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The Thinness of Oceanic Temperature Gradients

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A test of the scaling of the extent of the thinnest vertical temperature gradients, in the near-bottom boundary layer on the Oregon shelf, shows that the Batchelor wave number determines the cutoff wave number in vertical temperature gradient spectra. In combination with previous results, in other words, this test shows that the smallest scale at which significant temperature variance due to turbulence exists at any given point in the ocean is determined by the Batchelor scale, $(\nu D^2/\epsilon)^{1/4}$, ν being the kinematic viscosity, D the thermal diffusivity, and ϵ the kinetic energy dissipation per unit mass. Stress measurements in the viscous sublayer provide estimates of ϵ.

If the theory put forward by *Batchelor* [1959] concerning small-scale variations in the spatial distribution of a passive contaminant in a turbulent fluid applies to the distribution of heat in the ocean, a smallest length scale exists on which appreciable temperature gradients will be found. This scale depends on the kinetic energy dissipation rate ϵ as $(\nu D^2/\epsilon)^{1/4}$, where ν is the kinematic viscosity and D is the thermal diffusivity. Because this theory was derived for high Reynolds number, isotropic, homogenous turbulence, its relevance to the ocean must be shown. A decrease in the spectral intensity with wave number is indeed usually found in vertical temperature gradient spectra near the highest wave number observable. If a 'cutoff wave number' k_c is defined as the wave number at which a given spectrum has dropped to 10% of its peak value, we can ask whether k_c is related to the wave number $k_B = (2\pi)^{-1}(\epsilon/\nu D^2)^{1/4}$, corresponding to the Batchelor scale. Two problems are presented in making this test: (1) A value for ϵ is required and (2) considerable uncertainty exists as to the value of the 'universal constant' q involved in the Batchelor form.

The first evidence we found, which rendered the connection between k_c and k_B at least plausible, came from data taken in August 1977 at ocean station P as part of the mixed layer experiment (MILE) [*Caldwell et al.*, 1980]. The temperature gradient profiles came from the mixed layer, the seasonal thermocline, and the halocline. Values of ϵ were estimated from the rate of work, $\hat{E}$, done against the density gradient. A formula for $\hat{E}$ was derived by following atmospheric work in relating $\hat{E}$ and the eddy diffusivity and then calculating the eddy viscosity by the usual Osborn-Cox method. The utility of $\hat{E}$ as an indication of ϵ is based on the hope that the efficiency of the mixing process does not vary greatly. The formulas involved are

$$k_c = \frac{2.28}{(2q)^{1/2}} k_B$$

derived from the Batchelor spectrum and the definition of k_c,

$$K_\rho = I \cdot D \cdot \text{Cox}$$

the Osborn-Cox relation (assuming the eddy diffusivity for buoyancy, K_ρ, is the same as that for heat), where Cox is defined as $\langle (dT/dz)^2\rangle/(\langle dT/dz\rangle)^2$ and I is the isotropy factor, which varies from 1 for vertical stratification to 3 for isotropy,

$$\hat{E} = N^2 k_\rho$$

the relation between rate of work and eddy diffusivity in the presence of stratification represented by buoyancy frequency N, and the definition of the efficiency of mixing

$$\lambda \equiv \frac{\hat{E}}{\epsilon + \hat{E}}$$

Combining all these, we obtain

$$k_c = 2.28 \left\{\frac{I(1-\lambda)/\lambda}{4q^2}\right\}^{1/4} \cdot \frac{1}{2\pi}\left(\frac{\hat{E}}{\nu D^2}\right)^{1/4}$$

so that, except as affected by variations in λ and I, k_c and $\hat{E}$ will be correlated. The conclusion reached was that this correlation was quite good, and that the coefficient $2.28 \cdot [I(1-\lambda)/4q^2\lambda]^{1/4}$ is near 1.2, for cases where the Cox number is less than 2500. Reasonable choices of the parameters might be $\lambda = 0.2$ [*Osborn*, 1980], $I = 2 \pm 1$, and $q = 3.9 \pm 1.5$ [*Grant et al.*, 1968]. With these choices we might have expected the coefficient to lie between 0.98 and 1.94, so the value of 1.2 is reasonable. The scatter may be caused by any of a number of factors but probably principally results from either the statistical nature of the Batchelor theory or local variations in λ or I. A segment represented by a dot in this figure uses data from only 60 cm of the water column, so the effective number of degrees of freedom represented in k_c is few. The decrease in k_c/k_B as the Cox number increases above 2500 can be ascribed to a decrease in mixing efficiency; for very small mean gradients, N becomes irrelevant, and our scheme breaks down.

We have also found that when the Cox number is large, the form of the Batchelor spectrum is followed [*Dillon and Caldwell*, 1980]. This led us to suspect that k_c is always related to k_B, even at large Cox numbers. After all, the large Cox number situation resembles more closely the condition assumed by Batchelor.

To test this conjecture and to provide a conclusive test of the Batchelor scaling, we require a situation of large Cox number where ϵ can be estimated by some method not dependent upon the assumptions used in the test quoted above. The microstructure instrument could then be dropped through the region, and the observed cutoff wave numbers could be compared with a Batchelor scale calculated from ϵ. The boundary layer near the seabed offers such a situation. Using profiling speed sensors on a bottom-mounted tripod, we have been able to make measurements within the viscous sublayer and thereby accurately determine the stress at the seabed [*Caldwell and Chriss*, 1979]. With the sublayer stress measurements and

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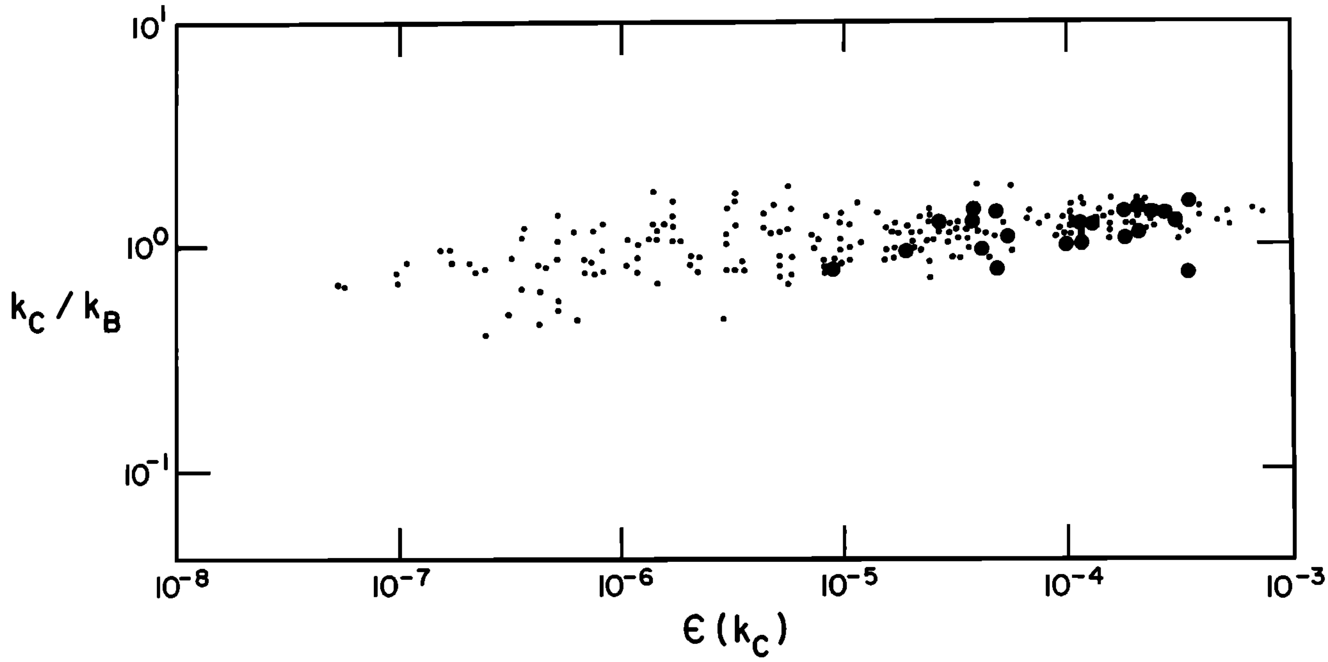

Fig. 1. Large dots represent the ratio of cutoff wave number in vertical temperature gradient spectra to the Batchelor wave number calculated from boundary layer assumptions and sublayer measurements, plotted versus dissipation. Small dots represent upper layer data with Cox number <1000, for which ϵ was calculated by an entirely different method [*Caldwell et al.*, 1980]. The units of ϵ are $cm^2\ s^{-3}$.

the assumption that the flow above the sublayer resembles the constant-stress portion of an unstratified boundary layer, the dissipation can be calculated as a function of distance from the boundary, yielding ϵ estimates independent of assumptions about transport in stratified fluids.

In June 1979 such an experiment was run in 100 m of water on the Oregon shelf, for the purpose of learning about the boundary layer flow. While the sublayer profiler was recording, a number of casts of the microstructure instrument were made, extending to the bottom within 1 km of the profiler. (The sampling of the two instruments is different, of course. A profiler record approximately 30 min long is compared to a microstructure traverse of the bottom 10 m requiring about 1 min.) The microstructure casts showed a well-mixed layer extending approximately 10 m upward from the bottom at this station, the profile resembling Figure 1*a* of *Newberger and Caldwell* [1981]. The mean temperature gradients were so small that accurate Cox numbers could not be calculated, but values were clearly greater than 10^4. The vertical temperature gradient spectra resembled the Batchelor form, and values of k_c could be determined.

Because the mean vertical temperature gradient is so small in the well-mixed region, it is plausible that the flow does resemble the constant-stress portion of an unstratified boundary layer. Stratification length scales are much larger than the layer thickness, so stratification is not likely to be important within the layer. Current velocities determined by the velocity profiler and by a rotor 0.6 m above the bed demonstrate the existence of a logarithmic layer. We have no current measurements above 1.6 m and so have no direct knowledge of the nature of the flow above. A test for consistency is discussed below.

In laboratory turbulent boundary layer flows, stress does not vary significantly with distance from the boundary, and the turbulence kinetic energy budget is dominated by a balance between the production of turbulence kinetic energy and its dissipation [*Hinze*, 1975, p. 649]. In the constant-stress log layer,

$$\frac{dU}{dz} = \frac{u_*}{0.41z}$$

and

$$\tau = \rho u_*^2$$

so

$$\text{production} = \tau \frac{dU}{dz} = \rho u_*^3/kz$$

Therefore if also production is assumed equal to dissipation,

$$\epsilon = u_*^3/0.41z \qquad (1)$$

where z is the height above the boundary, 0.41 is von Karman's constant, and u_* is the friction velocity $(\tau/\rho)^{1/2}$, τ being the bed stress and ρ the density. The wave number corresponding to the Batchelor scale is defined as

$$k_B = \frac{1}{2\pi}\left(\frac{\epsilon}{\nu D^2}\right)^{1/4} \qquad (2)$$

so if the assumptions about the boundary layer flow are correct,

$$k_B = \frac{1}{2\pi}\left(\frac{u_*^3/0.41z}{\nu D^2}\right)^{1/4} \qquad (3)$$

Using data from eight casts executed over a 5-day period, we compute the ratio k_c/k_B for 1-m segments of the water column. Points representing these ratios are indistinguishable from similar points representing data from the upper ocean for small Cox numbers (Figure 1). Numerical tests show that within the statistical variations of these data, the two samples could be drawn from the same parent distribution. That is, within the scatter of the points, similar in the two data sets, the relationship between k_c and k_B holds for this large Cox number case. Therefore increasing the Cox number does not break down the relationship. Our hypothesis that it is the assumptions required for the upper ocean test that failed is supported.

To render more plausible the assumption that the usual laboratory boundary layer assumptions apply well enough that ϵ can be estimated by u_*^3/kz, ϵ can be calculated from the microstructure data by assuming $k_c = k_B$ and solving for ϵ to yield

$$\epsilon = \nu D^2(2\pi k_c)^4 \quad (4)$$

When such values of ϵ for the bottom layer are plotted versus z, the z dependence expected from (3) is confirmed [*Newberger and Caldwell,* 1980, Figures 2 and 3]. Thus the boundary layer assumptions are confirmed, and we may now conclude that the extent of temperature gradients is controlled by the Batchelor scale; gradients thinner than $(\nu D^2/\epsilon)^{1/4}$ will not be found.

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