THIN-WALLED PLYWOOD CYLINDERS IN BENDING AND TORSION

Information Reviewed and Reaffirmed

January 1953

This Report is One of a Series Issued in Cooperation with the ARMY-NAVY-CIVIL COMMITTEE on AIRCRAFT DESIGN CRITERIA Under the Supervision of the AERONAUTICAL BOARD

No. 1501

UNITED STATES DEPARTMENT OF AGRICULTURE
FOREST SERVICE
FOREST PRODUCTS LABORATORY
Madison 5, Wisconsin
In Cooperation with the University of Wisconsin

THIN-WALLED PLYWOOD CYLINDERS IN BENDING AND TORSION[1]

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Summary

This report presents the results of tests conducted to obtain empirical relations between bending and torsional buckling stresses of thin-walled plywood cylinders under combined bending and torsion loads. Test results indicate that an interaction formula of the form $x^n + y^n = 1$ can be employed, where x and y are the ratios of the bending stress and torsional stresses in combined loading to the stresses in a cylinder at failure in pure bending and in pure torsion, respectively. The conclusion is that a value of 4/3 may be used for n.

Description of Specimens

Specimens were made of yellowpoplar plywood. All the veneer was of aircraft grade, rotary-cut at the Forest Products Laboratory. The testing program included two plywood constructions; three-ply with face plies of 1/100-inch veneer and core ply of 1/40-inch veneer, and five-ply with all plies of 1/100-inch veneers. Cylinders of three ply material were tested, some specimens with the face plies in an axial direction and some with the face plies in a circumferential direction. Cylinders having these same grain directions were tested in five-ply material and in addition five-ply cylinders having the face grain at +45° or -45° to the cylinder axis were tested. The cylinders having the grain direction of the face plies at -45° are those in which the compressive stress, due to torsion, is in the same direction as the grain of the face plies. In all cylinders, the grain of adjacent plies was at right angles. At least four specimens were prepared and tested for each plywood construction. The veneers in each group of four specimens were matched as closely as possible by cutting them from adjacent portions of the log.

All plywood was made flat, using a phenolic-resin film glue set in a hot press. The cylinders were formed by bending the flat plywood around a mandrel. Prior to forming, the plywood was scarfed, then moistened to facilitate bending, and finally bent around the mandrel. In this position the scarf joint was glued with a thermosetting synthetic-resin glue.

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[1] This mimeograph is one of a series of progress reports prepared by the Forest Products Laboratory to further the Nation's war effort. Results here reported are preliminary and may be revised as additional data become available.

All cylinders had a diameter of approximately 10-1/2 inches and were 45 inches in length. In all cylinders snugly fitting diaphragms turned from 3/4-inch plywood were placed at ends and at third points of the length (points of application of bending loads) with three additional diaphragms equally spaced in each outer third of the length (fig. 1). Diaphragms at ends and at third points were glued to the cylinder walls, others being held in place by friction.

No attempt was made to condition the specimens to a particular moisture content other than to allow them to remain in the testing Laboratory for about a week before testing. All specimens of a group were tested the same day.

Testing Methods

The apparatus used to test the cylinders is shown in schematic form in figure 1.

The end reactions for bending loads were provided by pins of 2-inch diameter fitted loosely into holes drilled into the end plates of the cylinders. The pins were supported by brackets fastened to the bottom head of the testing machine. One of the brackets contained a shaft and bearings, centered on the 2-inch pin, and was provided with a pulley and strap to apply torsion loads. The torque was transmitted to the cylinder by means of four pins fitted into holes in the cylinder end plates. The bracket at the other end was a rigid steel plate with steel pins to resist the torque applied. The end brackets were not placed close to the ends of the cylinder but were some distance away to allow for tilting of the ends due to bending. Bending loads were applied through thin steel straps passed around the cylinder at the third points. These straps passed over a beam supported at its center by a dynamometer.

The testing procedure was to test one cylinder in pure torsion, one in pure bending, and the remaining ones of the group in combinations of bending and torsion. The torsion load was applied and measured by means of a balance attached to the movable head of the testing machine. The bending load was applied and measured by means of a dynamometer attached to the movable head of the testing machine. When the specimen was subjected to combined loading, any selected torsion load was first applied and then the bending load was applied and increased until failure occurred. This was done to avoid including in the measured torque the friction load of the bending straps on the cylinder.

Test Results

Most of the cylinders failed by sudden buckling and breaking of the plywood. All those tested in pure bending or combined bending and torsion, failed by breaking of the plywood. The buckling and breaking occurred simultaneously and therefore no difference could be detected between the buckling load and the ultimate breaking load. Some of the cylinders subjected to pure torsion failed by buckling only. The load reached at the time of this buckling was the maximum load carried by the cylinder.

Computation of Results

The bending stresses at the maximum load (table 1) were computed by means of the formula

$$f_b = \frac{M\,(r + t/2)}{\pi r^3\, t}$$

where

f_b = stress in most remote fiber in pounds per square inch
M = bending moment in inch-pounds
r = mean radius of the plywood cylinder in inches
t = thickness of plywood in inches.

For cylinders tested in pure bending the buckling stress is denoted by f_{pb}.

The torsional stresses at the maximum load (table 1) were computed by means of the formula

$$f_t = \frac{T}{2\pi r^2\, t}$$

where

f_t = shear stress in pounds per square inch
T = torque in inch-pounds
r = mean radius of the plywood cylinder in inches
t = thickness of plywood in inches

For cylinders tested in pure torsion, the buckling stress is denoted by f_{pt}.

Ratios of buckling stresses in combined loading to buckling stresses of a matched specimen in pure loading, $(\frac{f_t}{f_{pt}})$ and $(\frac{f_b}{f_{pb}})$, were tabulated in table 1 and plotted in figure 2. The resulting distribution of points suggested an equation of the form $x^n + y^n = 1$. Cut-and-try methods led to 4/3 as the value of n. The resulting curve is shown as a solid line in figure 2.

Discussion of Results

In general, the test results show less variation than was obtained from tests on cylinders in compression.

Referring to figure 2, the two points representing a five-ply cylinder with the grain direction of the face plies at an angle of -45° are too high. This is due to the low buckling load for the matching cylinder tested in pure bending. The maximum bending load carried by this cylinder was less than that carried by the matched cylinder along with a torque of 40 percent of the maximum value in the cylinder that was tested in torsion alone. Each specimen included in this study had a length in which buckling could occur of about one and one-half times the diameter of the cylinder, It may be expected that shorter lengths will cause higher failing loads but it would be incorrect to assume that this effect of length upon load is the same for all plywood constructions.

Conclusions

The interaction between bending and torsion stresses at buckling loads for thin-walled plywood cylinders under combined bending and torsion loads is fairly represented, although somewhat conservatively, by the formula

$$\left(\frac{f_t}{f_{pt}}\right)^{4/3} + \left(\frac{f_b}{f_{pb}}\right)^{4/3} = 1$$

Table 1.--Results of tests of thin-walled plywood cylinders in combined bending and torsion.[1]

Specimen number	Number of plies	Grain direction of face plies[2]	Ply thicknesses: Faces	Ply thicknesses: Cross bands	Ply thicknesses: Core	Buckling stresses: Simple loading: Torsion f_{pt}	Buckling stresses: Simple loading: Bending f_{pb}	Buckling stresses: Combined loading: Torsion f_t	Buckling stresses: Combined loading: Bending f_b	Ratios: $\frac{f_t}{f_{pt}}$	Ratios: $\frac{f_b}{f_{pb}}$
		Degrees	Inch	Inch	Inch	Pounds per square inch					
1	3	0	0.010	.....	0.025	...	2170	...	....	.....	1.000
2	3	0	.010	.....	.025	430	....	...	....	1.000	.....
3	3	0	.010	.....	.025	...	....	200	1600	0.465	0.737
4	3	0	.010	.....	.025	...	....	270	1200	.628	.552
5	3	90	.010	.....	.025	...	2260	...	....	.....	1.000
6	3	90	.010	.....	.025	880	....	...	....	1.000	.....
7	3	90	.010	.....	.025	...	....	276	1440	.314	.638
8	3	90	.010	.....	.025	...	....	645	1160	.733	.514
9	3	90	.010	.....	.025	...	....	225	1880	.256	.832
10	5	0	.010	0.010	.010	...	3160	...	....	.....	1.000
11	5	0	.010	.010	.010	720	....	...	....	1.000	.....
12	5	0	.010	.010	.010	...	....	156	2770	.217	.876
13	5	0	.010	.010	.010	...	....	481	1780	.668	.563
14	5	0	.010	.010	.010	...	....	329	2160	.457	.684
15	5	0	.010	.010	.010	...	....	661	390	.918	.123
16	5	0	.010	.010	.010	...	....	413	1995	.574	.631
17	5	90	.010	.010	.010	....	2920	...	....	.....	1.000
18	5	90	.010	.010	.010	1030	....	...	....	1.000	.....
19	5	90	.010	.010	.010	....	....	390	2470	.378	.846
20	5	90	.010	.010	.010	....	....	546	2100	.530	.719
21	5	-45	.010	.010	.010	...	2950	...	....	.....	1.000
22	5	-45	.010	.010	.010	978	....	...	....	1.000	.....
23	5	-45	.010	.010	.010	...	....	397	3070	.406	1.040
24	5	-45	.010	.010	.010	...	....	810	1910	.828	.648
25	5	+45	.010	.010	.010	...	3050	...	....	.....	1.000
26	5	+45	.010	.010	.010	721	....	...	....	1.000	.....
27	5	+45	.010	.010	.010	...	....	468	1875	.649	.615
28	5	+45	.010	.010	.010	...	....	247	2900	.342	.950

[1]All specimens were 10-1/2 inches in diameter.

[2]This angle refers to the longitudinal axis of the cylinder.

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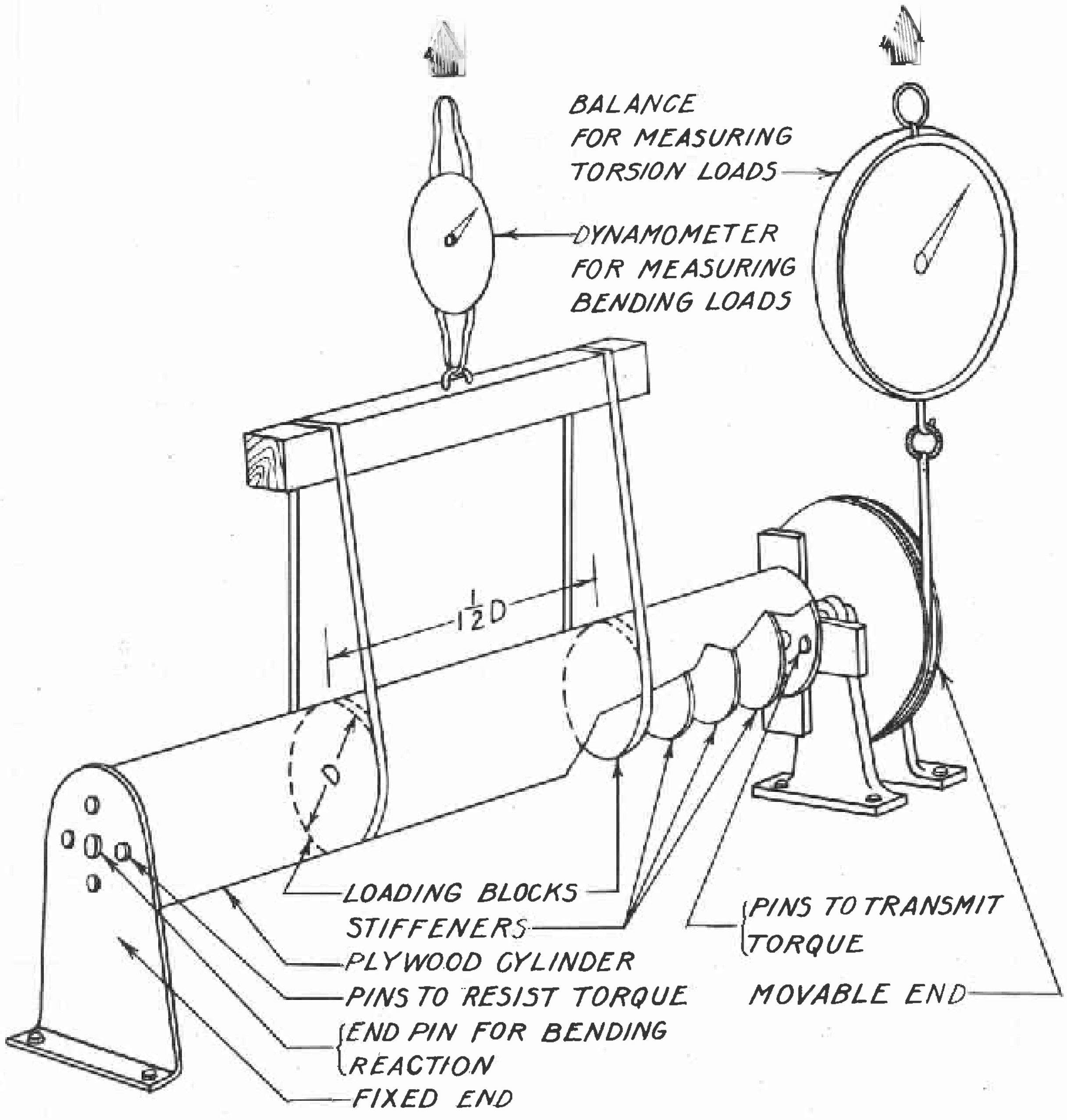

Figure 1.--Apparatus for testing thin-walled plywood cylinders in combined bending and torsion.

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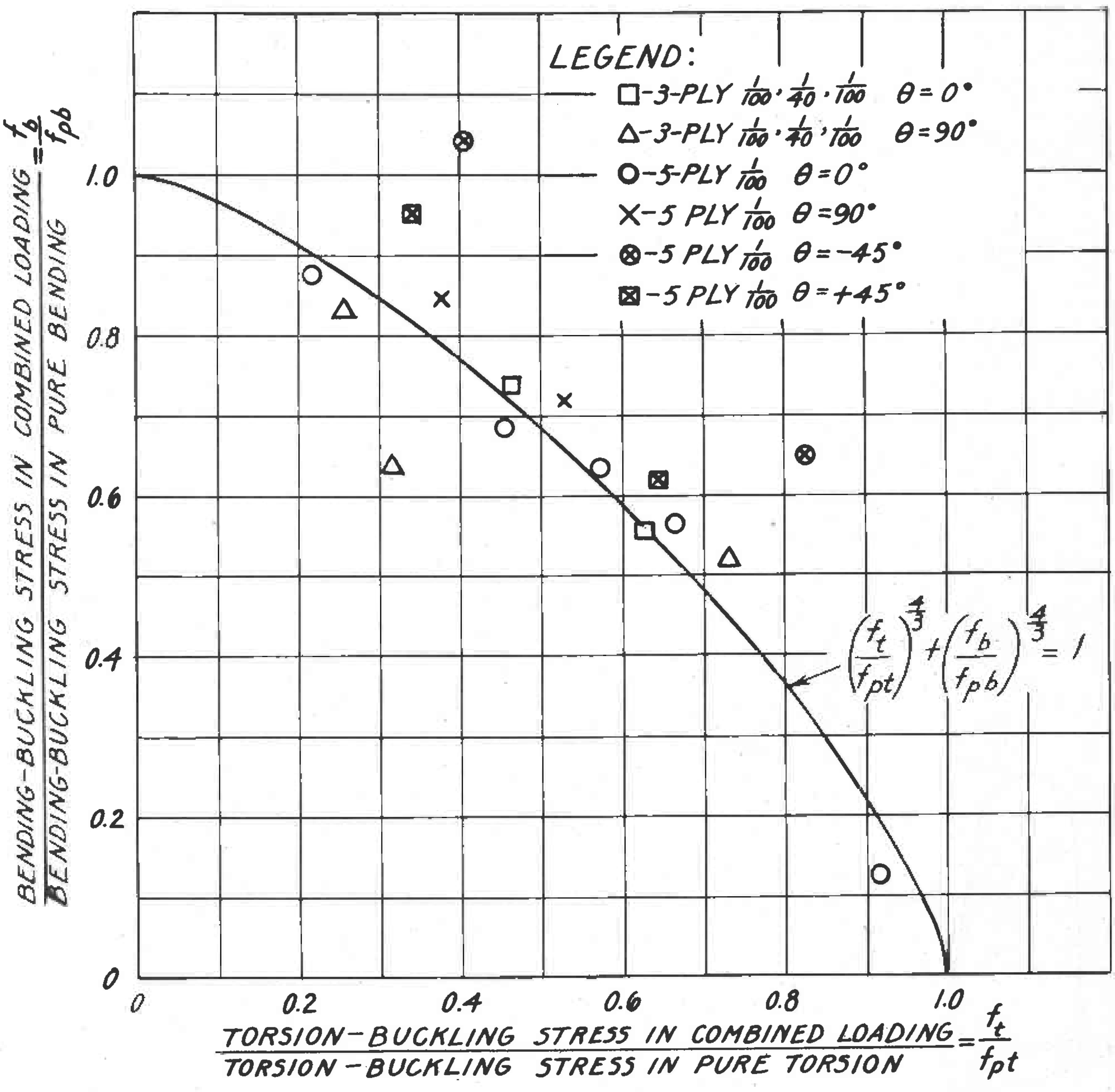

Figure 2.--Results of tests on thin-walled plywood cylinders under combined bending and torsion loads.

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