Wave Energy Converter Modeling in the Frequency Domain: A Design Guide

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Abstract—**Wave energy converter research continues to advance and new developers are continuing to emerge, leading to the need for a general modeling methodology. This work attempts to outline the design methodology necessary to perform frequency domain analysis on a generic wave energy converter. A two-body point absorber representing a generic popular design was chosen and a general procedure is presented showing the process to obtain first pass preliminary performance results. The result is a design guide that new developers can adapt to their particular design and wave conditions, which will provide the first steps toward a cost of energy estimate. This will serve the industry by providing a sound methodology to accelerate the new development of wave energy converters.**

I. INTRODUCTION

Wave Energy Converter (WEC) design is still in its infancy with significant research going into the design of new devices. Unlike the wind industry where a clear proven topology has been established, the wave energy industry is still looking for its first grid tied industrial scale wave generation site. As developers attempt to prove the merit of new designs a standard methodology is needed for the initial modeling of said devices.

Assuming that a rough physical WEC design has been chosen, frequency domain analysis is the first step in the validation of the merits of the design under operational sea conditions. In order to start a design, the detail of the end product does not need to be known, however the general shape and design philosophy is needed. The idea at this stage of the design process would be to create as simple a model as possible and start getting rough numbers for power output and performance characteristics. Several texts describing the theory behind ocean waves and structure interaction exist, which include [1],[2],[3]. Also, many papers on the subject exist, including [4],[5]. Development of a scaled wave energy converter is described in [6]. In addition, a numerical benchmark study of different wave energy converters is presented in [7].

Frequency domain analysis provides a good first step in the design process. Goals of frequency domain modeling include defining the parameters of the WEC, defining a mooring configuration and power take off system, and getting a first impression of how the device will perform. Because frequency domain analysis is intrinsically linear, many real nonlinear effects which become prominent under high and extreme sea conditions are not taken into account and results should be viewed with this in mind [5]. More detailed analysis will no doubt follow, but frequency domain analysis provides an insightful look at a preliminary design and WEC performance for normal energy conversion operation. Basic shape optimization, identification of resonant frequencies of the device, structure loading due to wave pressures, general frequency response characteristics, and power output characteristics are to be gained by this analysis.

During the process of developing a WEC model, many results are relevant. In particular Response Amplitude Operators (RAOs), Froude Krylov forces, diffraction forces, added mass, and radiation damping forces in the frequency domain are useful. Hydrostatic results can also provide insight into the design. An iterative approach to changing shape design and characterizing the system can help improve the design, providing greater power output. This would be a

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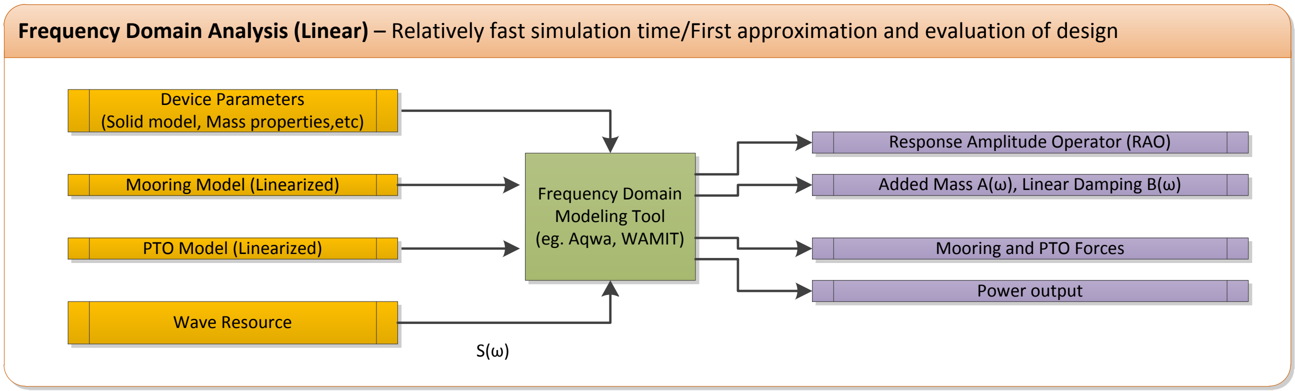

Figure 1. High level block diagram of inputs and outputs of frequency domain model.

first step in the process to estimate a cost of energy for the device.

For this paper a generic two-body point absorber was used as an example of the design process. This is a popular design which several companies have pursued or are pursuing. Two general-purpose representative software products, ANSYS Aqwa[8] and SolidWorks[9] were used in this modeling process, however several other modeling and hydrodynamics packages exist. Although these programs solve for motions (and forces) in six degrees of freedom, for convenience of exposition, focus in this presentation will be on the heave motion of a point absorber. Thus, only motion in the z-direction will be analyzed. A high level block diagram of inputs and outputs of the representative model is shown in Fig. 1.

This guide is targeted for researchers and wave energy converter companies that desire a methodology to follow in basic WEC design. It does not provide a novel design; rather it provides the background information and suggested techniques to expedite the process of designing a WEC.

A. *Linear Wave Theory*

A simple model of the waves is a good place to start defining the environment of a wave energy converter. Linear wave theory (also known as Airy wave theory) provides such a simple model, which assumes that the fluid flow is irrotational, incompressible, and inviscid, thus potential flow applies. It also assumes that the mean depth of water is uniform and that the wave amplitude is small. Airy waves can be modeled as the following

$$\eta(t) = A\cos(\omega t + kx) \tag{1}$$

where ω is the frequency in radians/sec and k is the wave number and $\eta(t)$ is the water surface elevation. The modeling at this stage utilizes these sinusoidal (also called harmonic) linear waves.

B. *Forces on Structures*

In order to ultimately define the potential power output from a wave energy device one must start with defining the forces acting on a structure. The governing equation of motion in the time domain is

$$M\ddot{z}(t) = F(t) \tag{2}$$

where M is the mass, z is the vertical displacement, and $F(t)$ are the total forces on the body. The types of wave forces on a body include viscous and non-viscous forces. The viscous forces include form drag and friction drag. Form drag is a function of the shape of the object. A body with a large apparent cross-section will have a larger drag than a body with a smaller cross-section, therefore presenting higher form drag. Friction drag is caused by the viscous drag present in the boundary layer around the body. The viscous forces including friction drag are usually relatively small in magnitude and neglected, or represented by a Morrison force term, and will not be discussed in this study.

Hydrodynamic forces exerted on the body, under the linear formulation, can be interpreted to include the Froude-Krylov force, diffraction force on a “fixed” body, and superposition of the radiation force due to the motion of the body. These forces arise from potential flow wave theory and linearization (which allows superposition of the linearized forces). The total (non-viscous) forces acting on a fixed floating body in regular waves consist of the sum of the diffraction forces and the Froude-Krylov forces and are denoted as the excitation force as shown in (3). The Froude-Krylov force is a wave induced force on the “fixed” body, and its specification does not account for the effects of the presence of the body. It is the incident wave force resulting from the pressure on the virtual fixed body in the undisturbed waves. The diffraction force is due to scattering which is a combination of wave reflection and diffraction.

$$F_e(\omega) = F_{FK}(\omega) + F_d(\omega) \tag{3}$$

where F_e is the excitation force, F_{FK} is the Froude-Krylov force and F_d is the diffraction force. In this theory, an assumption is made that the dimensions of the body are sufficiently large in comparison to the wavelength of the incoming wave such that the incoming waves acting on the body are diffracted by the presentation of the body. The interpretation of the Froude-Krylov force is that the pressure field of the wave is not affected by the presence of the body is purely for convenience and is an artifact of linearization.

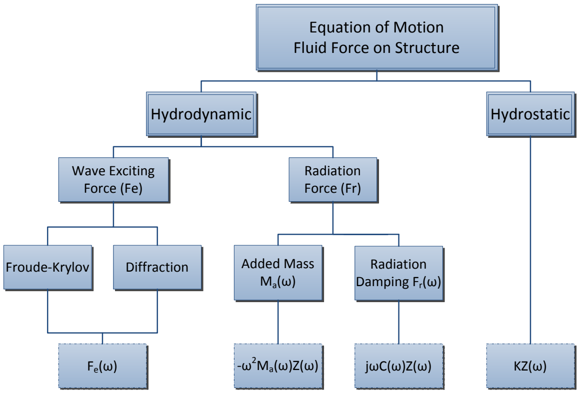

Figure 2. Equation of motion terms of fluid forces on a structure.

The important physics that is enforced is that there is no flow through the (fixed) rigid body.

The radiation force due to structure motion can be decomposed into an added mass and a radiation damping term as shown in (4). This hydrodynamic force is induced by the structure's oscillation, which in turn generates waves. The "added mass" force component is in phase and the "radiation damping" term is out of phase with the body motion. The added mass can be thought of as an added inertia on the body undergoing harmonic oscillation due to the presence of the surrounding fluid. The "radiation damping" is caused by the motion of the body in a fluid, generating out-going waves carrying energy to infinity and in phase with the body velocity thus acting as a velocity proportional "damping force".

$$\mathrm{F_r(\omega)} = -\left(-\omega^2 \mathrm{M_a(\omega)} + \mathrm{j\omega C(\omega)}\right)\mathrm{Z(\omega)} \tag{4}$$

where $M_a(\omega)$ is the added mass, $C(\omega)$ is the radiation damping, and $Z(\omega)$ is the heave motion and j is the imaginary unit.

The hydrostatic restoring or buoyancy force is exactly that, a force trying to return the structure to hydrostatic equilibrium. This force originates from the static pressure term because the wet surface of the body is exposed to varying hydrostatic pressures as a result of its oscillations. The hydrostatic stiffness is as follows

$$K = \rho g A' \tag{5}$$

where ρ is the density of the water, g is the acceleration of gravity, and A' is the cross-sectional area of the wetted surface. This leads to the definition of the hydrostatic force defined as

$$F_{hs}(\omega) = -KZ(\omega) \tag{6}$$

C. RAOs

Response Amplitude Operators (RAOs) are transfer functions which determine the effect a sea state will have on a structure in the water. This can be useful in determining the frequencies at which maximum amount of power can theoretically be extracted. In order to calculate the RAO for a given structure a general equation of motion is defined.

$$-\omega^2 MZ(\omega) = F_e(\omega) + F_r(\omega) + F_{hs}(\omega) \tag{7}$$

where M is the mass of the structure, $F_r(\omega)$ is the radiation force, F_{hs} is the hydrostatic force, $F_e(\omega)$ is the excitation wave force (both incident and diffracting forces) and $Z(\omega)$ is the heave response. Fig. 2 shows the relationship between the terms in (7). This equation then leads to the RAO given as

$$RAO(\omega) = \frac{F_e(\omega)}{K - \omega^2\left(M + M_a(\omega)\right) + j\omega C(\omega)} \tag{8}$$

The RAO is a complex quantity and it is common to define the RAO as the magnitude of (8) when the phase difference between the incident wave and motion of the device is not of interest. However, the phase can be important in optimizing power output as a 180 degree phase shift between the float and spar velocity will maximize the generator speed.

D. Workflow

The workflow required to achieve frequency domain analysis will be useful for the designer in the design process. The first task is to create a computer model of the device to feed

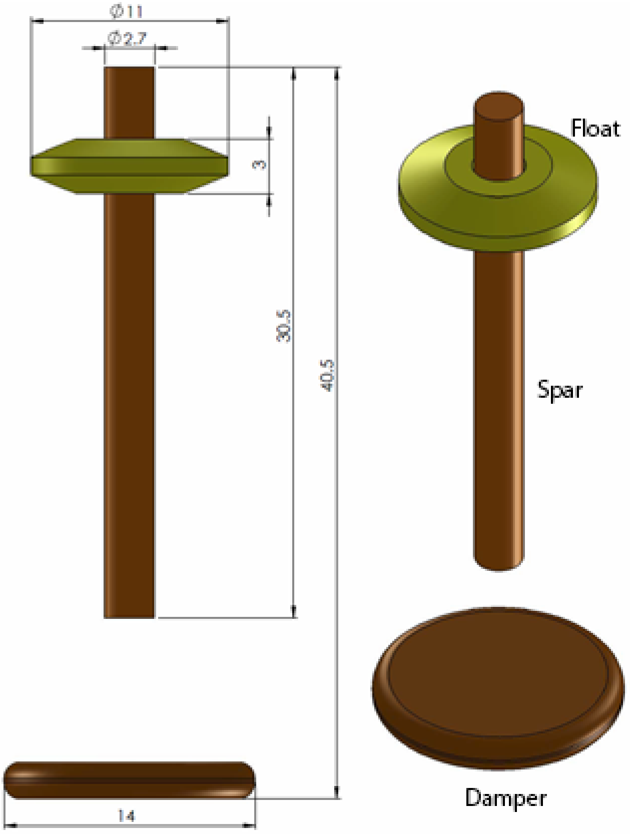

Figure 3. Generic WEC example. A massless, stiff connection between the spar and damper is present, but not shown. All dimensions in meters.

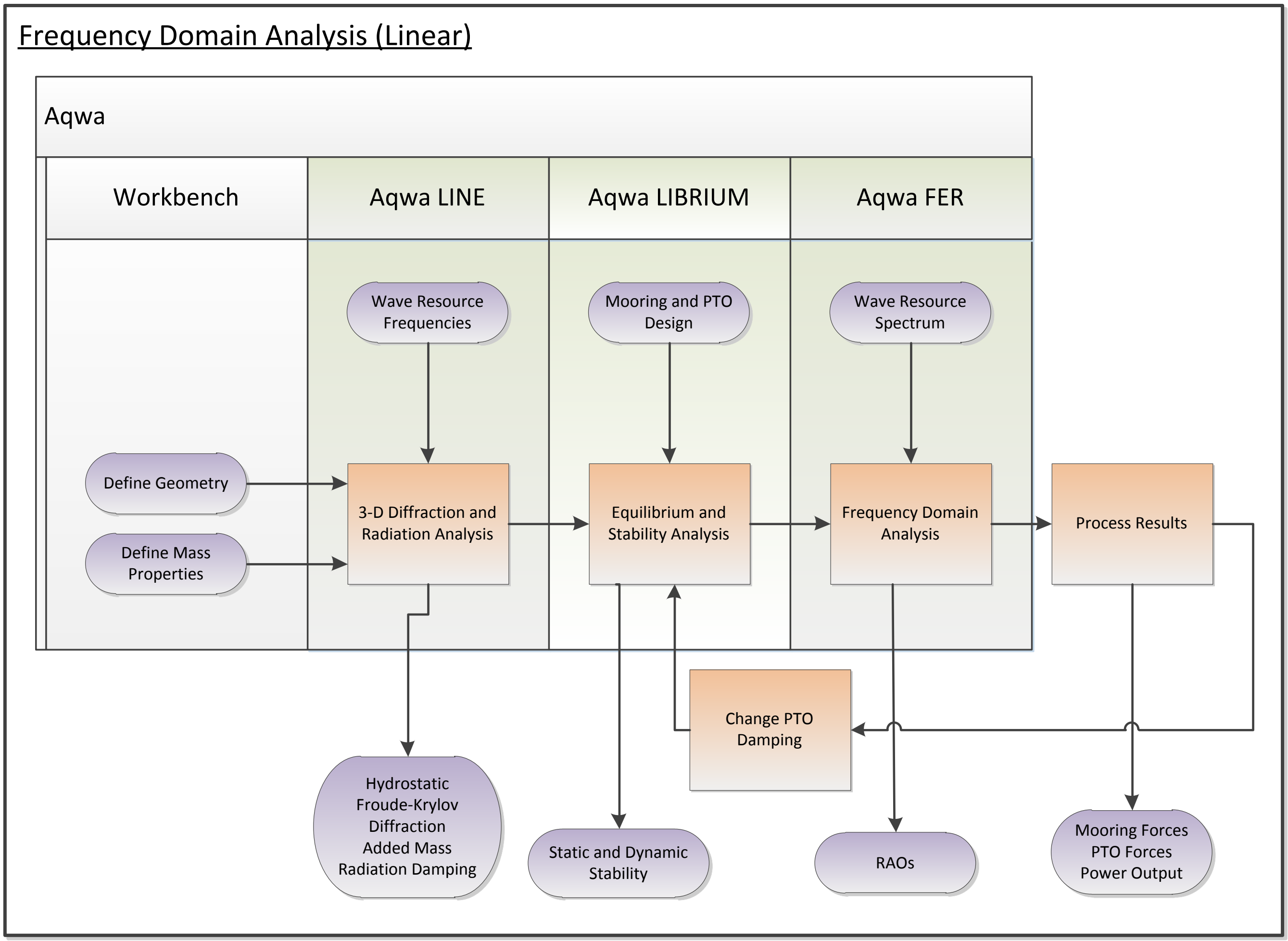

Figure 4. Example workflow for frequency domain analysis.

to the hydrodynamics package. Next, the hydrodynamics package would be applied. Although the approach outlined here is presented in the context of Aqwa, the general approach can be applied to any hydrodynamics program with minor modifications. Post processing can also be achieved with many different tools. As an example of the process, MATLAB and Excel were used to post process the data provided by Aqwa. The workflow decision tree for this design is shown in Fig. 4.

II. DEVICE GEOMETRY

Given an idea for a wave energy converter, a computer model representation of the device needs to be created at this stage. There are different methods of defining device geometry. The method chosen is dictated by the hydrodynamics package selected. In this example a solid model approach is chosen using SolidWorks to define the geometry. For those using WAMIT, as another example, a surface model needs to be created. Ultimately, whichever program is used, a mesh of the device geometry will be defined. Fig. 3 shows an example of a WEC geometry. There are three bodies for this model, including a float, center spar, and damping plate. The center spar and damping plate are locked together for the simulation. In a final design there would be a truss structure between the spar and damper but for initial simulation this is ignored.

A. Solid Model

Once a concept for a design is developed, the first step in analysis is creating a representation of the geometry of the device. This may take the form of a solid model, surface model, etc. In this example a solid model was used. There are several modeling packages available with which to develop a solid model. It is important to research the geometric formats required by the hydrodynamic tool that shall be used.

Beyond providing a convenient user interface for developing a model, packages such as Solidworks can provide valuable information for model simulation such as the center of gravity of bodies, inertia values associated with bodies, and mass properties. As an alternative, many of the hydrodynamic software packages including ANSYS Aqwa provide a simple modeling platform sufficient for creating basic shapes and devices.

1) Solidworks

Most WEC designs will be multi-bodied devices and in this case will be combined as an assembly of parts. In the generic point absorber example a separate body was created for the float, spar, and damper plate. The spar and damper are fixed together in the hydrodynamics package and the only relative motion is between the float and spar/damper. The reason for this has to do with the way that the hydrodynamic package deals with the interface between two bodies. If the proximity of two bodies is small enough, which is common in most WEC designs, the package requires time steps so small that it is not practical to run the simulation. Therefore, for this model, there will be three separate bodies, the float and damper plate will interact but there will be no interaction between the spar and the float.

Once the solid bodies have been modeled, characteristics such as inertia values and center of gravity can be gathered from the modeling program to be fed into the hydrodynamics program. When the geometry is imported into the solid modeling program the static water level needs to be defined and the body properties detailed. A point mass is defined for each body with the inertia values specified.

A big part of the modeling process includes generating a mesh with which the hydrodynamics program calculates the pressures and forces on each mesh element. This is where a tradeoff between model accuracy and simulation time and storage space needs to be made. Although a small mesh size would be preferred, a rougher mesh provides significantly shorter simulation times and smaller output files. Depending on the complexity of the bodies, and the patience for the simulation, a mesh size can be determined. The larger mesh size that is chosen, the faster results are obtained, however the smaller the mesh size, the more accurate the results will be. Take note that a single mesh size does not have to be used. A finer mesh can and was used near the interface between two bodies. This practice will lead to greater accuracy of results.

B. *Mass Properties*

1) *Moments of inertia*

Moments of inertia provide a measure of a bodies resistance to change in its state of rotation. For example, in a hydrodynamic package such as Aqwa, mass and inertia values can be specified using a point mass approach. Inertia values can either be specified directly or via knowledge of the radius of gyration. This is calculated as the root mean square distance of the bodies parts from its center of gravity. SolidWorks calculates inertia values for the bodies that can then be transferred to Aqwa. By default the mass definition is program controlled so there is no need to input mass values. Alternatively, manually specifying the mass is an option.

2) *Center of mass*

The center of mass is the weighted average location of the mass of the body. For example, the center of mass is communicated as an XYZ coordinate into Aqwa for each body. SolidWorks can be used to determine this center of mass using the mass properties dialog. This allows for simplified calculations by the hydrodynamics package by treating quantities as being referenced to the center of mass or treating a body as if its entire mass is concentrated at the center of mass.

Significant thought should be given to where the center of mass is located in the final design. For the device to be stable it needs to have a positive metacentric height. The metacenter is calculated as the ratio of the inertia resistance of the device divided by the volume of the device. At this stage however, device construction materials and all components are most likely not known. Therefore caution should be used to ensure that the center of mass is not in an unreasonable position.

III. WAVE RESOURCE DATA

In conducting frequency domain analysis a range and number of frequencies is specified for a simulation. Hydrodynamic packages often by default have a program controlled setting for frequency range which can be modified to target a specific site if necessary. Water depth is also a critical parameter to be defined in the design process.

A. *Range and number of frequencies*

By default the range of frequencies may be program controlled by the hydrodynamics package. This will provide a range that will include most if not all conditions that the device might encounter. A total number of frequencies should also be chosen where more frequencies provide more detailed results.

If one wishes to target a specific site with a targeted range of frequencies this is possible. For example, the National Data Buoy Center (NDBC)[10] provides historical data for many locations around the world. For example, if a site near the NDBC Stonewall Banks buoy was chosen, historical data shows that a range of frequencies of 0.125 Hz to 0.3 Hz would capture the wave frequencies found at that site.

B. *Wave directions*

Technically, if the device is symmetrical about the z-axis, there is no need to calculate multiple incident wave angles. In practice, however more accurate results will be obtained by choosing a number of incident wave angles. Increasing the number of angles has a relatively small impact on simulation time. A wave range, interval, and number of intermediate directions can be specified, all in degrees. For an asymmetrical device this can be a critical degree of freedom which further complicates the analysis results.

C. *Water depth*

Water depth is a very important parameter in influencing power extraction of a WEC device. Power output in shallow water can be different from that found in deep water. From an analysis perspective, in general, a deep water assumption will make for an easier process. However, if there is a desired location to target, or for specific scaled testing conditions it is important to use the depth in which the

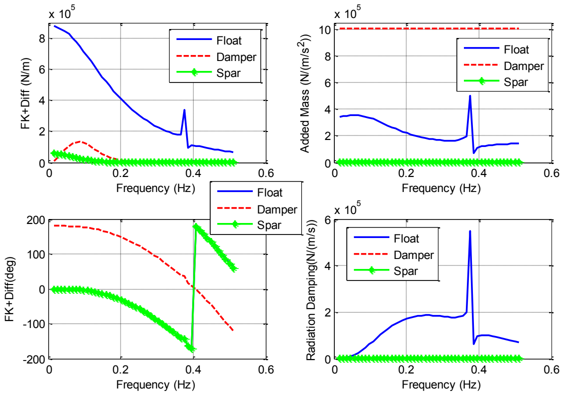

Figure 5.Example 3-D Diffraction and Radiation Analysis results.

device will operate. For this example a deep water application was used with a depth of 1000 m.

IV. HYDRODYNAMICS SOFTWARE PACKAGES

There are several industry standard hydrodynamic codes available for analysis of WECs. These include WAMIT[11], ANSYS Aqwa, and Orcaflex to name a few. There are also codes based on WAMIT such as WaveDyn and HydroD.

Both WAMIT and ANSYS Aqwa solve the linear water wave boundary value problem. They use the Boundary Element Method (BEM), also known as the panel method and the integral method, to find diffraction and radiation velocity potentials. In general, BEM apply source or dipole functions on the surfaces of submerged bodies, and solve for their strength so that all boundary conditions are met [12]. Once the diffraction and radiation velocity potential fields have been solved, excitation forces, added mass and damping matrices, as well as wave field pressure, velocity, and surface elevation can be found. Both WAMIT and Aqwa can also find hydrostatic forces and moments.

The primary difference between the two software packages is the data pre-processing, post processing, software interface, and supplementary calculations. WAMIT does not have a graphical user interface. Its inputs and outputs are text files that require pre-processing and post-processing by another program. Aqwa does have a limited graphical interface that aids in part of the setup and analysis of a problem but also uses text files for a significant portion of the modeling process. WAMIT users have high level of control over setup and access to a wide range of data outputs. Aqwa has more post-processing computational tools including frequency domain analysis, time domain analysis, and stability calculations.

In the end, at this stage, the excitation force, added mass matrix, damping matrix, and hydrostatic coefficients need to be computed. Both WAMIT and Aqwa compute these values with the same technique. It is left to the user to decide which software package he feels comfortable using, and whether he needs additional data or analysis that is provided by only one tool.

The current codes intended for WECs have been adapted from the ship industry where ship dynamics have been studied. In general these vessels are much larger, single bodied structures. Although improving, sometimes the application of these codes to smaller, multiple bodies interacting, can lead to issues in convergence and accuracy. This can be partially attributed to the nonlinear effects which have a greater influence on the performance of the system at a smaller scale.

Hydrostatic results are also included in this stage. Center of gravity, volumetric displacement, center of buoyancy, and metacentric heights are some of the parameters available. These will help with keeping the proper stability of the device.

A. 3-D Diffraction and Radiation Analysis

Once the solid model has been created and imported in the hydrodynamics package, the first step in analysis is a 3-D diffraction and radiation analysis. This will determine wave force and structure response calculations as well as hydrostatic analysis. Outputs include the Froude-Krylov forces, diffraction forces, added mass, and damping forces in the frequency domain as shown in Fig. 5.

Note, at this stage, there is no mooring or PTO defined and the structures are not connected. This is acceptable for the device specific characteristics, namely the hydrostatics, Froude-Krylov, added mass, and radiation parameters. However if we would like to get information regarding the response amplitude operators, power generation, or mooring characteristics we will have to run further analysis.

B. Equilibrium and Stability Analysis

Equilibrium and stability is the next step in the process. Static equilibrium and stability of a system under the influence of the following steady forces: gravity, buoyancy, wave drift force, steady wind force, current, thrusters, and mooring are considered.

This is the stage where the mooring and power take off

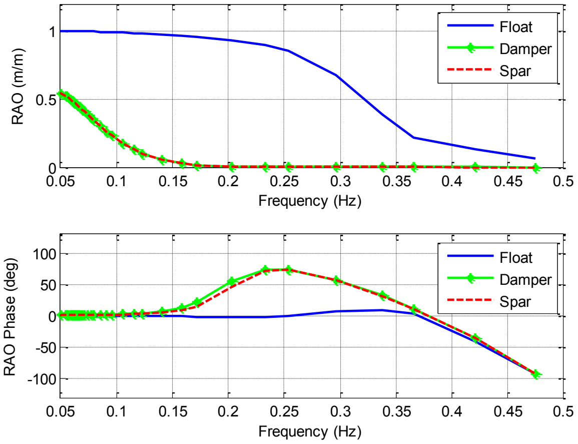

Figure 6. RAO amplitude and phase for each body.

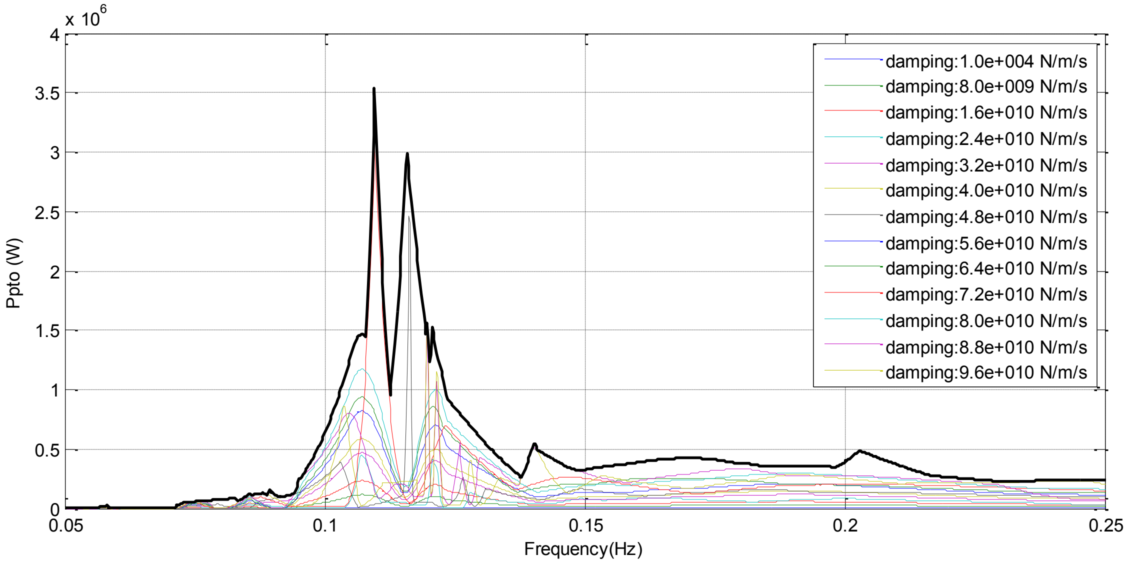

Figure 7. Power vs. Frequency plot for multiple damping values, including envelope (thick black line) showing max power output for each frequency which could be achieved using active damping.

model gets introduced into the system. Equilibrium and stability analysis is necessary to move on to both frequency and time domain modeling.

C. Frequency Domain Analysis

Frequency domain response allows insight into the RAOs and relative velocity measurements which lead to power output calculations based on power take off implementation and mooring application. The hydrodynamic package calculates the significant response of amplitudes in irregular waves. It utilizes a linearized stiffness matrix and damping to obtain the transfer function and response spectrum. The major benefit of this type of analysis is the ability to make a systematic parameter study while getting power predictions for the device. At this stage, a mooring model and power take off need to be defined to simulate a more complete system. The following sections describe the models used.

D. Mooring Model (Linearized)

The mooring configuration used in this example is a three point system. The lines are assumed to be a conventional linear elastic cable. The lines are assumed to have no mass and geometrically are represented as a straight line. The stiffness and the unstretched length are the two parameters that need to be specified.

Another option is a single point catenary mooring system. This can be modeled as a composite elastic catenary with weight. Mass per unit length and equivalent cross sectional area are properties that need to be defined for this type of mooring model.

There are many more possibilities for mooring a wave energy converter. The importance of mooring design and modeling should not be overlooked. Reference [13] shows many different mooring configurations with detailed analysis.

E. Power Take Off Model (Linearized)

One way to model a power take off (PTO) system is with an articulation allowing rotational motion. For this articulation a rotational damping term can be specified. Therefore, one must convert the rotational damping specified to an equivalent linear damping value to calculate the PTO force. Once that conversion has been made, calculating F_{pto} becomes

$$F_{pto}(\omega) = u_{rel}(\omega) B_l \tag{9}$$

where u_{rel} is the relative velocity between bodies and B_l is the linear damping value.

F. Response Amplitude Operators (RAOs)

The hydrodynamic package will provide a Response Amplitude Operator for each body. A sample output RAO magnitude and phase plot is shown in Fig. 6. As the plot shows the response drops off for waves with a period lower than around five seconds.

For example, if a monochromatic wave with an amplitude of 1 m and a frequency of 0.3 Hz is considered, the resulting heave motion of the float will have a magnitude of approximately 0.65 m and a phase shift of approximately 8 deg.

V. OUTPUT POWER COMPARISONS

Once the PTO forces are known the next natural step is to calculate the power output of the device as a function of frequency for different damping values. The power for each frequency was computed using the following equation

$$P_{PTO}(\omega) = u_{rel}(\omega)\ F_{PTO}(\omega) \tag{10}$$

where u_{rel} is the relative velocity between the float and the damper and F_{PTO} is the force applied to the power take off.

Fig. 7 shows a sample of this output run for 50 damping values. An envelope was then developed to show the maximum power output for each frequency. Theoretically, what this shows is the amount of power possible if able to apply any damping value in the range of damping values that is tested. This shows possible resonance peaks around eight or nine second waves.

VI. CONCLUSIONS

This paper presents an outline of the necessary steps to perform frequency domain analysis on concept wave energy devices. The result is a guide showing a path from idea to potential power output values in the frequency domain. Targeted toward wave energy converter device developers, it provides a clear methodology from concept to the first power predictions of a device.

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