# Fuzzy Logic and Non-market Valuation: A Comparison of Methods

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**Abstract:** In seeking to value environmental amenities and public goods, individuals often have trouble trading off the (vague) amenity or good against a monetary measure. Valuation in these circumstances can best be described as fuzzy—both in terms of the amenity valued, perceptions of property rights, and the numbers chosen to reflect values. In this paper, we review three approaches to fuzzy valuation, and compare results from fuzzy valuation with those obtained using usual techniques of valuation.

**Keywords**: fuzzy set theory; contingent valuation method; pairwise comparisons

## 1. Introduction

Changes in well being associated with changes in the availability of many environmental amenities (and public goods) cannot be traced through market transactions, because, in technical terms, the utility function is separable in the amenity. In this case, the approach to the valuation of environmental amenities is often to employ the contingent valuation method (CVM). This approach simply asks survey respondents how much they are willing to pay (WTP) for hypothetical (hence the term "contingent") increments or decrements in the availability of the amenity.[1] WTP can be elicited either using an open-ended question format (letting respondents provide any value they please) or a dichotomous choice format (providing a value and having the respondent answer yes or no to whether they would pay the "bid" amount). The former suffers from anchoring problems, while the latter often results in "yea-saying". CVM has been criticised by many researchers, with a major criticism being the large and irreconcilable difference between WTP and WTA (see Hausman 1993; Knetsch 2000). As a result, economists have begun to search for other methods of valuation.

Adamowicz and his colleagues have proposed using choice experiments (CE) or stated preferences, an approach rooted in the marketing literature (Adamowicz 1995; Adamowicz et al. 1998; Hanley et al. 1998). While the methodology has been used primarily to value

recreational sites, Adamowicz et al. (1998) apply CE to the estimation of nonuse values. Unlike CVM, CE does not require survey respondents to place a direct monetary value on a contingency. Rather, individuals are asked to make pairwise comparisons among environmental alternatives, with the environmental commodity (alternatives) characterised by a variety of attributes. For example, a survey respondent is asked to make pairwise choices between alternative recreational sites or activities, with each distinguished by attributes such as the probability of catching a fish, the type of fish, site amenities (e.g., availability of boat rentals), distance to the site, and so on. It is the attributes that are important, and it is these that are eventually assigned monetary value. In order to do so, one of the attributes must constitute a monetary touchstone (or proxy for price). Distance to a recreational site might constitute the proxy for price, but, more generally, one of the attributes will be an entry fee or an associated tax (etc.). Once the values of all attributes are known (from the value of the one and the pairwise rankings), the overall value of the amenity is determined by assuming additivity of the attributes' values. Of course, it is possible that the total value of the amenity is greater than the sum of its components, so proper design of the choice experiment is crucial.

Hanley et al. (1998) point out a number of advantages of the CE approach. First, it enables one to value the attributes that comprise an environmental commodity, which is important as many policy decisions involve changing attributes rather than the total gain or loss of an environmental commodity. For example, when a wilderness area is developed as a result of timber harvest, not all of its attributes are lost. Attribute valuation is also important because of its use in prediction. Second, choice experiments avoid the "yea-saying" problem of dichotomous choice surveys as respondents are not faced with the same "all-or-nothing" choice. Third, CE may offer advantages over CVM when

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[1] In practice and partly as a result of the nature of the interview procedure, respondents to questionnaires are generally asked to value a non-marginal change in availability (of wilderness area, a wildlife species, etc.). Further, WTP is employed because it appears to result in "better" estimates than elicitation of compensation demanded or willingness to accept compensation (WTA), which can be unbounded.

it comes to the transfer of benefits (e.g., transfer of estimated benefits for water quality improvements in one jurisdiction to those in another). Fourth, repeated sampling in CE enables consistency testing that is not possible in CVM. Fifth, CE may be a means of getting around the embedding problem (see Knetsch 2000). Finally, in the case of nonuse benefits estimation, by allowing some attributes to take on levels both above and below the status quo level, it is possible to estimate both WTP and WTA compensation (Adamowicz et al. 1998).

Gregory et al. (1993) propose a multiattribute-utility-theory approach to address the inability of respondents in a contingent valuation exercise to make holistic assessments about environmental resources. Individuals do not know the values of the resources they are asked to value, but construct them, "with whatever help or cues the circumstances provide" (p.181). In practice, workshops are used to "help" stakeholders construct values (preferences). One problem with this approach is that workshop participants (stakeholders) may not reflect the values of the broader society. Another is that the valuation process itself influences participants' responses. As a result, this method has not caught on in the economics literature.

In this paper, we address the potential of approaches rooted in fuzzy logic for evaluating nonuse amenity values. We investigate fuzzy approaches that complement CVM and pairwise rankings. Fuzzy logic has particular promise, in our view, when preferences are uncertain, with uncertainty originating for two reasons. First, people may have trouble valuing an environmental amenity because they are unfamiliar with the commodity, having little experience with it, or because the commodity is not readily describable in "crisp" language. How does one place a value on an "ecosystem" or on an "old-growth forest" when ecologists and foresters are unable to provide unambiguous definitions of these systems? Clearly, if we ask people in a CVM survey to place value on "old growth" or on an "ecosystem", or on caribou or minke whales, different people will have different images of each of these "commodities". Unlike market goods, environmental amenities cannot always be defined; they are best described as being a vague commodity.

Second, even if respondents are completely familiar with the environmental amenity, they may have trouble expressing tradeoffs between the amenity and monetary value; they are uncertain about the monetary value to attach to a change in the availability of an environmental amenity and this shows up in preference uncertainty (Hanemann and Kriström 1995). Several authors have adopted varying but ad hoc approaches for dealing with preference uncertainty of this kind (Li and Mattsson 1995; Ready et al. 1995; Loomis and Ekstrand 1998), but these rely on probabilistic interpretations of uncertainty. We contend that the apparent precision of standard WTP estimates (even as a mean value with confidence interval) masks the underlying vagueness of preferences and may lead to biased outcomes (Barrett and Pattanaik 1989).

It is likely impossible to separate the two types of uncertainty, but we argue in this paper that a fuzzy approach may lead to improved valuation of non-market amenities. We begin in the next section by providing a brief introduction to fuzzy logic. In section 3, we apply fuzzy logic to a Swedish survey that asked respondents to value forest protection, interpreting CVM responses in the same manner as those conducting the survey (Li and Mattsson 1995). We also summarise the results of an alternative fuzzy method for analysing this same data using a different interpretation of responses (van Kooten et al. 2000). Finally, we describe the method of fuzzy pairwise comparisons in section 4, and apply it to the valuation of water quality in British Columbia. The results of the fuzzy pairwise comparison are contrasted with those obtained from an open-ended CVM question embedded in the same questionnaire and a separate dichotomous-choice CVM study for the same region. The conclusions follow.

## 2. Brief Introduction to Fuzzy Logic

Multivalued or "fuzzy" logic was first introduced in the 1930s to address indeterminacy in quantum theory, with the quantum philosopher Max Black using the term "vagueness" to refer to uncertainty and introducing the notion of a membership function (Kosko 1992, pp.5-6). Subsequently, Lofti Zadeh (1965) introduced the term "fuzzy set" and the fuzzy logic it supports.

Zadeh's concern was with the ambiguity and vagueness of natural language, and the attendant inability to convey crisp information linguistically. The word "hot", for example, may be used to communicate many things; the information it imparts is context dependent and, thus, the term itself may be considered ambiguous. "Hot" may refer to temperature, spiciness or trendiness. Once the frame of reference is identified to be temperature, the information conveyed is still not clear, as the subjective perception of heat by one person is not necessarily congruent with the perception of heat by another person. There is no absolute temperature at which a thing may be said to have attained membership in the set of things that are "hot" and at which it may be said to have ceased to be merely "warm". Subjective interpretations of the term will allow for an overlap of temperature ranges. Thus, an object may be said to be "warm" by some while it is judged "hot" by others. In essence, it is accorded partial membership in both of the sets—it displays some of the requirements for being a "hot" thing while retaining some of the requirements for being a "warm" thing. It is this concept of partial membership that is central to the theory of fuzzy sets.

Now consider the idea of partial membership more formally. An element $x \in X$ is assigned to an ordinary (crisp) set $A$ via the characteristic function $\mu_A$, such that:

(1)
$$\mu_A(x) = 1 \qquad \text{if } x \in A.$$
$$\mu_A(x) = 0 \qquad \text{otherwise.}$$

The element has either full membership ($\mu_A(x)=1$) or no membership ($\mu_A(x)=0$) in the set $A$. The function takes on one of two possible values, $\{0,1\}$. A fuzzy set $\tilde{A}$ is also described by a characteristic function, the difference being that the function now maps over the closed interval [0 1]. Thus, an element may be assigned a value that lies between 0 and 1 and is representative of the degree of membership that $x$ has in the fuzzy set $\tilde{A}$.[2] If $\mu_A(x) \in (0\ 1)$, element $x$ has only some but not all of the attributes required for full membership in a set. A membership function describes the grade or degree of membership, with the membership function viewed as a representation of a fuzzy number (Klir and Folger 1988, p.17). It is in this form that fuzzy set theory is used to deal with vague preferences. Then, $\mu_A(x)=1$ means that the decision maker is very satisfied, while $\mu_A(x)=0$ indicates that the decision maker is completely unsatisfied, with intermediate values indicating degrees of partial satisfaction.

Membership functions are crucial to fuzzy set calculus. Set-theoretic operations for fuzzy sets were originally proposed by Zadeh (1965), including the intersection of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ as:

(2) $\qquad \mu_{\tilde{A} \cap \tilde{B}}(x) = \min\{\mu_A(x), \mu_B(x)\} \ \forall\ x \in X,$

and union as:

(3) $\qquad \mu_{\tilde{A} \cup \tilde{B}}(x) = \max\{\mu_A(x), \mu_B(x)\} \ \forall\ x \in X.$

Hence, the intersection $\tilde{A} \cap \tilde{B}$ is the largest fuzzy set that is contained in both $\tilde{A}$ and $\tilde{B}$, and the union $\tilde{A} \cup \tilde{B}$ is the smallest fuzzy set containing both $\tilde{A}$ and $\tilde{B}$. Both union and intersection of fuzzy sets are commutative, associate and distributive as is the case for ordinary or crisp sets. Further, the complement $\tilde{A}^c$ of fuzzy set $\tilde{A}$ is defined as:

(4) $\qquad \mu_{\tilde{A}^c}(x) = 1 - \mu_{\tilde{A}}(x).$

Fuzzy logic deviates from crisp or bivalent logic because, if we do not know $\tilde{A}$ with certainty, its complement $\tilde{A}^c$ is also not known with certainty. Thus, $\tilde{A}^c \cap \tilde{A}$ does not produce the null set as for crisp sets (where $A^c \cap A = \phi$), so fuzzy logic violates the "law of noncontradiction". It also violates the "law of the excluded middle" because the union of a fuzzy set and its complement does not equal the universe of discourse—the universal set. $\tilde{A}$ is properly fuzzy if and only if $\tilde{A}^c \cap \tilde{A} \neq \phi$ and $\tilde{A}^c \cup \tilde{A} \neq X$, where $X$ is the universal set (Kosko 1992, pp.269-72; also Zimmermann 1996).

Fuzzy numbers are used to describe fuzziness and subsume membership functions. We distinguish two types of fuzzy numbers that are important for evaluating environmental resources. First is the notion of fuzzy class or category that is most frequently associated with fuzzy sets and membership functions (Cox 1994). Linguistic descriptors are often used but, no matter how well one is able to describe a particular resource aspect, it will always remain vague. As an example, consider the set of "ponds". A "pond" ceases to be one when it becomes so large that it is conceived of as a "lake", or when it becomes so small that it is better thought of as a "puddle". But all three concepts—puddle, pond and lake—are fuzzy and dependent on the surface area of the water body to be classified, although other factors might enter into the classification, such as the water body's permanency or suitability for certain activities. If surface area is the distinguishing feature, then the fuzzy sets might look like those in Figure 1. Fuzzy sets can take a variety of functional forms—they can be linear, piece-wise linear, one-sided (as in the case of "puddles" and "lakes"), two-sided (as for "ponds"), bell-shaped, triangular, symmetric, asymmetric, and so on. Fuzzy sets can also overlap. Hence, a body of water can be classed as both a "puddle" and a "pond" at the same time, although (usually) with differing degrees of membership. It is the researcher's task to construct the relevant parameters that characterise the fuzzy sets "puddles", "ponds" and "lakes", although surveys of experts, say, can be used to specify the forms of the fuzzy sets.
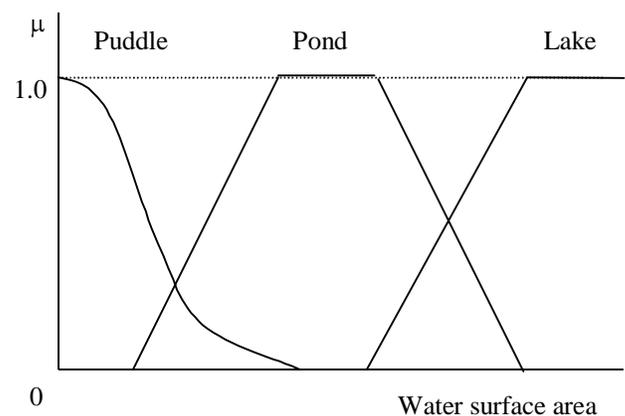


**Figure 1: Comparison of fuzzy sets**

It is the concept of fuzzy category that is most relevant to the situation where the amenity to be valued is

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[2]Generally membership functions are normalised so that there exists at least one $x \in X$ such that $\mu_A(x)=1$, and $0 \leq \mu_A(x) \leq 1 \ \forall\ x \in X$.

not well defined or known. We have vagueness in classification: What exactly is the amenity to be valued? While additional information about the amenity to be valued is helpful, it cannot eliminate all uncertainty. Linguistic terms and fuzzy classification are one means for dealing with some of the confusion about what exactly is to be valued.

The second type of fuzzy number is associated with fuzzy variables. In this case, the problem is not that of fuzzy set—the extent to which an element is a member of a vague set; rather, it is imprecision in value. For example, "pond" is a fuzzy class or quantity because it is not clear what surface area (or degree of permanency) is required to be sure that a "pond" is not a "lake" or "puddle". In contrast, if a waterfowl biologist estimates the size of a body of water from a satellite photograph to be 2.0 hectares, it could just as well be 1.5 ha or even 3.0 ha. This is uncertainty about its exact size, and an example of a fuzzy variable. In this case, fuzzy numbers represent approximations of a central value and can be described by distributions about that value. Such distributions can be symmetric or asymmetric. They are constructed as membership functions, which should not be confused with probability distributions as has been demonstrated by Kosko (1992, pp.263-94).

The fuzzy approach provides an alternative to the random utility maximisation model used to analyse dichotomous choice CVM responses (Hanemann 1984; Hanemann and Kriström 1995). It can be used to address preference uncertainty, or imprecision about the value of a particular variable (Cox 1994, pp.351-53). Fuzzy numbers can be used to deal with imprecision in the values that are elicited, although some of this imprecision will be related to fuzzy classification (vagueness about the amenity itself).

Finally, a concept required for working with fuzzy numbers is that of the $\alpha$-*level set*. The $\alpha$-level set $A_\alpha$ is simply that subset of $\tilde{A}$ for which the degree of membership exceeds the level $\alpha$, and is itself a crisp set (an element either meets the required level of $\alpha$ or it does not).

(5)    $A_\alpha = \{x \mid \mu_A(x) \geq \alpha\}, \alpha \in [0\ 1]\}.$

$A_\alpha$ is an upper level set of $\tilde{A}$. The use of $\alpha$-level sets provides a means of transferring information from a fuzzy set into a crisp form. Defining an $\alpha$-level set is referred to as taking an $\alpha$-cut, cutting off that portion of the fuzzy set whose members do not have the required membership or possibility value. It can be argued that the level of the $\alpha$-cut is a measure of the reliability of the imprecise coefficient. The trustworthier the central value of the fuzzy set, the higher is the $\alpha$-cut.

## 3. Fuzzy Contingent Valuation: Forest Preservation in Sweden

To illustrate how fuzzy membership might be used to estimate non-market values, we employ data from a CVM survey of Swedish residents undertaken during the summer of 1992 (Li and Mattsson 1995, hereafter L&M). The survey asked respondents whether they would be willing to pay a given amount "to continue to visit, use, and experience the forest environment as [they] usually do". Bid amounts took one the following values: 50, 100, 200, 400, 700, 1000, 2000, 4000, 8000 and 16000 SEK. Since the authors were interested in preference uncertainty, they elicited post-decisional confidence (using a graphical scale with 5% intervals) by asking, "How certain were you of your answer to the previous [dichotomous choice] question?" The authors interpreted responses as the subjective probabilities that the individual's true valuation is greater (for a 'yes' answer) or less (for a 'no' answer) than the bid. L&M also assumed that an individual might give different 'yes/no' answers to the same bid because of the randomness of her preferences. Finally, the researchers also collected data on household income, the respondent's age, gender, education level, and average annual number of forest visits. We employed the same criteria as L&M to obtain the same number of observations.

The format of the confidence question posed by L&M allows for different interpretations. The interpretation used by L&M is that the 'yes' and 'no' responses are complementary, which leads to the construction of a single WTP function. The other interpretation is that responses are not complementary and thus separate functions for WTP and willingness not to pay (WNTP) need to be considered. We illustrate the use of fuzzy logic for each of these interpretations, although the fuzzy approaches for dealing with preference uncertainty in these cases are quite different. Compared to L&M, both approaches lead to lower estimates of non-market value.

### Fuzzy Complementarity: Single WTP Function

First assume that the individual's response to the question of how certain she is about her answer to the choice problem is a measure of the "fuzziness" of the WTP response and not the utility function itself nor of the amenity to be valued. We construct a (single) fuzzy set of acceptable bids with various degrees of membership. Although choice of form of the membership function affects the results, this is no different than choice of the cumulative distribution function in the random utility maximisation model. If a respondent answers 'yes' to the dichotomous choice (DC) question, it is assumed she would then be willing to pay any lesser amount, so that we construct a one-sided membership function that is also a measure of the fuzziness of the WTP estimate. The bid

amounts are first converted to a proportion of respondents' income. We assume that all respondents are willing to pay zero percent of their income to preserve the forest; we assign the maximum value of 1 to a bid of zero, interpreted as full acceptability in the fuzzy set of "acceptable bid values as a proportion of income", denoted $\tilde{W}$. For respondents who accept a bid, their response to the certainty question, $\mu_{\tilde{W}}(x) \in [0\ 1]$, denotes the degree of membership of the associated bid in $\tilde{W}$. For the person who rejects a bid, the complementary fuzzy number is used, via equation (4), to indicate the degree of acceptability. Since no respondent was willing to pay 10% or more of household income towards forest protection, fuzzy numbers were truncated at the "tithe amount". Measuring bid as a proportion of income along the abscissa and $\mu_{\tilde{W}}(x)$ along the ordinate, we can construct a fuzzy number for WTP for each respondent.

The approach is explained with the aid of Figure 2. Person #1 accepts with 90% certainty a bid that is 2% of household income. The fuzzy number takes on a value of 1 if there is no cost for forest protection. The degree of certainty in her willingness to pay declines linearly at a rate of 0.05 for each percent of income that must be contributed, until at 10% it falls to zero. Person #2 rejects with 95% certainty a bid that constitutes 4% of income, which is interpreted as having a degree of membership of 0.05 in the set of acceptable bids (due to fuzzy complementarity). Again, assuming linearity, the person would be totally certain ($\mu_{\tilde{W}}(x)=0$) that they would not contribute if asked to contribute anything above 4.21% of income (the point where the membership function intersects the horizontal axis).
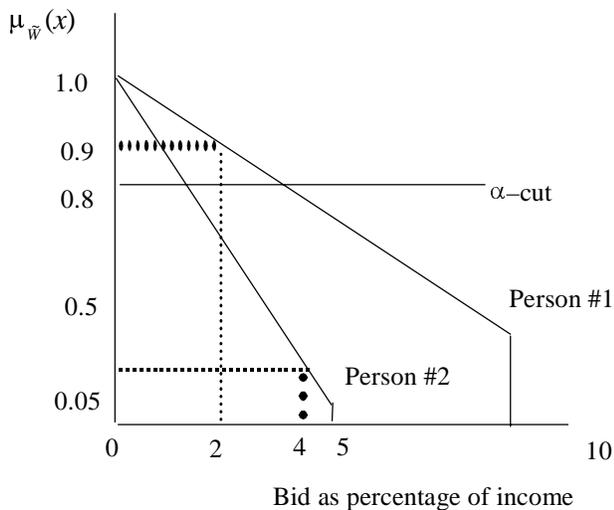


**Figure 2: Membership functions for bids as a proportion of income**

Also indicated in Figure 2 is an $\alpha$-cut of 0.8. For that value of certainty, person #1 would contribute 4% of income to protect the forest, while person #2 would only contribute 0.842% of income. Alternatively, suppose that #1 had an income of $40,000, while #2 had an income of $50,000. Then, person #1 accepts a WTP offer of $800 with 0.90 certainty, and based on the fuzzy number that was constructed in Figure 2, a request to pay $1,600 for forest protection with membership of degree 0.8. Person #2, on the other hand, rejects paying $2,000 (4% of income) to protect forests with 0.95 certainty (implying acceptance with 0.05 degree of certainty). Based on the derived membership function for person #2 in Figure 2, this person would accept paying 0.842% of their income, or $421, with degree of membership of 0.8. The total amount the two would pay together would be $2,021 (=$1,600+$421), or an average of $1,010.50 per household, with 0.8 degree certainty. For each respondent, we can plot the actual dollar amount on the abscissa and $\alpha$ on the ordinate. For each level of $\alpha$, then, we determine an average WTP.

The results of applying this approach to the Swedish data are summarised in Figure 3. Our calculations provide a fuzzy WTP number that ranges from 0 to 7,300 SEK, below the average values obtained by L&M, who estimate four different (but crisp) mean WTPs for their uncertainty-adjusted model. Their values range from a low of 7,352 SEK to a high of 12,817 SEK depending on the version of the model used and whether or not responses are truncated at 16,000 SEK. Our crisp representation of WTP depends on the degree of membership or $\alpha$-cut chosen. While respondents would certainly be willing to pay nothing to guarantee that the forest environment will be preserved, from Figure 3 they would be willing to pay on average about 1,800 SEK, with membership value 0.90, for forest protection, but 5,000 SEK with membership value 0.50. Thus, the higher the $\alpha$-cut or membership value for the fuzzy WTP number, the greater is our confidence that respondents consider it to be an acceptable amount to pay for forest protection.

In Figure 2, the membership function was assumed to take on only positive WTP values. This was done for convenience only because we constructed membership functions for every respondent and, for each respondent, we had but one observation on which to base the membership function. An alternative approach is to construct <u>and</u> estimate aggregate WTP and WNTP functions, which we do in the next subsection.
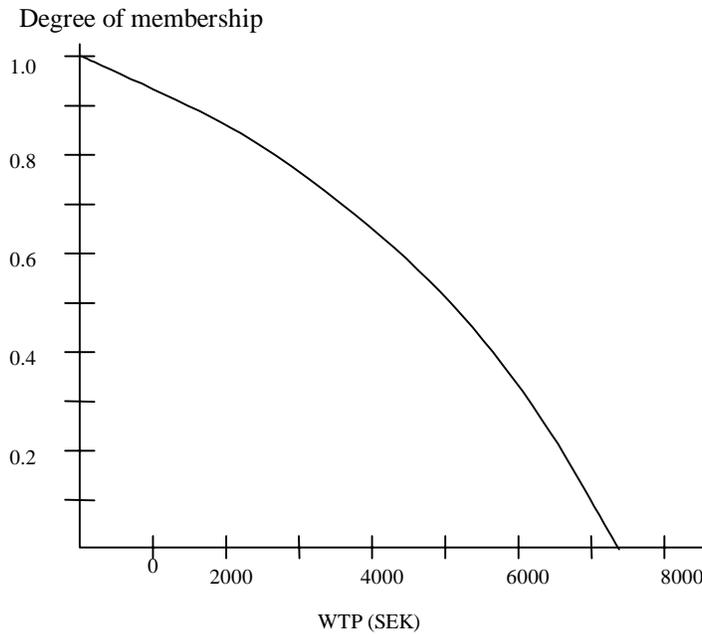
**Figure 3: Fuzzy WTP for continued use of forest environment**

## Membership Functions for Willingness to Pay and Willingness Not to Pay[3]

It is known that people have problems interpreting measures of uncertainty even when these are defined as probabilities (see, e.g., Knetsch 2000). We assume that an individual always provides the same 'yes/no' answer whenever the same bid is offered. Along with the 'yes/no' answer, she provides a post-decisional number between 0 and 1 that we interpret as a measure of her comfort, enthusiasm or inclination toward the given answer, or the degree of membership of the bid in the fuzzy sets of "acceptable bids" and "unacceptable bids", respectively. Classical CVM requires only one value, maximum willingness to pay (denoted $M$), to define a *crisp choice* rule: Accept a bid $W$ ('yes' answer) if $W \leq M$; do not accept a bid $W$ ('no' answer) if $W > M$. The corresponding crisp choice function is $C_{yes}(W) = 1$, if $W \leq M$ and $C_{yes}(W) = 0$, otherwise.

Here we formulate the choice criteria using the notion of fuzzy preference, so that $C_{yes}(W)$ is interpreted as a membership function for fuzzy WTP, and $C_{no}(W)$ a membership function for fuzzy WNTP. Define fuzzy sets $\tilde{M}$ and $\tilde{N}$ such that fuzzy number $\tilde{M}$ is the maximum WTP for an increase in the environmental amenity, and fuzzy number $\tilde{N}$ is the minimum WNTP for preservation of the amenity. For very high or very low bids, the respondent has little (if any) uncertainty about the response. She rejects or accepts bids with a high level of comfort and consistency.

The membership of WTP equals 1 for low bid values (the respondent is likely willing to "pay" negative amounts[4] and may also be willing to pay small positive amounts) and then declines as the bid increases above $w_1$ (Figure 4). A bid $w_0$ is the maximum value that a respondent would be WTP with a membership $\mu(w_0)$. The same $w_0$ is the minimum bid that a respondent would not be willing to pay at the comfort level $\mu(w_0)$. As the bid amount $W$ increases, membership in WNTP increases (as bids increase they become less acceptable for the respondent) and reaches 1 at $w_2$ (Figure 4).
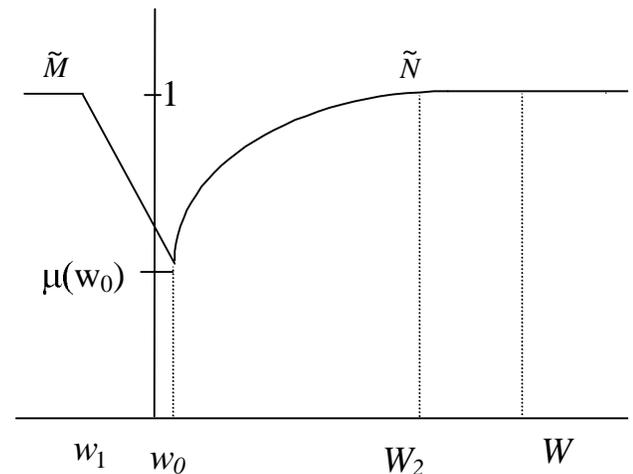


**Figure 4: Membership functions for WTP and WNTP as represented by the fuzzy numbers, $\tilde{M}$ and $\tilde{N}$, respectively.**

Three components of the fuzzy numbers $\tilde{M}$ and $\tilde{N}$ correspond to different aspects of a respondent's preference uncertainty in the non-market valuation context. First, the shape of the membership curves of $\tilde{M}$ and $\tilde{N}$ may depend on the respondent's attitude toward risk, thus explaining the asymmetrical feature of the two curves. Second, $\mu(w_0)$ reflects the strength of a respondent's preference uncertainty regarding valuation of the environmental amenity. A higher $\mu(w_0)$ corresponds to weaker preference uncertainty. Finally, the width of the interval $[w_1, w_2]$ relates to the range of the bid values over which a respondent's preferences are uncertain.

For a bid $w_0$, a respondent is indifferent, at the

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[3] Material in this section summarises van Kooten et al. (2000).

[4] Kriström (1997) and Loomis and Ekstrand (1998) also permit negative WTP values.

comfort level $\mu(w_0)$, between two alternatives: accepting or rejecting the bid. When a respondent is certain of her preferences, then $\mu(w_0)=1$ and $w_0=w_1=w_2$. Thus, our approach to CVM with vague preferences includes preference certainty as a special case. Another extreme value, $\mu(w_0)=0$, corresponds to the situation of strongest preference uncertainty. In this case, there is no single bid in the range of non-intersection that could be reported as a maximum value (with reasonable comfort) that a respondent is WTP. This occurs in Figure 4 if $\tilde{M}$ and $\tilde{N}$ do not intersect, in which case the degree of uncertainty is so great as to prevent a decision. This represents the situation where respondents register protest votes by not answering the valuation question.

Despite similarities to the classical method, our approach to CVM with vague preferences is peculiar. Classical CVM requires one value (maximum WTP, denoted $M$) to define a crisp choice function. The choice rule is far more complex when vague preferences are considered and more information is required. This should not be treated as a disadvantage of the proposed methodology, but rather as a way of incorporating real-life complexity.

Define the membership function of WTP be as follows:

(6)
$$\mu_{WTP}(W) = 1, \qquad W < w_1$$
$$\mu_{WTP}(W) = C_{yes}(W), \qquad w_1 \le W \le w_0$$

where $C_{yes}(W)$ is monotonically decreasing for $W \in [w_1, w_0]$. The degree of membership in WNTP is

(7)
$$\mu_{WNTP}(W) = C_{no}(W), \qquad w_0 \le W \le w_2.$$
$$\mu_{WNTP}(W) = 1, \qquad W > w_2,$$

where $C_{no}(W)$ is a monotonically increasing function for $W \in [w_0, w_2]$ (see Figure 4). Once the membership functions for WTP and WNTP are determined, the point of their intersection $(w_0, \mu(w_0))$ will be used to formulate an operational choice rule:

*Fuzzy choice rule.*
*(a)* Accept the bid $W \le w_0$ with comfort $C_{yes}(W)=\mu(W) \ge \mu(w_0)$, where $\mu(W)=\mu_{WTP}(W)$.
*(b)* Reject the bid $W > w_0$ with comfort $C_{no}(W)=\mu(W) \ge \mu(w_0)$, where $\mu(W)=\mu_{WNTP}(W)$.

The Swedish data are used to estimate fuzzy WTP and WNTP numbers.

We first assume that an individual's response to the question of how certain she is about her answer to the DC question is a measure of the uncertainty of WTP and WNTP in the case of 'yes' and 'no' responses, respectively. If a respondent answers 'yes' with a comfort $C_{yes}(W)$ to the DC question at the bid value $W$, it is

assumed she would then be willing to pay any lesser amount than $W$ with a comfort at least as high as $C_{yes}(W)$. It is also assumed that a maximum WTP value greater than $W$ may exist, but with a comfort level lower than $C_{yes}(W)$. Similar logic holds for 'no' answers and minimum WNTP.

The sample data were divided into two groups according to the respondents' answers to the DC contingent question. To estimate the membership function for WTP, we regress comfort level for the 'yes' answer on the relative bid expressed as a percent of the respondent's income. Similarly, we regress comfort level for the 'no' answer on the respondent's relative bid to estimate the WNTP membership function. Functional forms for fitting the sample data must satisfy conditions (6) and (7).

Membership functions for aggregated WTP and WNTP are estimated from available data using a statistical approach for constructing membership functions (Chameau and Santamarina 1987). Instead of individual WTP and WNTP, estimated membership functions of aggregate WTP and WNTP are developed. For data $(W_i, \mu_i)$, i=1,2, ..., $n$, and choice of a suitable functional form, membership functions can be estimated using the method of least–squares. Once the parameter values $(a, b, ...)$ are determined, then

(8) $\qquad \mu(W) = \max[0, \min(1, f(W, a, b, ...)]$ , $\quad \forall W$.

Different classes of functional forms are used in the literature to construct membership functions (Turksen 1991), but it is clear that estimating the contingent value is sensitive to the form of membership function chosen.

We selected two non-linear forms of the membership function that can cover a broad range of applications (Sakawa 1993). The functional form used for 'yes' responses is:

(9) $\qquad \mu_{\tilde{M}}(x) = a\tanh^{-1}(bW+k) + \dfrac{1}{2}$ , $\qquad a,$

$b, k \in \Re$ and $a > 0$ .

The minimum of the sum of squared deviations of the respondents' post–decisional comfort levels is reached for estimated parameter values, $a = 1.775$, b = $-0.026$ and k= 0.187.

The functional form employed for 'no' responses is:

(10) $\qquad \mu_{\tilde{N}}(x) = \dfrac{1}{2}\tanh(dW+e) + \dfrac{1}{2}$ , $\quad d, e \in \Re$ and

$d > 0$.

The minimum of the sum of squared deviations of the respondents' post–decisional comfort levels is obtained for $d = 0.044$ and $e = 0.466$.

Our estimate of the intersection of the membership of maximum WTP and minimum WNTP occurs at a comfort level of 74.9% and is associated with the relative bid of 1.82% of income.[5] With the average

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[5] This is found by solving: $1.77\tanh^{-1}(-0.026W+0.187) = 0.5\tanh(0.044W+0.466)$.

income in the given sample of 171,190 SEK, the intersection of two membership functions is associated with the value of 3116 SEK. This value may be interpreted as the respondents' WTP with a comfort of 74.9%, but it is also the respondents' WNTP with 74.9% comfort. It is thus the largest estimated value of the amenity for which there is an aggregate indifference between WTP and WNTP. Other measures of welfare may be reported if higher comfort levels than 0.749 are applied. In that case, we can report the WTP at the comfort level $c > 0.749$ (which will be below 3116 SEK) and the WNTP at the level $c$ (which will be above 3116 SEK) (see Figure 5). The range of the values between WTP and WNTP could be interpreted as the aggregated indifference at the comfort level $c$.

Again, our estimates of WTP are lower than those of L&M. The estimate of maximum WTP (3116 SEK) at 74.9% comfort is less than half the magnitude of L&M's lowest estimates—7352 SEK or 8578 SEK depending on what estimator is used.[6] Several explanations for the difference between L&M's and these results are possible. The one that accounts for the major difference is that L&M use mean WTP as a measure of welfare. If we assumed complementarity of the 'yes' and 'no' answers, i.e., $C_{yes}(W)=1-C_{no}(W)$, then the membership functions of WTP and WNTP would intersect at $(w_0, 0.5)$ and the value $w_0$ would correspond to the median WTP (which L&M do not report). In that sense, it would be more appropriate to compare our measure with the median WTP. Further, different assumptions are made about the nature of preference uncertainty. Finally, unlike L&M, we do consider either the analyst's uncertainty explicitly or other factors that may influence the response to the DC question.

The form of fuzzy numbers for WTP and WNTP may be explained by different attitudes towards acceptance and non-acceptance of a particular bid. Complete certainty of a 'no' answer occurs only for very high bid values. A respondent chooses <u>not</u> to accept a wide range of bid values including low ones. At the same time, she indicates her uncertainty through an expressed comfort level that is below 1. Respondents indicated preference uncertainty even at low positive bid values. The membership function for WTP has its highest value in the negative domain. This is consistent with the results of Kriström (1997) and Loomis and Ekstrand (1998). For a particular bid value, the membership values of WTP and WNTP add to one only in extreme cases of very high or very low bid values. This indicates that preference uncertainty exists for a wide range of bid values.

We now consider a means of using fuzzy logic that is closer to the approach of choice experiments because it relies on pairwise comparisons.

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[6] These are the truncated means, which are significantly lower than the overall means.

## 4. Fuzzy Pairwise Comparisons: Valuing Water Quality Improvements

Livestock wastes are a major source of groundwater pollution in the Abbotsford region of south-western British Columbia, Canada. The Abbotsford aquifer covers approximately 100 square km in BC and an additional 100 square km in the state of Washington. It is an important source of residential, industrial and agricultural water. Nitrate-nitrogen concentrations in the aquifer (as determined from well samples) have often exceeded the Canadian government's drinking water quality standard of 10 parts per million by volume (ppmv), as spelled out in its *Guidelines for Canadian Drinking Water Quality*. The nitrate-nitrogen concentration limit is meant to prevent adverse health impacts, including "blue baby syndrome" and possibly cancer.

In order to determine the viability of measures to reduce pollution of the aquifer from livestock wastes, it is necessary to value the benefits. Since the benefits of improved water quality are non-market in nature, it needs to be shown that these are significant. Three studies have been conducted to determine the social benefits of improving water quality in the Abbotsford region. One study employed fuzzy pairwise comparisons, while the others elicited WTP using open-ended and dichotomous-choice CVM formats.

### Fuzzy Pairwise Comparisons

Fuzzy pairwise comparisons were first used by van Kooten et al. (1986) to study farmers' goal hierarchies for use in multiple-objective decision making, and the approach relies on a market touchstone much like in choice experiments. The fuzzy pairwise method results in a ratio scale that can then be used to value non-market goods and services if one of the items in the set has a known market value. Fuzzy pairwise comparisons require that, if there are $k$ items, all are compared in pairwise fashion; thus, there are $k(k-1)/2$ pairwise comparisons that need to be made. Items can then be ordered. Respondents are asked not only to choose between two items, but to indicate an intensity of preference between the items.

A measure of the intensity of preference between two items, $A$ and $B$, is made by marking on a line, with endpoints denoted $A$ and $B$, the degree of preference for one over the other; a mark placed at the centre of the line indicates indifference. A measure of the intensity of the preference of item $A$ over item $B$ is determined by measuring the normalised distance from the left endpoint (where $A$ is assumed to be located) to the respondent's mark, where the line is of unit length after normalisation. Denote this distance by $r_{AB}$. If $r_{AB} < 0.5$, then $A$ is preferred to $B$; if $r_{AB} > 0.5$, $B$ is preferred to $A$; if $r_{AB} = 0.5$, $A$ is equally preferred to $B$; and $r_{AB} = 1 - r_{BA}$.

Van Kooten (1998) develops a measure indicating the intensity of preference among items. This

concept can be understood as the degree of membership of a fuzzy number. Once all of the pairwise measures of preference of item $i$ over $j$, $r_{ij}$, are obtained, the aggregated normalised measure of intensity for item $j$, $m_j$, is:

$$(11) \qquad m_j = 1 - \sqrt{\frac{\sum\limits_{i=1}^{k} r_{ij}^2}{k-1}}, \qquad j = 1, 2, \ldots, k$$

where $k$ is the number of items that are ranked by the fuzzy pairwise comparisons. Assume that, as a result of fuzzy pairwise measures, we obtain the following matrix of normalised distances:

| Item | 1 | 2 | 3 | 4 |
|------|--------|--------|--------|--------|
| 1 | 0 | 0.2121 | 0.9697 | 0.1212 |
| 2 | 0.7879 | 0 | 0.5606 | 0.4242 |
| 3 | 0.0303 | 0.4394 | 0 | 0.3485 |
| 4 | 0.8788 | 0.5758 | 0.6515 | 0 |

The matrix indicates that $1P2$, $1P4$, $2P4$, $3P1$, $3P2$ and $3P4$, where $P$ denotes "preferred to". Using the above formula, the preference intensity scores are as follows: $m_1 = 0.4227$, $m_2 = 0.3904$, $m_3 = 0.6757$ and $m_4 = 0.2863$. Further suppose that item 3 is valued at \$100. Then, by independence of irrelevant alternatives (one's preference between oranges and apples does not depend on whether or not a grapefruit exists in the choice set), item 4 is valued at \$42.37 ($= \$100 \times 0.2863/0.6757$).

## Comparing Methods for Valuing Water Quality Improvements

Van Kooten (1998) employed the results of a mail-out survey of households in the Abbotsford area (see Hauser et al. 1994) to determine the benefits of water quality improvements using fuzzy pairwise comparisons. From the predicted preference intensities for two ranked items (water quality and a 33-inch colour television—the market touchstone), respondents' intensity of preference for water quality relative to the colour television was determined using relation (11). Intensity of preference depended on whether the respondent owned land in the agricultural land reserve (ALR) and on whether they owned or rented their residence. Those owning both land and their residence valued improvements in water quality by a factor of 1.836 over the television, or about \$165 per year (if the television is valued at \$900). Those who owned their place of residence but did not own land in the ALR valued improvements in water quality at \$128/year, while those who owned no property whatsoever valued it at \$161/year. In general, improvements in water quality are valued higher by those with ALR land. These values are contrasted with those obtained using a traditional CVM instrument.

Hauser et al. (1994) used an open-ended contingent valuation question imbedded in the same mail-out survey to obtain estimates of WTP for improving water quality to at least the Canadian standard from whatever the respondents perceived it to be (they were only told that some well samples indicated it was above the standard). Their estimates of WTP ranged from \$55.35 to \$114.71 annually (depending on the regression model used for the bid functions) for those with ALR land, and \$80.00-\$114.71 for those with no land in the land reserve.

Finally, van Kooten et al. (1998) conducted a telephone survey in which they elicited WTP for water quality improvements in the Abbotsford region using a DC format. Three levels of improvement were presented to respondents—eliminating the problem entirely, reducing the pollution so that the Canadian drinking water standard was always met (from an assumed 12 to 10 ppmv), and a reduction of water pollution by half (from an assumed 12 to 6 ppmv). Mean annual household WTP (truncated at \$300) was estimated from the logit models to be \$160.54-\$209.54, depending on the proposed reduction in water quality and the regression model employed. Median values ranged from \$8.18 to \$161.51.

A comparison of the three studies suggests that the method of fuzzy pairwise comparisons provides results that are "in line" with those obtained by more traditional methods. A comparison of the open-ended and dichotomous choice approaches lends some support to the hypothesis that dichotomous choice formats lead to higher values of WTP because of "yea-saying". However, given that the approaches used in the two studies differ substantially, it is difficult to draw a definitive conclusion from these results. Further, the fuzzy approach supports the values from the dichotomous choice instrument.

## 5. Discussion

Preservation of environmental goods such as wildlife and ecosystems, and the valuation of such amenities, is fraught with vagueness and uncertainty. While many environmental goods cannot be described in a crisp fashion (they are vague by nature), providing greater information will also not reduce the associated uncertainty. Thus, if someone is uncertain about how to trade off an environmental amenity against income, providing more knowledge about the trade off or the commodity will not reduce the person's uncertainty. Uncertainty is not always associated with stochasticity. It is precisely in these instances that crisp forms of analysis fail, and that includes analyses that employ a probability approach. An alternative means of analysis is to use fuzzy logic.

While fuzzy techniques might be considered somewhat ad hoc, they are not less so than is the case with other valuation methods. Choice of a membership function is no different than choice of functional form for the distribution of WTP, no different than choice of a Weibull or log-logistic distribution function in standard

CVM. While function (11) is a particular aggregation, and others exist, the choice used to aggregate across individuals is no different from choice of the functional form for a bid function in analysis of open-ended CVM responses, or aggregation in the case of choice experiments.

In this paper, we demonstrate how several, different fuzzy approaches can be used to value environmental amenities. A summary of the comparisons between fuzzy and more traditional approaches is provided in Table 1. It is clear that fuzzy methods provide values that, on the whole, cannot be considered worse in some sense than those obtained by traditional CVM. This suggests that, at the very least, fuzzy analysis should be seriously considered in the valuation of that which is by nature fuzzy.

**Table 1: Comparison of Fuzzy and CVM Measures of Non-market Values**

| Item | Traditional CVM | Fuzzy Approach |
|---|---|---|
| Forest Preservation (SEK per year) | | |
| – single WTP membership function | 7,352 - 12,817 | 1,800 - 5,000 |
| – WTP and WNTP membership functions | 7,352 | 3,116 |
| Water Quality Improvements (C$ per year) | | |
| – open-ended CVM (mail survey) | 55 - 115 | 128 - 165 |
| – dichotomous-choice CVM (telephone) | 160 - 210 | 128 - 165 |

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