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The flux of tidal energy across latitude 60°S

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Abstract. How and where the ocean tides dissipate their energy are longstanding questions with both oceanographic and astronomical implications. Two decades ago, Doake suggested that flexing of Antarctic ice shelves by the underlying ocean tide is an important energy sink, perhaps accounting for over half the global dissipation rate. Observational constraints on Antarctic dissipation have been scarce. Here two new and complementary ocean-tide models, both derived from Topex/Poseidon satellite altimeter measurements, are used to determine the flux of tidal energy across 60°S toward the Antarctic coastline. Our results show relatively small fluxes and they therefore rule out Doake's suggestion: Antarctica is an insignificant sink in the global tidal energy budget.

Introduction

For well over a century, the secular slowing of the earth's rotation and the corresponding acceleration of the moon's mean motion have been attributed to "tidal friction" [*Munk and MacDonald*, 1960]. Clarifying exactly what that term entails—what physical mechanism is at work and where it occurs—contributes not only to our knowledge of ocean dynamics but also to our ability to decipher the history of the lunar orbit. An important constraint in resolving these questions is the total global rate of the tidal energy loss, and this is now well established: 2.5 ± 0.1 terawatts (TW) for the principal lunar semidiurnal tide (M_2) [e.g., *Ray*, 1994]. This rate is deduced from observations of the secular acceleration of the moon [*Dickey et al.*, 1994], from observations of long-period perturbations in the orbits of artificial satellites [e.g., *Cheng et al.*, 1992], and from direct calculations of the rate of working of tidal forces on the oceans [*Cartwright and Ray*, 1991]. Note that the latter tells us the rate at which energy is input to the oceans, but it tells us nothing about how or where that energy is consumed.

The leading candidate for the dissipation mechanism has long been bottom friction in the shallow seas, partly because friction in the deep ocean and the solid earth are apparently one or more orders of magnitude too small [*Munk and MacDonald*, 1960; *Ray et al.*, 1996]. But the question remains open: a rough estimate of the total dissipation in all shallow seas by *Miller* [1966] falls well short of the required 2.5 TW, while conversion of energy into internal tides may [*Sjöberg and Stigebrandt*, 1992; *Morozov*, 1995] or may not [*Bell*, 1975] be important.

It is with this background that *Doake* [1978] suggested that flexing of Antarctic ice shelves by the underlying ocean tide may alone consume 2 TW (± 50%) of tidal power. The observational evidence to support Doake's hypothesis is meagre and ambiguous. Supporting it are a few spot tidal observations near the ice shelves which display large tidal "ages" (large phase differences between principal lunar and solar tides) and abnormal nonlinear tides [*Cartwright*, 1979; *Pedley et al.*, 1986]. Both, particularly the latter, suggest large frictional dissipation in the region. Discounting the hypothesis are recent geodetic measurements on Ronne Ice Shelf by *Vaughan* [1995] who finds that measured ice flexures can be modeled by an elastic, as opposed to Doake's viscous or creep, rheology, with reduced energy consumption.

If dissipation near Antarctica is important, then there must exist a large flux of tidal energy crossing, say, the 60°S parallel. This is because the greatest part of the (semidiurnal) tidal energy enters the ocean in the lowest latitudes where the astronomical tidal potential reaches its maximum and where large expanses of the ocean have high tides in quadrature with the potential [e.g., *Cartwright and Ray*, 1991]. In contrast, the (semidiurnal) potential drops to zero at the poles and the direct working on the ocean in polar regions must be relatively small. If tidal energy is consumed at ice shelves, it must cross the 60° S parallel to get there. We here estimate the flux across 60°S from two recent global tidal models, both derived from Topex/Poseidon satellite altimeter measurements. Satellites provide the only comprehensive source of data in this remote location; the Topex data extend to 66° S, providing a wide band of accurate elevation measurements around 60° S, mostly in deep water where energy fluxes can be reliably inferred. In Miller's global compilation (referred to above) the Antarctic coastline was omitted, presumably because of scarcity of reliable data.

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Although diurnal tides are strongly enhanced around Antarctica, their known global dissipation rates [*Cartwright and Ray*, 1991] are too small to affect the argument of this paper. It suffices to examine M_2 only. We do, nevertheless, give some figures for O_1 and K_1.

Two Topex/Poseidon Models

Topex/Poseidon [*Fu et al.*, 1994] is the most accurate satellite altimeter mission yet flown. Owing to its dual-frequency altimeter, multi-channel radiometer, and multiple tracking systems, the satellite acts as an accurate tide gauge, yielding a precise time history of the height of the ocean. Harmonic decompositions of these measurements have resulted in a number of new global tidal elevation models [*Le Provost, Bennett, and Cartwright*, 1994], which have been shown to surpass any pre-Topex global model in accuracy [e.g. *Shum et al.*, 1997]. The two models used here are from an empirical response analysis by *Ray et al.* [1994] (version 941230; hereinafter denoted Model A) and from a generalized inverse analysis by *Egbert et al.* [1994] (version TPXO.3; hereinafter Model B). Model B is an 'assimilation model,' in which altimetry is incorporated into a numerical hydrodynamic model in a way that explicitly accounts for errors in both the satellite observations and the assumed model dynamics. The grid resolution of Model A is 1°; Model B slightly smaller. Comparisons to tide gauges suggest that model accuracies in the Southern Ocean are comparable with the global accuracies reported by *Shum et al.* [1997].

For both models tidal current velocities must be determined by assumed dynamical relationships, but because these relationships and their implementations are so different, the two models are quite complementary in the present application. Currents for Model A are determined (in deep ocean only) from tidal height gradients by solving the momentum equations of Laplace's tidal equations, with rigorous accounting for ocean loading and self-attraction but with no dissipation terms and no explicit enforcement of mass conservation (for further details, see *Cartwright et al.* [1992]). Currents for Model B are determined simultaneously with the heights through the assimilation method; this method also invokes the Laplace equations, including the continuity equation, but with simple scaling approximations for ocean loading and self-attraction and with frictional dissipation parameterized linearly in the current velocity.

Conceivably, energy fluxes from Model B might be biased by prior assumptions about the nature of dissipation at the Antarctic coastline, so that there could be some circularity in our deductions from this model. But such circularity is manifestly absent for Model A: those currents are estimated, at each desired location, strictly from the local elevation gradients; the only relevant dynamical assumption is that friction is zero at such location. Results obtained from the two models are quite similar, suggesting that in practice the accurate elevation measurements near 60°S tightly constrain tidal currents and fluxes there. We are thus confident that our final conclusions are not dependent on the dynamical assumptions of Model B at the Antarctic coastline.

Energy Balance

Below we shall ignore the relatively small effects of ocean loading and self-attraction. Then at any location the tidal energy balance is given by

$$D = W - \nabla \cdot \mathbf{P}$$

where the mean (integrating over a tidal cycle) oceanic dissipation D is balanced by the rate of working W on the ocean, less the flux divergence $\nabla \cdot \mathbf{P}$. The work rate W includes both gravitational tidal forces from the moon and mechanical forces from the body tide [e.g., *Garrett*, 1975]:

$$W = \rho \langle \mathbf{U} \cdot \gamma_2 \nabla \Phi \rangle$$

$$\mathbf{P} = g\rho \langle \mathbf{U}\zeta \rangle ,$$

where g is gravitational acceleration, ρ the density of seawater, ζ the tidal height, $\mathbf{U}$ the tidal transport vector (current velocity times ocean depth), γ_2 a combination of Love numbers that accounts for the body tide's influence on the tidal potential Φ, and brackets $\langle \cdot \rangle$ denote averaging over one tidal cycle. (Note that Garrett discusses a fourth term in the balance, an "equilibrium flux," that must be invoked when the work integral is computed in the more traditional manner: $W = \rho \langle \gamma_2 \Phi \, \partial\zeta/\partial t \rangle$. This additional flux term is unnecessary when W is computed as above.)

The mean horizontal energy flux across any section S (oriented by unit vector $\hat{\mathbf{n}}$) may be calculated as a line integral:

$$F = \int_S \mathbf{P} \cdot \hat{\mathbf{n}} \, dS = \int_S \rho g \langle \zeta \mathbf{U} \cdot \hat{\mathbf{n}} \rangle \, dS$$

This formula was originally derived by *Taylor* [1919] in his study of the flux into the Irish Sea. For our two adopted models, integration along all longitudes at 60°S yields −42 GW (Model A) and −1 GW (Model B), with the negative signs implying a total flux *away from* Antarctica. That the total integrated fluxes should be northward clearly implies that direct working by the moon on the Antarctic tides, although expected to be small, is nonetheless nonzero. In fact, Model B may be used to estimate $\int W \, dS$ integrated over the complete area south of 60°S (Model A cannot be used since it does not extend past the Topex limits of 66°S); this estimate yields 25 GW for the rate of working on the ocean in that area, giving a total dissipation south of 60°S of −17 GW (Model A) and 24 GW (Model B), the former being physically implausible.

Although there are differences between models, the

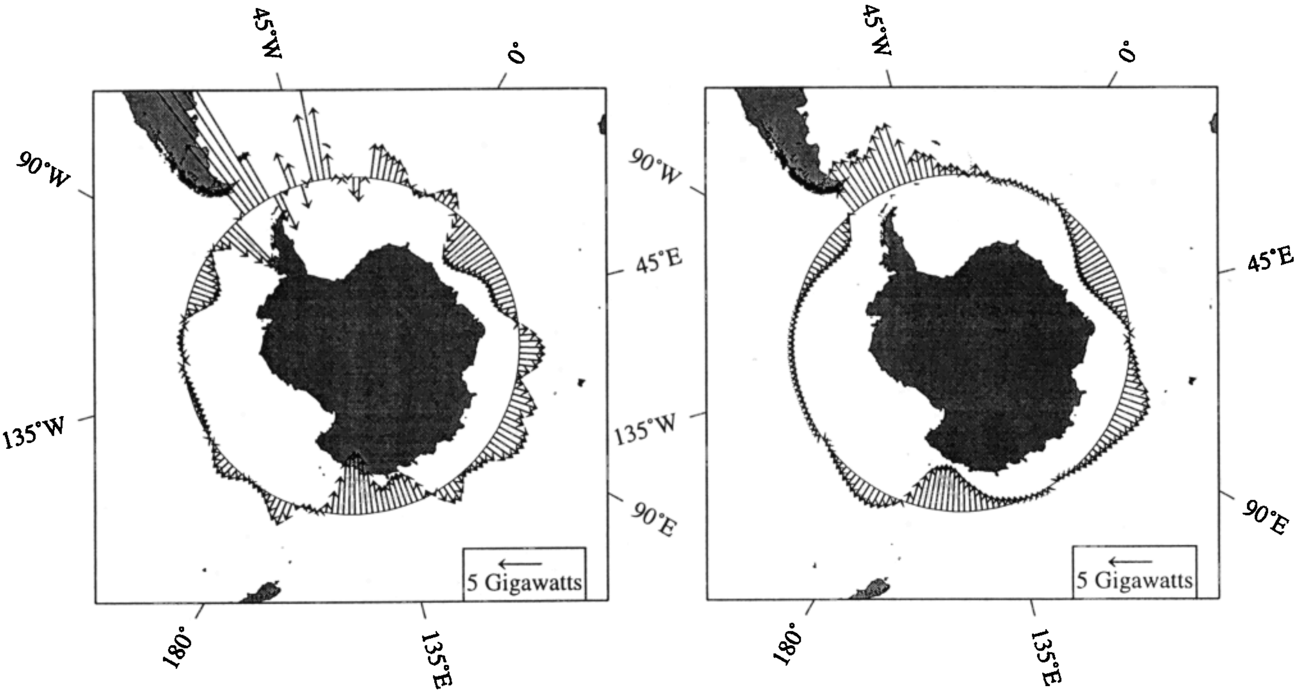

Figure 1. Mean meridional fluxes of M_2 tidal energy crossing latitude 60° S, integrated along 2°-wide longitude sections for Model A (left) and Model B (right). The noiselike behavior of Model A between 55° W and 30° W is suspect and may be due to inadequate spatial resolution near rugged topography.

important point is that they both rule out Doake's very large +2000 GW (± 50%) flux.

Detailed examination of the model flux estimates (Figure 1) shows good agreement except between 80° W and 50° W where Model A indicates larger flux toward the Patagonian Shelf off Argentina; this accounts for most of the 40 GW difference in the flux totals. Model A also appears somewhat noisy in character over the 55°–30° W band and is probably suspect there owing to inadequate spatial resolution near the rugged topography between the South Shetland and South Sandwich Island chains, and also perhaps to the assumption of no friction, which is unlikely in this region. On the other hand, *Dushaw et al.* [1997] have shown that current vectors of Model B are, in general, overly smooth. *Cartwright and Ray* [1989], using methods similar to

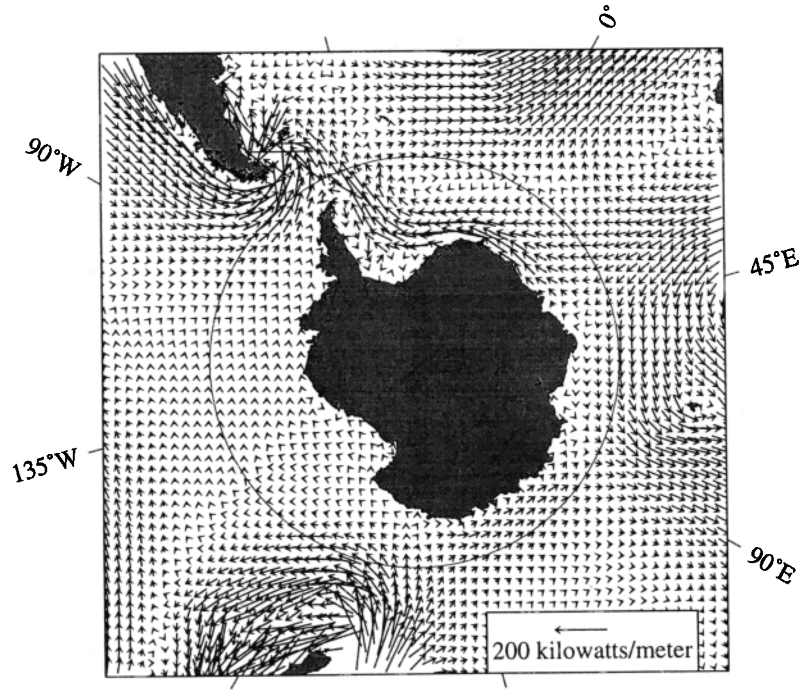

Figure 2. Mean M_2 tidal energy flux vectors from Model B (an update of the work described by *Egbert et al.*, 1994), evaluated on a stereographic grid. The faint circle denotes latitude 60° S.

those of Model A, previously inferred large northward fluxes onto the Patagonian Shelf, although their section was across 55°S.

In keeping with their relative amplitudes near Antarctica, the diurnal tides are found to exhibit comparable dissipation rates. Across 60°S Model B yields energy fluxes of 13 GW (O_1) and 19 GW (K_1). The work rates integrated over the area south of 60°S give 18 GW (O_1) and 16 GW (K_1), implying dissipation rates of 31 GW (O_1) and 35 GW (K_1), comparable to M_2.

A more complete picture of the M_2 energy flux vectors $\mathbf{P}$ throughout this region of the globe is given in Figure 2, based on Model B. Here we see that most of the patterns across 60°S evident in Figure 1 are part of larger circulation features, particularly the fluxes across 160°W–140°E, which capture part of the large circulation of energy flux rounding New Zealand, and the fluxes across 90°W–70°W, which capture part of the large flux that runs along the coast of Chile and then turns through the Drake Passage. Energy is seen to enter the outer Weddell Sea from the east along the Antarctic coast, to circulate through toward the west, and to leave near the Antarctic Peninsula where the flux merges with the Drake Passage flow and enters, and presumably dissipates on, the Patagonian Shelf.

To the eye, Figure 2 suggests that there is little dissipation occurring near Antarctica and our detailed integrated flux estimates confirm this. Compared to the total global M_2 power consumption of 2500 GW, it appears that dissipation near Antarctica plays a very minor role in the global tidal energy budget.

It is now of some importance to determine the flux of tidal energy entering all coastal and shallow-water regions of the globe. After 30 years, Miller's rough *in situ* estimates are in dire need of reevaluation. This task will be more difficult than the present determination, where our evaluation of heights ζ and transports $\mathbf{U}$ could be confined (except near 50°W) to deep-ocean areas. As the shallows are approached, both elevations and currents grow larger and spatially complex, making mapping more difficult. Moreover, in lower latitudes where the tidal forces are larger, direct tidal working on the shallow-water tide must be carefully separated from the deep-ocean working, because some significant fraction of the flux across any section may be due (as here at 60°S) to energy flowing away from the shallows. Improved accuracies also require accounting for the fluxes and work rates associated with the self-attraction potential and with ocean loading. Finally, a careful evaluation of uncertainties in all estimated terms of the energy balance is required, including an analysis of dynamical errors from faulty or overly simplistic hydrodynamical assumptions, all of which call for, in our opinion, a rigorous use of inverse methods. We anticipate important results along these lines in the coming few years, stimulated in no small measure by the accurate tidal observations from Topex/Poseidon.

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