Investment and Development of Fishing Resources: A Real Options Approach*

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Abstract

The valuation of the opportunity to either invest or exploit a fishery is particularly difficult because of the high uncertainty concerning the resource price. The traditional net-present-value (NPV) and other discounted-cash-flows (DCF) methods cannot properly capture the management's flexibility and strategic value aspects of a fishery, thus they may understate its value. The rationale for using an option-based approach to capital budgeting arises from its potential to conceptualize and quantify this flexibility, as new information arrives, to alter its operating strategy, to defer investments, to shut down (and restart) fishery development. Real Options valuation has traditionally been applied in the area of natural resource investments different from fishing resources. This paper presents a general bioeconomic model for the value of a fishery. It suffices to determine not only the value of the fishery when open and closed, but also the optimal policy for opening, closing and setting the harvest rate. Moreover, the paper turns to the valuation of a fishery investment opportunity and the optimal investment rule. The natural growth rate of fishing resource stock and the production function are those of the Schaefer model. Finally, results for the Pacific Yellowfin Tuna are presented.

keywords: Fishing Resources, Management's flexibility, Real Options.

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1. Introduction: The Real Options Theory.

The basic inadequacy of the net-present-value (NPV) approach and other discounted-cash-flow (DCF) approaches to capital budgeting is that they ignore, or cannot properly capture, management's flexibility to adapt and revise later decision (i.e.,review its implicit operating strategy). The traditional NPV approach, in particular, makes implicit assumptions concerning an "expected scenario" of cash flows and presumes management's commitment to a certain "operating strategy". Typically, an expected pattern of cash flows over a prespecified project life is discounted at a risk-adjusted rate to arrive at the project's NPV. Treating projects as independent investment opportunities, an immediate decision is then made to accept any project whose NPV is positive.

In the real world of uncertainty the realization of cash flows will probably differ from what management originally expected. In particular, the operating flexibility and strategic value aspects of a fishery cannot be properly captured by traditional DCF techniques, because of their dependence on the high degree of uncertainty attaching to the price of the fishing resource. Nevertheless, we can properly analyze these important aspects by thinking of investment and development opportunities as options on the fishing resource through the options-based technique of contingent claims analysis. Just as the owner of an American call option on a financial asset has the right -but no the obligation- to adquire the asset by paying a predetermined price on or before a predetermined date, and will exercise the option if and when it is in his/her best interest to do so, so will the holder of an option on the fishing resource. In the real life, a fishery does not have to be operated (i.e., harvest fishing resource) in each and every period. In fact, if the price of the fishing resource is such that cash revenues are not sufficient to cover variable operating costs, it might be better not to operate temporarily. If the price rises sufficiently, operation can be restarted. Thus, operation in each year may be seen as a call option to acquire that year's cash revenues by paying the variable costs of operating as exercise price (option to shut down and restart operations). Similarly, the optimal timing of investment in a fishery is analogous to the optimal exercise of an American call option on the gross present value of the completed fishery's expected operating cash flows, with an exercise price equal to the required investment expenditures. Management will invest such expenditures only if the price of the fishing resource increases sufficiently, but will not commit to the project if prices decline (option to defer investment). This option to wait is particularly valuable in fishing resource exploitation because the investment expenditures are largely irreversible; that is, they are mostly sunk costs that cannot be recovered (the capital is fishing firm or industry specific and it cannot be used productively by a different firm or in a different industry). Irreversibility makes investment especially sensitive to the fishing resource price risk.

Section 2 develops a general model of a fishery. A general bioeconomic model for the value to exploit a fishery is presented in section 3. Section 4 considers a particular version of the general model when the harvest rate equals the resource growth (i.e. under a sustainable development of the fishing resource). Section 5 shows a numerical application of the development valuation model to the Pacific Yellowfin Tuna fishery. Section 6 turns to the valuation of the fishery investment opportunity, and the optimal investment rule. Section 7 considers this model when the harvest rate is equal to the resource growth. Section 8 discusses a numerical application of the investment valuation model. Finally, the main conclusions are provided.

2. A model of the fishery.

This section presents a fishing resource model by defining all the variables which adequately describe the state of the resource at any point in time.

With harvesting, the rate of change in the resource stock must reflect growth and harvest; thus the dynamics of the resource stock might be described by the difference equation:

$$dX(t) = [F(X(t)) - h(t)]dt. \tag{2.1}$$

where, $h(t)$ is the production function; $F(X(t))$ stands for the instantaneous rate of growth in the biomass of the fish population.

Therefore, the model describes the dynamics of the stock in a deterministic context.[1]

The firm develops the fishery with the following total average cost function: $C = C(X)$.

On the other hand, it is assumed that the firm faces a competitive market for its output, with a spot price S that follows a Brownian motion:[2]

$$\frac{dS}{S} = \mu dt + \sigma dZ, \tag{2.2}$$

where, μ : local trend in the price; may be stochastic; σ : instantaneous standard deviation of the spot price, assumed to be known; dZ : increment to a standard Gauss-Wiener process.

3. A general model for valuing the opportunity to exploit a fishery.

The theory of real options is based on the Black, Merton and Scholes' models which were initially developed for financial options[3]. This approach has several advantages over the NPV model. Two essentially distinct approaches may be taken to the general problem of valuing the uncertain cash-flow stream generated by an investment project by using the Real Options Theory. On the one hand, it can be suppposed the existence of a futures market in the output commodity (Brennan y Schwartz (1985), Cortazar y Schwartz (1993)). On the other hand, it would be straighforward to derive an analogous model in a general equilibrium context similar to the

[1] Although a deterministic process is adopted, it is introduced a boundary condition that limits the harvest rate when the resource stock falls below a minimum biological stock (for example, notice a natural catastrophe). The stochastic character is not as important in fishing resources as in others (as forestry resources) because these natural catastrophes cannot been avoided and advanced.

[2] This stochastic process is used in the area of natural resources: Brennan and Schwartz (1985), Bjerksund and Ekern (1990), Pindyck (1988), Paddock, Siegel and Smith (1988), Cortazar, Schwartz and Salinas (1998), Cortazar and Schwartz (1993), Morck, Schwartz and Stangeland (1989).

[3] Black and Scholes (1973), Merton (1973).

previous one (Cortazar, Schwartz y Salinas (1998)). There is no futures market for the fishing resource[4] in this paper, and hence the last approach will be adopted to obtain the value of the fishery.

3.1. Notation.

Q : Value of the opportunity to exploit a fishery (or value of the fishery, for short);
K : convenience yield on holding one unit of output. It is the flow of services accruing to the owner of the physical commodity but not to the owner of a derivative contract on that commodity;
ρ: risk free rate of return, assumed constant;
t : calendar time.

3.2. Assumptions.

(i)The option to exploit is valued by risk-averse investors who are well diversified and need only be compensated for the systematic component of the risk;
(ii) There are not arbitrage opportunities;
(iii) The exchange of assets takes place continuously in time;
(iv) There exist neither transaction costs nor taxes betwen the assets exchanged in the market, and all of the assets are perfectly divisible;
(v) Markets are sufficiently complete; stochastic changes in S are spanned by existing assets. Specifically, it must be possible to find an asset (or construct a dynamic portfolio of assets), whose price is perfectly correlated with S. Let Y be the price of this "twin" asset:

$$\frac{dY}{Y} = \mu_y dt + \sigma_y dZ, \tag{3.1}$$

where μ_y denotes its expected return; it is further assumed that $\mu_y > \mu$ (because if $\mu_y = \mu$ there would be no opportunity cost to keeping the option alive, and one would never invest, no matter how high the NPV of the project).
(vi) There is no cost of closing and opening the fishery[5];
(vii) The option to exploit the fishery is perpetual, i.e.it has no expiration date;
$(viii)$ The convenience yield is assumed to be proportional to the current spot price: $K = kS$ (Brennan and Schwartz, (1985); Cortazar and Schwartz, (1993); Cortazar, Schwartz and Salinas, (1998)).

3.3. The partial differential equation for the value of the fishery, Q.

The value of the fishery, Q, will depend on the current commodity price, S, the fishing stock, X, and the calendar time, t:

$$Q = Q(S, X, t). \tag{3.2}$$

The opportunity to exploit the fishery is envisaged as a derivative asset. Applying Itô's lemma and using (2.1) and (2.2), the instantaneous change in the value of the fishery is given by:

$$\begin{aligned} dQ &= \{Q_S S\mu + Q_t + \frac{1}{2}Q_{SS}\sigma^2 S^2 \\ &\quad + Q_X [F(X) - h]\}dt + Q_S S\sigma dZ. \end{aligned} \tag{3.3}$$

In order to derive the differential equation governing the value of the fishery, let us consider the return to a portfolio consisting of a long position in the fishery and a short position in $\left(\frac{S\sigma Q_S}{Y\sigma_y}\right)$ twin contracts. The return on this portfolio is nonstochastic and to avoid riskless arbitrage opportunities it must be equal to the riskless return $\rho\left(Q - \left(\frac{S\sigma Q_S}{Y\sigma_y}\right)Y\right)dt$. On the other hand, consider a firm who has the right to harvest the fishing resource (exploit the fishery). The firm's cash flow from fishing activity production is $(S - C(X))hdt$. It is assumed that there exists a property tax rate, λ, so that the firm's after-tax cash flow is $[(S - C(X))h - \lambda Q]dt$. Combining these and using (3.1), (??) and the Capital Asset Pricing Model, the following differential equation is obtained:

$$\frac{1}{2}Q_{SS}\sigma^2 S^2 + Q_S S(\rho - k) - (\lambda + \rho)Q + Q_t + Q_X [F(X) - h] + (S - C(X))h = 0.$$

3.4. The general model.

The values of the fishery when open, $V(S, X, t)$, and closed, $W(S, X, t)$, are:

[4]Fama and French (1987) show a list of commodities for which there exist futures markets.

[5]It is assumed there are no such costs for the sake of simplicity. However, in fact, there exist several costs of opening and closing a fishery . Brennan and Schwartz (1985) consider explicitly the cost of opening and closing a mine. They show how sunk costs of opening and closing can explain the "hysteresis" often observed in extractive resource industries. During periods of low prices, managers often continue to operate unprofitable mines that had been opened when prices were high; at other times managers fail to reopen seemingly profitable ones that had been closed when prices were low.

$V(S,X,t) \equiv \max_\phi Q(S,X,t;1,\phi)$ and $W(S,X,t) \equiv \max_\phi Q(S,X,t;0,\phi)$

The value of the fishery under the value-maximizing policy $\phi^* = \{\overline{h}, \widehat{S}, X^*\}$, satisfy:

$$\max_{h\epsilon(0,\overline{h})} \left[\begin{array}{c} \frac{1}{2}V_{SS}\sigma^2 S^2 + V_S S(\rho - k) - V(\lambda+\rho) + V_t \\ +V_X\left[F(X) - h\right] + (S - C(X))h = 0 \end{array} \right],$$

$$\left[\frac{1}{2}W_{SS}\sigma^2 S^2 + W_S S(\rho - k) - W(\lambda+\rho) + W_t = 0\right]. \quad (3.4)$$

where:

$\overline{h}$: maximum harvest rate;

$\widehat{S}$: critical price at which it is optimum for a firm to exploit the fishery. It is chosen so as to maximize the value of the fishery.

X^* : optimal resource population level; if $X < X^*$ the fishery will be closed down completely. It is chosen so as to maximize the value of the fishery.

It may be verified that the deflacted (in small letters) value of the fishery satisfies:

$$\max_{h\epsilon(0,\overline{h})} \left[\begin{array}{c} \frac{1}{2}v_{ss}\sigma^2 s^2 + v_s s(r-k) - v(\lambda + r) \\ +v_X(F(X) - h) + (s - c(X))h = 0 \end{array} \right]; \quad (3.5)$$

$$\left[\frac{1}{2}w_{ss}\sigma^2 s^2 + w_s s(r-k) - w(\lambda + r) = 0\right]. \quad (3.6)$$

The following boundary conditions must also be satisfied:

$$w(0,X) = 0. \quad (3.7)$$

$$v(s,0) = w(s,0) = 0. \quad (3.8)$$

$$w(\widehat{s},X) = v(\widehat{s},X); \quad (3.9)$$
$$w_s(\widehat{s},X) = v_s(\widehat{s},X).$$

$$\lim_{s\to\infty} \frac{v(s,X)}{s} < \infty. \quad (3.10)$$

$$\frac{\partial v(s,X)}{\partial X}\,|_{X=M} = 0; \quad (3.11)$$

$$\frac{\partial w(s,X)}{\partial X}\,|_{X=M} = 0. \quad (3.12)$$

$$h(X,E) = 0, \quad if\ X < X^*; \quad (3.13)$$

$$h(X,E) = 0, \quad if\ X < X_{\min}. \quad (3.14)$$

$X_{\min}$, is assumed to be the minimum inventory in the fishery exogenously given. Equations (3.5) to (3.8) constitute the general model for the value of a fishery. They suffice to determine not only the deflacted value of the fishery when open and closed, but also the optimal policies for opening, closing, and setting the harvest rate. In general there exists no analytic solution to the valuation model, though it is straightforward to solve it numerically.

4. A particular valuation case: a sustained yield harvest.

Fish can be a sustainable natural resource and have long been an important source of food and other products for people and animals. An interesting case is that in which the harvest rate exactly equals the natural growth, so that the stock size will remain unchanged over time (a sustained yield harvest). This is the so-called steady-state equilibrium in the fishery (Perman, Ma and McGilvray (1996); Caddy and Griffiths (1996)). Now, a model that is analytically tractable is obtained because this sustained yield harvest enables to replace the partial differential equations for the value of the fishery with ordinary differential equations.

The value of the fishery when closed and open must satisfy the following equations:

$$\frac{1}{2}w_{ss}\sigma^2 s^2 + w_s s(r-k) - w(\lambda + r) = 0. \quad (4.1)$$

$$\frac{1}{2}v_{ss}\sigma^2 s^2 + v_s s(r-k) + (s - c(X))h - v(\lambda + r) = 0. \quad (4.2)$$

The boundary conditions that the value of the fishery must satisfy are the same as before. Following Ingersoll (1987), the critical price $\widehat{s}$ at which the option to exploit should be optimally exercised is given by:

$$\widehat{s} = c(X)\left(\frac{d_1}{d_1 - 1}\right), \quad (4.3)$$

where d_1 is defined as:

$$d_1 = \alpha_1 + \alpha_2,\ d_2 = \alpha_1 - \alpha_2, \quad (4.4)$$

$$\alpha_1 = \frac{1}{2} - \frac{r-k}{\sigma^2},\ \alpha_2 = \sqrt{\alpha_1^2 + \frac{2(\lambda + r)}{\sigma^2}}. \quad (4.5)$$

The complete solutions to equations (4.1) and (4.2) are:

$$w(s,X) = c_1 s^{d_1}, \quad (4.6)$$

$$v(s,X) = c_4 s^{d_2} + \frac{hs}{\lambda + k} - \frac{c(X)h}{\lambda + r}. \quad (4.7)$$

The constants, c_1 and c_4, are determined by the boundary conditions, which imply that:

$$c_1 = \frac{A\widehat{s}(d_2 - 1) + Bd_2}{\widehat{s}^{d_1}(d_2 - d_1)},\ c_4 = \frac{A\widehat{s}(d_1 - 1) + Bd_1}{\widehat{s}^{d_2}(d_2 - d_1)} \quad (4.8)$$

$$\text{where} : \ A = \frac{h}{\lambda + k},\ B = -\frac{hc(X)}{\lambda + r} \quad (4.9)$$

Sensitivity analysis of the critical price. In this subsection the partial derivatives of $\widehat{s}$ in (4.3) with respect to the tax rate, λ, the conveniece yield, k, the risk-free rate, r, and the volatility of the price, σ^2, are obtained.

$$\frac{\partial \widehat{s}}{\partial \lambda} < 0; \ \frac{\partial \widehat{s}}{\partial k} < 0; \ \frac{\partial \widehat{s}}{\partial r} > 0; \ \frac{\partial \widehat{s}}{\partial \sigma^2} \gtrless 0 \qquad (4.10)$$

These derivatives will be studied in detail in the following numerical application.

5. A numerical application.

5.1. The Schaefer model

Up to now, the model has included general functions of costs, harvest and growth of the resource. Now, it is interesting to specify those functional forms so as to obtain the value of the fishery according to some known parameters. The net natural growth in the biomass ($F(X)$) of fish population is often represented by the logistic function (Schaefer (1957)):

$$F(X) = \gamma X \left(1 - \frac{X}{M}\right) \qquad (5.1)$$

where: γ : intrinsic instantaneous growth rate of the biomass; M : carrying capacity of the habitat. It can be thought of M as the maximum population.
On the other hand, it is usually assumed that the harvest function depends on two inputs: the current size of the stock, X, and the fishing effort, E:

$$h = bEX$$

where b is the catchability coefficient, it is assumed to be constant,

5.2. Data for the fishery.

To illustrate the nature of the solution, a case based on the "Pacific Yellowfin Tuna" fishery (Conrad and Clark (1987)) is considered. However, the economic data are hypothetical.

γ, intrinsic instantaneous growth rate	2.6
M, maximum biomass sustainable	250,000
b, capturability coefficient	0.000038
r, risk-free rate	2% annual
k, convenience yield	1% annual
σ^2, volatility	6% annual
λ, tax rate	5% annual

5.3. The general model

Equations (3.5) - (3.8) comprise the general model for the value of a fishery. In general there exists no analytic solution to this valuation model, though it can solved numerically. There are several numerical procedures that can be used to price value derivative securities when exact formulas are not available. Some of then are the finite difference methods (Hull, (1997); Cortazar, Schwartz and Löwener, (1998); Brennan and Schwartz (1978); Courtadon (1982); Schwartz (1977); Geske and Shastri (1985); Hull and White (1990); Majd and Pindyck (1987)). In this work a numerical implementation of the implicit finite difference method is adopted as opposed to the explicit finite difference method, because of its robustness ans superior stability properties (Geske and Shastri (1985)).

As expected, an increase in the resource price increases the value of the fishery. However, this value is not always increased as the stock size increases, due to the shape of the growth function. An increase in the stock size will increase the value of the fishery if the stock size is on the growing section of the natural growth function; conversely, an increase in the stock size will decrease the value of the fishery if the stock size is on the decreasing section of the natural growth function

Along the left hand side of the growth function, the higher the stock size the higher the growth and the lower the costs, so that, both $v_X(F(X) - h)$ and $(s - c(X))h$ are higher. However, over the right hand side of the growth function, the higher the stock size the higher the economic term $(s - c(X))h$ because of the reduction in cost, whereas the lower the biological term $v_X(F(X) - h)$ because of the decrease in the growth function. On the other hand, the value of the fishery in the right hand side of the growth function decreases faster as the harvest policy becomes the more aggressive.[6]

[*Table* 1].

5.4. A particular valuation case: a sustained yield harvest.

This section uses the complete solutions (4.6) and (4.7) so as to obtain the value of the fishery when closed and open. The value of the fishery is obtained

[6] Numerical results have also been computed for other harvest policies less aggressive than the one presented in the paper: $h \in \{0, 53.000\}$, $h \in \{0, 55.000\}$, $h \in \{0, 56.000\}$ *tons*. They are available from the author upon request.

at each point on the growth curve that represents a sustainable yield of fish for a given stock, X, by going through the same model as in the previous section, that is, the Schaefer model.

As expected, the higher the resource price the higher the value of the fishery. On the other hand, the higher the resource stock (and therefore the lower the unitary costs), the higher the value of the fishery. Besides, not only the value of the fishery is higher but also the opportunities to exploit the fishery since the critical price decreases. As before, the value of the fishery is bound to the form of the growth function. It can be shown that each sustained harvest rate can be obtained with two different stock sizes, one of which is stable and the other one which is not (Romero, 1994). However, the value of the fishery in the stable stock size is higher than in the unstable one.

[*Table* 2].

5.4.1. Sensitivity Analysis.

The comparative statics for changes in the main parameteres are of some interest. The following tables report the values of the fishery (dollars per harvested ton) obtained by changing some of the key parameters of the model (λ, k, r, σ^2) in isolation, for different spot prices. This analysis is made for $X = 175.000$.[7]

Table: Value of the fishery and changes in the tax rate.

price ($/ton)	$\lambda = 0,02$ $\widehat{s}$=1078.389	$\lambda = 0,07$ $\widehat{s}$=718.585
600	9979.354	3079.252
700	12643.812	4254.455
800	15520.523	5556.662
900	18596.656	6847.943
1000	21861.554	8129.416

The higher the tax rate, the lower the value of the fishery because the after-tax benefits decrease. However, although an increase in λ reduces the value of the fishery, it also reduces the critical price at which it is optimal to exploit the fishery, and hence, there are more opportunities to exploit it.

Table: Value of the fishery and changes in the convenience yield.

price ($/tn)	$k = 0.005$ $\widehat{s}$ =847,132	$k = 0.015$ $\widehat{s}$=751,879
600	4.938,350	3.780
700	6.515,414	5.145
800	8.283,287	6.694,421
900	10.206,707	8.264,043
1000	12.125,389	9.826,973

Notice that an increase in the convenience yield results in a decrease in the value of the fishery, and also results in a decrease of the critical price. The reason is that as k becomes larger, the expected rate of growth of the resource price falls, and hence the expected appreciation in the value of the fishery. In the limit as $k \to \infty, v \to 0$, and $\widehat{s} \to c(X)$.

An increase in the convenience yield means that the scarcity expectatives of the resource will be greater in the future and it has already been shown that the lower the stock, the lower the value of the fishery. This result has also been found for other resource developments (Cortazar, Schwartz and Löwener (1998)).

Table: Value of the fishery and changes in the risk-free rate.

price ($/ton)	$r = 0.015$ $\widehat{s}$=773.144	$r = 0.025$ $\widehat{s}$=817.815
600	4148.516	4449.604
700	5600.184	5918.684
800	7254.454	7578.017
900	8957.393	9350.943
1000	10652.712	11102.048

If the risk-free rate is increased the value of the fishery increases, and so does the critical price. The reason is that the present value of the exploitation costs is $c(X)e^{-rt}$, whereas the present value of the fishery is ve^{-kt}, hence with k fixed, an increase in r reduces the present value of the cost but does not reduce the value of the exploitation.

Besides, the higher the risk-free rate, the higher the expected return rate of the price, and hence the higher the value of the fishery is.

Table: Value of the fishery and changes in volatility.

price ($/ton)	$\sigma^2 = 0,04$ $\widehat{s}$=706.448	$\sigma^2 = 0,08$ $\widehat{s}$=877.192
600	4159.145	4465.327
700	5782.292	5848.037
800	7549.315	7387.475
900	9288.504	9074.259
1000	11009.194	10796.443

Finally, it is observed that as volatility becomes higher so does the value of the fishery when closed, but conversely when open because the option to shut down the fishery is less valuable.

[7]The same steps can be followed for $X = 75.000$; $X = 100.000$; $X = 150.000$ and for any price.

6. A general model for the valuation of the opportunity to invest in a fishery.

For the moment the value of the option to exploit the fishery including the shut down (and restart) option and the optimal exploitation rule have been derived. Now, in this section, the question is when (at which price level) it is optimun for a firm to invest in a fishery. By going through the same steps as in the previous model, the paper turns to the valuation of a fishery investment opportunity including the delay option, and the optimal investment rule.

In the previous model, the value of the fishery has been obtained by assuming that the firm had the property right on the fishery. However, it may be interesting to know how much it would pay for that property right.

This irreversible investment opportunity is much like a financial call option. It gives the firm the right (which needs not be exercised) to make an investment expenditure, I, and receive the value of the fishery, Q. As with the financial call option, this option to invest is valuable in part because the future value of the fishery obtained by investing is uncertain (McDonald and Siegel (1986), Pindyck (1991)).

6.1. Assumptions.

The assumptions $(i)-(v)$ and $(viii)$ in section 3 are also considered here. The assumptions (vi) and (vii) are replaced with the following:
(vi) There exists no cost of temporarily delay and abandone the investment opportunity;
(vii) The option to invest in the fishery is perpetual, it has no expiration date;

6.2. The partial differential equation for the value of the opportunity investment, F(Q).

The value of the firm's option to invest in the fishery, F, depends on the value of the fishery, Q.
$F = F(Q)$. It is known that $Q = Q(S, X, t)$;Thus, $F(Q)$ can be expressed as:

$$F = F(S, X, t).$$

By going through the same steps as in the previous model of valuation, it can be easily checked that the value of the deflacted investment opportunity, f, must satisfy the following differential equation:

$$\frac{1}{2} f_{ss}\sigma^2 s^2 + f_X [F(X) - h] + f_s s (r-k) - rf = 0. \tag{6.1}$$

In addition, $f(s, X)$ must also satisfy the following boundary conditions:

$$f(0, X) = 0, \tag{6.2}$$

$$f(s, 0) = 0. \tag{6.3}$$

$$f(s^*, X) = q(s^*, X) - I, \tag{6.4}$$

$$f_s(s^*, X) = q_s(s^*, X) \tag{6.5}$$

where s^* is the critical price at which it is optimum to invest in the fishery and I is the investment expenditure.

The boundary conditions (6.2) and (6.3) establish that if Q goes to zero, so the option to invest will be worthless. (6.4) just says that upon investing, the firm receives a net payoff $q(s^*, X) - I$. The condition (6.5) is called the "smooth pasting" condition; if f were not continuous and smooth at the critical price, one could do better exercising at a different point.

To find f, equation (6.1) must be solved subject to the boundary conditions (6.2)-(6.5). In general there is no analytic solution to the valuation model, though it can be solved by means of numerical procedures.

7. A particular investment valuation case: a sustained yield harvest.

As in the previous model of fishery valuation, this section deals with the case in which the net natural growth function equals the harvest rate, that is, $h = F(X)$.
Under the previous assumptions, the value of the investment opportunity must satisfy the following differential equation:

$$\frac{1}{2} f_{ss}\sigma^2 s^2 + f_s s (r-k) - rf = 0; \tag{7.1}$$

$f(s, X)$ must also satisfy the boundary conditions (6.2)-(6.5).
The complete solution to (7.1) using the boundary condition (6.2) is:

$$f(s, X) = \left\{ \begin{array}{ll} c_5 s^{d_1}, & s \leq s^* \\ v(s, X) - I, & s > s^* \end{array} \right\}, \tag{7.2}$$

where d_1 is known, (4.4), and $v(s, X)$ is the value of the fishery corresponding to (4.6). In order to compute the constant c_5 and the critical price s^*, boundary conditions (6.4) and (6.5) are used:

$$c_5 s^{*d_1} = c_4 s^{*d_2} + \frac{hs^*}{\lambda + k} - \frac{c(X)h}{\lambda + r} - I; \qquad (7.3)$$

$$c_5 d_1 s^{*(d_1-1)} = d_2 c_4 s^{*(d_2-1)} + \frac{h}{\lambda + k}. \qquad (7.4)$$

Solving for c_5 in the previous equation (7.4) yields:

$$c_5 = \frac{d_2 c_4}{d_1} s^{*(d_2-d_1)} + \frac{h}{d_1(\lambda + k)} s^{*(1-d_1)}. \qquad (7.5)$$

Substituting c_5 into (7.3):

$$\frac{c_4(d_2-d_1)}{d_1} s^{*d_2} + \frac{h(1-d_1)}{d_1(\lambda+k)} s^* + \frac{c(X)h}{\lambda+r} + I = 0. \qquad (7.6)$$

The optimal investment rule (critical price) can be derived from the previous equation.

8. A numerical application.

This section illustrates the nature of the model solution. The data for the "Pacific Yellow Fin Tuna" and the Schaeffer model remain unchanged. However, it is necesary to know the total investment expenditure I, so the one corresponding to the "South Pacific Yellowfin Tuna" fleet is adopted(Wesney, Waugh (1989)): $I = 28,000000$ \$. On the other hand, given the empirical evidence, this fleet harvests 135.000 tons on a yearly basis.

8.1. The general and particular models.

Since the general model cannot be solved analytically, a numerical solution is obtained. Thus, as in the previous valuation model the implicit finite difference method is used. For the sustained yield harvest case; the values are derived from(7.2) using (7.5). On the other hand, the optimal investment rule $s*$ is derived from (7.6).

Assuming a resource stock of 200.000 tons, the value of the investment opportuniy in the fishery are the following (in dollars per harvested ton).

models	general	particular
price(\$/ton)	$X = 200,000$	$X = 175,000$
700	132.441	5606.030
800	331.125	7221.956
900	442.875	8958.081
1000	554.625	10681.422
1100	666.376	12393.398
1200	778.128	14097.037
critical price, s^*	724.415	817.38
$F(X)$	104,000	136,500
costs, $c(X)$	375.939	375.939
ratio, $\widehat{s}/c(X)$	1,9269498	2.1742357

The above values for the general model are relatively low because the harvest policy is too aggressive. Note that the resource population is on the right hand side of the natural growth function; moreover the "perpetual" harvest rate is higher than the growth rate.

It can be observed that the value of the investment opportunity is always lower than the value of the fishery. Otherwise, the opportunity cost of investing increases even more than the value of the fishery, and hence, there will be less incentive to exercise the investment option. On the other hand, the optimal exploitation rule, $\widehat{s}$ is higher than the optimal investment rule, s^*; this is so because the latter one includes the investment expeditures I, and therefore, a firm will exercise the investment option if and only if the resource price is at least as high as the investment and exploitation costs.

9. Conclusions.

In this paper the Real Options approach has been applied to the valuation of a renewable natural resource: a fishery. This theory is prefered to the traditional discounted cash flows methods, because the cash flows will probably differ from what management expected initially due to the high volatility of the fishing resource price; in fact, investors or managers may have valuable flexibility to alter the exploitation and investment policy in the fishery. The paper presents several models to value the opportunity to either invest or exploit a fishery.

In both cases, the solution of the general models is given by a partial differential equation that must be satisfied, and several boundary conditions. In general, there is no analytic solution to these valuation models. A particular case in which the harvest rate equals the natural net growth function (the resource stock is sustained) is also presented, which can be analytically tractable, that is, a closed solution that depends on the resource stock, the critical price and other key parameters of the model can be derived.

The above models suffice to determine not only these valuations, but also the optimal policy for opening, closing, delaying and setting the harvest rate. In particular, for the sustained stock model, a closed expression for either the optimal exploitation and investment rule (critical price) can be obtained. For the exploitation model, the critical price is proportional to the costs; the proportion depends on the risk-free rate, the convenience yield and the volatility of the resource price. The sensitivity analysis of it shows, that the higher the tax rate and the convenience yield, the more there is incentive to exercise the option because the critical price decreases. On the contrary, the higher the risk-free rate is the higher the critical price is.

The numerical application for the exploitation valuation models shows that either in the general model or in the particular case of a sustained stock, the higher the resource stock on the growing section of the natural growth function, the higher the value of the fishery. However, the biological state of the resource stock affects differently in both valuation models when the stock is on the decreasing section of the natural growth function. On the one hand, although the resource stock is on the decresing section, the higher the stock, the higher the value of the fishery in the case of sustained stock, because the resource stock is sustained and it is associated with low costs. On the other hand, the higher the resource stock, the lower the value of the fishery in the general model, because a new term appears in the differential equation (dependent on the growth function) so that, an increase in the resource stock reduces the value of the fishery. The higher the resource price, the higher the value of the fishery in both models (general and particular). As can be expected, the sensitivity analysis for the particular case shows that the lower the tax rate and the convenience yield and the higher the risk-free rate, the higher the value of the fishery.

The numerical application for the investment valuation models shows that the value of the investment opportunity in the fishery is always lower than the value of the fishery. Otherwise, the opportunity cost of investing increases even more than the value of the fishery, and hence, there will be less incentive to exercise the investment option. On the other hand, the optimal exploitation rule, $\widehat{s}$ is higher than the optimal investment rule, s^*; because the last one includes the investment expenditures I, and therefore, a firm will exercise the investment option if and only if the resource price is at least as high as the investment and exploitation costs

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Table 1: Value of the fishery (dollars per harvested ton) for a harvest policy of $h \in \{0, 158.000\}\, tn$.

price ($\$/ton$)	X =100,000	X =125,000	X =150,000	X =175,000	X =200,000	X =225,000
600	1193.211	2970.169	2824.188	471.903	201.104	77.353
700	1598.462	3978.930	3783.369	632.176	271.739	102.461
800	2059.215	5125.848	4873.917	814.271	342.474	127.598
900	2574.709	6409.027	6094.028	996.349	413.208	152.734
1000	3144.280	7826.816	7442.135	1178.428	483.943	177.871
1100	3767.343	9377.758	8687.944	1360.506	554.677	203.007
1200	4443.372	11060.546	9983.509	1542.585	625.411	228.143
1300	5171.894	12874.000	11279.025	1724.663	696.145	253.279
1400	5952.477	14817.044	12574.301	1906.742	766.880	278.415
1500	6784.723	16888.691	13870.603	2088.818	837.614	303.552
1600	7668.266	19088.026	15164.898	2270.901	908.348	328.688
1700	8602.764	21414.201	16460.764	2452.976	979.082	353.824
1800	9587.901	23866.426	17759.629	2635.049	1049.816	378.960
1900	10623.377	26443.958	19045.147	2817.146	1120.550	404.096
2000	11708.913	29146.099	20358.259	2999.193	1191.285	429.232
critical price,$\widehat{s}$	1069.740	1044.716	885.7159	798.8186	601.402	604.6641
cost,$c(X)$	657.894	526.315	438.596	375.939	328.947	292.397
ratio $\widehat{s}/c(X)$	1.6260075	1.9849646	2.0194346	2.1248624	1.8282664	2.0679560

Table 2: Value of the fishery (dollars per harvested ton) at several points on the growth curve for the sustained case: $h = F(X)$.

price($\$/ton$)	$X = 100,000$	$X = 125,000$	$X = 150,000$	$X = 175,000$	$X = 200,000$	$X = 225,000$
600	2604.368	3181.353	3746.468	4301.899	4849.174	5389.414
700	3488.891	4261.837	5018.883	5762.956	6495.873	7126.941
800	4494.557	5490.303	6465.565	7423.854	8233.969	8848.537
900	5619.702	6864.718	8084.122	9163.210	9954.604	10556.705
1000	6862.879	8383.3147	9823.434	10886.551	11663.349	12255.729
1100	8222.809	10044.530	11553.998	12598.527	13363.655	13948.265
1200	9698.348	11788.120	13272.806	14302.166	15057.772	15636.041
1300	11288.460	13519.778	14982.751	15999.520	16747.223	17320.228
1400	12991.746	15241.171	16685.860	17692.028	18433.076	19001.649
1500	14735.151	16954.492	18383.594	19380.724	20116.099	20680.893
1600	16467.939	18661.358	20077.030	21066.373	21796.858	22358.397
1700	18192.113	20362.987	21766.978	22749.548	23475.782	24034.490
1800	19909.209	22060.313	23454.061	24430.692	25153.197	25709.422
1900	21620.422	23754.063	25138.763	26110.146	26829.358	27383.390
2000	23326.698	25444.810	26821.465	27788.184	28504.467	29056.548
critical price, $\widehat{s}$	1391.492	1113.194	927.661	795.138	695.746	618.441
$F(X)$	156,000	162,500	156,000	136,500	104,000	58,500
cost, $c(X)$	657.894	526.315	438.596	375.939	328.947	292.397
ratio, $\widehat{s}/c(X)$	2.115070	2.115070	2.115070	2.115070	2.115070	2.115070