Remotely sensed reflectance and its dependence on vertical structure: a theoretical derivation

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An exact expression for the remotely sensed reflectance (RSR, upwelling radiance divided by downwelling scalar irradiance) just beneath the surface of the ocean is derived from the equation of radiative transfer. It is shown that the RSR at a given depth in the ocean depends only on the inherent optical properties, the attenuation coefficient for upwelling radiance, and two shape factors that depend on the radiance distribution and volume scattering function. The shape factors are shown to be close to unity. An exact expression for the RSR just beneath the surface as a function of the vertical structure of inherent and apparent optical properties is derived. This expression is solved for an N-layered system, which presents the possibility of inverting remotely sensed reflectance data to obtain the vertical structure of chlorophyll in the ocean.

I. Introduction

The most important parameter in the determination of oceanic properties from remotely sensed radiance is the reflectance just beneath the sea surface. In the literature this reflectance has usually been given as the ratio of upwelling and downwelling vector irradiance. If the underwater radiance distribution at a given wavelength (in this paper we will assume monochromatic light) is given by $L(\theta,\phi,z)$ (where z is positive downward, θ is the zenith angle, and ϕ is the azimuth), the irradiance reflectance ratio is given by

$$R(z) = \frac{E_u(z)}{E_d(z)} = \frac{\int_0^{2\pi}\int_{\pi/2}^{\pi} L(\theta,\phi,z)\cos\theta\sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} L(\theta,\phi,z)\cos\theta\sin\theta d\theta d\phi}. \quad (1)$$

Expressions for this ratio as a function of the inherent optical properties (absorption and scattering) and apparent optical properties (parameters that depend in some way on the radiance distribution) have been derived by various authors. Preisendorfer[1] obtained various expressions using the two-flow or Schuster method.

The expression of most interest here is given by

$$R(z) = \frac{b_{bd}(z)}{a_u(z) + b_{bu}(z) + K(z)}, \quad (2)$$

where $b_{bd}(z)$ and $b_{bu}(z)$ are the backscattering coefficients for the diffuse downwelling light stream and the diffuse upwelling stream, respectively, a_u is the absorption coefficient of the upwelling stream, and K_u is the attenuation coefficient of the upwelling irradiance defined by

$$K_u(z) = -\frac{1}{E_u(z)}\frac{dE_u(z)}{dz}. \quad (3)$$

While Eq. (2) is exact, the parameters b_{bd} and b_{bu} are difficult to determine.

Kozlyaninov and Pelevin[2] have used the same theory with further approximations to obtain

$$R(z) = \frac{b_b(z)}{2[a(z) + b_b(z)]}, \quad (4)$$

where $b_b(z)$ is the backscattering coefficient defined by

$$b_b(z) = 2\pi \int_{\pi/2}^{\pi} \beta(\theta,z)\sin\theta d\theta, \quad (5)$$

$\beta(\theta,z)$ is the volume scattering function, and $a(z)$ is the absorption coefficient. A similar result was obtained by Duntley *et al.*[3]

Gordon *et al.*[4] used the Monte Carlo technique to analyze a number of theoretical cases. A least-squares analysis of these results yielded

$$R(\tau) = \sum_{n=0}^{n=3} r_n(\tau)x^n, \quad (6)$$

where $x = (b_b)/(a + b_b)$, and τ = optical depth = $\int_0^z c(z)dz$, where c is the beam attenuation coefficient and r_n is the coefficient obtained from the least-squares analysis. Finally, Morel and Prieur[5] reported results of earlier work in which the following expression was obtained:

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$$R(z) = 0.33 \frac{b_b(z)}{a(z)} (1 + \Delta), \quad (7)$$

where Δ depends on the volume scattering function and the radiance distribution. As Δ is normally <0.05, this is equivalent to Eq. (4) provided $2b_b = a$.

The above expressions have been shown to be useful in the analysis of ocean color. The dependence of the reflectance just beneath the surface on the structure of optical properties at depth cannot be determined from these expressions, however. Furthermore, the remotely sensed parameter is radiance not irradiance so that the appropriate parameter to be studied for the inversion of ocean color data is the remotely sensed reflectance (RSR) defined by

$$\mathrm{RSR}(z) = \frac{L(\theta,\phi,z)}{E_{od}(z)} . \quad (8)$$

For the purpose of this paper we will use nadir radiance ($\theta = \pi$). Furthermore the scalar irradiance (E_o) will be used rather than the vector irradiance (E), as the former is much less dependent on solar elevation than the latter. The scalar downwelling irradiance is defined by

$$E_{od}(z) = \int_0^{2\pi} \int_{\pi/2}^{\pi} L(\theta,\phi,z) \sin\theta d\theta d\phi. \quad (9)$$

The definition of the remotely sensed reflectance for the remainder of this paper is thus given by

$$\mathrm{RSR}(z) = \frac{L(\pi,z)}{E_{od}(z)} , \quad (10)$$

where $L(\pi,z)$ is used rather than $L(\pi,\phi,z)$ as ϕ is immaterial for $\theta = \pi$. An exact expression for RSR(z) as a function of inherent and apparent optical properties will now be derived from the equation of radiative transfer.

II. Remote Sensing Reflectance

The equation of radiative transfer for a medium without internal sources and for which the horizontal gradients are negligible compared with the vertical ones is given by

$$\cos\theta \frac{dL(\theta,\phi,z)}{dz} = -c(z)L(\theta,\phi,z) + L^*(\theta,\phi,z), \quad (11)$$

where

$$L^*(\theta,\phi,z) = \int_0^{2\pi} \int_0^{\pi} \beta(\gamma,z)L(\theta',\phi',z) \sin\theta d\theta d\phi,$$

and

$$\cos\gamma = \cos\theta \cos\theta' - \sin\theta \sin\theta' \cos(\phi - \phi'). \quad (12)$$

Let $\theta = \pi$ so that

$$-\frac{dL(\pi,z)}{dz} = -c(z)L(\pi,z) + L^*(\pi,z), \quad (13)$$

where

$$L^*(\pi,z) = \int_0^{2\pi} \int_0^{\pi} \beta(\pi - \theta',z)L(\theta',\phi',z) \sin\theta' d\theta' d\phi'. \quad (14)$$

The integral over the zenith angle θ in Eq. (14) is now divided into a forward component $(0,\pi/2)$ and a backward component $(\pi/2,\pi)$. It is assumed that the volume scattering function as well as the radiance distribution are continuous in the region $(\pi/2,\pi)$. Equation (14) can then be rewritten in the following form:

$$L^*(\pi,z) = \beta(\xi,z) \int_0^{2\pi} \int_0^{\pi/2} L(\theta',\phi',z) \sin\theta' d\theta' d\phi' + L(\eta,z) \int_0^{2\pi} \int_{\pi/2}^{\pi} \beta(\pi - \theta',z) \sin\theta' d\theta' d\phi', \quad (15)$$

where $\pi/2 \le \xi \le \pi$ and $\pi/2 \le \eta \le \pi$. Hence

$$L^*(\pi,z) = \beta(\xi,z)E_{od}(z) + L(\eta,z)b_f(z), \quad (16)$$

where b_f is the forward scattering coefficient defined similarly to b_b in Eq. (3). $E_{od}(z)$ was defined in Eq. (9).

To proceed we need to relate $\beta(\xi,z)$ and $L(\eta,z)$ to more readily measured quantities. This is done by setting

$$\beta(\xi,z) = f_b(z) \frac{b_b(z)}{2\pi} \text{ and } L(\eta,z) = f_L(z)L(\pi,z), \quad (17)$$

thereby introducing the two shape parameters $f_b(z)$ and $f_L(z)$, which are anticipated to be close to unity as both the volume scattering function and the radiance distribution are relatively uniform in the region $(\pi,\pi/2)$ compared with $(0,\pi/2)$.

Substitution of Eqs. (16) and (17) into Eq. (11) yields

$$\frac{-dL(\pi,z)}{dz} = -c(z)L(\pi,z) + f_b(z) \frac{b_b(z)}{2\pi} E_{od}(z) + b_f(z)f_L(z)L(\pi,z). \quad (18)$$

The attenuation coefficient for nadir radiance, $k(\pi,z)$, is defined in a similar way as the attenuation coefficient for irradiance:

$$k(\pi,z) = \frac{1}{-L(\pi,z)} \frac{dL(\pi,z)}{dz} , \quad (19)$$

from which it follows that

$$L(\pi,z) = L(\pi,0) \exp[- \int_0^z k(\pi,z')dz']. \quad (20)$$

Substitution of Eq. (19) into Eq. (18) and factoring yield the desired exact equation for the remote sensing reflectance:

$$\mathrm{RSR}(z) = \frac{L(\pi,z)}{E_{od}(z)} = \frac{f_b(z)b_b(z)/2\pi}{k(\pi,z) + c(z) - b_f(z)f_L(z)} , \quad (21)$$

which is valid at all depths.

III. Dependence of RSR(o) on Vertical Structure

Of special interest is the RSR at the surface and its dependence on the vertical structure of the inherent and apparent optical properties. To find the vertical dependence, consider the equation of radiative transfer (11) into which Eqs. (16) and (17) have been substituted:

$$\frac{-dL(\pi,z)}{dz} = L(\pi,z)[f_L(z)b_f(z) - c(z)] + \frac{f_b(z)b_b(z)E_{od}(z)}{2\pi} . \quad (22)$$

Equation (22) is a linear differential equation of first order, the solution of which is given by

$$L(\pi,z)\exp\left\{\int_0^z -[c(z') - f_L(z')b_f(z')]dz'\right\} - L(\pi,o) = - \int_0^z \frac{f_b(z')b_b(z')}{2\pi} E_{od}(z') \times \exp\left\{\int_0^{z'} -[c(z'') - f_L(z'')b_f(z'')]dz''\right\}dz'. \quad (23)$$

At very great depth in the ocean, the radiance vanishes. By setting $z = \infty$, we thus get

$$L(\pi,o) = \int_0^\infty \frac{f_b(z')b_b(z')}{2\pi} E_{od}(z') \times \exp\left\{\int_0^{z'} -[c(z'') - f_L(z'')b_f(z'')]dz''\right\}dz'. \quad (24)$$

We now substitute

$$E_{od}(z) = E_{od}(o)\exp\left[-\int_0^z K_{od}(z')dz'\right], \quad (25)$$

where $K_{od}(z)$ is the attenuation coefficient of the downward scalar irradiance, into Eq. (24) and divide by $E_{od}(o)$ to obtain RSR(o):

$$\frac{L(\pi,o)}{E_{od}(o)} = \mathrm{RSR}(o) = \int_0^\infty \frac{f_b(z')b_b(z')}{2\pi} \exp\left\{\int_0^{z'} -[c(z'') - f_L(z'')b_f(z'') - K_{od}(z'')]dz''\right\}dz'. \quad (26)$$

Equation (26) shows the dependence of the remotely sensed reflectance on the vertical structure of the inherent optical properties $b_b(z)$ and $c(z)$, the apparent optical property $K_{od}(z)$, and the shape parameters $f_b(z)$ and $f_L(z)$.

IV. Shape Factors

The shape factors f_b and f_L depend on both the volume scattering function and the radiance distribution. Even in a homogeneous ocean these parameters will thus change as a function of depth. While Eqs. (21) and (26) are exact they are not very useful unless some *a priori* limits can be put on the range of values of f_b and f_L.

Near the surface of the ocean radiance distributions tend to be sharply peaked in the refracted direction of the sun. With increasing depth or increasing cloudiness, the maximum radiance shifts toward the vertical and the radiance distribution becomes less sharply peaked. For the analysis of the probable range of values of the shape factors in oceanic conditions a sharply peaked radiance distribution obtained[6] near the surface during bright sunshine (I in Fig. 1) was chosen as well as a diffuse radiance distribution that was obtained[6] during overcast conditions. Volume scattering functions in the ocean also vary greatly in shape. Again, extremes were chosen for the assessment of f_b and f_L. The first, labeled (1.25,3) in Fig. 1, is the volume scattering function obtained[7] from Mie theory for a collection of particles with a slope of 3 for the Junge-type cumulative particle size distribution and with a relative index of refraction of 1.25. This scattering function is representative of largely inorganic material, such as might be found off the mouth of a river. Two scattering functions were used that are representative of largely organic material, with low relative indices of refraction (1.02) and with cumulative size distribution slopes of 3 and 5. These are labeled (1.02,3) and (1.02,5), respectively. The scattering functions of the organic material are much more sharply peaked in the forward direction, while the scattering function for the inorganic material is relatively sharply peaked in the backward direction. Radiance distributions and volume scattering functions in the ocean tend to be less extreme than the ones chosen here for analysis. It is therefore anticipated that values for f_b and f_L obtained from the combination of the functions chosen would cover the range of values to be expected in the ocean. The values calculated for f_b and f_L are shown in Table I.

V. Discussion

An exact equation that describes the dependence of the remotely sensed reflectance solely on the vertical structure of the inherent optical properties can be obtained only by formally solving the equation of radiative transfer. Any equation for the dependence of radiance,

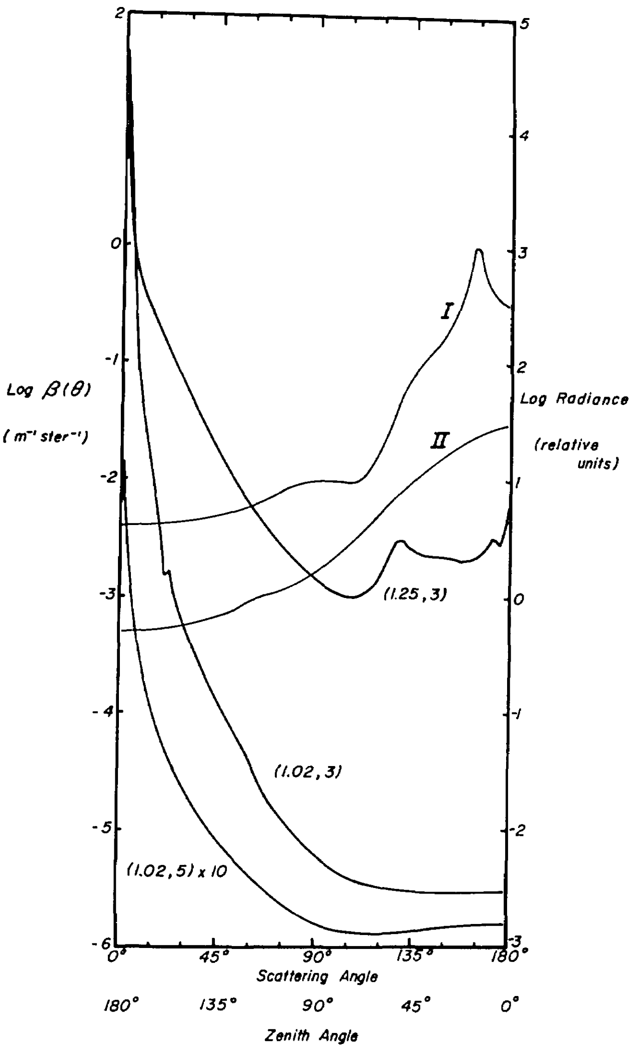

Fig. 1. Radiance distributions[6] (I and II) and the volume scattering functions[7] (1.02,5; 1.02,3; and 1.25,3) used to calculate the probable range of values of the shape factors f_b and f_L.

Table I. Values of f_L and f_b as Defined in Eq. (17) for the Two Radiance Distributions and the Three Volume Scattering Functions Shown in Fig. 1

Radiance distribution	Scattering function	f_L	f_b
I	(1.02,3)	1.001	0.810
I	(1.02,5)	1.074	1.028
I	(1.25,3)	1.064	1.267
II	(1.02,3)	1.002	0.852
II	(1.02,5)	1.111	0.996
II	(1.25,3)	1.099	1.202

irradiance, or their ratios on the inherent optical properties that is not obtained from a formal solution of the equation of radiative transfer will always contain apparent optical properties.[8] In this paper we have sought to limit this dependence on apparent optical properties to readily measureable quantities such as $k(\pi,z)$ and $K_{od}(z)$ and on the shape parameters $f_b(z)$ and $f_L(z)$, which have a small range of values in the ocean and can be approximated by unity without great loss of accuracy.

Table I shows that the shape factors f_b and f_L depend only weakly on the shape of the radiance distribution, the variation being <5% for the two radiance distributions with a constant volume scattering function. This is due to the shape of the radiance distribution in the upward direction being relatively constant even though the shape in the downward direction varies greatly. In a homogeneous water mass f_b and f_L could thus be expected to vary <5% from their surface values.

A larger variation of the shape factors f_b and f_L was observed as a function of the shape of the volume scattering function. Nevertheless for the extreme cases analyzed here, f_L was covered by a range from 1.00 to 1.12 and f_b was covered by a range from 0.80 to 1.27.

Equation (21) shows that any error in f_b is translated immediately into an error in RSR, but an error in f_L has a much smaller influence. Using the above argument and the calculated ranges of f_b and f_L for extreme cases of the radiance distribution and the volume scattering function, it is concluded that the expression for the RSR(z) can, in virtually all cases, be approximated to within 30% by setting $f_b(z)$ and $f_L(z)$ equal to 1. The resultant equation is

$$\mathrm{RSR}(z) = \frac{b_b(z)/2\pi}{k(\pi,z) + c(z) - b_f(z)} = \frac{b_b(z)/2\pi}{k(\pi,z) + a(z) + b_b(z)}. \quad (27)$$

At first glance it might appear that Eq. (27) ignores the dependence of RSR(o) on the vertical structure of the optical properties. This is not the case as $k(\pi,z)$ is an apparent optical property and therefore depends on the vertical structure of the radiance distribution, which in turn depends on the vertical structure of the inherent optical properties. This is seen by setting $z = 0$ in Eq. (27) and solving for $k(\pi,o)$ and substituting Eq. (26) for RSR(o).

If we set $f_L(z)$ and $f_b(z)$ equal to unity in Eq. (26), we obtain an equation that can be solved for an N-layered system. In such a system we assume that the optical properties are constant within each layer. K_{od} is an apparent optical property and could vary as a function of solar elevation even if the inherent optical properties are constant. Baker and Smith[9] have shown that K_d is relatively insensitive to solar elevation and have called K_d a "quasi-inherent" optical property. K_{od} should be even less sensitive to solar elevation than K_d. We assign the optical properties $c(l)$, $b_f(l)$, and $K_{od}(l)$ to the first layer with a depth range from 0 to z_1, similarly the depth range of z_{i-1} to z_i has optical properties $c(i)$, $b_f(i)$, and $K_{od}(i)$. The integral from 0 to ∞ in Eq. (26) is broken up into N segments z_{i-1} to z_i in which the optical properties are constant, so that

$$\mathrm{RSR}(o) = \sum_{i=1}^{N} \int_{z_{i-1}}^{z_i} \frac{b_b(i)}{2\pi} \exp\left[\int_0^{z} p(z'')dz''\right] dz', \quad (28)$$

where $p(z) = -c(z) + b_f(z) + K_{od}(z)$. We can similarly break up the integral in the exponent

$$\mathrm{RSR}(o) = \sum_{i=1}^{N} \int_{z_{i-1}}^{z_i} \frac{b_b(i)}{2\pi} \exp\left\{\sum_{j=1}^{i=1} \left[\int_{z_{j-1}}^{z_j} p(z'')dz''\right] + \int_{z_{i-1}}^{z'} p(z'')dz''\right\} dz'. \quad (29)$$

When the integrals in Eq. (28) are carried out, we obtain

$$\mathrm{RSR}(o) = \sum_{i=1}^{N} \frac{b_b(i)}{2\pi} \exp\left[\sum_{j=1}^{i-1} p(j)(z_j - z_{j-1})\right] \times \exp[-p(i)z_{i-1}] \left\{\frac{\exp[p(i)z_i] - \exp[p(i)z_{i-1}]}{p(i)}\right\}, \quad (30)$$

which can be rewritten as

$$\mathrm{RSR}(o)=\sum_{i=1}^{N}\frac{b_b(i)}{2\pi p(i)}\left\{\exp\left[\sum_{j=1}^{i}p(j)(z_j-z_{j-1})\right]-\exp\left[\sum_{j=1}^{i-1}p(j)(z_j-z_{j-1})\right]\right\}. \quad (31)$$

Equation (31) shows that the remotely sensed reflectance can be calculated if the optical properties of the layers and the depth of the bottom of the layers are known. A question of considerable interest is whether the above equation could be inverted if the remotely sensed reflectance was known at several wavelengths.

Morel[10] has indicated that, in areas where the suspended particulate matter is of biological origin (so-called case 1 waters), the inherent and apparent optical properties c, b_f, and K_{od} can be given as a function of the chlorophyll concentration only, so that only one independent parameter suffices to describe the bio-optical state of a layer. In that case each layer has only two unknown parameters, the chlorophyll concentration and the depth of the bottom of the layer. Since the depth of the bottom of the deepest layer is ∞, it is seen that for case 1 water with N layers there are $2N-1$ unknowns. In practice this approach would be limited by the degrees of freedom contained in the remotely sensed radiance data. Mueller[11] has shown that with current instrumentation there are at most five degrees of freedom. Case 1 waters typically display three distinct layers: a well-mixed surface layer, a layer of maximum chlorophyll concentration at the thermocline, and a clear layer beneath the chlorophyll maximum. It is obvious that the chlorophyll concentration in each layer as well as the depth range of each layer is described by a total of 5 unknowns. The remotely sensed reflectance at a given wavelength then depends on these five parameters as indicated in Eq. (31). If the remotely sensed reflectance were known at five wavelengths it would then be possible in theory to obtain the vertical structure of chlorophyll by inversion of five Eqs. (31). The above development thus opens up for the first time an approach toward sensing of vertical structure in the ocean by means of passive remote sensing.

Traditional theory refers to the irradiance reflectance ratio, so that it is of interest to see how $R(z)$ as defined by Eq. (1) relates to RSR(z) as defined by Eq. (21). To do so we define the distribution function[1]

$$D_d(z)=\frac{E_{od}(z)}{E_d(z)}. \quad (32)$$

We also define[12]

$$Q(z)=\frac{E_u(z)}{L(\pi,z)}. \quad (33)$$

Combining Eqs. (28) and (29) we get

$$\mathrm{RSR}(z)=\frac{R(z)}{Q(z)D_d(z)}. \quad (34)$$

The relationship of the remote sensing reflectance and the irradiance reflectance thus depends on the shape of the radiance distribution as well as on the vertical distribution of optical properties which is contained in the apparent optical properties.

VI. Conclusions

A new exact relationship for the remote sensing reflectance [Eq. (21)] has been derived from the equation of radiative transfer. This relationship contains two parameters, f_b and f_L, that are probably well within 30% of unity, so that the approximate expression Eq. (27) should be valid to within ~30%. An exact expression [Eq. (26)] for the depth dependence of the remote sensing reflectance on the inherent and apparent optical properties was also derived. This expression can be solved for multilayered systems when the shape factors are assumed to be unity. It is shown that for case 1 waters with N-distinct chlorophyll layers there are $2N-1$ independent parameters. The measurement of the remotely sensed reflectance at $2N-1$ wavelengths could then in theory be used to obtain the vertical structure of the chlorophyll.

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