551.521.3 : 551.584.31

The diffuse solar irradiance of slopes under cloudless skies

By M. D. STEVEN and M. H. UNSWORTH

Environmental Physics Unit, University of Nottingham, School of Agriculture, Sutton Bonington, Loughborough

SUMMARY

The diffuse irradiance of slopes relative to the diffuse irradiance of a horizontal surface is calculated by integrating the mean radiance distributions of cloudless skies in Britain for a wide range of solar elevations, azimuths and slope angles. There is reasonable agreement with measurements reported from a number of stations; discrepancies are probably due to errors in measurements and to the influence of aerosol on the radiance distributions. A simple model of the diffuse irradiance of slopes, which takes account of circumsolar radiation, is proposed, and agrees well with integrated values of irradiance.

1. INTRODUCTION

The diffuse irradiance of sloping surfaces may be either measured directly, or calculated from the angular distribution of radiance of the sky. Direct measurements, however, often include a component of radiation reflected onto the slope from neighbouring surfaces. This reflected radiation can be a large fraction of the diffuse irradiance, particularly on steep slopes and under cloudless skies when the ratio of diffuse to global radiation is small. On the other hand, separation of direct and diffuse solar radiation components by instruments with shade rings or discs excludes a fraction of circumsolar radiation which depends on shade-ring dimensions (Drummond 1956). Both these factors introduce large uncertainties in the measurement of the slope irradiance due to the sky alone, and may explain some of the discrepancies revealed by Robinson (1966) when he reviewed the literature.

The alternative approach of calculating the diffuse irradiance of a sloping plane by integrating the radiance distribution of the sky 'seen' by the plane, has been attempted less frequently but is attractive in allowing the calculation of irradiance of planes of any slope or orientation and of objects of complex shape, e.g. buildings, plants and animals. There are, however, few suitable sets of observations. Radiance measurements over a long period by Dines and Dines (1927) are limited to a narrow arc of the sky facing east; Tonne and Norman (1960) published luminance distributions for cloudless skies, but variations in the spectral composition of diffuse radiation preclude the use of these data in calculations of irradiance. Kondratyev and Manolova (1960) and Kondratyev and Fedorova (1977) calculated the solar irradiance of slopes on certain cloudless days from measurements of radiance distributions, and compared them with direct measurements, but it is not clear whether their observations are appropriate for other days, or for other regions where turbidity and surface reflection characteristics may differ. Steven (1977a) measured the radiance from cloudless skies at Sutton Bonington on a large number of days over a wide range of turbidities. When the measurements of radiance were normalized with respect to the diffuse irradiance of a horizontal surface, the normalized values at fixed points in the sky were, to a first approximation, independent of turbidity. On this basis, standard distributions of radiance from cloudless skies were proposed and tabulated for solar zenith angles from 35° to 65° (Steven 1977a).

The present paper presents values of the diffuse irradiance of slopes relative to horizontal diffuse irradiance, calculated from Steven's mean distributions. The values are compared with published measurements at other sites and a simple model of diffuse irra-

diance of slopes is developed, adequate for many purposes in agricultural meteorology and in building climatology.

2. THEORY

The diffuse irradiance $S_d(\alpha,\psi)$ of a slope, tilt α and azimuth ψ, is given by

$$S_d(\alpha,\psi) = \iint N(\theta,\phi)\,\mathbf{A}.d\Omega, \qquad . \qquad . \qquad . \qquad (1)$$

where the radiance, N, is a function of zenith angle θ and azimuth ϕ; $\mathbf{A}$ is the unit vector normal to the slope and $d\Omega$ is the solid angle of an element of the sky, the vector part denoting the direction of the element. Figure 1 shows the angular relationships for the case $\psi = 0$.

The vectors are given in Cartesian form by

$$\mathbf{A} = \begin{pmatrix} \sin\alpha\ \cos\psi \\ \sin\alpha\ \sin\psi \\ \cos\alpha \end{pmatrix} \text{ and } d\Omega = \begin{pmatrix} \sin\theta\ \cos\phi \\ \sin\theta\ \sin\phi \\ \cos\theta \end{pmatrix} (\sin\theta)\,d\theta\,d\phi$$

and $\mathbf{A}.d\Omega$ is their scalar product. The integration is performed over the region of sky that is exposed to the slope (Fig. 1). The limits are

$$0 \leqslant \theta \leqslant 90° \text{ for } \psi-90° < \phi \leqslant \psi+90° \text{ and}$$

$$0 \leqslant \theta \leqslant \theta_1 \text{ for } \psi+90° < \phi \leqslant \psi+270°.$$
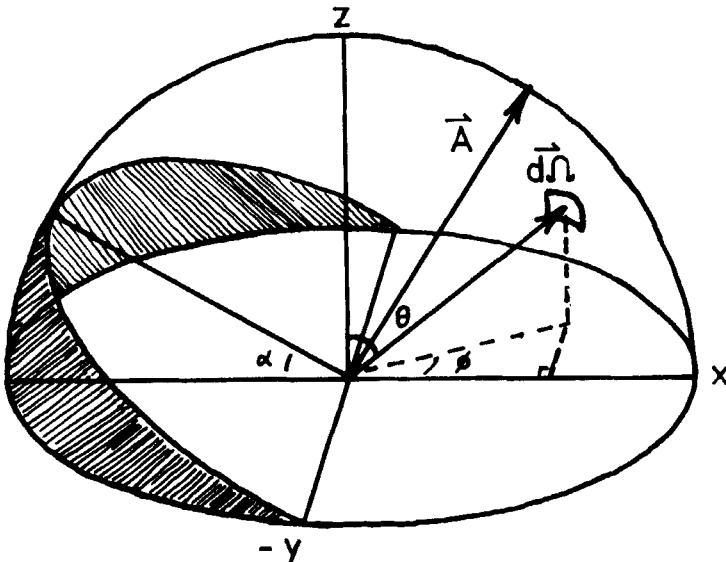

Figure 1. Angular relationships of slope to sky. The shaded area of sky is hidden from the slope.

The limit θ_1 is the zenith angle at the skyline and can be found by solving the equation $\mathbf{A}.d\Omega = 0$. The absorption coefficient of the surface as a function of angle may be introduced in Eq. (1) as a weighting function to calculate the radiation absorbed by the surface, but in this study absorption is not considered, and Eq. (1) defines the diffuse irradiance.

3. RESULTS

The standard distributions of clear sky radiance, $N(\theta,\phi)$, were fitted with two forms of analytic function, in which the coefficients were varied to find the best fit by the method of least squares (Steven 1977b). The first consisted of a series of orthogonal functions, f, of θ and ϕ, based on spherical harmonics, and an exponential function of ξ, the angle between the point (θ,ϕ) and the sun, to approximate the circumsolar radiation,

i.e.
$$N(\theta,\phi) = \pi^{-1} \left\{ \sum_{i=1}^{10} c_i f_i(\theta,\phi) + c_{11}\exp(-c_{12}\sin\xi) \right\}, \qquad (2)$$

where the functions f_i and values of the coefficients c_i at the best fit are given in appendix 1. Values of N calculated from Eq. (2) agreed well with measured values, the residual standard deviation of the relative differences being 0·04 and the maximum error of any individual fitted value being 8%, well within uncertainties in the measurements. Dogniaux (1975) proposed a simpler form of expression for relative radiance distributions as a function of θ, ξ and z (solar zenith angle). When applied to measurements for all ranges of z, Dogniaux's expression was too inaccurate for practical use, with a residual standard deviation 0·12 and maximum error in fitted values of 25% However, the Dogniaux expression was modified for application to each range of z separately, viz.

$$N(\theta,\xi) = \pi^{-1}\{d_1 + d_2\exp(d_3\,\xi) + d_4\cos^2\xi\}\{1 - \exp(d_5\sec\theta)\}. \qquad (3)$$

This equation fitted the measurements nearly as well as Eq. (2), with residual standard deviation 0·04 and maximum errors of 10%. Values of the coefficients d are tabulated in appendix 2.

The functions given in Eq. (2) were integrated (Eq. (1)) to give irradiance values $S_d(\alpha,\psi)/S_d$ relative to a horizontal surface, for a range of angles of tilt and azimuth. To account for minor inaccuracies introduced by the function-fitting and integration procedures,

TABLE 1. RELATIVE DIFFUSE IRRADIANCE OF PLANES WITH SLOPE α AND AZIMUTH ψ FOR SOLAR ZENITH ANGLE $z = 35°$

| $\psi(°)$ | $\alpha(°)$ | | | | | |
| | 15 | 30 | 45 | 60 | 75 | 90 |
	$S_d(\alpha,\psi)/S_d$					
0	1·12	1·19	1·19	1·12	0·98	0·79
30	1·10	1·15	1·14	1·06	0·92	0·73
60	1·05	1·05	1·00	0·90	0·75	0·58
90	0·98	0·92	0·83	0·70	0·56	0·42
120	0·92	0·80	0·66	0·53	0·41	0·32
150	0·87	0·71	0·56	0·43	0·34	0·28
180	0·85	0·68	0·52	0·40	0·32	0·27

TABLE 2. AS TABLE 1, BUT $z = 45°$

| $\psi(°)$ | $\alpha(°)$ | | | | | |
| | 15 | 30 | 45 | 60 | 75 | 90 |
	$S_d(\alpha,\psi)/S_d$					
0	1·16	1·25	1·28	1·23	1·12	0·94
30	1·13	1·21	1·22	1·16	1·04	0·86
60	1·07	1·09	1·05	0·96	0·83	0·66
90	0·98	0·93	0·84	0·72	0·58	0·44
120	0·90	0·78	0·65	0·52	0·41	0·32
150	0·84	0·68	0·54	0·42	0·34	0·27
180	0·82	0·64	0·50	0·40	0·32	0·26

TABLE 3. As TABLE 1, BUT $z = 55°$

| $\psi(°)$ | $a(°)$ | | | | | |
| | 15 | 30 | 45 | 60 | 75 | 90 |
	$S_d(a,\psi)/S_d$					
0	1·20	1·33	1·40	1·39	1·29	1·12
30	1·17	1·28	1·32	1·29	1·19	1·02
60	1·09	1·13	1·11	1·04	0·92	0·76
90	0·98	0·93	0·85	0·74	0·61	0·48
120	0·88	0·76	0·63	0·52	0·42	0·34
150	0·81	0·65	0·52	0·43	0·36	0·29
180	0·79	0·61	0·49	0·41	0·35	0·28

TABLE 4. As TABLE 1, BUT $z = 65°$

| $\psi(°)$ | $a(°)$ | | | | | |
| | 15 | 30 | 45 | 60 | 75 | 90 |
	$S_d(a,\psi)/S_d$					
0	1·26	1·46	1·58	1·60	1·54	1·38
30	1·22	1·39	1·48	1·48	1·40	1·24
60	1·12	1·19	1·21	1·17	1·06	0·90
90	0·99	0·95	0·88	0·78	0·66	0·53
120	0·86	0·74	0·63	0·53	0·45	0·37
150	0·78	0·62	0·52	0·45	0·39	0·32
180	0·75	0·58	0·49	0·43	0·38	0·31

the values were slightly corrected by renormalization with respect to the integrated value of S_d on a horizontal surface; the renormalized values are shown in Tables 1 to 4.

4. COMPARISON WITH OTHER STUDIES

Several authors have published values for the irradiance of vertical or sloping surfaces below cloudless skies. Dogniaux (1975) derived the diffuse irradiance of vertical surfaces, $S_d(90)$, by integrations of radiance distribution functions. Parmalee (1954) and Valko (1975) measured the diffuse irradiance of vertical surfaces directly, but included a component of reflected radiation from the ground. All three studies provided auxiliary data on
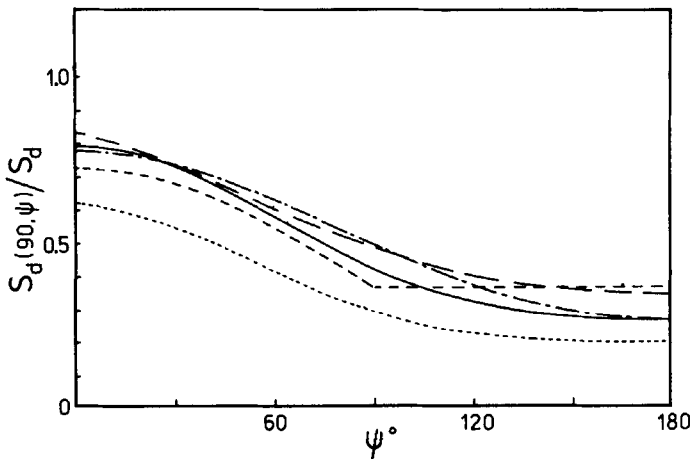

Figure 2. Comparisons of the relative diffuse irradiance of vertical surfaces under cloudless skies for solar zenith angle 35° according to various authors: ——— this work; —— – —— Valko; —— —— Dogniaux; ····· Parmalee; —————— this work, radiation model (Eq. (4)) with $s = 0·51$, $b = 0·87$.
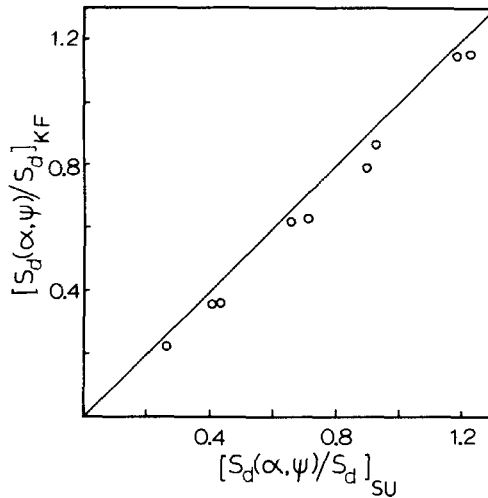
Figure 3. Comparison of the relative diffuse irradiance of a slope under cloudless skies when solar zenith angle was 42°, as measured by Kondratyev and Fedorova, and interpolated from Tables 1 and 2. The 1:1 line is shown.

the direct and diffuse components of horizontal irradiance as functions of atmospheric turbidity, but the definition of turbidity was different in each case. For comparison, values of vertical irradiance were selected where the corresponding irradiance of the direct beam at normal incidence was equal to the mean from the measurements of Steven (1977a), and consequently turbidities were approximately equal. With the results of Parmalee and Valko, the reflected component was subtracted from the diffuse irradiance, using data provided by the authors and assuming isotropic reflection. The values of $S_d(90)$ were normalized with respect to S_d and the resulting values of relative irradiance, $S_d(90)/S_d$, were compared with the present results. Figure 2 shows relative irradiance values for solar zenith angle 35°. The present results gave slightly lower values than those of Valko and Dogniaux for ψ between 40° and 140°, but there was close agreement with both authors near 0°, and with Valko near 180°. At $z = 45°$, 55° and 65° (not shown), there was similar general agreement with the shape of two curves (Dogniaux and Valko), but discrepancies between sets of results were sometimes up to 30%. The values calculated from Parmalee's data were consistently lower than the others.

Kondratyev and Fedorova (1977) tabulated measurements of the diffuse irradiance of slopes of different tilt and azimuth. Figure 3 compares their data for one occasion when z was 42° and turbidity was at the lower end of Steven's (1977a) range with linear interpolations from the integrated values of Tables 1 and 2. The figure shows that Kondratyev and Fedorova's values are on average 11% less than the integrated values.

Steven (1977a) found that radiance distributions varied very little with turbidity and consequently relative irradiances should be similarly invariant. Valko presented measured vertical irradiances for several turbidities and these were also compared with the present results using the method described earlier for subtracting reflected radiation. Figure 4 shows that there were considerable differences in Valko's data between values of $S_d(90)/S_d$ at different turbidities. Moreover, the variation with turbidity was irregular, some values of $S_d(90)/S_d$ first increasing and then decreasing with higher turbidities. We believe that this behaviour was due to uncertainties in the contribution of reflected radiation to the irradiance values. The subtraction of reflected radiation and renormalization with respect to S_d magnifies any errors in the values, and if the reflected radiation was not isotropic (Kondra-
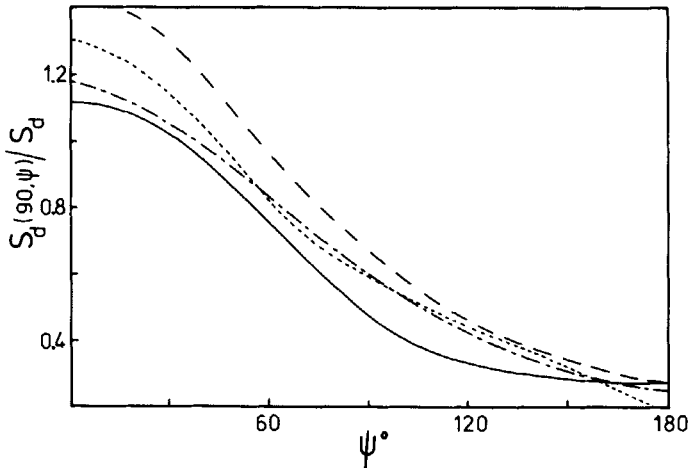
Figure 4. Comparisons of the relative diffuse irradiance of vertical surfaces under cloudless skies for solar zenith angle 55° according to this work (————), and Valko, at values of turbidity: $\tau = 0\cdot1$ ($\cdot\cdot\cdot$); $\tau = 0\cdot25$ (- - - - -); and $\tau = 0\cdot45$ (—— — ——).

tyev 1969) these errors may be very large. The values calculated from radiance distributions are about 10% lower than the mean of Valko's results at the highest and lowest turbidities.

As well as errors introduced in subtracting reflected radiation, uncertainties in the radiance distribution function may also be a cause of the discrepancies between the present results and those of other authors. The departures of the fitted functions from the radiance data were not found to be systematic. However, no measurements were taken at angles θ greater than 75° and the fitted functions were extrapolated there. While this region does not greatly affect horizontal irradiance, it has its maximum effect on vertical surfaces. This may be the reason for the disparity in irradiance values for ψ between 40° and 140°. Consequently, the comparisons in Figs. 2 and 4 represent an extreme case. The comparisons in Fig. 3 over a range of slopes and aspects, and based on a single set of observations in Russia, suggest that the integration of mean radiance distributions to predict radiation on slopes does not introduce large errors.

5. A MODEL OF DIFFUSE RADIATION UNDER CLOUDLESS SKIES

The integration of fitted functions that approximate the radiance distribution is too cumbersome for many practical purposes. Loudon (1965) and Robinson (1966) noted that much of the clear sky diffuse radiation comes from a narrow region close to the sun and they attempted to treat the diffuse radiation as the sum of an isotropic background component, S'_d, and a circumsolar component, S_{dc}, which is treated geometrically as part of the direct solar beam. The diffuse irradiance of a tilted plane exposed to the sun is then

$$S_d(\alpha,\psi) = S_{dc}(\cos\eta/\cos z) + S'_d\cos^2(\alpha/2), \qquad . \qquad . \qquad (4)$$

where η is the angle between the sun and the normal to the slope. When η exceeds 90° the circumsolar term in Eq. (4) is assumed zero.

Robinson suggested that S_{dc} could be expressed as a fraction, s, of the diffuse irradiance, S_d, of a horizontal surface, with $s = 0\cdot25$. Loudon claimed that the background diffuse radiation, S'_d, was virtually independent of turbidity and could be treated simply as a function of z. Since the circumsolar component is added to the direct beam, measurements of diffuse radiation would be, in theory, unnecessary if Loudon's hypothesis held. His data,

however, show considerable scatter in the determination of S'_d, and as they are based on vertical irradiance measurements alone, their applicability is limited.

Robinson's hypothesis was tested by substituting the integrated irradiance values (Tables 1 to 4) in Eq. (4). A function-fitting routine was used to find the best value of s by the method of least squares. The results were unsatisfactory. At the best value of s, the fitted values were almost all lower than the integrated values, by as much as 30% is some cases. Equation (4) is inadequate because the isotropic assumption in the background radiation is inconsistent with the common observation that cloudless skies are brighter towards the horizon, e.g. Dines and Dines (1927). To account for this variation, the isotropic assumption was abandoned and the background diffuse radiation was represented by a fictitious radiance distribution N' linear in $\cos \theta$:

$$N'(\theta) = N'(0)(1 + b\cos\theta)/(1 + b). \qquad . \qquad . \qquad . \qquad (5)$$

The 'standard overcast sky' (SOC), used for illumination calculations, has this form of distribution, with $b = 2$. Moon and Spencer (1942) integrated the SOC distribution analytically to derive expressions for the diffuse irradiance of any tilted plane. Extending their analysis to the general case where b can take any value, the relative background diffuse irradiance is

$$S'_d(\alpha)/S_d = \{\cos^2(\alpha/2)\} + 2b\{\pi(3+2b)\}^{-1}\{\sin\alpha - \alpha\cos\alpha - \pi\sin^2(\alpha/2)\}. \quad . \quad (6)$$

When $b = 0$, Eq. (6) reduces to the isotropic formula; and when $b = 2$ it is identical to the formula of Moon and Spencer.

When both s and b were allowed to vary in the function-fitting routine, the results improved considerably. The model was fitted to the irradiance values (Tables 1 to 4) in two ways: in one, data from all solar zenith angles were taken together; in the other they were separated into four groups according to z. The results are summarized in Table 5, where the 90% confidence limits on the values of s and b are ± 0.02 and ± 0.07, respectively.

TABLE 5. PARAMETERS s AND b FOR THE DIFFUSE RADIATION MODEL $S'_d (\alpha)/S_d = \cos^2(\alpha/2) + 2b/\{\pi(3+2b)\}$ $\{\sin\alpha - \alpha\cos\alpha - \pi\sin^2(\alpha/2)\}$ AT A RANGE OF SOLAR ZENITH ANGLES. SYMBOLS ARE EXPLAINED IN THE TEXT

z	s	b	Residual standard deviation	Maximum error
All z	0·51	−0·87	0·06	0·16
35°	0·63	−1·04	0·04	0·08
45°	0·60	−1·00	0·04	0·10
55°	0·53	−0·90	0·05	0·11
65°	0·46	−0·85	0·06	0·13

Values of b in Table 5 are sometimes less than -1, apparently implying 'negative radiance' in some parts of the sky (Eq. (5)). It must be stressed that the values of b and s were derived by fitting irradiance values (Tables 1–4) derived from the radiance distribution. It is invalid to use the model to represent the radiance distribution itself.

The differences between the model and the integrated irradiances are small but systematic, the largest differences occurring on planes facing away from the sun. This disagreement is inevitable because the model allows no azimuthal dependence of irradiance for planes which are not exposed to direct radiation, whereas in reality the azimuthal dependence is quite marked (e.g. Fig. 2). When the data were treated in four separate groups according to z, rather than together, the improvement in fitting accuracy was largely confined to planes facing the sun. For many practical purposes the model with

fixed values of s and b for all z is probably adequate; Fig. 2 also shows the irradiance values for vertical surfaces according to this model.

The diffuse radiation model is based on radiance distributions which cover almost the whole sky, and thus include circumsolar radiation. However, measurements of S_d by shade ring or shading disc methods usually exclude some circumsolar radiation and allot it to the direct beam, (Drummond 1956). In using the model with values of S_d measured by shading methods, radiation must not be counted twice, and the value of s must be reduced accordingly. If F is the fraction of S_d included in the direct beam, then the measured diffuse radiation is $(1-F)S_d$ and the corrected value of s is $(s-F)/(1-F)$. Calculations based on the standard radiance distributions indicated that when the measured direct beam includes circumsolar radiation from a zone of diameter $10°$, then F is approximately 0·03. There is at present no standardization in the methods used to measure or estimate S_d, and this may cause problems of comparison. Computations, however, show that estimates of the irradiance of slopes are not very sensitive to small errors in s or b.

6. DISCUSSION

It is surprising that, although turbidity strongly influences the diffuse irradiance of horizontal surfaces (Unsworth and Monteith 1972), it does not appear to have a large effect on the relative irradiance of slopes. The results of Unsworth and Monteith show that Mie scattering from aerosol produces most of the diffuse radiation at turbidities typical of central England, although molecular (Rayleigh) scattering is always important at short wavelengths (Bullrich 1964). In principle, both aerosol concentration and size distribution influence the radiance distribution. At large aerosol concentrations multiple scattering occurs, but Piaskowska-Fesenkova (Kondratyev 1969) found that multiple scattering had an insignificant effect on measured scattering functions. McCartney and Unsworth (1978) showed that the aerosol size distribution at Sutton Bonington in summer was relatively constant, and this may account for the approximate invariance of the normalized radiance values which were obtained at relatively large turbidities (mean $\tau_a = 0·3$, (Steven 1977a)). When radiance measurements were averaged over zones of the sky, Steven (1977a) found that extreme turbidities, $\tau_a = 0·1$ and 0·5, caused only 5 to 10% departure from the mean relative radiance of the zone. This indicates an upper limit on the variation due to turbidity from the mean integrated irradiance values, Tables 1 to 4, and suggests that the small disagreement between the present results and those of Kondratyev and Fedorova (Fig. 3) may be partly attributed to turbidity differences.

The data presented here show that the diffuse irradiance of slopes estimated from Tables 1–4 or from Eq. (4) is in reasonable agreement with observations from several sites when uncertainties in direct measurements, in observed radiance distributions, and in interpolation procedures are considered. Although there may be an overall uncertainty of 10–20% in estimated diffuse irradiance, the resulting uncertainty in global irradiance of sunlit slopes is much smaller; the procedures described here are adequate for many studies of irradiance in engineering and biology.

REFERENCES

Bullrich, K.	1964	Scattered radiation in the atmosphere and the natural aerosol, *Adv. Geophys.*, **10**, 99–260.
Dines, W. H. and Dines, L. H. G.	1927	Monthly mean values of radiation from various parts of the sky at Benson, Oxfordshire, *Mem. R. Met. Soc.*, **11**, 1–8.
Dogniaux, R.	1975	Variations geographiques et climatiques des expositions energetiques solaires sur des surfaces receptrices horizontales et verticales, *Inst. R. Met. Belg., Misc. Ser. B*, No. **38**.
Drummond, A. J.	1956	On the measurement of sky radiation, *Arch. Met. Geophys. Bioklm.*, **B7**, 413–436.
Kondratyev, K. Y.	1969	*Radiation in the atmosphere*, Academic Press, New York.
Kondratyev, K. Y. and Fedorova, M. P.	1977	Radiation regime of inclined surfaces, *WMO Tech. note 152*, WMO–No. 467, Geneva.
Kondratyev, K. Y. and Manolova, M. P.	1960	The radiation balance of slopes, *Solar Energy*, **1**, 14–19.
Loudon, A. G.	1965	The interpretation of solar energy measurements for building problems, *CIE, Proc. Conf. Sunlight in Buildings*, 111–118, Newcastle.
McCartney, H. A. and Unsworth, M. H.	1978	Spectral distribution of solar radiation. I: direct radiation, *Quart. J. R. Met. Soc.*, **104**, 699–718.
Moon, P. and Spencer, D. E.	1942	Illumination from a non-uniform sky, *Trans. illum. Engng. Soc., N.Y.*, **37**, 707–726.
Parmalee, G. V.	1954	Illumination of vertical and horizontal surfaces by diffuse solar radiation from cloudless skies, *Trans. Am. Soc. Heat. Vent. Engrs.*, **60**, 341–358.
Robinson, N.	1966	*Solar radiation*, Elsevier, Amsterdam.
Steven, M. D.	1977a	Standard distributions of clear sky radiance, *Quart. J. R. Met. Soc.*, **103**, 457–465.
	1977b	Angular distribution and interception of diffuse solar radiation, Ph.D. thesis, Nottingham University.
Tonne, F. and Norman, W.	1960	Die Berechnung der Sonnenwarmestrahlung auf senkrechte und beliebig geneigte Flachen unter Berucksichtigung meteorologischer Messungen, *Z. Met.*, **14**, 166–179.
Unsworth, M. H. and Monteith, J. L.	1972	Aerosol and solar radiation in Britain, *Quart. J. R. Met. Soc.*, **98**, 778–797.
Valko, P.	1975	Solar radiation impact on buildings of different shape and orientation, *Meteoplan* 1, Hallwag Verlag, Bern and Stuttgart.

APPENDIX 1

Values of the functions f and coefficients c in the fitted function

$$N(\theta,\phi) = \pi^{-1}\left\{\sum_{i=1}^{10} c_i f_i(\theta,\phi) + c_{11}\exp(-c_{12}\sin\xi)\right\}.$$

Symbols N, θ, ϕ and ξ are defined in section 2.

$$f_1 = 1\cdot 0$$
$$f_2 = \sin\theta\cos\theta$$
$$f_3 = \cos\theta$$
$$f_4 = \sin^2\theta\cos2\theta$$
$$f_5 = \sin\theta\cos\theta\cos\phi$$

$$f_6 = (3\cos^2\theta-1)/2$$
$$f_7 = \sin^3\theta\cos3\phi$$
$$f_8 = \sin^2\theta\cos\theta\cos2\phi$$
$$f_9 = \sin\theta\cos\theta(5\cos^2\theta-1)$$
$$f_{10} = (5\cos^3\theta-3\cos\theta)/2$$

Values of N are normalized with respect to S_d and the units are st^{-1}.

	$z(°)$			
	35	45	55	65
c_1	$1\cdot1 \;\pm0\cdot5$	$1\cdot0 \;\pm0\cdot5$	$0\cdot4 \quad\pm0\cdot05$	$1\cdot6 \;\pm0\cdot5$
c_2	$0\cdot12\pm0\cdot09$	$0\cdot7 \;\pm0\cdot1$	$0\cdot9 \quad\pm0\cdot2$	$1\cdot5 \;\pm0\cdot2$
c_3	$-2\cdot6 \;\pm0\cdot7$	$-0\cdot7 \;\pm0\cdot9$	$0\cdot7 \quad\pm0\cdot9$	$-1\cdot6 \;\pm0\cdot9$
c_4	$0\cdot04\pm0\cdot04$	$0\cdot26\pm0\cdot04$	$0\cdot38 \;\pm0\cdot05$	$0\cdot42\pm0\cdot06$
c_5	$-0\cdot04\pm0\cdot3$	$-0\cdot5 \;\pm0\cdot4$	$-0\cdot7 \quad\pm0\cdot5$	$-2\cdot0 \;\pm0\cdot4$
c_6	$1\cdot4 \;\pm0\cdot5$	$-0\cdot04\pm0\cdot7$	$-1\cdot3 \quad\pm0\cdot7$	$0\cdot2 \;\pm0\cdot6$
c_7	$0\cdot01 \pm0\cdot01$	$0\cdot04\pm0\cdot02$	$0\cdot09 \;\pm0\cdot02$	$0\cdot12\pm0\cdot02$
c_8	$-0\cdot1 \;\pm0\cdot1$	$-0\cdot3 \;\pm0\cdot1$	$-0\cdot3 \quad\pm0\cdot1$	$-0\cdot32\pm0\cdot08$
c_9	$-0\cdot16\pm0\cdot05$	$0\cdot01 \pm0\cdot06$	$0\cdot006\pm0\cdot07$	$0\cdot19\pm0\cdot07$
c_{10}	$-0\cdot6 \;\pm0\cdot2$	$0\cdot06\pm0\cdot2$	$0\cdot6 \quad\pm0\cdot2$	$0\cdot1 \;\pm0\cdot2$
c_{11}	$5\cdot6 \;\pm0\cdot6$	$5\cdot8 \;\pm0\cdot3$	$7\cdot0 \quad\pm0\cdot5$	$9\cdot8 \;\pm0\cdot8$
c_{12}	$2\cdot3 \;\pm0\cdot5$	$3\cdot9 \;\pm0\cdot6$	$4\cdot4 \quad\pm0\cdot6$	$5\cdot2 \;\pm0\cdot5$

The uncertainties given with the coefficients are standard errors. Some of the coefficient values are not significantly different from zero, but they are included because of correlations with other coefficients; if they were omitted, the optimum values of the other coefficients would change.

APPENDIX 2

Values of the coefficients d_1 to d_5 in the fitted function

$$N(\theta,\xi) = \pi^{-1}\{d_1 + d_2\exp(d_3\xi) + d_4\cos^2\xi\}\{1 - \exp(d_5\sec\theta)\}.$$

	$z(°)$			
	35	45	55	65
d_1	$0\cdot61\pm0\cdot02$	$0\cdot65\pm0\cdot02$	$0\cdot73\pm0\cdot02$	$0\cdot76\pm0\cdot03$
d_2	$11\cdot90\pm0\cdot60$	$10\cdot70\pm0\cdot50$	$11\cdot10\pm0\cdot40$	$13\cdot00\pm0\cdot50$
d_3	$-2\cdot97\pm0\cdot09$	$-2\cdot82\pm0\cdot06$	$-2\cdot97\pm0\cdot06$	$-3\cdot09\pm0\cdot07$
d_4	$-0\cdot12\pm0\cdot20$	$-0\cdot20\pm0\cdot10$	$-0\cdot07\pm0\cdot06$	$-0\cdot17\pm0\cdot05$
d_5	$-0\cdot45\pm0\cdot03$	$-0\cdot48\pm0\cdot03$	$-0\cdot48\pm0\cdot02$	$-0\cdot42\pm0\cdot02$

Comments in appendix 1 apply.