THE ELASTIC PROPERTIES OF WOOD

Young's Moduli and Poisson's Ratios

of Douglas-fir and Their Relations

to Moisture Content

Information Reviewed and Reaffirmed

March 1956

INFORMATION REVIEWED
AND REAFFIRMED
1962

No. 1528-D

THE ELASTIC PROPERTIES OF WOOD[1]

Young's Moduli and Poisson's Ratios of Douglas-fir and Their Relations to Moisture Content[2]

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Summary

This report presents data on the Young's moduli and Poisson's ratios of Douglas-fir as found from tests in compression at moisture content values of approximately 7, 13, and 20 percent. A summary table is presented showing average values for each of four planks, and general

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[1]This report is one of a series of progress reports prepared by the Forest Products Laboratory relating to the use of wood in aircraft. Results here reported are preliminary and may be revised as additional data become available.

[2]This report is the fifth of a series of reports presenting The Elastic Properties of Wood. Others in the series are: The Young's Moduli, Moduli of Rigidity, and Poisson's Ratios of Balsa and Quipo," Rept. No. 1528; "The Young's Moduli and Poisson's Ratios of Sitka Spruce and Their Relations to Moisture Content," Rept. No. 1528-A; "The Moduli of Rigidity of Sitka Spruce and Their Relations to Moisture Content," Rept. No. 1528-B; "Young's Moduli, Moduli of Rigidity, and Poisson's Ratios of Mahogany and Khaya," Rept. No. 1528-C.

[3]Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

averages for the three Young's moduli and their ratios, and for the six Poisson's ratios. Graphs are presented to illustrate the effects of moisture on the elastic constants. The Young's moduli and some of the Poisson's ratios are increased by a decrease in moisture content, while the reverse is true of other Poisson's ratios. The specimens covered only a small range in specific gravity, and no correlation of elastic properties with specific gravity was apparent.

Introduction

This report is the fifth of a series on "Elastic Properties of Wood" and presents data on Young's moduli and Poisson's ratios for Douglas-fir, and their relations to moisture content. Previous reports have been published for balsa and quipo, Sitka spruce, and mahogany and khaya.[2] Symbols and terminology used in the present report conform to the definitions and nomenclature presented in the first report of the series, No. 1528.[2]

Description of Material

Type of Specimen

Standard 2- by 2- by 8-inch compression prisms oriented with their long dimensions in the directions of each of the three axes of elastic symmetry, longitudinal, radial, and tangential, were used. A total of 48 specimens were tested, including 30 longitudinal, 6 glued radial, 6 glued tangential, and 6 solid tangential prisms.

Source and Selection of Material

Material for these tests was obtained from four planks selected from Douglas-fir on hand at the Laboratory. No record of the source of this material is available, and it is not known whether these planks were of coast, intermediate, or Rocky Mountain type. All planks were flat-sawed, about 3 inches thick, and 8 to 12 inches wide. A section about 6 feet long with straight grain and a minimum of knots and other defects was selected from each plank. The material was in an air-dry condition when selected, having originally been kiln-dried and then stored in the open. No serious checks or shrinkage defects were present in the portions used for specimens.

Preparation of Planks and Test Pieces

From three of the 6-foot plank sections in which the ring curvature was comparatively sharp, a stick about 2-1/2 by 2-1/2 inches in cross section was cut parallel to the grain and with growth rings as nearly as possible parallel and perpendicular to the edges of the stick. Test pieces 10 inches in length were cut in succession along the stick, thus placing them end-to-end in the same band of growth rings. Minor changes in direction were made in the final cutting so that alinement of the faces parallel to the grain and perpendicular to the rings would be most favorable. The upper diagram of figure 2 in Forest Products Laboratory Report No. 1528-A[2] illustrates the cutting procedure. The test pieces in the positions designated by Nos. 13, 14, 15, and 17 were used to make glued radial and tangential specimens by gluing together four 2-inch ring-matched cubes stacked in the radial and tangential directions. The remaining pieces were reduced to final size for longitudinal specimens.

From the fourth plank, in which the growth rings were sufficiently flat to justify solid tangential specimens, 6 transverse and 18 longitudinal pieces were cut as shown in the lower diagram of figure 2 in Forest Products Laboratory Report No. 1528-A.[2] The individual pieces were reduced to final size, making 18 longitudinal and 6 solid tangential specimens. All specimens were surfaced to true parallel and perpendicular faces, and stored in a 75° F., 65 percent relative humidity room for conditioning. Equilibrium moisture content of approximately 13 percent was attained prior to test.

Marking of Specimens

The specimens from each of the four planks were referred to as a series. Each specimen carried the series designation by means of the first digit of its number, and a final digit which showed its location in the plank. The digit following the decimal in the fourth series differentiated between the three specimens cut from the same longitudinal piece.

In series 10, 20, and 30, the arrangement for matching and marking was as shown in the upper diagram of figure 2 and table 2 of Forest Products Laboratory Report No. 1528-A.[2] In series 40 the marking was as shown in the lower diagram of figure 2 in the same report.[2] The six transverse specimens, 42.1, 42.2, 42.3, 46.1, 46.2, and 46.3, were solid tangential prisms, and the remainder of the specimens were longitudinal.

Testing Procedure

Instruments and their arrangement, methods of computation, and general testing procedure were as described in Forest Products Laboratory Report No. 1528.[2]

Longitudinal specimens were loaded to approximately 1,250 pounds per square inch; radial and tangential specimens to about 125 pounds per square inch.

Three to five independent sets of values (each from two or more runs at each instrument setting) were made at the initial moisture content of about 13 percent. Specimens were then conditioned successively at 80° F., 30 percent relative humidity (to approximately 7 percent moisture content), and at 80° F., 97 percent relative humidity (to approximately 20 percent moisture content), and one or two independent sets of values were obtained with each instrument arrangement at each moisture content. The maximum load at each of these runs was kept well within the proportional limit of the wood. All specimens were finally reconditioned to approximately 13 percent moisture content and then tested to failure. In these final tests, measurements for Young's modulus were made on one pair of faces, and one Poisson's ratio was obtained.

Explanation of Tables and Figures

Table 1 presents a summary of the average Young's moduli and Poisson's ratios at each of the three moisture content values for each of the four planks and, in addition, general averages for all specimens of each type. In forming the general averages, values from individual specimens were given equal weight.

Table 2 shows ratios of the various elastic constants calculated from the averages listed in table 1.

Table 3 presents a comparison of the average measured values for the two small Poisson's ratios, μ_{RL} and μ_{TL}, with values determined from elastic-constant equations by two methods: by use of average values for related constants as shown in table 2, and by use of values obtained from regression lines representing the effect of moisture on these constants (figs. 1 and 2).

Table 4 gives the average values for properties of Douglas-fir at ultimate strength compared to results of previous tests.

Figure 1 shows the variation of the three Young's moduli and their ratios $\frac{E_R}{E_L}$ and $\frac{E_T}{E_L}$, with moisture content. The individual points represent average values from the same 30 longitudinal, 6 radial, and 12 tangential specimens tested at each of the three values of moisture content. The values for the moduli and ratios are plotted on a logarithmic scale as ordinates, and the moisture content on a uniform scale as abscissas. The curves shown are the best fit linear relationships calculated by the method of least squares (regression of log E or R on moisture content). The figures in parentheses show the percentage increase in the moduli or the ratios for a 1 percent decrease in moisture content.

Figure 2 shows the variation of the Poisson's ratios with moisture content. The individual points represent average values from the same 30 longitudinal, 6 radial, and 12 tangential specimens tested at three moisture contents. The Poisson's ratio values are plotted to a logarithmic scale as ordinates and the moisture content to a uniform scale as abscissas. The curves shown are the best fit linear relationships for each series of points determined by the method of least squares (regression of log μ on moisture content). The figures in parentheses represent the percentage increase (positive values) or decrease (negative values) in the ratios for a 1 percent decrease in moisture content. The dashed lines represent values of μ_{RL} and μ_{TL} computed from the elastic-constant relationships using values of the other four Poisson's ratios and of Young's moduli as read from the straight lines shown in figures 1 and 2.

Figure 3 shows the effect of moisture on the ratios $\frac{E_R}{E_L}$, $\frac{E_T}{E_L}$, $\frac{E_T}{E_R}$, $\frac{\mu_{RL}}{\mu_{LR}}$, $\frac{\mu_{TL}}{\mu_{LT}}$, and $\frac{\mu_{TR}}{\mu_{RT}}$. Individual points on each curve represent the average ratios for all specimens included in the study (table 1). The ratios are plotted to a logarithmic scale as ordinates and the moisture content to a uniform scale as abscissas. The curves shown are the best fit linear relationships determined by the method of least squares (regression of log R on moisture content). The figures in parentheses show the percentage increase in the ratios for each 1 percent decrease in moisture content.

Variability of Young's Moduli and Poisson's Ratios

The range in specific gravity for specimens included in this study was insufficient to permit an analysis of the variation of Young's moduli or Poisson's ratios with specific gravity.

The values for the elastic constants in table 1 represent average values for the four planks and species, but it should be pointed out that all elastic constant values of individual specimens were quite variable for a particular moisture content. For example, at about 13 percent moisture content the measured values for Young's moduli ranged as follows: E_L from 1,843,000 to 2,516,000 pounds per square inch, E_R from 123,000 to 209,000 pounds per square inch, E_T from 93,000 to 141,000 pounds per square inch.

Individual values of the Poisson's ratios at 13 percent moisture content ranged as follows: μ_{LR} from 0.181 to 0.391; μ_{LT} from 0.286 to 0.598; μ_{RL} from 0.028 to 0.052; μ_{RT} from 0.348 to 0.492; μ_{TL} from 0.020 to 0.046; and μ_{TR} from 0.278 to 0.471. As with the averages shown in table 1, comparisons from the same specimens or from closely matched specimens show μ_{LT} consistently greater than μ_{LR}, and μ_{RT} greater than μ_{TR}.

Relations Among Elastic Constants

With wood considered as an orthotropic material, the following relations among the elastic constants should exist:

$$\frac{\mu_{LR}}{E_L} = \frac{\mu_{RL}}{E_R} \tag{1}$$

$$\frac{\mu_{LT}}{E_L} = \frac{\mu_{TL}}{E_T} \tag{2}$$

$$\frac{\mu_{RT}}{E_R} = \frac{\mu_{TR}}{E_T} \tag{3}$$

A consideration of table 2 shows that none of these relations holds for the data in this study. The limited number of radial and tangential specimens indicated the probability that average values of the elastic constants determined on specimens having their long axis in the radial or tangential directions are probably less dependable than those determined on specimens having their long axis parallel to the grain.

Values for the two small Poisson's ratios, μ_{RL} and μ_{TL}, are frequently calculated from the elastic constant relations shown in equations (1) and (2). Table 3 shows that such calculated values do not agree well with the measured values. These relations are discussed further in the next paragraph.

Effect of Moisture

Figure 1 shows that elastic moduli conform closely to straight-line relationships with moisture content when the moduli are plotted to a logarithmic scale as ordinates and the moisture content to an arithmetic scale as abscissas, thereby further substantiating the exponential theory[4] for strength-moisture relations.

To maintain equality in equations (1), (2), and (3) the Poisson's ratios must be influenced by moisture. Figure 2 shows that the Poisson's ratios when plotted as previously indicated do conform reasonably well to straight-line relationships with moisture, except for μ_{RT} and the two small ratios, μ_{RL} and μ_{TL}, which are difficult to measure accurately.

The elastic-constant relations previously given may be rearranged as follows:

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[4]"Wood Handbook," U. S. Department of Agriculture, revised 1955.

$$\frac{E_R}{E_L} = \frac{\mu_{RL}}{\mu_{LR}} \tag{4}$$

$$\frac{E_T}{E_L} = \frac{\mu_{TL}}{\mu_{LT}} \tag{5}$$

$$\frac{E_T}{E_R} = \frac{\mu_{TR}}{\mu_{RT}} \tag{6}$$

The ratios in equations (4) and (5) should be influenced by moisture because of the difference in the effect of moisture on E_R and E_T compared with E_L, whereas the ratios in equation (6) should be affected little because the effect of moisture on E_R is similar to its effect on E_T. The ratios in equations (4), (5), and (6) are plotted in figure 3 together with best-fit lines determined by the method of least squares. For perfect equality in these equations, the lines representing ratios of E and μ should coincide. This condition is not satisfied in any of the three cases. The lines for equations (4) and (6) are parallel but not coincident and the lines for equation (5) intersect. Figure 3 shows graphically what table 2 presents in tabular form, namely, that the theoretical relations between the elastic constants did not hold for this study at any moisture content.

On the other hand, figures 1 and 2 show that values of E_L, E_R, E_T, μ_{LR} and μ_{LT} vary uniformly with moisture; whereas values for μ_{RL} and μ_{TL} are much more erratic. Calculated values for these two small constants, based on the assumption that lines in figure 3 representing the variation of $\frac{\mu_{RL}}{\mu_{LR}}$ and $\frac{\mu_{TL}}{\mu_{LT}}$ should coincide with the corresponding ratios of E, agree closely with values of these constants calculated directly from the elastic equations (table 3). Such calculated values are generally less than the measured values. This is shown graphically in figure 2 where the dashed lines represent the relations with moisture for μ_{RL} and μ_{TL}, which are required in order to have the ratios for μ coincide with those for E in figure 3.

The assumptions made in determining values for the two small Poisson's ratios as outlined in the foregoing discussion are the same as those used in analyzing the data for Sitka spruce.[4] In this instance, however, the variability of other elastic constants used in establishing these lines is considerably greater than it was for spruce, and the probability of μ_{RL} and μ_{TL} being the principal sources of error in failure of the elastic equations to show equality is not as clear as it was for spruce. As was pointed out for Sitka spruce, it must be recognized that the effect of moisture may, to a large extent, be obscured by the great variability existing in measured values of the elastic constants and their ratios.

Ultimate Strength

Average values from the ultimate strength tests for the fiber stress at proportional limit and maximum crushing strength are shown in table 4.

The Douglas-fir used in these tests appears to be somewhat lower in maximum crushing strength than average material reported in Technical Bulletin No. 479, "Strength and Related Properties of Woods Grown in the United States." While the tests were not adequate to provide a relationship between strength and specific gravity, it is reasonably certain that adjustment of strength to 0.46 specific gravity (average reported for Douglas-fir, three types, in Bulletin No. 479) would not be compensated for by the decrease in moisture content from 13.4 to 12 percent.

No definite maximum load could be determined in compression perpendicular to grain. The average stress at proportional limit for radial specimens was 429 pounds per square inch and for tangential specimens, 387 pounds per square inch.

Conclusions

1. The maximum crushing strength for the material used in this series of tests appears to be lower than the species average for Douglas-fir.

2. Both Young's moduli and Poisson's ratios were variable in these tests. The average values shown in table 1 must be considered as tentative, and should be adjusted when further data are available. It should be recognized that values for individual specimens may differ considerably from the averages.

Report No. 1528-D   -9-

3.   The data from these tests are not sufficient to establish definite rela-
tionships between specific gravity and the elastic constants.

4.   The three Young's moduli are affected by moisture, but to a different
degree.   The moduli in the direction perpendicular to the grain (E_R and
E_T) change with moisture at about 8 to 10 times the rate for the modulus
parallel to the grain (E_L).

5.   The six Poisson's ratios are likewise affected by moisture, each to a
different degree.   Three of the four larger ratios, μ_{LT}, μ_{RT}, and μ_{TR},
tend to become smaller with a decrease in moisture, while μ_{LR} and the
two smaller ratios, μ_{RL} and μ_{TL}, increase with a decrease in moisture,
though not consistently.

6.   Use of average ratios for $\dfrac{E_R}{E_L}$ and $\dfrac{E_T}{E_L}$ to calculate Young's moduli per-
pendicular to the grain is not recommended if direct measurements can
be made, because such ratios are greatly influenced by moisture.

Table 1.—Summary of Young's moduli and Poisson's ratios for Douglas-fir[1]

Series Number (1)	Average moisture content[1] (2)	Average specific gravity[1,2] (3)	Compression longitudinal (E_L) Compressometer on LR faces (4)	Compressometer on LT faces (5)	Average (6)	Compression radial (E_R) Compressometer on LR faces (7)	Compressometer on RT faces (8)	Average (9)	Compression tangential (E_T) Compressometer on LT faces (10)	Compressometer on RT faces (11)	Average (12)	E_R/E_L [4] (13)	E_T/E_L [5] (14)	μ_{LR} (15)	μ_{LT} (16)	μ_{RL} (17)	μ_{RT} (18)	μ_{TL} (19)	μ_{TR} (20)
	Percent		1,000 lb. per sq. in.	1,000 lb. per sq. in.	1,000 lb. per sq. in.	1,000 lb. per sq. in.	1,000 lb. per sq. in.	1,000 lb. per sq. in.	1,000 lb. per sq. in.	1,000 lb. per sq. in.	1,000 lb. per sq. in.								
10	7.5		2,332	2,296	2,314	220	235	224	158	162	160	0.097	0.069	0.316	0.412	0.037	0.519	0.027	0.379
	12.9	0.507	2,382	2,260	2,321	138	139	148	112	128	120	.059	.052	.280	.416	.036	.420	.031	.379
	20.3		2,150	2,282	2,216	114	110	112	76	77	76	.051	.034	.255	.519	.015	.566	.015	.405
20	7.3		2,413	2,297	2,355	233	209	221	172	182	177	.094	.075	.284	.461	.025	.356	.026	.334
	12.6	.537	2,317	2,274	2,296	192	178	185	128	144	136	.081	.059	.236	.504	.042	.399	.038	.367
	19.3		2,282	2,276	2,279	135	134	134	83	84	84	.059	.037	.295	.608	.021	.554	.025	.406
30	7.4		2,493	2,361	2,427	192	200	196	148	158	153	.081	.063	.341	.435	.027	.464	.022	.334
	12.7	.516	2,230	2,232	2,261	145	130	138	119	132	126	.061	.056	.318	.431	.030	.351	.034	.343
	20.8		2,184	2,178	2,181									.276	.472				
40	7.5		2,409	2,337	2,373				118	128	123		.052	.283	.444			.021	.386
	13.0	.492	2,313	2,227	2,270				100	99	100		.044	.226	.450			.024	.392
	19.8		2,183	2,200	2,192				62	63	62		.028	.282	.476			.019	.388
Average[6]	7.5		2,410	2,329	2,370	212	216	214	139	148	144	.090	.061	.295	.441	.030	.447	.023	.368
	12.9	.506	2,320	2,239	2,280	158	149	154	110	116	113	.068	.050	.292	.449	.036	.390	.029	.374
	19.9		2,189	2,217	2,203	125	122	124	70	71	70	.056	.032	.274	.496	.018	.560	.019	.396

[1] Values recorded for each series are the averages of from 6 to 24 specimens of all types.

[2] Specific gravity based on weight when oven dry and volume at approximately 12 percent moisture content.

[3] Values recorded for each series are the averages of from 2 to 12 specimens of each type.

[4] Column (9) divided by column (6).

[5] Column (12) divided by column (6).

[6] Values shown are averages for individual specimens (30 longitudinal, 6 radial, and 12 tangential).

Report No. 1528-D.

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Table 2. --Relations between the elastic constants of Douglas-fir[1]

Moisture content :	$\dfrac{\mu_{LR}}{E_L}$	:	$\dfrac{\mu_{RL}}{E_R}$	:	$\dfrac{\mu_{LT}}{E_L}$	:	$\dfrac{\mu_{TL}}{E_T}$	:	$\dfrac{\mu_{RT}}{E_R}$	:	$\dfrac{\mu_{TR}}{E_T}$
(1)	(2)	:	(3)	:	(4)	:	(5)	:	(6)	:	(7)
Percent :	Sq. in. per lb. $\times 10^{-6}$	:	Sq. in. per lb. $\times 10^{-6}$	:	Sq. in. per lb. $\times 10^{-6}$	:	Sq. in. per lb. $\times 10^{-6}$	:	Sq. in. per lb. $\times 10^{-6}$	:	Sq. in. per lb. $\times 10^{-6}$
7.5 :	0.124	:	0.140	:	0.186	:	0.160	:	2.09	:	2.56
12.9 :	.128	:	.234	:	.197	:	.257	:	2.53	:	3.31
19.9 :	.124	:	.145	:	.225	:	.271	:	4.52	:	5.66

[1] From theoretical relations between the constants, the values in column (2) should equal those in column (3); similarly, column (4) should equal (5), and column (6) should equal (7).

Table 3. --Comparison of measured values of μ_{RL} and μ_{TL} with values calculated by use of elastic-equations and with values determined from moisture relations

Average moisture content	Measured values (from table 1)		Calculated values (from elastic equations)[1]		Calculated values (from moisture relations)[2]	
	μ_{TL}	μ_{RL}	μ_{TL}	μ_{RL}	μ_{TL}	μ_{RL}
7.5	0.023	0.030	0.027	0.027	0.027	0.026
12.9	.029	.036	.022	.020	.022	.021
19.9	.019	.018	.016	.015	.016	.016

[1] $\mu_{RL} = \mu_{LR}\dfrac{E_R}{E_L}$, $\mu_{TL} = \mu_{LT}\dfrac{E_T}{E_L}$

[2] Values shown were determined from regression lines representing the relations of the elastic constants with variation in moisture. Values for μ_{LR}, μ_{LT}, E_R, E_T, and E_L were taken from curves shown in figures 1 and 2, and values for μ_{RL} and μ_{TL} calculated by means of the elastic relations shown in footnote 1.

Table 4. --Average values for properties of
Douglas-fir at ultimate strength

Source of data	Moisture content	Specific gravity[1]	Maximum crushing strength
	Percent		P.s.i.
Present tests	13.4	0.50	6,535
Bulletin 479[2]	12.0	.46	6,980

[1] Based on weight when oven dry and volume at test.

[2] "Strength and Related Properties of Woods Grown in the United States," Technical Bulletin No. 479, table 1, average of three types of Douglas-fir.

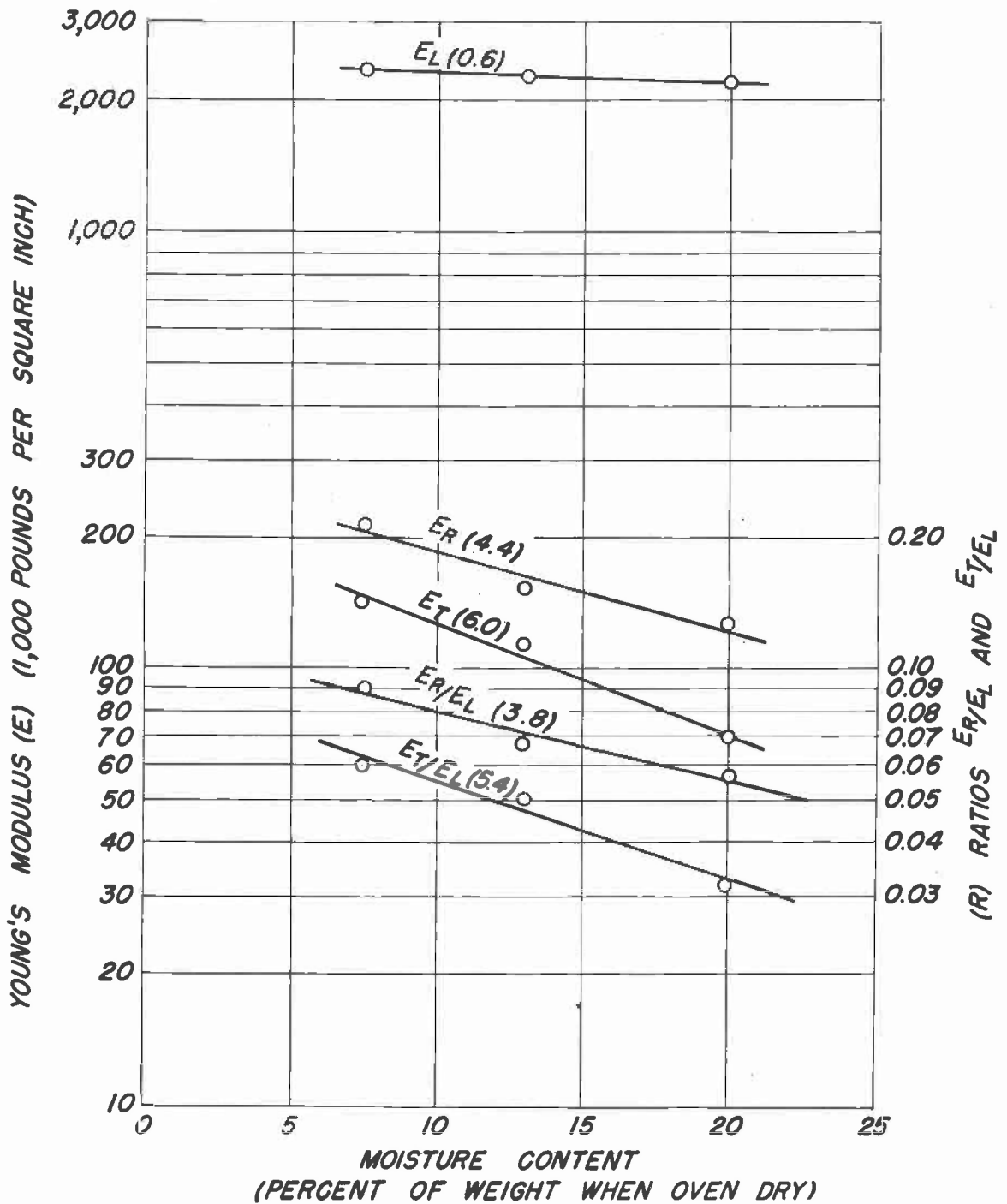Figure 1.--Average values of Young's moduli and their ratios, E_R/E_L, and E_T/E_L, plotted against the average moisture content for Douglas-fir specimens tested in compression. Figures in parentheses are the percentage increase in the moduli or ratios for 1 percent decrease in moisture content.
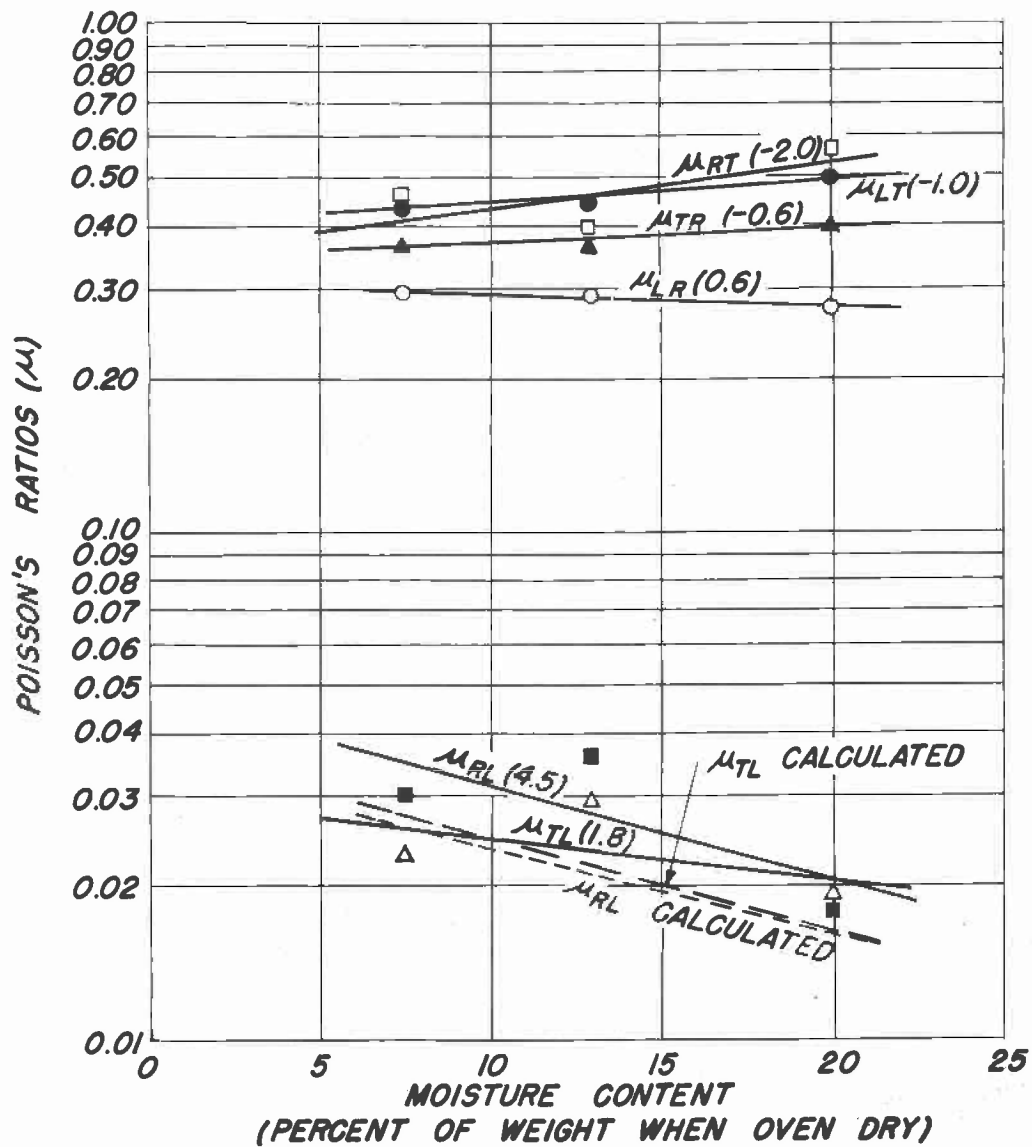
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Figure 2.--Average values of Poisson's ratios plotted against the average moisture content for Douglas-fir specimens tested in compression. Figures in parentheses are the percentage increase (positive values) or decrease (negative values) in the ratios for a 1 percent decrease in moisture content.
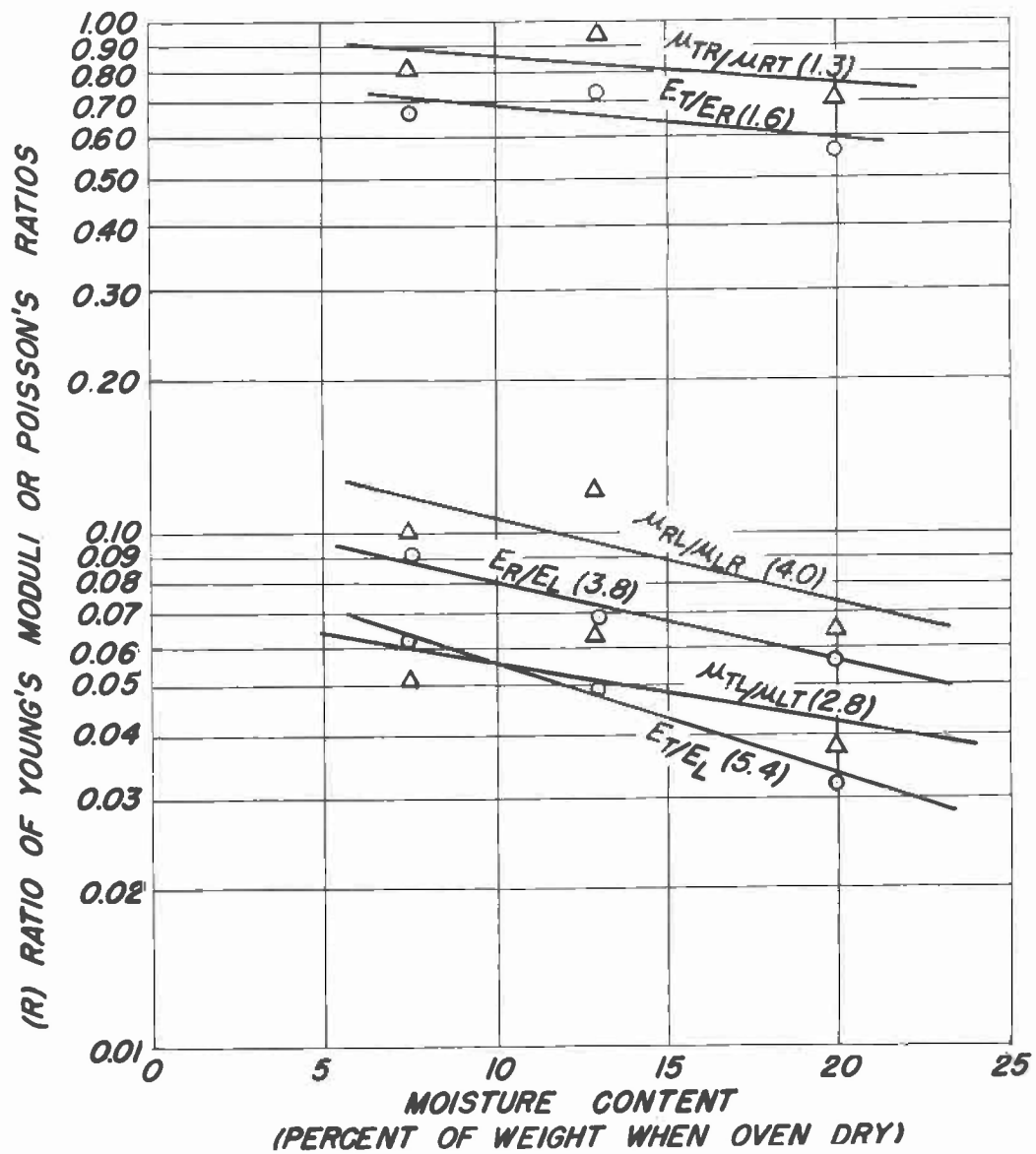
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Figure 3.--Relation of average values for ratios of Young's moduli
and Poisson's ratios to moisture content for Douglas-fir speci-
mens. Figures in parentheses are the percentage increase in the
ratios for a 1 percent decrease in moisture content.

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