calculations were made with a 3/4 inch guyline. An additional set was made for a 16 inch DIB alder with a 5/8 inch guyline, and a set was made with a 7/8 inch guyline. The results for the guyline sizes are displayed in Figure 27.

The amount of sag in the guyline increases with size for the same value of pretension, so very little slack needs to be taken up in the 5/8 inch line, and it is more effective in controlling stress during the beginning phase of skyline tensioning.

As we might expect, increasing guyline size increases the amount of skyline tension needed to create 1360 psi of stress. However, we encounter diminishing returns very rapidly. The skyline tensions to the assumed allowable stress for the given line sizes are: 5/8 inch - 56.3 kips, 3/4 inch - 60.5 kips, 7/8 inch - 57.4 kips.

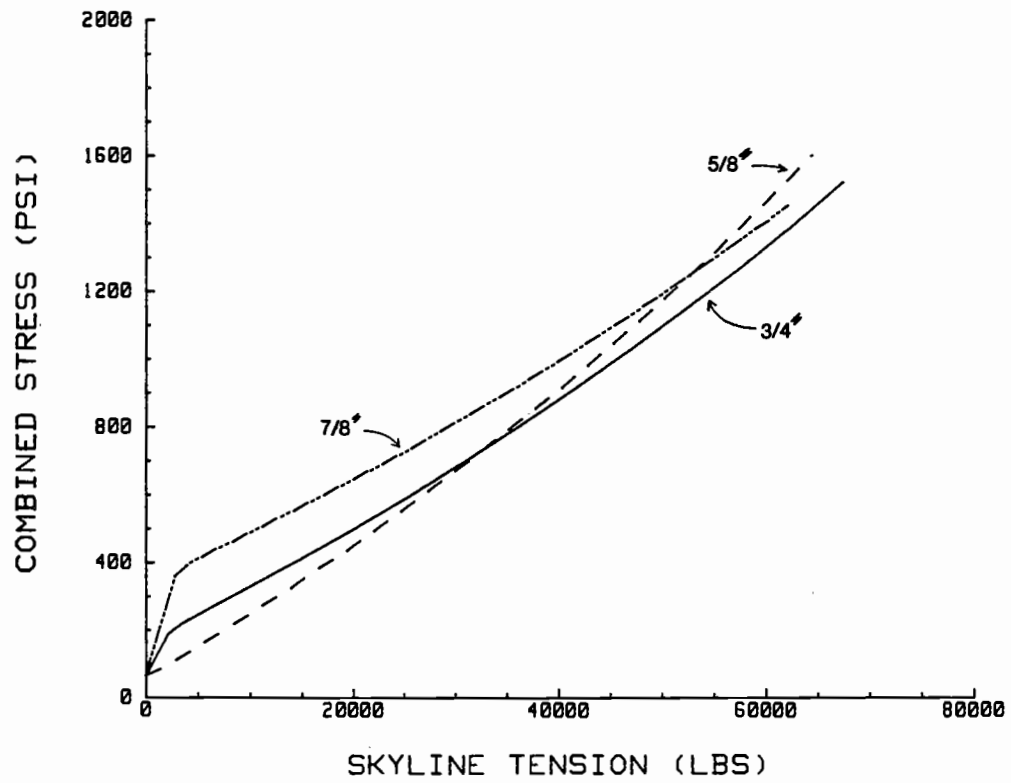


Figure 27. Combined stress in a 16 Inch DIB Alder vs Skyline Tension for Various Guyline Diameters

d. Rigging Height

Figure 28 displays combined stress in a 16 inch DIB hemlock versus skyline tension for the control height of 30 feet, and for heights of 25 and 35 feet. Increasing the height from 30 to 35 feet results in a 4% reduction in skyline tension (70.7 to 67.8 kips) to the allowable stress of 1546 psi, and decreasing the height from 30 to 25 feet results in a 2% increase in allowable skyline tension (70.7 to 72.4 kips).

Figure 29 shows stress in a 16 inch DIB spruce for 25, 30 and 35 feet. Increasing height from 30 to 35 feet reduces skyline tension to the allowable stress of 1228 psi by 13% (52.3 to 45.4 kips), and decreasing height from 30 to 25 feet increases allowable tension by 10% (52.3 to 57.7 kips).

Stress in a 16 inch DIB alder versus skyline tension for varying heights is shown in Figure 30. Increasing load height from 30 to 35 feet decreases allowable skyline tension by 11% (60.3 to 53.8 kips), and decreasing load height from 30 to 25 feet increases allowable tension by 9% (60.3 to 65.7 kips).

For the geometry and other set inputs, it appears that hemlock is the least sensitive to changes in rigging height.

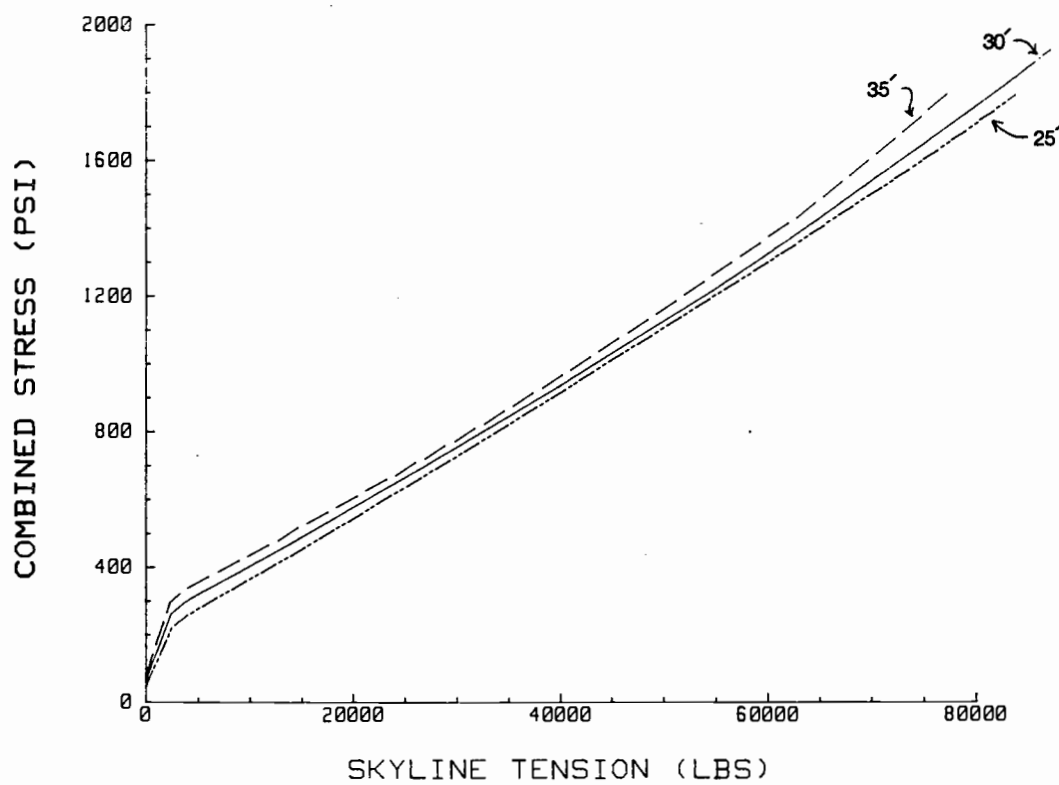


Figure 28. Combined stress in a 16 Inch DIB Hemlock vs Skyline Tension for Various Rigging Heights

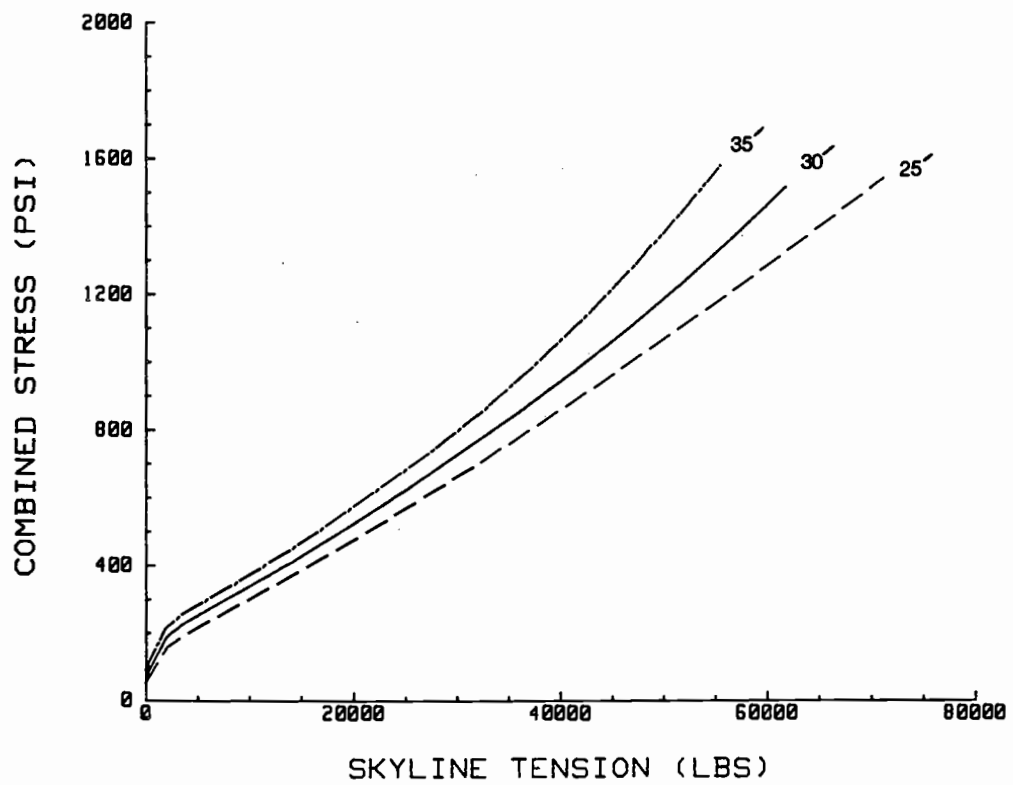


Figure 29. Combined stress in a 16 Inch DIB Spruce vs Skyline Tension for Various Rigging Heights

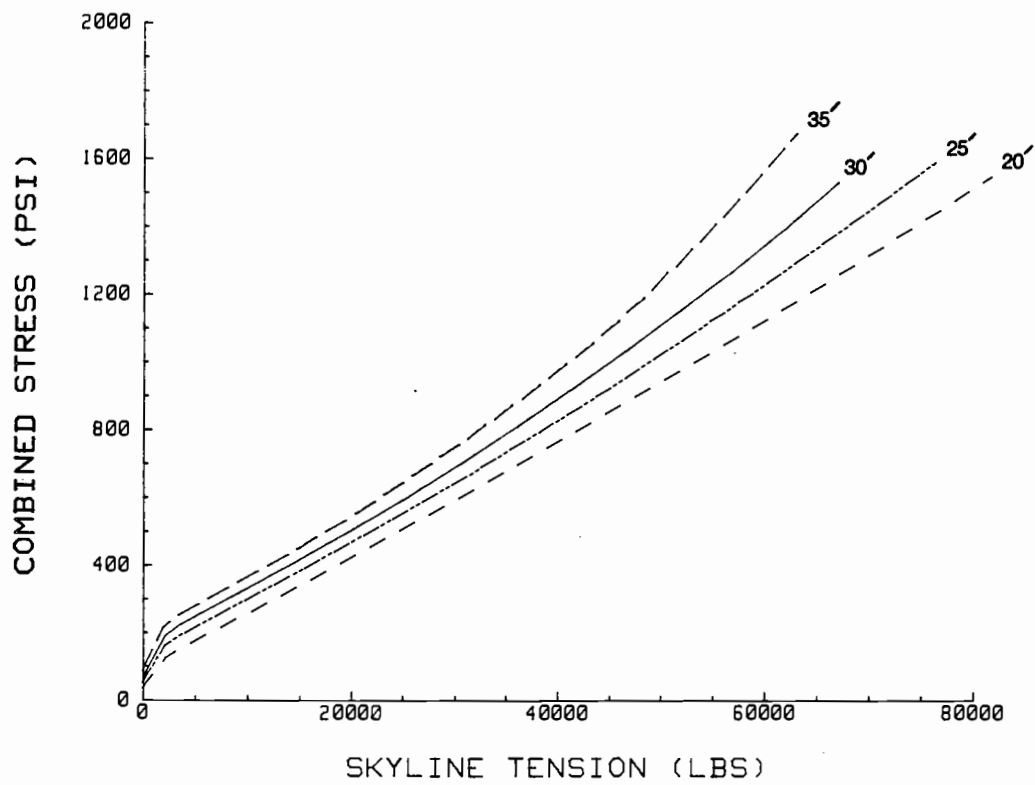


Figure 30. Combined stress in a 16 Inch DIB Alder vs Skyline Tension for Various Rigging Heights

### 5. Comparison with Douglas-fir

As mentioned in the introduction, the Oregon State Safety Code (1984) says to add 2 inches to recommended diameters for Douglas-fir tail trees when using other coniferous species.

At the end of Hemlock Analysis, the three species tested were ranked according to the skyline tension required to create the maximum allowable stress in a 16 inch tree. The values were:

SPECIES	SKYLINE TENSION (lbs) TO MAXIMUM ALLOWABLE STRESS
Hemlock	70,700
Alder	60,300
Spruce	52,300

A bending analysis was done for a 14 inch (inside bark) Douglas-fir. The same geometry was used as for the other species. A skyline angle of (-) 10 degrees ( $\beta = 80^\circ$ ) was found to maximize stress per pound of skyline tension.

The Wood Handbook (1974) average, green wood modulus of elasticity of 1,560,000 psi was used. A maximum allowable stress of 1740 psi was calculated exactly as it was done for the other species. The base stiffness and moment of inertia functions for Douglas-fir were developed by Pyles (1984).

A plot of skyline tension versus combined stress for the Douglas-fir is shown in Figure 31. Also shown are the curves for the 16 inch alder, spruce and hemlock.

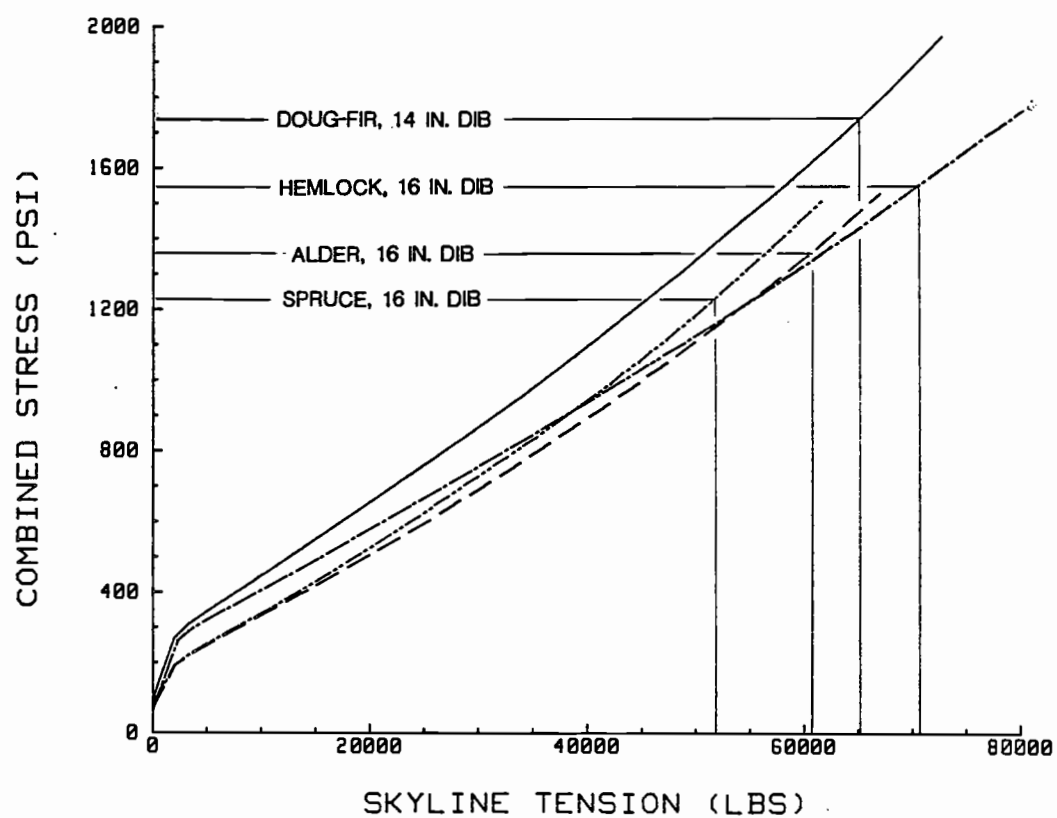


Figure 31. Combined stress vs Skyline Tension for a 14 Inch DIB Douglas-fir, and a 16 Inch DIB Alder, Spruce and Hemlock



The 14 inch Douglas-fir reaches its allowable stress of 1740 psi at about 65,000 pounds of skyline tension. The 16 inch hemlock reaches its maximum at 70,700 pounds, so the State's recommendations may be slightly conservative for hemlock.

The spruce reaches its maximum stress at 52,300 pounds. Referring to Figure 21, it appears that even an 18 inch spruce could not withstand 65,000 pounds of skyline tension without exceeding its maximum allowable stress.

Figure 19 indicates it would take an alder with an inside bark diameter of just over 17 inches to withstand 65,000 pounds of skyline tension.

The above comparisons are for a particular rigging height, geometry, pretension, etc. The relationships between the species may not be the same for all cases.

## V. SUMMARY

One objective of this study was to determine if the published values for modulus of elasticity could be used to predict cable loadings for red alder, Alnus rubra Bong., Sitka spruce, Picea sitchensis (Bong.) Carr, and western hemlock, Tsuga heterophylla (Raf.) Sarg., support trees. Field measurements indicated published values obtained from tests on small wood specimens could be used. For the rigging height and geometry evaluated, sensitivity analysis indicates that a difference in modulus of plus or minus 44% would affect load bearing capacity by a maximum of 17%.

Base stiffness values were developed for each species. A relatively large amount of variation was found with pull direction, and it appears that base stiffness may increase as load height decreases.

Normalized moment of inertia functions were developed for each species. A statistically based, repeatable procedure for forcing regression and confidence interval lines through a known point was documented.

Some of the violations inherent in analyzing support trees in terms of buckling were reviewed, and a case was analyzed wherein a buckling/axial stress analysis predicted a significantly larger allowable vertical load than did a combined stress analysis.

For the size classes and rigging height tested, alder was found to be between hemlock and spruce in support capacity. All three species are able to support relatively high skyline tensions before reaching conservative estimates of maximum combined stress.

The load bearing capacity of a 14 inch DIB (diameter inside bark) Douglas-fir, Pseudotsuga menziesii (Mirb.) Franco, was compared to those of a 16 inch DIB alder, spruce and hemlock. It was found that the 16 inch DIB hemlock could support about 9% more skyline tension to its allowable stress than the 14 inch Douglas-fir. An alder of at least 17 inches DIB would be needed to support the same load as the Douglas-fir, and a spruce with a DIB over 18 inches, which is outside the range of the field data, would be needed.

The effect of tree lean on combined stress was not determined. Any effects would probably vary with the direction of pull relative to the lean. Intuitively, we could expect an increase in combined stress for most cases.

The importance of quantifying the effects of all the variables that make up a tail tree system was shown in the limited parameter analysis. Of the variables tested, guyline pretension seemed to have the greatest effect on total stress.

## VI. SUGGESTIONS FOR FURTHER RESEARCH

1. Values for base stiffness over a greater range in tree diameters is needed for hemlock and spruce.
2. Base stiffness values for a greater range in stand characteristics, topography, etc. are needed for all species.
3. Better understanding of base stiffness variation with pull direction is needed.
4. Final determination of the relationship between base stiffness and load height should be made. If base stiffness does increase with decreasing load height, the importance to support capacity must be determined.
5. Values for stump pullout resistance need to be determined for alder, hemlock and spruce.
6. The importance of the eccentricity of the skyline load, due to its application through a block on the side of the tree, needs to be determined.
7. Inclusion of the effect of tree lean on support capacity is needed.
8. Additional guylines must be included in the model to more completely simulate actual support conditions.
9. Dynamic loads may affect tail tree support capacity, and a study of the effects is in progress.

## VII. REFERENCES

- Neter, J., W. Wasserman and M. Kutner. 1983. Applied Linear Regression Models. Illinois: Richard D. Irwin, Inc., 1983. pp. 160-163.
- Oregon Workers' Compensation Dept. 1982. Oregon Occupational Safety and Health Code, Division 80, Logging. Amended Sept. 1, 1982.
- Pyles, M. R. 1984. Behavior of second growth Douglas-fir trees in column action: Final report to Pacific Northwest Forest and Range Experiment Station. USDA Forest Service Research agreement suppl. PNW-82-324. PNW For & Range Exp Stn, Portland, OR.
- Sessions, J., M. Pyles, and J. Mann. 1985. Structural Analysis of Tail Spars, in Proceedings of Sixth Pacific Northwest Skyline Symposium, Vancouver, B.C.
- Stoupa, J. 1984. Behavior and Load Carrying Capacity of Anchors. Master of Science Thesis. Oregon State University, Corvallis, OR.
- Stuck, R. D. II. 1974. Predicting Diameter Inside Bark of Various Upper Stem Heights for Several Coniferous Species Along the Western Slope of the Cascade Mountains. Master of Science Thesis. Oregon State University, Corvallis, OR.
- USDA, Forest Service. 1974. Wood Handbook: Wood as an engineering material. Agriculture Handbook No. 72, Forest Products Laboratory, pp. 4-2 to 4-22.

**APPENDICES**

**APPENDIX A**  
**Method for Forcing a Regression**  
**Line Through a Point**

## Reference:

Neter, J., W. Wasserman and M. Kutner.  
Applied Linear Regression Models. Illinois:  
 Richard D. Irwin, Inc., 1983. pp. 160-163.

The desire was to force a regression line of the general form  $Y = x^b$ , and also the lower limit for expected value through the same point.

More specifically, the dependent variable was the ratio of moment of inertia at a given height,  $I_h$ , to the moment of inertia at 5 feet height,  $I_5$ . This ratio,  $I_h/I_5$  was to be a function of height,  $h$ . The form of the equation was  $I_h/I_5 = h^b$ . Logically, and physically  $I_h/I_5$  must equal 1 when  $h = 5$  feet, ie.  $I_5/I_5$  will equal 1 for all trees.

Since we wanted to force the regression line through the point (5,1), ie. at 5 feet  $I_h/I_5 = 1$ , we set up a program that would minimize the sums of  $(I_h/I_5 - 1)^2$  and  $(\text{height} - 5)^2$ .

A log transformation as follows is required to use the least squares method for determining the b-coefficient:

$$\text{if } I_h/I_5 = h^b, \text{ then } \log(I_h/I_5) = b \log h$$

To force the regression line through (5,1), the following relation was used:

$$\begin{aligned} x_i &\text{ was defined as } \log(h_i) - \log(5), \\ y_i &\text{ was defined as } \log(I_{h_i}) - \log(1), \\ &\text{and } b \text{ was determined by:} \end{aligned}$$



$$b = \frac{\sum x_i y_i}{\sum x_i^2} \quad (5.17 \text{ Neter, et al.})$$

the variance,  $S(Y_i) = \sqrt{\frac{x_i(\text{MSE})}{\sum x_i^2}}$  (5.17 Neter, et al.)

where  $\text{MSE} = \frac{\sum (Y_i - bx_i)^2}{n-1}$  (5.17 Neter, et al.)

and finally the lower limit for the prediction interval was obtained by:

$$Y - t(s)(Y_i)$$

where  $t$  is a 2-tail value with  $n-1$  degrees of freedom.

```

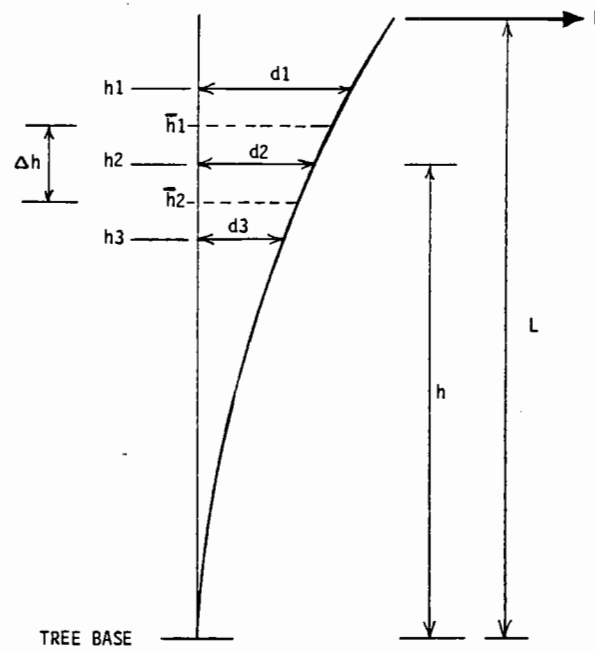
2 Jun 1985      11:21:12

1  | ----- PROGRAM "STATS" -----
2  |
3  |
4  |
5  |
6  |      Forces line of the form  $Ih/15=a(\text{HEIGHT})^b$ 
7  |      through the point (5,1)
8  |      where HEIGHT=5 feet and  $Ih/15=1$ 
9  |
10 |
11 |
12 |
13 |
14 |
15 |
16 | INPUT "ENTER FILE NAME",F$
17 | INPUT "ENTER THE NUMBER OF ROWS OF DATA",N
18 | F$="I5DATA2"
19 | N=39
20 |
21 |
22 |
23 |
24 |
25 |
26 | T=8
27 |
28 |
29 |
30 |
31 |
32 |
33 |
34 |
35 |
36 |
37 |
38 |
39 |
40 |
41 |
42 |
43 |
44 |
45 |
46 |
47 |
48 |
49 |
50 |
51 |
52 |
53 |
54 |
55 |
56 |
57 |
58 |
59 |
60 |
61 |
62 |
63 |
64 |
65 |
66 |
67 |
68 |
69 |
70 |
71 |
72 |
73 |
74 |
75 |
76 |
77 |
78 |
79 |
80 |
81 |
82 |
83 |
84 |
85 |
86 |
87 |
88 |
89 |
90 |
91 |
92 |
93 |
94 |
95 |
96 |
97 |
98 |
99 |
100|
101|
102|
103|
104|
105|
106|
107|
108|
109|
110|
111|
112|
113|
114|
115|
116|
117|
118|
119|
120|
121|
122|
123|
124|
125|
126|
127|
128|
129|
130|
131|
132|
133|
134|
135|
136|
137|
138|
139|
140|
141|
142|
143|
144|
145|
146|
147|
148|
149|
150|
151|
152|
153|
154|
155|
156|
157|
158|
159|
160|
161|
162|
163|
164|
165|
166|
167|
168|
169|
170|
171|
172|
173|
174|
175|
176|
177|
178|
179|
180|
181|
182|
183|
184|
185|
186|
187|
188|
189|
190|
191|
192|
193|
194|
195|
196|
197|
198|
199|
200|
201|
202|
203|
204|
205|
206|
207|
208|
209|
210|
211|
212|
213|
214|
215|
216|
217|
218|
219|
220|
221|
222|
223|
224|
225|
226|
227|
228|
229|
230|
231|
232|
233|
234|
235|
236|
237|
238|
239|
240|
241|
242|
243|
244|
245|
246|
247|
248|
249|
250|
251|
252|
253|
254|
255|
256|
257|
258|
259|
260|
261|
262|
263|
264|
265|
266|
267|
268|
269|
270|
271|
272|
273|
274|
275|
276|
277|
278|
279|
280|
281|
282|
283|
284|
285|
286|
287|
288|
289|
290|
291|
292|
293|
294|
295|
296|
297|
298|
299|
300|
301|
302|
303|
304|
305|
306|
307|
308|
309|
310|
311|
312|
313|
314|
315|
316|
317|
318|
319|
320|
321|
322|
323|
324|
325|
326|
327|
328|
329|
330|
331|
332|
333|
334|
335|
336|
337|
338|
339|
340|
341|
342|
343|
344|
345|
346|
347|
348|
349|
350|
351|
352|
353|
354|
355|
356|
357|
358|
359|
360|
361|
362|
363|
364|
365|
366|
367|
368|
369|
370|
371|
372|
373|
374|
375|
376|
377|
378|
379|
380|
381|
382|
383|
384|
385|
386|
387|
388|
389|
390|
391|
392|
393|
394|
395|
396|
397|
398|
399|
400|
401|
402|
403|
404|
405|
406|
407|
408|
409|
410|
411|
412|
413|
414|
415|
416|
417|
418|
419|
420|
421|
422|
423|
424|
425|
426|
427|
428|
429|
430|
431|
432|
433|
434|
435|
436|
437|
438|
439|
440|
441|
442|
443|
444|
445|
446|
447|
448|
449|
450|
451|
452|
453|
454|
455|
456|
457|
458|
459|
460|
461|
462|
463|
464|
465|
466|
467|
468|
469|
470|
471|
472|
473|
474|
475|
476|
477|
478|
479|
480|
481|
482|
483|
484|
485|
486|
487|
488|
489|
490|
491|
492|
493|
494|
495|
496|
497|
498|
499|
500|
501|
502|
503|
504|
505|
506|
507|
508|
509|
510|
511|
512|
513|
514|
515|
516|
517|
518|
519|
520|
521|
522|
523|
524|
525|
526|
527|
528|
529|
530|
531|
532|
533|
534|
535|
536|
537|
538|
539|
540|
541|
542|
543|
544|
545|
546|
547|
548|
549|
550|
551|
552|
553|
554|
555|
556|
557|
558|
559|
560|
561|
562|
563|
564|
565|
566|
567|
568|
569|
570|
571|
572|
573|
574|
575|
576|
577|
578|
579|
580|
581|
582|
583|
584|
585|
586|
587|
588|
589|
590|
591|
592|
593|
594|
595|
596|
597|
598|
599|
600|
601|
602|
603|
604|
605|
606|
607|
608|
609|
610|
611|
612|
613|
614|
615|
616|
617|
618|
619|
620|
621|
622|
623|
624|
625|
626|
627|
628|
629|
630|
631|
632|
633|
634|
635|
636|
637|
638|
639|
640|
641|
642|
643|
644|
645|
646|
647|
648|
649|
650|
651|
652|
653|
654|
655|
656|
657|
658|
659|
660|
661|
662|
663|
664|
665|
666|
667|
668|
669|
670|
671|
672|
673|
674|
675|
676|
677|
678|
679|
680|
681|
682|
683|
684|
685|
686|
687|
688|
689|
690|
691|
692|
693|
694|
695|
696|
697|
698|
699|
700|
701|
702|
703|
704|
705|
706|
707|
708|
709|
710|
711|
712|
713|
714|
715|
716|
717|
718|
719|
720|
721|
722|
723|
724|
725|
726|
727|
728|
729|
730|
731|
732|
733|
734|
735|
736|
737|
738|
739|
740|
741|
742|
743|
744|
745|
746|
747|
748|
749|
750|
751|
752|
753|
754|
755|
756|
757|
758|
759|
760|
761|
762|
763|
764|
765|
766|
767|
768|
769|
770|
771|
772|
773|
774|
775|
776|
777|
778|
779|
780|
781|
782|
783|
784|
785|
786|
787|
788|
789|
790|
791|
792|
793|
794|
795|
796|
797|
798|
799|
800|
801|
802|
803|
804|
805|
806|
807|
808|
809|
810|
811|
812|
813|
814|
815|
816|
817|
818|
819|
820|
821|
822|
823|
824|
825|
826|
827|
828|
829|
830|
831|
832|
833|
834|
835|
836|
837|
838|
839|
840|
841|
842|
843|
844|
845|
846|
847|
848|
849|
850|
851|
852|
853|
854|
855|
856|
857|
858|
859|
860|
861|
862|
863|
864|
865|
866|
867|
868|
869|
870|
871|
872|
873|
874|
875|
876|
877|
878|
879|
880|
881|
882|
883|
884|
885|
886|
887|
888|
889|
890|
891|
892|
893|
894|
895|
896|
897|
898|
899|
900|
901|
902|
903|
904|
905|
906|
907|
908|
909|
910|
911|
912|
913|
914|
915|
916|
917|
918|
919|
920|
921|
922|
923|
924|
925|
926|
927|
928|
929|
930|
931|
932|
933|
934|
935|
936|
937|
938|
939|
940|
941|
942|
943|
944|
945|
946|
947|
948|
949|
950|
951|
952|
953|
954|
955|
956|
957|
958|
959|
960|
961|
962|
963|
964|
965|
966|
967|
968|
969|
970|
971|
972|
973|
974|
975|
976|
977|
978|
979|
980|
981|
982|
983|
984|
985|
986|
987|
988|
989|
990|
991|
992|
993|
994|
995|
996|
997|
998|
999|
1000|

```

**APPENDIX B**  
**Variables in Modulus of**  
**Elasticity Calculations**

VARIABLES IN  
MODULUS OF ELASTICITY CALCULATIONS



Note: All deflections are those remaining after base rotation has been subtracted.

$$D1 = d1 - d2$$

$$D2 = d2 - d3$$

$$H1 = h1 - h2$$

$$H2 = h2 - h3$$

$$\theta1 = D1/H1$$

$$\theta2 = D2/H2$$

$$\Delta h = \bar{h}_1 - \bar{h}_2$$

$$\Delta \theta = \theta1 - \theta2$$