JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 85, NO. C9, PAGES 4919–4922, SEPTEMBER 20, 1980

An Arc Tangent Model of Irradiance in the Sea

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The solar energy flux as a function of depth (the irradiance profile) in the ocean is an important function. It influences the dynamics of the mixed layer via the heat budget as well as the biology of the euphotic zone. The following three-parameter model can take into account the very rapid decrease near the surface due to absorption of long-wavelength radiation by water as well as the eventual exponential decrease at great depths. $E(z) = E(0)e^{-K_1 z}(1 - K_2 \arctan K_3 z)$, where $E(z)$ is the irradiance at depth z and K_1, K_2, and K_3 are constants that depend on the optical properties of the water. The constants are determined for Jerlov's water types. The constants can readily be calculated from any irradiance profile.

INTRODUCTION

Solar radiation in the region of 300–2500 nm is a source of energy for the upper ocean. This energy influences the dynamics of the ocean and is the ultimate power source of the entire oceanic ecosystem. Much of the light energy (with wavelengths of 800–2500 nm) is absorbed in the first meter. The profile of energy flux (irradiance) in the first meter is governed primarily by the absorption properties of pure water. At greater depths and for the shorter wavelengths, scattering and absorption by suspended particles becomes the dominant process. The spectrum of light continuously changes as a function of depth, since each wavelength has a different rate of attenuation. Heat budget and ecosystem modeling both require a simple mathematical model of irradiance as a function of depth. This paper describes a three-parameter model that can take account of the rapid decrease of solar energy near the sea surface as well as the eventual near-asymptotic exponential decrease at great depths.

In a homogeneous ocean, irradiance penetration for a given wavelength λ is given by

$$E_\lambda(\lambda, z) = E_\lambda(\lambda, 0) \int_0^z e^{-K(\lambda, z)}\, dz \tag{1}$$

where $E_\lambda(\lambda, z)$ is the irradiance per unit wavelength for wavelength λ at depth z; $K(\lambda, z)$ is the irradiance attenuation coefficient for wavelength λ at depth z. The irradiance for all wavelengths, $E(z)$, is then governed by

$$E(z) = \int_{\lambda=0}^{\infty} E_\lambda(\lambda, z)\, d\lambda = \int_{z=0}^{z}\int_{\lambda=0}^{\infty} E_\lambda(\lambda, 0) e^{-K(\lambda, z)z}\, d\lambda\, dz \tag{2}$$

or

$$E(z) = E(0) \int_{z=0}^{z}\int_{\lambda=0}^{\infty} \frac{E_\lambda(\lambda, 0)}{E(0)}\, e^{-K(\lambda, z)z}\, d\lambda\, dz$$

We can thus define

$$g(z) = \int_{z=0}^{z}\int_{\lambda=0}^{\infty} \frac{E_\lambda(\lambda, 0)}{E(0)}\, e^{-K(\lambda, z)z}\, d\lambda\, dz \tag{3}$$

as the function which determines the penetration of solar energy into the ocean. $E_\lambda(\lambda, 0)/E(0)$ is the normalized spectral distribution of solar energy just beneath the ocean surface. The functional form of $g(z)$ has considerable bearing upon the absorption of energy as a function of depth, but it is not easy to model, since the attenuation coefficients $K(\lambda, z)$ decrease sharply with decreasing wavelength down to a minimum for $\lambda \approx 475$ nm for the clearest ocean water. For more turbid waters the wavelength of maximum transmission may shift within the spectrum. All light with $\lambda \geq 800$ nm is attenuated mostly in the first meter of the sea surface [*Jerlov*, 1976]. A simple exponential model as is commonly used thus underestimates the attenuation near the surface. At much greater depths the irradiance becomes nearly monochromatic, and the exponential model is valid. In that case we can write [*Jerlov*, 1976; *Tyler and Preisendorfer*, 1962]

$$E(z) = E(z') \exp\,[(-K_1(z - z')] \tag{4}$$

where $E(z)$ is the received irradiance at depth z (in W/m^2); $E(z')$ is the irradiance at the reference depth z' (in W/m^2); and K_1 is the irradiance attenuation coefficient (in m^{-1}).

The value of K_1 obtained from this expression is usually obtained from the straight-line portion of the curve of log I versus z [e.g., *Smith et al.*, 1973].

Recently, the irradiance-depth curve has been modeled by means of a bimodal exponential expression [*Kraus*, 1972; *Paulson and Simpson*, 1977]. The expression used,

$$E(z) = E(0)[R \cdot \exp(-C_1 z) + (1 - R) \exp(-C_2 z)]$$

employs one exponential decay term for an upper layer where absorption of long-wave light is important and a second exponential decay term for greater depths where the light which is attenuated is in the blue-green region. The greatest errors in this expression occur in the surface layer, especially above 10 m, since the light there is attenuated faster than an exponential.

In the model developed herein, which is also a three-parameter model, the curve of the log of irradiance versus depth is assumed to be a product of the original irradiance decay expression (1) and a form of the arc tangent curve. This permits the near-surface irradiance to be attenuated very rapidly. By using the surface irradiance values and values of the irradiance at three other depths (two deep and one very shallow) the complete profile of irradiance with depth can be obtained. The four irradiance values permit one to calculate the three parameters which define the irradiance attenuation properties of a water mass. These parameters have been determined for all of the water types as classified by *Jerlov* [1976]. We are considering the total irradiance field (300–2500 nm), as does *Jerlov* [1976]. In addition, determinations of these parameters have been made for irradiance profiles from the eastern tropical Pacific [*Spinrad et al.*, 1979], the Atlantic Ocean near the

Paper number 9C1277.
0148-0227/79/009C-1277$01 00

TABLE 1. Calculated Values of K_1, K_2, K_3, and K_2 arctan K_3 for Each of *Jerlov's* [1976] Water Types

Water Type	K_1, m^{-1}	K_2	K_3, m^{-1}	K_2 arctan K_3
I	0.0440	0.3963	4.4547	0.535
IA	0.0490	0.3981	4.4236	0.537
IB	0.0574	0.4103	4.0725	0.546
II	0.0670	0.4158	3.9865	0.551
III	0.1250	0.4234	3.7062	0.554
1	0.1360	0.4500	3.3772	0.577
3	0.2231	0.4495	3.7049	0.588
5	0.3541	0.4626	3.6806	0.604
7	0.5028	0.4789	3.7150	0.626
9	0.5913	0.5247	3.6026	0.682
Eastern tropical Pacific	0.0637	0.6280	0.4896	0.286
Eastern tropical Atlantic	0.0691	0.3650	0.6580	0.186
Lake Tahoe	0.0680	0.251	0.622	0.140

Congo River [*Zaneveld et al.,* 1979], and Lake Tahoe [after *Smith et al.,* 1973].

The Model

In homogeneous water we may remove the depth dependence of K, and therefore

$$g(z) = \int_{z=0}^{z} \int_{\lambda=0}^{\infty} \frac{E_\lambda(\lambda, 0)}{E(0)} e^{-K(\lambda)z} \, d\lambda \, dz$$

$$= -\frac{1}{K} \int_{\lambda=0}^{\infty} \frac{E_\lambda(\lambda, 0)}{E(0)} [e^{-K(\lambda)z} - 1] \, d\lambda$$

If the wavelength of minimum attenuation is given by λ_m, then

$$g(z) = \frac{-e^{-K(\lambda m)z}}{K} \int_{\lambda=0}^{\infty} \frac{E_\lambda(\lambda, 0)}{E(0)} [e^{-[K(\lambda)-K(\lambda m)]z} - 1] \, d\lambda \quad (5)$$

or

$$g(z) = e^{-K(\lambda_m)z} f(z) \quad (6)$$

Now $f(z)$ must approach a constant value at a large depth at which the irradiance becomes nearly monochromatic. Also, $f(z)$ must decrease much faster than a simple exponential. Such a function can be modeled by means of an arc tangent function, retaining a three-parameter model but permitting the energy to be attenuated rapidly in the surface layer. We thus set

$$f(z) = 1 - K_2 \arctan K_3 z \quad (7)$$

so that at $z = 0$, $f(z) = 1$, and at $z = \infty$, $f(z) = 1 - (K_2\pi/2)$. The rate at which $f(z)$ decreases from 1 to $[1 - (K_2\pi/2)]$ is determined by K_3. Our complete model then becomes

$$E(z)/E(0) = \exp(-K_1 z)(1 - K_2 \arctan K_3 z) \quad (8)$$

where we have set $k(\lambda_m) = K_1$.

The parameters K_1, K_2, and K_3 can be determined by using irradiance values at the surface (z_0), at a shallow depth (z_1), and at two large depths (z_2, z_3). The value of K_1 is obtained by using only the exponential factor of (8) with the irradiances at the two great depths. That is,

$$E(z_3)/E(z_2) = \exp[-K_1(z_3 - z_2)] \quad (9)$$

or

$$K_1 = \frac{-[\ln E(z_3) - \ln E(z_2)]}{(z_3 - z_2)} \quad (10)$$

K_2 is then calculated by assuming that at the great depth (z_3), arctan K_3z_3 can be approximated by $\pi/2$ so that

$$K_2 = \frac{2}{\pi}\left[1 - \frac{E(z_3)}{E(z_0)} \exp(K_1 z_3)\right] \quad (11)$$

Then to solve for K_3, K_1 and K_2 are substituted into (8), using the shallow irradiance value:

$$K_3 = \left\{\tan\left[\frac{1}{K_2}\left(1 - \frac{E(z_1)}{E(z_0)}\right) \exp(K_1 z_1)\right]\right\} z_1^{-1} \quad (12)$$

In the solution of (11) the assumption that arctan $K_3z_3 = \pi/2$ is true within 1% only when $K_3z_3 \geq 64.5$. In the case of coastal water measurements it may be impossible to obtain

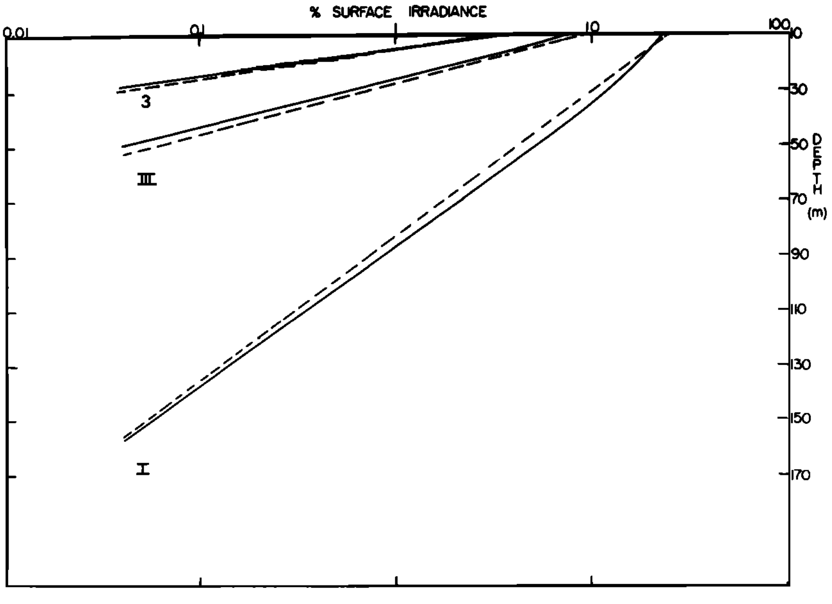

Fig. 1. Depth profiles of downward irradiance (below 10 m) in percent of surface irradiance for water types I, III, and 3. Solid curves are from *Jerlov* [1976]. Dashed curves are as computed from the arc tangent model.

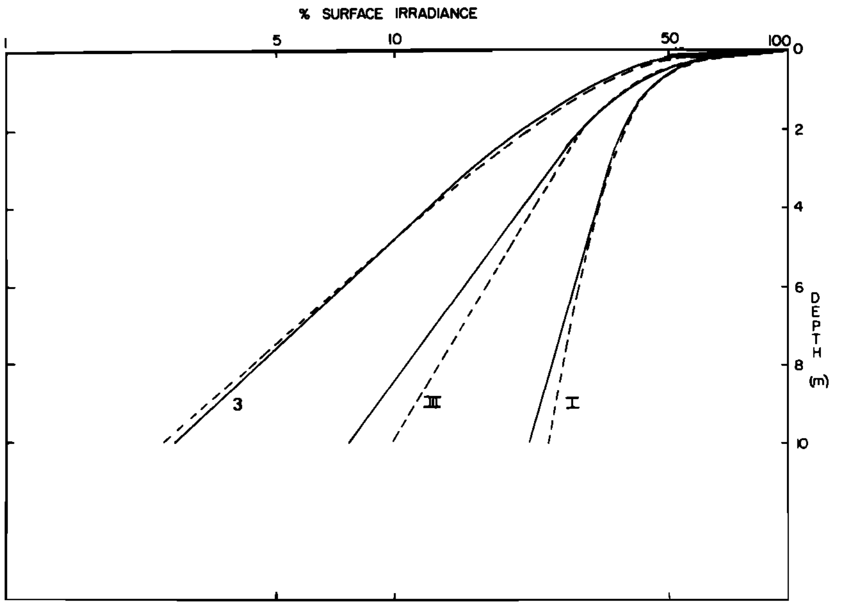

Fig. 2. Depth profiles of near-surface downward irradiance in percent of surface irradiance for water types I, III, and 3. Solid curves are from *Jerlov* [1976]. Dashed curves are as computed from the arc tangent model.

light levels adequate to measure at depths which satisfy this approximation. Consequently, an iterative calculation is carried out in which (11) is first solved for K_2, assuming $K_3 z_3 = \pi/2$. Equation (12) is then solved as before, and the solution for K_3 (along with the calculated value of K_1) is used in (8) to resolve the new value of K_2. This iterative process is used until the variation in K_2 with the previous iteration is less than 5%.

Results and Observations

The calculated values of K_1, K_2, K_3, and K_2 arctan K_3 are shown in Table 1 for each of *Jerlov*'s [1976] water types and for stations in the eastern tropical Pacific [*Spinrad et al.*, 1979], the eastern Atlantic [*Zaneveld et al.*, 1979], and Lake Tahoe [see *Smith et al.*, 1973].

Figure 1 shows the levels of total irradiance versus depth below 10 m for several of *Jerlov*'s [1976] water types and as computed from the arc tangent model. Similar results for the upper 10 m of ocean surface are shown in Figure 2. The arc tangent model fits the actual irradiance profile within 5% of the surface irradiance value for all depths. The model also approximates both coastal and oceanic irradiance profiles equally well.

The value of K_1 indicates the exponential slope of the irradiance profile with depth in the deep ocean, where the irradiance is nearly monochromatic. Generally, the wavelength of maximum irradiance transmission increases with increasing values of K_1. Clean ocean water (such as type I) with a value of K_1 from 0.03 to 0.05 has a peak transmission at a wavelength of approximately 475 nm; water type 9 has a peak transmission at 575 nm [*Jerlov*, 1976].

At great depths the value of arctan $K_3 z$ approaches $\pi/2$, which means that the deep irradiance is described as

$$\frac{E(z)}{E(0)} \cong \exp(-K_1 z)\left(1 - K_2 \frac{\pi}{2}\right) \tag{13}$$

As K_2 increases, the amount of irradiance at a given depth decreases. Since the irradiance at great depths is nearly monochromatic (as determined by K_1), it is seen that K_2 indicates the relative amounts of monochromatic light available for each water type. Generally, after the surface attenuation of long-wave irradiance in the ocean there is more blue light available than green. Since clean ocean water has its maximum transmission in the blue light region, the value of K_2 should be lower for clean ocean water than for the more turbid coastal waters.

The magnitude of K_2 arctan $K_3 z$ describes the strong near-surface attenuation of long-wavelength irradiance. The values as indicated for *Jerlov*'s [1976] water types are higher than those measured in the Pacific, the Atlantic, and Lake Tahoe. This is due to the low response of the irradiance meters used in the latter examples for long-wave irradiance (infrared). The meter used in the Atlantic and Pacific does not detect any irradiance at wavelengths greater than 1000 nm. The maximum wavelength of detection for the meter used in Lake Tahoe is 750 nm. The determinations of K_3 for *Jerlov*'s [1976] water types are for measurements of irradiance up to 2500 nm. The irradiance between 1000 and 2500 nm is attenuated within the first meter of seawater, so consequently, the values of K_3 are much higher for *Jerlov*'s [1976] water types.

The importance of this arc tangent model lies in its accuracy at shallow depths. There is a strong attenuation of long-wave solar radiation in the upper 10 m of the sea surface. Accurate modeling of the irradiance profile in this layer is important for oceanic heat budget studies and photosynthesis determinations.

Acknowledgments. The authors wish to thank Gail Henwood for typing the manuscript. Support from the Office of Naval Research through contract N00014-76-C-0067 under project NR 083-102 is gratefully acknowledged.

References

Jerlov, N. G., *Marine Optics*, 231 pp., Elsevier, New York, 1976.
Kraus, E. B., *Atmosphere-Ocean Interaction*, 275 pp., Oxford University Press, New York, 1972.
Paulson, C. A., and J. J. Simpson, Irradiance measurements in the upper ocean, *J. Phys. Oceanogr.*, *7*, 953–956, 1977.
Smith, R. C., J. E. Tyler, and C. R. Goldman, Optical properties and

color of Lake Tahoe and Crater Lake, *Limnol. Oceanogr.*, *18,* 189–199, 1973.
Spinrad, R. W., J. R. V. Zaneveld, and H. Pak, Irradiance and beam transmittance measurements off the west coast of the Americas, *J. Geophys. Res.*, *84,* 355–358, 1979.
Tyler, J. E., and R. W. Preisendorfer, Transmission of energy within the sea, in *The Sea,* vol. 1, edited by M. N. Hill, pp. 397–451, Interscience, New York, 1962.
Zaneveld, J. R. V., R. W. Spinrad, and D. W. Menzies, Hydrographic and optical measurements in the Congo River and Angola Basin during May 1978, *Data Rep. 79-3,* Sch. of Oceanogr., Oreg. State Univ., Corvallis, 1979.

(Received May 7, 1979;
revised August 8, 1979;
accepted August 22, 1979.)