Supplement to

ELASTIC BUCKLING OF A SIMPLY SUPPORTED RECTANGULAR SANDWICH PANEL SUBJECTED TO COMBINED EDGEWISE BENDING AND COMPRESSION

Results for Panels with Facings of Either Equal or Unequal Thickness and with Orthotropic Cores

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Results for Panels with Facings of Either Equal or Unequal Thickness and with Orthotropic Cores[1]

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Summary and Conclusions

A previous analysis (2)[3] of the problem of simply supported rectangular sandwich panels subjected to any combination of edgewise bending and compression is extended. Literal equations are derived that are used for the construction of design curves for the prediction of buckling loads on panels with facings of either equal or unequal thickness and with orthotropic cores. The design curves are believed sufficiently accurate for use in design of sandwich panels.

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[2] Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

[3] The underlined numbers in parentheses refer to Literature Cited, page 22.

Introduction

This report is a supplement to a previous report (2) which contains a theoretical analysis for the determination of critical loading at which buckling occurs for a simply supported rectangular sandwich panel subjected to any combination of edgewise bending and compression. Details of mathematical development are given in the section entitled Mathematical Analysis.

In the previous report (2) it was assumed that the core was orthotropic and subjected only to anti-plane stress.[4] The facings were assumed to be homogeneous and isotropic and were analyzed by the use of conventional thin plate theory in accordance with the Bernoulli-Navier hypothesis.[5] The theory was developed for panels having facings of unequal thickness, t and t'. It was shown that the simultaneous solution of a set of linear, homogeneous equations[6] determines the critical load on the sandwich panel. A simplified literal solution of this linear, homogeneous set of equations was obtained on the basis of the following further assumptions:

1. The transverse modulus of elasticity, E_c, of the core is infinite.
2. The moduli of rigidity, G_{yz} and G_{xz}, of the core are equal.
3. The facing thicknesses, t and t', are equal.

Design curves[7] for the prediction of critical loads were constructed (see figs. 5, 6, and 7 of (2)) from this literal solution for panels subjected to pure edgewise compression, and for panels subjected to pure edgewise bending.

In this supplementary report, additional simplified literal solutions of the linear, homogeneous set of equations[6] are presented. These

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[4]See Literature Cited (2), p. 4.

[5]Plane sections perpendicular to the median plane of the plate remain plane during deformation.

[6]Equations (173), (174), (175), (176), (177), and (178) of Literature Cited (2).

[7]These design curves were based on an additional assumption involving the flexural rigidity of the facings. See Literature Cited (2), pp. 78 and 79.

solutions define the critical load for a panel made of facings with unequal thickness, t and t', and made of a core with moduli of rigidity, G_{yz} and G_{xz}. Design curves for the prediction of critical load are constructed for panels subjected to pure edgewise bending. These curves are for use in designing sandwich panels with orthotropic cores and with facings of either equal or unequal thickness. The additional solutions and design curves presented in this report are based on continued use of the assumption of infinite transverse modulus of elasticity of the core, E_c, and on a modified flexural rigidity of the spaced facings.

Notation

a	length of sandwich in direction of loading (see figs. 1, 2, and 3)
b	width of sandwich in direction perpendicular to loading (see figs. 1, 2, and 3)
β	$\frac{a}{b}$
c	thickness of core (see fig. 1)
t	thickness of upper facing (see fig. 1)
t'	thickness of lower facing (see fig. 1)
E	modulus of elasticity of facings
E_c	transverse modulus of elasticity of core
μ	Poisson's ratio of facings
G_{xz}	modulus of rigidity of core in xz plane
G_{yz}	modulus of rigidity of core in yz plane
r	$\frac{G_{xz}}{G_{yz}}$

N_b	maximum value of loading (load per unit width, b, of panel) due to pure edgewise bending (see fig. 3)
N_c	value of loading (load per unit width, b, of panel) due to pure edgewise compression (see fig. 3)
N_o	$N_b + N_c$ (see figs. 2 and 3)
N_{ocr}	value of N_o at buckling
α	$\frac{2N_b}{N_o}$ (see figs. 2 and 3)
m, n	integers
$k_{m\alpha}$	critical load factor corresponding to loading defined by α
I_F	$\frac{t^3 + (t')^3}{12}$
I_T	$\frac{tt'}{t + t'}\left(c + \frac{t + t'}{2}\right)^2$
I	$I_F + I_T$
I_M	$\frac{tt'c^2}{t + t'}$
D	$\frac{EI}{1-\mu^2}$
D_T	$\frac{EI_T}{1-\mu^2}$
D'_{mn}	$\frac{E}{1-\mu^2}\left[I_F + \frac{I_T}{1 + \frac{W}{\beta^2}\frac{\Phi^2_{mn}}{\Lambda_{mn}} + \frac{n^2 m^2 rW\left(1 - \frac{1}{r}\right)^2}{\Lambda_{mn}\left(1 + \Lambda_{mn}\frac{W}{\beta^2}\frac{1-\mu}{2}r\right)}}\right]$

W	$\frac{ctt'}{t + t'} \frac{\pi^2}{b^2} \frac{E}{1-\mu^2} \frac{1}{G_{xz}}$
δ	$\frac{E\pi^2}{c(1-\mu^2)}$
ξ_{mn}	$D'_{mn} \pi^4 \theta^2_{mn}$
Γ	$N_{ocr} \frac{m^2\pi^2}{a^2} (1 - \frac{\alpha}{2})$
θ_{mn}	$\frac{m^2}{a^2} + \frac{n^2}{b^2}$
Φ_{mn}	$m^2 + n^2\beta^2$
Λ_{mn}	$m^2 + \frac{n^2\beta^2}{r}$
Ω_{mn}	$\frac{\Phi^2_{mn}}{1 + \frac{W}{\beta^2} \frac{\Phi^2_{mn}}{\Lambda_{mn}} + \frac{n^2 m^2 r W (1 - \frac{1}{r})^2}{\Lambda_{mn} (1 + \Lambda_{mn} \frac{W}{\beta^2} \frac{1-\mu}{2} r)}}$
ρ_{mn}	$1 + \frac{1}{r} \frac{a^2}{m^2} \frac{n^2}{b^2}$

Results and Discussion

All the results presented in this report refer to a sandwich panel composed of elements with the following properties:

1. The core is orthotropic and is capable of resisting only anti-plane stress.[4] The transverse modulus of elasticity of the core, E_c, is infinite.
2. The facings are isotropic and are of unequal thickness.

1. Case $\alpha = 0$, $E_c = \infty$

The loading on the panel for this case is pure edgewise compression.

The critical load for this case may be obtained from the equation

$$N_{ocr} = \frac{\pi^2}{m^2 a^2} \Phi^2_{ml} D'_{ml} \qquad (1)$$

where

$$D'_{mn} = \frac{E}{1-\mu^2} \left[I_F + \frac{I_T}{1 + \frac{W}{\beta^2} \frac{\Phi^2_{mn}}{\Lambda_{mn}} + \frac{n^2 m^2 r W \left(1 - \frac{1}{r}\right)^2}{\Lambda_{mn} \left(1 + \Lambda_{mn} \frac{W}{\beta^2} \frac{1-\mu}{2} r\right)}} \right] \qquad (2)$$

$$\beta = \frac{a}{b} \qquad (3)$$

$$\Phi_{mn} = m^2 + n^2 \beta^2 \qquad (4)$$

$$\Lambda_{mn} = m^2 + \frac{n^2 \beta^2}{r} \qquad (5)$$

$$I_F = \frac{t^3 + (t')^3}{12} \qquad (6)$$

$$I_T = \frac{tt'}{t + t'} \left[c + \frac{t + t'}{2} \right]^2 \qquad (7)$$

$$r = \frac{G_{xz}}{G_{yz}} \qquad (8)$$

and

$$W = \frac{ctt'}{t + t'} \frac{\pi^2}{b^2} \frac{E}{1-\mu^2} \frac{1}{G_{xz}} \quad . \qquad (9)$$

When $r = 1$ and $t = t'$, equation (1) reduces to equation (181) in reference (2). Equation (1) is identical to a result obtained by Ericksen and March by use of the so-called "tilting" method in reference (1). Thus, the assumptions inherent in the "tilting" method applied to the flat panel evidently include the following:[8]

1. The core is capable of resisting only anti-plane stress.[4]
2. The transverse modulus of elasticity of the core, E_c, is infinite.

When I_F is much less than I_T, as is the case in most sandwich constructions, equation (1) may be written in the form[9]

$$N_{ocr} = \frac{\pi^2}{b^2} D_T \, k_{m0} \qquad (10)$$

where

$$D_T = \frac{EI_T}{1-\mu^2} \qquad (11)$$

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[8] See reference (2), pp. 83 and 84.

[9] Note that when $t = t'$, D_T of this report becomes D_{BM} of reference (2).

$$k_{m0} = \frac{\Omega_{mn}}{m^2 b^2} \qquad (12)$$

and

$$\Omega_{mn} = \frac{\Phi_{mn}^2}{1 + \frac{W}{\beta^2} \frac{\Phi_{mn}^2}{\Lambda_{mn}} + \frac{n^2 m^2 r W \left(1 - \frac{1}{r}\right)^2}{\Lambda_{mn} \left(1 + \Lambda_{mn} \frac{W}{\beta^2} \frac{1-\mu}{2} r\right)}} . \qquad (13)$$

When $\underline{r} = 1.0$ and $\underline{t} = \underline{t'}$, equation (10) reduces to equation (188) in reference (2).[10]

For $\underline{r} = 1.0$, equation (10) is presented in the form of design curves in reference 1. For $\underline{r} = 0.4$, $\underline{r} = 1.0$, and $\underline{r} = 2.5$, equation (10) is presented in the form of design curves in reference (3). The aforementioned design curves present the critical load factor, k_{m0},[11] as a function of the panel parameters $\frac{a}{b}$ and $\underline{W}$.[11]

2. Case α General, $E_c = \infty$

The loading on the panel for this case is any combination of edgewise bending and compression defined by α.

The critical load for this case may be obtained from the equation

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[10] It is noted that equation (10) reduces to equation (188) in reference (2) except that D_T appears instead of D_M. D_T is a better approximation for the flexural rigidity of the spaced facings than is D_M. For many panels, these approximations are essentially equivalent.

[11] The critical load factor, k_{m0} in this report, is designated k_m in reference (3). The parameter $\underline{W}$ in this report is designated $\underline{V}$ in reference (3). There are further differences in notation which are not noted here.

$$\xi_{m1}\,\xi_{m2}\,\xi_{m3} - \Gamma\,(\xi_{m1}\,\xi_{m2} + \xi_{m1}\,\xi_{m3} + \xi_{m2}\,\xi_{m3}) + \Gamma^2\,(\xi_{m1}$$

$$+ \xi_{m2} + \xi_{m3}) - \Gamma^3 - (\alpha N_{ocr} \frac{m^2}{a^2})^2 \left[(\frac{48}{25})^2 (\xi_{m1} - \Gamma) \right.$$

$$\left. + (\frac{16}{9})^2 (\xi_{m3} - \Gamma) \right] = 0 \qquad (14)$$

where

$$\xi_{mn} = D'_{mn}\,\pi^4\,\theta^2_{mn} \qquad (15)$$

and

$$\Gamma = N_{ocr} \frac{m^2 \pi^2}{a^2} (1 - \frac{\alpha}{2}) \qquad (16)$$

When $\underline{r} = 1$ and $\underline{t} = \underline{t'}$, equation (14) reduces to equation (202) of reference (2).

Design curves can be made from equation (14). These design curves could be constructed to present the critical load factor, $\underline{k_{m\alpha}}$, as a function of $\frac{a}{b}$, $\underline{\alpha}$, and $\underline{W}$.

3. Case $\alpha = 2$, $E_c = \infty$

The loading on the panel for this case is pure edgewise bending.

The critical load for this case may be obtained from the equation

$$N_{ocr} = \frac{1}{2} \frac{a^2}{m^2} \sqrt{\frac{\xi_{m1}\,\xi_{m2}\,\xi_{m3}}{(\frac{48}{25})^2 \xi_{m1} + (\frac{16}{9})^2 \xi_{m3}}} \qquad (17)$$

where ξ_{mn} is given in equation (15).

When $r = 1.0$ and $t = t'$ equation (17) reduces to equation (197) of reference (2).

Since $I_F \lessgtr I_T$ in most sandwich panel constructions, equation (17) may be written in the approximate form

$$N_{ocr} = \frac{\pi^2}{b^2} D_T \, k_{m2} \tag{18}$$

where D_T is defined in equation (11),

$$k_{m2} = \frac{\pi^2}{m^2 \beta^2} \sqrt{\frac{\Omega_{m1} \Omega_{m2} \Omega_{m3}}{(\frac{96}{25})^2 \Omega_{m1} + (\frac{32}{9})^2 \Omega_{m3}}} \, , \tag{19}$$

and Ω_{mn} is defined in equation (13).

When $r = 1.0$ and $t = t'$, equations (18) and (19) reduce, respectively, to equations (198)[12] and (199) of reference (2).

Equation (18) was used to prepare design curves for this report. These design curves are based on values of $r = 0.4$, $r = 1.0$, and $r = 2.5$ and are presented in figures 4, 5, and 6, respectively. The Poisson's ratio of the facings, μ, was assumed equal to 0.333[13] for the curves in which $r = 0.4$ and $r = 2.5$. Because of the nature of equation (18) when $r = 1.0$, it was not necessary to assume a value of μ for this particular design curve. The designer is free to choose any value of μ in this case. It is believed that the assumption of $\mu = 0.333$ in the curves for $r = 0.4$

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[12] D_T is a better approximation for the flexural rigidity of the spaced facings than is D_M. Note that when $t = t'$, D_T of this report becomes D_{BM} of reference (2).

[13] Poisson's ratio, μ, of the facings was also assumed equal to 0.333 for the design curves in reference (3).

and $r = 2.5$ will introduce no significant error in critical load factor for the ranges of μ which exist in the properties of sandwich facings.

It is of interest to note that each of the curves for $W > 0$ in figures 4, 5, and 6 exhibit finite values of k_{m2} and hence finite values of N_{ocr} when the panel ratio, $\frac{a}{b}$, is zero.[14] This apparent inconsistency may be attributed to the fact that the actual flexural rigidity of the spaced facings, D, has been approximated by the flexural rigidity, D_T. If the exact value, D, is used, the critical load, N_{ocr}, is found to approach infinity as $\frac{a}{b}$ approaches zero. Thus, the assumption that

$$D_T \doteq D$$

is inaccurate when $\frac{a}{b}$ is very small. For values of $\frac{a}{b}$ greater than, say 0.2, the curves are believed sufficiently accurate for use in design. The curves give conservative values of k_{m2} and hence of N_{ocr}, of course, when $\frac{a}{b} < 0.2$.

Mathematical Analysis

The mathematical developments in this report are based on a sandwich panel composed of elements with the following assumed properties:

1. The core is orthotropic and is capable of resisting only anti-plane stress.[4] The transverse modulus of elasticity of the core, E_c, is infinite.
2. The facings are isotropic and are of unequal thickness.

1. Case $\alpha = 0$, $E_c = \infty$

The loading on the panel for this case is pure edgewise compression. The critical load for this case is defined in equation (180) of reference (2). The determinant of order 6 in this equation can be immediately

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[14] This characteristic is also exhibited by all the curves in reference (3).

reduced to a determinant of order 5 by expanding it by row 2,[15] that is, by $R_2^{(1)}$. The resulting determinant of order 5 may be reduced to a determinant of order 1 by the following successive operations:

$$\left.\begin{aligned} R_2^{(2)} &= R_2^{(1)} + R_3^{(1)} \\ R_4^{(2)} &= R_4^{(1)} + R_5^{(1)} \end{aligned}\right\} \qquad (20)$$

$$\left.\begin{aligned} C_2^{(3)} &= C_2^{(2)} + C_3^{(2)} \\ C_4^{(3)} &= C_4^{(2)} + C_5^{(2)} \end{aligned}\right\} \qquad (21)$$

$$\left.\begin{aligned} C_5^{(4)} &= C_5^{(3)} - \frac{1}{2} C_4^{(3)} \\ C_1^{(4)} &= C_1^{(3)} - \frac{1}{G_{xz}\,\rho_{mn}} \frac{a^2}{m^2\pi^2} C_3^{(3)} \end{aligned}\right\} \qquad (22)$$

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[15] The letter R signifies "row" and the letter C signifies "column." The numerical subscript denotes the particular row or column involved. The numerical superscript denotes a specific determinant. Thus, $R_3^{(2)}$ denotes row 3 of a 2nd determinant found from additions of rows and/or columns.

$$\left.\begin{aligned}
R_1^{(5)} &= R_1^{(4)} + \frac{c}{2\pi^2 \theta_{mn} G_{xz}} R_4^{(4)} \\
R_2^{(5)} &= R_2^{(4)} - \frac{c}{2} R_4^{(4)} \\
R_5^{(5)} &= R_5^{(4)} - \frac{m^2}{a^2 \theta_{mn}} R_4^{(4)}
\end{aligned}\right\} \qquad (23)$$

$$R_1^{(6)} = R_1^{(5)} + \frac{1}{2} \frac{a^2 c}{m^2 \pi^2} \frac{1}{G_{xz}} R_5^{(5)} \qquad (24)$$

$$C_5^{(7)} = C_5^{(6)} - \frac{1}{2\theta_{mn}} \left(\frac{m^2}{a^2} - \frac{n^2}{b^2}\right) C_4^{(6)} \qquad (25)$$

$$\left.\begin{aligned}
R_3^{(8)} &= R_3^{(7)} - \left(1 - \frac{G_{yz}}{G_{xz}}\right) \frac{c}{\theta_{mn}} \frac{n^2}{b^2} R_4^{(7)} \\
R_2^{(8)} &= R_2^{(7)} - \frac{c + t'}{2} R_4^{(7)} \\
R_1^{(8)} &= R_1^{(7)} + \frac{a^2}{m^2 \pi^2} \frac{c}{2} \frac{1}{G_{xz}} R_5^{(7)}
\end{aligned}\right\} \qquad (26)$$

$$\left.\begin{aligned} R_3^{(9)} &= R_3^{(8)} + c\rho_{mn} R_5^{(8)} \\ R_1^{(9)} &= R_1^{(8)} + \frac{c}{2\pi^2 \theta_{mn}} \frac{1}{G_{xz}} R_4^{(8)} \end{aligned}\right\} \tag{27}$$

and

$$C_3^{(10)} = C_3^{(9)} + \frac{t'c}{t + t'} \rho_{mn} C_5^{(9)} \quad . \tag{28}$$

At this point, the determinant of order 5 may be expanded using a Laplace expansion by $\underline{R_5^{(10)}}$ to give a determinant of order 4. Further operations on the resulting determinant of order 4 are as follows:

$$\left.\begin{aligned} R_1^{(11)} &= R_1^{(10)} - \frac{a^2}{m^2\pi^2} \frac{1}{G_{xz} \rho_{mn}} R_3^{(10)} \\ C_2^{(11)} &= C_2^{(10)} + \frac{t}{2} C_4^{(10)} \end{aligned}\right\} \tag{29}$$

and

$$R_4^{(12)} = R_4^{(11)} + \frac{\dfrac{t'\delta c\theta_{mn}}{G_{xz}}\left[1 + \dfrac{a^2}{m^2} \dfrac{n^2}{b^2} \dfrac{1}{\rho_{mn}} (1 - \dfrac{G_{yz}}{G_{xz}})\right]}{\dfrac{1}{G_{xz}} \dfrac{a^2}{m^2\pi^2} (1 - \dfrac{G_{yz}}{G_{xz}} \dfrac{1}{\rho_{mn}} \dfrac{a^2}{m^2} \dfrac{n^2}{b^2})} R_1^{(11)} . \tag{30}$$

At this point, the determinant of order 4 may be expanded using a Laplace expansion by $\underline{C_1^{(12)}}$ to give a determinant of order 3. Further operations on the determinant of order 3 are as follows:

$$R_3^{(13)} = R_3^{(12)} - \frac{t' c \delta \theta_{mn} \frac{n^2 \pi^2}{b^2} \left(1 - \frac{G_{yz}}{G_{xz}}\right)}{\frac{n^2 \pi^2}{b^2} G_{yz} \rho_{mn} + \frac{tt'}{t + t'} c^2 \delta \rho_{mn}^2 \pi^2 \frac{m^2}{a^2} \frac{n^2}{b^2} \frac{1-\mu}{2}} R_2^{(12)} . \tag{31}$$

The determinant of order 3 may now be expanded by $\underline{C_2^{(13)}}$ to give a determinant of order 2. A final operation on the resulting determinant of order 2 is

$$R_1^{(14)} = R_1^{(13)} + \frac{\frac{t^2}{2} + ct + \frac{tt'}{2}}{t + t' + \frac{tt' \delta c^2}{G_{xz}} \frac{m^2}{a^2} \frac{\left[1 + \frac{a^2}{m^2} \frac{n^2}{b^2} \frac{1}{\rho_{mn}} \left(1 - \frac{G_{yz}}{G_{xz}}\right)\right]^2}{\left[1 - \frac{G_{yz}}{G_{xz}} \frac{1}{\rho_{mn}} \frac{a^2}{m^2} \frac{n^2}{b^2}\right]} + \frac{tt' \delta c^2 \frac{n^2}{b^2} \left(1 - \frac{G_{yz}}{G_{xz}}\right)^2}{G_{yz} \rho_{mn} + \frac{tt'}{t + t'} \delta c^2 \rho_{mn}^2 \frac{m^2}{a^2} \frac{1-\mu}{2}}} R_2^{(13)} . \tag{32}$$

A literal expansion of the resulting determinant of order 2 yields

$$N_{ocr} = c\delta\pi^2\theta^2_{mn} \frac{a^2}{m^2\pi^2} \left\{ \frac{t^3}{12} - \frac{(t')^3}{6} - ct'(c + t') + \frac{t'}{\theta^2_{mn}} \left\{ \left(\theta_{mn} \frac{t'}{2} + \frac{m^2}{a^2} \rho_{mn}\, c\right)^2 + \frac{n^2}{b^2}\, c\left(1 - \frac{G_{yz}}{G_{xz}}\right) \left[\theta_{mn}(c + t') + \frac{m^2}{a^2}\, c\,\rho_{mn}\right] \right\} \right.$$

$$\left. + \frac{\left[\frac{t^2}{2} + ct + \frac{tt'}{2}\right]\left[\frac{(t')^2}{2} + ct' + \frac{tt'}{2}\right]}{t + t' + \frac{tt'\delta c^2}{G_{xz}} \frac{m^2}{a^2} \frac{\left[1 + \frac{a^2}{m^2}\frac{n^2}{b^2}\frac{1}{\rho_{mn}}\left(1 - \frac{G_{yz}}{G_{xz}}\right)\right]^2}{\left[1 - \frac{G_{yz}}{G_{xz}}\frac{1}{\rho_{mn}}\frac{a^2}{m^2}\frac{n^2}{b^2}\right]} + \frac{tt'\delta c^2 \frac{n^2}{b^2}\left(1 - \frac{G_{yz}}{G_{xz}}\right)^2}{G_{yz}\,\rho_{mn} + \frac{tt'}{t + t'}\,\delta c^2 \rho^2_{mn} \frac{m^2}{a^2}\frac{1-\mu}{2}}} \right\} \tag{33}$$

Equation (33) may be algebraically simplified to the form given in equation (1) of this supplementary report.

2. Case $\alpha = 2$, $E_c = \infty$

The loading on the panel for this case is pure edgewise bending. The critical load for this case is defined in equation (194) of reference (2). The determinant of order 18 in equation (194) of reference (2) can be reduced[16] to the determinant of order 3 shown in the following equation:

$$\begin{vmatrix} \xi_{m1} & -\frac{32}{9}\frac{m^2}{a^2}N_{ocr} & 0 \\ -\frac{32}{9}\frac{m^2}{a^2}N_{ocr} & \xi_{m2} & -\frac{96}{25}\frac{m^2}{a^2}N_{ocr} \\ 0 & -\frac{96}{25}\frac{m^2}{a^2}N_{ocr} & \xi_{m3} \end{vmatrix} = 0 \qquad (34)$$

where

$$\xi_{mn} = D'_{mn}\,\pi^4\,\theta^2_{mn} .$$

Expansion of the determinant in equation (34) leads directly to equation (17).

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[16] The method of reduction in this case closely parallels the method of reduction detailed previously in this report for the case $\alpha = 0$ and $E_c = \infty$.

3. Case α Any Value, $E_c = \infty$

The loading on the panel for this case is any combination of edgewise bending and compression defined by α. A reduced form of the characteristic equation formed by again setting the determinant of the coefficients A_{mn}, B_{mn}, . . . , G_{mn} equal to zero in the linear set of equations (173) through (178) of reference (2) was found to be:

$$\begin{vmatrix} \xi_{m1} - \Gamma & -\frac{16}{9}\frac{m^2}{a^2} N_{ocr}\,\alpha & 0 \\ -\frac{16}{9}\frac{m^2}{a^2} N_{ocr}\,\alpha & \xi_{m2} - \Gamma & -\frac{48}{25}\frac{m^2}{a^2} N_{ocr}\,\alpha \\ 0 & -\frac{48}{25}\frac{m^2}{a^2} N_{ocr}\,\alpha & \xi_{m3} - \Gamma \end{vmatrix} = 0 \tag{35}$$

where

$$\xi_{mn} = D'_{mn}\,\pi^4\,\theta^2_{mn}$$

$$\Gamma = N_{ocr}\,\frac{m^2\pi^2}{a^2}\left(1 - \frac{\alpha}{2}\right) .$$

Expansion of the determinant in equation (35) leads directly to equation (14).

Use of Design Curves

To calculate the buckling load, $\underline{N_{ocr}}$, of a specific sandwich panel, it is first necessary to calculate the value of $\underline{r}$. From this calculated value of $\underline{r}$ it is possible to select which one of figures 4, 5, and 6 is applicable. Next, it is necessary to calculate the parameter $\underline{W}$ for the panel under consideration. The value of $\underline{W}$ designates which one of the families[17] of curves is applicable. Corresponding to the ratio $\frac{a}{b}$ of the panel, the designer then selects the value of the critical load factor $\underline{k_{m2}}$. Finally, the critical load, $\underline{N_{ocr}}$, may be computed from the equation

$$N_{ocr} = \frac{\pi^2 D_T}{b^2} k_{m2} \tag{36}$$

where

$$D_T = \frac{E}{1-\mu^2} \frac{tt'}{t + t'} \left[c + \frac{t + t'}{2}\right]^2 . \tag{37}$$

The critical edgewise moment, $\underline{M_{ocr}}$, on the panel may then be computed from the equation

$$M_{ocr} = N_{ocr} \frac{b^2}{6} . \tag{38}$$

The integer, $\underline{m}$, identified with the curve from which $\underline{k_{m2}}$ is read, indicates the number of half sine waves into which the panel will buckle in the direction of loading when the edge load assumes the value $\underline{N_{ocr}}$.

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[17] A family of curves is here defined as that set of curves derived for various values of $\underline{m}$ and corresponding to a particular value of $\underline{W}$.

If the facings are very thin in comparison with the core thickness as is the case with many modern sandwich constructions so that $\frac{t + t'}{2} < < c$, then the flexural rigidity of spaced membrane facings, D_M, may be used for D_T in equation (36).

$$D_M = \frac{E}{1-\mu^2} \frac{tt'c^2}{t + t'} \quad . \qquad (39)$$

As the parameter W increases from zero, the curve for $m = \infty$ approaches a straight horizontal line. When W equals or exceeds a value, say W_s, for which $m = \infty$ is a straight horizontal line, then the critical load factor, k_{m2}, may be computed directly from the equation

$$k_{m2}\Big|_{W \geq W_s} = \lim_{a \to 0} k_{m2} = \frac{\pi^2}{W_s} \sqrt{\frac{1}{\left(\frac{96}{25}\right)^2 + \left(\frac{32}{9}\right)^2}}$$

$$= \frac{1.886}{W_s} \quad . \qquad (40)$$

The value of N_{ocr} associated with k_{m2} in equation (40) is known as the shear instability value.[18] The value of W_s for a particular panel may be estimated[19] from a study of figures 4, 5, and 6.

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[18] See reference (2), p. 82.

[19] When $r = 1.0$ (fig.5), $W_s = 0.215$. A literal expression for W_s when $r \neq 1$ has not been derived.

Various schemes of interpolation will occur to the designer in using these curves. One such scheme is for use in obtaining k_{m2} for values of r other than 0.4, 1.0, and 2.5 given in figures 4, 5, and 6, respectively. Values of k_{m2} may be read for r = 0.4, r = 1.0, and r = 2.5 from which a graph of k_{m2} versus r may be constructed. A value of k_{m2} for a specific value of r may then be estimated from this graph.

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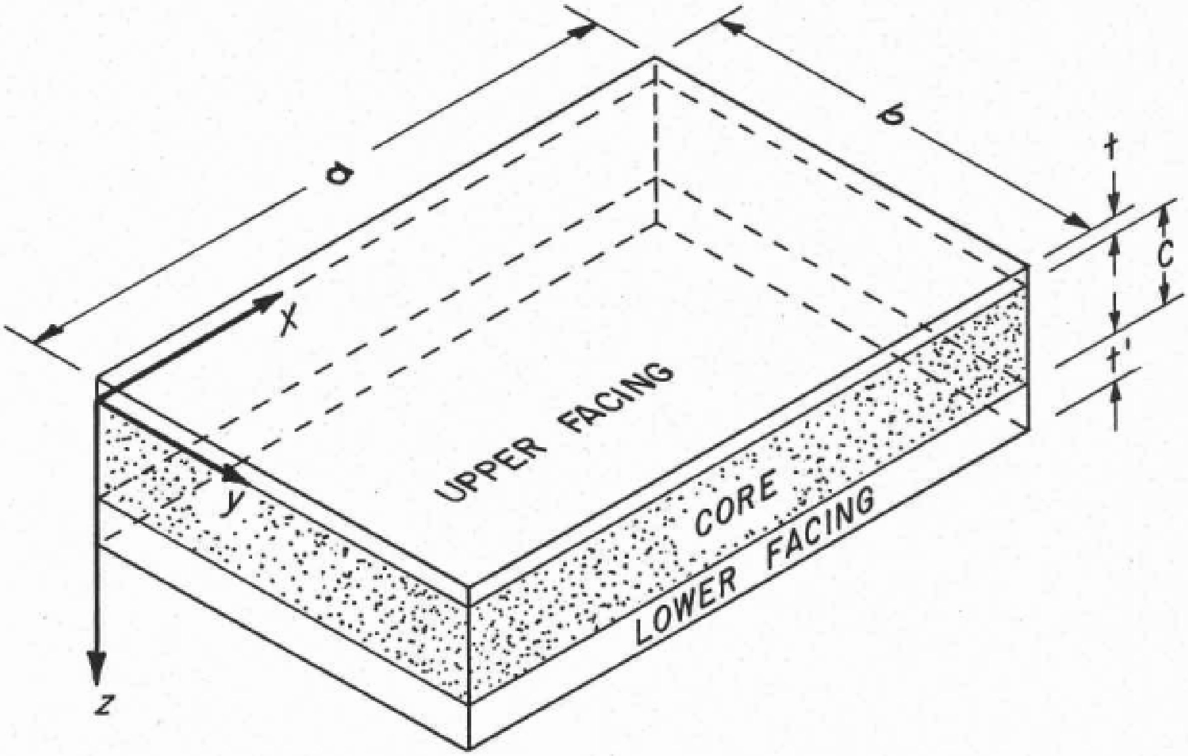
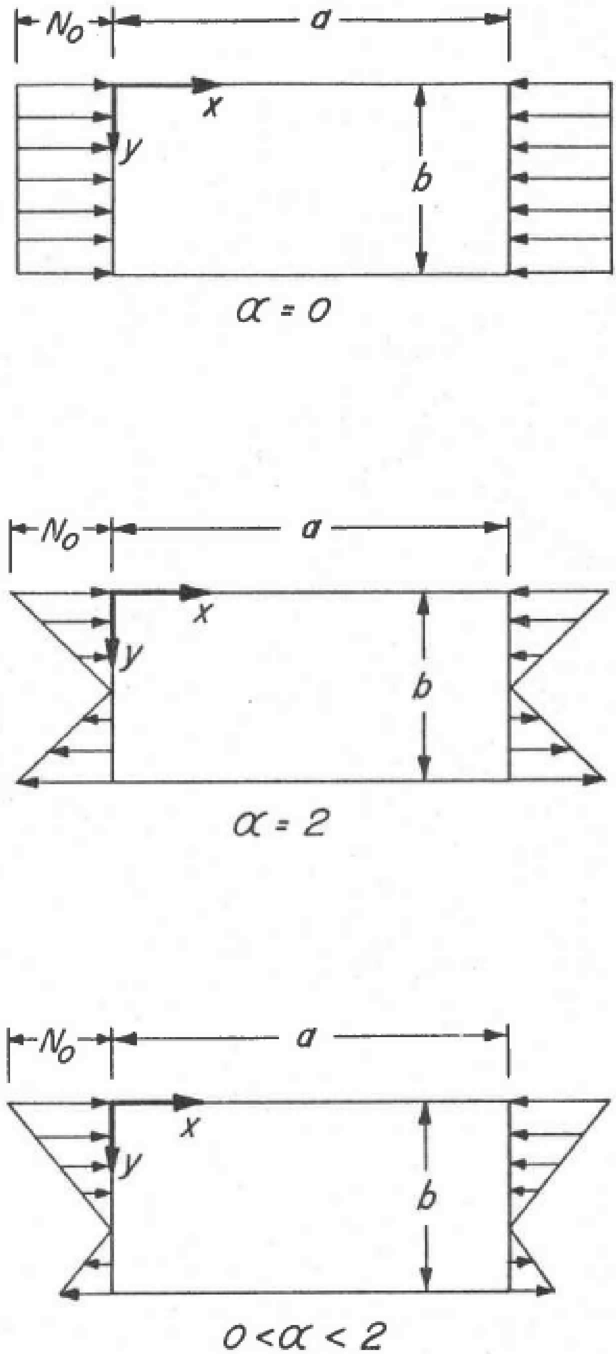


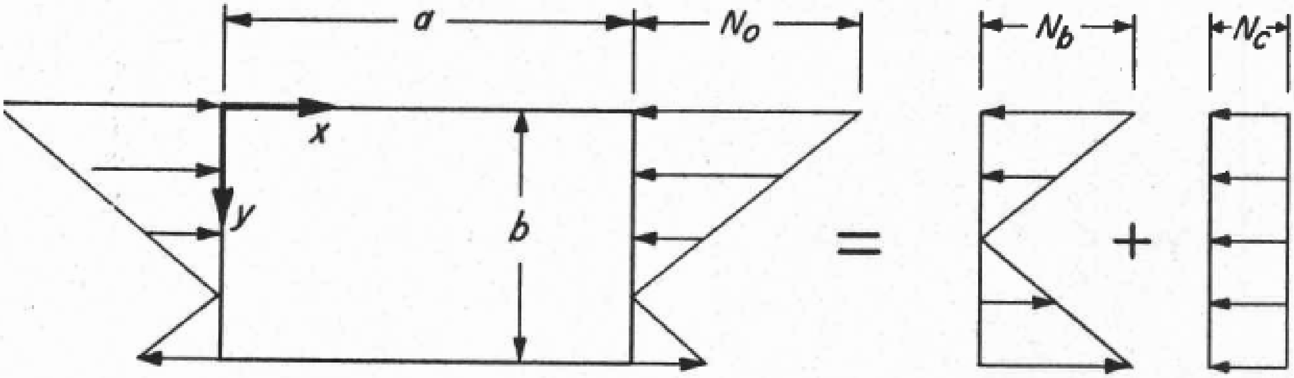
M 108 739

Figure 1.--Isometric drawing of a sandwich panel.



M 108 736

Figure 2.--Top view of sandwich panel showing different combinations of edgewise bending and compression as defined by α. N_o is the load per unit width, b, of panel.

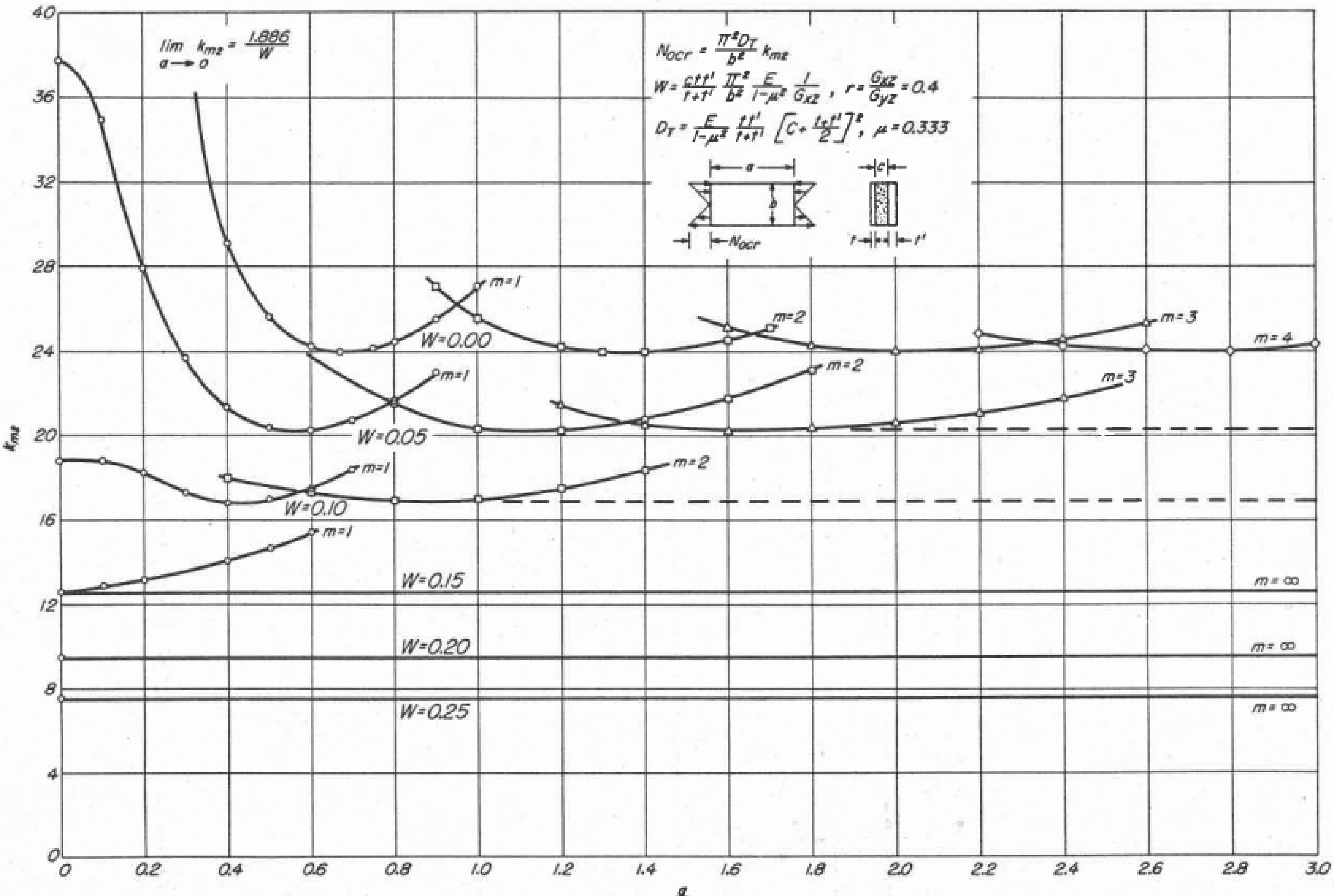


$$\alpha = \frac{2N_b}{N_b + N_c} = \frac{2N_b}{N_o}$$

$$N_o = N_b + N_c$$

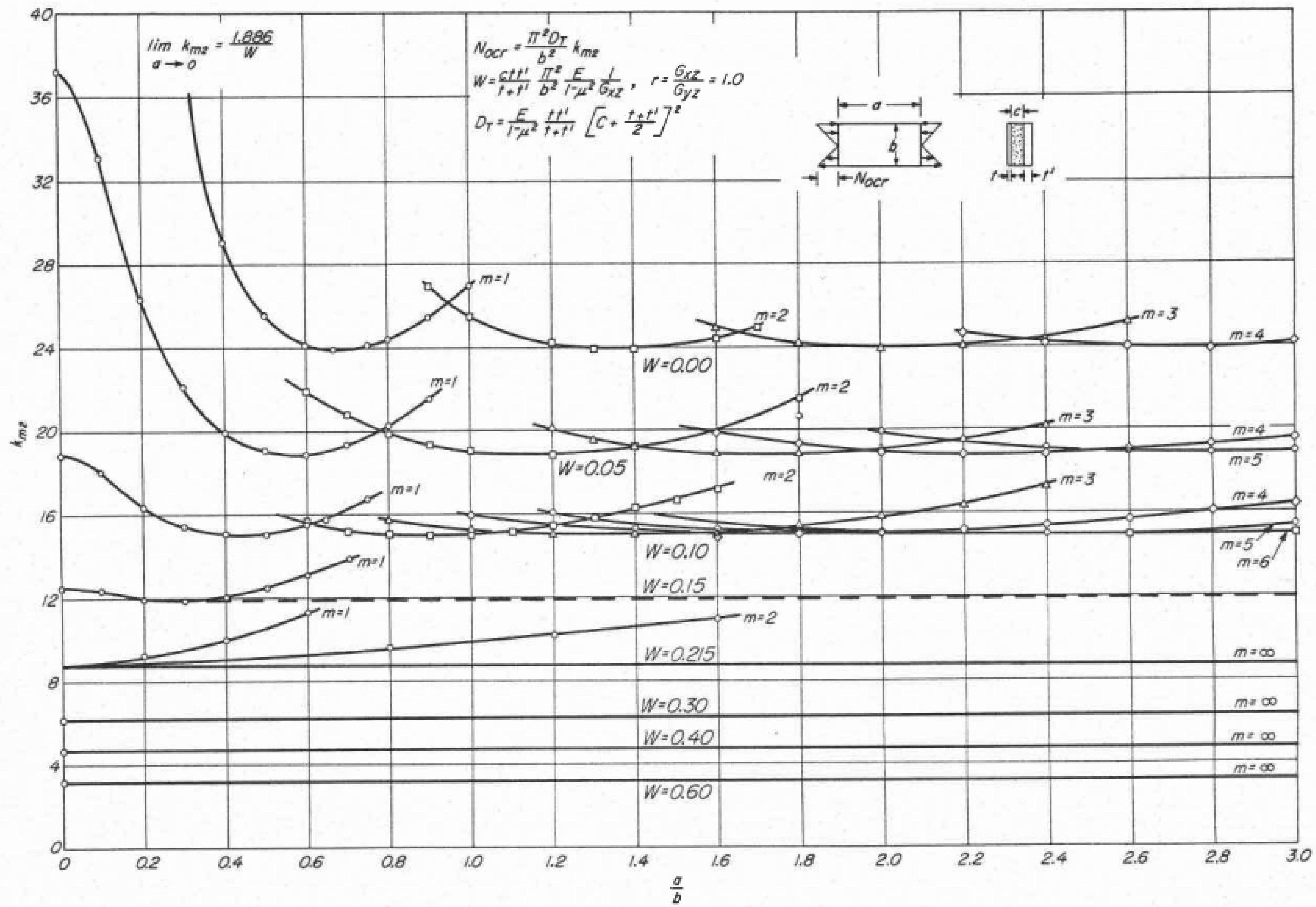
M 108 740

Figure 3. --Pictorialized definition of α, N_o, N_b, and N_c.



M 108 928

Figure 4. --Critical load factor $\underline{k_{m2}}$ versus $\frac{a}{b}$ for $\underline{\alpha}$ = 2 (pure edgewise bending), $\underline{r}$ = 0.4.



M 108 930

Figure 5. --Critical load factor $\underline{k_{m2}}$ versus $\frac{a}{b}$ for $\underline{\alpha}$ = 2 (pure edgewise bending), r = 1.0.

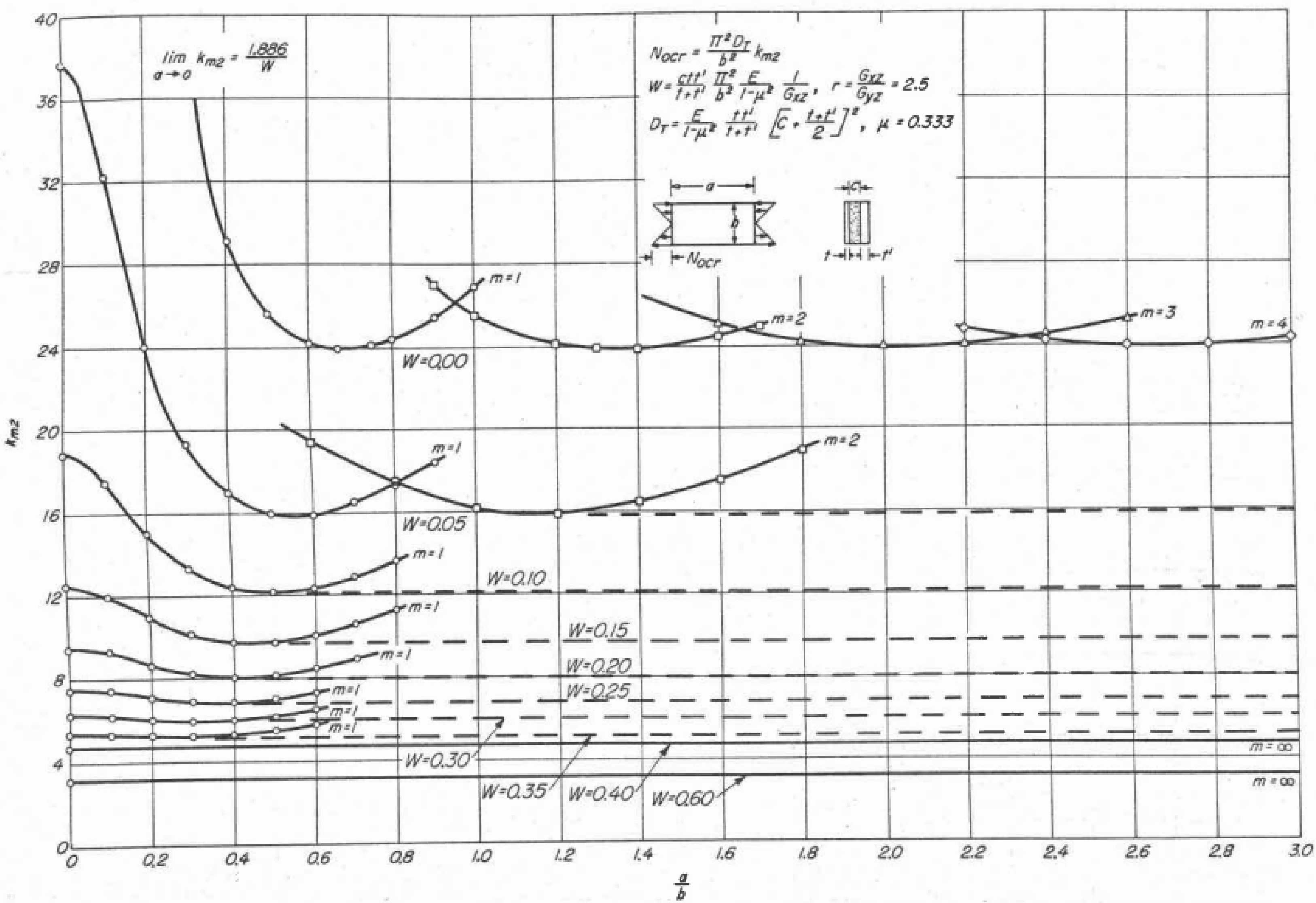


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Figure 6. --Critical load factor k_{m2} versus $\frac{a}{b}$ for $\alpha = 2$ (pure edgewise bending), r = 2.5.

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