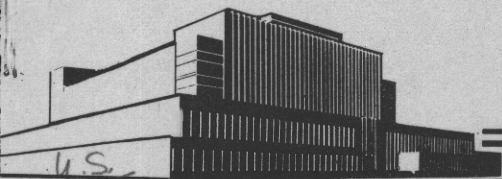
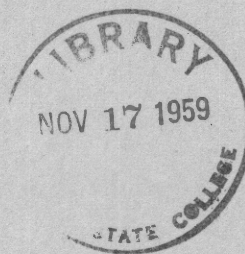


STRUCTURAL SANDWICH DESIGN CRITERIA

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STRUCTURAL SANDWICH DESIGN CRITERIA¹

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Abstract

This report presents design criteria for structural sandwich constructions under various types of loading. Included are a brief historical background, a discussion of what constitutes a structural sandwich construction, and basic design philosophy. Detailed design procedures are presented for determining deflections of sandwich beams or panels and buckling of sandwich columns and simply supported panels under edge load, including formulas for calculating facing stresses and core shear stresses. Formulas presented are general and can be applied to a wide range of materials if properties of the materials are known. Methods of test for evaluating certain material properties are discussed briefly. Modes of failure of sandwich under various loadings are illustrated.

Introduction

A structural sandwich is a layered construction formed by bonding two thin facings to a thick core. It is a "stressed-skin" construction similar to some prefabricated house constructions in which facings are bonded to spaced stringers or studs. The core of the sandwich is continuous, or so nearly continuous that much thinner facings can be used than in stressed-skin construction.

The basic design concept is to space the strong, thin facings far enough apart with a thick core to assure that the construction will be stiff, to provide a core that is stiff and strong enough to hold the facings flat through a bonding medium such as an adhesive layer, and to provide a core of sufficient shearing resistance.

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²Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

The advantage of spaced facings to provide greater stiffness without much increase in amount of material needed was investigated in about 1820 by a Frenchman named Duleau (7)³ who tested spaced bolted bars and found the stiffness varied as the difference between the cube of the total thickness and the space if the bolts were very tight and not placed too far from each other. This discovery eventually led to the design of I beams and other stiff structural shapes. The structural sandwich is analogous to an I beam, with the facings carrying direct compression and tension loads, as do I-beam flanges, and the core carrying shear loads, as does the I-beam web.

Thus, a structural sandwich is a definite, purposeful construction, carefully designed to have certain characteristics. In order to prevent confusion as to what was considered a structural sandwich, the American Society for Testing Materials adopted the following tentative definition in 1951 and made it standard in 1953:

"ASTM C274-53. Structural Sandwich Construction.

A laminar construction comprising a combination of alternating dissimilar simple or composite materials assembled and intimately fixed in relation to each other so as to use the properties of each to attain specific structural advantages for the whole assembly."

Sandwich Stiffness

Stiffness is a prime consideration for most structural design. The stiffness D of a rectangular beam having thickness h, width b, and modulus of elasticity E is given by the formula

$$D = E \frac{bh^3}{12} \quad (1)$$

If a sandwich is constructed with the same overall dimensions but with a core of a different material of thickness c, the stiffness is given by

$$D = \frac{E_f b(h^3 - c^3)}{12} + E_c \frac{bc^3}{12} \quad (2)$$

where E_f is the modulus of elasticity of the facings and E_c is the modulus of elasticity of the core. For simplicity here, it has been assumed that

³Underlined numbers in parentheses refer to literature cited at the end of this report.

the facings are of equal thickness. Mathematical formulas have been derived for constructions with facings of different materials and thicknesses (2).

Since the core of the sandwich is to be extremely lightweight compared to the facings, it will probably be a weak, limber material whose stiffness can be neglected in considering the sandwich stiffness. Thus, the last term of the preceding formula may be omitted, and the sandwich stiffness given simply by

$$D = \frac{E_f b(h^3 - c^3)}{12} \quad (3)$$

which can be rewritten as

$$D = \frac{E_f b h^3}{12} \left(1 - \frac{c^3}{h^3}\right) \quad (4)$$

The amount of facing material in a piece of sandwich is given by

$$A_f = 2fb \quad (5)$$

where f is the facing thickness and b is the sandwich width. This may also be expressed by $A_f = (h - c)b$ or

$$A_f = hb \left(1 - \frac{c}{h}\right) \quad (6)$$

The term $(1 - c/h)$ of formula (6) represents the proportion of facing material in a sandwich as compared to total material in a rectangular section of area hb . Similarly, the term $(1 - c^3/h^3)$ of formula (4) represents the proportion of stiffness of the sandwich as compared to the stiffness of a rectangular section given by formula (1). Then, for example, if a sandwich has one-eighth the amount of facing material in a solid section, $(1 - c/h) = 1/8$ and $c/h = 7/8$. Entering formula (4) with $c/h = 7/8$, we find that

$c^3/h^3 = 2/3$ (approximately) and $(1 - c^3/h^3) = 1/3$, or the sandwich has one-third the stiffness of a solid section of the same width and depth as the sandwich and of the same material as the facing. Therefore, a sandwich having one-eighth the weight (excluding the light-weight core) of solid material can be made to have one-third the stiffness of solid material. If the sandwich is three times wider than solid material, it will have the same stiffness as the solid material and still be only three-eighths as heavy.

Obviously, the gain in stiffness-weight ratio possible with structural sandwich might be offset by reductions in shear stiffness and strength. As Duleau discovered, it was necessary that the bars be held tightly by the bolts to allow shears to be transmitted. By the same token, the

sandwich facings must also be securely bonded to the core, otherwise the stiffness will be no more than the sum of the stiffnesses of each of the sandwich parts.

Thus far the design discussion has been limited to determining the flexural stiffness \underline{D} or in the more usual terminology the \underline{EI} of the sandwich. This stiffness is used in the design of beams, particularly in regard to limitations on deflections. The deflection of a sandwich beam, however, is not given entirely by the flexural stiffness. It must also include the deflection due to shearing deformations, because the core material may have a low shear modulus. Thus, in solving for deflections of sandwich beams, the following expression must be integrated (2).

$$\frac{d^2 y}{dx^2} = -\frac{M_x}{D} + \frac{1}{N} \left(\frac{dV_x}{dx} \right) \quad (7)$$

where y is deflection, x is distance along the beam, M_x is moment at point x , V_x is shear load at point x , D is flexural stiffness given by formula (3), and N is shear stiffness of sandwich. For sandwich having thin facings,

$$N = \frac{(h + c)b}{2} G_c \quad (8)$$

where G_c is the core shear modulus.

Integration of formula (7) leads to the following general expression for deflection of a sandwich beam or panel,

$$y = \frac{k_B Pa^3}{D} + \frac{k_S Pa}{N} \quad (9)$$

where y is deflection, P is total load, a is span, D is flexural stiffness given by formula (3), N is shear stiffness given by formula (8), and k_B and k_S are constants dependent on the beam loading. Formula (9) gives deflection in terms of the usual bending deflection (first term) and the shear deflection (second term). For a long span, the first term of formula (9) will be of more consequence than the second term. This is also true in dealing with other materials where it is usually necessary to consider shearing deflections only if the span is very short. Values for the constants k_B and k_S for several beam loadings are given in table 1.

Strength of Sandwich

The strength of a sandwich beam under bending and shear loads is determined by the ability of the facings to resist compression or tension and

that of the core and adhesive bond to resist shear. The stresses produced in the facings by bending moment applied to the sandwich are given by the formula

$$F = \frac{2M}{f(h + c)b} \quad (10)$$

where F is the mean compressive or tensile stress, M is the bending moment, f is the thickness of one facing, h is the total sandwich thickness, c is the core thickness, and b is the sandwich width. The shear stress in the core is given by

$$S = \frac{2V}{(h + c)b} \quad (11)$$

where S is the core shear stress, and V is the shear load on the sandwich.

Formulas (10) and (11) are approximate and can be used for most sandwich with thin facings and moderately rigid and thick cores. If facings are thick and cores fairly thin and of very low rigidity, more accurate analyses must be used to take into account the shear load carried by the facings (4).

Sandwich construction may also be used effectively for carrying edge loads, as might be required if it were used for bearing walls in buildings. Such design must be dependent upon the buckling resistance of a sandwich column or upon its ability to resist direct compression in the facings, whichever is the lesser. Compressive stresses in the facings are given by the formula

$$S = \frac{P}{2fb} \quad (12)$$

If the panel is simply supported at its ends, the column buckling load is given by

$$P = \frac{\pi^2 D}{a^2 \left(1 + \frac{\pi^2 D}{a^2 N}\right)} \quad (13)$$

where P is total load, f is facing thickness, b is column width, a is column length, D is defined by equation (3), and N is defined by equation (8). The second term in the denominator of formula (13) accounts for possible shearing deformation in the core.

If the load-bearing wall panel is held in line at its vertical edges, the buckling load of the panel is given approximately (3) by

$$P = \frac{4\pi^2 D}{b^2 \left(1 + \frac{\pi^2 D}{b^2 N}\right)^2} \quad (14)$$

for panels that are at least as long as they are wide and for which the second term in the bracket of the denominator is less than or equal to unity. In formula (14), P is total load, b is panel width, D is given by equation (3), and N is given by equation (8).

The preceding design criteria for sandwich stiffness and strength are suitable for sandwich with thin isotropic facings and isotropic cores and are approximate for orthotropic materials. More exact analyses have been made for sandwich with orthotropic facings and cores and for moderately thick facings. Such analyses have been prepared in general form for design of aircraft sandwich components and are available in report form from the U. S. Forest Products Laboratory. Much of the design information and fabrication techniques has been summarized for the aircraft industry (10).

Design Example

In order to illustrate a design procedure, consider a design of a floor panel of sandwich construction. The panel is to be simply supported at its ends on a 12-foot span. Its width is to be 4 feet. The design load is to be 40 pounds per square foot and the maximum deflection under this load shall be no more than 0.20 inch. The facings are to be of plywood having an effective compressive and tensile modulus of elasticity of 1,000,000 pounds per square inch and a compressive design stress of 1,200 pounds per square inch; tensile stress exceeds this, so the design must be based on compressive stress. The core is to be of a material having a shear design stress of 20 pounds per square inch and a shear modulus of 8,000 pounds per square inch.

Because the panel is fairly long, the design will probably depend upon deflection rather than shear or facing stress; and the core thickness will be found by assuming a facing thickness, solving the deflection equation for the core thickness, and then checking the core shear stress and facing compression stress.

From the prescribed loading and equation (9), the maximum deflection of the panel is given by

$$y = \frac{5 Pa^3}{384 D} + \frac{Pa}{8N} \quad (15)$$

By substituting $h = c + 2f$ in equation (3), the expression for D becomes

$$D = \frac{E_f b}{12} (6 c^2 f + 12 c f^2 + 8 f^3)$$

which can be written as follows if a $2f^3$ term is neglected

$$D = \frac{E_f f (c + f)^2 b}{2} \quad (16)$$

Also, formula (8) for N can be written as

$$N = G_c (c + f) b \quad (17)$$

Substituting (16) and (17) into (15) gives

$$y = \frac{5Pa^3}{192 E_f b f (c + f)^2} + \frac{Pa}{8G_c b (c + f)} \quad (18)$$

which can be written as

$$(c + f)^2 - \frac{Pa}{8G_c b y} (c + f) - \frac{5Pa^3}{192 E_f b f y} = 0 \quad (19)$$

and solving (19) for c finally results in

$$c = \frac{Pa}{16 G_c b y} \left[1 + \sqrt{1 + \frac{20 G_c^2 b y a}{3 P E_f f}} \right] - f \quad (20)$$

Since the panel is to be a floor panel, the facing must be thick enough to withstand some concentrated and localized impact loads; hence a facing 1/2 inch thick will be used. (For purposes of this paper, both facings are to be of the same thickness. It might, however, be advantageous to use a thinner lower facing, in which case other mathematical formulas must be used (9).) No rational design criteria are available for considering concentrated and impact loads, and the designer must judge from the intended use whether thick facings are needed.

Solving equation (20) gives

$$c = \frac{1920 \times 144}{16 \times 8000 \times 48 \times 0.20} \left[1 + \sqrt{1 + \frac{20 \times 8000 \times 48 \times 0.20 \times 144}{3 \times 1920 \times 1,000,000 \times 0.50}} \right] - 0.50$$

$$c = 0.225 \left[1 + \sqrt{615.4} \right] - 0.50 = 5.3 \text{ inches}$$

From this, the total thickness of the panel will be 6.3 inches and $(h + c) = 11.6$ inches, and from equation (10) the facing stress will be

$$F = \frac{2 \times 34,560}{0.50 \times 11.6 \times 48} = 248 \text{ p.s.i.}$$

and the core shear stress

$$S = \frac{2 \times 960}{11.6 \times 48} = 3.44 \text{ p.s.i.}$$

Thus, the facing stress is about 20 percent and the core stress about 17 percent of design values.

Modes of Failure

Tests of sandwich to failure give much useful information, particularly if the sandwich is to be redesigned to increase its load-carrying capacity. Figure 1 illustrates the possible modes of failure that may occur in addition to failures of compression or tension in the facings and shearing of the bond between facing and core.

General buckling of sandwich under edge load occurs as shown in figure 1,A. This type of failure is predictable by use of formulas (13) and (14). Increase of the load at which general buckling occurs can be obtained by increasing the core thickness or increasing the facing stiffness. Similar failure of sandwich in flatwise flexure is characterized by increasing deflection without much increase in load.

Shear crimping of sandwich under edge load, as shown in figure 1,B occurs because of low sandwich shear stiffness. The failure is predictable by use of formulas (13) and (14) when the second term of the denominator becomes large. Load-carrying capacity can be improved by increasing the core shear modulus or using a thicker core. Failure of sandwich under flatwise bending and shear can have a similar appearance. Such sandwich can be improved best by using a thicker core or a core of greater shear strength.

Dimpling of facings can occur in sandwich with thin facings on honeycomb cores. The type of failure is shown in figure 1,C. The facing stress at which dimpling occurs can be predicted (6). Dimpling can also occur in the compression facing of a sandwich subjected to flatwise bending. Improvement in the sandwich, if necessary, can be attained by decreasing the honeycomb cell size or by increasing the facing stiffness.

Wrinkling of sandwich facings can occur as shown in figure 1,D. Wrinkling of the facings can result in local separation of the facing from the core, and if the bond between facings is strengthened, can result in local crushing of the facing into the core. Analyses have been made of

failures of this type (5), and predictions of critical facing stresses involve parameters combining core elastic properties and initial facing waviness. Determinations of facing stresses at wrinkling are best made by testing small specimens and applying the results, along with core and sandwich properties, with the theory to predict performance of similar constructions. Wrinkling of the compression facing of sandwich in flatwise bending can also occur.

Methods of Test for Sandwich and Its Components

Often, in order to prove a design, it is necessary to test complete sandwich panels or small specimens, and it may even be necessary to evaluate core properties by some tests. Tests of complete panels for buildings can be conducted according to standard procedures published by the American Society for Testing Materials (1). The tests covered by this procedure include the following:

Tests of walls: axial loads (compressive and tensile), transverse load, concentrated load, impact load, and racking load.

Tests of load-bearing partitions: general requirements.

Tests of non-load-bearing partitions: impact load and concentrated load.

Tests of floors: transverse load, concentrated load, and impact load.

Tests of roofs: transverse load and concentrated load.

Test methods for sandwich construction and for core materials have been published by the American Society for Testing Materials and the U. S. Department of Defense (8). Of particular interest here are the flatwise flexure test, edgewise compression test, and flatwise tension test for sandwich, and the shear test for core material.

The flatwise flexure test (see ASTM C393-57T or MIL-STD-401A) can be designed to determine flexural and shear stiffness of the construction, approximate core shear modulus and core shear strength, and moment-carrying capacity of the sandwich, by use of rather small sandwich specimens. Apparatus for testing flexure specimens is shown in figure 2.

Essential features of the apparatus include one movable reaction point and load applied through a pivot point so that slight twist of a specimen is easily accommodated, and measurement of deflection with a dial gage supported on pins placed in the core at reactions for measuring midspan deflection of a pin placed in the core. This means of measuring deflection assures that apparatus deflection or deformations will not be included as specimen deflection.

The edgewise compression test (see ASTM C364-57 or MIL-STD-401A) employs small specimens to determine ultimate load-carrying capacity of the sandwich construction and modes of failure such as those shown in figure 1. Important to the test are precautions that must be taken to avoid eccentricity of load application. In order to accomplish this, it is necessary to observe deformations in each facing in the initial loading stage and to adjust loading until deformations are alike in each facing before loading to failure. Apparatus shown in figure 3 has been found to perform satisfactorily. Facing deformations are measured with Marten's mirror compressometers, and any necessary adjustment to equalize deformations is done by turning a screw under the movable base at the lower end of the specimen.

The flatwise tensile test of sandwich construction (see ASTM C297-55 or MIL-STD-401A) is designed to determine the strength normal to the sandwich facings. Load is applied through blocks bonded to the sandwich facings. Failure occurs in the core material or in the bond between the facing and core. The strength values obtained are useful in predicting wrinkling of the facing of the sandwich, as discussed previously, especially if data are available on core-flatwise properties.

The shear test for core material (see ASTM C273-53 or MIL-STD-401A) determines core shear modulus and core shear strength--data that are useful for computing shear deflection and strength of sandwich in bending and for determining values of the parameters, using shear modulus in computing buckling of columns and panels under edge load. Apparatus for conducting the test under tensile load is shown in figure 4. The test specimen is at least 12 times as long as its thickness, and load is applied along a line connecting diagonally opposite ends of the specimen through fittings attached to plates bonded to the specimen. Shear deformations are measured with a dial gage arranged to measure slip of one loading plate with reference to the other. In order to prevent excessive bond deformation, the adhesive used to join the loading plates to the core should be rigid and as thin as possible.

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Table 1.--Values of k_B and k_S for several sandwich beam loadings

Loading	Beam ends	Deflection: at	k_B	k_S
Uniformly distributed	Both simply supported	Midspan	$5/384$	$1/8$
Uniformly distributed	Both clamped	do	$1/384$	$1/8$
Concentrated at midspan	Both simply supported	do	$1/48$	$1/4$
Concentrated at midspan	Both clamped	do	$1/192$	$1/4$
Concentrated at outer quarter points	Both simply supported	do	$11/768$	$1/8$
Concentrated at outer quarter points	Both simply supported	Load point	$1/96$	$1/8$
Uniformly distributed	Cantilever, 1 free, 1 clamped	Free end	$1/8$	$1/2$
Concentrated at free end	do	do	$1/3$	1

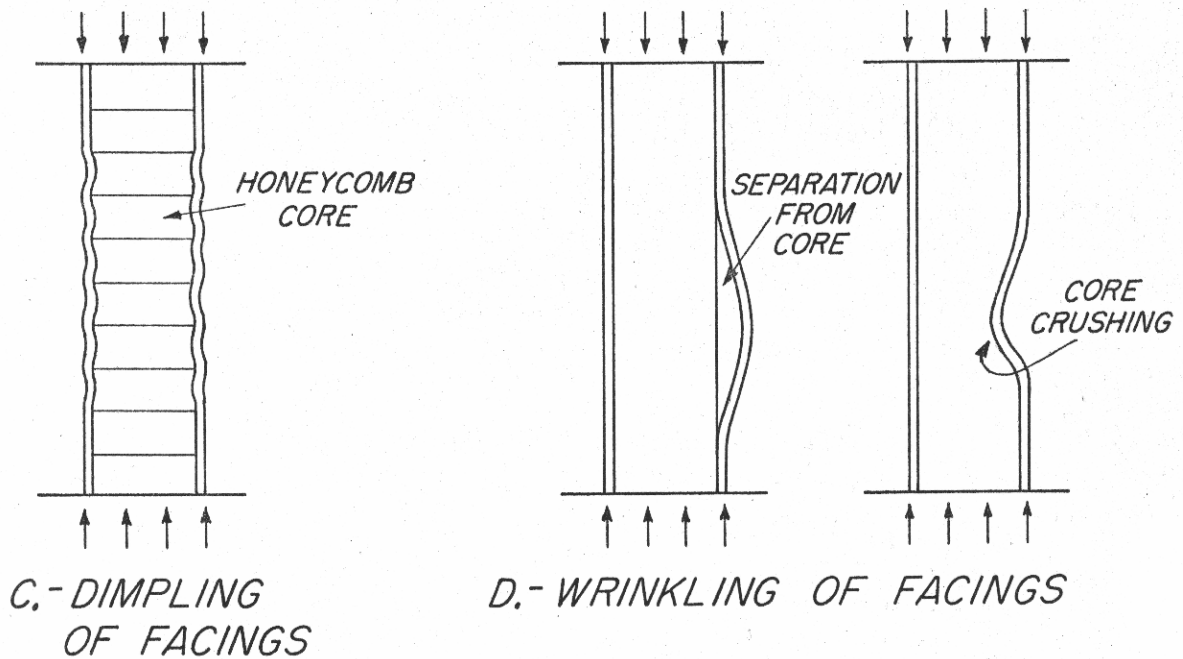
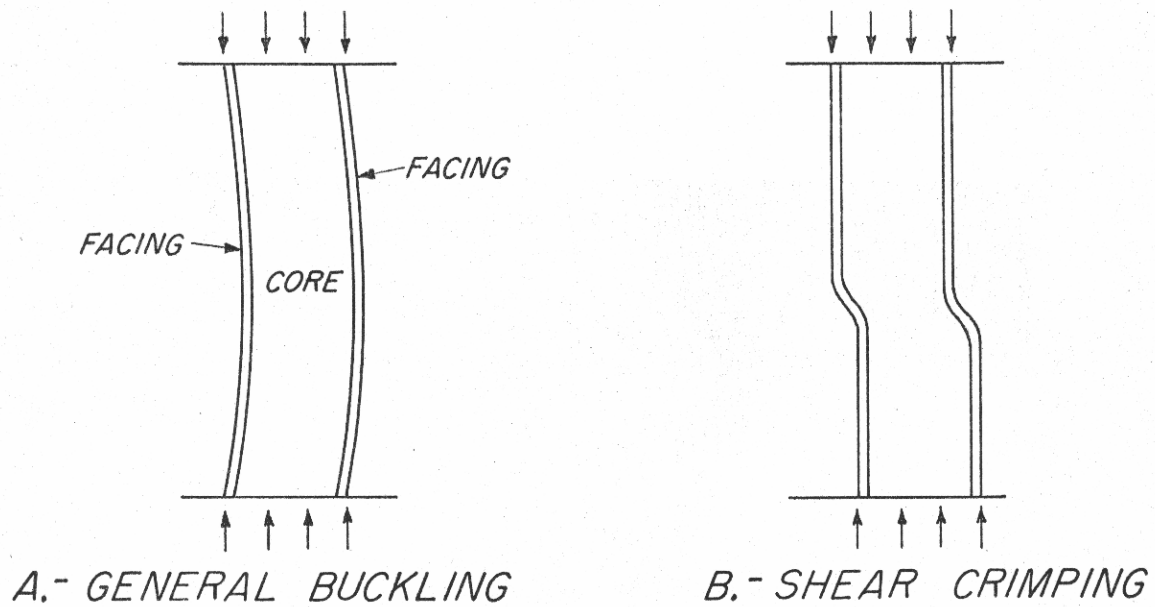


Figure 1. -- Modes of failure of sandwich construction under edgewise loads. A, General buckling; B, shear crimping; C, dimpling of facings; D, wrinkling of facings either away from or into core.

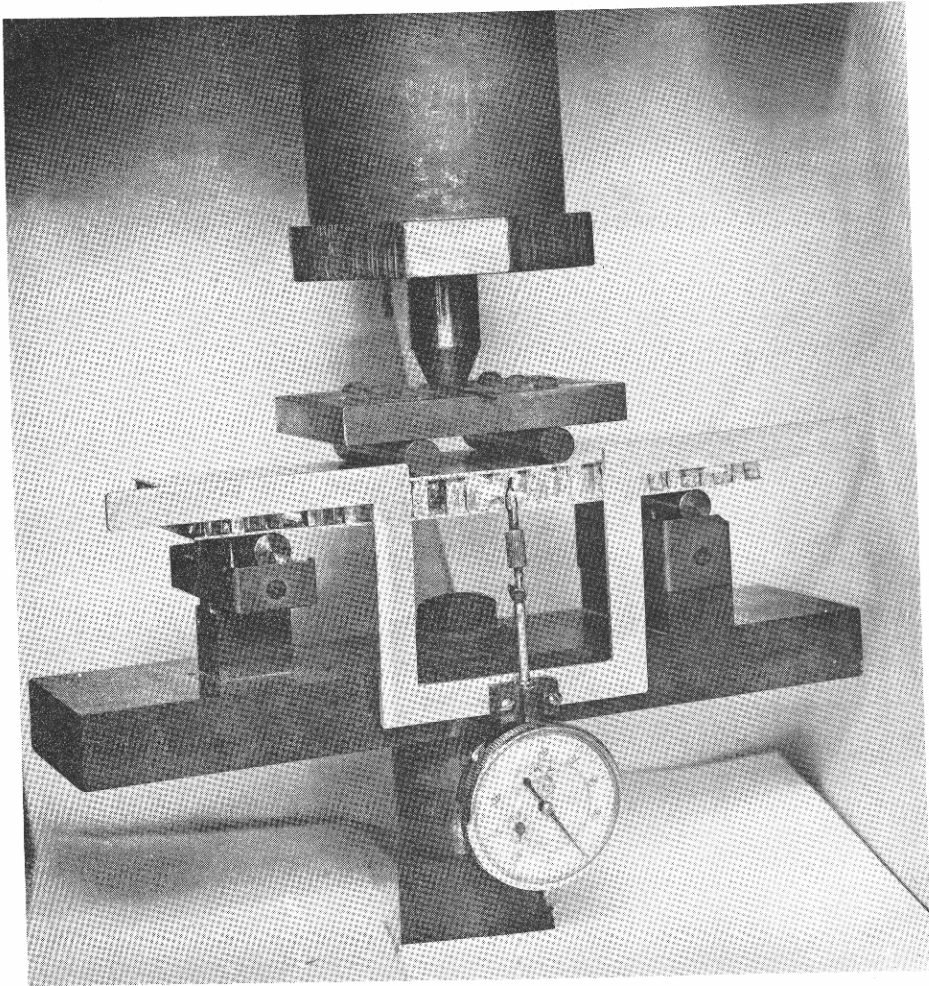


Figure 2. --Apparatus for conducting flexure test of sandwich construction.

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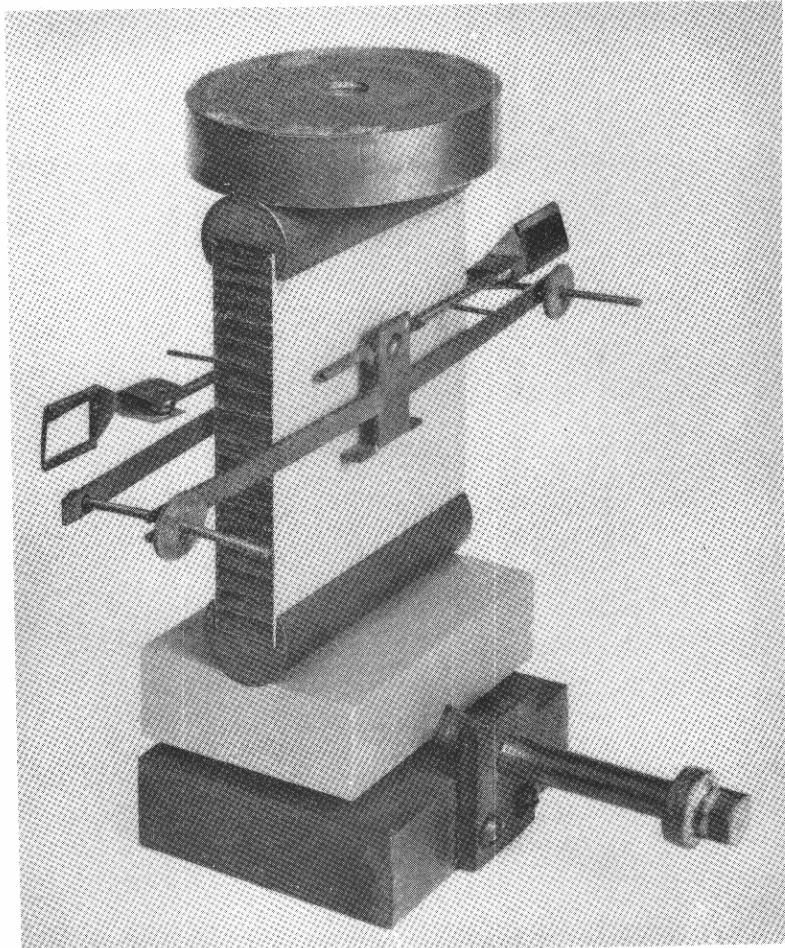


Figure 3. --Apparatus for testing sandwich construction
in edgewise compression.

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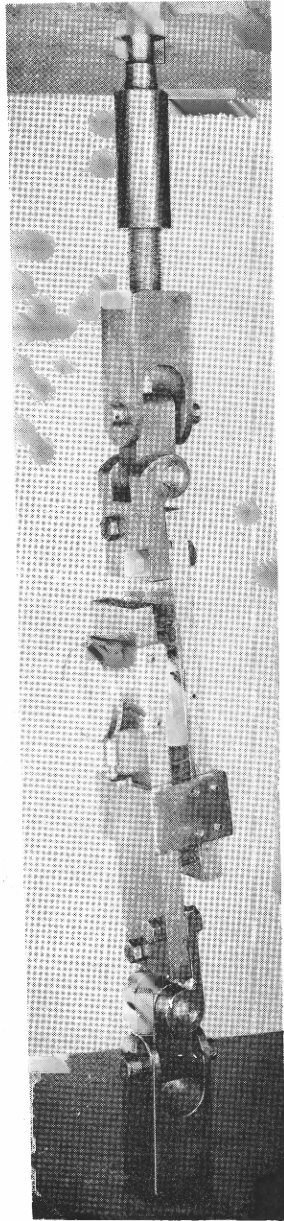


Figure 4. --Apparatus for determining shear characteristics of sandwich cores.

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