BUCKLING OF THIN, CURVED, PLYWOOD PLATES IN AXIAL COMPRESSION

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BUCKLING OF THIN, CURVED, PLYWOOD PLATES IN AXIAL COMPRESSION[1]

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Summary

This report presents the results of tests made to determine the buckling stress of thin, curved, plywood plates under axial compression. The tests were made in an apparatus that was arranged to prevent change of curvature at the ends of the plates and to provide guides, not clamps, for the edges. The plates can thus be thought of as being simply supported. While it is apparent that a plate supported in this manner is not found in industrial or commercial products where the edges are usually fastened to stiffeners, this method of testing provides means of determining the effectiveness of thin plates in carrying loads without the complicated analysis that would be necessary if stiffeners were incorporated.

The conclusions are briefly that the buckling stresses of thin, curved plywood plates in axial compression are approximately equal to those of thin-walled plywood cylinders provided the width and length of the plates are each greater than their radius of curvature. It seems likely that the length effect on the buckling stress of such curved plates is the same as for thin-walled cylinders in compression. Plates of smaller widths and lengths exhibit higher buckling stresses.

Scope of the Work

The major portion of the testing was conducted on thin, curved, plywood panels of widths from 1/2 to 4 and lengths from 1 to 6 times the radius of curvature of the specimen. Both axial and circumferential directions of the face plies were used in the principal series of tests, and a few check tests were made on specimens having the grain direction of the plywood at an angle of 45° with the vertical.

As a check on the effect of the edge guides a few tests made on cylinders fitted with internal "spiders" to restrain buckling were compared with tests on curved panels of matched material.

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[1] This mimeograph is one of a series of progress reports prepared by the Forest Products Laboratory to further the Nation's war effort. Results here reported are preliminary and may be revised as additional data become available

Buckling stresses of restrained cylinders were also compared with theoretical buckling stresses of unrestrained cylinders.

Description of Specimens

All specimens were made of yellow birch or yellow-poplar aircraft grade veneers, rotary cut at the Forest Products Laboratory. The plywood was glued into flat sheets in a hot press using Tego film glue. To form these flat sheets to the proper radius of curvature they were wet and bent over a hot mandrel. Prior to forming the cylinders the sheets were scarf cut. The scarf gluing was accomplished on the hot mandrel.

Sheets of matched material were pressed flat and subsequently cut into minor coupons for testing in bending, compression, and tension. The matching was accomplished by using veneers obtained from the same portion of the log as were the veneers in the curved plates and cylinders.

While none of the material was conditioned prior to testing, it was allowed to remain in the testing laboratory for about a week and then tested.

The major portion of the plywood consisted of 3-ply material with faces of 1/100-inch veneers and cores of 1/40-inch veneers. Some of the plywood used in the restrained cylinder study was of 5-ply material with all veneers 1/100-inch thick.

All specimens in this study, curved plates and cylinders, were formed to a 5-1/4-inch radius of curvature.

Apparatus and Testing Methods

The apparatus used to test the curved plates was arranged to simply support the vertical edges of the plate (fig. 1). The top of the specimen was allowed to bear on the edge guide support plate fastened to the upper head of the testing machine, and the bottom rested on a flat plate mounted on a spherical bearing on the lower head of the machine. The bottom plate was adjusted to assure uniform bearing, and screw jacks were placed to prevent tilting. A strap of webbing was passed around the ends of the cylinder to hold the ends tightly to the end plates. The separation of the edge guides to receive the specimen was adjusted by means of feeler gages to provide a uniform clearance of about 0.002 inch along the entire length of the specimen. The edge guides were such as to restrain the curved plates in the radial direction only and afforded no restraint in the circumferential direction. The load was applied at a slow uniform rate until failure occurred.

The cylinders that were fitted with internal restraining spiders were loaded in the same manner as the curved plates. The internal spider was arranged to prevent the cylinder from buckling in certain places by holding 6 or 12 equally spaced radial vanes against the inner wall of the cylinder.

These vanes were not glued to the specimen. The vanes were made about 1/4 inch shorter than the length of the specimen so that they would carry no load.

Methods of testing minor coupons in bending, compression, and tension are described in the appendix.

Results of Tests

All specimens (curved plates, cylinders, or restrained cylinders) failed by buckling of the plywood. While this was usually a sudden failure. all buckles appearing instantly, it was occasionally of a progressive nature, that is, an initial buckle would develop, then a second, third, and so on until maximum load was reached. The progressive buckling occurred rapidly, and the difference between the load at the initial buckle and the maximum load was usually indiscernible.

In general, the shape of the buckles (figs. 2, 3, 4, and 5) was essentially the same as those found in cylinders under compression, although the size and the aspect ratio changed when the width of the plate was decreased below the radius of curvature of the specimen. The size of the buckle appearing in the narrower plates was smaller than in cylinders, and the aspect ratio approached unity.

Long specimens, 1/2 radian wide, of plywood with the face grain direction axial, flattened when the load was applied because of the small stiffness of the panel in its circumferential direction.

Cylinders fitted with restraining spiders failed by buckling. The shape of the buckles was the same as that of the curved panels of comparable widths. It was difficult to adjust the vanes on the spider to fit the exact inside of the cylinder. Evidence substantiating this was seen occasionally when the width of the buckle was greater than the spacing of the vanes. This also may have been due to the fact that buckles in adjacent bays caused some increase in diameter of the specimen thus allowing the buckle to increase in width.

Computation of Results

The buckling stresses were computed in the usual manner, by dividing the buckling load by the loaded area. From previous work[2] the buckling stress for thin-walled plywood cylinders in axial compression was found to be expressed by the formula:

$$p = kE_L \frac{h}{r}$$

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[2] Mimeo. No. 1322, "Buckling of Long, Thin, Plywood Cylinders in Axial Compression."

where:

p = buckling stress
k = a constant depending on the ratio $\frac{E_1}{E_1 + E_2}$ and defined in Mimeo. 1322
E_L = modulus of elasticity of the veneers measured parallel to the grain
h = thickness of the plywood
r = radius of curvature of the plywood

Values of E_L were determined from bending, tension, and compression tests on minor specimens of the plywoods. The method is discussed in the following:

For any plywood construction having the grain of the face plies either parallel or perpendicular to the span, the modulus of elasticity (E_1 or E_2) in bending is related to E_L by the formula:

$$E_1 \text{ or } E_2 = \frac{\sum_{i=1}^{i=n} \frac{E_i}{E_L} I_i}{I} E_L = KE_L$$

where:

I = moment of inertia of the entire cross section about its centerline ($I = \frac{bh^3}{12}$)

I_i = moment of inertia of the i^{th} ply about the neutral axis of the cross section

E_i = modulus of elasticity of the i^{th} ply, measured parallel to the span (for plies having the face grain parallel to the span $E_i = E_L$; for plies having face grain perpendicular $E_i = 0.045E_L$ for yellow birch and $E_i = 0.037E_L$ for yellow-poplar)

The values of E_1 and E_2 were determined from the test data on a plywood beam simply supported at the ends and loaded at the center, by using the formula $E = \frac{Pl^3}{48 \Delta I}$. The values of E_L were then computed by use of the values of K given in table 1.[3]

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[3] Table 1 is the same as Table 7 of Forest Products Laboratory Mimeo. 1322-B.

For any plywood construction having the grain of the face plies either parallel or perpendicular to the direction of the applied load the modulus of elasticity (E_a or E_b) in tension or compression is related to E_L by the formula:

$$E = \frac{\sum_{i=1}^{i=n} \frac{E_i}{E_L} A_i}{A} E_L = CE_L$$

where:

A = area of the entire cross section (A=bd)

A_i = area of the i^{th} ply

Values of E_a and E_b were obtained from compression and tension test data by using the formula $E = \frac{Pl}{Ae}$. The values of E_L were then computed by use of the values of C given in table 1.

Values of E_L as found from several tests of each kind (bending, tension, and compression) and in each direction on coupons from the matched flat plates of plywood were averaged to get the value applicable to a particular curved plate or cylinder. The values of E_2 as found from the bending tests of the plywood were erratic, and many were for this reason omitted from the averages. Furthermore, the valid values were too few to be used as a basis for computing the ratio $\frac{E_1}{E_1 + E_2}$ for use in finding the value of k to be used in the buckling formula. Consequently, the equivalent ratio $\frac{K_1}{K_1 + K_2}$ was in each instance computed from the data of table 1 which have been derived from more adequate tests and are hence considered more appropriate for the purpose. Using these computed values of $\frac{K_1}{K_1 + K_2} = \frac{E_1}{E_1 + E_2}$ values of k for use in the buckling formula were read from the graph in figure 6.

Discussion of Results

Experimental values of k as computed from the results of tests on curved plates using the relation $k = \frac{pr}{E_L h}$ with values of p as found in these tests and values of E_L as derived from coupons taken from matched plywood panels are listed in column 7 of tables 2 and 3. Column 8 lists for comparison theoretical values of k as found by the use of figures 6 from the ratio

$\frac{K_1}{K_1 + K_2}$. The actual buckling stress as found in test is shown in column 4 and the theoretical value in column 9. The theoretical value was found from the buckling formula ($p = kE_L \frac{h}{r}$) using the theoretical values of k and the values of E_L derived from tests on the coupons from matched panels.

Curved Plates -- Face Grain Axial and Circumferential

The effect of length of specimen as shown in figure 7 is greater for specimens with face grain direction axial than for specimens with face grain direction circumferential. In both instances the length effect is not significant until the length is shorter than twice the radius of curvature of the specimen. A decrease in length from two radii to one radius increases the k about 20 percent.

The effect of width of specimen as shown in figure 8 is not apparent until the width is less than one radian. A decrease in width from one radian to one-half radian increases the k about 10 percent.

The points plotted in figure 9 show about the same scatter as points in a similar plot for thin plywood cylinders in compression (fig. 10).[4]

Curved Plates -- Face Grain at 45 Degrees

Figure 11 presents a comparison between curved panels of plywood with the grain directions of the veneers at 45° to the axis of the plate and cylinders of matched material. Experimentally determined values of k are plotted. The points representing specimens of lengths greater than the radius of curvature agree closely with theoretical values inasmuch as k is essentially the same for plates and cylinders. The points representing shorter specimens show an increase in the buckling stress of the curved plate as compared to that of the long cylinder. This increase is about 20 percent, or about the same as for specimens having face grain directions axially or circumferentially.

Restrained Cylinders -- Face Grain Axial and Circumferential

Table 4 presents the data obtained from tests of restrained cylinders. The experimental results agree fairly well with theoretical values.

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[4] Figure 3 from Mimeo. 1322, "Buckling of Long, Thin Plywood Cylinders in Axial Compression."

Curved Plates Compared with Restrained Cylinders -- Face Grain Axial and Circumferential

Table 5 and figure 12 present the results of tests on curved plates compared with restrained cylinders in compression. Due to the difference in edge conditions, the restrained cylinders exhibit buckling stresses about 20 percent greater than the curved panels. The curved plate edge is free to hinge and free to move circumferentially, while in a cylinder the continuation of material over a restraining vane provides more of a fixed edge condition.

In general, much of the variation in test results is accredited to the fact that small initial imperfections may cause buckling. This variation may also be due to differences in the dimensions of the specimen and in the mechanical properties of the materials. Factors affecting thicknesses may be due to difficulties encountered in manufacture, such as the cutting of the veneer, shrinkage during drying, pressing the plywood, and many others. Because of the variation of the mechanical properties of the material it is conceivable that even though a minor specimen was cut from a portion of the log adjacent to the major specimen and had a similar specific gravity and moisture content, the mechanical properties might differ to some degree.[5]

Figure 13 presents a design curve for long, thin-walled plywood cylinders and can be used for designing curved plates.

Conclusions

The buckling stresses of thin, curved, plywood plates simply supported at the edges and loaded in axial compression are approximately equal to those of thin-walled plywood cylinders provided that the width and length are each greater than the radius of curvature of the specimen. A decrease in width from one radian to one-half radian increases the k about 10 percent. A decrease in length of specimen from two radians to one radian increases the k about 20 percent.

APPENDIX

Tests of Minor Specimens

Flat plates of plywood that matched the material of the cylinders and curved plates furnished specimens for minor tests in bending, tension, and compression.

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[5] See points on curves for strength-moisture and strength-specific gravity relations for wood as given in Technical Bulletin 479, "Strength and Related Properties of Woods Grown in the United States."

Bending specimens were 1 inch wide and were tested on a span length 48 or more times the thickness of the specimen for face grain direction parallel to the span and 24 or more times the thickness for face grain direction perpendicular to the span, thus eliminating the necessity of correcting for shear deformation. Specimens were cut so that the face grain direction was parallel to the span and perpendicular to the span. Specimens rested on end supports rounded to about 1/16-inch radius. The load was applied at the center with a bar of about 1/8-inch radius. The stiffer specimens were tested on an apparatus that measured the load by means of a platform scale reading to 1/100 pound. The rate of loading and the driving force was obtained from a testing machine on which the scale was mounted (fig. 14). Specimens not stiff enough to test on the platform scale apparatus were tested on an apparatus which used water as a load. The load increments could be read to 10 grams, and the rate of loading was controlled by the size of nozzle (fig. 15). Immediately after testing, a sample was cut from the specimen for moisture content determination. The specific gravity was obtained from measurements, and weights were taken before testing and subsequently corrected for moisture content. During the test, load-deflection curves were plotted. Properties computed from the test data were the modulus of elasticity, the fiber stress on the most remote fiber at the proportional-limit load, the modulus of rupture, the moisture content, and the specific gravity. The modulus of rupture could not always be computed because the specimen sometimes was pulled down between the supports and wood failure did not occur.

The tension specimens were 1 inch wide and 11 inches long with face grain directions parallel and perpendicular to the direction of the load. The ends of the specimens were gripped in Templin grips fastened to the testing machine by bolts passed through spherical bearings. The strain over a 2-inch gage length was measured at the center of the specimen with a Tripolitis extensometer reading to 0.0001 of an inch (fig. 16). The rate of loading was 0.05 inch per minute. During the test, load-deformation curves were plotted. Properties computed from the test data were the modulus of elasticity, the tensile stress at the proportional-limit load, the tensile strength, the moisture content, and the specific gravity. Moisture content and specific gravity were determined in the same manner as they were for the bending specimens.

The compression specimens were 1 inch wide and 4 inches long with face grain directions parallel and perpendicular to the direction of the load. Because of the thinness of the specimen an apparatus consisting of thin spring fingers was used to give lateral support (fig. 17). The strain was measured over a 2-inch gage length with a Marten's mirror apparatus. Load-deformation curves were plotted as the test proceeded. Properties computed from the test data were the modulus of elasticity, the compressive stress at the proportional-limit load, the compressive strength, the moisture content, and the specific gravity. The moisture content and specific gravity were determined in the same manner as for the bending specimens.

Table 1.--Values of K and C used for computing E_L

Plywood construction	Bending parallel		Bending perpendicular		Tension or compression parallel		Tension or compression perpendicular	
	Yellow-birch K_1	Yellow-poplar K_1	Yellow-birch K_2	Yellow-poplar K_2	Yellow-birch C	Yellow-poplar C	Yellow-birch C	Yellow-poplar C
3-ply (1:1:1)	0.965	0.965	0.080	0.072	0.682	0.679	0.363	0.358
3-ply (1:1.25:1)	.945	.944	.100	.093	.632	.629	.413	.408
3-ply (1:1.67:1)	.912	.910	.133	.127	.566	.562	.479	.475
3-ply (1:2.5:1)	.836	.834	.209	.203	.470	.465	.575	.572
5-ply (1:1:1:1:1)	.800	.800	.245	.237	.618	.614	.427	.423
5-ply (1:1.25:1.25:1.25:1)	.744	.742	.301	.295	.584	.582	.461	.456
5-ply (1:1.67:1.67:1.67:1)	.664	.661	.381	.276	.545	.541	.500	.496
5-ply (1:2.5:2.5:2.5:1)	.547	.544	.498	.493	.498	.493	.547	.544
5-ply (1:1.25:1.67:1.25:1)	.724	.722	.321	.315	.613	.610	.432	.427
7-ply (1:1:1:1:1:1:1)	.724	.722	.321	.315	.590	.588	.455	.449
7-ply (1:1.67:1.67....:1)	.604	.600	.441	.437	.538	.534	.507	.503
9-ply (1:1:....:1)	.681	.678	.364	.359	.575	.572	.470	.465
9-ply (1:1.25:....:1)	.632	.629	.413	.408	.556	.552	.489	.485

Table 2.--A comparison of experimental and theoretical buckling constants for thin, curved plates of 3-ply yellow birch plywood in axial compression (face grain direction axial).[1]

Specimen number	Length	Width	Buckling stress p_T	Compressive stress at proportional limit load[2] $\sigma_{P.L.}$	Modulus of elasticity[2] E_L	Experimental $k = \frac{pr}{E_L h}$	Theoretical[2] k	Theoretical buckling stress p_c	$\frac{p_T}{\sigma_{P.L.}}$	$\frac{p_c}{\sigma_{P.L.}}$	Moisture content	Specific gravity
	Inches	Radians	Lb. per sq. in.	Lb. per sq. in.	1,000 lb. per sq. in.			Lb. per sq. in.			Per-cent	
[3]41	30	0.5										
[3]43	20	0.5										
[3]45	10	0.5										
47	5	0.5	2,528	4,270	2,746	0.1134	0.1020	2,274	0.592	0.533	4.7	0.62
1	30	1	1,402	3,840	2,605	.0628	.1020	2,272	.365	.593	4.6	.55
5	20	1	1,688	3,800	2,487	.0787	.1020	2,188	.444	.576	4.0	.54
9	15	1	1,828	4,200	2,380	.0904	.1020	2,063	.435	.491	3.5	.55
13	10	1	2,202	2,990	2,616	.1037	.1020	2,166	.736	.724	4.9	.60
17	5	1	2,330	3,680	2,291	.1234	.1020	1,926	.633	.523	4.4	.57
2	30	2	2,107	3,920	2,327	.1082	.1020	1,986	.538	.507	4.4	.56
6	20	2	1,879	3,530	2,501	.0891	.1020	2,151	.532	.609	3.5	.56
10	15	2	1,771	3,880	2,292	.0923	.1020	1,957	.456	.504	3.7	.54
14	10	2	2,035	2,710	2,645	.0941	.1020	2,206	.751	.814	4.4	.59
18	5	2	2,368	4,200	2,330	.1233	.1020	1,959	.564	.466	4.5	.56
3	30	3	1,782	3,860	2,518	.0846	.1020	2,149	.462	.557	4.0	.55
7	20	3	1,780	3,260	2,367	.0919	.1020	1,976	.546	.606	3.8	.55
11	15	3	1,740	4,160	2,247	.0919	.1020	1,931	.418	.464	3.9	.54
15	10	3	1,877	3,000	2,612	.0878	.1020	2,181	.627	.727	4.3	.59
19	5	3	2,295	----	2,290	.1264	.1020	1,852	----	----	3.9	.57
4	30	4	1,756	4,820	2,619	.0802	.1020	2,233	.364	.463	3.3	.56
8	20	4	1,494	4,660	2,650	.0679	.1020	2,244	.321	.482	3.8	.56
12	15	4	1,835	3,480	2,243	.0949	.1020	1,972	.527	.567	4.0	.54
16	10	4	2,104	3,640	2,318	.1101	.1020	1,949	.578	.535	4.7	.55
20	5	4	2,436	4,160	2,377	.1292	.1020	1,923	.586	.462	4.0	.58

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[1] All specimens were formed to a radius of 5-1/4 inches. The face plies were 0.010 inch thick. The core ply was 0.025 inch thick. Each value represents the average of three tests.

[2] These values were obtained from test data of minor coupons.

[3] These specimens were so narrow that they would not maintain their formed radius but flattened after a small load was applied.

Table 3.--A comparison of experimental and theoretical buckling constants for thin, curved plates of 3-ply yellow birch plywood in axial compression (face grain direction circumferential).[1]

Specimen number	Length	Width	Buckling stress p_T	Compressive stress at proportional limit load[2] $\sigma_{P.L.}$	Modulus of elasticity[2] E_L	Experimental $k = \frac{pr}{E_L h}$	Theoretical[2] k	Theoretical buckling stress p_c	$\frac{p_T}{\sigma_{P.L.}}$	$\frac{p_c}{\sigma_{P.L.}}$	Moisture content	Specific gravity
	Inches	Radians	Lb. per sq. in.		1,000 lb. per sq. in.			Lb. per sq. in.			Percent	
42	30	0.5	2,659	4,880	2,556	0.1281	0.1122	2,329	0.545	0.477	4.2	0.62
44	20	0.5	2,407	3,560	2,465	.1212	.1122	2,228	.676	.626	4.3	.62
46	10	0.5	2,601	4,080	2,833	.1131	.1122	2,580	.638	.632	4.2	.61
48	5	0.5	2,814	5,040	2,556	.1346	.1122	2,346	.558	.465	3.7	.63
21	30	1	2,046	3,340	2,476	.0952	.1122	2,411	.613	.722	4.1	.53
25	20	1	2,372	2,320	2,386	.1153	.1122	2,308	1.022	.995	3.4	.54
29	15	1	2,332	3,340	2,147	.1270	.1122	2,060	.698	.617	3.6	.53
33	10	1	2,457	3,480	2,622	.1154	.1122	2,389	.706	.686	3.8	.61
37	5	1	2,436	3,420	2,379	.1247	.1122	2,192	.712	.641	4.4	.57
22	30	2	2,183	3,020	2,279	.1128	.1122	2,171	.723	.719	4.0	.54
26	20	2	2,111	2,620	2,327	.1092	.1122	2,169	.806	.828	3.8	.56
30	15	2	2,001	2,550	2,653	.0895	.1122	2.509	.785	.984	3.9	.55
34	10	2	2,298	2,010	2,831	.1008	.1122	2,558	1.143	1.273	4.7	.60
38	5	2	2,312	2,920	2,275	.1243	.1122	2,087	.792	.715	4.1	.55
23	30	3	2,040	2,660	2,388	.1036	.1122	2,209	.767	.830	4.0	.55
27	20	3	1,927	2,730	2,416	.0946	.1122	2,286	.706	.837	3.8	.55
31	15	3	2,145	3,450	2,583	.0992	.1122	2,426	.622	.703	4.1	.54
35	10	3	2,235	2,120	2,600	.1059	.1122	2,368	1.054	1.117	3.8	.60
39	5	3	2,280	3,920	2,174	.1312	.1122	1,950	.582	.497	4.0	.58
24	30	4	2,154	3,680	2,331	.1130	.1122	2,139	.585	.581	3.3	.57
28	20	4	1,856	1,760	2,368	.0944	.1122	2,206	1.055	1.253	3.6	.55
32	15	4	1,995	3,520	2,460	.0969	.1122	2,310	.567	.656	4.7	.51
36	10	4	2,280	3,000	2,402	.1151	.1122	2,223	.760	.741	4.3	.54
40	5	4	2,261	2,840	2,283	.1220	.1122	2,079	.796	.732	3.7	.57

[1] All specimens were formed to a radius of 5-1/4 inches. The core ply was 0.025 inch thick. The face plies were 0.010 inch thick. Each value represents the average of three tests.

[2] These values were obtained from test data of minor coupons.

Table 4.--A comparison of experimental and theoretical buckling constants for restrained plywood cylinders 10-1/2 inches in diameter in axial compression.[1]

Specimen number	Face grain direction	Number of vanes on spider	Length	Buckling stress p	[2]E_L	Experimental $k = \frac{pr}{E_L h}$	Theoretical[2] k	$\frac{\text{Exp.k}}{\text{Theo.k}}$
			Inches	Lb. per sq. in.	1,000 lb. per sq. in.			
6	Axial	0	24	1,745	2,740	0.0785	0.1022	0.768
1	Axial	6	24	1,860	2,370	.0958	.1022	.937
2	Axial	12	24	1,732	2,770	.0763	.1022	.746
3	Axial	12	24	2,052	2,560	.0967	.1022	.946
4	Axial	12	24	2,156	2,490	.1068	.1022	1.045
5	Axial	12	24	2,107	2,810	.0913	.1022	.894
9	Circumferential	0	24	2,270	2,630	.1064	.1115	.954
10	Circumferential	6	24	2,103	2,750	.0924	.1115	.828
7	Circumferential	12	24	2,493	2,630	.1171	.1115	1.050
8	Circumferential	12	24	2,617	2,570	.1259	.1115	1.129
11	Circumferential	12	24	2,524	2,700	.1151	.1115	1.031
12	Circumferential	12	24	2,293	2,540	.1109	.1115	.994

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[1] Restraint was provided by vanes of an internal spider touching but not glued to the plywood. All plywood was of 3-ply yellow birch, faces 0.010 inch and core 0.025 inch.

[2] These values were obtained from test data of minor coupons.

Table 5.--A comparison of the buckling stress of thin, curved, plywood plates in axial compression with the buckling stress of restrained plywood cylinders in axial compression.[1]

Specimen number	Species	Number of plies	Ply thicknesses			Modulus of elasticity[2]	Curved plate				Restrained cylinder		
			Face	Cross	Core	E_L	Width	Length	Buckling stress p	$k = \frac{pr}{E_L h}$	Number of vanes on spider	Buckling stress p	$k = \frac{pr}{E_L h}$
			Inch	Inch	Inch	1,000 lb. per sq. in.	Radians	Inches	Lb. per sq. in.			Lb. per sq. in.	
						Face grain direction axial							
B48	Yellow birch	3	0.0100	---	0.0250	2,413	1	30	1,811	0.0911	12	2,106	0.1043
B50	Yellow birch	3	.0100	---	.0250	2,150	0.5	5	1,508	.0808	12	2,087	.1161
B53	Yellow birch	5	.0100	0.0125	.0125	2,542	0.5	30	2,300	.0867	12	3,585	.1338
P48	Yellow-poplar	3	.0100	---	.0250	1,634	1	30	1,156	.0815	6	1,476	.1076
P50	Yellow-poplar	3	.0100	---	.0250	1,717	0.5	5	1,559	.0963	12	1,597	.1138
P53	Yellow-poplar	5	.0100	.0125	.0125	1,918	0.5	30	2,043	.0947	12	2,621	.1277
						Face grain direction circumferential							
B49	Yellow birch	3	.0100	---	.0250	2,245	1	30	2,202	.1146	12	2,424	.1262
B51	Yellow birch	3	.0100	---	.0250	2,292	0.5	5	1,865	.0951	12	2,196	.1132
B52	Yellow birch	3	.0100	---	.0250	2,330	0.5	30	2,185	.1088	12	2,257	.1142
B54	Yellow birch	5	.0100	.0125	.0125	2,615	0.5	30	2,739	.0970	12	3,643	.1296
P49	Yellow-poplar	3	.0100	---	.0250	1,785	1	30	1,259	.0795	6	1,366	.0940
P51	Yellow-poplar	3	.0100	---	.0250	1,818	0.5	5	1,264	.0807	12	1,638	.1062
P52	Yellow-poplar	3	.0100	---	.0250	1,942	0.5	30	1,387	.0817	12	1,684	.1088
P54	Yellow-poplar	5	.0100	.0125	.0125	1,932	0.5	30	1,772	.0834	12	2,333	.1134

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[1] All specimens were formed to a 5-1/4-inch radius. All restrained cylinders were 30 inches long. Each value represents the average of three tests.

[2] These values were obtained from test data of minor coupons.

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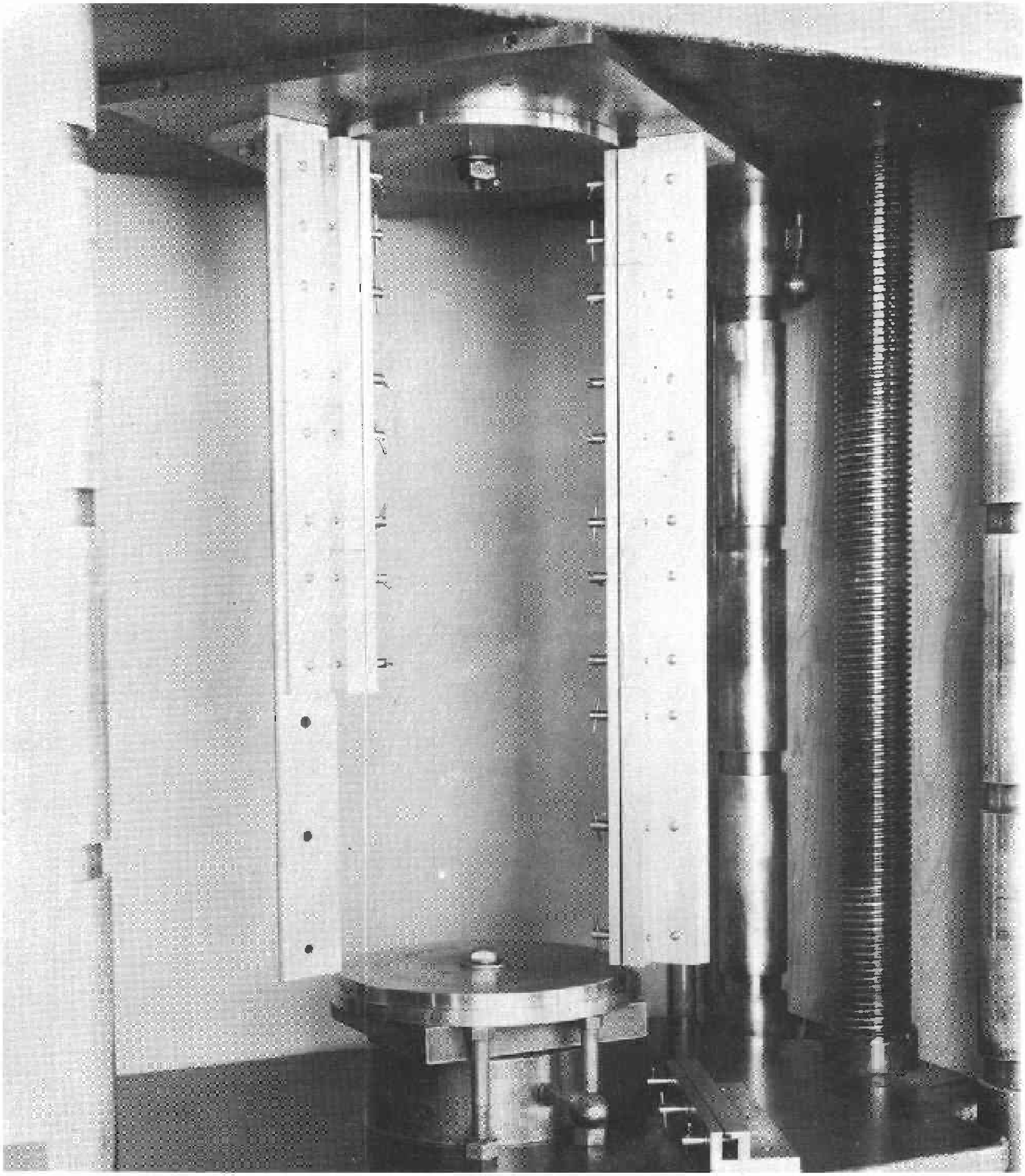

Figure 1.--Apparatus for testing curved plates in axial compression. The edge guides can be removed and the bottom head blocked so that short specimens can be tested. Width adjustment is obtained by fastening the edge guide supports at different positions in the upper plate.

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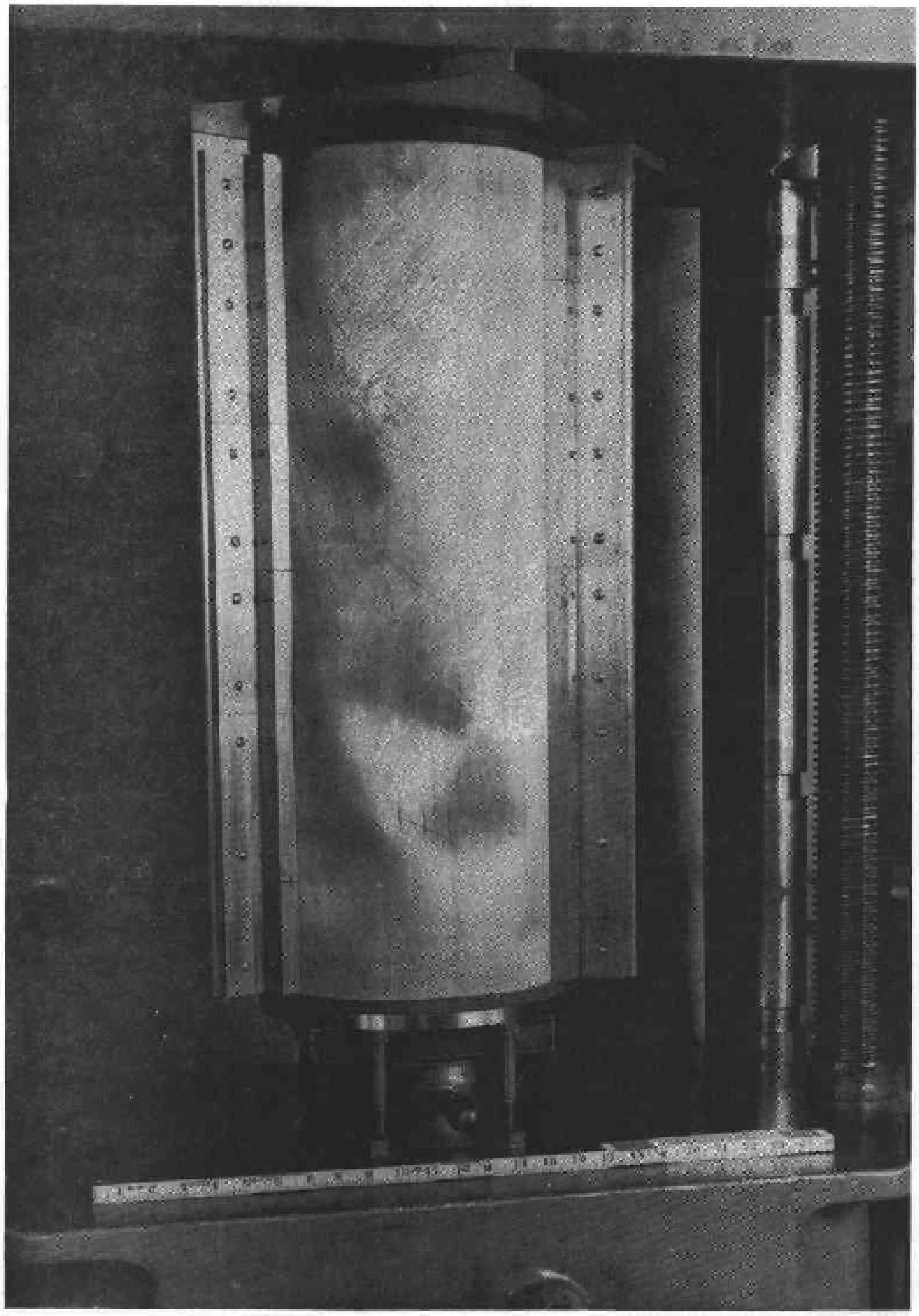

Figure 2.--Buckling failure of a curved plywood plate in axial compression. The plywood is 3-ply yellow birch. The faces are of 0.010-inch veneers, and the core is of 0.025-inch veneers. The face grain direction is axial. The specimen is formed to a 5-1/4-inch radius and is 2 radians wide and 30 inches long.

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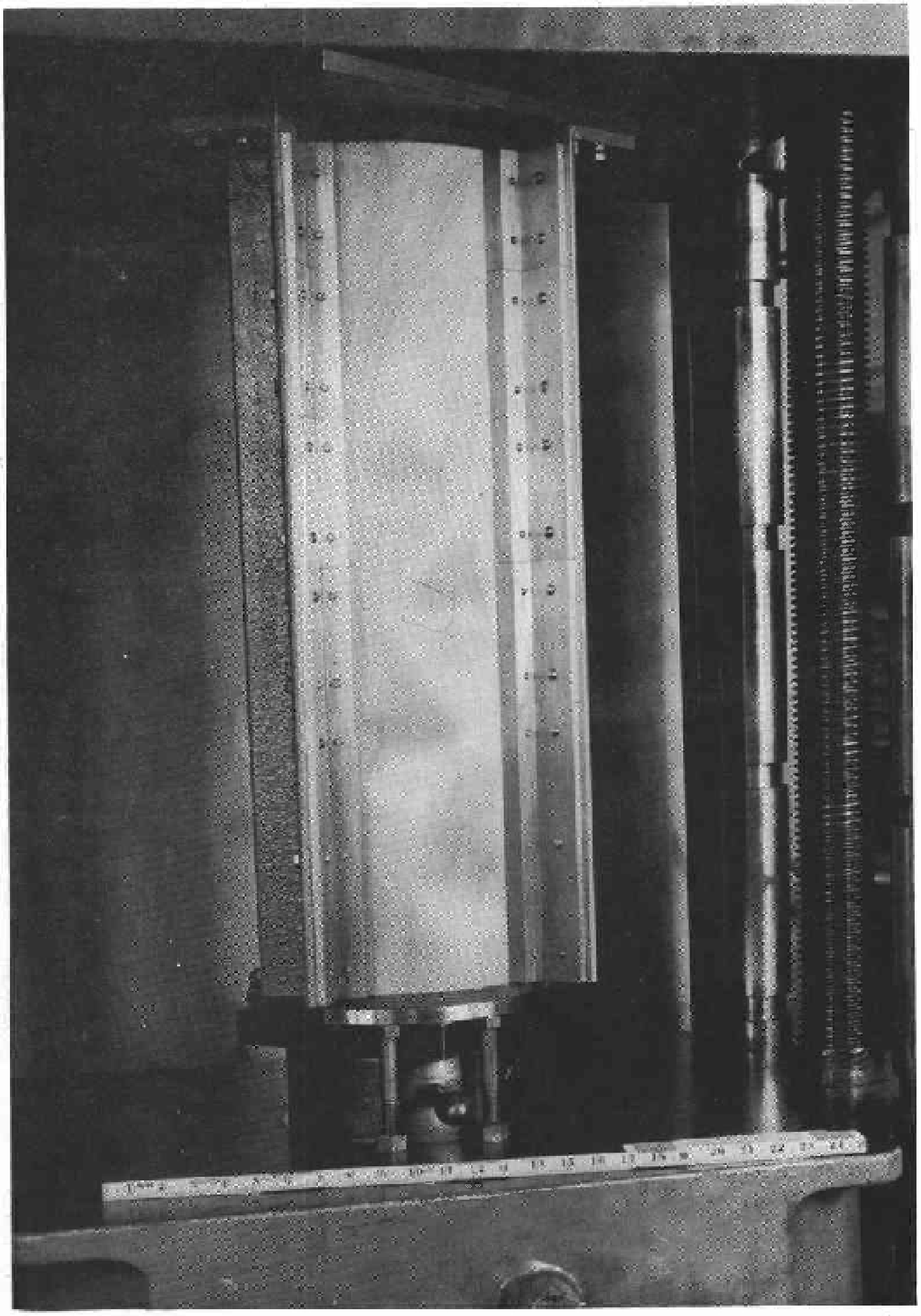

Figure 3.--Buckling failure of a curved plywood plate in axial compression. The plywood is 3-ply yellow birch. The faces are of 0.010-inch veneers, and the core is of 0.025-inch veneers. The face grain direction is axial. The specimen is formed to a 5-1/4-inch radius and is one radian wide and 30 inches long.

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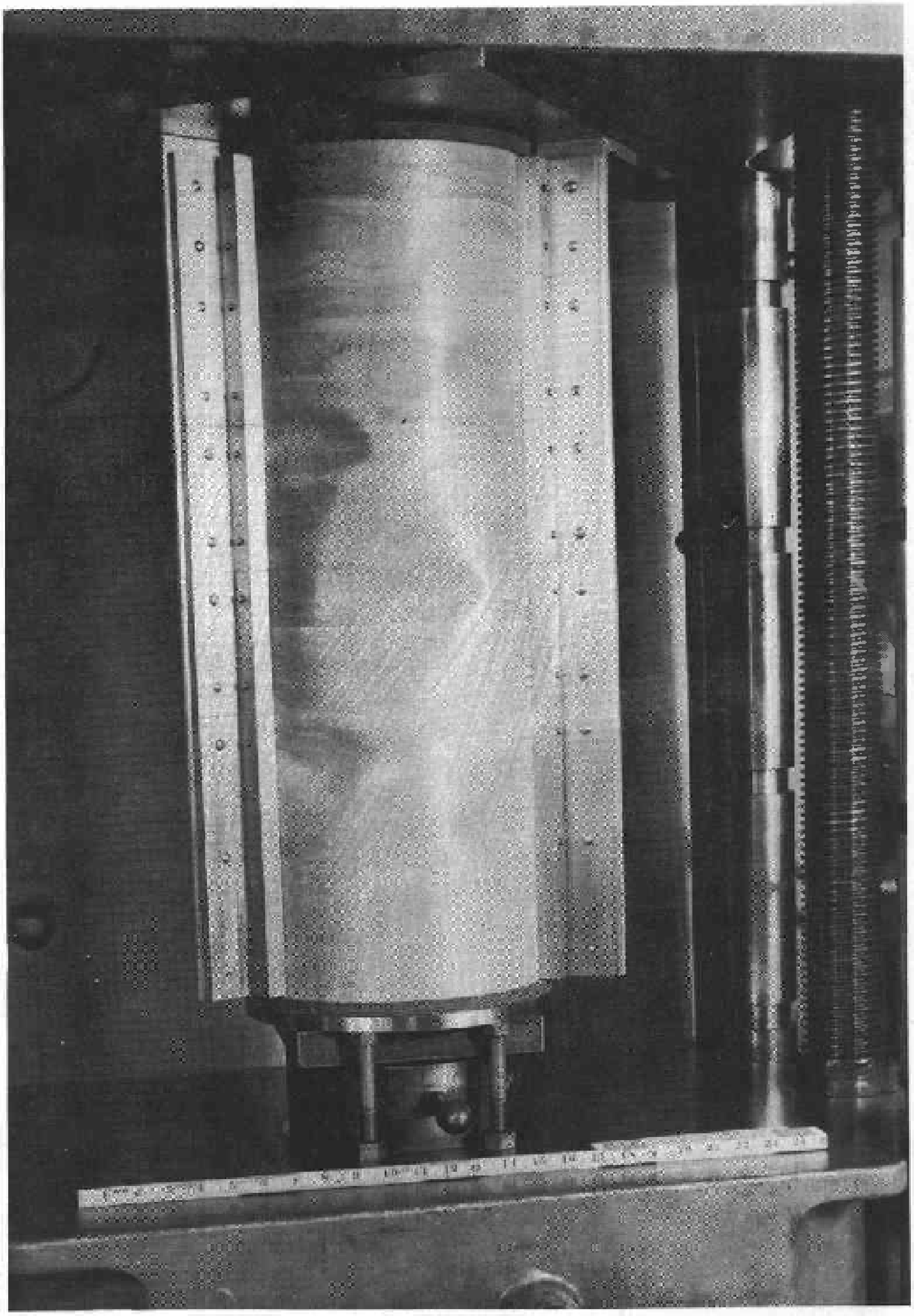

Figure 4.--Buckling failure of a curved plywood plate in axial compression. The plywood is 3-ply yellow birch. The faces are of 0.010-inch veneers, and the core is of 0.025-inch veneers. The face grain direction is circumferential. The specimen is formed to a 5-1/4-inch radius and is 2 radians wide and 30 inches long.

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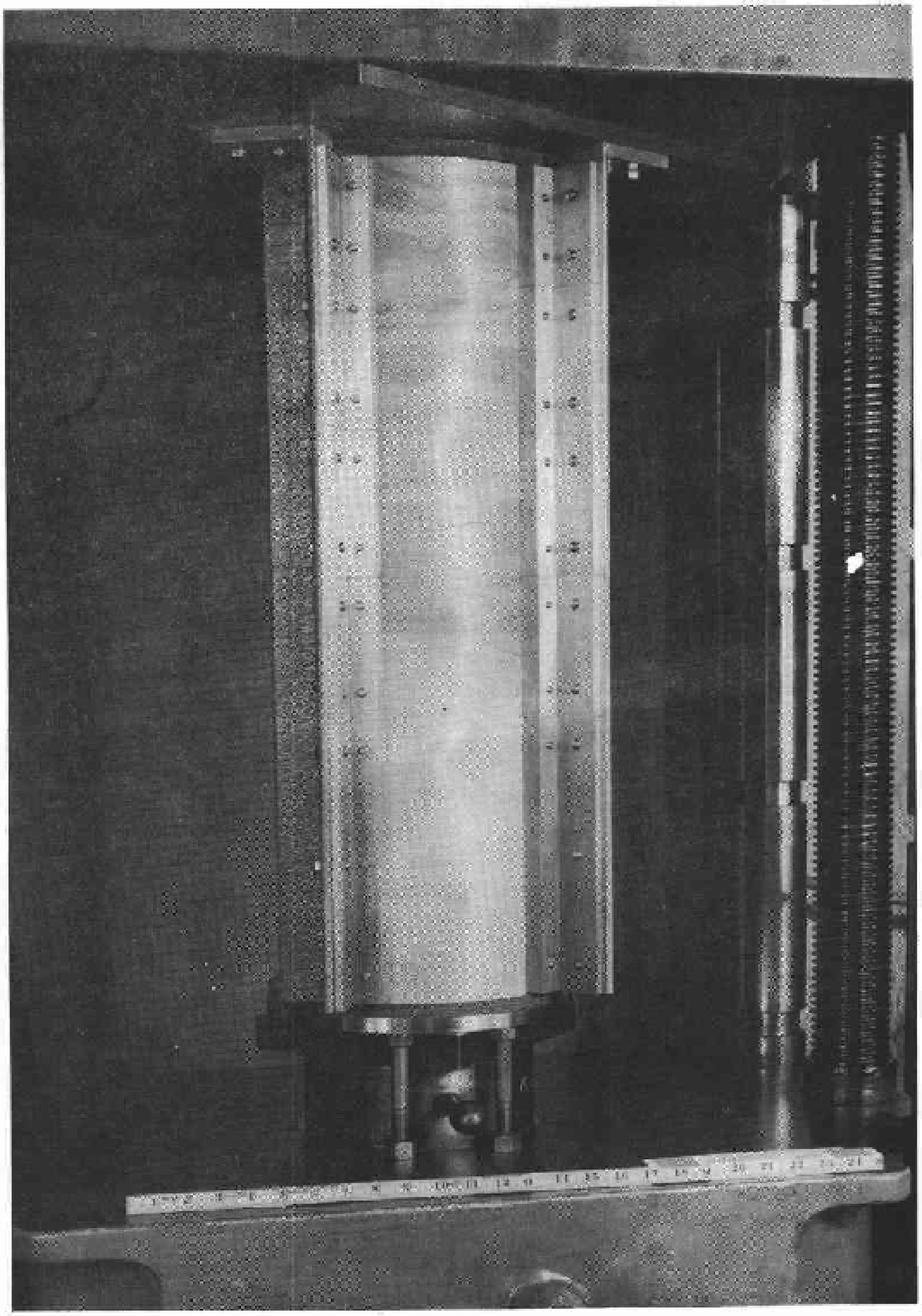

Figure 5.--Buckling failure of a curved plywood plate in axial compression. The plywood is 3-ply yellow birch. The faces are of 0.010-inch veneers, and the core is of 0.025-inch veneers. The face grain direction is axial. The specimen is formed to a 5-1/4 inch radius and is one radian wide and 30 inches long.

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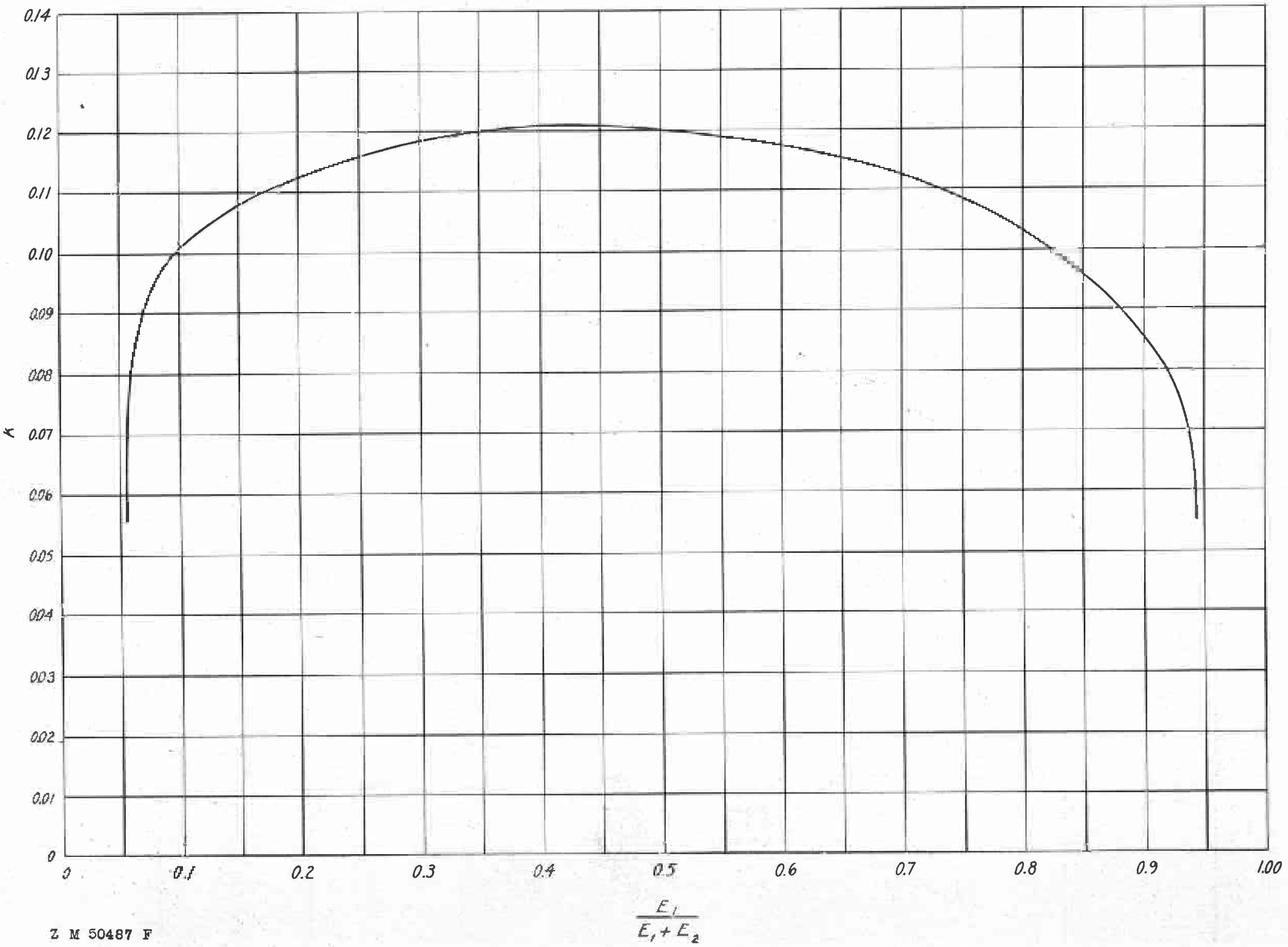

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Figure 6.--Theoretical curve for long, thin, plywood cylinders in axial compression. The ordinates represent k in the formula $p = kE_L\frac{h}{r}$, where p is the buckling stress. The abscissas represent the ratio $E_1/(E_1 + E_2)$, where E_1 and E_2 are proportional to the stiffness of the plywood in the axial and circumferential directions, respectively.

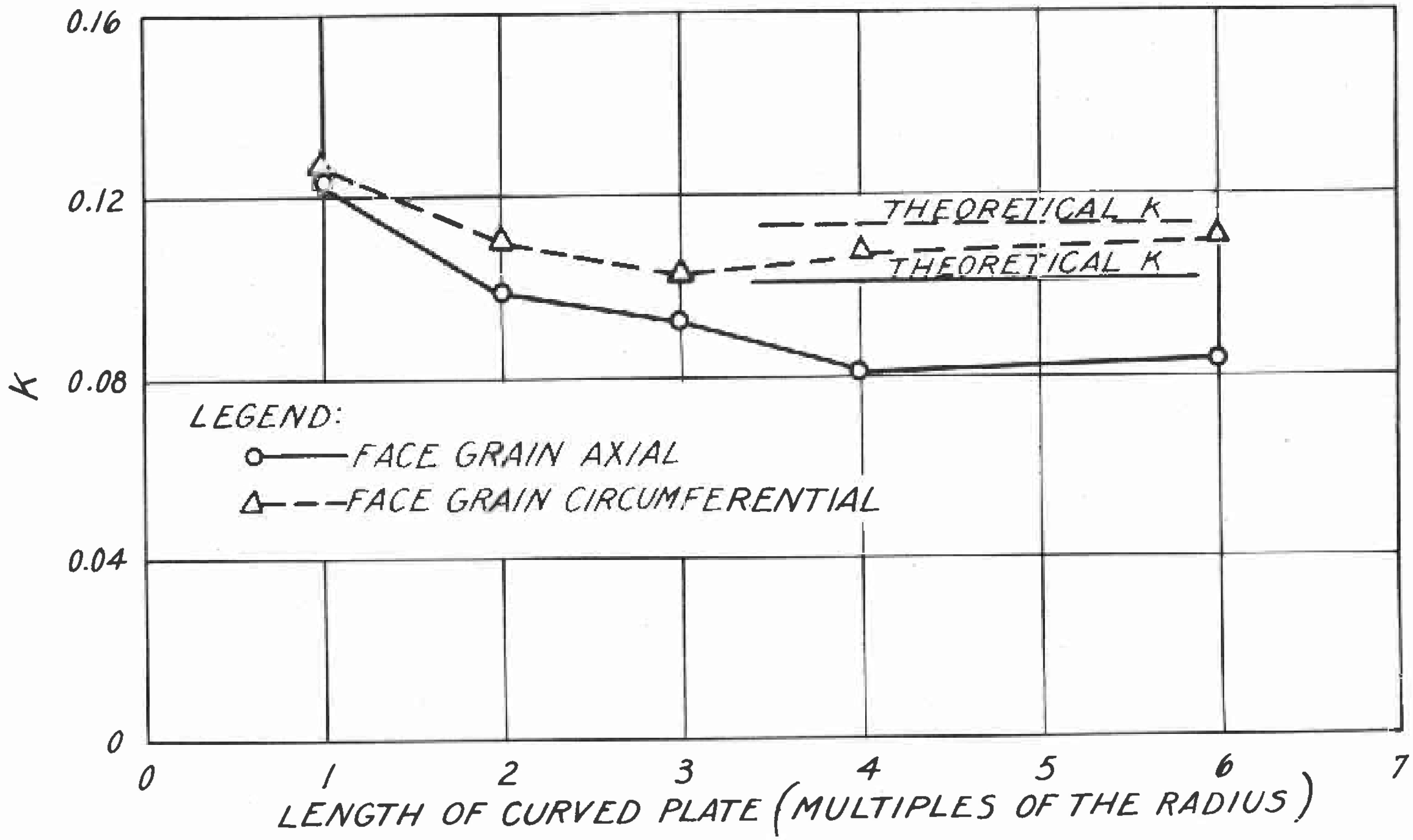

Figure 7.--The effect of length of specimen on the buckling constant ($k = pr/E_L h$) for thin, curved, yellow birch plywood plates under axial compression loads. All plates were 3-ply, with faces of 0.010-inch and cores of 0.025-inch veneers, and were formed to a 5-1/4-inch radius. Each point represents the results of three major tests.

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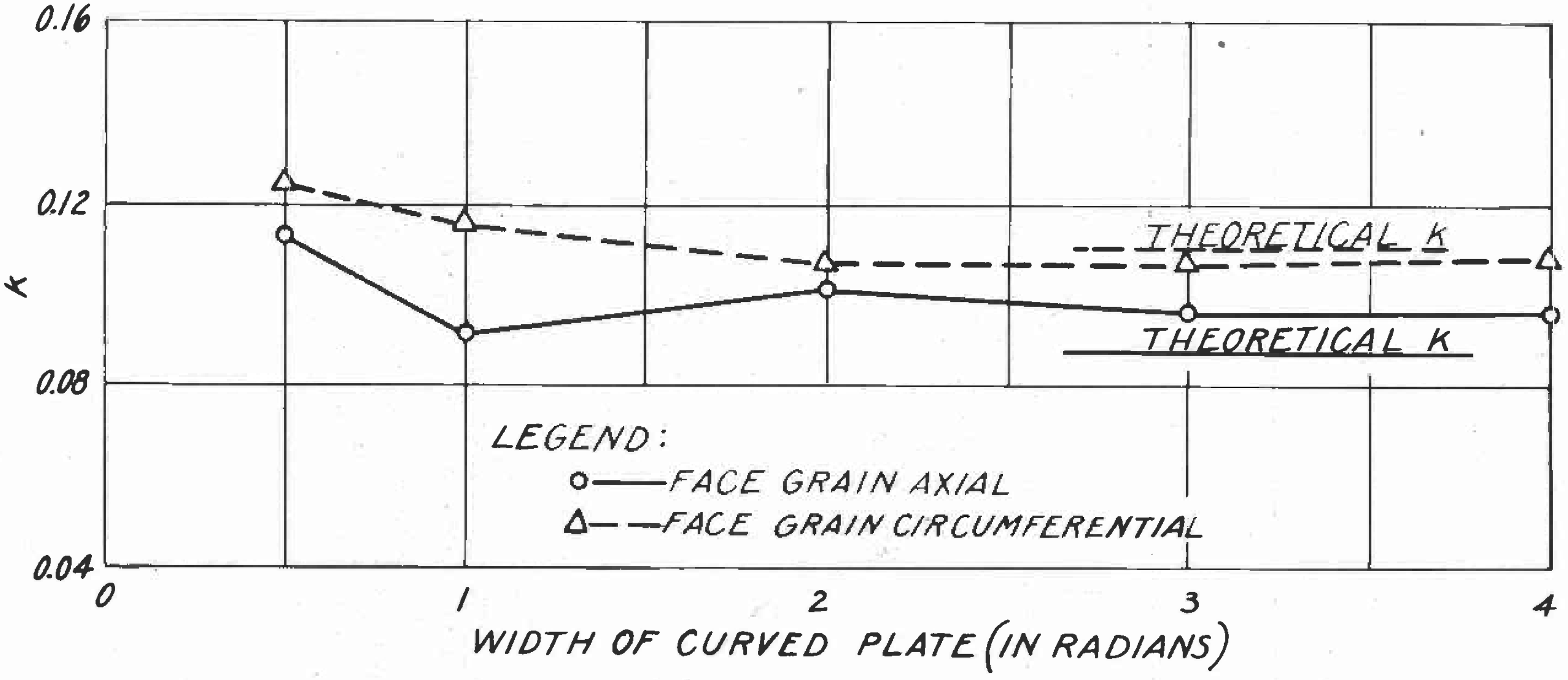

Figure 8.--The effect of width of specimen on the buckling constant ($k = pr/E_Lh$) for thin, curved, yellow birch plywood plates under axial compression loads. All plates were 3-ply with faces of 0.010-inch and cores of 0.025-inch veneers, and were formed to a 5-1/4-inch radius. Each point represents the results of three major tests.

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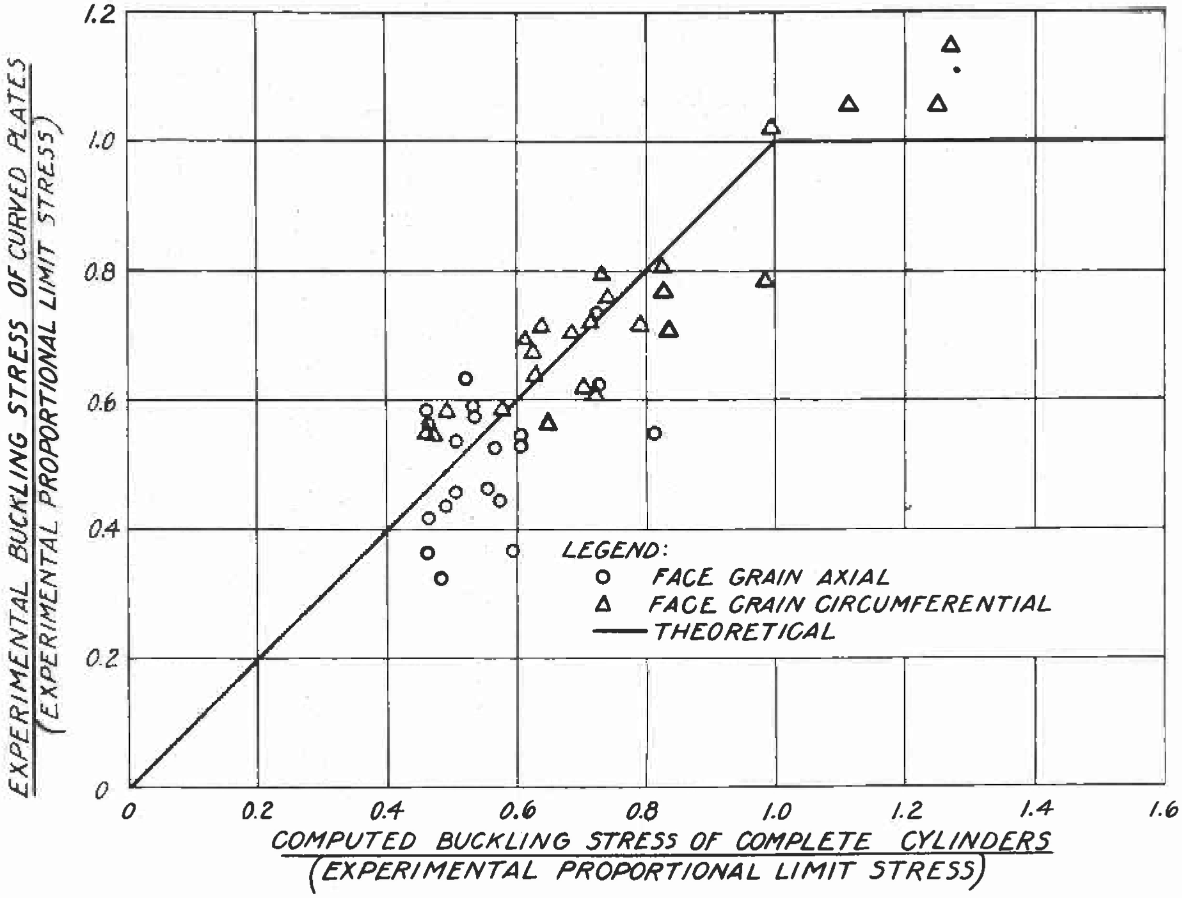

Figure 9.--Comparison of test and computed buckling stress to show effect of approach of buckling stress to stress at proportional limit in compression for thin, curved, yellow birch plywood plates under axial compression loads. All plates were 3-ply with faces of 0.010-inch and cores of 0.025-inch veneers, and were formed to a 5-1/4-inch radius. Each point represents the results of three major tests.

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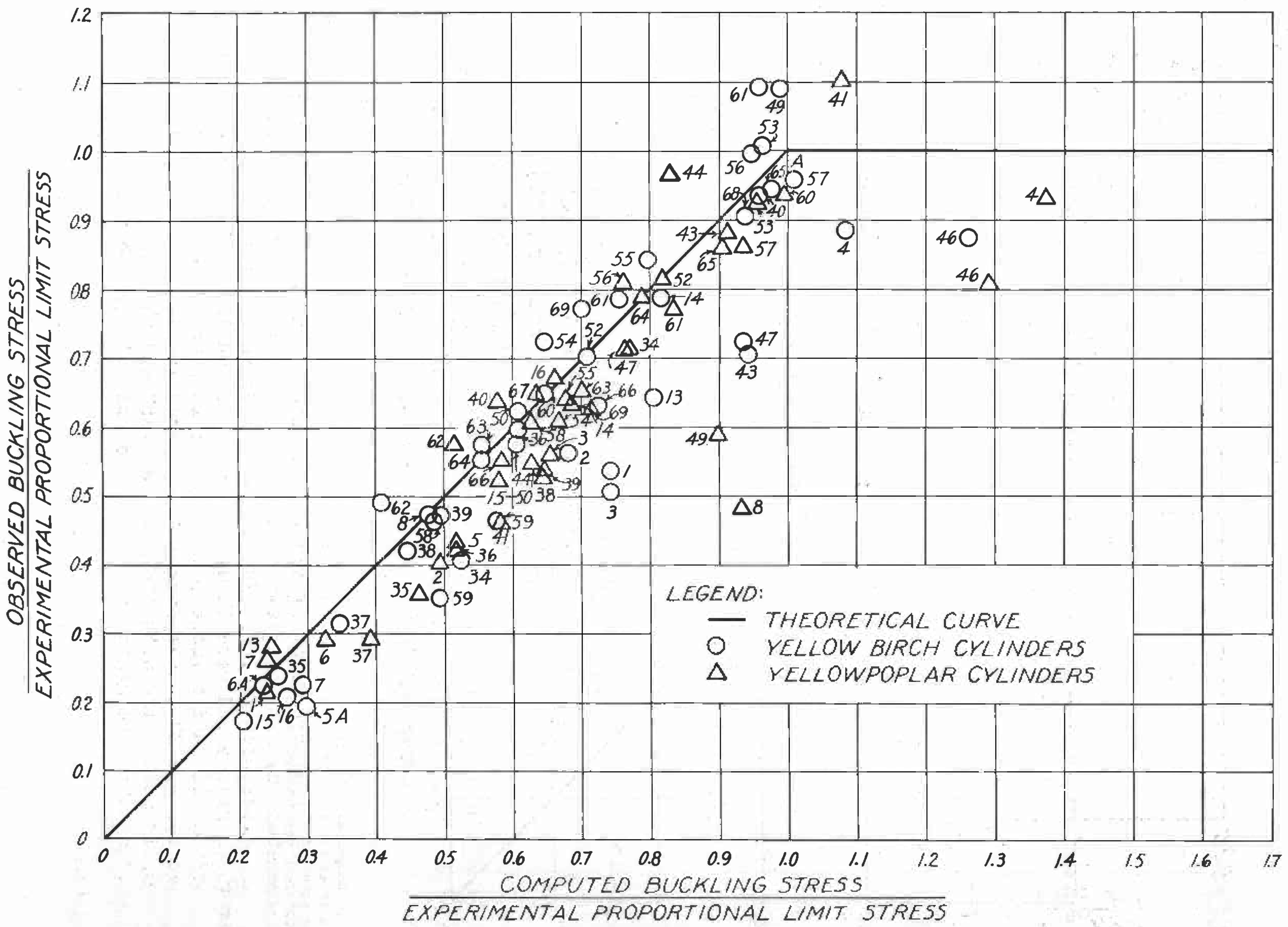

Figure 10--Comparison of test and computed buckling stress to show effect of approach of buckling stress to stress at proportional limit in compression. Direction of grain of face plies either axial or circumferential. Each point represents the average result of a group of tests. Each group of cylinders is designated by a number written adjacent to the corresponding point. (This graph is the same as figure 3, Forest Products Laboratory Mimeo. No. 1322.)

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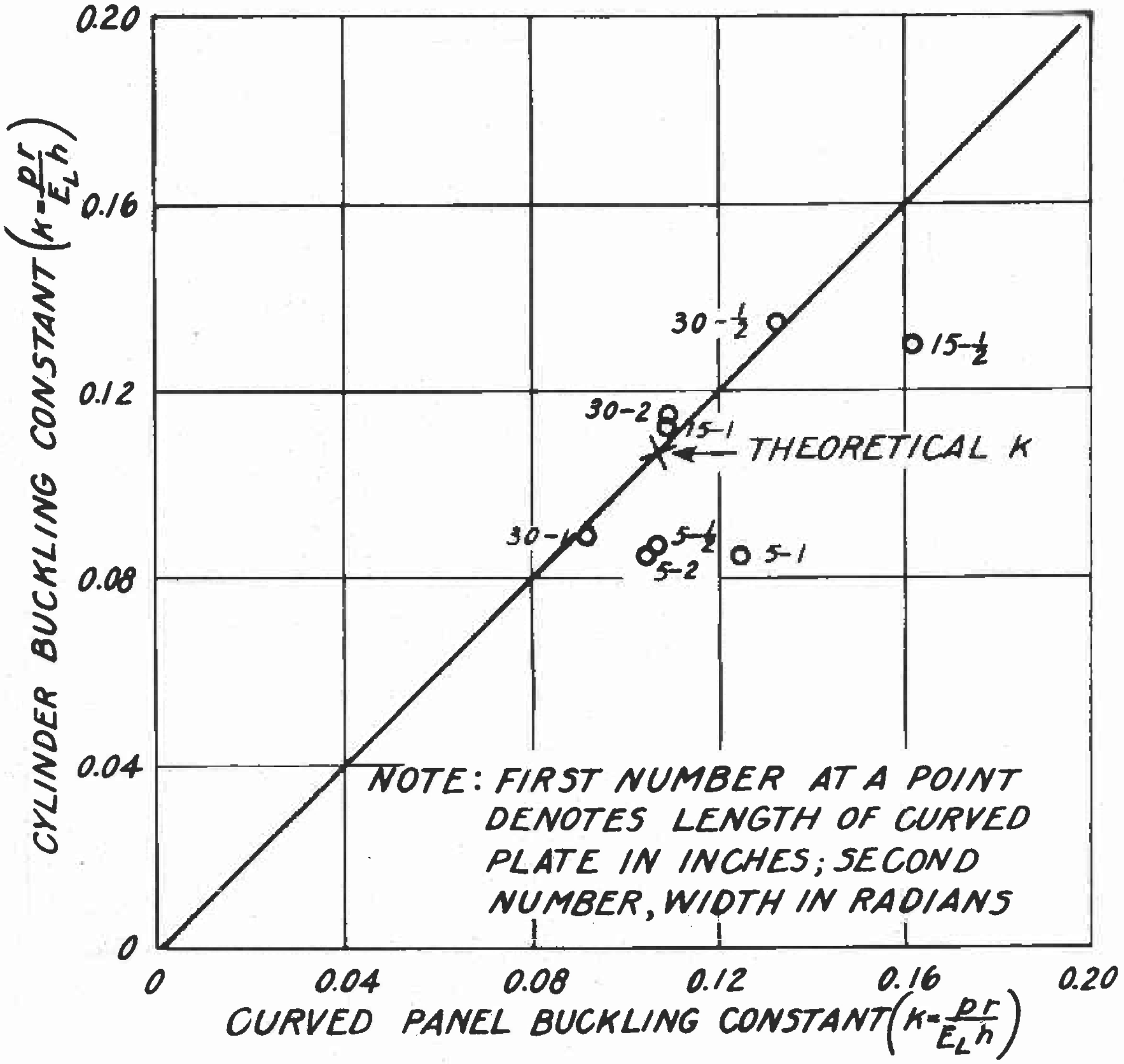

Figure 11.--Comparison of buckling constants of thin, curved, plywood plates in axial compression with those of thin, plywood cylinders in axial compression. The curved plates were formed to a 5-1/4-inch radius and were in widths of 2, 1, and 1/2 radians and lengths of 30, 15, and 5 inches. The cylinders were 10-1/2 inches in diameter and 30 inches long. All plywood was 3-ply. The faces were 0.010 inch thick with the grain at -45° and the cores 0.025 inch thick with the grain at +45° to the axis of the specimen.

Z M 54168 F

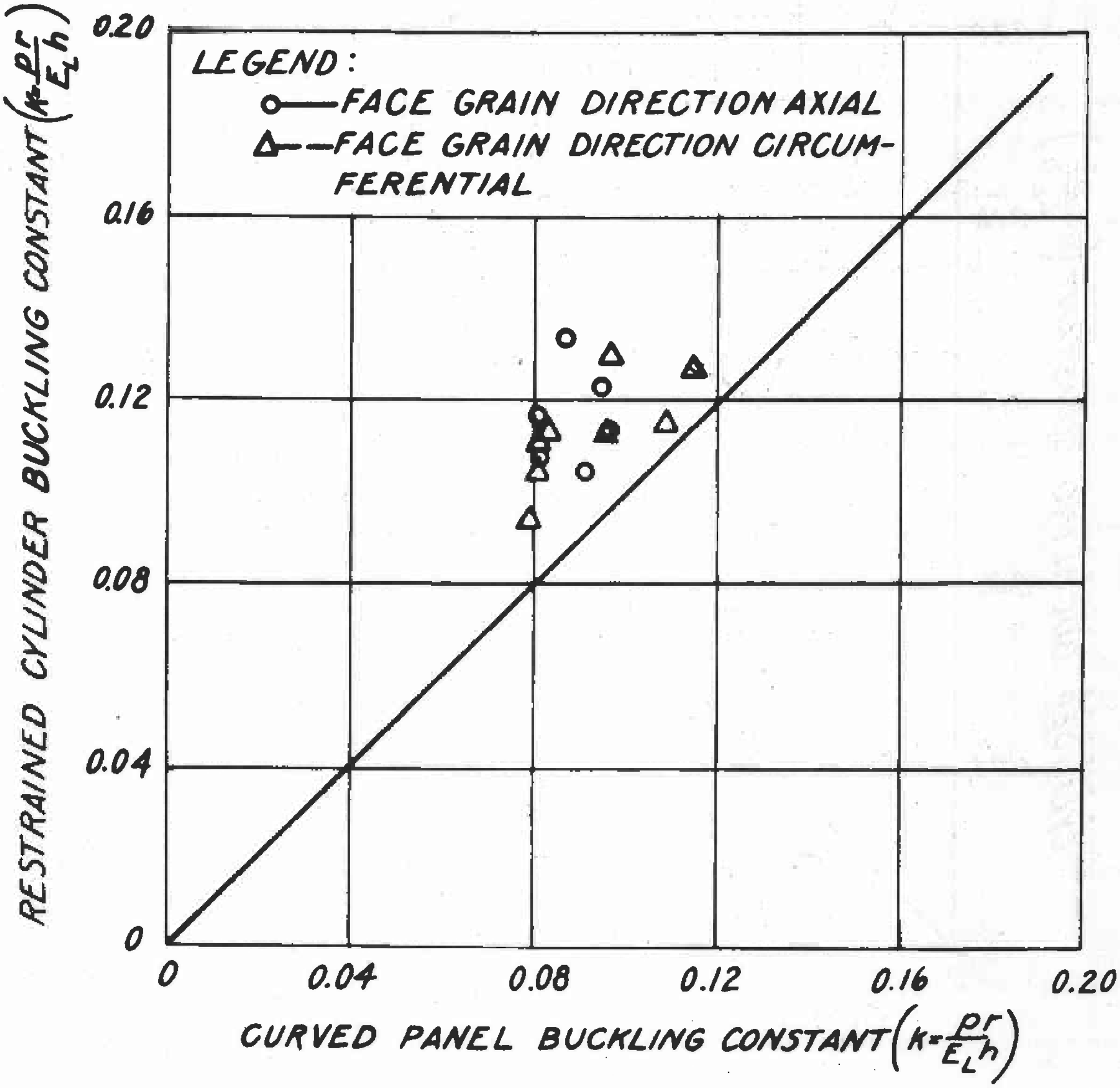

Figure 12.--A comparison of buckling constants of thin, curved plywood plates in axial compression with those of restrained plywood cylinders in axial compression. The curved plates were formed to a 5-1/4-inch radius and were in widths of one or one-half radians and lengths of 30 inches or 5 inches. The cylinders were 10-1/2 inches in diameter and 30 inches long. Some were restrained with 6 vanes and some with 12 vanes. For details as to plywood and sizes see table 5.

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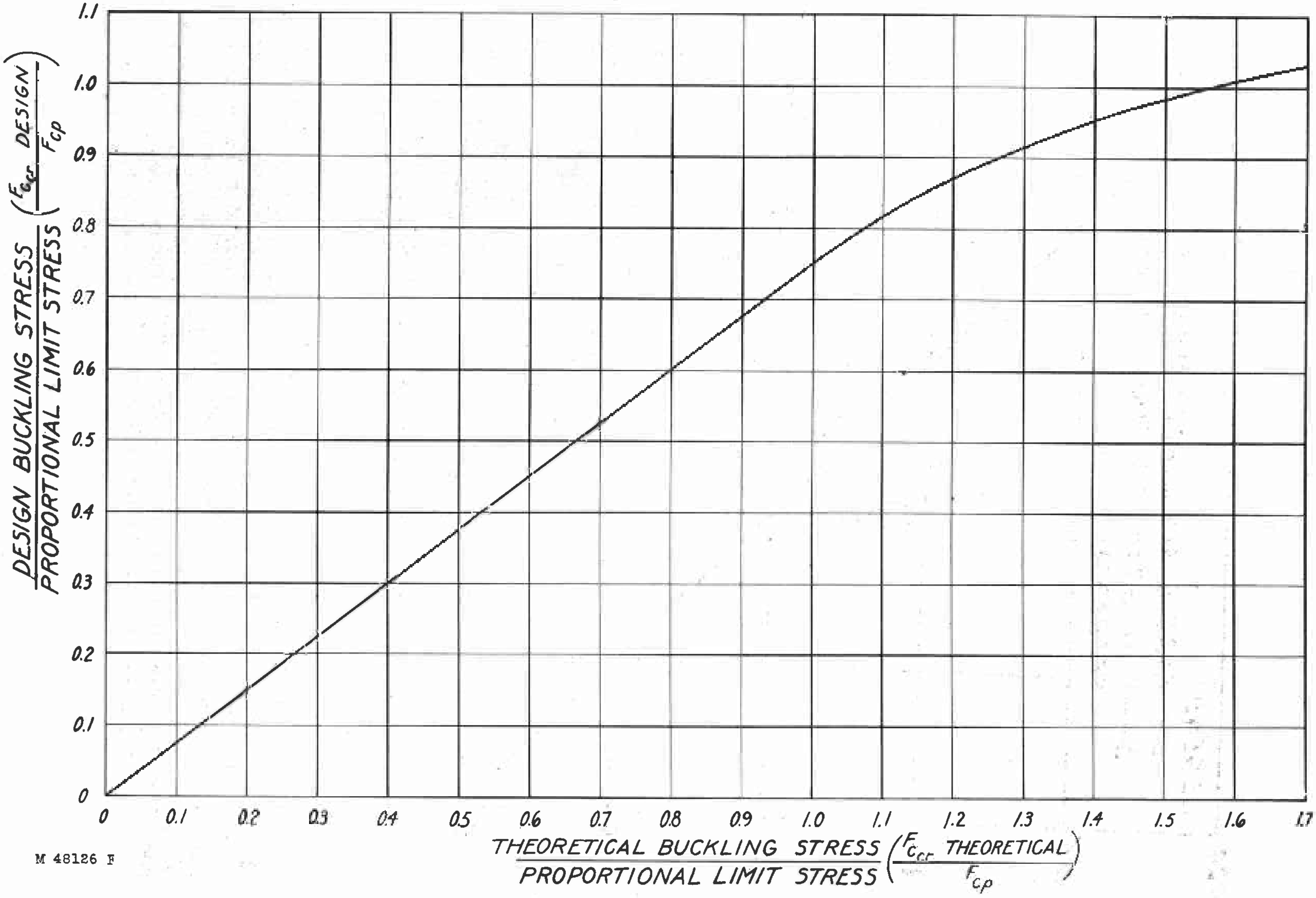

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Figure 13.--Design curve for long thin-walled plywood cylinders. This curve can also be used for designing thin, curved, plywood plates.

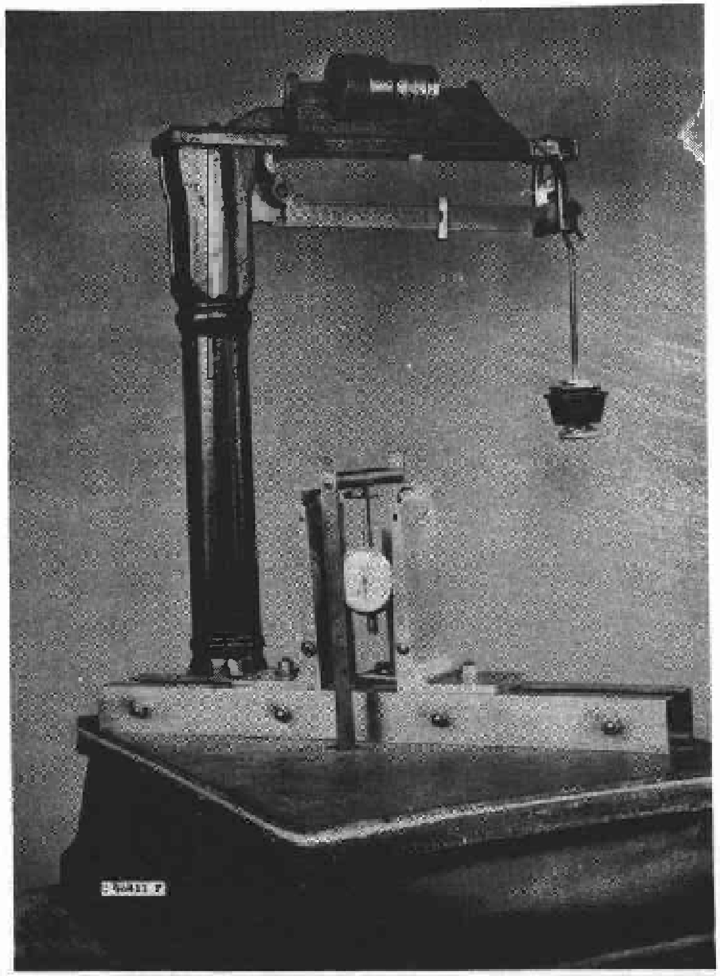

Figure 14.--Bending test of a plywood specimen on a platform-scale apparatus.

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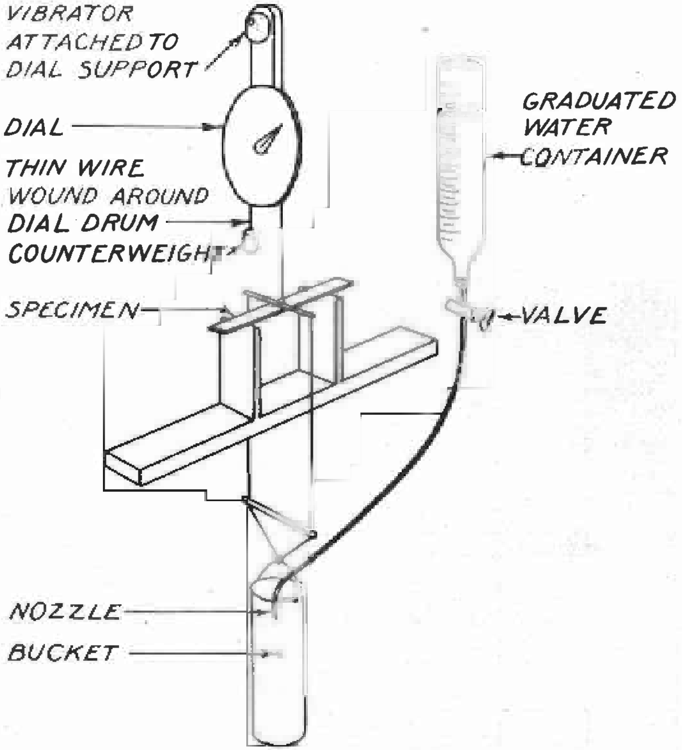

Figure 15.--Apparatus for bending tests of plywood specimens not stiff enough to be tested on the platform-scale apparatus of figure 14.

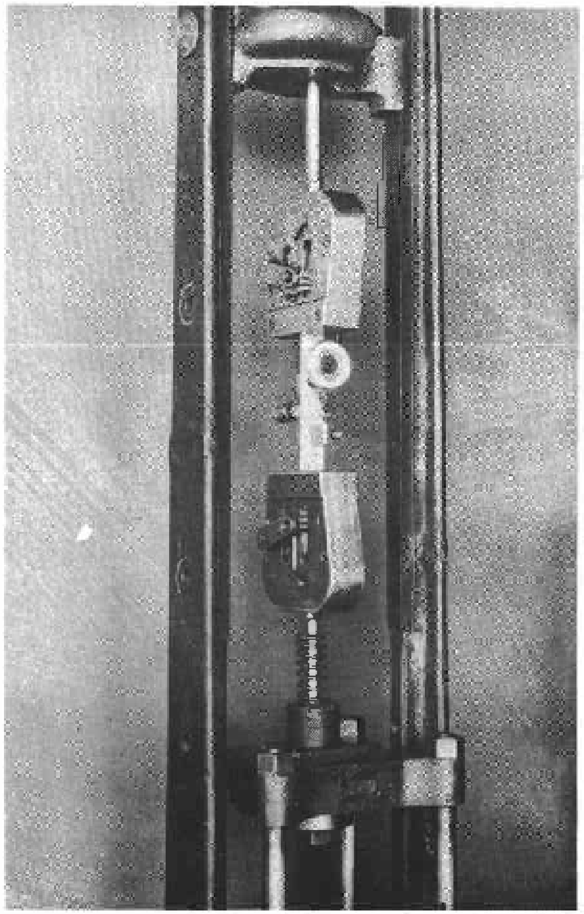

Figure 16.--Tension test of plywood.

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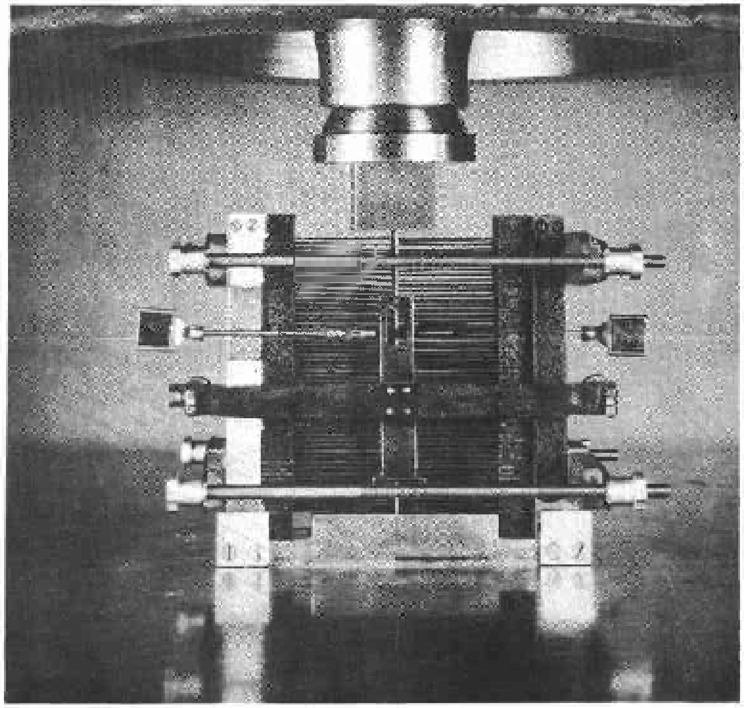

Figure 17.--Compression test of thin plywood.