WRINKLING OF THE FACINGS OF SANDWICH CONSTRUCTION SUBJECTED TO EDGEWISE COMPRESSION

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WRINKLING OF THE FACINGS OF SANDWICH CONSTRUCTION SUBJECTED TO EDGEWISE COMPRESSION[1]

Sandwich Constructions Having Honeycomb Cores

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Summary

The wrinkling of the facings of sandwich construction is influenced by the cell size of the honeycomb core. Equations governing this wrinkling and a method of design are presented. Results of tests illustrating the accuracy of the equations are reported. It is shown that sandwich constructions having honeycomb cores with thick walls may be designed by either this method or the one previously presented in Forest Products Laboratory Report No. 1810.

Introduction

Forest Products Laboratory Report No. 1810 (of which this report is a supplement) deals with the compressive wrinkling stresses in the facings of sandwich constructions having continuous core materials. When a sandwich construction is subjected to edgewise loads, the facings act similarly to elastically supported columns and, therefore, have definite critical loads. Also, these facings are never perfectly flat, so that deflections occur at small loads and increase more and more rapidly as the load approaches the critical load. The core restrains

[1]This progress report is one of a series prepared and distributed by the Forest Products Laboratory under U. S. Navy, Bureau of Aeronautics No. NAer 01336 and USAF No. 33(038)-51-4326-E, Amend. 2(53-131). Results here reported are preliminary and may be revised as additional data become available.

[2]Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

these deflections, thus tensile and shear stresses develop in the core and in the bond between the core and the facings. These stresses increase very rapidly as the critical load is approached, and thus failure takes place in the core or bonds at loads less than the critical value. Report No. 1810 presents formulas for the maximum loads and a method of design for sandwich constructions that may fail by this method. These formulas and this method will not apply to certain sandwich constructions involving honeycomb core materials because the size of the cells in the core material will influence the length of the waves into which the facings wrinkle. This supplement to Report No. 1810 presents equations and a design method that do apply to sandwich constructions having cores of honeycomb materials.

Nomenclature

The nomenclature is identical with that of Report No. 1810.

c thickness of core

f thickness of facings

t tensile strength of core (or bond) in the direction perpendicular to the facings

E_f modulus of elasticity of facing material

E_x modulus of elasticity of core in direction of load

E_y modulus of elasticity of core in direction perpendicular to facings

E_z modulus of elasticity of core parallel to facings and perpendicular to the direction of the load

K_o ratio of wave amplitude to half-wave length at no load

L half-wave length of wrinkles

ℓ length of the column

P compressive load per inch of edge on each facing

p stress in facings

$$Q_p = p\left(\frac{\lambda_f}{E_f E_y \mu_{xy}}\right)^{\frac{1}{3}}$$

$$q = \frac{c}{f}\mu_{xy}\left(\frac{\lambda_f}{E_f E_y \mu_{xy}}\right)^{\frac{1}{3}}$$

λ_f one minus square of Poisson's ratio for facings

σ_{yx} Poisson's ratio of core. The ratio of the strain in the x direction to the strain in the y direction due to a stress in the y direction. Similarly σ_{xy}, σ_{xz}, σ_{zx}, σ_{yz}, and σ_{zy}

$$A = \frac{\pi^2}{12} \frac{f^2}{L^2} \frac{E_f}{\lambda_f}$$

$$a = \frac{24}{\pi^4} \frac{E_y \lambda_f}{E_f} \frac{L^4}{f^3}$$

$$b = \frac{2}{\pi} \frac{E_y L K_o}{t}$$

Discussion of Mathematical Analysis

Critical Stresses in Facings

In Report No. 1810 the critical stress in the facings were found to be insensitive to changes in β and σ (nomenclature in Report No. 1810) over certain ranges of these properties. The values $\beta = 0.8$ and $\sigma = 0.25$ were chosen as values that yield sufficiently accurate results for many sandwich constructions. It was pointed out, however, that these values may not apply to sandwich panels having honeycomb cores; and that for use of the curves given for design purposes:

$$K = \frac{1}{2\beta} - \sigma \tag{1}$$

should be greater than 0.5.

By the use of the definitions in Report No. 1810 and Maxwell's relations, the value of K may be expressed by:

$$K = \frac{\sqrt{E_x E_y}}{2\,\mu_{xy}} - \sigma_{yx} \sqrt{\frac{E_x}{E_y}} \tag{2}$$

for plane stress, and:

$$K = \frac{\sqrt{E_x E_y}}{2\,\mu_{xy}(1 - \sigma_{xz}\,\sigma_{zx})} - \frac{\sigma_{yx} + \sigma_{zx}\,\sigma_{yz}}{1 - \sigma_{zy}\,\sigma_{yz}} \sqrt{\frac{E_x}{E_y}} \tag{3}$$

for plane strain; in which y is always associated with the L direction (fig. 1) and x and z are associated with the T and R directions, respectively, or the R and T direction, respectively. The values of σ_{yx} and σ_{yz} are substantially equal to the value of Poisson's ratio for the material of which the cell walls are made and may be taken to be about 0.3.

Values of K were computed by means of equation (2) for a few honeycomb core materials for which appropriate elastic properties were available. These values are given in table 1. It is found that for most of these materials K is very small compared with 0.5. This occurs because the values of E_x are usually very small compared with the values of E_y and μ_{xy}.

From this it was assumed that usable equations for the critical stress in the facings of sandwich panels having honeycomb cores could be obtained from equations 1.9.1 (for antisymmetrical wrinkling) and 1.10.1 (for symmetrical wrinkling) of Report No. 1810 by letting the ratio $\frac{E_x}{E_y}$ approach zero. This was done by first letting $\sigma = 0$ and then taking the limit of these equations.

By taking this limit, equations 1.9.1 and 1.10.1 become:

$$Q = \frac{\pi^2 c^2}{12 L^2 q^2} \frac{\mu_{xy}}{E_y} + \frac{q}{2\left[1 + \frac{\pi^2 c^2}{12 L^2} \frac{\mu_{xy}}{E_y}\right]} \tag{4}$$

(for antisymmetrical wrinkling) and (for symmetrical wrinkling)

$$Q = \frac{\pi^2 c^2}{12 L^2 q^2} \frac{\mu_{xy}}{E_y} + \frac{2 L^2 q}{\pi^2 c^2} \frac{E_y}{\mu_{xy}} \tag{5}$$

Thus there is a critical value of Q associated with each half-wave length (L) that may occur. The minimum values are:

$$Q_f = \sqrt{\frac{2}{q} - \frac{1}{q^2}} \tag{6}$$

for antisymmetrical wrinkling, and

$$Q_f = \sqrt{\frac{2}{3q}} \tag{7}$$

for symmetrical wrinkling.

In equation (6) q cannot be less than the cube root of 2 if β is to remain positive, as it must. (Because it is an elastic property.)

The curves of equations (6) and (7) are plotted in figure 2 together with the curve of shear instability, $Q = \frac{1}{2} q$. It may be noted that outside of the range of q in which the value of Q is controlled by shear instability, symmetrical wrinkling leads to lesser values of Q than antisymmetrical wrinkling. This leads to the conclusion that sandwich panels with honeycomb cores will wrinkle symmetrically.

Stresses in Facings at Failure

As discussed in Report No. 1810, failure may be expected to occur before the critical stress is reached, due to failure in the core, or in the bond between the core and facings, because of the stresses built up associated with the waves originally present in the facings. In Report No. 1810 the data indicated that the amplitude of these original waves was proportional to the core thickness; thus the waves were probably created in the manufacture of the panel because of the presence of hard and soft spots in the core material. This assumption may not be valid in connection with honeycomb cores because these cores are very uniform in texture and thickness.

The sheets of facing material, as they are received by the panel manufacturer, are never perfectly flat, nor are they perfect ruled surfaces. As they are pressed to the honeycomb cores, buckles form that break into shorter and shorter buckles as the pressure is increased. When the pressure is sufficient to effect good bonding between the facing material and the core, the size of the buckles is coincident with the size of the cells of the core material and their amplitude is consistent with the amount of excess facing material over that required to make the facing sheets perfect ruled surfaces.

Now, if these same facing sheets were pressed to a core having a larger cell size than the previous one, buckles will be formed consistent with the same amount of excess material but of a greater length. It follows that the ratios of the amplitude to the wave lengths of these two sizes of buckles are identical.

This argument leads to William's assumption (ref. 9, Report No. 1810) that the amplitude of the original wrinkles is proportional to their wave lengths. Therefore, the limit form of an equation is required that is similar to equation 1.14.8 or 1.14.10 of Report No. 1810 but applying to symmetrical wrinkling. Such equations were derived, and it was found that the equation based on shear failure (not reported) is independent of the shear strength, which implies that shear stress at the bond between the facings and core is not developed. The one based on tension failure is:

$$Q_p = \frac{\dfrac{\pi^2 c^2}{12L^2 q^2} \dfrac{\mu_{xy}}{E_y} + \dfrac{2L^2 q}{\pi^2 c^2} \dfrac{E_y}{\mu_{xy}}}{1 + \dfrac{2L\,E_y}{\pi c\,t} K_o} \tag{8}$$

In this equation the parameters:

$$Q_p = p\left(\frac{\lambda_f}{E_f E_y \mu_{xy}}\right)^{\frac{1}{3}}$$

$$q = \frac{c}{f}\,\mu_{xy}\left(\frac{\lambda_f}{E_f\,E_y \mu_{xy}}\right)^{\frac{1}{3}}$$

do not exclusively contain the construction and elastic properties of the sandwich panel. It might better be written in the form:

$$p = \frac{\pi^2}{12}\,\frac{f^2}{L^2}\,\frac{E_f}{\lambda_f}\;\frac{1 + \dfrac{24}{\pi^4}\dfrac{E_y \lambda_f}{E_f}\dfrac{L^4}{cf^3}}{1 + \dfrac{2}{\pi}\dfrac{E_y}{t}\dfrac{L}{c} K_o} \tag{9}$$

or in the abbreviated form:

$$p = \frac{A}{L^2}\,\frac{c + aL^4}{c + bL} \tag{10}$$

In equation (10) the thickness of the facings is included in the symbols A and a. This was done because it is not expected that equation (9) will show the variation of wrinkling stress with that of facing thickness. There is little assurance that the same amount of excess material will occur in facing materials of different thickness, and, therefore, the factor K_o may readily be an unknown function of this thickness.

This equation may be used in a number of ways. It is assumed that the information required for the computation of A and a is at hand. If K is very small (in the hundredths), L becomes the cell size of the core material and the value of b may be found from tests of specimens having a certain core thickness (c). It is then possible to compute, by use of the equation, the wrinkling stresses of specimens having other core thicknesses. If K is not small, equation (10) does not apply accurately. A reasonable approximation may be obtained by assuming both b and L are unknown. Thus tests of specimens having two different core thicknesses are required to obtain these values. The following equations, obtained from equation (10), are useful for this purpose.

$$b = \frac{Ac_1}{L^3 p_1} + \frac{AaL}{p_1} - \frac{c_1}{L} = \frac{Ac_2}{L^3 p_2} + \frac{AaL}{p_2} - \frac{c_2}{L} \qquad (11)$$

$$Aa\left[\frac{1}{p_1} - \frac{1}{p_2}\right] L^4 + \left[c_2 - c_1\right] L^2 + A\left[\frac{c_1}{p_1} - \frac{c_2}{p_2}\right] = 0 \qquad (12)$$

Equation (10) may be handled in still another way, with b and L assumed to be unknown. The value of L is assumed to be that which will cause the wrinkling stress (p) to be a minimum. By placing the derivative of p with respect to L equal to zero and solving for b:

$$b = \frac{2c}{L} \; \frac{c - aL^4}{aL^4 - 3c} \qquad (13)$$

and directly from equation (10):

$$b = \frac{Ac}{L^3 p} + \frac{AaL}{p} - \frac{c}{L} \qquad (14)$$

These two equations, simultaneous in b and L, may be solved by cut-and-try methods. The solutions obtained will depend upon the core thickness (c) assumed.

It will be shown that this method of solution does not agree well with experimental results and the previous ones do. It follows that the value of the half-wave length (L) is associated with the cell size of the honeycomb core material.

Description of Specimens

It was desired to obtain face wrinkling at stresses in the facings well below the proportional limit value of the facing material. Tempered spring steel 0.01 inch thick was, therefore, chosen for the face material. Edgewise compression tests of this material supported between lubricated plates indicated that the proportional limit stress was above 200,000 pounds per square inch. The modulus of elasticity was very near to 30 million pounds per square inch, and the Poisson's ratio was assumed to be 0.3. The ratio $\frac{E_f}{\lambda_f}$ was, therefore, assumed to be 33 million pounds per square inch.

Two resin-treated paper-honeycomb-core materials were used. According to tests, subsequently described, they were found to have the following properties:

Core type	A	B
Modulus of elasticity along the flutes (E_L) . . . p.s.i. --	16,700	68,600
Modulus of elasticity across the flutes (E_R) . . p.s.i. --	0.99	309
Modulus of rigidity, strains in LR plane (G_{LR}) . . . . . . p.s.i. --	1,560	6,700
Value of κ computed according to equation (2)	0.039	0.32
Cell size in the R direction . . . . in. --	0.24	0.17

Other properties of core B were measured and will be found in the appendix.

Core A was used in six thicknesses, 3/8, 1/2, 3/4, 1, 1-1/2, and 2-3/8 inches. Five specimens of sandwich construction were made of each thickness. The specimens were 4 inches long and 2 inches wide. The core material was oriented in these specimens with the R direction (fig. 1) parallel to the length of the specimen. The ends of the facings were ground smooth and parallel to each other. Plaster ends were cast on the specimens as described in Report No. 1556[3] and illustrated in figure 4 of that report.

Core B was used in seven thicknesses, 3/8, 1/2, 5/8, 3/4, 7/8, 1, and 1-1/4 inches. Three specimens of sandwich construction were made of the 3/8-inch thickness, five each of the 1/2- and of the 5/8-inch thickness, two of the 7/8-inch thickness, and six each of the remaining thicknesses. The width of the specimens was 1-1/4 inches, and their lengths varied roughly with their thicknesses and were 2-1/2, 3, 3-1/2, 4, 4-1/2, 5, and 5 inches. Their ends were ground and cast in plaster as described for core A.

Methods and Results of Tests

Sandwich Specimens

The sandwich specimens were tested according to the method described in Report No. 1556[3] on pages 4 and 5 and illustrated in figure 4 of that report. The results of these tests are given in table 2 for sandwich constructions having core A and in table 3 for those having core B. In these tables the distance between the plaster ends (unsupported length) is given in column 1, the width of the specimen in column 2, the core thickness in column 3, the load at failure in column 4, and the stress in the facings at failure in column 5. In computing these stresses it was assumed that the core does not carry an appreciable portion of the load.

In table 2, column 6 lists the average dimension of the cells in the direction of the load for each specimen. Column 7 lists the moduli of rigidity of the core material calculated from the maximum loads of the specimens that were sufficiently thin to fail by shear instability. The formula used for these calculations is:

$$\mu_{xy} = \frac{c}{\dfrac{(f+c)^2}{2P - \dfrac{2}{3}\dfrac{\pi^2}{\ell^2}\dfrac{E_f}{\lambda_f} f^3} - \dfrac{1}{\dfrac{1}{8}\dfrac{\pi^2}{\ell^2}\dfrac{E_f}{\lambda_f} f}}$$

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[3]Methods for Conducting Mechanical Tests of Sandwich Construction at Normal Temperatures. Forest Products Laboratory Report No. 1556. Revised February 1950.

This formula was obtained from formula 3.211 (A) of ANC bulletin 23[4] by assuming the rectangular sandwich panel to be infinitely wide and to be clamped at its loaded ends. The average value (1,560 lbs. per sq. in.) of the thinner specimens that failed by shear instability was used in the calculations subsequently described, because the thicker specimens that failed in this way yielded erratic values.

Similar values of modulus of rigidity are given in table 3, column 6, for core material B. The dimension of the cells in the direction of the load for this core material was measured on one specimen only and was found to be 0.17 inch.

Specimens of Core Material

The moduli of elasticity in the direction of the flutes (E_L) was determined by the method described in Report No. 1555,[5] pages 5 and 6, and illustrated in figure 3 of that report. Two specimens of core A were tested and yielded the values 13,800 and 19,600 pounds per square inch, with an average of 16,700 pounds per square inch. Four specimens of core B were tested and yielded the values 65,300, 67,900, 68,400, and 72,900 pounds per square inch, with an average of 68,600 pounds per square inch.

The moduli of elasticity in the other two directions (E_R and E_T) were measured for core A on specimens 4 inches long and 2 by 2 inches in cross section. Light loads were applied in a testing machine, and the deformation between the heads of the machine were read. Two specimens were tested for the determination of each modulus. The values obtained for E_R were 0.909 and 1.070 pounds per square inch, with an average of 0.99 pounds per square inch. The values for E_T were 4.47 and 4.59, with an average of 4.53 pounds per square inch.

Similar values were obtained for core B by the method described in the appendix. Four specimens were tested. The individual values and their averages are given in table 4.

Discussion of the Results of Tests

The results of the tests are given in tables 2 and 3, and the average values of the stresses at failure are plotted, as circles, against the average core thicknesses

[4]Sandwich Construction for Aircraft. ANC Bulletin 23 published by the Muntions Board Aircraft Committee.

[5]Methods of Test for Determining Strength Properties of Core Material to Sandwich Construction at Normal Temperatures. Forest Products Laboratory Report No. 1555. Revised September 1950.

in figures 3, 4, and 5. In these figures the maximum and minimum values are plotted as dashes. It will be seen that there is considerable variation within a single group of tests. This variation is probably due to variations in the tensile strength of the bond between the facings and the core. It is assumed, however, that the average value obtained for each group of tests reflects the average value of the tensile strength of the bond.

The value of κ for core A was 0.039, as previously indicated, so that equation (10) of the mathematical analysis should apply. Referring to figure 3, the straight line to the left is the shear instability curve given by:

$$p_f = \frac{c}{2f} \mu_{xy}$$

Examination of the failures of the two groups of thinnest specimens indicated that they failed by shear instability; and the modulus of rigidity of the core material (μ_{xy}) was calculated by a formula similar to that above, but taking the stiffnesses of the individual facings into account, as previously indicated.

The remaining plotted values vary only slightly with the core thickness. It follows that the right-hand fraction in equation (10) must be nearly unity. If it is assumed that this fraction is unity, the value of the half-wave length (L) may be calculated by using the average value (43,900) of the stress. This computed half-wave length is found to be 0.249 inch, which agrees very well with the measured cell size of 0.24 inch. This agreement indicates that the half-wave length is associated with the cell size.

The average stress at failure for one of the groups of specimens tested may be used for the determination of $\frac{t}{K_o}$ by the method previously described, by using the cell size (0.24 in.) as the half-wave length. By using the fourth point from the left in figure 3 for this purpose, the value of $\frac{t}{K_o}$ is found to be 5,270, and the curve shown is obtained. The experimental points roughly fall on this curve except for those indicating failure due to shear instability.

Several other computed curves are given in figure 3 for comparison with this one. The dimpling stress, as computed according to Report No. 1817,[6] was found to be greater than the proportional limit of the steel facings and, thus, does not influence the results. The critical curves for antisymmetrical and

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[6] Short-column Compressive Strength of Sandwich Constructions as Affected by the Size of the Cells of Honeycomb-core Materials. Forest Products Laboratory Report No. 1817.

symmetrical wrinkling are given according to equations (6) and (7). It is evident that the experimental points are not related to these curves. A curve is also shown that was calculated from equations (13) and (14), so as to pass nearly through the third and fourth points. For this purpose a value of $\frac{t}{K_o}$ of 8,170 was chosen. The half-wave lengths associated with this curve are also shown in figure 3. It is evident that the experimental points are not associated with this curve. All these curves do not take into account the influence of the cell size on the half-wave length, and therefore support the theory that this influence should be taken into account.

The value of κ for core B was 0.32, which is too large for the present theory and too small for the theory given in Report No. 1810. It will be shown, however, that both theories apply to this core material.

Figure 4 shows the experimental points plotted on the graph of figure 16 of Report No. 1810. It may be seen that the points roughly follow the curve for a value of κ of 0.05, except for those groups of specimens that failed because of shear instability. Thus the positions of the points could readily be estimated by the use of this curve sheet if the position of one of them is known. The use of the curve sheet leads to reasonable estimates.

Figure 5 is a plot for this data similar to figure 3. Because of the large value of κ(0.32) it could be guessed that the use of a single point and a half-wave length equal to the cell size (0.17 in.) would not be suitable. This may be illustrated by assuming that the second fraction in equation (10) is unity and by calculating the half-wave length by using the average value of the stress at failure (165,000 lbs. per sq. in.). The value of the half-wave length resulting from this calculation is 0.128 inch, which does not compare well with the measured value of 0.17 inch. If, however, equations (11) and (12) are used with the experimental values for the second point from the left and the average values for the sixth and seventh points, a value for the half-wave length of 0.1377 and a value for $\frac{t}{K_o}$ of 94,500 are obtained. The resulting curve is shown in figure 5, and it may be seen that the remaining experimental points, except the first one for which shear instability controls, fall roughly on the curve. Used in this way, the theory predicts reasonable values for the remaining points.

Several other computed curves are given in figure 5 for comparison with this one. The dimpling stress, as computed according to Report No. 1817,[6] was found to be greater than the proportional limit of the steel facings and, thus, does not influence the results. The critical curves from equations (6) and (7) show little relation to the data. Critical stresses for various values of the half-wave length may be obtained from equation (10) by equating b to zero (tensile strength of bond is infinite). The data are, of course, related to the critical stress for L = 0.1377, as is seen from the figure, but are not related to the critical stress for a half-wave length equal to the cell size (0.17). The half-wave lengths associated with the symmetrical critical stresses are also shown in the figure. It is seen that the curves associated with a constant half-wave length of 0.1377 inch are related to the experimental data. It seems, therefore, that the half-wave length is limited by the cell size of the core but is not equal to it. It is possible, however, that all the theory does, used in this way, is to put a smooth curve through the experimental points; but even if this is the case, it does not detract from the usefulness of the theory. It suggests, however, that the values of the half-wave length and $\frac{t}{K_o}$ obtained, may be fictitious.

Conclusions

It is concluded from these tests that:

1. If the value of κ for the honeycomb core material is sufficiently small (near zero), the theory developed applies.
2. If the value of κ is larger, equations (11) and (12) may be used with reasonable accuracy.
3. The theory of Report No. 1810 may be used for still greater values of κ.
4. There is an intermediate region of the values of κ where the theory of Report No. 1810 and equations (11) and (12) both apply.

Appendix

Elastic Properties of Honeycomb Core B

In this report it is assumed that the honeycomb core materials are orthotropic in the RT plane (fig. 1). It is evident from the geometry of the figure that the R and T axes are orthogonal axes of symmetry and, therefore, that the material, in mass, is orthotropic; but that undoubtedly this orthotropicity does not hold over dimensions about equal to, or less than, the dimensions of the individual cells.

A few tests were made in which strains were measured over a one-inch gage length that included about six cells. Although these tests were inadequate, they do indicate that the material is orthotropic over dimensions of this size. They are reported here for what they are worth.

If the material is orthotropic Maxwell's relation:

$$\frac{E_T}{\sigma_{TR}} = \frac{E_R}{\sigma_{RT}}$$

should hold and equations 2.1 (A) of ANC Bulletin 23[2] should hold. Two axes exist to which stress may be applied without causing shear strain. Their positions are given by placing a_{31} of these equations equal to zero. The equation obtained by this process is:

$$\tan^2\phi_D = \frac{\dfrac{2}{E_R} + \dfrac{2\sigma_{RT}}{E_R} - \dfrac{1}{\mu_{RT}}}{\dfrac{2}{E_T} + \dfrac{2\sigma_{RT}}{E_R} - \dfrac{1}{\mu_{RT}}} \qquad (15)$$

Also, if the material is loaded at some angle (ϕ) to the T direction, a_{11} becomes the reciprocal of the modulus of elasticity (E_ϕ) in that direction leading to the equations:

$$\frac{2\sigma_{RT}}{E_R} = \frac{1}{E_T \tan^2\phi} + \frac{\tan^2\phi}{E_R} + \frac{1}{\mu_{RT}} - \frac{1}{E_\phi \sin^2\theta \cos^2\theta} \qquad (16)$$

These equations were checked by the substitution in them of measured values of the various elastic properties. The apparatus used is illustrated in figure 6. Four specimens were used. Their dimensions were $\frac{1}{2}$ inch in the L direction and 2 inches in both the T and R directions. A specimen, A in figure 6, was placed between the support, B, and the loading bar, C, on 1/8-inch-diameter rollers as shown. A wire yoke connected the loading pan, E, to the loading bar, C, and the specimen was loaded by placing weights on the pan. Deformation of the specimen was measured by means of an autocollimated optical-type compressometer, of one-inch gage length, placed on top of the specimen. Each specimen was tested in eight positions: (1) the force applied and the strains measured in the T direction, (2) the force applied in the T direction and the strains measured in the R direction, (3) the force applied and the strain measured in the R direction, (4) the force applied in the R direction and the strains measured in the T direction. The specimen was then turned the other side up and tests in these four positions were repeated. The loading consisted of increasing the load by equal increments and then decreasing it to zero. Thus the effect of the slight amount of friction present in the apparatus was eliminated by averaging these two curves. The stress-strain curves were plotted, and their slopes were computed. The values of these slopes are given in table 4. From these slopes values of E_T, E_R, σ_{TR}, σ_{RT}, $\frac{E_T}{\sigma_{TR}}$, and $\frac{E_R}{\sigma_{RT}}$ were computed. These values are listed in table 4.

The average value of $\frac{E_T}{\sigma_{TR}}$ was found to be 440, and that of $\frac{E_R}{\sigma_{RT}}$ was found to be 380. According to Maxwell's relations, these values should be identical, and they probably are, within the accuracy of the tests. Their average value of 410 was used for comparison with subsequent calculations.

The cells in the core material seem to be arranged in rows running at an angle of 32° to the T direction. It would seem that the two axes previously referred to should be about parallel to these rows. A single value of μ_{RT} was obtained from a test similar to that described in Report No. 1555,[3] pages 13 and 14, and illustrated in figures 12 and 13. The value obtained was 2,490 pounds per square inch. This value, with the others already obtained, was substituted in equation (15), and ϕ_D was found to be 41°, which shows that the two axes are roughly parallel to the rows of cells.

Two specimens similar to those tested in the apparatus illustrated in figure 6 were cut from the core material so that the load could be applied in the direction of the rows of the cells ($\phi = 32°$). The values of E_ϕ obtained were 5,300 and 2,640, or an average of 3,970 pounds per square inch. This value, with the values of E_T, E_R, and μ_{RT} already obtained, were substituted in equation (16), and a value of $\frac{E_R}{\sigma_{RT}}$ of 420 was obtained, which compares well with the average experimental value of 410.

The results of these tests indicate that the honeycomb core material is, substantially, an orthotropic material and may be treated as such in calculations that involve the average properties of the material.

Table 1.--Values of *K* for a few honeycomb core materials

Material	Cell size	Density	Modulus of elasticity			Modulus of rigidity		*K*	
			L	R	T	LR	LT	R	T
	In.	Lbs./ cu.ft.	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.		
4-oz. duck (U.S. Plywood)	7/16	3.7	24.7	0.011	0.025	2.8	5.6	0.086	0.061
Westplack B-6	1/4	6.0	82.8	.045	.054	2.7	9.7	.345	.104
Westplack C	3/8	3.8	51.5	.007	.017	2.4	6.7	.125	.063
Paper	0.265	4.4	47.2	.042	.064	8.9	15.7	.072	.044
N.A.C.A. T.N.	.085	5.0	48.5	.044	.067	9.2	15.9	.074	.044
1529	....	6.2	72.5	.101	.111	11.7	17.7	.104	.066
	....	6.6	71.5	.087	.108	16.7	18.2	.063	.066
	....	8.6	100.5	.357	.467	18.5	33.8	.144	.082

Table 2.--Dimensions and results of tests of sandwich constructions having type A cores

Column length	Width	Core thickness	Load at failure	Stress in facings	Cell height	Modulus of rigidity of core
(1)	(2)	(3)	(4)	(5)	(6)	(7)
In.	In.	In.	Lb.	P.s.i.	In.	P.s.i.
1.36	2.00	0.323	1,225	30,600	....	1,555
1.37	2.00	.325	1,310	32,800	....	1,537
1.47	2.01	.327	1,330	33,300	....	1,620
1.45	2.00	.327	1,330	33,300	....	1,618
1.44	2.01	.327	1,240	31,000	....	1,478
Av.	....	.327	....	32,200	....	[1]1,562
2.07	2.00	.503	1,380	34,500	....	1,226
2.00	2.03	.507	1,020	25,500	....	853
2.00	2.00	.501	1,595	39,900	....	1,425
1.92	2.00	.501	1,405	35,100	....	1,241
1.97	2.01	.505	1,255	31,400	....	1,090
Av.	....	.503	....	33,300	....	1,159
2.28	2.00	.705	1,950	48,800	0.245	....
2.22	2.00	.702	1,950	48,800	.248	....
2.10	2.00	.707	1,925	48,200	.247	....
2.22	2.00	.705	1,540	38,500	.251	....
2.19	2.00	.704	1,360	34,000	.243	....
Av.	....	.705	....	36,200	.247	....
2.08	2.00	1.024	1,865	46,600	.246	....
2.16	2.02	1.021	2,100	52,500	.235	....
2.11	2.00	1.020	1,595	39,900	.241	....
2.07	2.01	1.024	1,620	40,500	.233	....
2.06	2.00	1.020	1,450	36,300	.250	....
Av.	....	1.022	....	43,200	.241	....

Table 2.--Dimensions and results of tests of sandwich constructions having type A cores (Continued)

Column length	Width	Core thickness	Load at failure	Stress in facings	Cell height	Modulus of rigidity of core
(1)	(2)	(3)	(4)	(5)	(6)	(7)
In.	In.	In.	Lb.	P.s.i.	In.	P.s.i.
2.01	2.00	1.446	1,775	44,400	0.238	....
2.22	2.00	1.445	1,575	39,400	.241	....
2.16	2.01	1.448	2,200	55,000	.243	....
2.18	2.01	1.450	2,075	51,900	.221	....
2.23	2.01	1.449	1,455	36,400	.238	....
Av.	....	1.448	....	45,400	.236	....
2.26	2.00	2.390	1,775	44,400	.240	....
2.19	2.00	2.391	1,940	48,500	.235	....
2.29	2.00	2.399	1,525	38,100	.220	....
2.28	2.00	2.396	1,355	33,900	.241	....
2.17	2.00	2.396	1,545	38,600	.245	....
Av.	....	2.394	....	40,700	.236	....
Grand av.	....	....	....	....	.240	....

[1] Modulus of rigidity of 1,560 from first group of specimens used in computations.

Table 3.--Dimensions and results of tests of sandwich constructions having type B cores

Column length	Width	Core thickness	Load at failure	Stress in facings	Modulus of rigidity of core
(1)	(2)	(3)	(4)	(5)	(6)
In.	In.	In.	Lb.	P.s.i.	P.s.i.
1.10	1.28	0.378	3,530	141,200	6,500
1.15	1.26	.379	2,570	102,700	4,700
1.13	1.26	.378	4,620	185,000	8,800
Av.	....	.378	....	142,900	6,700
2.16	1.27	.522	3,440	137,600	....
2.04	1.25	.506	4,440	177,600	....
2.09	1.26	.511	4,335	173,500	....
2.08	1.27	.503	4,975	199,000	....
2.03	1.27	.503	4,610	184,500	....
Av.	....	.507	....	174,400	....
2.42	1.27	.625	4,190	167,600	....
2.46	1.26	.622	3,810	152,500	....
2.51	1.28	.622	3,950	158,000	....
2.52	1.27	.621	4,830	193,300	....
2.52	1.26	.622	4,085	163,500	....
Av.	....	.622	....	166,500	....
2.95	1.26	.741	3,975	159,000	....
3.15	1.26	.749	4,295	172,000	....
2.98	1.26	.747	4,240	169,600	....
3.08	1.26	.747	4,430	177,200	....
3.14	1.27	.742	4,100	164,000	....
3.08	1.27	.756	3,400	136,000	....
Av.	....	.747	....	162,900	....
3.65	1.25	.894	3,855	154,000	....
3.76	1.25	.896	4,200	168,000	....
Av.	....	.895	....	161,000	....

Table 3.--Dimensions and results of tests of sandwich constructions having type B cores (Continued)

Column length	Width	Core thickness	Load at failure	Stress in facings	Modulus of rigidity of core
(1)	(2)	(3)	(4)	(5)	(6)
In.	In.	In.	Lb.	P.s.i.	P.s.i.
4.20	1.25	1.008	4,140	165,700	....
4.20	1.25	1.006	3,800	152,100	....
4.22	1.25	1.017	4,890	195,600	....
4.20	1.25	1.017	3,720	149,000	....
4.08	1.25	1.006	4,150	166,000	....
4.20	1.25	1.001	3,690	147,600	....
Av.	....	1.009	....	162,500	....
4.23	1.25	1.250	4,300	172,000	....
4.31	1.25	1.248	3,480	139,200	....
4.01	1.25	1.247	4,335	173,500	....
4.26	1.25	1.246	3,350	134,000	....
4.34	1.25	1.246	3,770	150,700	....
4.16	1.25	1.253	3,635	145,500	....
Av.	....	1.247	....	152,500	....

Table 4.--Elastic properties of core B

Spec. No.	$\frac{f_T}{e_T}$	$\frac{f_T}{e_R}$	$\frac{f_R}{e_R}$	$\frac{f_R}{e_T}$	σ_{TR}	σ_{RT}	$\frac{E_T}{\sigma_{TR}}$	$\frac{E_R}{\sigma_{RT}}$
	E_T		E_R					
1	580	385	253	394				
	510	410	320	454				
Av.	545	397	286	424	1.37	0.68	400	420
2	760	675	337	480				
	860	424	446	480				
Av.	810	549	391	480	1.48	.82	550	480
3	570	455	285	253				
	442	352	260	396				
Av.	506	403	272	325	1.25	.84	400	320
4	581	348	321	279				
	442	298	252	343				
Av.	511	323	286	311	1.58	.92	320	310
Av.	593		309				440	380

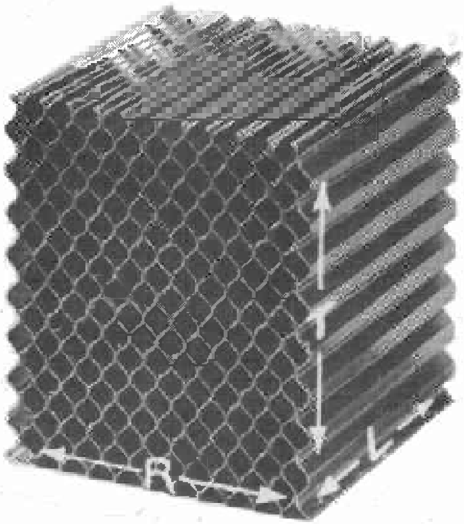

Figure 1. --Cross-sectional view of a honeycomb core material, showing the directional notation as used in this report.

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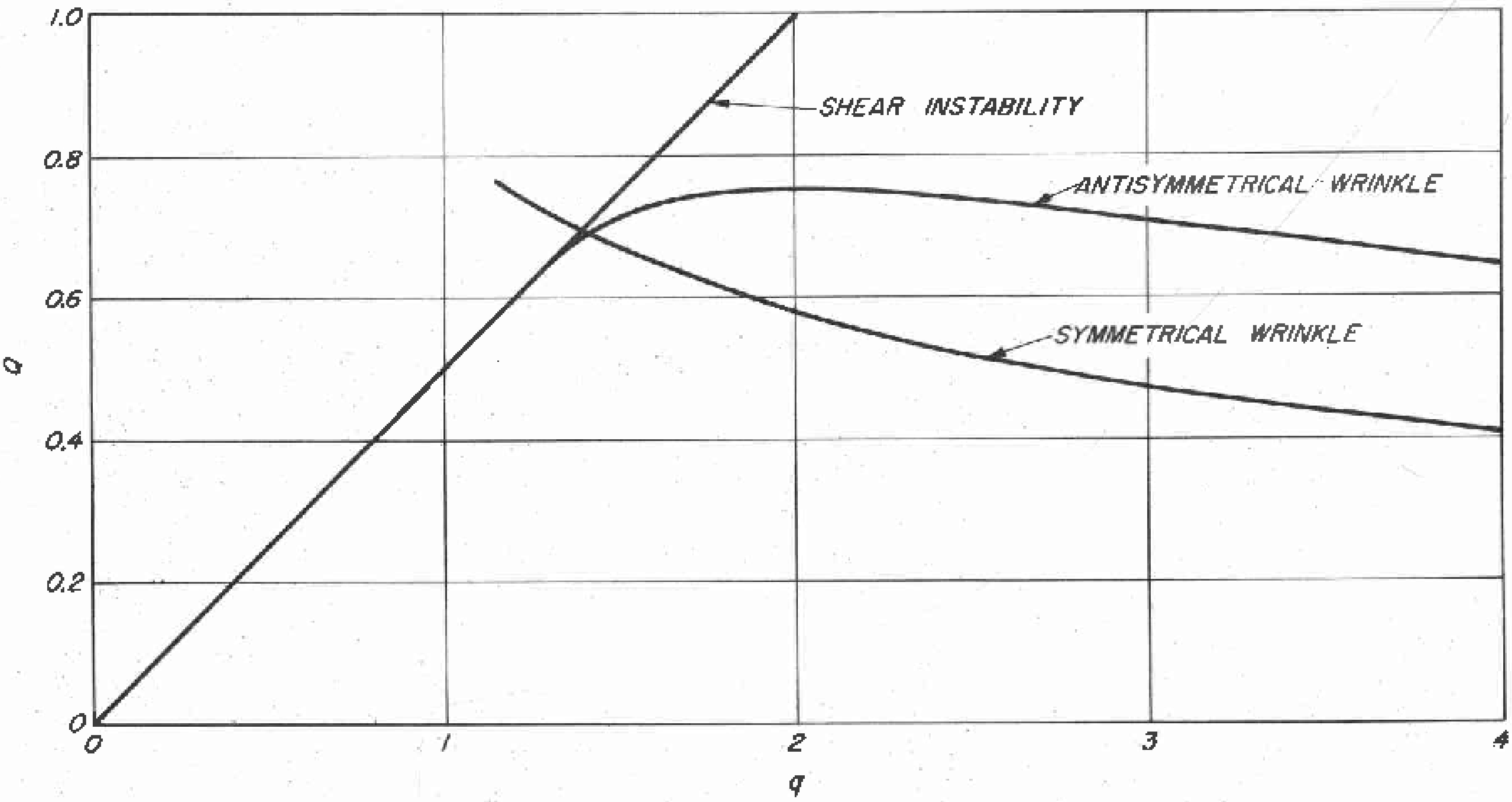

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Figure 2. --Critical wrinkling curves for sandwich constructions involving core materials having small values of κ.

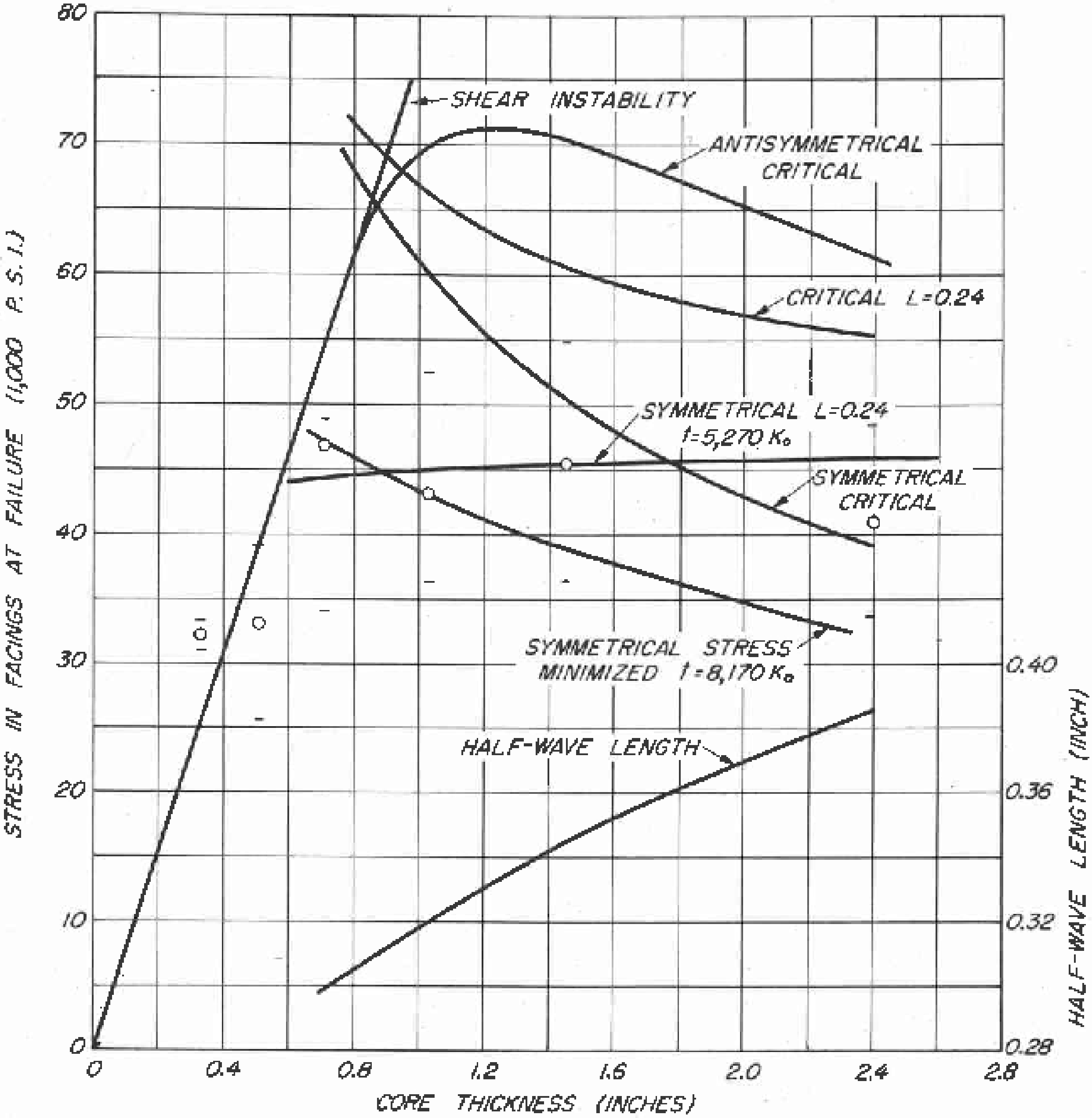

Figure 3.--Analysis of results from sandwich constructions having core A.

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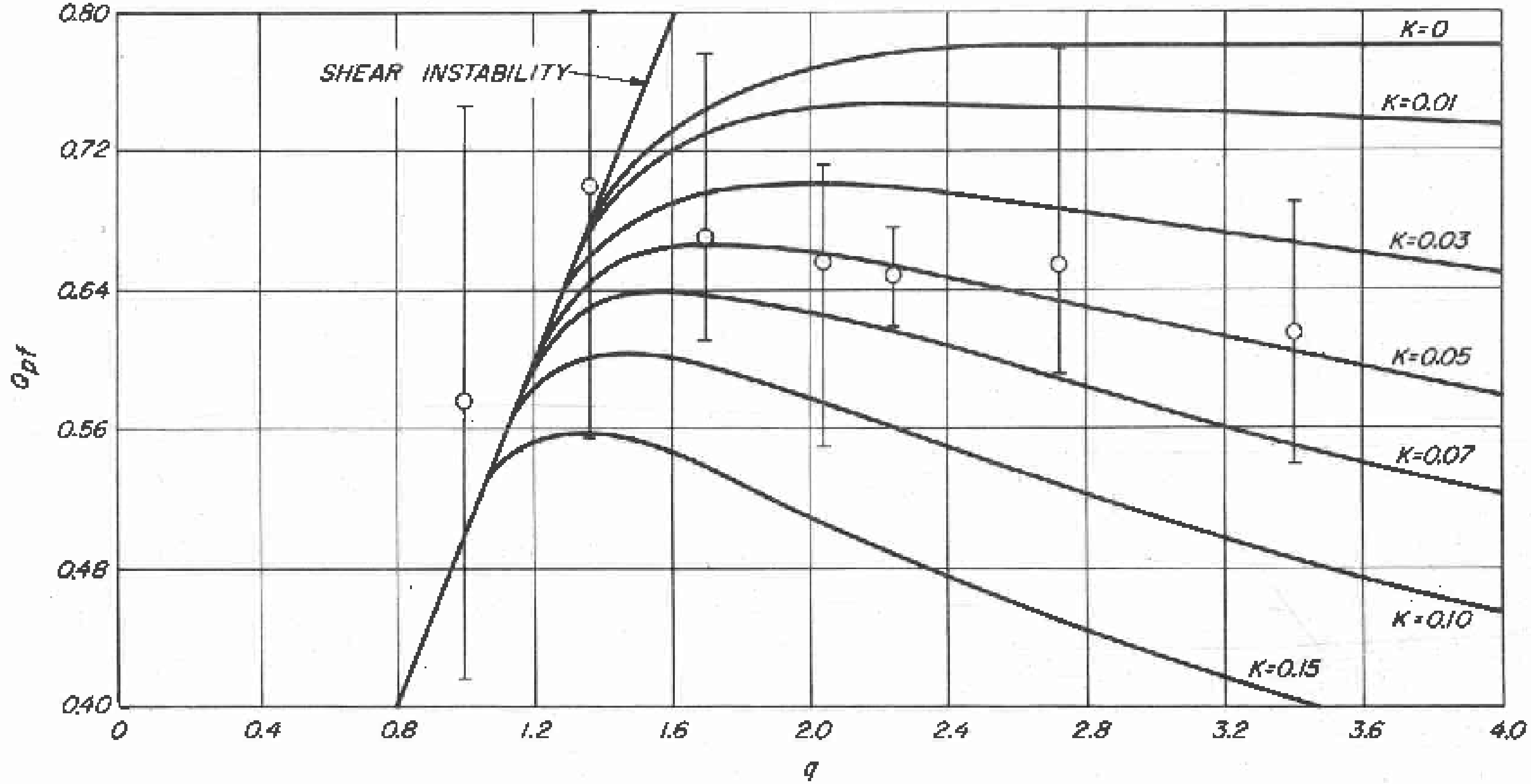

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Figure 4. --Data of sandwich specimens having core B plotted on curves of figure 16 of Forest Products Laboratory Report No. 1810.

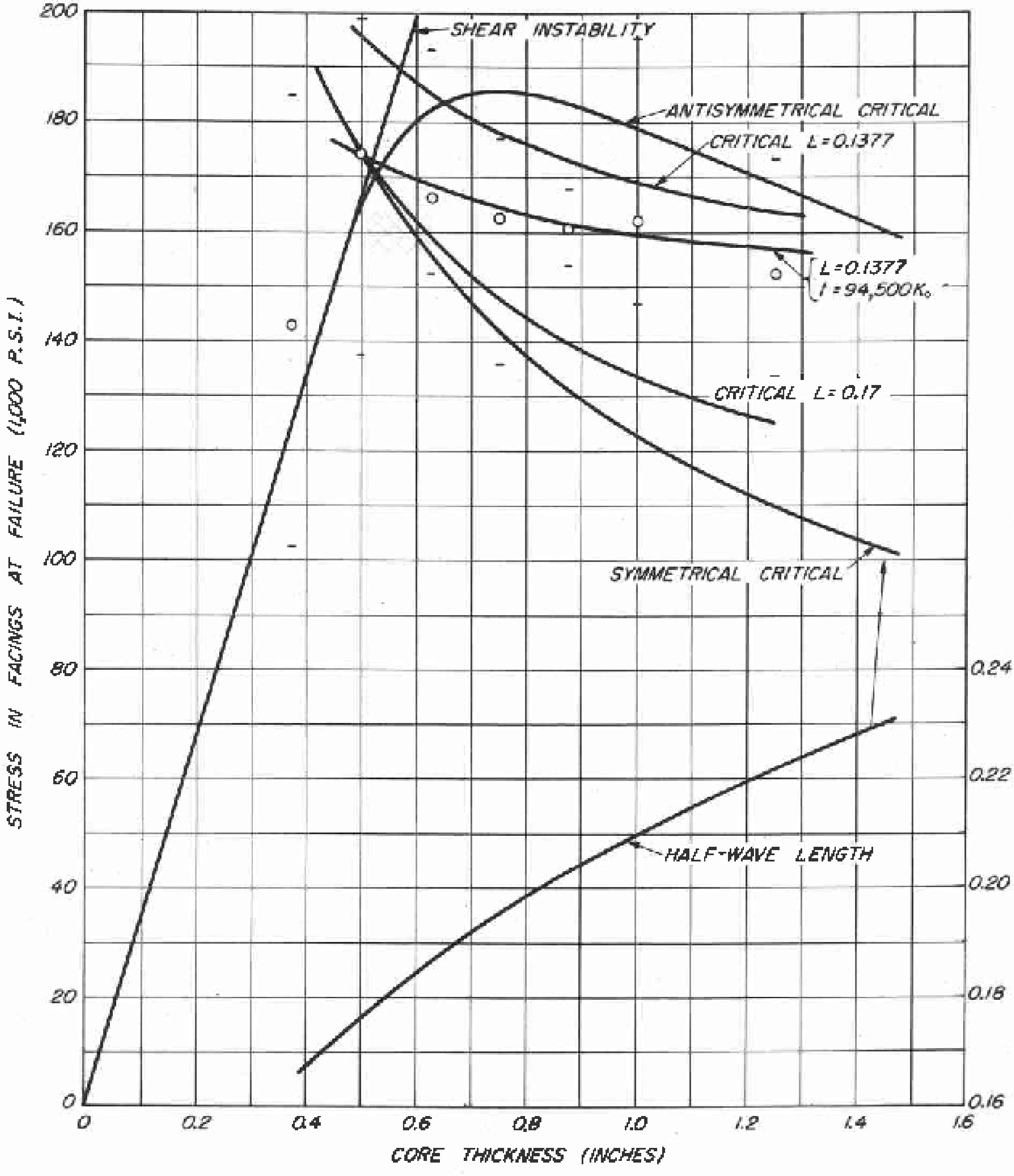

Figure 5. --Analysis of results from sandwich constructions having core B.

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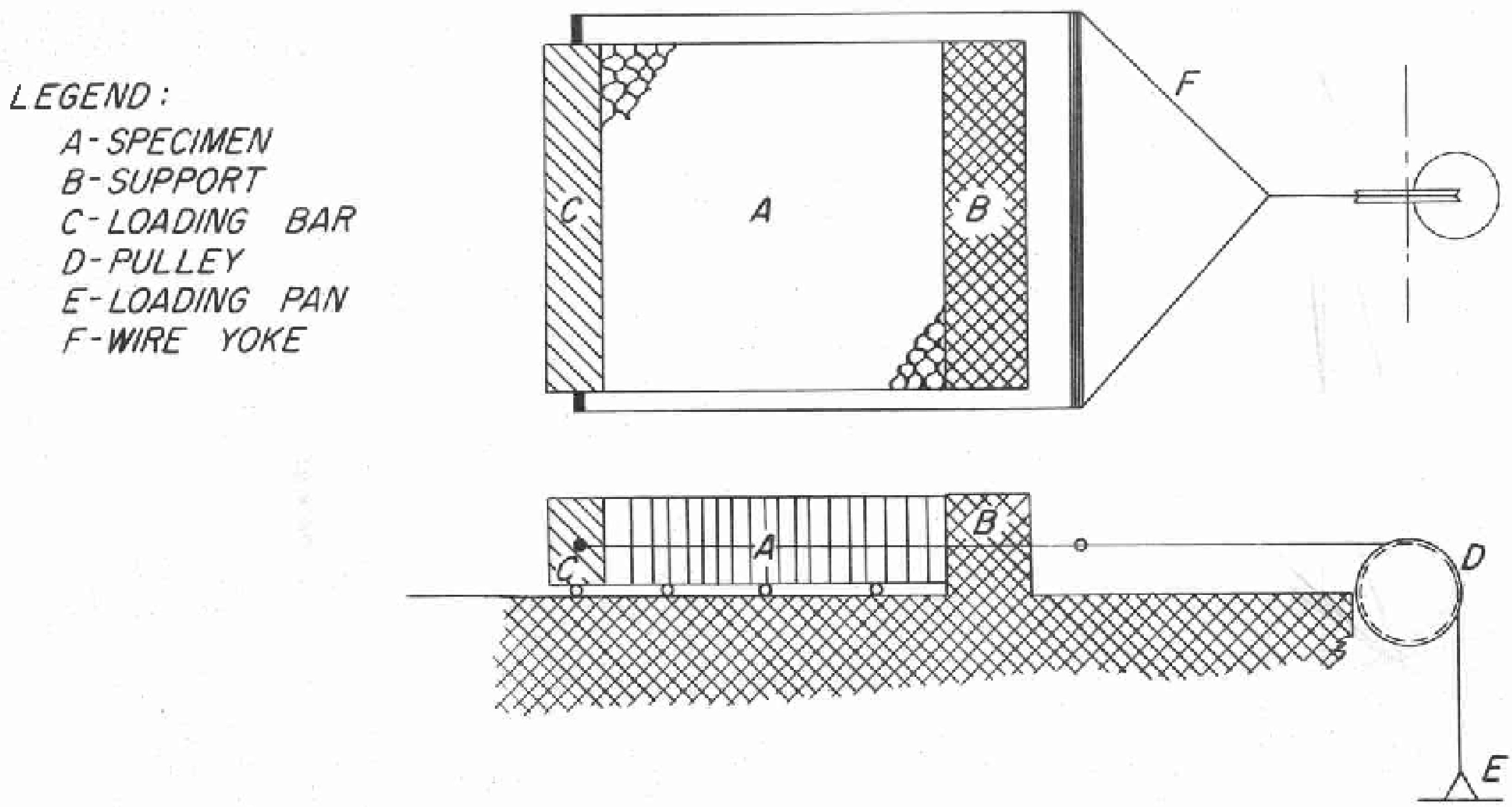

Figure 6. --Schematic diagram of test apparatus for determining Poisson's ratio for 1/2- by 2- by 2-inch honeycomb core specimens.

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