PERCENTAGE RENEWALS AND AVERAGE LIFE OF RAILWAY TIES

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Studies of Records of 127,500 Treated and Untreated Test Ties Support Use of Law of Probability -- Charts Show Average Life and Probable Remaining Life at Any Given Period

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For a number of years the Forest Products Laboratory, in cooperation with the railroads, has been collecting tie-service records from test sections of track in various parts of the country. These records comprise both treated and untreated ties of different species. Inspections are made at regular intervals (usually annually) after the ties are placed in service and renewals started. When all the ties in a test section have been removed, the average life of the ties in the group is computed.

Obviously the average life will vary widely for different groups. Nevertheless, throughout all of the groups there is a remarkably constant correlation between the proportion of renewals made and the corresponding elapsed proportion of the average life. To illustrate, in a group of ties having an average life of 12 years, about 22 percent would be renewed in 9 years, and in another group having an average life of 8 years about 22 percent would be renewed in 6 years, or at 75 percent of the average life in each case.

Renewal Curves

Several years ago the Curve A in figure 1 was prepared by M. E. Thorne, showing the percentage of tie renewals plotted against percentage of average

¹Published in Engineering News-Record, August 26, 1926.

² Maintained at Madison, Wisconsin, in cooperation with the University of Wisconsin.

life elapsed (see Proceedings of the American Wood-Preservers' Association, 1918). This graph takes the form of the "cumulative frequency" or "summation" curve derived from the familiar probability type, and is based on completed service test records of 12, 185 untreated ties from 26 groups, and 30,751 treated ties from 17 groups, or 42,936 ties in all. A number of railroads in using this curve have found that it indicates very closely the relation between renewals and average life of ties in their own practice, especially when the number of ties involved is fairly large.

Since the preparation of this curve, a large number of other service tests have been completed, and curves based upon these indicate clearly the operation of the law of probability in the replacement required annually. The additional records include 44,970 untreated and 39,646 treated ties, making a total of 57,155 and 70,397, respectively, or a grand total of 127,552 ties. There are 58 different groups of untreated and 37 groups of treated ties, or 95 records altogether. The records, which include renewals for all causes, are probably the most extensive thus far obtained from the standpoint of numbers, distribution, and reliability. An effort was made to eliminate all data not considered entirely reliable.

For each group of ties the average life, the percentage of renewals, and the percentage of average life elapsed at each renewal period were computed. Curves were drawn for the individual groups, showing the relation between the total percentage of tie renewals and the percentage of average life elapsed. Readings were then taken from each of the curves (for definite percentages of renewals) and a weighted average value of percentage of average life elapsed was computed. Weighted averages were determined for untreated ties alone, treated ties alone, and for all ties including both treated and untreated. The data thus obtained are shown in table 1.

Summation Curves

The information given in this table is plotted in figures 1 and 2. Figure 1 shows both the original curve, A, mentioned above the corresponding new curve, B, derived from the more complete data on 127,552 ties. It will be observed that the results indicated by the former are fairly well checked, over the more important range, by the latter curve.

Figure 2 shows the summation data of the table plotted on "probability" paper. This paper is so constructed that when data which vary in accordance with the normal law of probability are plotted in the summation form as functions

of their probability of occurrence, the points will lie on a straight line. With the data thus plotted it is possible to extend the curve and study the results that may be expected beyond the range over which data are obtained. It is also more convenient to use the straight line than the curve plotted on rectangular coordinate paper, as in figure 1, especially at the upper and lower limits.

Separate and Combined Curves

There is apparently some difference in the relation of renewals to average life for the treated and untreated ties, shown in figure 2, more particularly in the range where the renewals are small. In the more important working range, however, the difference is only slight. It is possible that the variation arises on account of decay being the more important influence affecting renewals of untreated ties, whereas mechanical wear is the more potent factor with treated ties. In any event the small difference in the two curves makes it possible to employ a combined curve with sufficient accuracy.

In the combined curve for all 127,552 ties the points fall practically along a straight line through the coordinate point representing 50 percent renewals and 94 percent of average life. The results are strikingly close to a true probability relation, as proved by the fact that a straight line can be drawn through a majority of the plotted points.

By extending the line of the combined curve it will be found to intersect the horizontal top line (99.99 percent replacements) at about 181 percent of the average life. In other words, practically all ties are replaced when about 181 percent of the average life has elapsed. Computations made from the data to determine the percentage of average life at which all ties were removed gave about 180 percent, which checks the curve very closely. An examination of the lower end of the combined curve shows that nearly 40 percent of the computed average life elapses before 1 percent of the ties are replaced. Renewals then become more frequent, reaching a maximum rate between 90 and 100 percent of the average life.

Use of Curve Diagram

The following example will illustrate the use of the single straight-line combined graph. Assume that in a homogeneous group of ties, all placed in service at the same time, 25 percent have been removed in 6 years.

Using the graph, it will be noted that 25 percent would normally be removed when about 78 percent of the average life has elapsed. Hence, 6 years is 78 percent of the average life to be expected, which is thus figured at about 7.7 years.

If it is assumed that the group contained 1,500 ties and it is desired to estimate the removals expected during the succeeding year, take 7 years as a percentage of 7.7 years, the average life. The result is 91 percent, and the curve shows that a total of 45 percent or 675 ties will have been replaced at that period. Renewals to be expected during the seventh year, therefore, amount to 300 ties. The renewals for subsequent years can be readily computed in the same manner.

If in the above case the curve for untreated ties had been used, the computed average life would have been about 7.5 years. The curve for treated ties would have given about 7.8 years. In either case the variations from the figures obtained by using the combined curve would be less than 3 percent. Since the graph is based on extensive data from test groups of ties placed in many different sections of the country and under widely varying traffic conditions, it is felt that it furnishes a reliable guide for estimating the average life and the rate of tie renewals when the group of ties under consideration is large enough to be representative.

Chart for Simplified Computation

In order to avoid the computations necessary in using the curves shown in figures 1 and 2, the chart figure 3 has been prepared, which is based on the curve, for all ties in figure 2. Probability paper was used for this chart also. However, in this case definite average life values were assumed and the percentage of ties replaced (corresponding to different proportions of the average life) was plotted against years in service instead of percentage of average life, as in figure 2. The diagonal lines are marked with the average life to be expected for various percentages of replacements after a given period of years in service.

Method of Using Chart

The previous example will illustrate the use of this chart. For a group of ties, 25 percent are assumed to be removed after 6 years. The horizontal line representing 25 percent replacements intersects the vertical 6-year

line at about seven-tenths of the horizontal distance between 2 diagonals, 1 representing 7 and the other 8 years average life. The point of intersection, therefore, represents an average life of 7.7 years, checking the value previously determined from figure 2.

To determine subsequent renewals, let it be assumed again that there are 1,500 ties in the group and that 25 percent are removed after 6 years of service. The total percentage of renewals to be expected by the end of the seventh year can be found by reading upwards on the 7-year line to a point about 0.7 of the horizontal distance between the diagonals representing an average life of 7 and 8 years, respectively. This is about 45 percent, or a total of 675 ties, checking the result found previously from figure 2. 675 less 375 gives 300, the number to be replaced during the seventh year. Likewise, reading up the line of 8 years service to a point 0.7 of the horizontal distance between the 7- and 8-year average life diagonals, it is found that about 65 percent, or 975 ties, are removed by that time. Similarly, it will be found that a total of about 84 percent or 1,260 are removed in 9 years. The time at which practically all ties are out can be determined by reading on the horizontal scale directly below the point on the top line taken 0.7 the distance between the 7- and 8-year diagonals. Projecting from this point to the lower scale it will be found that practically all ties of this group, having a computed average life of 7.7 years, would be removed in slightly over 13.9 years, or approximately 14 years.

Frequency Curve

If a frequency curve (showing percentages of ties removed at successive time intervals) is plotted from the combined data with the origin of coordinates of the curve at 100 percent average life, as in figure 4, a certain degree of positive skewness will be found. This is because more than 50 percent of the ties have been replaced before 100 percent of the average life is reached, as already indicated by the cumulative curve in figure 2. It is very likely that the following are factors of track maintenance which tend to produce skewness of the curve with respect to the origin taken at 100 percent of the computed average life:

- (a) In the early removals, consideration of economy may make it desirable to remove a certain number of ties that would normally be serviceable for a longer period.
- (b) When a large proportion of the ties have been replaced, those surviving have no noteworthy influence on the track conditions. For that reason some of them may be left in service a considerably longer time than would be permissible if ties of the same condition were present in large numbers. Even

a few ties left in service several years longer than the majority of those in the group may have considerable weight toward increasing the computed average life. This would tend, of course, to produce an unsymmetrical frequency curve when the origin is taken at 100 percent average life.

(c) Other factors which doubtless increase the early mortality of ties are fires, washouts, wrecks, etc.

To eliminate skewness in the graph, the origin may be taken at the "mode" (94 percent average life) instead of the arithmetic mean (100 percent average life). By this means a normal frequency curve applying correctly to the data will be obtained. Such a curve is given in figure 5, which shows the rate or replacements for 10 percent intervals of average life. The renewals for successive 10 percent intervals are represented by rectangles, and a smooth curve is drawn through the midpoints. It will be noted that between 29 and 39 percent of the average life only 0.70 percent of the ties are removed whereas between 89 and 99 percent of the average life (the period of maximum replacements) 17 percent are removed.

In the equation for the symmetrical form of the frequency curve in figure 5 as given below Y represents the frequency or percentage of replacements for a given value \overline{x} ; x is the distance to the right or left of the origin measured in intervals of $\overline{10}$ percent average life for the group considered; \underline{e} is the base of the Naperian system of logarithms.

$$-\frac{x^2}{10.95}$$

$$Y = 17e$$

General Considerations

In the use of all the foregoing curves the following considerations should be borne in mind: (1) The larger and more homogeneous the group of ties under consideration, the greater the likelihood that the curve of renewals will closely represent the results that will be obtained. (2) When the number of renewals is very small, the average life estimated from the curves will not be as reliable as that indicated when larger proportions of renewals have been made. When, however, the number involved is sufficient to furnish a representative sample, the chart or curves provide a means of closely estimating the rate of renewals and average life to be expected, which will hold for groups of ties having either a long or a short average life.

Table 1.--Data for cumulative frequency curves

Untreated 57,155 ties		:: Treated			:: ::Treated and untreate			:: d::Percentage
		70,39		7 ties			52 ties	: average
of	of aver-	::	of	: of aver-	::	of	:Percentage : of aver- : age life	
5	60.9	::	5	56.7	::	5	58.5	:: 55.0
10	67.3	::	10	63.1	::	10	: 64.8	:: 63.8
20	76.3	::	20	72.1	::	20	: 73.9	:: 73.6
30	82.7	::	30	79.3	::	30	80.7	:: 81.5
40	88.8	::	40	85.3	::	40	86.8	:: 88.0
50	94.5	::	50	91.9	::	50	93.1	:: 94.0
60	99.9	::	60	98.8	::	60	99.2	:: 100.0
70	105.6	::	70	106.5	::	70	: 106.0	:: 106.0
80	111.7	::	80	116.0	::	80	: 114.1	:: 113.5
90	119.1	::	90	125.7	::	90	122.7	:: 123.5
95	124.5	::	95	134.5	::	95	130.0	:: :: 132.0

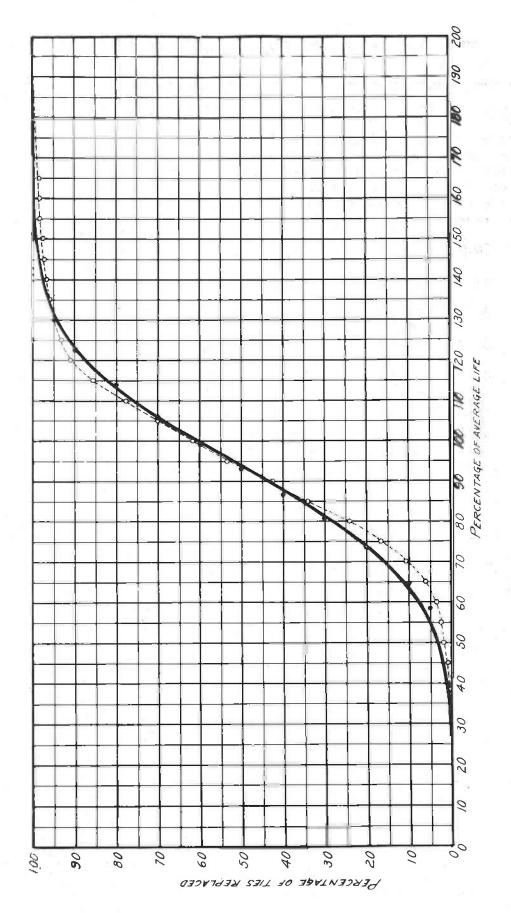


Figure 1. -- (a) Curve of total replacements (1918 studies); (b) Curve of total replacements (all studies).

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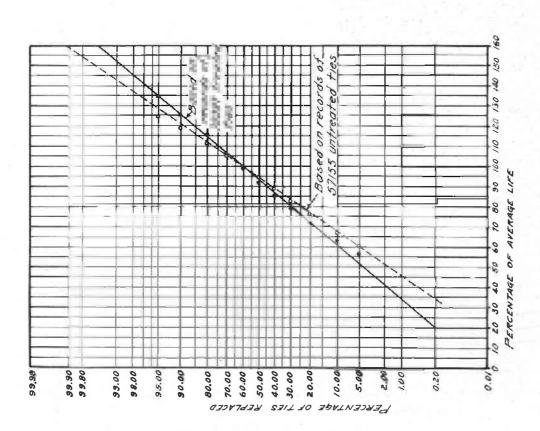


Figure 2. -- Relation of tie replacements of average life.

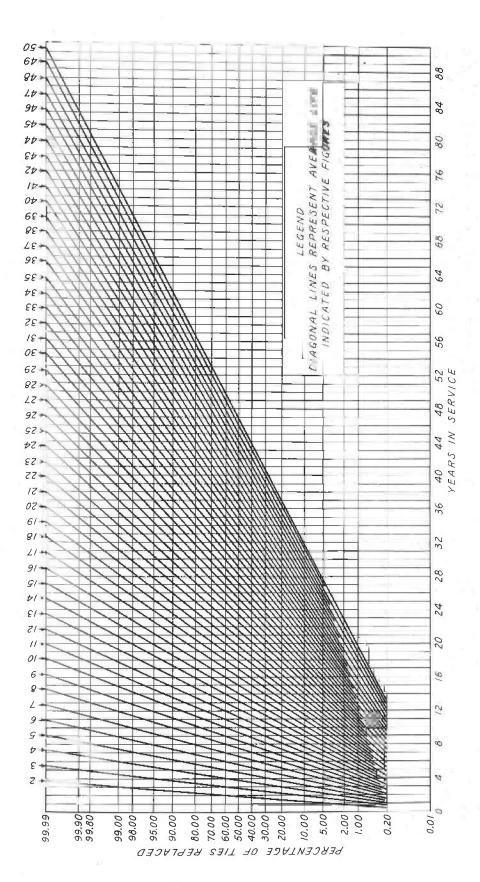


CHART FOR DETERMINING PROBABLE LIFE OF TIES

Figure 3. -- Chart for determining probable life of ties.

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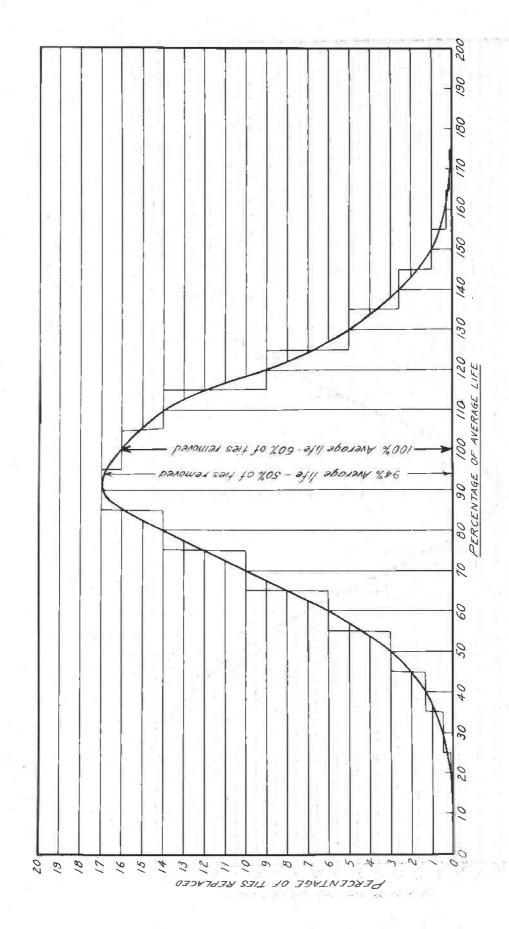


Figure 4. -- Frequency curve showing successive percentage tie replacements for 10 percent Unsymmetrical form - Origin taken at 100 percent average life. intervals of average life.

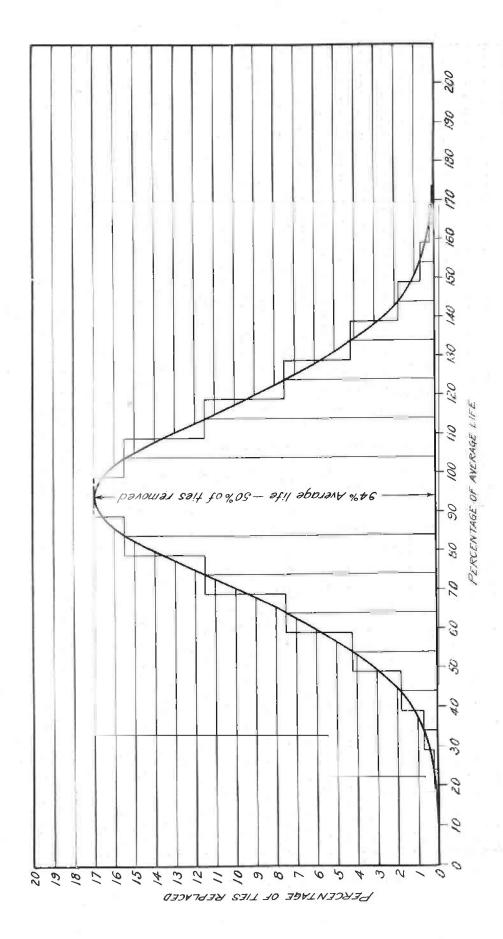


Figure 5. --Frequency curve showing successive percentage tie replacements for 10 percent Symmetrical form - Origin taken at 94 percent. intervals of average life.

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