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SAMPLING BIAS IN VGP LONGITUDES

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Abstract. I derive probability densities for virtual geomagnetic pole (VGP) longitudes for a general statistical model of local magnetic field variations. I show that even for very simple statistically homogeneous models of secular variation, the distribution of VGP longitudes is peaked 90° away from the sampling longitude. Thus, when sites are distributed unevenly, a non-uniform overall distribution of VGP longitudes is to be expected. Analysis of the recent geomagnetic field indicates that this bias can be significant, particularly when random errors in paleomagnetic data are allowed for. It is possible that some of the deviations from uniformity seen in recent compilations of paleomagnetic reversal and secular variation data are a statistical artifact resulting from the distortion of the VGP transformation, and the non-uniformity of paleomagnetic sampling sites.

Introduction

Recently several groups of researchers have noted systematic patterns in virtual geomagnetic pole (VGP) longitudes during reversals, which they have interpreted in terms of core-mantle dynamics [*Tric et al.*, 1991; *Clement*, 1991]. However, the significance of these observations is in some dispute [*Valet et al.*, 1992; *Langeris et al.*, 1992]. Using the highest quality data from a limited number of reversals, *Valet et al.* [1992] tested the statistical significance of these claims, and concluded that the evidence for VGP path systematics is not yet compelling. Subsequently, *Laj et al.* [1992] have pointed out that when more sophisticated methods of analysis are applied to a slightly larger set of reversal data, the VGP systematics become statistically significant.

VGPs have also been used in studies of paleo-secular variation (PSV) data. *Constable* [1992] found that the histogram of VGP longitudes computed for a set of 2,244 PSV data from the last 5 My has two approximately antipodal peaks, near the longitudes which seem to be preferred for the reversal VGP paths. Here statistical tests demonstrated unequivocally that the VGP longitudes deviated significantly from a uniform distribution, and *Constable* concluded that the PSV data provide evidence for persistent non-zonal components of the geomagnetic field. In all of these statistical tests, a uniform distribution of VGP longitudes was taken as the null hypothesis.

For both the reversal and the PSV data, sampling sites are very unevenly distributed. For instance, for the PSV data set used by *Constable* there are two large holes (75° and 45° wide) in longitudinal coverage. The potential biasing of VGP statistics by such uneven sampling is often mentioned as a potential uncertainty in these studies [e.g., *Valet et al.*, 1992], and some efforts have been made to allow for this complication [*Constable*, 1992]. Here I explicitly address the question of whether a uniform distribution of VGP longitudes should be expected in PSV or reversal data when the distribution of field sampling sites is non-uniform.

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Directional Densities and the VGP Transformation

The VGP transformation is a one-to-one mapping $\hat{\mathbf{b}}' \rightarrow \hat{\mathbf{v}}$ from the local magnetic field direction ($\hat{\mathbf{b}}'$) to the direction ($\hat{\mathbf{v}}$) of a geocentric dipole which would cause this field. It has long been appreciated that the scatter of local field directions measured in a series of lava flows is distorted (both in shape and magnitude) by this transformation [*Cox*, 1970]. Here I derive expressions for VGP directional densities for a general statistical model of variations in local field directions.

To set notation I begin with a formal development of the VGP transformation. Since all statistical models considered here are invariant under rotations about the Earth's axis, it suffices to consider observations at longitude $\phi = 0$, for a range of colatitudes θ_0. Two right handed Cartesian coordinate systems are considered. In the "standard coordinate system" $\hat{\mathbf{z}}$ is aligned with the rotation axis, and $\hat{\mathbf{x}}$ points in the direction ϕ=0. The local coordinate system at the sampling site ($\mathbf{r}_0 = r(\sin\theta_0, 0, \cos\theta_0)$ in the standard system) is obtained by rotating the standard coordinate frame an angle θ_0 along the meridian ϕ=0 so that the $\hat{\mathbf{z}}' = \hat{\mathbf{r}}_0$. I will use primes to indicate that the components of a vector should be interpreted in the local coordinate system, and "hats" to denote unit vectors (thus $\hat{\mathbf{v}} = \mathbf{v}/\|\mathbf{v}\|$). The components in the standard and local coordinate systems of a fixed vector are related via $\mathbf{x}' = \mathbf{U}\mathbf{x}$ and $\mathbf{x} = \mathbf{U}^T\mathbf{x}'$, where

$$\mathbf{U} = \begin{bmatrix} \cos\theta_0 & 0 & \sin\theta_0 \\ 0 & 1 & 0 \\ -\sin\theta_0 & 0 & \cos\theta_0 \end{bmatrix}.$$

It is readily verified that the magnetic fields observed at $\mathbf{r}_0$ for a dipole at the origin with moment $\mathbf{d}'$ are $\mathbf{b}' = \mathbf{V}\mathbf{d}'$, where (with all vectors expressed in the local coordinate system) $\mathbf{V} = c\ \mathbf{diag}[-1, -1, 2]$. Here c is a constant which, because we are ultimately interested only in directions, can be set to 1. Thus, when expressed in the standard coordinate system, the VGP is

$$\hat{\mathbf{v}}(\mathbf{b}') = \mathbf{U}^T\mathbf{V}^{-1}\mathbf{b}'/\|\mathbf{U}^T\mathbf{V}^{-1}\mathbf{b}'\| \qquad (1)$$

With this formulation VGPs are expressed in Cartesian coordinates as unit vectors which are simply related to the local magnetic field vectors. In fact, with paleomagnetic data only the directional vector $\hat{\mathbf{b}}' = \mathbf{b}'/\|\mathbf{b}'\|$ is generally available for computation of the VGP. However

$$\hat{\mathbf{v}}(\hat{\mathbf{b}}') = \hat{\mathbf{v}}(\mathbf{b}') \tag{2}$$

so this complication is only apparent. The statistical properties of VGPs are insensitive to when or how magnitude information is lost. In the following it will often be convenient to apply the VGP transformation (1) directly to the unnormalized local fields; this is justified by (2).

In general the magnetic field observed at $\hat{\mathbf{r}}_0$ can be written

$$\mathbf{b}' = \mathbf{b}_{AD}' + \mathbf{b}_{SV}' = d\mathbf{VU}\hat{\mathbf{z}} + \mathbf{b}_{SV}' ,$$

where d is a scalar giving magnitude and polarity of the dominant axial dipole $\mathbf{b}_{AD}'$. The remainder of the field, $\mathbf{b}_{SV}'$ (the "secular variation"), includes both dipole wobble and the non-dipole fields. Our goal is to derive the statistical distribution of VGP directions for some simple, but in fact fairly general statistical models for d and $\mathbf{b}_{SV}'$.

First, I consider the simpler case of the *marginal* distribution of VGP longitudes. Let $\mathbf{P_H}$ be the matrix which projects onto the horizontal components of the VGP. Then, with the VGP as defined in (1), the VGP longitude ϕ can be represented in Cartesian coordinates by the unit two-vector

$$\hat{\mathbf{v}}_L = \begin{bmatrix}\cos\phi\\ \sin\phi\end{bmatrix} = \frac{\mathbf{P_H}\hat{\mathbf{v}}(\mathbf{b}')}{\|\mathbf{P_H}\hat{\mathbf{v}}(\mathbf{b}')\|} = \frac{\mathbf{P_H}\mathbf{v}(\mathbf{b}')}{\|\mathbf{P_H}\mathbf{v}(\mathbf{b}')\|} = \frac{\mathbf{v}_L}{\|\mathbf{v}_L\|} .$$

In the following I refer to $\hat{\mathbf{v}}_L$ as the VGP longitude, and $\mathbf{v}_L$ as the "unnormalized VGP longitude". Since

$$\mathbf{v}_L = \mathbf{P_H}\mathbf{v}(\mathbf{b}') = \mathbf{P_H}\mathbf{U}^T\mathbf{V}^{-1}\mathbf{b}_{SV}' , \tag{3}$$

the *marginal* VGP longitude distribution can depend only on the statistical properties of $\mathbf{b}_{SV}'$, not on the model assumed for the axial dipole part of the field. Under mild assumptions concerning the random vector $\mathbf{b}_{SV}'$, an exact expression for the probability density of $\hat{\mathbf{v}}_L$ can be given.

First, assume that $\mathbf{b}_{SV}'$ has zero mean with covariance

$$E[\mathbf{b}_{SV}'\mathbf{b}_{SV}'^T] = \Sigma_{SV} = \Sigma_{SV}^{1/2}\Sigma_{SV}^{1/2\,T}$$

(where $\Sigma_{SV}^{1/2}$ is the lower triangular part of the Cholesky decomposition of Σ_{SV}). Then the vector $\mathbf{u}' = \Sigma_{SV}^{-1/2}\mathbf{b}_{SV}'$ necessarily has an isotropic covariance, with $E[\mathbf{u}'\mathbf{u}'^T] = \mathbf{I}$. Assume further that the unit vectors $\hat{\mathbf{u}}' = \mathbf{u}'/\|\mathbf{u}'\|$ have a uniform distribution. This will hold when $\mathbf{b}_{SV}'$ has an ellipsoidal density centered at the origin (i.e., when the joint density of $\mathbf{u}'$ is radially symmetric). A special case which satisfies this condition is the Gaussian model for $\mathbf{b}_{SV}'$ of *Constable and Parker* [1988].

The unnormalized VGP longitude vector $\mathbf{v}_L$ given in (3) has 2×2 covariance matrix $\Sigma_L = \mathbf{P_H}\mathbf{U}^T\mathbf{V}^{-1}\Sigma_{SV}\mathbf{V}^{-1}\mathbf{U}\mathbf{P}_\mathbf{H}^T$. Furthermore, $\Sigma_L^{-1/2}\mathbf{v}_L/\|\Sigma_L^{-1/2}\mathbf{v}_L\|$ is uniformly distributed on the unit circle, so $\mathbf{w} = \mathbf{v}_L/\|\Sigma_L^{-1/2}\mathbf{v}_L\|$ is a linear transformation of a uniformly distributed random direction. But $\hat{\mathbf{w}} = \mathbf{w}/\|\mathbf{w}\| = \hat{\mathbf{v}}_L$, so the transformation approach outlined by *Watson* [1983, pp 109-110] can be used to compute the density of the random longitudinal direction vector $\hat{\mathbf{v}}_L = \hat{\mathbf{w}}$. From Eq. (3.6.7) of *Watson*, this is

$$p_L(\hat{\mathbf{v}}_L) = [2\pi|\Sigma_L|^{1/2}\,\mathbf{v}_L^T\Sigma_L^{-1}\mathbf{v}_L]^{-1} \tag{4}$$

where $|\Sigma_L|$ denotes the determinant of Σ_L.

The density has a particularly simple form when $\Sigma_{SV} = \mathbf{diag}[\sigma_{xx}, \sigma_{yy}, \sigma_{zz}]$. In this case, $\Sigma_L = \sigma_{yy}\,\mathbf{diag}[D^2, 1]$ where $D^2 = \cos^2\theta_0\sigma_{xx}/\sigma_{yy} + \sin^2\theta_0\sigma_{zz}/4\sigma_{yy}$. By writing $\mathbf{v}_L = (\cos\phi, \sin\phi)$, and simplifying (4) we find

$$p_L(\phi) = [2\pi D(D^{-2}\cos^2\phi + \sin^2\phi)]^{-1} . \tag{5}$$

For an isotropic local field covariance ($\sigma_{xx} = \sigma_{yy} = \sigma_{zz}$), $p_L(\phi)$ is sharply peaked 90° away from the sampling site, with the non-uniformity greatest at low-latitudes (Figure 1). The plotted density is perhaps unrealistic, since the scatter in local field directions is not generally isotropic [*Cox*, 1970; *Harrison*, 1980; *Merrill and McElhinny*, 1983]. Nonetheless, this example illustrates the central thrust of this note: isotropic variations in local field directions will result in VGP longitudes which tend to be offset 90° from the sampling longitude. In the next section I will consider some more plausible statistical models of secular variation scatter.

First I sketch a derivation of the full VGP directional density. Even when the ultimate interest is in VGP longitudes, the full VGP distribution may be required. For example, this would be necessary to compute the distribution of the *weighted* average VGP longitude statistic MVL $= \tan^{-1}(\sum v_{yi} / \sum v_{xi})$ used by *Valet et al.* [1992] to average VGP longitudes during a reversal transition.

We assume $\mathbf{b}' = d\mathbf{VU}\hat{\mathbf{z}} + \mathbf{b}_{SV}'$, where d is a fixed constant and $\mathbf{b}_{SV}'$ is Gaussian with covariance matrix Σ_{SV}. Then the unnormalized VGP $\mathbf{v}(\mathbf{b}') = d\hat{\mathbf{z}} + \mathbf{U}^T\mathbf{V}^{-1}\mathbf{b}_{SV}'$ has mean $d\hat{\mathbf{z}}$ and covariance matrix $\Sigma_V = \mathbf{U}^T\mathbf{V}^{-1}\Sigma_{SV}\mathbf{V}^{-1}\mathbf{U}$. Let $\mathbf{u} = \Sigma_V^{-1/2}\mathbf{v}(\mathbf{b}') = d\Sigma_V^{-1/2}\hat{\mathbf{z}} + \mathbf{e}$. Then the random vector $\mathbf{e}$ is Gaussian with covariance $\mathbf{I}$, and the corresponding unit vector $\hat{\mathbf{u}} = \mathbf{u}/\|\mathbf{u}\|$ has an *angular Gaussian* distribution, with parameters $\hat{\mathbf{s}} = \Sigma_V^{-1/2}\hat{\mathbf{z}}/\|\Sigma_V^{-1/2}\hat{\mathbf{z}}\|$ and $m = \|\Sigma_V^{-1/2}\hat{\mathbf{z}}\|$. *Watson* [1983; sec. 3.7] shows that this distribution is well approximated by a Fisher distribution with location parameter $\hat{\mathbf{s}}$, and a dispersion parameter k which is a function of m. *Watson* [1983, pp 117-118] gives asymptotic forms for small and large m

$$k(m) \approx 4(2\pi)^{-1/2}m \quad m \ll 1 \qquad k(m) \approx m^2 \quad m \gg 1 ,$$

and a brief table for intermediate values.

The density of the VGP $\hat{\mathbf{v}} = \Sigma_V^{1/2}\hat{\mathbf{u}}/\|\Sigma_V^{1/2}\hat{\mathbf{u}}\|$ can be calculated using a slight generalization of the transformation approach used above [*Watson*, 1983; Eq. 3.6.4]. In terms of the Fisher density $p_F(\hat{\mathbf{u}};\hat{\mathbf{s}},k)$ and $\hat{\mathbf{u}} = \Sigma_V^{-1/2}\hat{\mathbf{v}}/\|\Sigma_V^{-1/2}\hat{\mathbf{v}}\|$, the result is

$$p_V(\hat{\mathbf{v}}) = |\Sigma_V|^{-1/2}p_F(\hat{\mathbf{u}};\hat{\mathbf{s}},k(m))\,\|\hat{\mathbf{v}}^T\Sigma_V^{-1}\hat{\mathbf{v}}\|^{-3/2} \tag{6}$$

$$= k(m)\exp[k(m)\rho]\,[4\pi\sinh k(m)\,|\Sigma_V|^{1/2}\,\|\hat{\mathbf{v}}^T\Sigma_V^{-1}\hat{\mathbf{v}}\|^{3/2}]^{-1}$$

where $\rho = \hat{\mathbf{v}}(\mathbf{b}')^T\Sigma_V^{-1}\hat{\mathbf{z}}(\hat{\mathbf{v}}^T\Sigma_V^{-1}\hat{\mathbf{v}})^{-1/2}(\hat{\mathbf{z}}^T\Sigma_V^{-1}\hat{\mathbf{z}})^{-1/2}$.

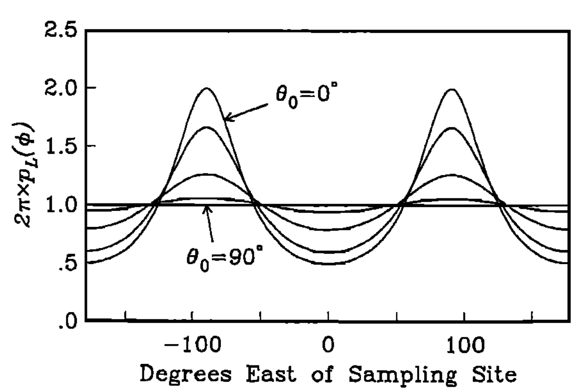

Fig. 1. Normalized VGP longitude densities, for an isotropic distribution of local magnetic field variations. The density is sharply peaked 90° away from the sampling site.

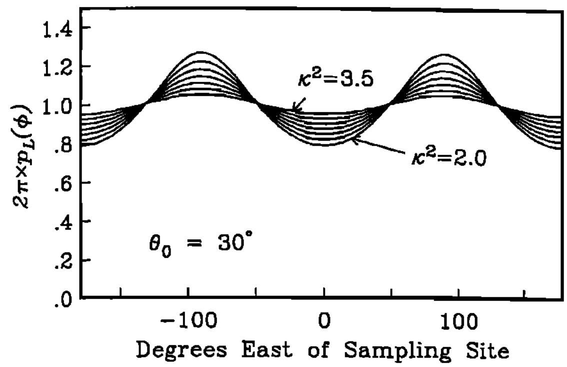

Fig. 2. Densities for $2 \le \kappa^2 \le 3.5$ in steps of 0.25 for a site at 30° latitude.

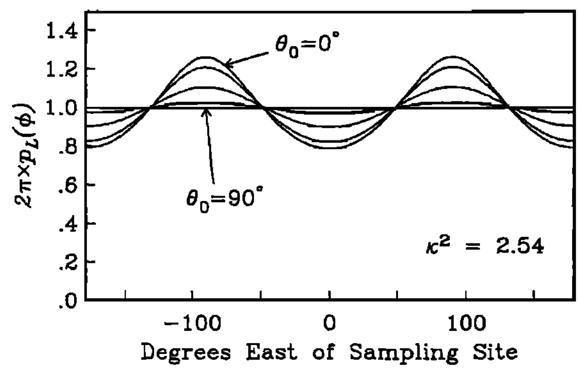

Fig. 3 Densities for $\kappa^2 = 2.54$ (the value suggested by the recent geomagnetic field, when isotropic random and systematic errors are allowed for), for five latitudes (0°, 22.5°, 45°, 67.5° and 90°).

Eq. (6) is actually quite general, since it can be used to compute the density of VGP directions for any mean and covariance of the local field with a Gaussian distribution. In fact, this distributional assumption is much stronger than required. The result holds (approximately) provided the Fisher distribution is a reasonable approximation to the (necessarily isotropic) scatter in $\hat{\mathbf{u}}$ about its mean. When $d=0$, $p_F(\hat{\mathbf{u}};\hat{\mathbf{s}},k)$ reduces to a uniform density, and (6) reduces to

$$p_V(\hat{\mathbf{v}}) = [4\pi |\Sigma_V|^{1/2} (\hat{\mathbf{v}}^T \Sigma_V^{-1} \hat{\mathbf{v}})^{3/2}]^{-1} \quad . \tag{7}$$

Eq. (7) could be used for modeling the marginal distribution of VGPs during a reversal, by including random axial dipole variations in Σ_{SV}.

Statistical Models for $\mathbf{b}_{SV}'$

The example of Figure 1 demonstrated that an isotropic distribution for $\mathbf{b}_{SV}'$ resulted in a highly non-isotropic distribution of VGPs. However, *Cox* [1970] has pointed out that VGPs are more nearly isotropic than local field directions. Furthermore, it has frequently been argued [e.g., *Harrison*, 1980; *Merrill and McElhinny*, 1983; *McFadden and McElhinny*, 1984] that secular variation necessarily should lead to isotropic scatter in VGPs. This would suggest that the bias evident in Figure 1 may not be relevant to real paleomagnetic VGP distributions.

It is instructive to consider the statistical model for the geomagnetic field of *Constable and Parker* [1988]. In this model $\mathbf{b}_{SV}'$ is a sample from an homogeneous (i.e., rotationally invariant in a statistical sense) random process on the sphere. It might well be argued that this model is overly simplistic. However, it has the maximum conceivable symmetry among all models dominated by an axial dipole, and thus seems a reasonable null hypothesis model when one seeks to demonstrate that paleomagnetic data require some further breakdown of symmetry.

TABLE 1. Covariance Σ_{SV} Estimated from DGRF-85

	B_x	B_y	B_z
B_x	1.10		
B_y	0.31	0.91	
B_z	0.18	0.04	3.16

All entries are normalized by the average horizontal field variance $(\sigma_{xx} + \sigma_{yy}) = 3.97\times10^7 \mathrm{nT}^2$.

As the gradient of a random scalar potential arising from internal sources, $\mathbf{b}_{SV}'$ can be expanded in spherical harmonics as

$$\mathbf{b}_{SV}'(\mathbf{r}) = -\nabla \sum_{l=1}^{\infty} \sum_{m=-l}^{l} (a/r)^{l+1} a_{lm} Y_{lm}(\hat{\mathbf{r}}) \ .$$

Since the covariance of the random potential is assumed invariant under all rotations (i.e., $\boldsymbol{Cov}[\phi(\mathbf{r}_1), \phi(\mathbf{r}_2)] = \boldsymbol{Cov}[\phi(\mathbf{W}\mathbf{r}_1), \phi(\mathbf{W}\mathbf{r}_2)]$ for all rotations $\mathbf{W}$) the random coefficients a_{lm} must have zero mean and diagonal covariance $Ea_{lm}a_{l'm'} = \sigma_l^2 \delta_{ll'} \delta_{mm'}$ [e.g., *Yaglom*, 1961]. Furthermore, the covariance matrix of the field components has the form [*Constable and Parker*, 1988]

$$\boldsymbol{E}\mathbf{b}_{SV}'^T \mathbf{b}_{SV}' = \Sigma_{SV} = \sigma_H^2 \,\mathbf{diag}[1, 1, \kappa^2] \tag{8}$$

$$\sigma_H^2 = \sum_{l=1}^{\infty} \frac{l(l+1)}{2} \sigma_l^2 \qquad \sigma_Z^2 = \sum_{l=1}^{\infty} (l+1)^2 \sigma_l^2 \qquad \kappa^2 = \sigma_Z^2 / \sigma_H^2 .$$

By setting $\kappa_l^2 = 2+2/l$ we can write

$$\kappa^2 = \sum w_l \kappa_l^2 \quad \text{where} \quad w_l = l(l+1)\sigma_l^2 \left[\sum l(l+1)\sigma_l^2\right]^{-1} \tag{9}$$

are weights which sum to one. Thus κ^2 is the weighted average of $\kappa_l^2 = 2+2/l$. It follows that $2 < \kappa^2 \le 4$ and that $\kappa^2 = 4$ only if $\sigma_l^2 = 0$ for all $l > 1$ - i.e., only if the field is always purely dipolar. Conversely, κ^2 approaches 2 for fields which are dominated by short wavelength features.

The density of VGP longitudes for this model is given by (5) with $D^2 = \cos^2\theta_0 + (\kappa^2/4)\sin^2\theta_0$. For $\kappa^2 = 4$, or for $\theta_0 = 0°$ or 180° (sampling at the pole), $D = 1$ and the density is uniform. Otherwise it is peaked 90° away from the sampling site. The deviation from uniformity is greatest for small κ^2 (Figure 2), and at low latitudes (Figure 3).

κ^2 can be estimated for the recent geomagnetic field using the DGRF models [e.g., *Langel* 1992]. One way to do this is to use the Gauss coefficients to estimate σ_l^2 and then use (9). For DGRF-85 this yields an estimate of $\kappa^2 = 3.30$. An alternative approach, which also allows a check on the form of the covariance matrix predicted by the homogeneous statistical model, is to use the DGRF models to directly compute the average of Σ_{SV} across the globe. The results for DGRF-85 (Table 1) are reasonably consistent with (8), and suggest a value of $\kappa^2 = 3.16$. The deviations (i.e., $\sigma_{xx} \ne \sigma_{yy}$; nonzero off-diagonal elements) are of questionable significance, considering that only a single realization of the field is available.

For $\kappa^2 = 3.16$ - 3.3 the deviation of VGP longitude density from uniform is ±10% or less (Figure 2). However, random errors in the paleomagnetic observations, which are likely to be more isotropic [e.g., *Merrill and McElhinny*, 1983], can increase this bias significantly. The summary of data from the past 5 my given by *McElhinny and Merrill* [1975; Table 3] suggests that within-site variability increases the scatter in average field directions for individual lava flows by approximately 10%. Assuming this represents isotropic measurement error, and that the covariance of the local field due to PSV is of the form $\Sigma_{SV} = \sigma_H^2 \mathbf{diag}[1, 1, \kappa^2]$, the covariance of the recorded fields will be approximately $1.2\times\sigma_H^2 \mathbf{diag}[1, 1, \kappa_e^2]$ where $\kappa_e^2 = (\kappa^2+.2)/1.2$ is the effective value of κ^2. For $\kappa^2 = 3.16$, $\kappa_e^2 = 2.80$. Allowing for the possibility of correlated or systematic errors which do not affect within-site scatter (e.g., tectonic rotation or tilting, local magnetic anomalies, secondary overprinting, or rock magnetic complications), reduces κ^2 further. For instance, assuming that these errors are comparable to those estimated from the within-site scatter (so that total errors increase between flow scatter by 20%) yields $\kappa_e^2 \approx (\kappa^2+.4)/1.4 = 2.54$. For this value of κ^2 deviations from uniformity reach ±25% at low latitudes (Figure 3).

The results given here are not really inconsistent with the conclusions of previous workers. Even with $\kappa^2 = 2.5$, the VGPs are indeed more nearly isotropic than local field directions. However, as these results suggest, neither distribution is likely to be purely isotropic.

Conclusions

I have shown that even for simple spherically symmetric statistical models for secular variation of local magnetic fields, VGP longitude densities will be peaked 90° away from the sampling site. This departure from uniformity should be most severe at low latitudes, and when κ^2 is small. For plausible effective values of κ^2 (which allow for measurement errors) the bias can be substantial.

A 90° offset from the concentration of sampling sites is very clear in the PSV VGP longitude histograms presented by *Constable* [1992], and was also noted by *Valet et al* [1992] in mean VGP longitudes computed during reversals. While it is by no means clear that this bias effect can explain the systematics seen in paleomagnetic VGPs, this possibility should be carefully considered before more exciting physical models are endorsed.

The models given here make explicit predictions (e.g., concerning the latitudinal dependence of the bias) which could be tested. This might help to establish how important this effect is. In any event, future efforts to rigorously test the statistical significance of VGP systematics should probably adopt the general approach elucidated here and begin from more realistic null hypothesis models of the actual magnetic field variations, so that they can incorporate possible bias due to uneven sampling on the Earth's surface.

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References

Clement, B.M., Geographical distribution of transitional VGPs: evidence for non-zonal equatorial symmetry during the Matuyama-Bruhnes geomagnetic reversal, *Earth planet. Sci. Lett., 104*, 48-58, 1991.

Constable, C.G., Link between geomagnetic reversal paths and secular variation of the field over the past 5Myr, *Nature, 358*, 230-233, 1992.

Constable, C.G. and R.L. Parker, Statistics of the geomagnetic secular variation for the past 5m.y., *J. Geophys. Res., 93*, 11,569-11581, 1988.

Cox, A., Latitude dependence of the angular dispersion of the geomagnetic field, *Geophys. J. R. Astron. Soc., 20*, 253-269, 1970.

Harrison, C.G.A., Secular variation and excursions of the geomagnetic field, *J. Geophys. Res., 85*, 3511-3522, 1980.

Laj, C., A. Mazaud, R. Weeks, M. Fuller, and E. Herrero-Bervera, Geomagnetic Reversal Paths, *Nature, 359*, 111-112, 1992.

Langel, R.A., International Geomagnetic Reference Field 1991 revision, *Geophysics, 57*, 956-959, 1992.

Langeris, C.G., A.A.M. van Hoof, and P. Rochette, Longitudinal confinement of geomagnetic reversal paths as a possible sedimentary artifact, *Nature, 358*, 226-230, 1991.

McFadden, P.L. and M.W. McElhinny, A physical model for paleosecular variation, *Geophys. J. R. Astron. Soc., 78*, 809-830, 1984.

McElhinny, M.W. and R.T. Merrill, Geomagnetic secular variation over the last 5my, *Rev. Geophys. Space Phys., 13*, 687-708, 1975.

Tric, E., C. Laj, C. Jehano, J.-P. Valet, C. Kissel, A. Mazaud, and S. Iccarino, High resolution record of the upper Olduvai transition from Po Valley (Italy) sediments: support for dipolar transition geometry?, *Phys. Earth planet. Inter., 65*, 319-336, 1991.

Valet, J.-P., P. Tucholka, V. Courtillot, and L. Meynadier, Paleomagnetic constraints on the geometry of the geomagnetic field during reversals, *Nature, 356*, 400-407, 1992.

Merrill R.T. and M.W. McElhinny, *The Earth's Magnetic Field*, Academic Press, London, 401 pp., 1983.

Watson, G.S., *Statistics on Spheres*, John Wiley and Sons, New York, 238 pp., 1983.

Yaglom, A.M., Second order homogeneous random fields, in *Proceedings of the Forth Berkeley Symposium on Mathematical Statistics and Probability, vol. 2*, pp. 593-622, University of California Press, Berkeley, 1961.

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