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Substitutability among undesirable outputs

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In recent years, economists have started to move beyond calculating regulatory effects on a pollutant-by-pollutant basis since their interaction is important. In this study, we take up this issue. To allow for joint production of multiple pollutants and marketable output, we specify our technology using a directional distance function. This allows us to treat pollutants as joint outputs, yet accounts for their 'undesirability'. We estimate the distance function for a sample of coal-fired electric power plants from 1985 to 1998, which includes the first 4 years of Phase I of the Clean Air Act Amendments of 1990. We focus on the interaction between SO_2 and NO_x, as they became more highly regulated and estimate shadow prices of the pollutants and the Morishima elasticity of transformation between two pollutants, NO_x and SO_2, as well as with respect to the desirable output, kilowatt-hours of electricity. As expected, we find that power plants increase NO_x emissions as they decrease SO_2, i.e. they are substitutes.

I. Introduction

Technologies like coal-fired electric utilities and steel plants produce, in general, more than one pollutant or bad (undesirable) output. Early efforts to evaluate regulatory impacts proceeded on a pollutant-by-pollutant basis. In recent years, however, economists have moved beyond calculating the effect of single pollutants in recognition of the fact that the interaction among pollutants is important, especially from a policy perspective.[1] For instance, increasingly tight regulations on SO_2 (sulphur dioxide) emissions might cause electric power plants to substitute (i.e. assign fewer inputs to abate) a less regulated pollutant, such as NO_x (nitrogen oxides), or reduce the production of electricity.

Researchers have used a variety of strategies to calculate whether reducing the production of one bad output may result in the increase of another bad output (Greenstone, 2003; Gamper-Rabindran, 2006). As an example, Burtraw *et al.* (2003) calculate the reduction in undesirable or bad output

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[1] Montero (2001) and Schmieman *et al.* (2002) developed theoretical models of joint abatement.

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production (e.g. SO_2 and NO_x emissions from electric power plants) as an ancillary effect of regulations to reduce production of another bad output, i.e. carbon emissions. In another approach, Considine and Larson (2006) treat emissions as an input, rather than an undesirable output, in their study of the effects of sulphur dioxide permit trading and banking. They estimate Morishima elasticities of substitution and find – not surprisingly – that emissions and high sulphur coal are complements, but emissions and low sulphur coal are substitutes in the production of electricity. In our study, we treat pollutants as outputs produced as byproducts of electricity generation and estimate the elasticity of substitution among these outputs.

In this study, we apply the model by Färe *et al.* (2005) and estimate the Morishima elasticity of transformation among pollutants and between pollutants and electricity. We apply the model to coal-fired electric power plants over the period 1985 to 1998. During this period, SO_2 emissions came under increasingly stringent regulations with emission caps imposed and allowances granted in Phase I generating plants *via* the 1990 Amendments to the Clean Air Act. The new regulatory framework was fully implemented in 1995 and the trading of emissions allowed the power plants to seek the most profitable method for meeting the regulations. We have data on the emissions of two pollutants, SO_2 and NO_x, together with the desirable output of electric power plants, electricity (kWh).

The model we estimate is based on an environmental technology. Such technology has outputs that are weakly disposable together with good and bad outputs null-joint. We define these concepts in Section II and apply them to the directional output distance function. This function is defined on the environmental technology and parameterized by a quadratic function. We estimate the parameters *via* stochastic frontier methods and calculate the Morishima elasticities of transformation using the estimates; we find that SO_2 and NO_x are substitutes, which implies that gains in welfare from the reduction of SO_2 are being at least partially offset by increases in NO_x.[2]

The remainder of this study is organized in the following manner. Section II specifies a model of the joint production of good and bad outputs. Section III discusses the data and empirical results, and in Section IV, we summarize our study and discuss possible extensions.[3]

II. Environmental Directional Distance Function

A technology is characterized in terms of its output sets as[4]

$$P(x) = \{(y, b) \colon x \,\text{can produce}\, (y, b)\} \qquad (1.1)$$

where $x \in \Re_+^N$ denotes inputs, $y \in \Re_+^M$ a good (desirable) output vector and $b \in \Re_+^J$ the vector of pollutants (bad or undesirable outputs). We assume that $P(x)$ meets the standard axioms for a technology (Färe and Primont, 1995). In addition, we assume outputs (y, b) are weakly disposable (Shephard, 1970), and the good and bad outputs are null-joint (Shephard and Färe, 1974). Weak disposability allows firms to proportionally reduce all outputs, $(y, b) \in P(x) \Rightarrow (\theta y, \theta b) \in P(x)$ if $0 \leq \theta \leq 1$. We assume that null-jointness indicates bad outputs are by-products of the good outputs if $(y, b) \in P(x)$ and $b = 0$, then $y = 0$, i.e. no good output can be produced without some bad output production.

Here, we model the technology with the directional output distance function introduced by Chung *et al.* (1997).[5] Let $g \in \Re_+^M x \Re_+^J$ be a directional vector. This distance function is defined as

$$\vec{D}_o(x, y, b; g_y, g_b) = \max\{\beta : (y + \beta g_y, b - \beta g_b) \in P(x)\} \qquad (1.2)$$

where $g = (g_y, g_b)$.

Färe *et al.* (2005) derive relative shadow prices for bad output given good output prices using the duality between the revenue function and the directional output distance function. Let $p = (p_1, \ldots, p_M)$ represent a vector of desirable output prices and let $q = (q_1, \ldots, q_J)$ represent a vector of undesirable output prices which might be unobservable. Relative shadow prices are then found as

$$\frac{q_j}{p_m} = -\frac{\partial \vec{D}_o(x, y, b; g)/\partial b_j}{\partial \vec{D}_o(x, y, b; g)/\partial y_m} \qquad (1.3)$$

For any point interior to the production technology, the relative shadow price is evaluated on its corresponding boundary point. Note that we can

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[2] While it is possible to use these elasticities to calculate welfare gains associated with allowing electric power plants to substitute between SO_2 and NO_x emissions, we focus our analysis on modelling the joint production technology.

[3] All appendices and data are available from the corresponding author upon request.

[4] For a more in-depth discussion of this topic, see Färe *et al.* (2005).

[5] This function is a variation of Luenberger's (1992, 1995) shortage function.

solve for relative shadow prices without information on observed prices of undesirable outputs. If as is likely, p_m is known, it may be used to solve for the typically unobserved q_j.

We employ the Morishima output elasticity of transformation to investigate the ease with which good outputs can be substituted (reduced) when bad outputs are reduced and the ease with which bad outputs are substituted for each other in production (see Blackorby and Russell (1989) for a discussion on the Morishima elasticity of substitution). The Morishima elasticity of transformation gives the percent change in the shadow price ratio between two outputs due to a percent change in the output ratio and takes the general form

$$M_{jm} = \frac{\mathrm{d}\ln(q_j/p_m)}{\mathrm{d}\ln(y_m/b_j)} \quad \text{and} \quad M_{jj'} = \frac{\mathrm{d}\ln(q_j/q_{j'})}{\mathrm{d}\ln(b_{j'}/b_j)} \qquad (1.4)$$

Using the directional output distance function, the Morishima output elasticity of transformation between bad output j and good output m and between bad outputs j and j' are

$$M_{jm} = y_m^* \left\{ \frac{\vec{D}_{jm}}{\vec{D}_j} - \frac{\vec{D}_{mm}}{\vec{D}_m} \right\} \quad \text{and} \quad M_{jj'} = b_{j'}^* \left\{ \frac{\vec{D}_{jj'}}{\vec{D}_j} - \frac{\vec{D}_{j'j'}}{\vec{D}_{j'}} \right\} \qquad (1.5)$$

where $y_m^* = y_m + \beta g_y$ and $b_j^* = b_j - \beta g_b$ are frontier outputs, $\vec{D}_m$ and $\vec{D}_j$ are first-order derivatives, and $\vec{D}_{mm}$ and $\vec{D}_{jm}$ are second-order derivatives of the distance function. In general, these elasticities are not symmetric, so $M_{jm} \neq M_{mj}$ and $M_{jj'} \neq M_{j'j}$.

We parameterize $\vec{D}_{\mathrm{o}}(x, y, b;\, 1, 1)$ by a quadratic function, i.e.[6]

$$\begin{aligned}
&\vec{D}_{\mathrm{o}}(x, y, b;\, 1, 1)\\
&= \alpha_{\mathrm{o}} + \sum_{n=1}^{3} \alpha_n x_n + \frac{1}{2}\sum_{n=1}^{3}\sum_{n'=1}^{3} \alpha_{nn'} x_n x_{n'}\\
&\quad + \beta_1 y_1 + \frac{1}{2}\beta_{11} y_1^2 + \sum_{j=1}^{2} \gamma_j b_j + \frac{1}{2}\sum_{j=1}^{2}\sum_{j'=1}^{2} \gamma_{jj'} b_j b_{j'}\\
&\quad + \sum_{n=1}^{3} \delta_{n1} x_n y_1 + \sum_{n=1}^{3}\sum_{j=1}^{2} \eta_{nj} x_n b_j + \sum_{j=1}^{2} \mu_{1j} y_1 b_j
\end{aligned} \qquad (1.6)$$

While Färe *et al.* (2005) used the goal programming approach of Aigner and Chu (1968) to estimate the parameters of a quadratic directional distance function, we use a stochastic approach. Appending a random error term, $v \sim N(0, \sigma_v^2)$, to our estimating equation and invoking the translation property, we have

$$\begin{aligned}
&\vec{D}_{\mathrm{o}}(x, y, b;\, 1, 1) - \alpha\\
&= \alpha_{\mathrm{o}} + \sum_{n=1}^{3} \alpha_n x_n + \frac{1}{2}\sum_{n=1}^{3}\sum_{n'=1}^{3} \alpha_{nn'} x_n x_{n'}\\
&\quad + \beta_1 (y_1 + \alpha) + \frac{1}{2}\beta_{11}(y_1 + \alpha)^2 + \sum_{J=1}^{2} \gamma_j (b_j - \alpha)\\
&\quad + \frac{1}{2}\sum_{j=1}^{2}\sum_{j'=1}^{2} \gamma_{jj'} (b_j - \alpha)(b_{j'} - \alpha) + \sum_{n=1}^{3} \delta_{n1} x_n (y_1 + \alpha)\\
&\quad + \sum_{n=1}^{3}\sum_{j=1}^{2} \eta_{nj} x_n (b_j - \alpha) + \sum_{j=1}^{2} \mu_{1j} (y_1 + \alpha)(b_j - \alpha) + v
\end{aligned} \qquad (1.7)$$

In general, we do not directly observe $\vec{D}_{\mathrm{o}}(x, y, b;\, 1, 1)$ but must estimate it. Subtracting $\vec{D}_{\mathrm{o}}(x, y, b;\, 1, 1) = \mu$ from both sides of (1.7) yields

$$\begin{aligned}
-\alpha &= \alpha_{\mathrm{o}} + \sum_{n=1}^{3} \alpha_n x_n + \frac{1}{2}\sum_{n=1}^{3}\sum_{n'=1}^{3} \alpha_{nn'} x_n x_{n'} + \beta_1 (y_1 + \alpha)\\
&\quad + \frac{1}{2}\beta_{11}(y_1 + \alpha)^2 + \sum_{J=1}^{2} \gamma_j (b_j - \alpha)\\
&\quad + \frac{1}{2}\sum_{j=1}^{2}\sum_{j'=1}^{2} \gamma_{jj'} (b_j - \alpha)(b_{j'} - \alpha)\\
&\quad + \sum_{n=1}^{3} \delta_{n1} x_n (y_1 + \alpha) + \sum_{n=1}^{3}\sum_{j=1}^{2} \eta_{nj} x_n (b_j - \alpha)\\
&\quad + \sum_{j=1}^{2} \mu_{1j} (y_1 + \alpha)(b_j - \alpha) + v - \mu
\end{aligned} \qquad (1.8)$$

where the term μ is the inefficiency component of the error term, $\varepsilon = v - \mu$. To recover the inefficiency component of the composite error term, ε, one needs to assume a distribution structure for μ. Two potential candidates for the inefficiency distribution are the half-normal distribution and the exponential distribution. In the empirical section, we report estimates of (1.8) for the half-normal distribution of the inefficiency term[7] and explain our choice for α.

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[6] Chambers (1998) proposed using a quadratic form to parameterize the directional distance function, while Tran and Smith (1983) used a translog function to analyse the joint production of four undesirable outputs. While the parameters of the translog function can be restricted to satisfy a homogeneity property, the directional distance function has the translation property. A quadratic form can be restricted to satisfy the translation property, while the translog function cannot be restricted to satisfy the translation property.

[7] The estimates for the two cases were not significantly different. Results for both are available in Appendix B.

III. Data and Results

The technology modelled in this study consists of one good output, net electrical generation in kWh (y_1) and two bad outputs – SO_2 (b_1) and NO_x (b_2). The inputs consist of the Capital Stock (CS, x_1), the number of employees (x_2) and the heat content (in Bituminous, Btu) of the coal, oil and natural gas consumed at the plant (x_3).[8] The Federal Energy Regulatory Commission (FERC) Form 1 survey collects information on the cost of plant and equipment and the average number of employees for each electric power plant. While the FERC 1 survey collects data on the historical cost of plant and equipment, it does not collect data on investment expenditures or depreciation costs. As a result, we assume changes in the cost of plant and equipment reflect Net Investment (NI). Next, we convert the historical cost data into constant (1973) dollar values using the Handy–Whitman Index (HWI) (Whitman, Requardt and Associates, LLP, 2002). This is the same procedure employed by Yaisawarng and Klein (1994, p. 453, footnote 30) and Carlson *et al.* (2000, p. 1322). The net constant dollar CS for year n is calculated in the following manner:

$$\mathrm{CS}_n = \sum_{t=1}^{n} \frac{\mathrm{NI}_t}{\mathrm{HWI}_t} \tag{1.9}$$

In the first year of its operation, the NI of a power plant is equivalent to the total value of its plant and equipment. Appendix A contains a detailed discussion of the derivation of the CS.

The US Department of Energy (DOE) Form Energy Information Administration-767 (EIA-767) survey is the source of information about fuel consumption, fuel quality and net generation of electricity, which the US DOE uses to derive its emission estimates of SO_2 and NO_x.[9] Our panel consists of 76 coal-fired power plants for 1985–1998. While the plants may consume coal, oil or natural gas, in order to model a homogeneous production technology, coal must provide at least 95% of the Btu of fuels consumed by each plant.[10] In addition, some plants consume miscellaneous fuels such as: petroleum coke, blast furnace gas, coal–oil mixture, fuel oil #2, methanol, propane, wood and wood waste, refuse, bagasse and other nonwood waste. Although a number of plants consume fuels other than coal, petroleum and natural gas, these miscellaneous fuels represent very small percentages of fuel consumption (in Btu). In deriving our sample, we exclude a plant when its consumption of miscellaneous fuels represented more than 0.0001% of its total consumption of fuel (in Btu). For a plant whose consumption of miscellaneous fuels is less than 0.0001% of its total fuel consumption, the former consumption is ignored.[11]

In 1995, a total of 263 generating units at 110 power plants were subject to Phase I regulations, with an additional 175 units classified as substitution units. Substitution units enter Phase I as part of a multi-unit plan to assist Phase I units in attaining emission reduction targets. The number of generating units participating as substitution units declined between 1995 and 1998. In 1995, of the 76 power plants in our sample, 29 include 81 generating units, classified as original Phase I units, while another nine power plants include 19 units, classified as substitution units. In addition, some of the 29 power plants with Phase I units also had substitution units. As a result, 38 of the power plants in our sample had no Phase I or substitution units, i.e. none of their generating units were subject to these regulations. Finally, when comparing Phase I and non-Phase I plants, we assume that the 1995 classifications are valid throughout 1995–1998.

Table 1 reports descriptive statistics for the outputs and inputs for the pooled data which includes 76 electric power plants over 14 years. To estimate the directional output distance function, we divide each output and input by its mean value reported

[8] Because depreciation is not modelled when calculating the capital stock, vintage effects are not captured by the capital stock data used to estimate the production frontiers employed to derive the elasticities of transformation. Another potential source of error in the results is associated with treating all coal consumed as having the same quality (i.e. sulphur content). Hence, as currently specified, our model does not account for fuel switching as a strategy to reduce SO_2 emissions.

[9] In 1995 the US Environmental Protection Agency (EPA) initiated a programme to measure SO_2 emissions. In order to maintain consistency with pre-1995 emission data, we continue to employ the US DOE engineering estimates of SO_2 emissions for 1995–1998.

[10] It is possible to expand our sample to include power plants whose primary fuel is oil or natural gas. This can be accomplished in one of two ways. First, the plants can be added to the sample with their heat input listed as Btu's in the same manner in which we treat the coal-fired power plants in our sample. The drawback in this strategy is that it treats all power plants, regardless of fuel, as having identical production technologies. A second approach consists of modelling separate technologies for coal, natural gas and oil power plants, and calculating separate elasticities of transformation for each of the three production technologies. This approach would permit some insights into the emission consequences of switching from coal-fired power plants to either natural gas or oil power plants.

[11] Appendix A contains additional information about the data.

Table 1. Summary statistics (76 coal-fired power plants, 1985–1998)

	Units	Mean	SD	Minimum	Maximum
Electricity	kWh (millions)	4760.3	3876.8	45.4	20 654.6
SO_2	Short tons	53 069.2	65 853.1	704.2	401 136.6
NO_x	Short tons	18 875.9	15 975.1	243.9	80 138.3
CS	Millions of 1973$	226.1	136.1	34.3	752.7
Employees	Workers	193.6	116.3	24.0	700.0
Heat	Btu (billions)	48 352.5	38 753.6	687.9	199 891.3

in Table 1. To implement the translation property, we imposed on our directional output distance function, we choose α for each power plant to equal the index value of SO_2.[12] We employ a window approach to estimate the distance function by pooling consecutive 2-year periods of data so that we estimate (1.8) for 1985–1986, 1987–1988, …, 1997–1998.[13] The periods chosen correspond to periods before, during and after implementation of the 1990 Amendments to the Clean Air Act. In 1996, NO_x reductions under Phase I started. A one-to-one correspondence does not exist between SO_2 Phase I units and NO_x Phase I units.

Parameter estimates are provided in Appendix B. We report two sets of parameter estimates for each 2-year period. The first set of estimates assumes that the inefficiency component of the composite error term has a half-normal distribution, and the second set of estimates assumes that the inefficiency component has an exponential distribution. The estimates for 1989–1990 did not converge for the exponential distribution. The estimated parameters from the two methods have the same signs except for η_{12} in 1985–1986, and α_2, α_{11} and β_1 in 1997–1998. In addition, the results of a t-test for differences in the parameter estimates revealed no significant difference in the parameters estimated in any year for the half-normal distribution versus the exponential distribution except for 1989–1990. Hereafter, we report only results for the half-normal distribution.

The stochastic method allows us to test whether the distribution of inefficiencies is different from zero. Using a likelihood ratio test, we reject the null hypothesis $\sigma_u = 0$ in every set of 2 years except 1995–1996 for the exponential distribution. Therefore, these tests indicate electric power plants are not producing on the frontier of $P(x)$.[14]

Table 2. Estimates of inefficiency: means (SD)

Year	$\vec{D}_o(x, y, b; 1, 1)$
1985–1986	0.033 (0.024)
1987–1988	0.043 (0.034)
1989–1990	0.054 (0.048)
1991–1992	0.047 (0.036)
1993–1994	0.042 (0.025)
1995–1996	0.001 (0.000)
1997–1998	0.001 (0.000)

Note: Since the data are mean deflated, the values are the number of mean values that goods can be increased and bads reduced.

In Table 2, we report the inefficiency estimates for each period. The inefficiency estimates are $\vec{D}_o(x, y, b; 1, 1) = E(u|\varepsilon)$. Following Kumbakhar and Lovell (2000), the estimates of $E(u|\varepsilon)$ are obtained as

$$E(u|\varepsilon) = u_{*i} + \sigma_* \left\{ \frac{\phi(-u_{*i}/\sigma_*)}{\Phi(u_{*i}/\sigma_*)} \right\} \tag{1.10}$$

where $\phi(\cdot)$ is the density of the standard normal distribution, $\Phi(\cdot)$ the cumulative distribution function of the standard normal distribution, $\sigma = (\sigma_u^2 + \sigma_v^2)^{1/2}$, $u_{*i} = -\varepsilon_i \sigma_u^2/\sigma$ and $\sigma_* = \sigma_u \sigma_v/\sigma$ for the normal/half-normal model, and $u_{*i} = -\varepsilon_i - \sigma_v^2/\sigma_u$ and $\sigma_* = \sigma_v$ for the exponential model.

To interpret the numbers in the table, consider the mean estimate of the half-normal model – $\vec{D}_o(x, y, b; 1, 1) = 0.033$ for 1985–1988. Given our normalization of the data, the desirable output could be expanded by 0.033 times (3.3%) its mean value, and SO_2 and NO_x could be reduced by 0.033 times (3.3%) its mean value before the hypothetical

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[12] The index value of SO_2 for an observation is its SO_2 production normalized by the mean SO_2 production of all observations.

[13] We experienced convergence problems for the half-normal distributions when attempting to estimate the directional distance function on a year-by-year basis. All estimates were calculated using Stata.

[14] The exception is when the directional distance function is estimated for 1995–1996 with an inefficiency term that follows an exponential distribution.

Table 3. Violations of monotonicity and shadow price ratios (SDs)

	Violations of monotonicity			Shadow price ratios (kWh per ton of bad output)	
	Number of observations for which			SO_2	NO_x
Year	$\partial\vec{D}_o(\cdot)/\partial y_1 > 0$	$\partial\vec{D}_o(\cdot)/\partial b_1 < 0$	$\partial\vec{D}_o(\cdot)/\partial b_2 < 0$	$-\frac{\partial\vec{D}_o(\cdot)/\partial b_1}{\partial\vec{D}_o(\cdot)/\partial y}$	$-\frac{\partial\vec{D}_o(\cdot)/\partial b_2}{\partial\vec{D}_o(\cdot)/\partial y}$
Half-normal distribution of inefficiency					
1985–1986	0	1	51	75 954 (81 761)	21 494 (25 378)
1987–1988	0	2	37	104 797 (102 504)	30 453 (33 867)
1989–1990	0	21	35	68 302 (64 333)	40 117 (21 092)
1991–1992	0	0	72	63 542 (54 643)	22 145 (22 605)
1993–1994	0	0	81	148 602 (155 410)	48 320 (69 037)
1995–1996	0	0	74	90 796 (83 798)	26 140 (33 567)
1997–1998	0	0	85	127 535 (174 947)	43 087 (80 024)

Table 4. Morishima elasticities of transformation: means (SDs)

	1985–1986	1987–1988	1989–1990	1991–1992	1993–1994	1995–1996	1997–1998
Panel A: All observations, half-normal distribution of inefficiency							
$M_{SO_2,kWh}$	−0.061	−0.183	−0.423	−0.062	−0.335	−0.265	−0.446
	(0.377)	(0.230)	(4.131)	(0.106)	(0.360)	(0.233)	(0.503)
M_{kWh,SO_2}	−0.088	−0.049	−0.307	−0.006	0.003	0.077	0.145
	(0.122)	(0.087)	(3.729)	(0.042)	(0.009)	(0.065)	(0.121)
$M_{NO_x,kWh}$	−1.997	−2.677	−1.228	−0.344	−1.460	−2.033	−2.566
	(11.052)	(24.761)	(15.391)	(26.078)	(10.018)	(14.025)	(35.979)
M_{kWh,NO_x}	2.003	2.259	1.407	0.245	1.036	1.722	1.396
	(16.687)	(32.765)	(20.748)	(33.118)	(9.314)	(16.446)	(34.399)
M_{SO_2,NO_x}	1.868	2.047	0.760	0.183	0.755	1.573	1.183
	(16.666)	(32.759)	(21.860)	(33.117)	(9.278)	(16.440)	(34.399)
M_{NO_x,SO_2}	0.864	0.433	0.799	0.595	0.111	0.326	0.203
	(7.793)	(3.622)	(9.301)	(7.542)	(1.098)	(3.068)	(5.301)
Panel B: Observations satisfying monotonicity, half-normal distribution of inefficiency							
$M_{SO_2,kWh}$	−0.021	−0.174	0.024	−0.097	−0.525	−0.350	−0.700
	(0.282)	(0.244)	(0.557)	(0.117)	(0.429)	(0.260)	(0.634)
M_{kWh,SO_2}	−0.085	−0.046	0.044	−0.016	−0.002	0.086	0.178
	(0.124)	(0.091)	(0.195)	(0.049)	(0.011)	(0.063)	(0.123)
$M_{NO_x,kWh}$	−5.972	−7.315	−4.953	−9.956	−7.670	−8.865	−14.013
	(10.907)	(25.033)	(13.881)	(23.202)	(9.429)	(14.928)	(41.697)
M_{kWh,NO_x}	7.246	8.632	6.504	12.558	6.794	9.298	12.523
	(17.665)	(31.074)	(19.153)	(31.638)	(9.445)	(18.498)	(37.336)
M_{SO_2,NO_x}	7.153	8.449	6.568	12.466	6.408	9.122	12.207
	(17.616)	(31.064)	(19.067)	(31.656)	(9.464)	(18.501)	(37.374)
M_{NO_x,SO_2}	2.153	1.355	2.638	3.498	0.718	1.382	1.329
	(9.267)	(3.143)	(10.077)	(8.647)	(1.141)	(3.633)	(6.010)

average power plant becomes efficient (i.e. produces on the frontier of $P(x)$). Mean inefficiency rises from 1985–1986 to 1989–1990 and then falls through 1997–1998 for the half-normal estimates.

The theoretical technology of production implies monotonicity conditions such that decreases in the desirable output or increases in the undesirable output that are still feasible do not reduce inefficiency. In Table 4, we report the number of times our results violate the monotonicity assumptions. The greatest number of violations is for NO_x, especially during the periods 1991–1992 to 1997–1998. All observations satisfy the monotonicity condition for the desirable output of electricity. Between 1 and 21 observations fail to satisfy monotonicity for SO_2 during the first three periods, 1985–1986 to 1989–1990, but during the final four periods, 1991–1992 to 1997–1998, all observations satisfy the monotonicity condition.

We also report the relative shadow prices of NO_x and SO_2 in terms of foregone electricity in Table 3.

To reduce 1 ton of SO_2 costs 75 954 kWh or 76 mWh of electricity during 1985–1986 for the estimates derived from the half-normal and about 128 mWh of electricity during 1992–1995. Multiplying the price of electricity times the shadow price ratio yields the opportunity cost of abating 1 ton of SO_2. For instance, at a price of \$35/mWh of electricity, the shadow price of SO_2 is \$2660 = \$35 × 76 in 1985–1986 and \$4480 in 1997–1998 given the half-normal estimates. To reduce 1 ton of NO_x, the shadow price ratio given the half-normal estimates indicates that 21 494 kWh or 21 mWh of electricity must be foregone during 1985–1986 and about 43 mWh of electricity in 1997–1998. For the half-normal estimates and assuming electricity prices of \$35/mWh, the shadow cost of reducing 1 ton of NO_x is \$735 = \$35 × 21 in 1985–1986 and \$1510 in 1997–1998.

Interestingly, the permit price for 1 ton of SO_2 reported for 2001 by Lutter and Burtraw (2002) is between \$152 and \$211, and the permit price for NO_x is about \$2000. Our estimates, which reflect the opportunity cost of abatement, are higher for SO_2, but lower than the permit price for NO_x. However, by our calculations, the relative shadow price of SO_2 to NO_x is about 3.2, which is in line with the estimates of relative damages by Lutter and Burtraw. They state that damages from 1 ton of SO_2 are 3–5 times the damages of 1 ton of NO_x.

In Table 4, we report the Morishima elasticities of transformation for all observations and for only those observations satisfying monotonicity. We focus our discussion on those observations that satisfy the monotonicity conditions. As expected, $M_{\mathrm{SO_2,kWh}} < 0$ and becomes more negative in the latter period, 1997–1998, than in the beginning period, 1985–1986. This result indicates that it is becoming more costly to reduce SO_2 in terms of foregone kWh and is consistent with that found by Färe *et al.* (2005). The elasticity of transformation, $M_{\mathrm{kWh,SO_2}}$, is negative in the beginning periods, 1985–1986 and 1987–1988, but turns positive in the two latter periods, 1995–1996 and 1997–1998. The elasticity $M_{\mathrm{NO}_x,\mathrm{kWh}}$ is negative in every year for both estimation methods.

Turning to the elasticity of transformation for the two undesirable outputs, both $M_{\mathrm{NO}_x,\mathrm{SO_2}}$ and $M_{\mathrm{SO_2},\mathrm{NO}_x}$ are positive in every year, indicating that the two undesirable outputs are substitutes in the production process. Thus, regulatory efforts that limit emissions of one pollutant such as SO_2 will have an unintended consequence of increasing the other pollutant. The elasticity declines from 1991–1992 to 1993–1994 before increasing throughout the remainder of the periods. This pattern indicates that it is becoming more difficult to substitute the less regulated pollutant, NO_x, for the more regulated pollutant – SO_2 emissions. The elasticity $M_{\mathrm{NO}_x,\mathrm{SO_2}}$ increases from 1985–1986 to 1991–1992, reaches a minimum in 1993–1994 and then increases, but its 1997–1998 value is less than its 1985–1986 value. This indicates that declines in the relative intensity of NO_x to SO_2 are easier to achieve in 1997–1998 than they were in the beginning periods 1985–1986.

As a robustness check, we also calculated shadow prices and elasticities for plants subject to Phase I restrictions versus plants that were not subject to Phase I. With one notable exception, we found no significant difference between the two groups. The exception was for the elasticities of substitution for 1995–1996 and 1997–1998 with values $M_{\mathrm{kWh,SO_2}}$ of 0.101 and 0.175 for Phase I plants and 0.061 and 0.125 for the non-Phase I plants, respectively. We interpret this result as follows: for both types of plants, the opportunity cost of reducing SO_2 increases between these two periods. The fact that the values are higher for the Phase I plants in each period than for the non-Phase I plants suggests that it would be less costly to get further reductions in SO_2 by targeting the non-Phase I plants. For a more detailed discussion of these results, see Appendix C.

IV. Conclusions

First, we introduced an environmental directional distance function that models the joint production of good and bad outputs. After parameterizing our directional output distance function as a quadratic function and estimating it using stochastic frontier methods, we demonstrated how the distance function parameters are used to calculate Morishima elasticities of transformation among undesirable outputs. Hence, we have demonstrated the practicality of specifying a parametric directional distance function to obtain estimates of the ease of substituting among bad outputs. Our estimated model indicates that it is becoming more difficult to substitute the less regulated undesirable output (NO_x emissions) for the more regulated undesirable output (SO_2 emissions) with the longer history of regulations, although the opposite is not true (see Burtraw and Evans, 2004, for a discussion of NO_x regulations for coal-fired power plants).

The primary contribution of this study is demonstrating the practicality of calculating elasticities of transformation among undesirable outputs within a formal production model instead of relying on ad hoc methods to discern the association among undesirable outputs. While the empirical findings of this study may not be surprising to some researchers, the main purpose of the empirical portion of the study was demonstrating how to implement calculating the elasticities of transformation of undesirable outputs.

Knowledge of the ease of substitutability is useful information to possess when determining least-cost strategies of obtaining desired levels of environmental quality. For example, various combinations of SO_2 and NO_x can yield the same level of environmental quality (i.e. benefits). Information on the ease of substitutability would inform decision makers about whether the optimal regulatory strategy might involve allowing firms to trade off reductions in NO_x emissions for SO_2 emissions as proposed by Lutter and Burtraw (2002). Our results suggest that NO_x emissions are relatively cheaper to reduce than SO_2.

While the analyses of Lutter and Burtraw (2002) and Greenstone (2003) depend on pollutants being substitutes, ancillary benefit calculations (Burtraw *et al.*, 2003) depend on polluting outputs being complements. Therefore, having information about the type and degree of substitutability among pollutants is essential to recommending the appropriate policy.

Finally, the data employed in this study can be expanded to include bad output production reported by the Toxic Release Inventory. This would permit an analysis of the extent to which reductions in toxic air emissions shifted bad output production from one media (i.e. air) to other media (i.e. water and ground).

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