TEMPERATURES OBTAINED IN TIMBERS WHEN THE SURFACE TEMPERATURE IS CHANGED AFTER VARIOUS PERIODS OF HEATING

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TEMPERATURES OBTAINED IN TIMBERS WHEN THE SURFACE TEMPERATURE IS CHANGED AFTER VARIOUS PERIODS OF HEATING*

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Introduction

Extension studies have been made at the Forest Products Laboratory to determine the rate of temperature change in both round and sawed timbers when they are heated in different mediums and also the more important variables that affect the rate of heat transfer. Results of these investigations have been discussed in various publications (1 to 11).[2]

In previous papers (1-6, 8-11) discussing the rate of temperature change in wood, time-temperature curves have been presented that show the approximate temperature obtained at different points within a timber when a given surface temperature is applied for any definite heating period. It is often desirable, however, to know what temperatures will be obtained at any given time when the surface has been subjected to a higher or lower temperature during the period under consideration.

The purpose of this paper is to show how the approximate wood temperature may be determined at any time after the surface temperature is changed and to discuss the various fields in which this information may be used.

Factors Affecting the Rate of Temperature Change

The following factors were found to have an important effect on the rate of temperature change: (1) heating medium, (2) moisture content; (3) direction of grain in which heat movement takes place, and (4) density or specific gravity.

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[2] Numerals underlined in parentheses refer to publications named in the list of references at the end of this paper.

Experiments showed that under the same temperature, steam heated faster than liquids; liquids heated faster than hot plates, and hot plates heated faster than dry air. Water heated faster than oils, and the rate of heating in air was increased as the humidity was increased. These differences were apparently caused by variations in surface contact, specific heat of the heating medium, and similar factors.

Wood seasoned below the fiber saturation point (about 30 percent moisture) heated slower than green material. At and above the fiber saturation point, however, variations in moisture content do not appear to cause any further important changes in the rate of heating. It may, therefore, be assumed that green timbers will heat at about the same rate (for any particular species) when the moisture content is at or above the fiber saturation point. For practical purposes in making temperature calculations, timbers with an average moisture content below 20 percent may be considered as seasoned, while those with a moisture content above 20 percent may be considered as green. In general, green material will heat about 10 to 25 percent faster than seasoned wood under the same conditions of heating.

There is apparently no significant difference in the rate of heating in the radial and tangential directions, but the rate of heating is about 2-1/4 to 2-3/4 times as fast in the longitudinal direction, or along the fibers, as in the transverse direction. Longitudinal heating does not need to be taken into consideration, however, unless the timbers are exceptionally short and the cross-sectional area is large in proportion to the length. For example, the effect of end heating on the temperature at the mid-portion of a timber 4 by 4 inches in cross-section and 4 feet long would not need to be considered; but if a timber of the same length and having cross-sectional dimensions of 12 by 12 inches were heated, the effect of end heating would be important, except for short heating periods.

The density, or specific gravity, of the wood affects both the conductivity (rate at which heat units pass through the wood) and the diffusivity, which is a measure of the rate of temperature change. The diffusivity, which may be considered a constant over normal ranges of temperature, varies inversely as the product of the density and specific heat, and directly as the conductivity. Heat conductivity is, in general, analogous to electrical conductivity, while temperature difference is analogous to voltage. Through a misunderstanding of the differences between heat conductivity and the rate of temperature change, it has been assumed by some that the heavier woods that conduct heat more rapidly will also heat faster or cool faster than the lighter woods that are known to have better insulating properties. This is contrary to what actually takes place, for although the lighter or low density woods are not such good heat conductors as the heavier woods, the rate of temperature rise when the wood is heated, or the rate of temperature drop in cooling, is more rapid in timbers of the lower density species.

Temperature Changes That Occur When Wood Is Cooled After Heating or When the Heating Medium Temperature Is Increased During the Heating Period

When the temperature of the central portion of a timber is lower than the surface temperature applied during the heating period, a variable amount of temperature rise occurs at the interior (a) when the material is removed from the heating medium and exposed in the air at atmospheric temperature, (b) when it is in contact with the same medium as that used in heating but at a lower temperature, or (c) when it is placed in some other medium at a lower temperature. For example, in the application of the hot and cold bath treatment the partially-heated timbers are either cooled in a bath of the same preservative maintained at a lower temperature, or allowed to remain in the preservative while both the wood and preservative cool. When they are allowed to remain in the preservative the rate of cooling is slower and the average cooling temperature, over a given period, is higher than when the hot preservative is replaced with preservative at a lower temperature. Under these conditions the temperature rise at the interior of the timber would be higher than when the wood is cooled at a lower surface temperature for the same period.

On the other hand, when timbers are heated in steam, liquids, or in air as in the dry kiln, a variable amount of time is usually required to raise the heating medium temperature to the maximum. Again in some heating operations it may be desirable to maintain a lower heating temperature for a time and then increase the temperature for a final period of heating. This will necessarily have a variable effect on the final wood temperature depending on the time the different temperatures are applied.

Field of Application

In the preservative treatment of wood it is important to know whether the temperatures and heating periods employed have been sufficient to sterilize the interior of the timbers if there is danger that infection is present.

Laboratory experiments have shown that the temperature required to kill wood destroying fungi will depend both on the temperature and on the time the temperature is applied (12). As a result of these experiments it was concluded that it is not practical to attempt to sterilize wood by heating it to temperatures lower than 150° F., since the most resistant fungus tested was not killed even after being heated at 140° F. for 12 hours. All specimens were sterilized, however, when the wood was maintained at a temperature of 150° F. for about 1-1/4 hours, and progressively shorter periods were required as the temperature was increased.

When laminated timbers are glued they are commonly heated in kilns to a temperature that is sufficient to set the glue on all surfaces to which the glue is applied. Under these circumstances it is necessary to consider both the temperature and the time that the temperature is maintained. For such conditions it is of interest to know what temperature should be obtained at the center before heating is discontinued, or to determine the rate of cooling (if the wood is heated somewhat above the specified minimum temperature) so that the length of the required heating period can be determined. The laminated timbers are sometimes heated throughout to the temperature of the heating medium and when this is done it may be desirable to compute the approximate rate of cooling so that sufficient time can be allowed for setting of the glue.

Again, in the steaming and vacuum process the time the vacuum is applied should be limited by the temperature of the wood in the region where moisture is to be removed. When the temperature is too low to evaporate moisture effectively at the corresponding vacuum temperature, the vacuum should be discontinued to avoid unnecessary cooling of the wood, since the higher wood temperatures are more favorable for treatment. The approximate temperature changes that take place at any given distance from the surface may be readily computed for various vacuum periods, by the methods to be discussed.

Factors Affecting Temperature Change During Cooling Period

The amount of temperature rise that occurs at the interior when partially heated timbers are cooled will depend upon such factors as the minimum temperature reached at the center before cooling starts, the temperature of the heating medium (assumed as the surface temperature), the average cooling temperature, and whether the cooling medium is the same as that used in heating. Under the same temperature conditions, timbers heated or cooled in air at a normal humidity will generally change temperature more slowly than in a liquid or in an atmosphere at high humidity, such as can be maintained in a dry kiln. Radiation and evaporation of moisture will naturally have an important bearing on the rate of cooling in gases or in a vacuum. Cooling will also progress more slowly in still air or in a liquid that is not circulated during the cooling period since the heat is carried away more slowly under such conditions. For the same reason, timbers will cool more slowly when close piled while cooling as, for example, when left on trams after removal from the treating cylinder.

A slow rate of cooling because of a gradual drop in the surface temperature, or cooling at high surface temperatures, naturally favors a greater rise in temperature at the interior during the cooling period when the timbers have not been heated throughout to the maximum surface temperature.

Timbers allowed to cool in the heating medium to any given surface temperature while the temperature of the medium is gradually reduced may, for practical purposes, be assumed to be cooled at a temperature that is the average between the initial and final temperature of the heating medium for the cooling period in question.

If timbers are first heated at a surface temperature U_b for a given time T_1 and are then further heated at a lower temperature U_2 for a time $T-T_1$ (where T is the total heating period in hours), it is evident that the temperature U_2 may be considered as a cooling temperature of U_2 degrees applied at the surface.

Temperature Computations

The same procedure can be followed in making temperature calculations whether the final temperature applied at the surface is higher or lower than the initial heating temperature. The method of making these computations after the surface temperature has been changed will be discussed in the following section.

Method of Computing Temperatures

As has been mentioned in earlier papers (1 to 6 and 8 to 11, inclusive) the rate of temperature change in a timber of any particular dimensions depends upon the diffusivity, which may be considered a constant for normal ranges of temperature. Average values of this factor have been determined in the laboratory experiments for woods of different densities heated in different heating mediums.

Figure 1 shows the relation of diffusivity and specific gravity for green wood heated in steam and for air-seasoned and green wood heated in creosote. Data for wood heated in water were not complete enough for plotting curves on this figure, but the data obtained indicate that it should be sufficiently accurate to assume green wood heated in water will heat about 90 percent as fast as when steam is used as the heating medium. In other words, the heating periods should be increased about 11 to 12 percent over those required for heating with steam at the same temperature, to obtain a given temperature at any particular point in the timber.

In previous publications (1 to 6 and 8 to 11 inclusive) time-temperature curves have been shown for both round and sawed timbers when heated by different methods with the surface temperatures assumed as constant during the heating period. The basic formulas for making these computations were given in several of these publications (1,2,10 and 11). The present paper, however, discusses the rate of temperature change that occurs at any time after the surface temperature is changed from the initial heating temperature U_b to a different surface temperature U_2 where U_2 may be either lower or higher than U_b.

Equation 3, shown in the appendix, was derived for computing the temperature at any particular point in round timbers. Similarly, equation 6 was derived for computing the temperature in sawed timbers.

Definitions of the symbols used and a list of simple algebraic formulas derived from equations 3 and 6, that can be used with data from the curves, are also given in the appendix.

Calculations involving the use of the basic equations 3 and 6 are tedious and require a large amount of work because of the type of equations employed and the number of variables that must be taken into consideration. The simple algebraic formulas given in the appendix and indicated by letters for the purpose of reference, may be readily used in conjunction with temperature data taken from the proper time-temperature curves shown in figures 2 to 5, inclusive. This provides a simple method of making temperature computations and avoids the more difficult calculations from the basic equations 3 and 6. For example, by using equation (A) (Appendix) only a simple arithmetical calculation is needed to find the temperature U_c at any time T, where U_c is the temperature at some particular point within a timber and T is the total time period under consideration. In using equation (A) it is merely necessary to read temperatures from the proper time-temperature curve for the timber of the cross section under consideration (figs. 2 to 5, inclusive).

It may be noted that equations 3 and 6 include the following factors:

(1) Initial wood temperature = U_0 (Taken as 60° F. in computing data for figures 2 to 5, inclusive)

(2) Temperature of the heating medium during the initial heating period = U_1 (Taken as 200° F. in computing data for figures 2 to 5, inclusive)

(3) Diffusivity factor for transverse heating = h^2 (Taken as 0.00025 in computing data for figures 2 to 5, inclusive)

(4) Heating period t_1 (in seconds) = first heating period when surface temperature U_1 is applied.

(5) Total heating period t (in seconds), which includes both the time that the first surface temperature U_1 and the time that the second surface temperature U_2 is applied. The surface temperature U_2 is, therefore, applied for the time $t-t_1$.

(6) a = radius of round timbers (equation 3), a and b = width and thickness of sawed timbers (equation 6).

(7) r = distance of point p from the center of round timbers and (x,y) = coordinates of point p in sawed timbers. In both cases, p is the point at which the temperature is to be determined.

Equations 3 and 6 each contain two converging series. The first series within the brackets is the same as that used in computing the time-temperature curves in figures 2 to 5, inclusive, for any period T when the surface temperature is U_1 (1, 2). Similarly, the second series within the brackets is the same as that used in computing the data for the curves in figures 2 to 5, inclusive, when the heating period is $T-T_1$. If the two series enclosed within the brackets (which involve the variables, time, dimensions, and diffusivity) are designated as (x_1) and (x_2), respectively, then (x_1) and (x_2) may be computed from the data plotted in figures 2 to 5 when the time T_1 and T are assumed. It is from this relation that equation (A), appendix, is derived. If (x_1) and (x_2) are substituted for these convergent series, equation (3) for round timbers could be written,

$$U_c = U_2 + 2(U_a - U_b)(x_1) + 2\cdot(U_b - U_2)(x_2)$$

Similarly, equation (6) for sawed timbers could be written,

$$U_c = U_2 + (U_a - U_b)\frac{16}{\pi^2}(x_1) + (U_b - U_2)\frac{16}{\pi^2}(x_2) \ .$$

In this case, $(x_1) = \left[\frac{(U_m - U_1)}{K(U_0 - U_1)}\right]$ where U_m is the temperature read from the curve for the timber in question after heating for time T. $U_0 = 60°$ F., $U_1 = 200°$ F., and $K = 2$ for round timbers and $= \frac{16}{\pi^2}$ for sawed timbers. Substituting the numerical values given:

$$(x_1) = \left[\frac{(200 - U_m)}{K(200 - 60)}\right]$$

Likewise $(x_2) = \left[\frac{(U_n - U_1)}{K(U_0 - U_1)}\right] = \left[\frac{(200 - U_n)}{K(200 - 60)}\right]$ where U_n is the temperature read from the curve for the timber under consideration after heating for time $(T - T_1)$.

If (x_1) and (x_2) are substituted in equation (3) for round timbers,

$$U_c = U_2 + 2(U_a - U_b)\left[\frac{(200 - U_m)}{2(200-60)}\right] + 2(U_b - U_2)\left[\frac{(200 - U_n)}{2(200-60)}\right]$$

$$= U_2 + \left[\frac{(U_a - U_b)(200 - U_m) + (U_b - U_2)(200 - U_n)}{140}\right]$$

Substituting the corresponding values of (x_1) and (x_2) in equation (6) for sawed timbers,

$$U_c = U_2 + (U_a - U_b)\frac{16}{\pi^2}\left[\frac{(200 - U_m)}{\frac{16}{\pi^2}(200-60)}\right] +$$

$$(U_b - U_2)\frac{16}{\pi^2}\left[\frac{(200 - U_n)}{\frac{16}{\pi^2}(200-60)}\right] = U_2 + \left[\frac{(U_a - U_b)(200 - U_m) + (U_b - U_2)(200 - U_n)}{140}\right]$$

which is the same general form as that shown for round timbers.

Both of the foregoing expressions evidently reduce to the form,

$$U_c = U_2 + \left[\frac{U_m(U_b - U_a) - 200(U_2 - U_a) - U_n(U_b - U_2)}{140}\right] \quad \text{.........(A)}$$

which applies for both round and sawed timbers and requires only the substitution of numerical values of the temperatures indicated. It should be borne in mind that U_m and U_n are read from figures 2 to 5 for the heating periods of T and $T-T_1$, respectively, while U_2, U_a or U_b may be any assumed temperatures.

This formula (A) may be used in conjunction with the curves in figures 2 to 5 inclusive, for finding the wood temperature U_c when the initial wood temperature U_a, and surface temperatures U_b and U_2 are assumed. Illustrative examples showing the application of formula (A) will be given later. Figures 2 to 5 (from which values of U_m and U_n are determined for heating periods of T and $T-T_1$ hours) give time-temperature curves for the center and for a point midway between the center and surface of both round and sawed timbers of the dimensions indicated. These curves show the rate of heating when the surface temperature of 200° F. is applied for any time T. Additional time-temperature curves for distances of 1/2, 1, 2, 2-1/2, 3, 3-1/2, and 4 inches from the surface and at the center of round timbers and for various distances from the surface to the center of sawed timbers are given in a previous paper (6). Values of U_m and U_n may also be read directly from the curves in the latter paper for any of the distances from the surface mentioned, since they were prepared using the same diffusivity (0.00025), the same initial wood temperature (60° F.), and the same heating-medium temperature (200° F.) as were used for computing the curves shown in figures 2 to 5.

Rate of Cooling

Figures 6 to 19, inclusive, show values of U_c (the change in temperature during cooling) computed by means of equations (3) and (6) of the appendix, when the cooling temperature U_2 is assumed as 80° F. These curves have been plotted as a guide in showing what temperatures should be obtained at the center of both round and sawed timbers before cooling starts to permit a subsequent rise in temperature that will be sufficient to sterilize the wood or to insure that it will reach or stay above a given temperature for a definite period. The curves also show the time required to reach the maximum temperature at the central portion of timbers of different dimensions after the surface heating has been discontinued and cooling has started. In addition, they show the time the temperature is at or above any temperature below the maximum reached.

While it is desirable to consider the approximate diffusivity of the wood when calculations are made to determine the time T or T_1 required to heat any particular point in a timber to a given temperature, a consideration of diffusivity is usually of less importance when the wood is cooled. If for example, the center is initially heated to any particular temperature lower than the surface temperature, the maximum temperature rise that occurs will be the same, regardless of the diffusivity. The effect of differences in diffusivity in this case is merely to change the rate of heating and cooling at various points in the timber. In other words, variations in diffusivity will affect the time T and T_1 required to obtain any given temperature under the heating or cooling conditions assumed.

The curves shown in figures 6 to 19 were computed on the assumption that the heating and cooling mediums were the same. When the wood is cooled in air or under other conditions that are favorable for a slower rate of cooling, the rise in temperature should be somewhat higher than that indicated by the curves shown in the figures.

In general, it may be assumed that these curves represent the minimum increase in temperature that would occur during cooling when the center has been heated for the period T_1 with the surface maintained at the temperature U_b. The temperatures obtained in the heating period T_1 are shown at the point where the curves start, which is the time when the cooling period is zero. Although the curves in figures 6 to 19 were based on a cooling temperature of 80° F. a variation of a few degrees higher or lower than 80° would not have a significant effect on the temperature changes at various periods of cooling. Since the rate of cooling will in many cases be somewhat slower than indicated, this provides for a reasonable amount of variation in cooling temperatures. The effect of applying different cooling temperatures can be easily investigated by means of equation (A).

The purpose of figure 6 is to illustrate the temperature distribution between the center and surface of a timber after various periods of cooling and to show the effect of using different heating temperatures. A 10-inch diameter timber has been assumed with the center heated to 135° F. before cooling starts. In figure 6A the heating medium temperature U_b was taken as 200° F., while in figure 6B it was taken as 225° F. In both cases the initial wood temperature was taken as 60° F.

The temperature distribution from the surface to the center at the time cooling starts is shown by the broken line curve from the surface temperature to the center temperature of 135° F. The curves are shown for only one half the diameter, since the temperature distribution is considered symmetrical with respect to the center. Although the temperature curves in figure 6 are plotted for a 10-inch diameter timber, the temperature distribution would be the same, at the same proportional distance from the surface, for a timber of any other diameter. For example, the temperature at a distance of 1 inch from the surface of the 10-inch diameter timber is about 200° F. (fig. 6B), as indicated by the broken line curve. The same temperature would be obtained in a 16-inch diameter timber at the same proportional distance of $\frac{1}{10} \times 16$ or 1.6 inches from the surface, assuming the same wood and that the center temperatures are the same (in this case 135° F.). For a timber 8 inches in diameter the temperature of 200° F. (fig. 6B) would likewise be at a distance of $\frac{1}{10} \times 8$ or 0.8 inch from the surface when the center temperature is 135° F.

Temperature Changes in Round Timbers

When longitudinal heating can be neglected, as in most timbers of commercial size, the symmetrical form of round sections, such as poles and piling, makes it possible to compute the temperatures in timbers of any diameter when the temperature distribution is known for any one diameter.

The time required to reach the same temperature at the same proportional distance in any two timbers of diameters D_1 and D_2 inches will be directly proportional to the squares of the diameters or squares of the radii. To illustrate, assuming the same species and same heating conditions, if T_{10} represents the time required to reach a given temperature in a timber 10 inches in diameter and T_{14} is the time required to reach the same temperature at the same proportional distance from the surface of a timber 14 inches in diameter, then

$$\frac{T_{14}}{T_{10}} = \frac{196}{100} \text{ or } T_{14} = 1.96\, T_{10}.$$

This relation will hold for the data plotted in figures 2 and 3 and also for those in figures 6 to 14, inclusive.

From figure 6 it may be noted that the maximum temperature rise is at the center and the amount of temperature rise decreases rapidly as the surface is approached. The effect of using higher surface temperatures is shown by comparing the temperature distribution curves in figure 6A with those in figure 6B. Although the center temperature in both cases is the same, all intermediate temperatures are higher when the higher heating-medium temperature is employed. The rise in temperature during cooling is also higher when the higher heating-medium temperature is used.

Figure 7 shows cooling curves for the center of timbers 8, 10, 12, 14, 16, and 18 inches in diameter when heating temperatures of 200°, 225°, and 250° F. are used. The initial wood temperature was taken as 60° F. and the diffusivity as 0.00025, the values used for plotting the data in figures 2 to 5, inclusive. In computing the curves for figure 7 the center was assumed to be heated to a temperature that would rise (after cooling started) to slightly over 150° F. The required heating period was determined by assuming U_c = 151° F., by taking U_n at the time of cooling ($T-T_1$) when U_c would be approximately a maximum as indicated by figures 8 to 14, inclusive, and then solving for U_m from equation A. In this case, U_m is determined by using equation B derived from formula (A) (appendix). The method of finding U_m, the temperature U_x to which the point under consideration must be heated before cooling starts, and the corresponding heating period T_1 when the other temperature conditions are known or assumed, is illustrated in example 1.

Example 1

The following conditions will be assumed: diameter of timber = 15 inches; required temperature to be reached at the center after cooling starts = 155° F. = U_c; heating-medium temperature = 260° F. = U_b; initial wood temperature = 70° F. = U_a; cooling temperature = 65° F. = U_2. From figure 7C, in which the heating conditions are nearest to those assumed in this example, it is found that the maximum temperature for a timber 15 inches in diameter should be reached when the cooling period ($T - T_1$) is about 5 hours. Assuming first that ($T - T_1$) = 5 hours, U_n is found from figure 2 for a timber 15 inches in diameter to be about 71.5° F. Substituting in equation (B) of the appendix,

$$U_m = \left[\frac{200(65-70) + 71.5(260 - 65) + 140(155 - 65)}{(260 - 70)}\right]$$

$$= 135° \text{ F., approximately.}$$

From figure 2 it is found that a heating period (T) of about 13.4 hours is required to reach a temperature of 135° F. Since $T-T_1$ = 5 hours, T_1 = (13.4 - 5) or 8.4 hours. From figure 2 the temperature reached after heating for 8.4 hours is found to be about 99.5° F. when the surface temperature U_b = 200° F. and the initial wood temperature U_a = 60° F., which were the values used in computing the data for figures 2 to 5. The corresponding temperature U_x, obtained when U_b = 260° F. and U_a = 70° F. (assumed for this example) may be found from equation (C) of the appendix. From this equation,

$$U_x = 260 - \left[\frac{(260 - 70)(200 - 99.5)}{(200 - 60)}\right] = 124° \text{ F., approximately.}$$

The center would, therefore, reach a temperature of 124° F. after heating at 260° F. for 8.4 hours, and after cooling for 5 hours at 65° F. the center temperature would rise to 155° F. In order to determine whether approximately the maximum temperature is reached after cooling for 5 hours, cooling periods slightly less than and slightly more than 5 hours can be assumed, and the corresponding values of U_c can be determined in the same manner as for the 5-hour cooling period. For example, for a cooling period of 4-1/2 hours U_n = 68° F., approximately. Substituting in equation (B) gives U_m = 131° F. From figure 2 it is found that a heating period (T) of about 12.75 hours is required to reach this temperature. Substituting in equation (A) of the appendix,

$$U_c = 65 + \left[\frac{131(260 - 70) - 200(65 - 70) - 68(260 - 65)}{140} \right] = 155.4^\circ \text{ F.}$$

which is only slightly more than 155° F. obtained after 5 hours cooling. A check for a somewhat longer period than 5 hours would easily show whether the temperature might rise further with increase in cooling period. Since, however, there is only a slight variation in the values of U_c for cooling periods of 4.5 and 5 hours, it is evident that the maximum rise in temperature occurs after cooling for approximately 5 hours.

Since timbers of any diameter will have the same rise in temperature when the center temperature and the heating and cooling conditions are the same, curves similar to those shown in figure 7 could be plotted if desired, for various diameters with the curves starting at 124° F. The heating and cooling periods required for any given temperature will be proportional to the squares of the diameters as previously mentioned.

Figures 8 to 11, inclusive, show the computed rate of temperature change at the center and midway between the center and surface of a timber 10 inches in diameter when heated with surface temperatures of 200°, 220°, 240°, and 260° F. and then cooled at 80° F. after the center has been heated to different temperatures.

The approximate temperature changes for center temperatures within any of the 10-degree intervals shown may be easily determined by interpolating. For example, in figure 8 it is found that the maximum temperature rise is about 20° F. (140° - 120° F.) when the center has been heated to 120° F. When the center is heated to 130° F. the maximum temperature rise is about 17° F. (147° - 130° F.). If the center is assumed to be heated to 125° F., the maximum temperature during cooling at 80° F. would then be determined as about 1/2(147° + 140°) = 144° F., approximately. The rise in temperature that occurs after heating is discontinued naturally decreases as the difference between the heating temperature and interior temperature decreases. For example, when the center is heated to a temperature close to that of the heating medium, practically no rise in temperature will occur during the cooling period.

The heating periods required to reach the different center temperatures are shown on each figure. From the heating periods given, it is evident that the time required for heating the wood to any desired temperature is reduced rapidly as the surface temperature U_b is increased. It may also be noted that when the center of a timber is heated to a given temperature before cooling starts, by using different heating medium temperatures, the maximum rise in temperature during the cooling period is higher for the higher heating temperature.

The curves in figures 8 to 11 that are plotted for a timber 10 inches in diameter will apply equally well for any other timber of diameter D inches, since the time required to reach any temperature shown will be proportional to the squares of the diameters. For example, in figure 8 the temperature at the center of a 10-inch diameter timber before cooling starts is shown as approximately 135° F. after cooling for 3 hours when the center has been heated to 120° F. In order to reach the same temperature at the center of a timber 14 inches in diameter a cooling period of $\frac{196}{100} \times 3$ or 5.88 hours would be required for the same wood and same heating conditions. This is the temperature shown in figure 13 for a 14-inch diameter timber after cooling for 5.88 hours.

For convenience the multiplying factor is shown in table 1 for timbers ranging in diameter from 8 to 18 inches.

To illustrate, figure 9 shows that after the center of a timber 10 inches in diameter has been heated to 130° F. with the surface temperature of 220° F., the maximum rise in temperature is about 21° F., which is reached after cooling for about 2 hours. If a timber of the same wood and 16-1/2 inches in diameter were heated at the center to 130° F. under the same heating conditions and then cooled at 80° F., the time required to reach the same maximum temperature rise would be computed as $2 \times 2.72 = 5.44$ hours.

If the diffusivity is other than 0.00025, the required time is readily found from the proportional relation shown by equation (E) of the appendix where h_a^2 represents any given diffusivity. Table 2 has been prepared so that the ratios, $\frac{0.00025}{h_a^2} = F_1$ and $\frac{h_a^2}{0.00025} = F_2$, are multiplying factors. For example, $T_x = F_1 T_h$ and $T_h = F_2 T_x$ in equation (E) or $T_D = fF_1 T_d$ and $T_d = \frac{1}{f} F_2 T_D$ in equation (G) appendix. These factors will cover a sufficient range of diffusivities for general purposes.

The relation of diffusivity, diameter of timber, and heating period is expressed by equation (G) of the appendix. This equation will be found convenient to use with the multiplying factors "f" given in table 1 and F_1 or F_2 given in table 2.

Example 2 will illustrate how to compute the temperature at any time T when the heating conditions, cooling temperature, and diffusivity are all different from those assumed in computing the various time-temperature curves.

Example 2

The following conditions will be assumed: Diameter of timber, D = 16.5 inches. Center is to be heated to a temperature of 125° F. before cooling starts = U_x. Cooling temperature U_2 = 5° F. Diffusivity of wood h_a^2 = 0.00028. Initial wood temperature U_a = 70° F. Heating medium temperature U_b = 235° F. Cooling period $T - T_1$ = 5 hours.

Find the required heating period T_1 and the center temperature after cooling for 5 hours.

The first step is to find U from equation (D) of the appendix. Substituting in this expression gives,

$$U = 200 - \left[\frac{(235 - 125)(140)}{(235 - 70)}\right] = 106.7° \text{ F.}$$

This is the temperature that would be reached at the center in time T_1 if the heating conditions were the same as those assumed in computing the temperature curves plotted in figure 2. Although the time temperature curve for the center of a timber 16-1/2 inches in diameter is not plotted in figure 2, the data for the 10-inch diameter timber can be conveniently used. In this case the multiplying factor f is $\frac{(16.5)^2}{100}$ = 2.72 as shown in table 1. From figure 2 the time required to reach a temperature of 106.7° F. in a 10-inch diameter timber is found to be about 4.15 hours (= T_d in equation (G), appendix). This is the time required to reach a temperature of 125° F. at the center under the heating conditions assumed in the example when the diffusivity is 0.00025. Since, however, the diffusivity is taken as 0.00028, by substituting in equation (G) of the appendix

$$T_D = T_1 = (2.72)\left(\frac{0.00025}{0.00028}\right)(4.15) = (f)(F_1)(T_d) = 10.1 \text{ hours, approximately.}$$

This is the value of T_1 which was to be determined.

The next step is to find the temperature U_c after cooling for 5 hours at a surface temperature of 5° F. Before adjusting for differences in diffusivity, T_1 = (4.15)(2.72) = 11.3 hours. The cooling period of 5 hours is equivalent to a period of $\left(\frac{0.00028}{0.00025}\right)(5) = F_2(5)$ = 1.12(5) or 5.6 hours when the diffusivity is the lower value. Then T = 11.3 + 5.6 or 16.9 hours. The corresponding value of T for a timber 10 inches in diameter = $\frac{16.9}{2.72}$ = 6.21 hours. The value of U_m for this period is found from figure 2 to be about 139° F. Similarly, the value of U_n, for $\frac{5.6}{2.72}$ or 2.06 hours, = 69° F., approximately. Substituting in equation (A),

$$U_c = 5 + \left[\frac{139(235 - 70) - 200(5 - 70) - 69(235 - 5)}{140} \right] = 143° \text{ F.,}$$

approximately, which is the temperature to be found.

The following example will illustrate how to find U_m and the corresponding values of T_1 and U_x when U_c, U_2, U_b, U_a, the cooling period $(T - T_1)$, and h_a^2 are assumed.

Example 3

Assume the following conditions: U_c = 160° F.; U_b = 240° F.; U_a = 75° F.; $T - T_1$ = 6 hours; diameter of timber = 14.5 inches; U_2 = 10° F.; and h_a^2 = 0.00028.

Find T_1 and the temperature U_x to which the center must be heated before cooling starts.

The first step is to find the time T_h from equation (E) corresponding to a 6-hour heating period when h_a^2 = 0.00028. In this case, $6(0.00028) = T_h(0.00025)$ or $T_h = F_2(6) = 6.72$ hours. For a 10-inch diameter timber the corresponding period would be $\frac{6.72}{2.10} = 3.2$ hours. From figure 2 the temperature U_n for this heating period is found to be about 89° F. Substituting in equation (B), of the appendix,

$$U_m = \left[\frac{200(10 - 75) + 89(240 - 10) + 140(160 - 10)}{240 - 75} \right] = 172.5° \text{ F.}$$

From figure 2 it is found that a temperature of 172.5° F. is obtained in a 10-inch diameter timber in about 10.1 hours (=T) when the diffusivity is 0.00025. This = $(\frac{0.00025}{0.00028})(10.1) = 10.1(F_1) = 10.1(0.89)$ or 9 hours when the diffusivity is 0.00028. This time T must be multiplied by the factor "f" (table 1) which gives T = 2.1(9) or 18.9 hours for a timber 14.5 inches in diameter. Since $T - T_1$ = 6 hours, T_1 = 12.9 hours. The corresponding time T_1 for a 10-inch diameter timber = $\frac{12.9}{2.1}$ = 6.15 hours. This corresponds to a heating period of $6.15(\frac{0.00028}{0.00025}) = F_2(6.15)$ or 6.9 hours when the diffusivity is 0.00025. The temperature for this heating period is found from figure 2 to be about 146° F. This is the center temperature U, obtained in time T_1, which is 6.9 hours for a 10-inch diameter or (6.9) (2.1) = 14.49 hours for a 14.5-inch diameter timber when the surface temperature is taken as 200° F. and the initial wood temperature as 60° F.

For the heating conditions assumed, the temperature U_x is computed from equation (C) which gives, $U_x = 240 - \left[\frac{(240 - 75)(200 - 146)}{140}\right] = 176°$ F.

From these computations the required value of T for the 14.5-inch diameter timber is therefore 18.9 - 6 or 12.9 hours, and the temperature that must be obtained at the center before cooling starts is 176° F.

Since this temperature is higher than the temperature of 160° F. required at the center after cooling for 6 hours, it is evident that the center temperature is falling rather than rising after cooling for this period. An examination of figure 10 shows that the maximum temperature rise is reached in a 10-inch diameter timber in about 2 hours when the diffusivity is 0.00025 or $(2)(\frac{0.00025}{0.00028}) = F_1(2) = 0.893(2)$ or 1.79 hours when the diffusivity is 0.00028. For the 14.5-inch diameter timber the maximum temperature would then be reached after cooling for about (1.79)(2.1) or approximately 3.76 hours. Computations based on a cooling period of 2 hours for a 10-inch diameter timber (when the diffusivity = 0.00025) gives $U_m = 144°$ F. when $U_c = 160°$ F. The corresponding value of T = 6.65 hours and $T_1 = 4.65$ hours. For $T_1 = 4.65$ hours the temperature U = 115.5° F. and $U_x = 140°$ F. when $U_b = 240°$ F., the temperature assumed. For a diffusivity of 0.00028, $T_1 = 0.893(4.65)$ or 4.15 hours. When the timber is 14.5 inches in diameter $T_1 = (4.15)2.1$ or 8.72 hours.

Example 4

This example will illustrate the method of computing the temperature after various vacuum periods, when the steaming and vacuum treatment is employed. The following conditions will be assumed: steam temperature 260° F. = U_b. Steaming period 8 hours = T_1. Average temperature during vacuum period 150° F. = U_2. Length of vacuum period 2 hours = $T-T_1$. Diameter of timber 10 inches. Initial wood temperature = 60° F. Diffusivity factor $h_a^2 = 0.00030$.

Find the temperature U_c at a distance of 2-1/2 inches from the surface (which for this diameter is midway between the center and the circumference) at the end of the vacuum period.

The first step is to find the temperature obtained at 2-1/2 inches from the surface, at the end of the steaming period. Since the diffusivity = 0.00030, the equivalent steaming period T_1 will be $(\frac{0.00030}{0.00025})(8) = F_2(8)$ or 9.6 hours when the diffusivity is 0.00025, the value assumed in computing the temperature curves. From figure 3 it may be found that a temperature of about 180° F. would be obtained in 9.6 hours with the temperature conditions assumed in computing the plotted data. From equation (C) the corresponding temperature U_x is found to be about 231° F. when the surface temperature is 260° F.

The corresponding vacuum period $T-T_1$ is $\left(\frac{0.00030}{0.00025}\right)(2) = F_c(2) =$ 1.2(2) or 2.4 hours and the equivalent period T will be 9.6 + 2.4 = 12 hours. From figure 3 the temperature U_m, for T = 12 hours, is found to be about 188° F. and U_n, for $T-T_1$ = 2.4 hours, is 107° F. Substituting in equation (A)

$$U_c = 150 + \left[\frac{188(260 - 60) - 200(150 - 60) - 107(260 - 150)}{140}\right] = 206° \text{ F.},$$

which is the temperature desired. Points nearer the surface will, of course, lie between the surface temperature of 150° F. and the temperature of 206° F. The computed drop in temperature at a point midway between the center and surface at the end of the 2-hour vacuum period would then be 231° - 206° or 25° F.

Temperature Changes in Sawed Timbers

Figures 12 to 14, inclusive, show the computed rates of temperature change at the center of round timbers 8, 10, 12, 14, 16, and 18 inches in diameter when heated at 200° F. for various periods of time and then cooled at 80° F. Similar curves are shown in figures 15 to 18 for sawed timbers with cross-sectional dimensions of 7 by 9, 8 by 10, 10 by 10, 10 by 12, 12 by 12, 12 by 14, and 14 by 14 inches.

Figure 19 shows the effect of different diffusivity values on the rate of cooling. An 8- x 10-inch timber has been taken as an illustration when heating temperatures of 200°, 220°, 240°, and 260° F. are used. The center is assumed to be heated to 135° F. when cooling starts with the surface temperature maintained at 80° F. Although the rise in temperature is the same, for the same heating conditions, it may be noted that the rate of temperature change is more rapid as the diffusivity is increased.

Figures 4 and 5 show the rate of temperature change at the center and midway between the center and surface of rectangular timbers of various cross-sectional dimensions when the temperature of the heating medium is 200° F. As previously mentioned, temperature curves for various other distances from the surface to the center of sawed timbers are given in an earlier paper (6) and can be used in the same manner as illustrated for computations obtained at the center. While examples 1 to 4 illustrate the method of computing temperature changes in round timbers, the same procedure is to be used for rectangular timbers except that figures 4 and 5 are to be used for finding values of U_m, U_n, and the time T or T_1.

Since rectangular timbers vary in two dimensions, namely width and thickness, a proportional relation between the dimensions and the required heating period can be obtained only for square dimension timbers. In the case of square timbers the required heating period varies directly as the squares of the cross-sectional dimensions. If, for example, a timber "W" inches in width and thickness requires a heating period of T_W hours to reach a given temperature at any given point p inches from the surface, to obtain the same temperature at the same proportional distance from the surface of a timber "x" inches in width and thickness will require a heating period of $T_x = \frac{T_W(x^2)}{W^2}$ hours.

In some cases, as for example in gluing laminated timbers, the entire timber is heated to the temperature of the heating medium and is then gradually cooled over a period of time to allow the glue sufficient time to set properly at a temperature at or above the minimum required. Under such conditions the rate of cooling determines the time that the wood will be held at a temperature favorable for setting the glue.

When the entire timber is heated to the same temperature U_b throughout, the value of U_m that is to be substituted in equation (A) is the heating-medium temperature of 200° F. used in computing figures 2 to 5, inclusive. Equation (A) then becomes,

$$U_c = U_2 + \left[\frac{(200 - U_n)(U_b - U_2)}{140}\right]$$

This form will apply when a uniform temperature has been obtained at all points within the timber and is designated as equation (A_1) in the appendix. The following conditions will be assumed for an example illustrating the method of applying equation (A_1).

Example 5

Timber dimensions 8 by 10 inches. Wood temperature (entire timber) = 210° F. = U_b. Minimum temperature required for gluing 190° F. Diffusivity $h_a^2 = 0.00020$. Average temperature during cooling period = 150° F. = U_2. Find U_c after cooling for 2-1/2 hours. The diffusivity is 0.00020, hence the cooling period of 2-1/2 hours is equivalent to $2.5\left[\frac{0.00020}{0.00025}\right] = 2.5(F_2)$ or 2 hours when the temperature charts are used.

Since the center of the timber is at the lowest temperature during the heating period, the rate of cooling at this point is of principal interest. The rate of cooling at the center, however, will necessarily be the slowest, since this point is at the maximum distance from the surface.

From figure 4, U_n, for a 2 hour period, is found to be about 72° F. Substituting values in equation (A), which in this case takes the form shown in equation (A_1), $U_c = 150 + \left[\frac{(200 - 72)(210 - 150)}{140}\right] = 150 + 55$ or 205° F. which is 15 degrees higher than the minimum temperature of 190° F. necessary for setting the glue.

If, in examples such as the foregoing, it is desired to find the time that U_c will be any given temperature, it is merely necessary to solve for U_n in equation (A_1) and find $T-T_1$ from the proper chart among figures 2 to 5 for the timber under consideration. This gives

$$U_n = \left[\frac{200\ (U_b - U_2) - 140\ (U_c - U_2)}{U_b - U_2}\right] .$$

For example, assume it is desired to find the cooling period needed to reach a temperature of 190° F. (= U_c) when U_2 = 150° F. and U_b = 210° F. Substituting in the foregoing expression,

$$U_n = \left[\frac{200\ (210 - 150) - 140\ (190 - 150)}{(210 - 150)}\right] = 128° \text{ F., approximately.}$$

From figure 4 it is found that a temperature of 128° F. is reached at the center of an 8- by 10-inch timber in about 5.0 hours when the diffusivity is 0.00025. In example 5 the diffusivity h_a^2 was assumed as 0.00020. Substituting in equation (E) of the appendix,

$$T_x\ (0.00020) = 5.0\ (0.00025) = 5\ (F_1) \text{ or } T_x = 5(1.25) = 6.25 \text{ hours,}$$

which is the time to be determined. Since it was assumed that a temperature of 190° F. was necessary to set the glue, nothing would be gained by further cooling at 150° F.

Example 6

This example will illustrate an application of the data for computing the approximate temperature at any particular point in a timber when the temperature is gradually raised until the maximum is reached.

The following conditions will be assumed: Green Southern yellow pine timbers 10 by 12 inches in cross-section are steamed at a uniform temperature of 260° F. for 10 hours after the maximum temperature of 260°F. is reached. Two hours are taken to reach the maximum temperature; hence, the total heating period, T, is 12 hours. The initial wood temperature will be assumed as 75° F. = U_a. If a temperature recording instrument is used, the average temperature can be determined from the recording chart, but for convenience it will be assumed that the temperature is uniformly increased to the maximum. The average temperature over the 2 hour period will be taken as $\frac{75 + 260}{2}$ or about 168° F.

Since the wood is first assumed to be heated at 168° F. for 2 hours and then heated at 260° F. for 10 hours, U_b = 168° F., U_2 = 260° F., T_1 = 2 hours, T = 10 + 2 or 12 hours and $T-T_1$ = 10 hours.

The center will be taken as the point at which the temperature is to be determined at the end of the 12-hour period, and the diffusivity of the wood h_a^2 will be taken as 0.00030.

Since the curves in figures 2 to 5, inclusive are based on a diffusivity of 0.00025, the temperature U_m obtained at the end of 12 hours would be determined as that obtained in $12\left(\frac{0.00030}{0.00025}\right) = 12(F_2)$ or 12(1.2) = 14.4 hours when the diffusivity is 0.00025 (Equation (F), appendix). Similarly, the temperature obtained in 10 hours would be the same as that shown by the temperature curves for a heating period of $10\left(\frac{0.00030}{0.00025}\right) = 10(F_2)$ or 12 hours.

From figure 5, U_m for a heating period of 14.4 hours is found to be about 174° F. for a 10- by 12-inch timber, and the temperature U_n for a heating period of 12 hours is found to be about 163° F. Substituting in equation (A) of the appendix and solving for U_c gives

$$U_c = 260 + \left[\frac{174\ (168 - 75) - 200\ (260 - 75) - 163\ (168 - 260)}{140}\right] = 218° \text{ F.}$$

which is the temperature to be determined.

If the temperature of 260° F. had been applied for the full period of 12 hours (corresponding to 14.4 hours when the diffusivity is 0.00025), the temperature U_x would be computed from equation (C) of the appendix. In this case, U_b = 260, U_a = 75, and U = 174. Substituting these values in equation C,

$$U_x = 260 - \left[\frac{(260 - 75)\ (200 - 174)}{140}\right] = 226$$

or 8 degrees higher than when 2 hours are taken to reach the maximum temperature.

APPENDIX

Method of Computing Temperature Changes in Round and Sawed Timbers When the Surface Temperature is Changed After Any Given Heating Period

Round Timbers

In deriving an equation for computing the temperature U_c at any point r inches from the axis of round timbers it is necessary to solve the following partial differential equation:

$$\frac{\delta u}{\delta t} = h^2 \left[\frac{\delta^2 u}{\delta r^2} + \frac{1}{r} \frac{\delta u}{\delta r} \right] \quad \ldots\ldots\ldots\ldots\ldots (1)$$

In this equation h^2 is the diffusivity factor for heating in the transverse direction and t is the time in seconds.

The following notation will be used:

t_1 = time (in seconds) of first heating period when surface temperature is U_1.

t is the total time (in seconds) under consideration and includes both the time that the surface temperature U_1 is applied and the time that the second surface temperature U_2 is maintained. In this case $t-t_1$ will represent the second heating period when the surface temperature U_2, which may be either higher or lower than the first heating temperature U_1, is applied.

U_0 = initial wood temperature

U_c = temperature obtained at any given point p in a timber after time t.

a = radius of timber.

r = the distance from the center to point p.

Equation (1) must be solved to meet the following boundary conditions:

For first heating period when, $t \leq t_1$

Let $\theta = (U_c - U_1)$

$U_c = U_0$ when $t = 0$, then $\theta = (U_0 - U_1)$ when $U_c = U_0$.

$U_c = U_1$ when $r = a$ and $t \leqq t_1$, then $\theta = 0$ when $r = a$.

$$\theta = \sum_{n=1}^{\infty} A_n e^{-\lambda_n^2 h^2 t} J_o(\lambda_n r) \qquad \text{when } T \leqq T$$

$$A_n = \frac{2 (U_o - U_1)}{\lambda_n a J_1 (\lambda_n a)}$$

For second heating period

$\theta = (U_c - U_2)$ when $t > t_1$

$U_c = U_2$ when $r = a$ and $t > t_1$, then $\theta = 0$ when $r = a$. When $t > t_1$

$$\theta = \sum_{n=1}^{\infty} A_n e^{-\lambda_n^2 h^2 t} J_o(\lambda_n r) + \sum_{n=1}^{\infty} B_n e^{-\lambda_n^2 h^2 (t - t_1)} J_o(\lambda_n r) \quad \ldots\ldots (2)$$

where $B_n = \dfrac{2(U_1 - U_2)}{\lambda_n a J_1 (\lambda_n a)}$

By substituting values of A_n and B_n in (2), and $(U_c - U_2)$ for θ, the first few terms of the general equation for computing the temperature U_c at a given point r at any time t becomes,

$$U_c = U_2 + 2(U_o - U_1)\left[\frac{1}{\lambda_1 a J_1(\lambda_1 a)} e^{-\lambda_1 h^2 t} J_o(\lambda_1 r) + \frac{1}{\lambda_2 a J_1(\lambda_2 a)} e^{-\lambda_2^2 h^2 t} J_o(\lambda_2 r)\right.$$

$$\left. + \frac{1}{\lambda_3 a J_1(\lambda_3 a)} e^{-\lambda_3^2 h^2 t} J_o(\lambda_3 r) + \ldots\ldots\ldots\ldots\right]$$

$$+ 2(U_1 - U_2)\left[\frac{1}{\lambda_1 a J_1(\lambda_1 a)} e^{-\lambda_1^2 h^2 (t - t_1)} J_o(\lambda_1 r)\right.$$

$$\left. + \frac{1}{\lambda_2 a J_1(\lambda_2 a)} e^{-\lambda_2^2 h^2 (t - t_1)} J_o(\lambda_2 r) + \ldots\ldots\ldots\right] \ldots\ldots\ldots\ldots (3)$$

In the foregoing equations $J_o(\lambda_n r)$ and $J_1(\lambda_n a)$ are Bessel's functions of the zeroth and first order respectively. λ_n is a root of $J_o(\lambda_n a) = 0$, and e is the base of the Naperian logarithms.

Sawed Timbers

In the case of sawed timbers the following differential equation must be solved for computing the temperature change U_c at any point (x,y).

$$\frac{\delta u}{\delta t} = h^2 \left[\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} \right] \quad \text{.........................} \quad (4)$$

This is solved to meet the following boundary conditions:

For the first heating period when $t \leqq t_1$,

Let $Z = (U_c - U_1)$

$U_c = U_0$ when $t = 0$, then $Z = (U_0 - U_1)$

$U_c = U_1$ when $\left.\begin{matrix} x = 0, & y = 0 \\ x = a, & y = b \end{matrix}\right)$ and $t \leqq t_1$, then $Z = 0$ when $x = 0$ and a,

and $Z = 0$ when $y = 0$ and b.

When $t \leqq t_1$

$$Z = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{m,n} \left(\text{Sin}\,\frac{m\pi x}{a}\,\text{Sin}\,\frac{n\pi y}{b}\right) e^{-\pi^2 h^2 t \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)}$$

$$A_{m,n} = \frac{4(U_0 - U_1)}{ab} \int_0^a \int_0^b \text{Sin}\,\frac{m\pi\mu}{a}\,\text{Sin}\,\frac{n\pi\nu}{b}\,d\mu\,d\nu$$

$$= \frac{16(U_0 - U_1)}{mn\pi^2}$$

For second heating period

$Z = (U_c - U_2)$ when $t > t_1$

$U_c = U_2$ when $\left.\begin{matrix} x = 0, & y = 0 \\ x = a, & y = b \end{matrix}\right)$ and $t > t_1$, then $Z = 0$ when $x = 0$ and a,

and $Z = 0$ when $y = 0$ and b.

When $t > t_1$

$$Z = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{m,n} \left(\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\right) e^{-\pi^2 h^2 t \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)}$$

$$+ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{m,n} \left(\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\right) e^{-\pi^2 h^2 (t - t_1)\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)} \quad \ldots\ldots(5)$$

where $B_{m,n} = \frac{4(U_1 - U_2)}{ab} \int_0^a \int_0^b \sin \frac{m\pi\mu}{a} \sin \frac{n\pi\nu}{b} \, d\mu \, d\nu$

$$= \frac{16(U_1 - U_2)}{mn\pi^2}$$

Substituting values of $A_{m,n}$ and $B_{m,n}$ in (5), and $(U_c - U_2)$ for Z, the first few terms of the general equation for computing the temperature U_c at a given point (x,y) in a transverse section, at any time t becomes,

$$U_c = U_2 + (U_0 - U_1)\frac{16}{\pi^2}\left[\sin \frac{\pi x}{a} \sin \frac{\pi y}{b} e^{-\pi^2 h^2 t \left(\frac{1}{a^2} + \frac{1}{b^2}\right)}\right.$$

$$+ 1/3 \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} e^{-\pi^2 h^2 t \left(\frac{9}{a^2} + \frac{1}{b^2}\right)}$$

$$+ 1/3 \sin \frac{\pi x}{a} \sin \frac{3\pi y}{b} e^{-\pi^2 h^2 t \left(\frac{1}{a^2} + \frac{9}{b^2}\right)}$$

$$\left. + 1/5 \sin \frac{5\pi x}{a} \sin \frac{\pi y}{b} e^{-\pi^2 h^2 t \left(\frac{25}{a^2} + \frac{1}{b^2}\right)} + \ldots\ldots\right]$$

$$+ (U_1 - U_2)\frac{16}{\pi^2}\left[\sin \frac{\pi x}{a} \sin \frac{\pi y}{b} e^{-\pi^2 h^2 (t - t_1)\left(\frac{1}{a^2} + \frac{1}{b^2}\right)}\right.$$

$$+ 1/3 \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} e^{-\pi^2 h^2 (t - t_1)\left(\frac{9}{a^2} + \frac{1}{b^2}\right)}$$

$$+ 1/3 \sin \frac{\pi x}{a} \sin \frac{3\pi y}{b} e^{-\pi^2 h^2 (t - t_1)\left(\frac{1}{a^2} + \frac{9}{b^2}\right)}$$

$$\left. + 1/5 \sin \frac{5\pi x}{a} \sin \frac{\pi y}{b} e^{-\pi^2 h^2 (t - t_1)\left(\frac{25}{a^2} + \frac{1}{b^2}\right)} + \ldots\ldots\right] \quad ..(6)$$

In equations 4 to 6, inclusive, t, t_1, U_0, U_1, U_c, U_2, and h^2 are defined the same as for round timbers, while a and b are the width and thickness of a transverse section.

SYMBOLS USED IN FORMULAS FOR COMPUTING TEMPERATURES

IN ROUND AND SAWED TIMBERS

U_o = Initial wood temperature taken as 60° F. in computing figures 2 to 5.

U_1 = Heating medium temperature taken as 200° F. in computing figures 2 to 5.

U = Computed temperatures found from figures 2 to 5 after heating for time T_1 where T_1 is in hours.

U_a = Any assumed initial wood temperature.

U_b = Any assumed heating medium temperature applied for time T_1.

U_x = Computed temperature when the initial wood temperature U_a, the heating medium temperature U_b, or both are different from 60° and 200° F., the temperatures assumed in computing curves in figures 2 to 5.

U_m = Temperature found from figures 2 to 5 corresponding to a heating period of T hours.

U_n = Temperature from figures 2 to 5 corresponding to a heating period of $T-T_1$ hours.

U_c = Temperature at any point p in a timber after heating for T_1 hours at any surface temperature U_b and then cooling or heating at surface temperature U_2 for time $T-T_1$. In other words, U_c is temperature of the wood at point p, at end of time T.

U_2 = Surface temperature applied for $T-T_1$ hours.

T = Total period (hours) for which surface temperatures U_b and U_2 are applied.

T_1 = Time (hours) surface temperature U_b is applied.

$T-T_1$ = Time (hours) surface temperature U_2 is applied.

T_x = Time (hours) required to obtain a given temperature at point p in a timber when the diffusivity = h_x^2

T_h = Time (hours) required to obtain the same temperature at point p, as that obtained in time T_x, when the diffusivity = 0.00025

t and t_1 = Time in seconds.

h^2 = Diffusivity factor (square inch per second) taken as 0.00025 in computing data for figures 2 to 5 inclusive.

h_a^2 = Any diffusivity different from h^2 = 0.00025.

METHOD OF USING FIGURES WHEN BOTH DIFFUSIVITY AND HEATING CONDITIONS ARE DIFFERENT FROM THOSE USED IN COMPUTING DATA FOR TIME - TEMPERATURE CURVES

The following equations (A) to (G) are relations derived from the basic equations (3) and (6). These equations show the relation of time and diffusivity and methods of finding the corresponding wood temperature when the heating conditions are different from those assumed in computing the data for temperature curves. (For definition of symbols see list.)

$$U_c = U_2 + \left[\frac{U_m (U_b - U_a) - 200 (U_2 - U_a) - U_n (U_b - U_2)}{140} \right] \quad . . . (A)$$

When the entire timber is heated to the temperature of the heating medium

$$U_c = U_2 + \left[\frac{(200 - U_n) (U_b - U_2)}{140} \right] \quad . . . . (A_1)$$

$$U_m = \left[\frac{200 (U_2 - U_a) + U_n (U_b - U_2) + 140 (U_c - U_2)}{(U_b - U_a)} \right] \quad . . . . (B)$$

When U_a, U_b, and U are known it is necessary to determine U_x, the wood temperature obtained when U_a, U_b, or both are different from U_o and U_1. Solving for U_x in terms of U_b, U_a, U_1, U_o, and U (when $U_o = 60°$ and $U_1 = 200°$ F.) gives,

$$U_x = U_b - \left[\frac{(U_b - U_a) (200 - U)}{140} \right] \quad . . . . (C)$$

When U_x, U_a, and U_b are known it is necessary to solve for U to find the heating period T or T_1 from the proper figure. Solving for U in terms of U_a, U_b, U_x, U_o, and U_1 (when $U_o = 60°$ and $U_1 = 200°$ F.) gives,

$$U = 200 - \left[\frac{(U_b - U_x) (140)}{(U_b - U_a)} \right] \quad . . . . (D)$$

$$T_x h_a^2 = T_h (0.00025) \quad . . . . (E)$$

In the case of round timbers, if T_d is the heating period (hours) required to obtain a temperature U_d at a point r inches from the center of a timber d inches in diameter (or $(\frac{d}{2} - r)$ inches from the surface) when the diffusivity is 0.00025, and T_D is the heating period (hours) required to obtain the same temperature at the same proportional distance from the surface of a timber D inches in diameter,

$$T_D = \left(\frac{D^2}{d^2}\right)\left(\frac{0.00025}{h_a^2}\right)(T_d) \quad \ldots\ldots\ldots (G)$$

Let X_1 represent the distance from the surface of the timber of diameter d inches and X_2 the same proportional distance from the surface of the timber of diameter D inches. Then X_2 is evidently $(\frac{D}{d})(X_1)$ and $X_1 = (\frac{d}{D})\ X_2$.

If for convenience d is taken as 10 inches, $T_D = \left(\frac{D^2}{100}\right)\left(\frac{0.00025}{h_a^2}\right)(T_{10})$ where T_{10} is the time (hours) required to reach the temperature U_d at the point under consideration in a timber 10 inches in diameter. The time T_{10} may be determined from the temperature curve for the 10-inch diameter timber if U_d is assumed.

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Table 1.--Multiplying factors for timbers of different diameters[1]

Diameter of timber = D	Multiplying factor "f" = $\frac{D^2}{100}$	Diameter of timber = D	Multiplying factor "f" = $\frac{D^2}{100}$
Inches		Inches	
8	0.64	13	1.69
8-1/2	0.72	13-1/2	1.82
9	0.81	14	1.96
9-1/2	0.90	14-1/2	2.10
10	1.00	15	2.25
10-1/2	1.10	15-1/2	2.40
11	1.21	16	2.56
11-1/2	1.32	16-1/2	2.72
12	1.44	17	2.89
12-1/2	1.56	18	3.24

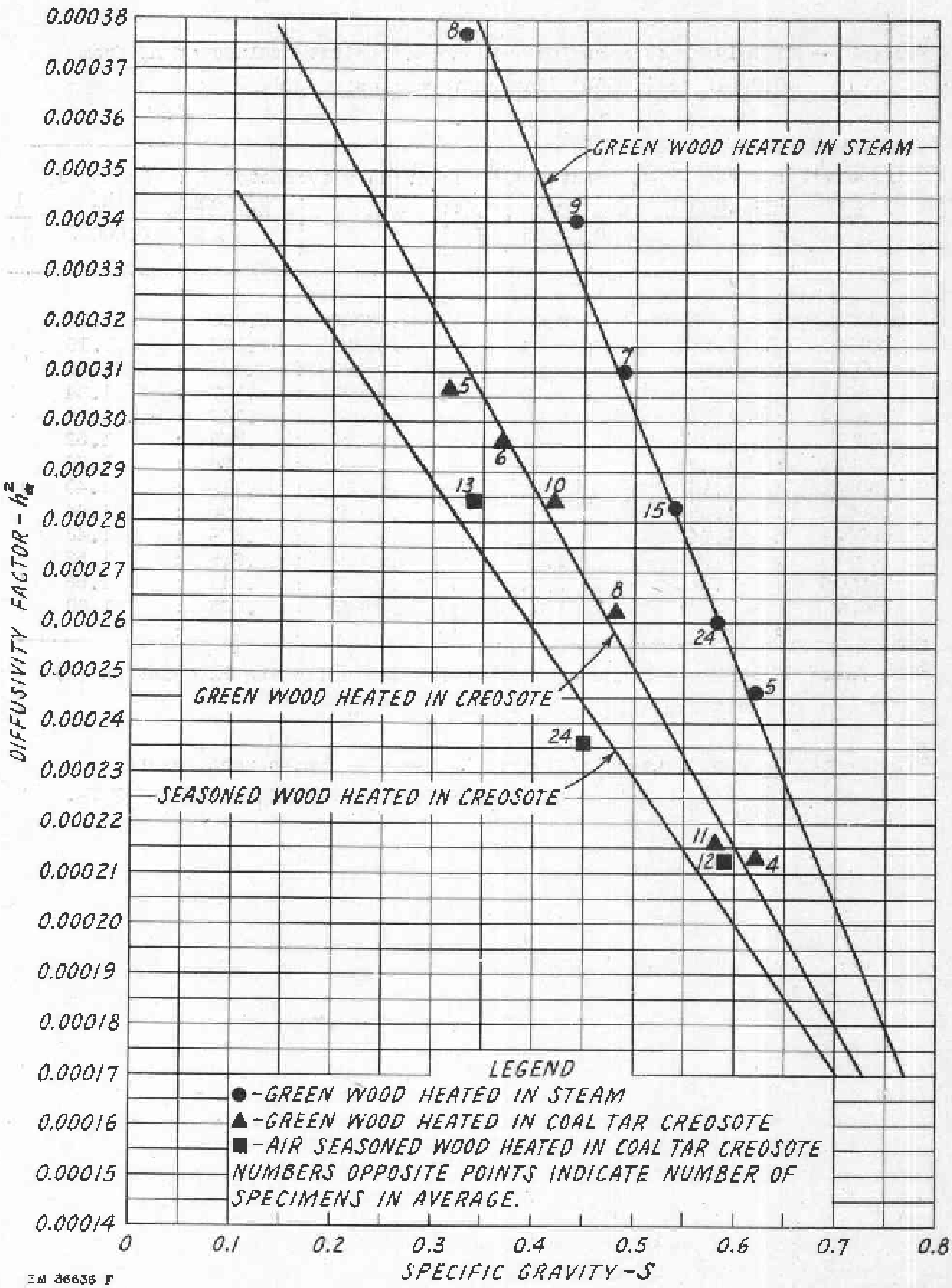
[1]The time required to reach a given temperature in a timber 10 inches in diameter multiplied by factor "f" gives the time required to obtain the same temperature for the diameter D, at the same proportional distances from the surface.

Table 2.—Multiplying factors for various diffusivity values of h_x^2 from 0.00015 to 0.00035 (ratios to nearest 0.01)

Diffusivity h_a^2	Factor F_1[1] $= \frac{0.00025}{h_a^2}$	Factor F_1[2] $= \frac{h_a^2}{0.00025} = \frac{1}{F_1}$	Diffusivity h_a^2	Factor F_1 $= \frac{0.00025}{h_a^2}$	Factor F_1 $= \frac{h_a^2}{0.00025} = \frac{1}{F_1}$
0.00015	1.667	0.60	0.00028	0.893	1.12
.00016	1.562	.64	.00029	.863	1.16
.00017	1.472	.68	.00030	.833	1.20
.00018	1.390	.72	.00031	.807	1.24
.00019	1.317	.76	.00032	.782	1.28
.00020	1.250	.80	.00033	.758	1.32
.00021	1.191	.84	.00034	.736	1.36
.00022	1.137	.88	.00035	.715	1.40
.00023	1.087	.92	.00036	.695	1.44
.00024	1.042	.96	.00037	.676	1.48
.00025	1.000	1.00	.00038	.659	1.52
.00026	.962	1.04	.00039	.641	1.56
.00027	.927	1.08	.00040	.625	1.60

[1] The factor F_1 will be found convenient to use in finding T_x equation (E) of the appendix where $T_x = \left(\frac{0.00025}{h_a^2}\right) T_h = F_1 T_h$.

[2] The factor F_2 can be used in finding T_h, when T_x is assumed, where $T_h = \left(\frac{h_a^2}{0.00025}\right) T_x = F_2 T_x$. In equation (G) $T_D = fF_1 T_d$ and $T_d = \frac{1}{f} F_2 T_D$.



Z M 36636 F

Figure 1.--Relation of diffusivity and specific gravity of wood.

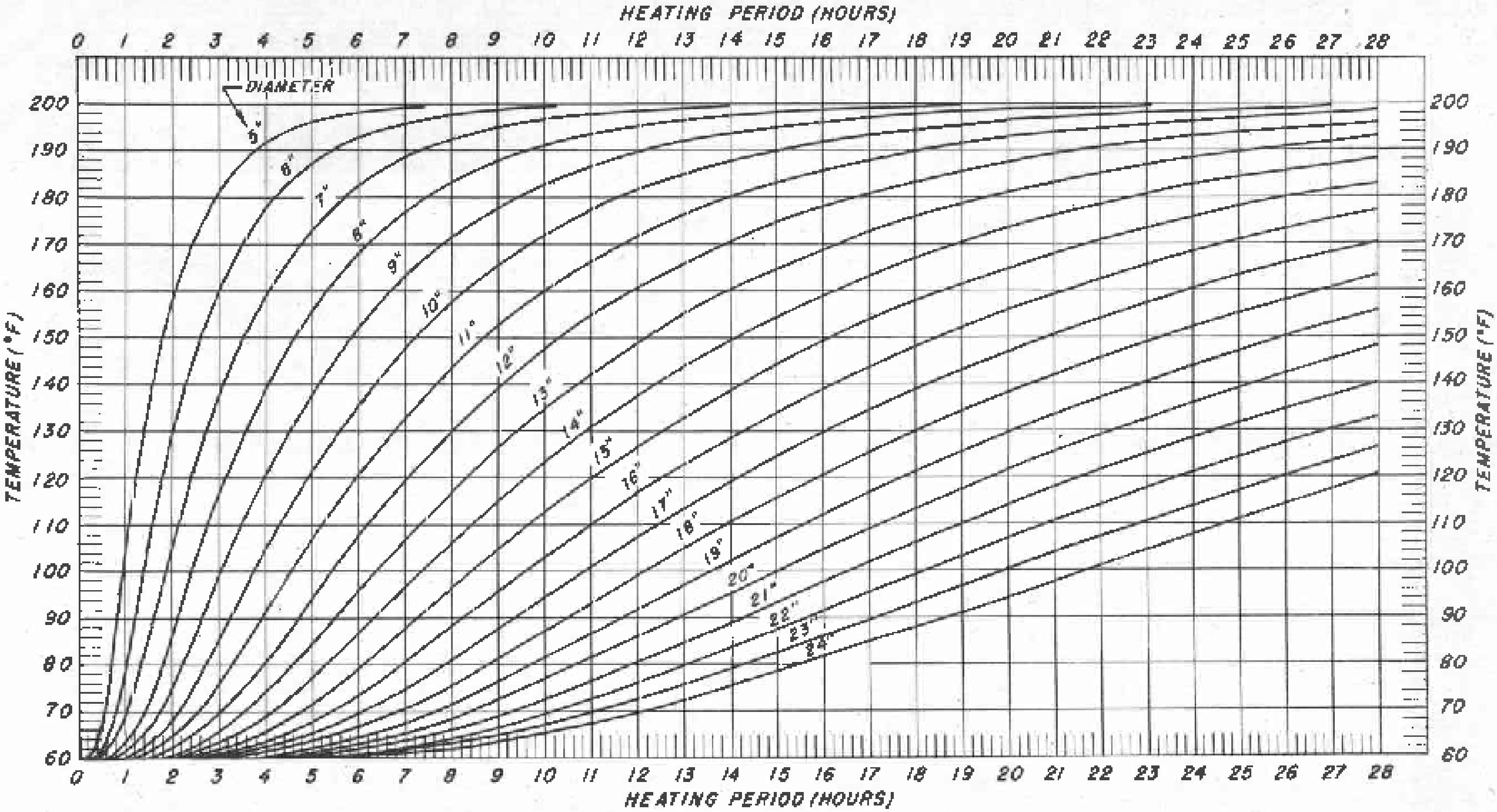


Figure 2.--Temperature at center of round timbers of various diameters heated at 200° F. for various periods of time. (Diffusivity = 0.00025; initial wood temperature = 60° F.)

Z M 67338 F

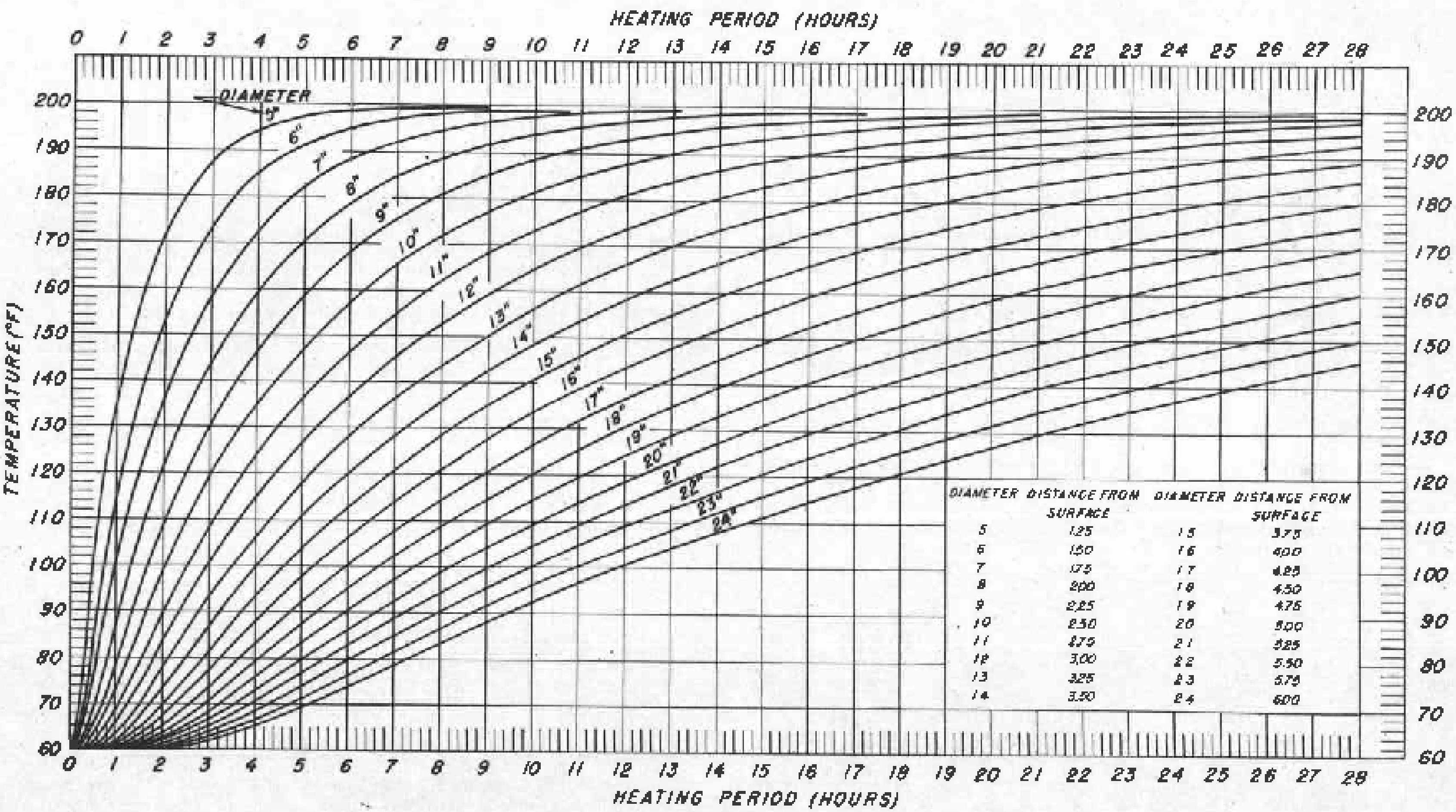


Figure 3.--Temperature midway between the surface and center of round timbers of various diameters heated at 200° F. for different periods of time. (Diffusivity = 0.00025; initial wood temperature = 60° F.)

Z M 67336 F

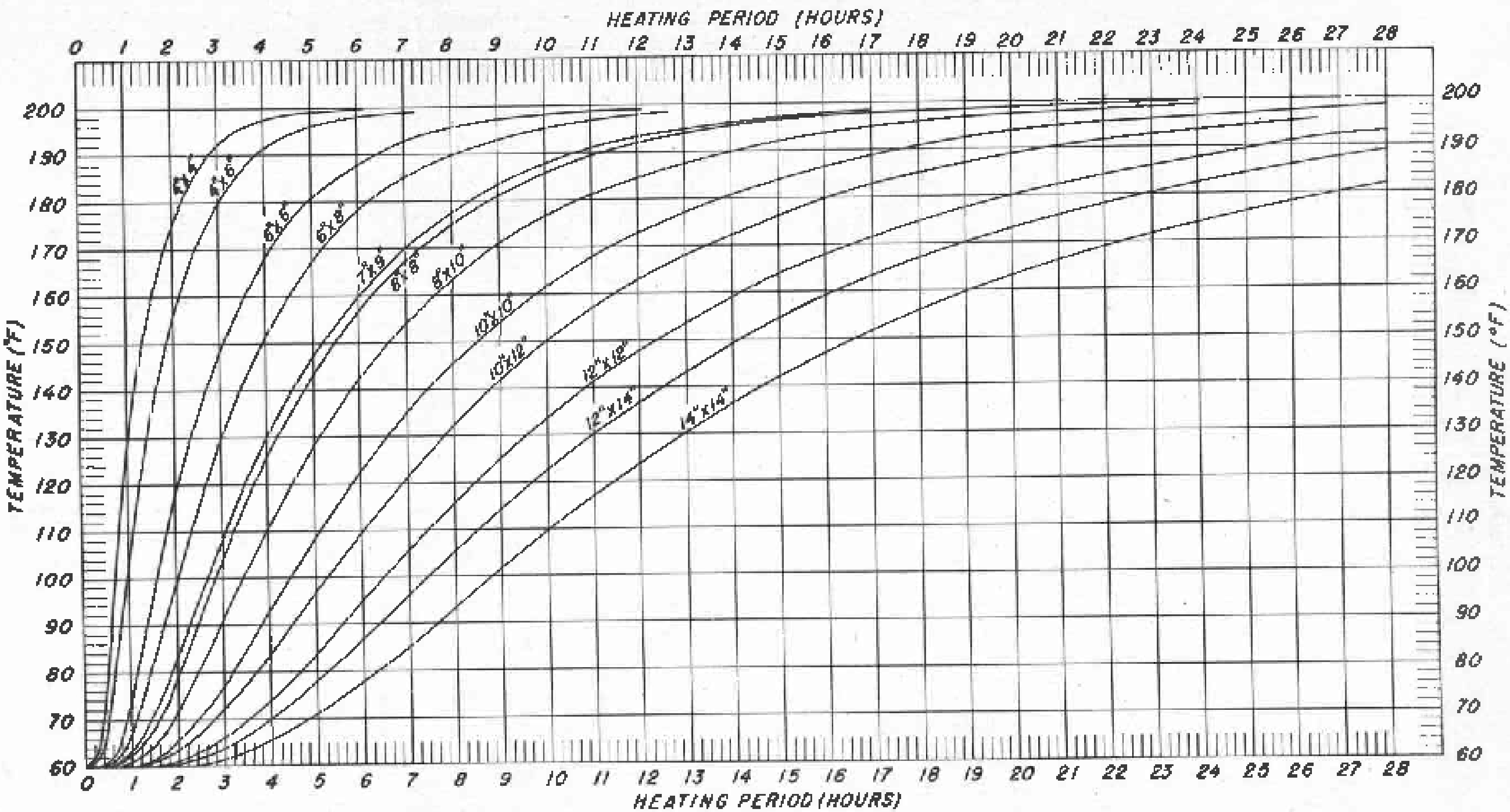


Figure 4.--Temperature at center or axis of sawed timbers of various cross-sectional dimensions heated at 200° F. for different periods of time. (Diffusivity = 0.00025; initial wood temperature = 60° F.)

Z M 67387 F

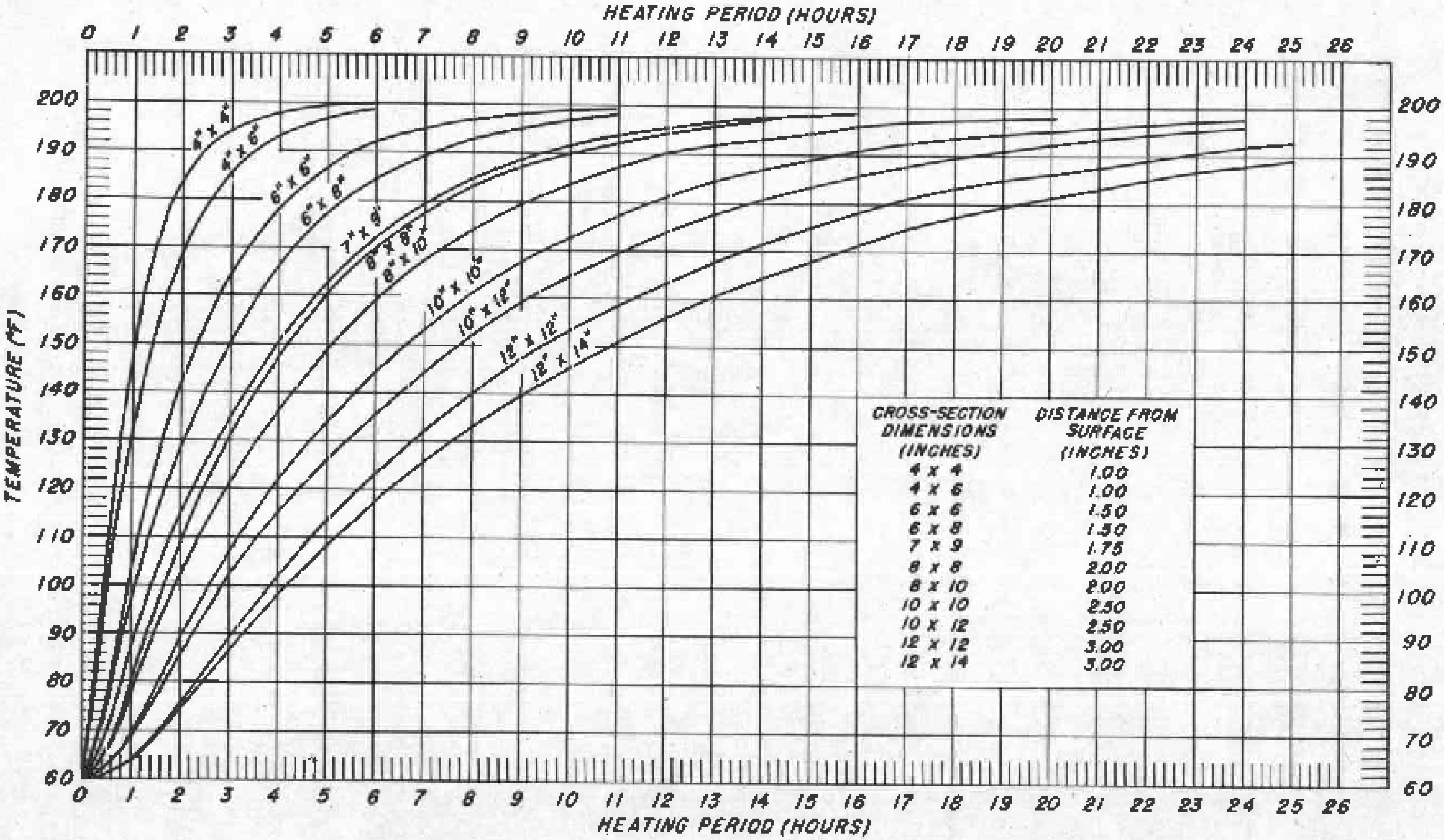


Figure 5.--Temperature midway between the surface and center of sawed timbers of various cross-sectional dimensions (distance measured on the short axis when width is greater than thickness) heated at 200° F. for different periods of time. (Diffusivity = 0.00025; initial wood temperature = 60° F.)

Z M 67538 F

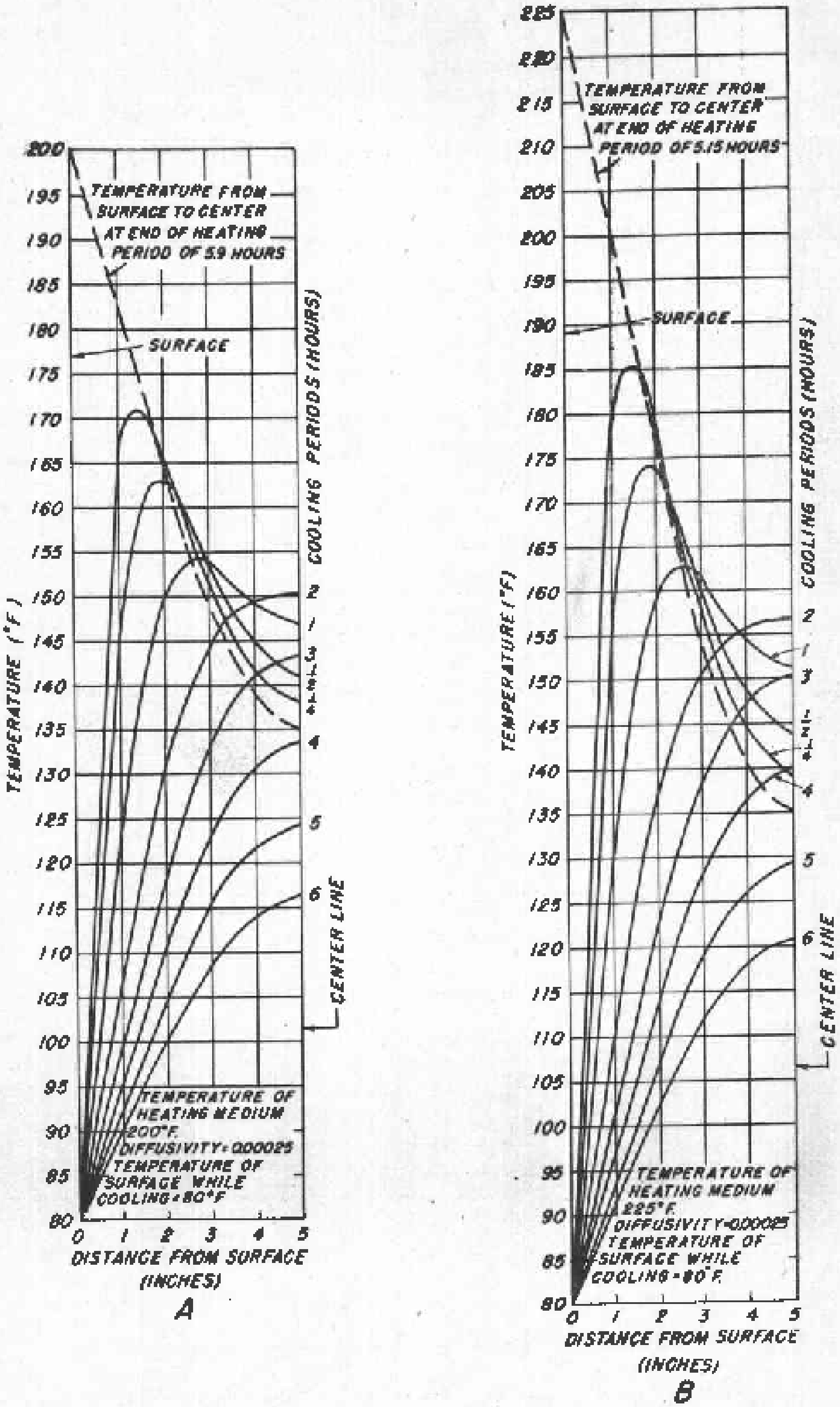


Figure 6.--Temperature distribution in timber 10 inches in diameter after cooling for various periods of time with the temperature of the surface at 80° F. Cooling started after center is heated to 135° F.

Z M 67339 F

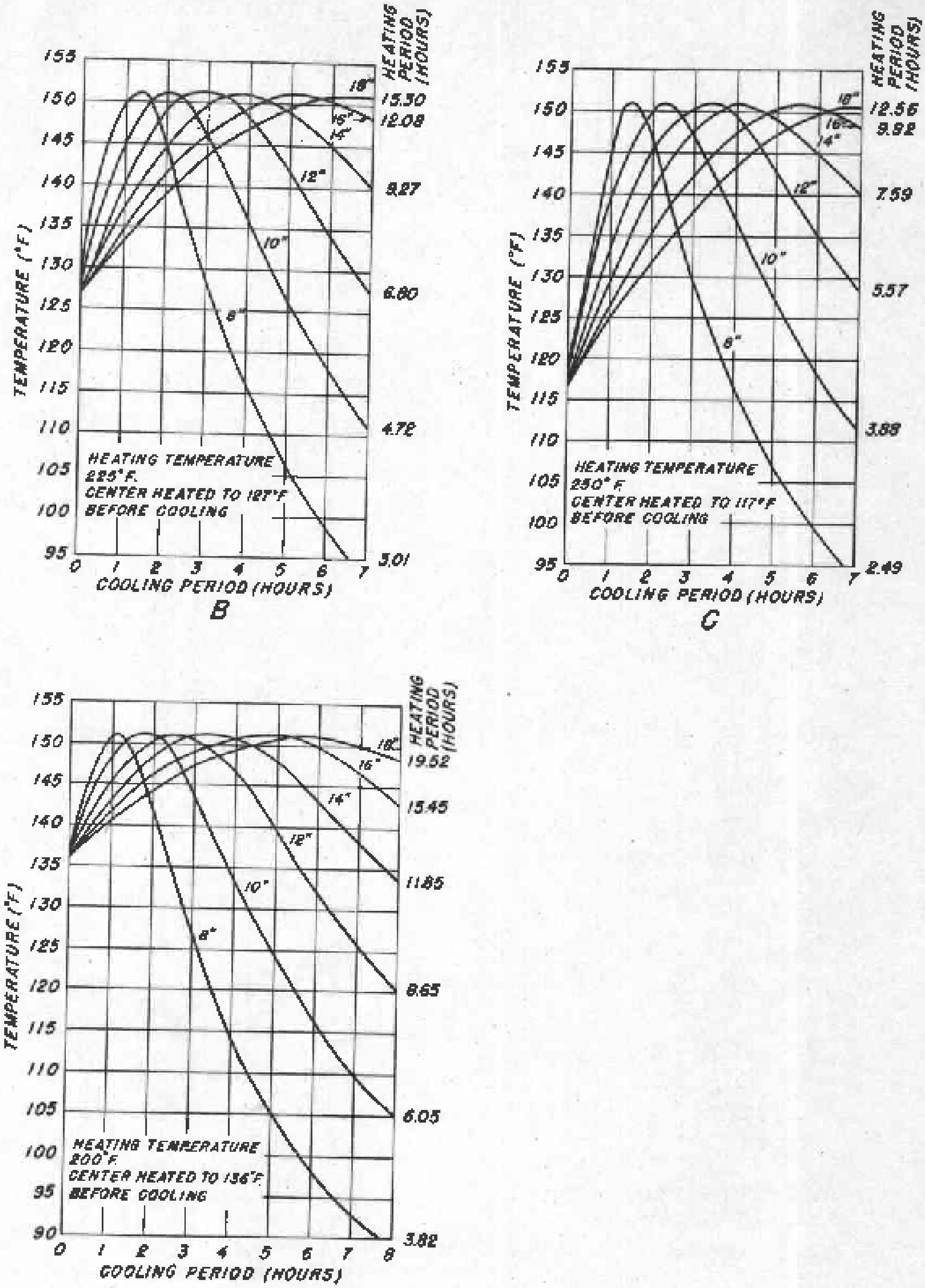


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Figure 7.--Temperature change at center of round timbers when cooled with the temperature of the surface at 80° F. after the center is heated to temperatures that will result in a maximum center temperature of 151° F.

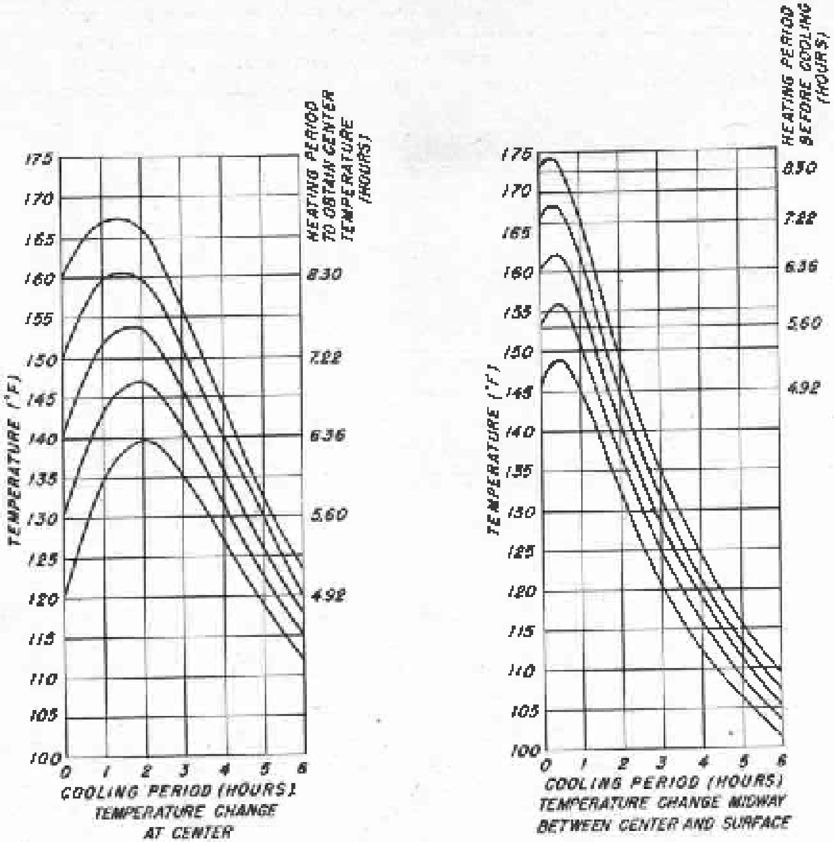

Figure 8.--Temperature change at center and midway between center and circumference of timbers 10 inches in diameter. Heating temperature = 200° F.; initial wood temperature = 60° F.; cooling temperature = 80° F.; and diffusivity = 0.00025.

Z M 67341 F

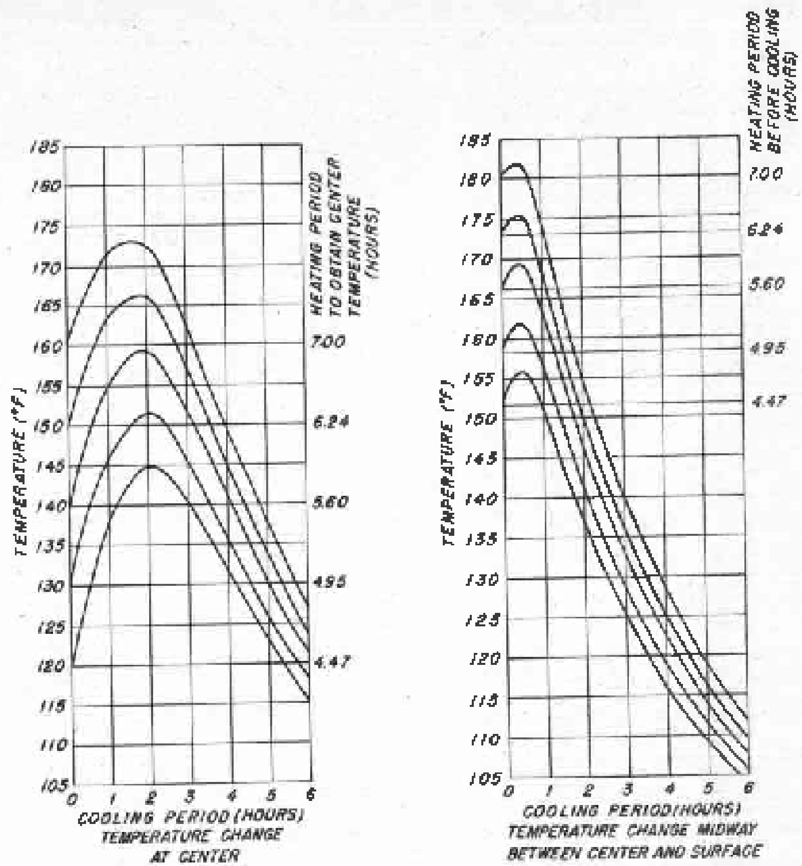

Figure 9.--Temperature change at center and midway between center and circumference of timbers 10 inches in diameter. Heating temperature = 220° F.; initial wood temperature = 60° F.; cooling temperature = 80° F.; and diffusivity = 0.00025.

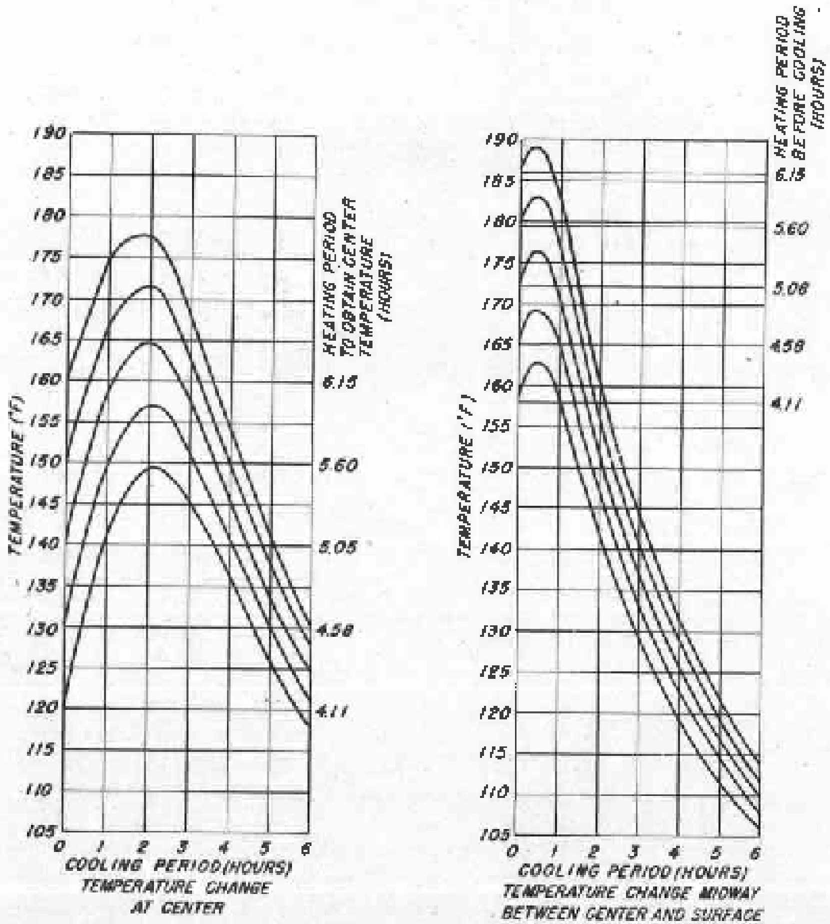

Figure 10.--Temperature change at center and midway between center and circumference of timbers 10 inches in diameter. Heating temperature = 240° F.; initial wood temperature = 60° F.; cooling temperature = 80° F.; and diffusivity = 0.00025.

Z M 67342 F

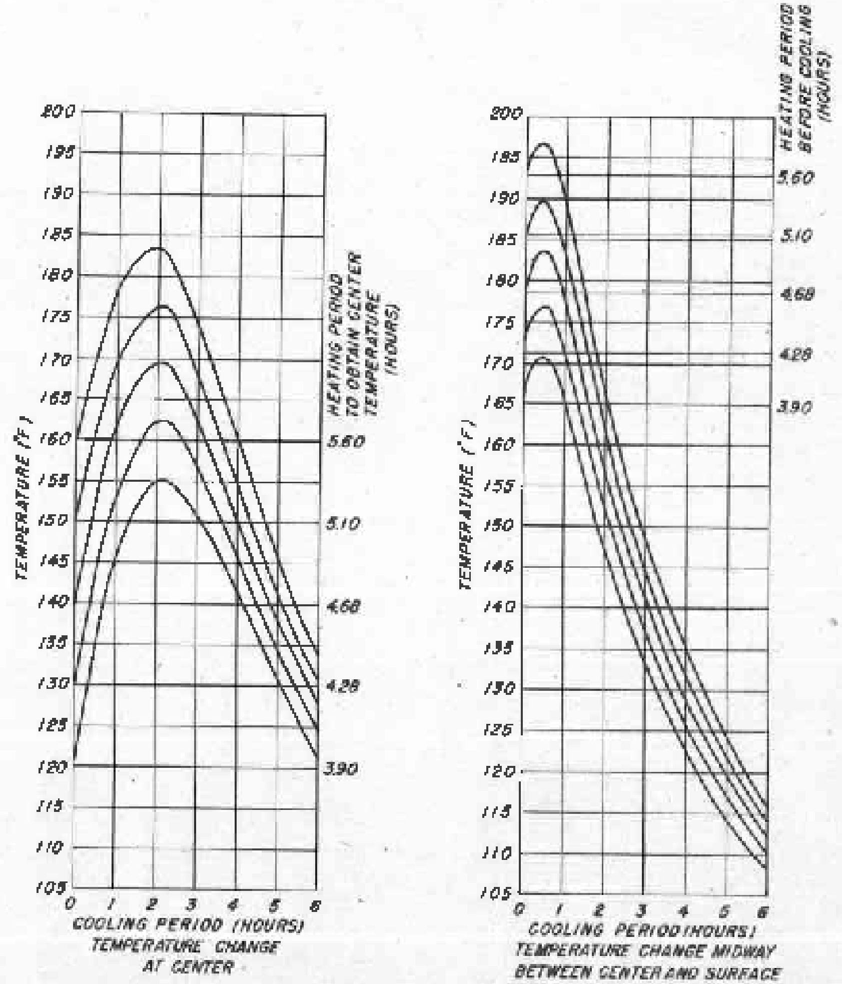

Figure 11.--Temperature change at center and midway between center and circumference of timbers 10 inches in diameter. Heating temperature = 260° F.; initial wood temperature = 60° F.; cooling temperature = 80° F.; and diffusivity = 0.00025.

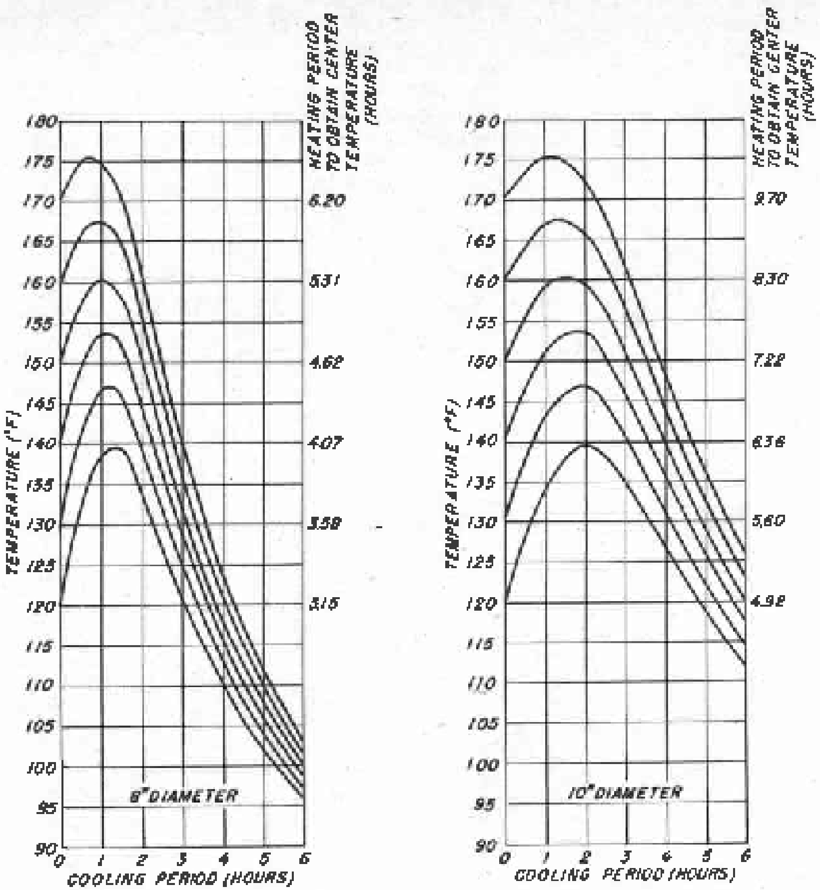

Figure 12.--Rate of temperature change at center of round timber 8 and 10 inches in diameter while cooling after center has been heated to different temperatures. Initial wood temperature = 60° F.; heating medium temperature = 200° F.; diffusivity = 0.00025; and cooling temperature = 80° F.

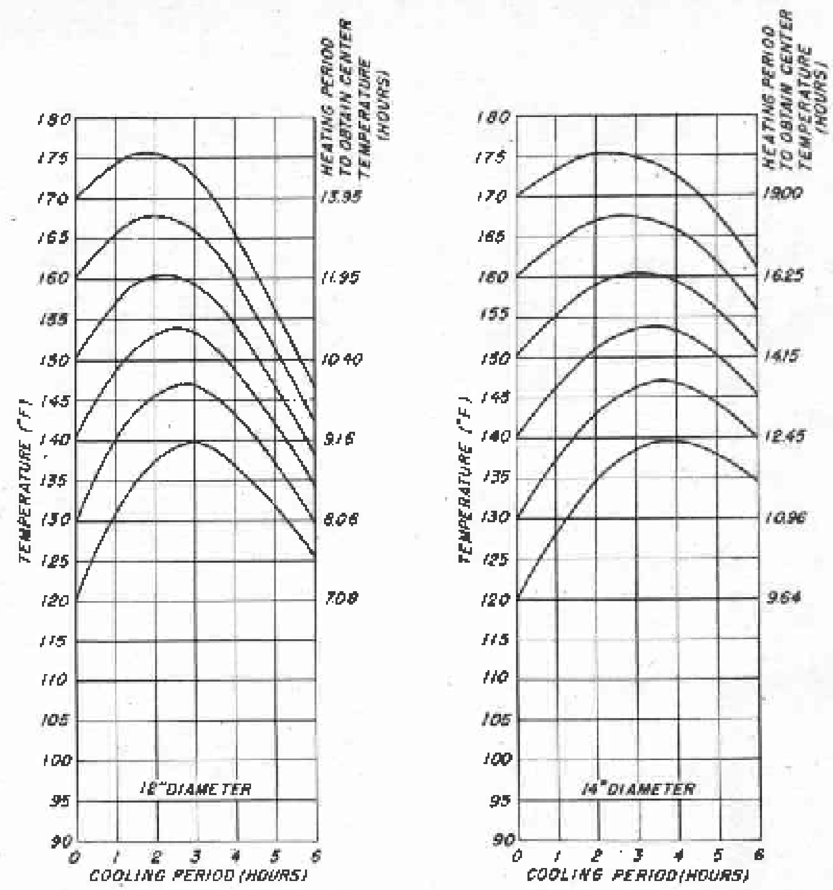

Figure 13.--Rate of temperature change at center of round timber 12 and 14 inches in diameter while cooling after center has been heated to different temperatures. Initial wood temperature = 60° F.; heating medium temperature = 200° F.; diffusivity = 0.00025; and cooling temperature = 80° F.

Z M 67343 F

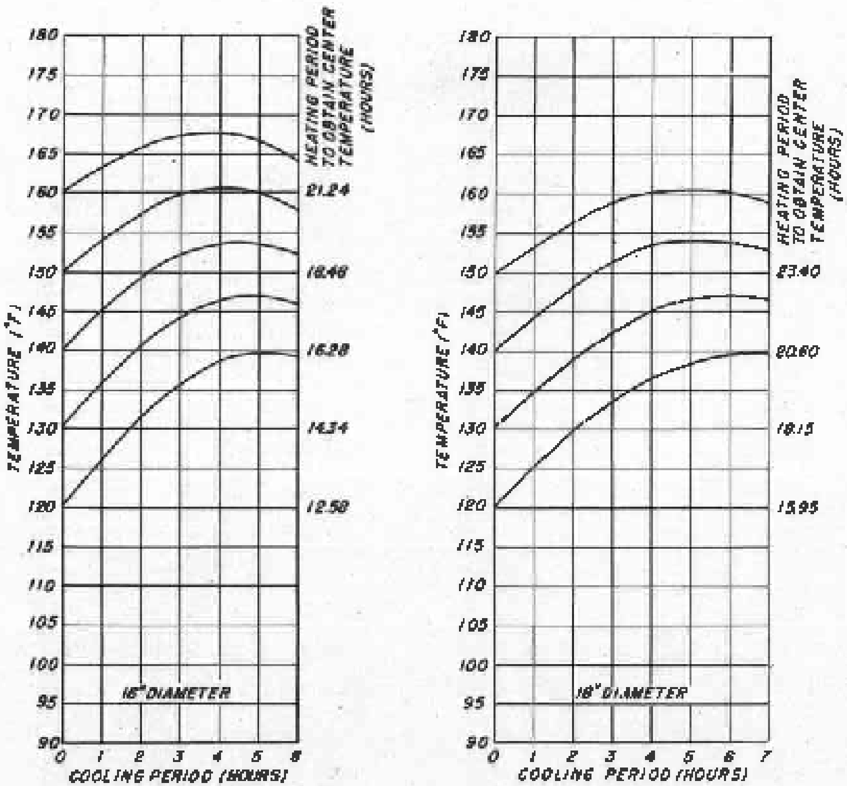

Figure 14.--Rate of temperature change at center of round timber 16 and 18 inches in diameter while cooling after center has been heated to different temperatures. Initial wood temperature = 60° F.; heating medium temperature = 300° F.; diffusivity = 0.00025; and cooling temperature - 80° F.

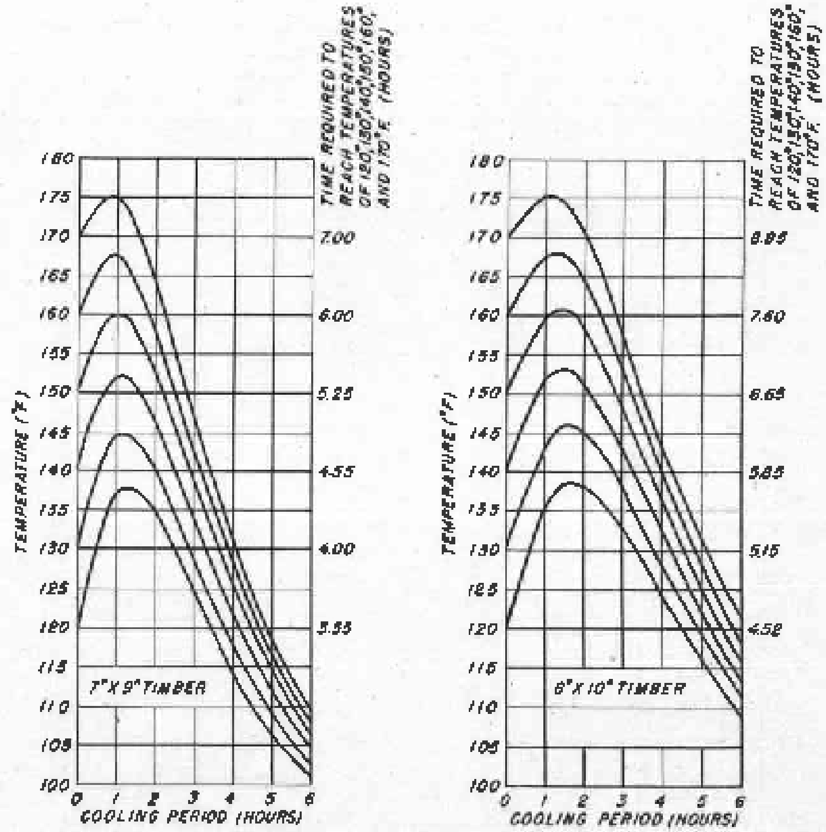

Figure 15.--Rate of temperature change at center of 7- by 9-inch and 8- by 10-inch sawed timbers while cooling after center has been heated to different temperatures. (Initial wood temperature = 60° F.; heating medium temperature = 200° F.; diffusivity = 0.00025; and cooling temperature = 80° F.)

Z M 67344 F

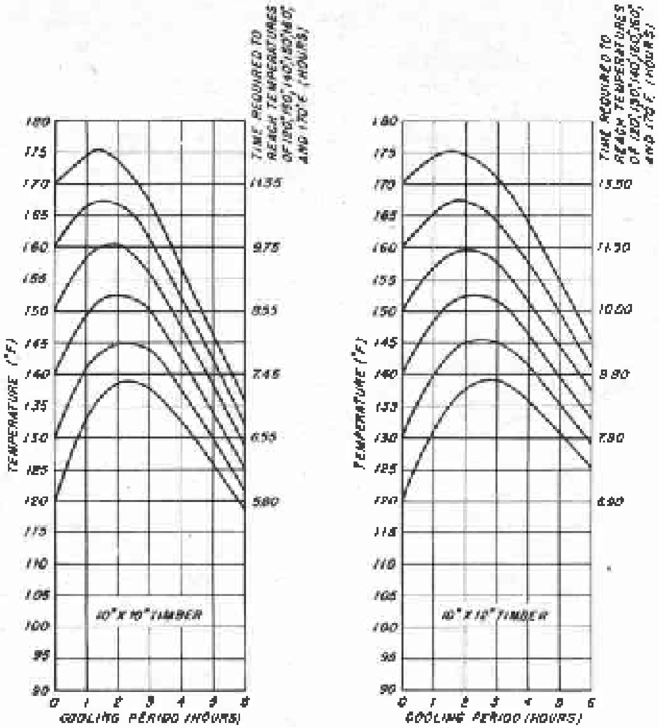

Figure 16.--Rate of temperature change at center of 10- by 10-inch and 10- by 12-inch sawed timbers while cooling after center has been heated to different temperatures. (Initial wood temperature = 60° F.; heating medium temperature = 200° F.; diffusivity = 0.00035; and cooling temperature = 80° F.)

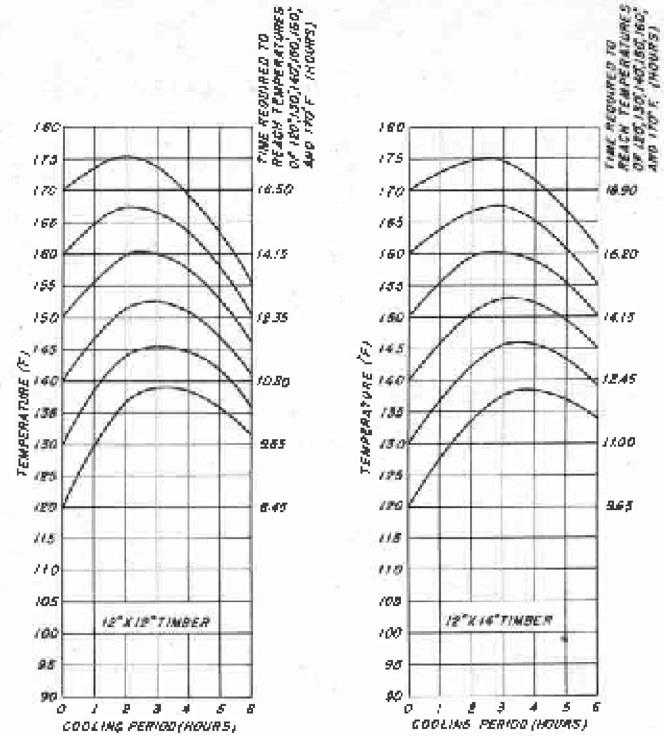

Figure 17.--Rate of temperature change at center of 12- by 12-inch and 12- by 14-inch sawed timbers while cooling after center has been heated to different temperatures. (Initial wood temperature = 60° F.; heating medium temperature = 200° F.; diffusivity = 0.00025; and cooling temperature = 80° F.)

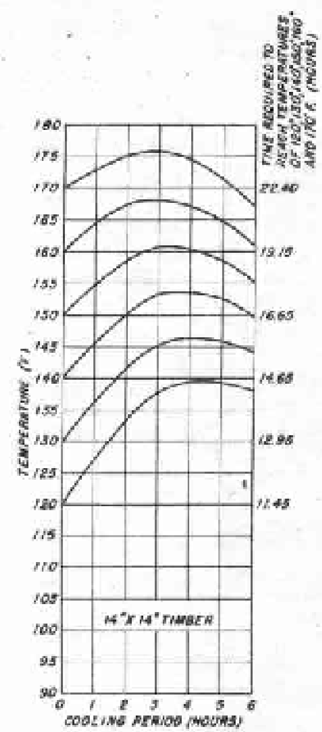

Figure 18.--Rate of temperature change at center of 14- by 14-inch sawed timbers while cooling after center has been heated to different temperatures. (Initial wood temperature = 60° F.; heating medium temperature = 200° F.; diffusivity = 0.00025; and cooling temperature = 80° F.)

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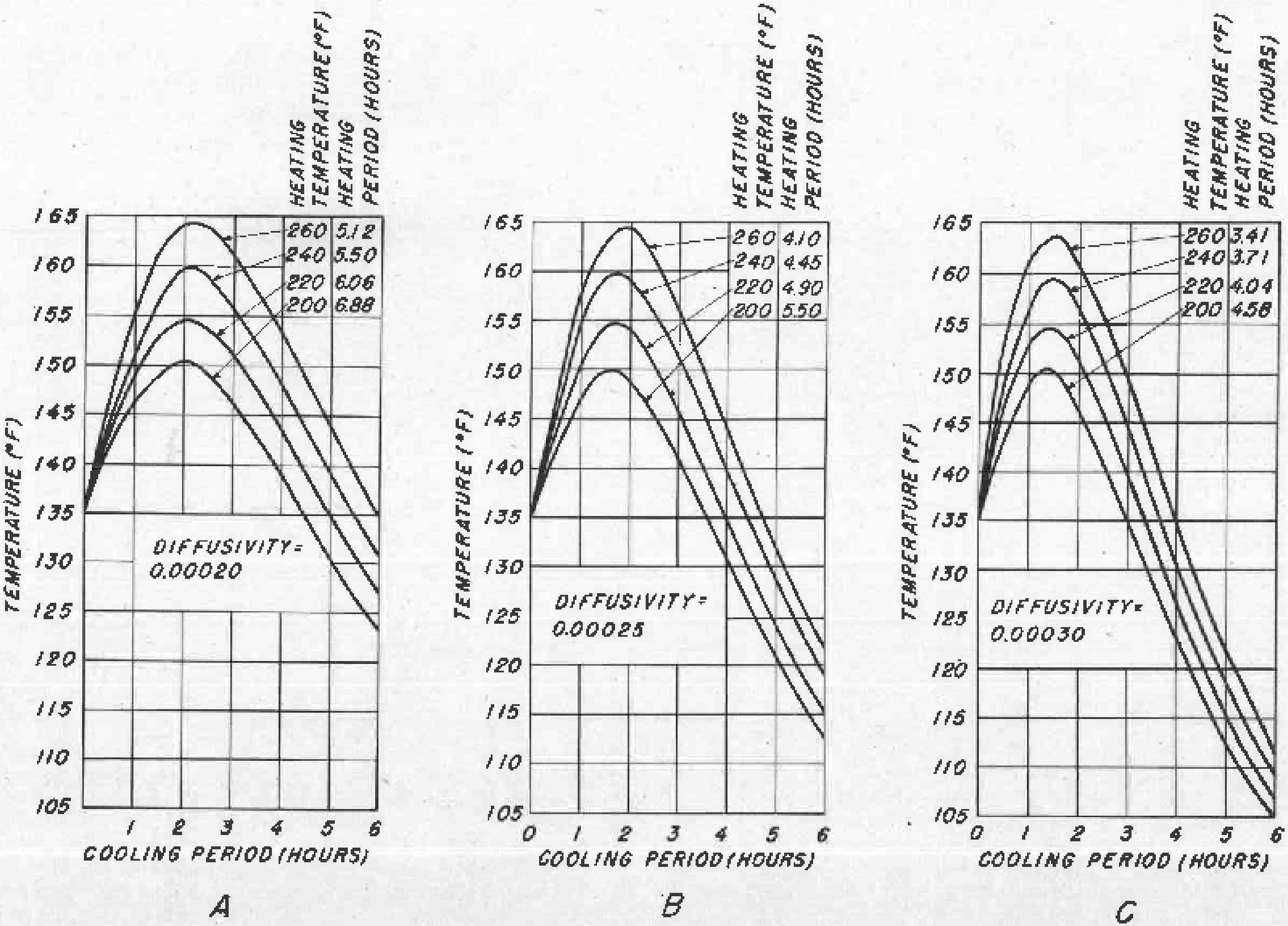

Figure 19.--Effect of different diffusivities on rate of temperature change at center of 8- by 10-inch sawed timber after center has been raised to 135° F. with different heating-medium temperatures. Surface temperature while cooling = 80° F.

2 M 57348 F