

Long-wave radiation at the ground

I. Angular distribution of incoming radiation

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SUMMARY

The apparent emissivity of the atmosphere ϵ , defined as the ratio of incoming long-wave radiation to black-body radiation at screen temperature T_a , was measured under clear skies in the English Midlands and in the Sudan. At a zenith angle Z the emissivity was given by

$$\epsilon(Z) = a + b \ln(\text{usec } Z)$$

where u is the reduced depth of precipitable water (cm). For a set of 46 scans in England, the mean values of a and b were 0.70 ± 0.05 and 0.090 ± 0.002 . Systematic deviations about these mean values could be ascribed to: (i) temperature gradients; (ii) aerosol. The Sudan measurements gave $a = 0.67 \pm 0.03$ and $b = 0.085 \pm 0.002$ consistent with the English results and observations already published. There is some evidence that minimum values of a have increased over the past 50 years.

Integration over the hemisphere gives the flux density (W m^{-2}) of atmospheric radiation as $1.06 \sigma T_a^4 - 119 (T \text{ in K})$, where σ is Stefan's constant, or $5.5 T_a + 213 (T \text{ in } ^\circ\text{C})$.

Radiation records by Dines and Dines (1927) for overcast skies are analysed to show that the angular distribution is the same as for cloudless skies; that the mean temperature of cloud base at their site was 11K below screen temperature; and that when the mean fraction of cloud cover is c , the apparent emissivity is

$$\epsilon_a(c) = (1 - 0.84c)\epsilon_a(0) + 0.84c.$$

1. INTRODUCTION

In conventional meteorology, the net income of radiation at a point on the earth's surface is defined as the difference between the flux density of incoming radiation of all wavelengths and the flux density of radiation which is reflected and emitted by the surface. To maintain a standard record of net radiation, the radiometer must be exposed above a uniform horizontal surface. Records of this type have been extensively used to explore differences in the heat balance of different types of surface and to assess the rate of evaporation from water, soil, and vegetation.

The radiation regime of surfaces which are not horizontal is more difficult to determine but is important in topographic studies, in building climatology, and in establishing the heat balance of animals and people in the open. Several analytical methods have been derived for calculating the interception of direct and diffuse solar radiation by inclined surfaces but corresponding calculations for long-wave radiation have a much more empirical basis. Geiger (1965) and Sellers (1965) reviewed a number of methods for estimating long-wave exchange in relation to topography and the attempt made by Priestley (1957) to determine the atmospheric radiation intercepted by a sheep, treated as a cylinder, is frequently quoted in the literature of climatic physiology.

As part of a wider study of radiation climate in Britain, the angular distribution of atmospheric radiation was measured on clear days over a period of 3 years. Older measurements by Dines and Dines (1927) provide an extension to cloudy skies and formulae based on both sets of observations are reported in this paper. A sequel presents an analytical method for calculating the long-wave radiation intercepted by sloping surfaces and solid objects.

2. EXPERIMENTAL DETAILS

Most of the measurements were taken at Sutton Bonington (52.8°N, 1.3°W), a rural site in the Midlands of England, but a few were made near Wad Medani (14.2°N, 33.3°E) in the Sudan Gezira, an irrigated region surrounded by semi-desert. All the readings were made with a Linke-Feussner radiometer between 1967 and 1970 at times when no cloud was visible. Linke (1933) described the instrument and Robinson (1947) its use to measure atmospheric radiation. The radiometer receives a cone of radiation of half-angle $5^{\circ} 6'$, i.e. $1/125$ of the radiation to a hemisphere.

The instrument was calibrated for long-wave radiation by exposure over a water bath at known temperatures, assuming an emissivity of unity for the water surface. The temperature of the instrument was measured with a thermometer fitted in good thermal contact with the thermopile. Additional precautions were taken to avoid heating of the thermometer and of the instrument terminals by short-wave radiation. Calibration in the short-wave spectrum was made by comparing the flux of direct solar radiation at normal incidence and the direct flux on a horizontal surface measured with a Kipp solarimeter, shaded and unshaded. The solarimeter was calibrated regularly against a standard at Kew Observatory using an integrating hemisphere.

Calibration factors obtained in these ways agreed within $\pm 2\%$ with each other and with the manufacturer's calibration. The factor did not change over the period of measurements.

Long-wave radiation in the presence of short-wave solar radiation is determined as the difference between measurements of long- plus short-wave radiation (no filter over the thermopile) and of short-wave radiation alone (using a clear glass filter opaque to long-wave radiation). A correction factor for absorption and reflection by the clear glass filter was determined from a long series of observations of the direct solar beam with and without the filter. Corrections were made for the temperature sensitivity of the instrument calibration factor, using figures given by the manufacturer.

The reproducibility of single measurements of atmospheric radiation with the instrument was usually about $\pm 5\text{W m}^{-2}$, but in strong winds fluctuations of the output due to air pressure changes in the instrument made this figure difficult to achieve.

3. METHODS

The quantity measured with a Linke-Feussner instrument is the energy received by the surface of the thermopile within a solid angle of $2\pi/125$ but it is usually expressed as an equivalent flux density \mathcal{L} . This quantity is π times the flux density of radiation received by the thermopile per unit solid angle, i.e. it is the flux density of radiation which the thermopile would receive from a hemisphere radiating uniformly like the part of the sky which the radiometer is pointing at. The symbol $\mathcal{L}(Z)$ will be used to specify the equivalent flux density at zenith angle Z .

The relation between \mathcal{L} and Z for cloudless skies was found by measuring \mathcal{L} at zenith angles of 0, 20, 30, 40, 50, 60, 70 and 80°. No significant variation of flux density with azimuth was found. At each angle, the instrument zero was checked using the built-in shutter both before and after measurements of long-wave and long- plus short-wave radiation. Corrections for drift of the zero were applied on the basis of these figures. The radiometer output was measured on a Comark portable microvoltmeter and the sensitivity of the scale used ($100\mu\text{V}$) was checked with a standard voltage source.

When long-wave radiation was measured during daylight, the direct component of solar radiation was measured at the same time so that atmospheric turbidity could be estimated using the method described by Unsworth and Monteith (1972).

4. ANALYSIS OF MEASUREMENTS

Elsasser (1942) showed that the isothermal emissivity of water vapour and carbon dioxide varies nearly linearly with the logarithm of the optical depth of water vapour, u' . Consequently, in an isothermal atmosphere the emissivity of a column at angle Z from the zenith should increase linearly with the logarithm of $\sec Z$, since $u' \propto \sec Z$. The results of Dines and Dines (1927), Robinson (1947, 1950) and others, show that the same relationship is valid for non-isothermal, real atmospheres, presumably because most of the atmospheric radiation originates within 100m of the ground, a layer in which the temperature may often be treated as uniform for the purposes of estimating a radiation flux. Robinson (1947) modified an arbitrary procedure of Brooks (1941) for reducing radiation measurements in non-isothermal atmospheres to emissivities and the modified method is used in this analysis. The apparent emissivity, $\epsilon(Z)$, of the column of length $u \sec Z$ is defined as $\mathcal{L}(Z)/\sigma T_a^4$ where u is the zenith optical water path, T_a is the ambient air temperature, and σ is Stefan's constant taken as $56.7 \times 10^{-9} \text{W m}^{-2} \text{K}^{-4}$. The results for each scan were reduced in this manner, adopting one further modification suggested by Robinson. For $Z = 80^\circ$ the radiometer receives radiation between 75° and 85° and $\sec 80^\circ$ is not a good mean value for the path length. A factor of $\sec 81.8^\circ = 7.0$, was used for this zenith angle.

The zenith optical water path, u , was obtained from the routine radiosonde ascents at noon or midnight from Crawley, Hemsby or Aughton depending on the synoptic situation and time of day. Precipitable water calculated for each layer from the radiosonde record was converted to an effective optical path using a square root pressure correction.

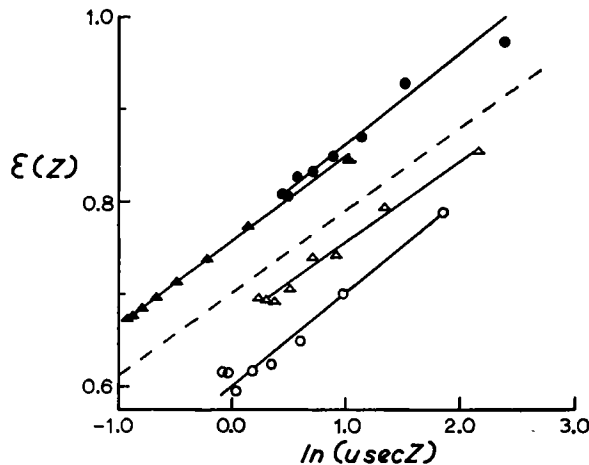


Figure 1. Dependence of emissivity $\epsilon(Z)$ on optical path length $u \sec Z$ for four representative scans
 ● 2112 GMT, 1 July 1968 △ 1017 GMT, 5 May 1970
 ▲ 2130 GMT, 9 February 1969 ○ 2300 GMT, 9 January 1968
 - - - - best straight line through all observations at Sutton Bonington (46 scans)

To illustrate extremes, Fig. 1 shows the variation of $\epsilon(Z)$ with the logarithm of optical water path, $\ln(u \sec Z)$, for four scans. Also shown is the mean straight line based on 46 scans selected for minimum scatter (the linear correlation coefficient for most scans was greater than 0.95). Scatter was almost always a result of large drifts of the instrument zero, or fluctuating output in strong wind. The variation of emissivity with zenith angle can be expressed as

$$\epsilon(Z) = a + b \ln(u \sec Z) \quad . \quad . \quad . \quad (1)$$

TABLE 1. VALUES OF THE COEFFICIENTS a AND b IN THE FORMULA $\epsilon(Z) = a + b \ln(\text{usec}Z)$

Site		$a \times 10^2$	$b \times 10^3$
Sutton Bonington	All occasions (46)	70 ± 5	90 ± 2
	Maximum a	82.7 ± 0.5	
	b		140 ± 20
	Minimum a	60.4 ± 0.6	
	b		68 ± 10
	Inversions (5)	71 ± 5	82 ± 4
	Maximum a	76.5 ± 0.8	
	b		97 ± 10
	Minimum a	65.5 ± 0.9	
	b		68 ± 10
	Lapse (11)	67 ± 3	94 ± 3
Maximum a	72 ± 2		
b		111 ± 8	
Minimum a	65 ± 1		
b		75 ± 12	
Sudan Gezira	All occasions (11)	67 ± 3	85 ± 2
	Maximum a	70.9 ± 0.1	
	b		99 ± 7
	Minimum a	65 ± 1	
b		73 ± 4	

A correlation analysis for the 46 sets of observations showed that values of the slope, b , were not significantly different between sets but differences in the intercept, a , were highly significant. Table 1 shows the value of b for the pooled regression and the mean value of a . Values of $\text{usec}Z$ ranged from 0.2cm to 15.4cm. The table also shows the ranges found for a and b .

Five of the 46 scans were made when strong inversions were likely, i.e. wind speeds less than 2m s^{-1} after dark or when the sun was close to the horizon. Eleven scans were made when strong lapse rates were likely, i.e. wind speed less than 2m s^{-1} , strong insolation, and dry ground. Table 1 shows mean values of a and b for the lapse and inversion situations along with the ranges of a and b measured for each situation.

Table 1 also contains ranges and mean values of a and b for 11 scans in the Sudan. Values of u were determined from the Khartoum radiosonde ascent and $\text{usec}Z$ ranged from 2.0 to 15.4cm. A small correction was made for the greater humidity of air in the irrigated Gezira. The histograms in Fig. 2 show the ranges of a and b found in all scans in England and in the Sudan.

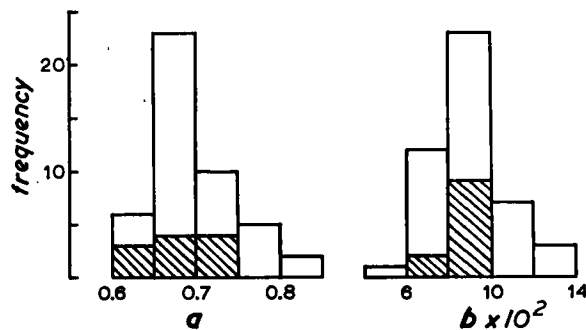


Figure 2. Frequency distribution of a and b in England (46 determinations) and in the Sudan (hatched sections - 11 determinations).

5. INTERPRETATION OF RESULTS

Part of the variation in a and b can be attributed to the effect of temperature gradients either in the horizontal or in the vertical. The measured value of T_a may not represent the average temperature at the surface because of local microclimatic or topographic effects. Below an inversion, T_a will be less than the mean temperature of the radiating column. As the apparent emissivity will be affected more at small zenith angles than at large zenith angles, b should then be smaller than average and a should be larger than average. Table 1 confirms this expectation and indicates the scale of changes in a and b . From the complementary argument, the value of b should be larger than average during a strong lapse and this was also found.

Robinson (1947, 1950) pointed out that $\epsilon(Z)$ may also be affected by the radiative behaviour of particulate material, e.g. pollution or haze near the ground or water droplets in high cloud. He distinguished two cases which may be described in terms of a and b . Values of b would be anomalously large as a result of pollution but not as a result of high cloud. Values of a would be anomalously large when high cloud was present, but b would be unaffected. The histograms in Fig. 2 show that there was a large range of b in England suggesting that pollution or haze often influenced the apparent emissivity. In the Sudan, local sources of man-made pollution are trivial but there is much natural dust well distributed through the atmosphere. Although fewer measurements were made, it is significant that a and b appear to be less variable than in England.

Atmospheric emission depends on the distribution of temperature and water vapour and may depend on radiative properties of aerosols. Values of a and of b showed no clear trends when plotted against T_a or u , but the range of a was larger at low temperatures than at high temperatures. In general a and b were also uncorrelated with turbidity. Turbidity is a measure of the attenuation of short-wave radiation by dust throughout the atmosphere whereas most of the long-wave radiation measured at the surface originates in the lower troposphere. As a significant exception to the general rule, Fig. 3 shows the variation of a turbidity coefficient τ_a and of a and b during a period of very cold weather in which the variation of τ_a was due to local pollution sources (Unsworth and Monteith 1972). The variation of a and b supports this conclusion and indicates that the aerosol influenced long-wave radiation as well as short wave. The three days bear some resemblance to a single case study by Robinson (1950) from which he drew similar conclusions.

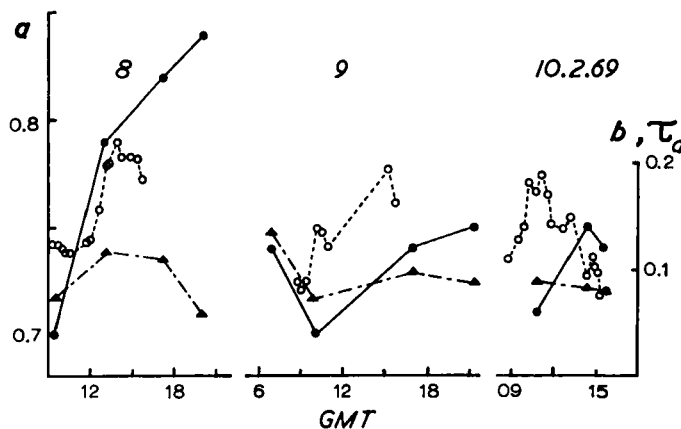


Figure 3. Diurnal changes of a , b and short wave turbidity parameter τ_a for period of northerly winds and local pollution at Sutton Bonington (Unsworth and Monteith 1972).

● a ; ▲ b ; ○ τ_a

A scan which produced one of the largest values for a ($a = 0.770$, $b = 0.090$, $u = 1.6\text{cm}$) was taken at 2112 GMT on 1 July 1968. Heavy falls of Saharan dust occurred in Southern England on the previous night (Stevenson 1969). The large value of atmospheric emissivity on this occasion may well have been due to the radiative effects of the dust layer at high level.

6. COMPARISON WITH PREVIOUS WORK

Dines and Dines (1927) made a long series of observations of the angular distribution of atmospheric radiation from clear and from overcast skies at Benson, Oxford, over the period 1922–1926. They quoted monthly mean values for the radiation from 6 sectors of the sky based on 515 scans of clear skies. Corresponding values of a and b can be derived using monthly mean values of air temperature and vapour pressure at the surface from Oxford and applying the empirical relationship

$$\log u = 0.295\sqrt{e} - 0.803 \quad . \quad . \quad . \quad (2)$$

to calculate u (cm H₂O) from vapour pressure (mb) (Monteith 1961). Table 2 shows the values of a and b derived in this way using the three-month mean values of emissivity and u . The average values of a are lower than at Sutton Bonington suggesting that occasions of very low emissivity may have been commoner fifty years ago because there was less haze, particulate pollution or high cloud. Robinson (1947, 1950) published results of 38 scans at Kew for which the mean values of a and b are shown in Table 2. The table also

TABLE 2. VALUES OF THE COEFFICIENTS a AND b FROM PREVIOUS WORK

Site		$a \times 10^2$	$b \times 10^3$
Benson, England	December–February (Dines and Dines 1927)	65.5	96
	March–May	66.0	87
	June–August	65.4	95
	September–November	64.1	96
	Annual average	65.4	92
Blue Hill, USA	(Brooks 1941)	72	87
Kew, England	All occasions (38) (Robinson 1947, 1950)	70	85
	Minimum emissivity (10)	69	70
	Maximum a	74	
	Minimum a	68	
Theoretical values for isothermal emissivity	(Yamamoto 1952)	73.2	71.7

shows corresponding mean values for ten occasions of ‘ minimum emissivity ’ and the total range of a . Robinson’s mean values for all scans are similar to ours although his range of a was much less. The difference between the very low value of b for his minimum emissivity scans and the average values of b implies that additional sources of radiation are often present near the ground.

An empirical curve derived by Brooks (1941) from observations at Blue Hill Observatory, Harvard, during a single week of anticyclonic weather corresponds to $a = 0.72$ and

$b = 0.087$. Reviewing all the measurements, values of a range from 0.60 to 0.83 and values of b range from 0.068 to 0.14. Theoretical values for the isothermal emissivity of water vapour (Yamamoto 1952) are $a = 0.73$, $b = 0.072$. Values of a in Tables 1 and 2 are consistent with isothermal atmospheres but values of b are frequently much larger, suggesting an additional source of radiation low in the atmosphere. The mean values $a = 0.70$ and $b = 0.090$ derived from 46 scans at Sutton Bonington appear to be appropriate for most sites.

Compared with Eq. (1), other types of formulae (Linke 1931; Falckenberg 1954; Bennett, Bennett and Nagel 1960; Kondratyev 1969) are more difficult to handle and predict values of emissivity which are less consistent with the measurements reported here.

Eq. (1) clearly does not represent the variation of emissivity at very large angles, since it fails to predict that emissivity tends to unity as Z approaches 90° . When $a = 0.70$ and $b = 0.090$, $\epsilon(Z) = 1$ when $u \sec Z = 28$, i.e. $Z = 89.5^\circ$ when $u = 2\text{mm}$ and $Z = 86^\circ$ when $u = 20\text{mm}$. This discrepancy will rarely be significant.

7. HEMISPHERIC RADIATION

(a) Representative angle

According to Eq. (1), the equivalent flux density at a zenith angle of Z is $\mathcal{L}(Z) = [a + b \ln(u \sec Z)] \sigma T_a^4$. The real flux density on a horizontal surface exposed to the whole sky can be calculated from the integral

$$L_d = \int_0^{2\pi} d\phi \int_0^{\pi/2} (\mathcal{L}(Z)/\pi) \cos Z \sin Z dZ \quad \cdot \quad \cdot \quad \cdot \quad (3)$$

$$= \sigma T_a^4 [a + b(0.5 + \ln u)] \quad \cdot \quad \cdot \quad \cdot \quad (4)$$

It follows that the real flux density on a horizontal surface must be equal to the equivalent flux density at an angle Z given by $\ln \sec Z = 0.5$ i.e. $L_d = \mathcal{L}(Z)$ for $Z = 52.5^\circ$. This result was demonstrated experimentally by Dines and Dines (1927), Robinson (1947) and others but it is not immediately obvious from their analyses that the representative angle of 52.5° is independent of a and b . The necessary and sufficient condition which establishes a representative angle of 52.5° is a linear relation between $\mathcal{L}(Z)$ and $\ln \sec Z$.

(b) Integrated results

Values of L_d were calculated from all scans from Eq. (3) by dividing the sky into seven zones corresponding to the zenith angles of observation and assuming no variation of $\mathcal{L}(Z)$ with azimuth within each zone.

Eq. (4) implies that the apparent atmospheric emissivity ϵ_a defined as $L_d/\sigma T_a^4$ should be a linear function of $\ln u$ but, in fact, our values of ϵ_a were poorly correlated with $\ln u$ ($r = 0.71$). When u is fixed, i.e. in a specific air mass, $\epsilon(Z)$ is strongly correlated with the logarithm of the optical pathlength $u \sec Z$. In different air masses, however, the integral of $\epsilon(Z)$, given by ϵ_a , is poorly correlated with $\ln u$, presumably because of the variation of a and b (illustrated in Table 1 and Fig. 2) and because T_a is not necessarily an appropriate reference temperature. The emissivity, ϵ_a , cannot be reliably predicted from Eq. (4) even by using mean values of a and b . A more accurate method for estimating L_d exploits the close relationship between u and T_a which exists at many sites (Deacon 1970).

The variation of L_d with σT_a^4 is shown in Fig. 4 which includes Sudanese as well as Sutton Bonington observations. All the results are well represented by a straight line, i.e.

$$L_d = c + d\sigma T_a^4 \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (5)$$

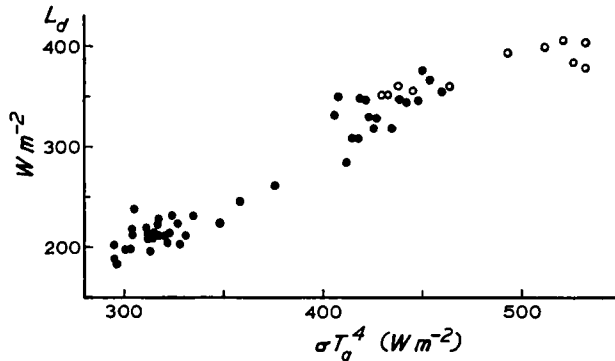


Figure 4. Relation between hemispherical incoming long-wave radiation L_d and black-body radiation at screen temperature

● Sutton Bonington ○ Sudan Gezira

The corresponding emissivity may be written as

$$\varepsilon_a = d + c/(\sigma T_a^4) \quad (5a)$$

For the English measurements alone, c was $-119 \pm 16 \text{ W m}^{-2}$ and d was 1.06 ± 0.04 . The uncertainty in a single estimate of L_d is about $\pm 30 \text{ W m}^{-2}$.

Swinbank (1963) analysed data from Aspendale and from Kerang and found $c = -171 \text{ W m}^{-2}$ and $d = 1.20$. He also analysed the average results of Dines and Dines (1927) and found $c = -113 \text{ W m}^{-2}$ and $d = 1.05$, agreeing closely with the figures for Sutton Bonington. It is significant that Swinbank's measurements were much less scattered ($\pm 5 \text{ W m}^{-2}$) than measurements in England, presumably because they were taken (i) in regions where aerosol is less variable and (ii) at night when the atmosphere was consistently stable. The relatively large values of c and d found at Aspendale and Kerang are consistent with inversion conditions prevailing at night.

For a restricted range of temperature, e.g. 268 to 298K, appropriate for mean values of T_a , black-body radiation and atmospheric radiation may both be treated as linear functions of temperature (Monteith 1973). If T_0 is chosen as a base temperature near the mid-point of the range, it can be shown that

$$\sigma T^4 \simeq e + f(T - T_0) \quad (6)$$

where $e = \sigma T_0^4$ and $f = 4\sigma T_0^3$. If $T_0 = 283\text{K}$, $e = 365 \text{ W m}^{-2}$ and $f = 5.2 \text{ W m}^{-2} \text{ K}^{-1}$. The first order error in this approximation is $6\sigma T_0^2(T - T_0)^2$ which is 1.7% of black-body radiation or 6 W m^{-2} for $T - T_0 = \pm 15\text{K}$.

By substituting Eq. (6) into Eq. (5), the corresponding relation for incoming long-wave radiation becomes

$$L_d = (c + de) + df(T_a - T_0)$$

$$\text{or} \quad L_d = 268 + 5.5(T_a - 283) \quad (7)$$

when $T_0 = 283\text{K}$. When temperature is expressed in $^{\circ}\text{C}$, the relation becomes even simpler, i.e.

$$L_d = 213 + 5.5T_a \quad (7a)$$

which at 10°C is identical to a similar formula derived from Swinbank's measurements (Monteith 1973). Eqs. (7) and (7a) contain two sources of error: the linear approximation to black-body radiation ($\pm 6 \text{ W m}^{-2}$ for σT_a^4); and uncertainty in L_d estimated from Eq. (5)

($\pm 30 \text{ W m}^{-2}$). The second error is much larger than the first and Eqs. (7) and (7a) therefore fitted the Sutton Bonington measurements as well as Eq. (5).

8. EMISSIVITY OF CLOUDY SKY

Beneath a cloudy sky, the flux of long-wave radiation received at the ground has two components: radiation emitted by water vapour and carbon dioxide below the cloud base; and radiation emitted by water droplets forming the base. As Swinbank (1963) emphasized, most of the atmospheric radiation reaching the earth's surface originates within a few hundred metres of the ground so when cloud has a base height exceeding 1000m, say, the gaseous component of the downward flux can be treated as if the sky were cloudless. Eq. (5a) may therefore be used to estimate this component, as $\epsilon_a \sigma T_a^4$.

If $(1 - \epsilon_a)$ is regarded as the apparent transmissivity of the atmosphere between the ground and cloud base, the second component of radiation from an overcast sky will be $(1 - \epsilon_a)\sigma T_c^4$ where T_c is the radiative temperature of the cloud base. If $T_a - T_c = \delta T$ and δT is small in comparison with T_a (K), the difference in black-body radiation at screen and cloud base temperatures may be approximated by $4\sigma T_a^3 \delta T$ so that the total incoming radiation

$$L_d = \epsilon_a \sigma T_a^4 + (1 - \epsilon_a)\sigma T_c^4 = \sigma T_a^4 [1 - (1 - \epsilon_a)4\delta T/T_a] \quad (8)$$

Distinguishing the apparent emissivity of a cloudless sky by $\epsilon_a(0)$ and of an overcast sky of $\epsilon_a(1)$, it follows that

$$[1 - \epsilon_a(1)]/[1 - \epsilon_a(0)] = 4\delta T/T_a \quad (9)$$

The argument used to derive this simple formula can also be applied to radiation measured at any zenith angle. A climatological mean value of δT can therefore be derived from measurements of $\epsilon(Z, 1)$ and $\epsilon(Z, 0)$ at a series of zenith angles, provided mean temperature and mean vapour pressure below the cloud layer are independent of cloud cover. Dines and Dines (1927) tabulated mean values of $\epsilon(Z, 0)$ and $\epsilon(Z, 1)$ for 6 zenith angles in each month of the year. For each angle, we calculated mean values of the two emissi-

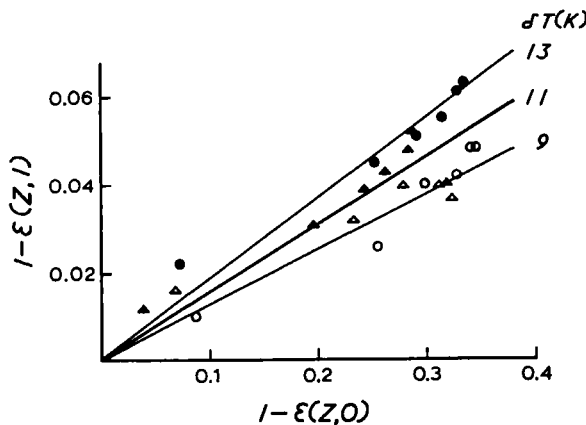


Figure 5. Relation between $1 - \epsilon(Z, 1)$ for overcast skies and $1 - \epsilon(Z, 0)$ for cloudless skies

- December-February
- March-May
- ▲ June-August
- △ September-November

TABLE 3. VALUES OF T_a AND δT IN EQ. (9) DERIVED FROM MEASUREMENTS BY DINES AND DINES (1927)

Period	Dec.-Feb.	Mar.-May	June-Aug.	Sep.-Nov.
T_a (K)	279	284	290	285
δT (K)	9.0	13.2	12.4	9.6

vities for three-month periods and Fig. 5 shows that the mean values of $1 - \varepsilon(Z, 1)$ were linearly related to $1 - \varepsilon(Z, 0)$ i.e. the angular distribution of radiation from overcast and from clear skies is experimentally indistinguishable. The straight lines drawn on the same graph were calculated from Eq. (9) for an appropriate mean value of $T_a = 284\text{K}$. Table 3 shows the values of T_a given by Dines and Dines and the values of δT calculated from mean value of $1 - \varepsilon(Z, 1)$ divided by the mean value of $1 - \varepsilon(Z, 0)$. The mean value of δT for the year is 11.0K and assuming that the temperature gradient in the lower atmosphere is close to the adiabatic lapse, the corresponding mean height of the cloud base is 1100m . This is a physically sensible figure consistent with the initial assumption about the origin of atmospheric radiation. From analysis of measurements with a net radiometer at Kew, Monteith (1961) derived a mean value of δT equivalent to $1.7/(1 - \varepsilon_a)$ or 6.8K when ε_a has a mean value of 0.75 . The new determination of δT is more accurate and is more rationally derived.

The annual variation in δT may be related to stronger surface heating in summer and to annual changes in the mean height of the cloud base. For calculating a mean monthly value of $\varepsilon_a(1)$, however, the variation is small enough to neglect because the cloud component of downward radiation is always much less than the gaseous components. With $T_a = 284\text{K}$, $\delta T = 11\text{K}$, the quantity $4\delta T/T_a$ can be taken as 0.155 throughout the year.

The effective emissivity of the atmosphere for an average overcast sky can now be written as

$$\begin{aligned}\varepsilon_a(1) &= 1 - 0.16[1 - \varepsilon_a(0)] \\ &= 0.16\varepsilon_a(0) + 0.84\end{aligned}\quad (10)$$

As a simple extension, the apparent emissivity when a fraction c of the sky is covered by cloud will be

$$\varepsilon_a(c) = (1 - 0.84c)\varepsilon_a(0) + 0.84c\quad (11)$$

A similar equation relates $\varepsilon(Z, c)$ to $\varepsilon(Z, 0)$. Eq. (11) may be used to determine average values of incoming radiation for periods of several weeks. On any specific occasion, however, the measured value of the apparent emissivity may be greater or less than $\varepsilon_a(c)$ depending on whether the appropriate value of δT is smaller or larger than 11K . In particular, $\varepsilon_a(c)$ may be close to $\varepsilon_a(0)$ when the sky is overcast by cirrostratus or some other form of high cloud. The flux of downward radiation below different types of cloud can be estimated from formulae reviewed by Kondratyev (1969).

The value of c used in Eq. (11) is an effective value of cloudiness for long-wave radiation, i.e. it is the fraction of sky obscured by cloud weighted by the angular distribution of the incoming flux. Because gaps between clouds appear to decrease from the zenith to the horizon, the appropriate value of c may be somewhat greater than the value which an observer would estimate from the pattern of cloud above him. However, it is encouraging to find that Eq. (11) with a theoretical value of c is consistent with a formula for net long-wave exchange developed by Ångström (1919), using measured values of cloudiness.

For the special case in which the radiative temperature of the ground is T_a , the ratio of net long-wave radiation under cloudy skies to the corresponding net flux under clear skies is $(1 - \varepsilon_a(c))/(1 - \varepsilon_a(0)) = 1 - 0.84c$ from Eq. (11). The relation given by Ångström is $1 - 0.9c$.

9. CONCLUSION

In principle, the flux density of atmospheric radiation received at the earth's surface can be measured in two ways: by determining the amount of radiant energy intercepted by a horizontal surface facing the whole hemisphere of the sky; or by determining the equivalent flux density of radiation as a function of zenith angle using a surface which receives radiation from a small fraction of a solid angle. Because the equivalent flux density is a strictly linear function of $\ln \sec Z$, these two methods are readily interchangeable. The flux density of radiation on a horizontal surface is uniquely determined by a single measurement of the equivalent flux density at $Z = 52.5^\circ$. Conversely, the distribution of flux with zenith angle, $\mathcal{L}(Z)$, can be calculated when the flux density on a horizontal surface, L_d , is known, using the relation

$$\mathcal{L}(Z) = L_d - b(0.5 - \ln \sec Z) \sigma T_a^4$$

derived from Eqs. (1) and (4).

For climatological studies, the mean radiation from clear skies and its angular distribution can be estimated from a mean screen temperature and the radiation from cloudy skies can be estimated from a simple extension of this method incorporating a factor for fractional cloudiness.

Knowledge of the angular distribution of atmospheric radiation on clear or cloudy nights allows the radiation intercepted by solid bodies and sloping surfaces to be calculated and the relevant formulae are presented in a companion paper.

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