Market fragility and international market crashes

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Abstract

We extend the Pukthuanthong and Roll (2009) measure of integration to provide an estimate of systemic risk within international equity markets. Our measure indicates an increasing likelihood of market crashes. The conditional probability of market crashes increases substantially following increases of our risk measure. High levels of our risk measure indicate the probability of a global crash is greater than the probability of a local crash. That is, conditional on high levels of systemic risk, the probability of a severe crash across multiple markets is larger than the probability of a crash within a smaller number of markets.

JEL classifications: G01; G15

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1. Introduction

International systematic risk exposure varies across countries and through time. Consequently, periods in which aggregate systematic risk exposure is high across multiple countries will correspond to periods in which the risk of a negative shock propagating internationally, and of multiple markets jointly crashing, is the greatest. To develop a time-varying measure of systemic risk within international equity markets, we extend the integration analysis of Pukthuanthong and Roll (2009). Their focus is measuring time-varying integration, however, their setting provides a unique framework in which an underlying world market factor is identifiable, and average loadings across countries on this factor can vary through time. From this unique setting, we aggregate time-varying loadings on the world market factor across countries to create a measure of systemic risk. Intuitively, a negative shock to the underlying world factor is likely (unlikely) to lead to severe market declines across multiple countries if the shock occurs during a period in which average exposure to this factor is high (low). From this intuition, we name our systemic risk measure the Fragility Index (FI), as periods in which the measure is high identify periods in which international equity markets are much more susceptible to a negative shock to the world market factor. We find that increases in systemic risk lead periods in which the probability of market crashes increases substantially. Further, conditional on high levels of risk, the probability of a global crash across multiple markets often exceeds the probability of local crashes confined within a smaller number of markets. This finding is very consistent with the concept of our risk measure, and indicates that if a shock occurs during periods in which multiple markets share a high risk exposure to a common factor, then these multiple markets will experience simultaneous market declines.

This study relates to research that considers the international propagation of financial shocks. Given that financial shocks, as well as contagion, may have significant impacts on investor wealth, these topics have received significant attention within the literature. In general, studies of contagion test if a shock in one market spreads to other markets, with multiple studies providing mixed results. The mixed results within the existing literature suggest that certain crises propagate internationally, while others remain local. As examples, Forbes and Rigobon (2001) study several emerging market crises and find evidence of high levels of interdependence, but not contagion. On the other hand, Asgharian and Nossman (2011) provide evidence of risk spillovers from the U.S. to European countries. Interestingly, Longstaff (2010) finds evidence of contagion within the subprime asset-backed derivatives market. Our study builds on the existing literature by identifying periods in which national stock markets exhibit a high degree of inter-relation, and consequently identifying periods in which a shock in one market may be more likely to spread internationally.

Existing research also considers the probability of poor returns across markets. For example, Christiansen and Ranaldo (2009) find that closer economic linkages following EU entry may increase the probability of co-exceedances. Markwat, Kole, and van Dijk (2009) find support for a domino effect, in which local and regional crashes increase the probability of a subsequent global crash. Our approach builds on this research by indicating periods of high systemic risk prior to an initial crash, and our robustness analysis shows FI contains significant predictive information regarding initial, unpreceded shocks. Kumar, Moorthy, and Perraudin (2003) find that economic and financial data can predict an increasing likelihood of emerging market currency crashes. Finally, Bartram, Brown, and Hund (2007) study risk within the global banking system during international crises. We extend the existing literature by presenting a parsimonious risk measure, in which aggregate systemic risk is captured by loadings on the underlying world market factor, and by considering an extensive sample of international markets.

Kritzman et al. (2011) also study market crashes. They create their absorption ratio (AR) from the proportion of return variance explained in a principal component (PC) framework. Our approach focuses on underlying factor loadings; a large portion of an asset's return may be explained by PCs, even if the asset has a small or negative exposure to the underlying economic factors (cf. Pukthuanthong and Roll, 2009). Relative to Kritzman et al. (2011), by extending the Pukthuanthong and Roll (2009) framework, our paper provides stronger economic justification for our systemic risk measure. Further, Kritzman et al. focus their results on five domestic markets and measures of systemic risk within each domestic market, and only briefly investigate international events by highlighting levels of their global AR prior to four identified international events. Contrasting their focus, our study considers international risk dynamics across a lengthy sample with a focus on joint co-exceedances across markets. Finally, in Section 4.4 we compare our results with a measure comparable to the AR. Our analyses that include both FI and AR show a strong and significant relation between FI and market crashes, but fail to provide evidence of a similar relation for AR. Within Section 4.4, we also show that the relation between

FI and future crashes is robust to controlling for conditional volatility, and within Section 4.5 we show that FI does not require an initial shock.

We find that high levels of FI precede dramatic increases in the conditional probability of severe international equity market declines. As examples of our results, the probability of the return to our world index falling below its fifth percentile is 3.8% and 4.4%, conditional on FI falling below its 90^{th} and 98^{th} percentiles, respectively. However, these conditional probabilities increase to 15.8% and 32.4% when FI exceeds the same thresholds in question. We further classify country market indexes into three cohorts that approximate levels of market development, and analyze the probability of joint co-exceedances. Conditional on extreme high realizations of FI, the probability of each of the three cohorts simultaneously exhibiting a return below their tenth percentile is 27%, while the corresponding probability is approximately equal to 4% when FI is below the same threshold. In general, conditional on high levels of systemic risk, we find the probability of severe global market crashes (crashes across a majority of cohorts) increases dramatically, and frequently is also significantly greater than the probability of minor or local market crashes (crashes confined within a smaller number of cohorts). Continuing the above example, while the probability of a simultaneous crash within all three cohorts conditional on high levels of fragility is 27%, the conditional probabilities of just one or two cohorts crashing are 11%, and 9%, respectively. As further examples of our results, in a logistic regression setting, the odds of all three cohort indexes simultaneously exhibiting a return that falls below their second percentile increase by three, and nine times, respectively, as FI increases by one and two standard deviations. Although crashes are a greater concern than ubiquitous increases in prices, we show in Section 4.3 that FI also measures a greater likelihood of simultaneous large price increases. Evidently, if a global factor drives the returns in many countries, then when those countries are more sensitive to that factor, both increases and decreases in the global factor should be reflected in larger returns (in absolute value.) That is, FI can predict both negative and positive extreme returns.[1]

Our study makes several important contributions. First, we present an ex ante measure that exhibits a strong and positive relation with the conditional probability of extreme market crashes, and with the probability of crashes propagating across markets. This measure has implications for portfolio management, as well as for policy makers. Second, we extend the contagion literature by identifying an important factor that relates to the likelihood of a shock in one market propagating internationally. Third, we extend the systemic risk literature by presenting a generalizable measure. Existing research focuses on crises stemming from specific risks, while our index is flexible and able to capture any economic variable that increases loadings on the world market factor, which also allows inclusion of a large international sample of countries in our study.

2. Economic framework

Our focus is creating a generalizable measure of systemic risk across countries. To create this measure, we extend the integration analysis of Pukthuanthong and Roll (2009) who regress country returns on ten global factors. These factors are estimated by out-of-sample PCs based on

1. We thank an anonymous referee for suggesting this analysis.

the covariance matrix in the previous calendar year computed with the returns from 17 major countries, the "pre-1974 cohort" described in their paper. In their analysis, the *R*-square from the regression provides a measure of world market integration. Further, they provide substantial empirical evidence indicating that the first PC, which explains the greatest proportion of variance, represents an underlying world market factor. Consequently, loadings on the first PC represent exposures to the world market factor. We extend their model to aggregate loadings on the world factor at a point in time as a measure of systemic risk. Arguably, periods in which exposure to the world factor across multiple markets is high, may precede crashes as a negative shock to the world factor would have relatively larger impacts across all of these markets, relative to periods in which average systematic risk exposure is low.[2] In this setting, the occurrence of negative shocks to the world factor may be unpredictable, but the impact and spillover effect of a given shock will vary with levels of systemic risk. Specifically, we assume that given forward-looking market participants, significant negative market shocks will occur randomly through time, but our approach will identify periods in which a shock of a given magnitude will have a greater likelihood of impacting multiple markets.

We explain the economic framework of our risk measure, and differentiate this from measures of integration. Pukthuanthong and Roll (2009) describe a model in which 'Salt' and 'Water' are two underlying economic factors. In this setting, two countries could exhibit high

2. To clarify terminology, we use 'systematic' risk to refer to loadings on the first PC which represent exposure to the world factor, and we use 'systemic' risk to refer to the probability of simultaneous market downturns across many countries. Consequently, periods in which average systematic risk across countries is high, correspond to periods of significant 'systemic' risk, as a shock to the world factor during these periods would cause market declines across all countries within the system.

integration if both share a high exposure to one factor, Salt, for example. Alternatively, two countries could also exhibit high integration if one exhibited a high exposure to Salt, and the other was exposed to Water, and even negatively related to Salt. In the first example, systemic risk is high because both countries have a high exposure to a common risk factor (Salt), and a negative shock to this risk factor would be expected to propagate across both countries, leading to simultaneous market declines. In the second example, although integration is high, the negative shock to Salt would impact only the first country, and may benefit the second country due to its negative exposure to the risk factor. From this example it is clear that general levels of integration do not distinguish between the two cases, and consequently do not provide a complete picture of systemic risk. Our study generalizes the above example by focusing on an important underlying factor, the world market factor identified by Pukthuanthong and Roll (2009), and then creating a risk measure by aggregating exposure to this factor across many markets.

To estimate loadings on the world market factor, we start with the Pukthuanthong and Roll (2009) framework, in which country returns are regressed on ten PCs. We specify

$$R_{j,t} = \sum_{i=1}^{10} \beta_{j,i} PC_{i,t} + e_{j,t}, \tag{1}$$

in which $R_{j,t}$ represents the US dollar-denominated return for index *j* during day *t*, and $PC_{i,t}$ represents the *i*th PC during day *t*. $PC_{i,t}$ is estimated based on Pukthuanthong and Roll (2009). Implementing the above specification requires some degree of estimation and parameterization. We now discuss how we specify our risk measure with respect to important parameters that define FI, including estimation of factor loadings, factor loading aggregation, and definition of co-exceedances. Importantly, in Section 4.5, we show that results of our study, and the related inferences, are robust to multiple alternative specifications for each of the parameters discussed.

To aggregate the estimated factor loadings, we take the cross-sectional average of loadings on the world market factor across countries at each point in time; we argue that this provides the measure of systemic risk that is most intuitive and consistent with the economic framework of Pukthuanthong and Roll (2009). We note that this approach would not be feasible in the restrictive international CAPM, as betas averaged across the component indexes of the world market portfolio would be constrained to equal one in all cases. The Pukthuanthong and Roll (2009) PC analysis allows flexibility and can select a subset of component portfolios, or even place extra weight on some portfolios, which allows average loadings on the first PC to vary though time. Further, the Pukthuanthong and Roll approach creates the PCs from the 17 countries that enter the database prior to 1974, and the CAPM cross-sectional average restriction only applies to assets that are included within the market portfolio. Therefore, while the first PC proxies for a world market factor, even in the restrictive CAPM setting, variability in average loadings across the remaining post-1974 countries would still be feasible. All in all, we argue that the framework of Pukthuanthong and Roll provides the flexible setting that is necessary to allow time-variation in average factor loadings as a means of measuring international systemic risk, which would not be possible in the more restrictive international CAPM setting. Importantly, our robustness results in Section 4.5 include two alternative approaches to factor loading aggregation that would be feasible under the restrictive international CAPM setting, and both approaches further indicate FI contains significant predictive information.

To estimate Eq. (1) and calculate average loadings, we use a 500-day rolling window for each country and place a decaying weighting scheme on previous daily observations such that the weight placed on daily observation *t*-x is equal to 0.995^{x-1}. This approach is similar to Kritzman et al. (2011), allows the impact of lagged days to decay through time, and places an approximate 50% weight on the observation halfway through the rolling window. Countries are excluded from the analysis at a given point if we have less than 100 usable daily observations for the country within the specific rolling window. Our approach, which allows time-varying loadings, is also similar to international asset-pricing studies of contagion (cf. Bekaert, Harvey, and Ng, 2005). For a given day *t*, we take the average of the loading on the first PC, which is estimated across days *t*-500 through day *t*-1. We define this variable as $\mu_{PC1,t}$, which represents FI. In our later analyses, we consider results across levels of market development (developed, developing, emerging, and frontier), and in these analyses, we define $\mu_{PC1,t}$ for specific cohort indexes such that these variables measure average exposure to the world factor within a specific market classification. We note that loadings on the additional PCs may also contain predictive information, however, we focus our measure and results on the first PC only, as this is identified by Pukthuanthong and Roll (2009) as the world factor.

Our analysis requires identification of risk states, and specification of market crashes. Initially, we specify full-sample percentiles of FI to identify risk states, and for notation, we define fragility based on $\mu_{PC1,t} > Pk(\mu_{PC1})$ in which $Pk(\mu_{PC1,t})$ represents the *k*th percentile of μ_{PC1}, while latter analyses implement logistic regressions that do not require this specification. Finally, we identify the crash subsample for index *j* as all days in which $R_{j,t} \leq Pk(R_j)$ for arbitrary return percentile threshold k. Within this setting, $R_{j,t}$ represents the return to index j during day t and $Pk(R_j)$ represents a specified threshold percentile of full-sample returns for index j. Our 'bad return' day is thus defined as a day that the index return falls below a given threshold percentile. This approach is consistent with Bae, Karolyi, and Stulz (2003) who identify contagion based on full-sample percentiles. Analyses across cohorts define return percentiles specific to each cohort.

A final consideration for the construction of FI is trading day synchronicity. In the international context, non-synchronous trading in markets across time zones creates a potential concern. By matching returns based on calendar days, we take a conservative approach to the non-synchronous trading issue in which the potential impact of non-synchronous trading would bias our results against the predictive ability of FI. To illustrate, if a shock occurs during trading hours early in the day (before western hemisphere markets open), and this shock propagates internationally, then we would expect the shock to manifest in the western markets when they open, and our methodology would capture this spillover. If a shock occurs later in the day (after eastern hemisphere markets close), then we would expect the shock to manifest during the next trading day within those markets, and the approach would not capture any potential spillover. However, potential lead/lag approaches attempting to capture these types of spillovers could also lead to a spurious relation across FI and crashes. Therefore, our results present a conservative measure, and may understate the true predictive ability of FI. Our analyses covering the world index as well as market classification indexes, which include countries across the globe and trade throughout the calendar day, further mitigate non-synchronous trading concerns, as a given shock may manifest within the markets open at the time of the shock.

3. Data

We consider a broad selection of national equity market indexes. Daily data are extracted for 82 countries from DataStream, a division of Thomson Financial. The data consist of broad country indexes converted into a common currency (the US dollar). The Appendix lists the countries, identifies the indexes, reports the time span of daily data availability, and provides the DataStream mnemonic indicator. If the mnemonic contains the symbol "RI," the index includes reinvested dividends; otherwise, the index represents an average daily price. Similar to Pukthuanthong and Roll (2009), we assign countries to three specific cohorts based on each country's initial appearance in the database. We define countries appearing prior to 1984 as Cohort 1, countries appearing from 1984 through 1993 as Cohort 2, and the remaining countries as Cohort 3.[3] In this classification, Cohort 1, Cohort 2, and Cohort 3 represent developed, developing, and emerging/frontier markets, respectively (cf. Pukthuanthong and Roll, 2009; Berger et al., 2011). Throughout the study we calculate equal-weighted returns to the all-country index, and to the three market classification cohorts using countries with available data at each point in time. The focus on equal-weighted returns provides a cleaner measure of joint co-exceedances, as equal-weighted indexes are more likely to detect crashes that occur across

3. Our cohort classification is similar in approach as Pukthuanthong and Roll (2009), but combines Cohorts 1 and 2 from their study into our Cohort 1. Consequently, our study considers three cohorts, while their paper uses four.

multiple countries. A focus on value-weighted indexes would lead to results that are driven by the largest markets within the sample.

To consider co-exceedances across all cohort groups, we start the sample on December 29, 1994 which corresponds to the entry of Cohort 3 countries. The sample ends on November 30, 2010. Although results are based on the sample above, calculation of FI for the initial part of this sample utilizes available observations that are prior to the start of the return analysis. Daily returns are calculated as log index relatives from valid index observations. An index observation is not used if it exactly matches the previous reported day's index. When an index is not available for a given trading day, DataStream inserts the previous day's value. This happens whenever a trading day is a holiday in a country and also, particularly for smaller countries, when the market is closed or the data are simply not available. Our daily returns are thus filtered to eliminate such invalid observations. This approach is consistent with Pukthuanthong and Roll (2009) and further mitigates concerns regarding non-synchronous trading.

We present a general picture of the relation between FI and returns in Table 1. The mean, median, and standard deviation of the world index, Cohorts 1, 2, and 3 are shown across the full-sample, as well as across deciles of fragility. As FI increases from the first to the tenth decile, mean returns tend to decline. A plunge in returns is most drastic in Cohort 3 where the mean returns appear negative starting from the seventh decile to the tenth decile of FI. Finally, the analysis suggests an increase in volatility as fragility increases. For example, the standard deviation of the all-country index conditional on fragility above the 80th or 90th percentiles is over twice the standard deviation conditional on fragility falling in the first, second, or third deciles. This provides support for the concept of FI. As risk exposure becomes concentrated on the world factor, diversification benefits will diminish, as returns reflect this common factor.

Insert Table 1 about here

Although not detailed in Table 1, when measured across all countries, the time-series median of FI is 0.137 with a standard deviation of 0.082. FI thresholds based on the 80^{th}, 90^{th}, 95^{th}, and 98^{th} percentiles are 0.170, 0.346, 0.372, and 0.387, respectively. Finally, we have argued that the flexible PC approach allows time-variation in cross-sectional average loadings within the 17 pre-1974 component which would not be possible in the restrictive international CAPM setting. We find that the time-series standard deviation of loadings on the first PC averaged across the 17 pre-1974 countries is 0.107. This value appears larger than the time-series standard deviation of 0.078 for the countries not included in the PC construction. We argue that this provides further support of our approach, as the PC estimation is empirically selecting weights that allow significant time-series variability in average loadings on the first PC, even within the countries that make up the PCs.

4. Fragility Index and probabilities of market crashes

4.1. Empirical crash probability conditional on FI

We analyze the conditional probability of market crashes across levels of FI. In the initial analysis, we consider returns to the world index, which is comprised of all countries within our sample. Considering the all-country index, fragility may manifest because this index likely becomes more volatile and prone to extreme realizations as all component countries share similar risk-exposure, and consequently, diversification benefits are dampened as FI increases. We consider various thresholds of FI and definitions of market crashes. Relatively low thresholds for FI provide a safety-first measure, which is likely to detect both major and minor events, while higher FI thresholds detect periods of extreme systemic risk. Similarly, various crash definitions consider the tradeoff between higher probability and lower impact events, with lower probability and higher impact events. Table 2 reports the expected number of crashes under the assumption that crashes are independent from FI, $Ex(X \mid \mu_{PC1,t} > Pi\%(\mu_{PC1}))$, the actual number of occurrences, $f(X \mid \mu_{PC1,t} > Pi\%(\mu_{PC1}))$, and the empirical probability of a crash, $f/n(X \mid \mu_{PC1,t} > Pi\%(\mu_{PC1}))$, conditional on FI exceeding the *i*th percentile. We also report the same statistics conditional on FI falling below the given thresholds. Low levels of FI may indicate periods of decreasing risk. Finally, we report *Z*-scores testing that the probability of a crash is constant across levels of FI, $H_0: d = 0$.

Insert Table 2 about here

Across all specifications considered, there is a strong and statistically significant relation between high values of FI and subsequent market crashes. For example, as fragility increases from below the 80th percentile of FI to above this threshold, the empirical probability of crashes increases from 16.6% to 33.4% (representing a 201% increase), 7.1% to 21.5% (287%), 3% to 13% (433%), and 0.9% to 6.3% (700%) for crashes defined as returns below the 20th, tenth, fifth, and second percentiles, respectively. Our approach also shows that for all high (low) risk states, the frequency of market crashes is higher (less) than the expected number of market crashes.[4] Highlighting the 95th percentile of FI as a measure of periods of extreme systemic risk, we observe 147 days in which the return falls below the fifth percentile and the preceding value of FI fell below its 95th percentile. This corresponds to an empirical frequency of 4.1%, and is slightly below the 177.7 observations we would expect if FI and crashes were independent. On the other hand, conditional on FI exceeding the 95th percentile, we would only expect 9.3 days in which the subsequent return fell below the fifth percentile, but instead observe 40 such occurrences, corresponding to an empirical frequency of 21.4%.

To analyze the predictive content of FI in the context of co-exceedances across markets, we present the empirical probabilities of multiple cohorts falling below crash thresholds on a given day. The potential number of cohort index returns falling below the return thresholds ranges from zero, for a day in which no cohort group crashes, to three, representing a day in which all three cohorts jointly crash. Table 3 presents the frequency and probability of market crashes for different thresholds of FI and bad returns, using the same notation as Table 2. The table also reports chi-squared statistics, χ^2, and associated *p*-values, for each event. Entries for each outcome in the χ^2cell report that occurrence's contribution to an overall chi-square statistic, and the *p*-value for the given chi-square statistic in isolation. We report only the expected number of crashes and the chi-square statistics for the high risk states.

4. Even with the assumption that crashes are independent from FI, the expected number of crashes will still vary with the threshold of FI specified. For example, we have 3,744 daily observations, and by definition 74 daily returns will fall below the second percentile (3,744 * 0.02 = 74). If FI was independent from subsequent daily returns, we would expect to have 7.4 (= 3,744 * 0.02 * 0.10) and 3.7 (= 3,744 * 0.02 * 0.05) observations in which the return during day *t* fell below the second percentile and FI calculated through day *t*-1 exceeded the 90th and 95th percentiles, respectively, simply due to chance. Showing that we actually have 27 and 18 of these observations, respectively, suggests a strong relation between FI and subsequent crashes.

*** Insert Table 3 about here ***

Consistent with the results in Table 2, the results in Table 3 show that the frequency of market crashes increases with high levels of FI. Further, high levels of FI precede increases in the probabilities of multiple cohorts jointly crashing. Frequently, conditional on high levels of FI, the probability of all three cohorts jointly crashing exceeds the probability of one or two cohorts crashing. For example, conditional on FI falling below the 95th percentile, the probability of one, two, and three cohorts jointly exhibiting returns below their tenth percentile are 10%, 4%, and 3%, respectively. The corresponding probabilities conditional on FI exceeding the 95th percentile are 11%, 7%, and 19%. Comparing probabilities of crashes across risk states indicates that conditional on high levels of FI, the probabilities of two or three cohorts jointly crashing increase substantially. Further, conditional on low levels of FI, the probability of just one cohort crashing (10%) exceeds the probability of two or three cohorts crashing combined (4% + 3%). However, conditional on high levels of FI, the probability of all three cohorts jointly crashing (19%) is greater than the probability of just one or two cohorts crashing (11% + 7%).[5]

4.2. Logistic regression models

This section presents logistic regression models that estimate the relation between FI and subsequent crashes. Existing research also utilizes logistic models to estimate the likelihood of market crashes (cf., Markwat, Kole, and van Dijk, 2009; and Christiansen and Ranaldo, 2009). In

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5. We conduct a similar analyses as above based on the proportion of country indexes that jointly crash, and continue to find similar results. For example, during our sample there are 22 days in which more than 50% of Cohort 3 countries experience a return below their fifth percentile. Of these 22 days, 20 follow days in which FI exceeds the 80th percentile, and only two follow days in which FI is below this cutoff. Therefore, conditional on fragility falling above (below) the 80th percentile, the probability of this level of severe joint crash across countries is 2.74% (0.07%). This equates to a difference of approximately 40 times in magnitude.

the initial logistic analysis, we define the dependent variable for various return thresholds and consider separate analyses across the world index, as well as across our cohort-specific indexes. In this analysis, the dependent variable takes the value of one for any day in which the return to the given index falls below the given return threshold, and takes the value of zero otherwise. Therefore, a positive coefficient on FI indicates a positive relation between fragility and subsequent crashes. Table 4 reports coefficient estimates on FI, as well as odds ratios which indicate the increase in the odds of a crash for a one and two standard deviation increase in FI. For notation, we use $\emptyset$ to represent the coefficient estimate on FI.

Insert Table 4 about here

The analysis in Table 4 presents how the risk state parameter impacts the probability of market crashes. The coefficients on the risk state parameter are positive and highly significant in every case considered. For example, considering Cohort 3 and returns below the fifth percentile, the coefficient estimate of 8.7 and associated odds ratios of 2.0 and 4.2 indicate that the odds of Cohort 3 crashing double, and more than quadruple, as FI increases by one and two standard deviations, respectively. The results suggest that the most dramatic relation between FI and market crashes exists for extreme crash definitions and emerging markets. That is, the increase in crashes conditional on an increase in FI is most dramatic for Cohort 3 and for returns lower than the second percentile.

We extend the logistic regression setting to examine how FI impacts the probability of simultaneous market crashes across cohort indexes, and present results in Table 5. Panels A through C present results from logistic regressions with dependent variable, $Y_t = I_{\sum X_i \geq n}$, which is an indicator variable equal to one for any day in which the number of market crashes across cohorts equals or exceeds the specified number *n*, and equal to zero otherwise. Panel D presents an ordered logit model in which the dependent variable, $Y_t = \sum X_i$, is equal to the number of cohort returns falling below the return threshold on given day *t*, and Panel E presents a similar analysis in a generalized logistic setting. Column headings of Table 5 identify the return thresholds.

Insert Table 5 about here

Logistic regressions in Table 5 indicate a strong relation between FI and the likelihood of international crashes across multiple markets. Coefficient estimates are again all positive and highly significant. The impact of FI is greatest when all three cohorts jointly crash. For example, defining crashes based on returns falling below the fifth percentile, coefficient estimates on FI monotonically increase from 6.6 to 10.3 as the dependent variable takes the value of one on a day in which at least one cohort crashes (Panel A), to Panel C in which the dependent variable takes the value of one on a day in which all three cohorts crash. Finally, Panel E presents generalized logistic regressions which compare the probability of *i* cohort indexes crashing, each relative to the state of the world in which no cohorts crash. For this analysis, we introduce the subscript '*i*' in which *i* represents the number of cohorts that crash on a specific day. As an example, $\emptyset_3$ represents the coefficient related to the chance of three cohorts jointly crashing, relative to the chance of no cohorts crashing. The results indicate that high levels of FI dramatically increase the chance of two, or all three, cohorts crashing, and either marginally increase, or in some cases even decrease, the chance of just one cohort crashing. For example, defining crashes as returns falling below the 20th percentile, the coefficient of 7.4 for $\emptyset_3$ indicates the likelihood of all three cohorts jointly crashing *increases* dramatically with FI, while the estimate of -1.8 for $\emptyset_1$ indicates that high FI actually *decreases* the chance that just one cohort will crash. In other words, when fragility is high, if a shock occurs, then it tends to propagate across a majority of markets. Specifications in which the threshold for returns is lower do indicate that high levels of FI predict an increasing risk of just one cohort crashing. For example, a one standard deviation increase in FI leads to a 1.8 times increase in the odds of one cohort exhibiting returns below its second percentile, while also leading to a 3.1 times increase in the odds of all three cohorts crashing. In the final three rows of Panel E, we present statistical tests, $\emptyset_i = \emptyset_j$, comparing coefficient estimates across levels of market crashes, testing that the impact of FI on *i* number of markets crashing is equal to the impact of FI on *j* number of markets crashing. For example, defining crashes based on the second percentile of returns, the statistic of 9.6 indicates that an increase in FI has a larger impact on the probability of all three cohorts jointly crashing, compared to its impact on the probability of just one cohort crashing. These tests indicate that in all cases, the increased probability of all three cohorts crashing together is greater than the increased probability of one or two cohorts crashing. This analysis supports the earlier results that high levels of FI increase the probability of severe crashes.[6]

6. In unreported analyses, we conduct similar logistic regressions across country indexes, rather than aggregate cohort indexes. The results are consistent with the two central previous findings. The first, increases in FI lead to increasing probabilities of market crashes. For example, defining crashes as returns below the tenth percentile, a one standard deviation increase in FI leads to a 3.98 times increase in the chance that over 75% of all countries will simultaneously crash. Second, we continue to find that conditional on high FI, global crashes are more likely than smaller crashes. Continuing the above example, a one standard deviation increase in FI only leads to an increase of

4.3. Fragility index as a predictor of positive extreme returns

Although crashes are a greater concern than ubiquitous increases in prices, we show in this section that FI also measures a greater likelihood of simultaneous large price increases. Evidently, if a global factor drives the returns in many countries, then when those countries are more sensitive to that factor, both increases and decreases in the global factor should be reflected in larger returns (in absolute value). To consider the relation between FI and positive joint co-exceedances, we perform logistic regressions similar to Table 5. In this analysis, the dependent variables are comparable to the earlier variables, but are based on cohort returns exceeding the 80^{th}, 90^{th}, 95^{th}, or 98^{th} percentiles. Table 6 reports results with return thresholds detailed in the column headings, and with the dependent variables identified in the panel headings. The results show there is a strong relation between FI and the upside, and similar to the earlier results, the relation between FI and the co-exceedances is strongest for all three cohorts jointly increasing, relative to just one or two cohorts increasing.

Insert Table 6 about here

4.4. Fragility index and volatility comparison

Within this section we compare FI to standard volatility estimates to ensure that FI contains predictive information beyond conditional volatility. Specifically, it is well-known that volatility is persistent, and volatility may also influence the PC coefficient estimates used to construct FI. For the comparison, we estimate the probability of a crash on day *t* as a function of FI and volatility, both calculated through day *t*-1. This analysis will reveal whether FI contains

1.60 times in the odds that between 25% and 50% of all countries will simultaneously crash, and an increase of 2.16 times in the odds that between 50% and 75% of all countries simultaneously crash.

predictive information regarding the likelihood of future market crashes, after controlling for conditional volatility. As an additional comparison, we also consider the predictive ability of FI relative to levels of international integration. To conduct the comparison of FI relative to volatility and integration, we conduct logistic regressions using several of the dependent variables defined in Table 5, with the fifth percentile of returns defined as the crash threshold. We then regress these dependent variables on FI and additional control variables. We present coefficient estimates and odds ratios in Table 7, with each panel containing an alternative control variable. Specifically, Panel A contains results in which we estimate a generalized autoregressive conditional heteroskedasticity (GARCH) model of conditional volatility for our all-country index return using observations through day t-1, and use these estimates to calculate a forecasted value of volatility for day t. Panels B and C contain results in which we use the same rolling-window and weighting scheme as FI calculation, and calculate the cross-sectional average of country index return standard deviations across all countries through day t-1, as well as the rolling-window standard deviation of the all-country index return through day t-1, respectively. Finally, following Pukthuanthong and Roll (2009), we measure integration as the cross-sectional average of the adjusted R-square from each PC regression used to calculate FI. This measure is also comparable to the absorption ratio, and is considered in Panel D of Table 7. The variance inflation factors (VIFs) between FI and the alternative variables all fall well below the acceptable benchmark of ten, indicating multicollinearity is not a concern. For notation, within this analysis we use subscripts, FI, σ, and AR to define coefficient estimates and odds ratios for FI, for the given volatility measure identified in the specific panel, and for the market integration measure, respectively.

*** Insert Table 7 about here***

Results in Table 7 reveal a strong relation between FI and subsequent market crashes that is robust to controlling for conditional volatility. All coefficient estimates on FI are large in magnitude, and with only one exception in which the coefficient estimate on FI is significant at the 5% level (row two, Panel C), all remaining coefficient estimates on FI are significant at the 1% level. For example, the estimate of 5.9 and associated odds ratio of 1.6 in Panel A indicates that, after controlling for forecasted volatility from the GARCH specification, a one standard deviation increase in FI leads to a 1.6 times increase in the odds that one or more cohort indexes will exhibit a subsequent return below the fifth percentile. Interestingly, this is comparable in magnitude to the value of 1.7 reported in Table 5 from the analysis that does not control for volatility. For comparison, a one standard deviation increase in the GARCH forecast only leads to a 1.2 times increase in the odds of one or more cohorts crashing. Interestingly, the comparison of FI relative to the volatility measures is most dramatic with the dependent variable that takes the value of one only on days in which all three cohorts jointly crash. In Panel A, the odds ratio of 2.2 indicates a one standard deviation increase in FI more than doubles the odds that all three cohorts will jointly crash, while a one standard deviation increase in the GARCH conditional volatility forecast only increases the odds by 1.3 times. Further, in Panels B and C, the odds ratios from FI remain approximately equal to 2.0 for the odds of all three cohorts jointly crashing, while the corresponding coefficient estimates for the alternative volatility measures are insignificant. This is very consistent with our concept of FI, as high volatility may precede isolated market crashes, but FI reveals periods in which risk is concentrated, and if a shock occurs, it would be expected to propagate across multiple markets. Finally, in all specifications considered with both FI and the adjusted *R*-square measure, the *R*-square coefficients are insignificant with associated odds ratios that are small in magnitude, while FI coefficients remain positive and highly significant. Unreported specifications using total *R*-square as well as *R*-square from regressions with only the first PC as an independent variable, yield qualitatively similar results.

Next, we perform an alternative robustness check comparing FI to volatility. We split our sample into three-month and six-month calendar periods, and assess if crashes during the current period relate to FI calculated through the previous period. This analysis will control for both previous and current volatility. For each period *T*, we estimate the PC regressions across each country with sufficient data during the period. The average loading on the first PC during period *T* provides a measure of fragility during the specific period; thus, we define this measure as FI_T. To compare FI to volatility, the realized volatility approach of Andersen et al. (2001, 2003) provides a good benchmark. They show that aggregating high-frequency squared returns creates a good measure of longer-horizon second moments. As an example, Berger and Turtle (2009) use daily cross-products to estimate quarterly market covariances for many portfolios. For each period *T*, we calculate the volatility of our all-country world index by summing daily squared returns within the period. We note that inferences are unchanged if the simple standard deviation of return replaces the realized volatility measure. From our approach, we have variables representing fragility and volatility during period *T*. To facilitate comparisons, both variables are standardized with zero mean and unit standard deviation. Finally, for each period *T*, we create three dependent variables, $Y_T = \sum I_{X \geq 1}$, $Y_T = \sum I_{X \geq 2}$, and $Y_T = \sum I_{X=3}$, which are count variables equaling the number of days within period *T* in which one, two, or three cohort daily returns fall below their fifth percentile, respectively. Table 8 presents Poisson regression results in which we regress the number of daily occurrences during each period *T*, on FI calculated during the previous period, FI_{t-1}. The regression analyses consider volatility during period *T*, σ_T, as well as during period *T*-1, σ_{T-1}. The analysis based on contemporaneous volatility indicates whether FI, calculated only through period *T*-1, contains predictive information concerning market crashes during period *T*, even after controlling for realized volatility during period *T*. This comparison may also be interpreted as an analysis of a naïve forecast of FI relative to a volatility forecast based on perfect foresight. The analysis based on lagged volatility during period *T*-1 provides an additional comparison of FI relative to volatility, and will indicate if FI estimation simply captures periods of high volatility.

Insert Table 8 about here

The results in Table 8 further verify that after controlling for volatility, FI contains significant predictive information regarding future market crashes. Given that both FI and volatility are standardized, comparisons across coefficient estimates indicate the differential impact of FI and volatility on crashes. Comparing lagged FI to contemporaneous volatility, we note that FI is significant, and large in magnitude in each possible case. For example, with three-month periods and the dependent variable based on the number of days in which all cohorts simultaneously crash, the coefficient estimate on FI, 0.6, is over twice the magnitude of the estimate on volatility, 0.2. Overall, these results indicate that even with perfect knowledge of future volatility, FI contains important predictive information. Finally, the results based on lagged FI and lagged volatility reveal a much stronger relation between FI and market crashes, relative to volatility. Specifically, we find a positive relation between lagged volatility and future market crashes only when using three-month periods and in the case in which the dependent variable is based on days in which at least one cohort index crashes. However, even in this case, the coefficient estimate is marginally significant, and a fraction of the size of FI coefficient. The remaining cases indicate an insignificant or even negative relation between lagged volatility and future crashes. Contrasting the insignificant relation between lagged volatility and future crashes, lagged FI remains positively and significantly related to future crashes. In general, the results based on FI and volatility suggest that FI is a better predictor of future crashes, even when compared to future volatility.[7]

4.5. Alternative specifications

Overall, we have shown a strong predictive relation between FI and subsequent market crashes, or international co-exceedances. However, several potential concerns regarding the implementation of our measure exist. In this section we show that the central results of our study are very robust to multiple alternative specifications that address key concerns. Panel D of Table

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7. As an additional analysis of volatility and FI, in results available upon request, we conduct Granger Causality tests using FI and volatility from the three-month and six-month periods. In each case, we consider up to four lags. In the case of six-month periods and four lags, we find that both FI influences volatility, as well as volatility influences FI. In every remaining case, we find that FI significantly influences volatility, but there is an insignificant impact of volatility on FI. These results are consistent with the concept of FI, as when FI is high, we could expect risk exposures to be concentrated and volatility to increase.

5, with negative co-exceedances defined based on the fifth percentile of returns, and a dependent variable set equal to the number of cohorts that fall below this threshold on each day, provides a good example of the central results of the study; therefore, we conduct our robustness analyses with this as a baseline specification. We note that in unreported analyses, we find that the results throughout the paper are also robust to the alternative specifications considered. In Table 9 we present results from multiple alternative specifications. Specifically, each row in Table 9 describes how the approach differs from the baseline specification, and presents the coefficient estimate on FI, associated *p*-value, and odds ratio, under the alternative approach.

Insert Table 9 about here

Results in Table 9 indicate that the earlier results are robust to all alternative specifications considered. We initially consider results relating to calendar periods and rolling-window estimation. First, our specific sample period may be of concern for two reasons. The global financial crisis beginning in 2008 could be one unique instance that is driving our results. Alternatively, data for our third cohort group are added to the sample as the data are available, therefore, data are relatively thin for Cohort 3 early in our sample period. However, the coefficient estimates of 15.3 and 7.6 for the subsample that ends on December 31, 2007 and the subsample that begins on December 1, 2000, respectively, alleviate these concerns, and show the relation between FI and subsequent crashes is robust to alternative samples. Furthermore, a potential concern might be that the decreasing weighting scheme used to estimate loadings on a 500-day rolling window artificially induces variability in average loadings that would not exist if we had weighted observations equally in our PC regressions. The significant result of 5.9 from the specification with an equal-weighting scheme reveals that variability in average loadings is not created by our weighting scheme. The significant coefficient estimates of 6.3, 3.8, and 4.0 indicate that FI is robust to a 60-day rolling-window calculation which measures FI as a short-run, rather than long-run measure, robust to comparing short-run relative to long-run FI which captures increases in short-run relative to long-run risk, and to creating FI such that the composition of the PCs, which is updated annually, is constant for each rolling window used to estimate FI, respectively.[8]

The robustness results in Table 9 also address concerns relating to aggregation of factor loadings. As discussed earlier, our approach to aggregating factor loadings would not be feasible in the restrictive international CAPM. As one alternative, we define FI as the cross-sectional 75th percentile of loadings on the world factor; this approach identifies periods in which 25% of the countries within our sample exhibit high loadings on the world factor, and would also be feasible in the international CAPM. The coefficient estimate of 4.1 indicates that high levels of this FI measure precede market crashes. Interestingly, we also find a positive coefficient estimate on dispersion of factor loadings, measured by cross-sectional standard deviation, which is equal to 6.4. Overall, the combined results based on the 75th percentile of loadings, and the standard deviation of loadings suggest that periods in which a number of countries exhibit extreme loadings on the underlying world factor reveal periods of high risk.[9]

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8. In the 60-day rolling-window specification, we maintain our decaying weighting structure such that the observation halfway through the rolling window is weighted approximately 50%.

9. In unreported results we regress the dependent variable on the average of factor loadings, the standard deviation of factor loadings, and an interaction term. The interaction term enters with a negative loading, indicating that when average exposure is already high, then high dispersion actually reduces the likelihood of a crash.

A final concern is that FI does not capture any initial market shock; rather, an initial unpredicted shock leads to an increase in volatility as well as an increase in FI. Table 9 presents results conditional on no cohort return falling below its fifth percentile in the previous ten, 20 and 50 trading days. In this way, we measure the relation between FI and co-exceedances that are not preceded by an initial crash. The coefficient estimates conditional on no preceding crash within the previous ten and 20 trading days of 5.9 and 6.8, respectively, are highly significant and similar to the full-sample benchmark of 6.8. Unconditionally, we would expect a cohort return to fall below its fifth percentile in one of every 20 trading days. Therefore, conditioning our results on no cohort experiencing a return below its fifth percentile in the preceding 50 trading days reduces the sample to only 821 trading days, but the coefficient estimate of 11.0, which is significant at the 5% level, indicates the predictive ability of FI does not require an initial shock.

5. Conclusions and discussion

In this paper we argue that the probability of a worldwide financial crash is highest during periods in which many countries share a high exposure to the world market factor. Specifically, we extend the Pukthuanthong and Roll (2009) integration analysis to develop FI. This risk measure is defined as the average loading on the world factor across countries at a point in time. There is substantial evidence that FI identifies periods of systemic risk. When a country has a high loading on the first PC from Pukthuanthong and Roll (2009), it is heavily exposed to the world factor and thus may not offer diversification. Consequently, when exposures are high across multiple countries and the world factor becomes volatile or is subject to negative shocks, multiple markets will simultaneously crash. The strongest relation between FI and subsequent crashes exists for crashes defined as simultaneous co-exceedances across all markets, for the emerging market cohort, and for extreme crash definitions.

The results are robust across multiple dimensions. We find that FI predicts crashes across each cohort index individually, as well as across all cohorts jointly. Further, robustness analyses reveal that FI does not simply capture volatility impacts, or levels of worldwide integration, and also does not require an initial shock. Finally, the relation between FI and subsequent crashes is robust to all of the alternative parameterizations for FI considered.

Our study lays down a fundamental for future studies. Policy makers should adopt FI to predict the period during which the economy is most fragile and systemic risk is high. Future researchers can explore why our fragility index is an example of systemic risks. Can it be explained by frictions as other papers on crises have argued? Greenwood and Thesmar (2011) study institutional ownership and “fragility” of individual stocks while Jotikasthira, Lundblad, and Ramadorai (2011) define “capital at-risk” and relate it to fire sales. There should be more studies showing the underlying asset pricing model and explanations behind the results of our study.

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Appendix. Country index sample periods and index identification

Eighty-Two countries have index data availability from DataStream, a division of Thomson Financial. Some countries have several indexes and the index chosen has the longest period of data availability. All index values are converted into a common currency, the US dollar. An index with the designation "RI" is a total return index (with reinvested dividends). The designation "PI" denotes a pure price index. When calculating log returns from the indexes, neither the beginning nor the ending index value can be identical to its immediately preceding index value; (this eliminates holidays, which vary across countries, and days with obviously stale prices). Cohorts 1, 2, and 3 include countries appearing in DataStream prior to 1984, from 1984 through 1993, and post-1993.

	DataStream availability				
Country	Begins	Ends	Cohort	Index identification	DataStream mnemonic
Argentina	2-Aug-93	30-Nov-10	2	ARGENTINA MERVAL	ARGMERV(PI)~U$
Australia	31-Dec-79	30-Nov-10	1	AUSTRALIA-DS MARKET	TOTMAU$(RI)
Austria	1-Jan-73	30-Nov-10	1	AUSTRIA-DS Market	TOTMKOE(RI)~U$
Bahrain	31-Dec-99	30-Nov-10	3	DOW JONES BAHRAIN	DJBAHR$(PI)
Bangladesh	1-Jan-90	30-Nov-10	2	BANGLADESH SE ALL SHARE	BDTALSH(PI)~U$
Belgium	31-Dec-79	30-Nov-10	1	BELGIUM-DS Market	TOTMKBG(RI)~U$
Botswana	29-Dec-95	30-Nov-10	3	S&P/IFCF M BOTSWA0.	IFFMBOL(PI)~U$
Brazil	7-Apr-83	30-Nov-10	1	BRAZIL BOVESPA	BRBOVES(PI)~U$
Bulgaria	20-Oct-00	30-Nov-10	3	BSE SOFIX	BSSOFIX(PI)~U$
Canada	31-Dec-79	30-Nov-10	1	S&P/TSX COMPOSITE INDEX	TTOCOMP(RI)~U$
Chile	2-Jan-87	30-Nov-10	2	CHILE GENERAL (IGPA)	IGPAGEN(PI)~U$
China	3-Apr-91	30-Nov-10	2	SHENZHEN SE COMPOSITE	CHZCOMP(PI)~U$
Colombia	10-Mar-92	30-Nov-10	2	COLOMBIA-DS Market	TOTMKCB(RI)~U$
Côte d'Ivoire	29-Dec-95	30-Nov-10	3	S&P/IFCF M COTE D'IVOIRE	IFFMCIL(RI)~U$
Croatia	2-Jan-97	30-Nov-10	3	CROATIA CROBEX	CTCROBE(PI)~U$
Cyprus	3-Sep-04	30-Nov-10	3	CYPRUS GENERAL	CYPMAPM(PI)~U$
Czech Republic	9-Nov-93	30-Nov-10	2	CZECH REP.-DS NON-FINCIAL	TOTLICZ(RI)~U$
Denmark	31-Dec-79	30-Nov-10	1	MSCI DENMARK	MSDNMKL(RI)~U$
Ecuador	2-Aug-93	30-Nov-10	2	ECUADOR ECU (U$)	ECUECUI(PI)
Egypt	2-Jan-95	30-Nov-10	3	EGYPT HERMES FINANCIAL	EGHFINC(PI)~U$
Estonia	3-Jun-96	30-Nov-10	3	OMX TALLINN (OMXT)	ESTALSE(PI)~U$
Finland	2-Jan-91	30-Nov-10	2	OMX HELSINKI (OMXH)	HEXINDX(RI)~U$
France	31-Dec-79	30-Nov-10	1	FRANCE-DS Market	TOTMKFR(RI)~U$
Germany	31-Dec-79	30-Nov-10	1	DAX 30 PERFORMANCE	DAXINDX(RI)~U$
Ghana	29-Dec-95	30-Nov-10	3	S&P/IFCF M GHA0.	IFFMGHL(PI)~U$

Greece	26-Jan-06	30-Nov-10	3	ATHEX COMPOSITE	GRAGENL(RI)~U$
Hong Kong	2-Jan-90	30-Nov-10	2	HANG SENG	HNGKNGI(RI)~U$
Hungary	2-Jan-91	30-Nov-10	2	BUDAPEST (BUX)	BUXINDX(PI)~U$
Iceland	31-Dec-92	30-Nov-10	2	OMX ICELAND ALLSHARE	ICEXALL(PI)~U$
India	2-Jan-87	30-Nov-10	2	INDIA BSE (100) NATIONAL	IBOMBSE(PI)~U$
Indonesia	2-Apr-90	30-Nov-10	2	INDONESIA-DS Market	TOTMKID(RI)~U$
Ireland	1-Jan-73	30-Nov-10	1	IRELAND-DS MARKET	TOTMIR$(RI)
Israel	23-Apr-87	30-Nov-10	2	ISRAEL TA 100	ISTA100(PI)~U$
Italy	31-Dec-79	30-Nov-10	1	ITALY-DS MARKET	TOTMIT$(RI)
Jamaica	29-Dec-95	30-Nov-10	3	S&P/IFCF M JAMAICA	IFFMJAL(PI)~U$
Japan	31-Dec-79	30-Nov-10	1	TOPIX	TOKYOSE(RI)~U$
Jordan	21-Nov-88	30-Nov-10	2	AMMAN SE FINANCIAL MARKET	AMMANFM(PI)~U$
Kenya	11-Jan-90	30-Nov-10	2	KENYA NAIROBI SE	NSEINDX(PI)~U$
Kuwait	28-Dec-94	30-Nov-10	3	KUWAIT KIC GENERAL	KWKICGN(PI)~U$
Latvia	3-Jan-00	30-Nov-10	3	OMX RIGA (OMXR)	RIGSEIN(RI)~U$
Lebanon	31-Jan-00	30-Nov-10	3	S&P/IFCF M LEBANON	IFFMLEL(PI)~U$
Lithuania	31-Dec-99	30-Nov-10	3	OMX VILNIUS (OMXV)	LNVILSE(RI)~U$
Luxembourg	2-Jan-92	30-Nov-10	2	LUXEMBURG-DS MARKET	LXTOTMK(RI)~U$
Malaysia	2-Jan-80	30-Nov-10	1	KLCI COMPOSITE	KLPCOMP(PI)~U$
Malta	27-Dec-95	30-Nov-10	3	MALTA SE MSE -	MALTAIX(PI)~U$
Mauritius	29-Dec-95	30-Nov-10	3	S&P/IFCF M MAURITIUS	IFFMMAL(PI)~U$
Mexico	4-Jan-88	30-Nov-10	2	MEXICO IPC (BOLSA)	MXIPC35(PI)~U$
Morocco	31-Dec-87	30-Nov-10	2	MOROCCO SE CFG25	MDCFG25(PI)~U$
Namibia	31-Jan-00	30-Nov-10	3	S&P/IFCF M NAMBIA	IFFMNAL(PI)~U$
Netherlands	31-Dec-79	30-Nov-10	1	NETHERLAND-DS Market	TOTMKNL(RI)~U$
New Zealand	4-Jan-88	30-Nov-10	2	NEW ZEALAND-DS MARKET	TOTMNZ$(RI)
Nigeria	30-June-95	30-Nov-10	3	S&P/IFCG D NIGERIA	IFGDNGL(PI)~U$
Norway	2-Jan-80	30-Nov-10	1	NORWAY-DS MARKET	TOTMNW$(RI)
Oman	22-Oct-96	30-Nov-10	3	OMAN MUSCAT SECURITIES MKT.	OMANMSM(PI)~U$
Pakistan	30-Dec-88	30-Nov-10	2	KARACHI SE 100	PKSE100(PI)~U$
Peru	2-Jan-91	30-Nov-10	2	LIMA SE GENERAL(IGBL)	PEGENRL(PI)~U$
Philippines	2-Jan-86	30-Nov-10	2	PHILIPPINE SE I(PSEi)	PSECOMP(PI)~U$
Poland	16-Apr-91	30-Nov-10	2	WARSAW GENERALINDEX	POLWIGI(PI)~U$
Portugal	5-Jan-88	30-Nov-10	2	PORTUGAL PSI GENERAL	POPSIGN(PI)~U$
Romania	19-Sep-97	30-Nov-10	3	ROMANIA BET (L)	RMBETRL(PI)~U$
Russia	1-Sep-95	30-Nov-10	3	RUSSIA RTS INDEX	RSRTSIN(PI)~U$
Saudi Arabia	31-Dec-97	30-Nov-10	3	S&P/IFCG D SAUDI ARABIA	IFGDSB$(RI)
Singapore	1-Jan-73	30-Nov-10	1	SINGAPORE-DS MARKET EX TMT	TOTXTSG(RI)~U$

Slovakia	14-Sep-93	30-Nov-10	2	SLOVAKIA SAX 16	SXSAX16(PI)~U$
Slovenia	31-Dec-93	30-Nov-10	2	SLOVENIAN EXCH. STOCK (SBI)	SLOESBI(PI)~U$
South Africa	31-Dec-79	30-Nov-10	1	SOUTH AFRICA-DS MARKET	TOTMSA$(RI)
South Korea	31-Dec-79	30-Nov-10	1	KOREA SE COMPOSITE (KOSPI)	KORCOMP(PI)~U$
Spain	31-Dec-79	30-Nov-10	1	MADRID SE GENERAL	MADRIDI(PI)~U$
Sri Lanka	2-Jan-85	30-Nov-10	2	COLOMBO SE ALLSHARE	SRALLSH(PI)~U$
Sweden	28-Dec-79	30-Nov-10	1	OMX STOCKHOLM (OMXS)	SWSEALI(PI)~U$
Switzerland	31-Dec-79	30-Nov-10	1	SWITZ-DS Market	TOTMKSW(RI)~U$
Taiwan	31-Dec-84	30-Nov-10	2	TAIWAN SE WEIGHTED	TAIWGHT(PI)~U$
Thailand	2-Jan-87	30-Nov-10	2	THAILAND-DS MARKET	TOTMTH$(RI)
Trinidad	29-Dec-95	30-Nov-10	3	S&P/IFCF M TRINIDAD & TOBAGO	IFFMTTL(PI)~U$
Tunisia	31-Dec-97	30-Nov-10	3	TUNISIA TUNINDEX	TUTUNIN(PI)~U$
Turkey	4-Jan-88	30-Nov-10	2	ISE TIOL 100	TRKISTB(PI)~U$
Ukraine	30-Jan-98	30-Nov-10	3	S&P/IFCF M UKRAINE	IFFMURL(PI)~U$
Utd. Arab Emirates	1-Jun-05	30-Nov-10	3	MSCI UAE	MSUAEI$
United Kingdom	31-Dec-79	30-Nov-10	1	UK-DS MARKET	TOTMUK$(RI)
United States	31-Dec-79	30-Nov-10	1	S&P 500 COMPOSITE	S&PCOMP(RI)~U$
Venezuela	2-Jan-90	30-Nov-10	2	VENEZUELA-DS MARKET	TOTMVE$(RI)
Zimbabwe	6-Apr-88	6-Oct-06	2	ZIMBABWE INDUSTRIALS	ZIMINDS(PI)

Table 1

Average returns across risk states

The table presents summary statistics of equal-weighted returns, in percentage form, to the all-country index, as well as to the Cohort 1, 2, and 3 indexes. Cohorts 1, 2, and 3 are stock indexes of countries that appear in DataStream prior to 1984, from 1984 through 1993, and post-1993, respectively. Panel A presents statistics based on full-sample returns, while Panel B presents returns across levels of fragility. Fragility is based on the coefficient, $\beta_{j,i,t}$, on the first PC according to Pukthuanthong and Roll (2009), in which country stock returns are regressed on ten PCs using daily observations from day *t*-500 through day *t*-1. We restrict the analysis to countries with at least 100 usable observations during any particular period, and use weighted least squares in which the weight placed on daily observation *t*-x is equal to 0.995^{x-1}. Panel B presents results across deciles of fragility formed from the mean of loadings on the first PC across countries at a given point in time. In Panel B, average loadings and subsequent deciles of fragility are specific to countries included within each cohort. The sample begins December 29, 1994 and ends November 11, 2010. Countries included within the sample, and their cohort assignments, are included in the Appendix.

	World index			*Cohort* 1			*Cohort* 2			*Cohort* 3		
	Mean	Median	Std	Mean	Median	Std	Mean	Median	Std	Mean	Median	Std
Panel A: *Full-sample summary statistics*												
	0.0254	0.0719	0.8046	0.0234	0.0875	1.1217	0.0210	0.0702	0.7731	0.0344	0.0569	1.1814
Panel B: *Statistics across deciles of fragility*												
1st decile	0.1075	0.1515	0.5164	0.0532	0.1343	0.9910	0.1208	0.1224	0.4605	0.0617	0.0591	0.5212
2nd decile	0.0235	0.0538	0.5153	0.0520	0.0875	0.5651	0.0191	0.0060	0.3860	0.0978	0.0896	0.4669
3rd decile	0.1052	0.1040	0.3931	0.0304	0.0686	0.6249	0.0286	0.0795	0.5357	0.2122	0.0689	2.2370
4th decile	0.0991	0.0737	0.7785	0.0436	0.1199	0.7184	0.0625	0.0801	0.5019	0.0777	0.0703	0.4693
5th decile	-0.0107	0.0404	0.4755	-0.0562	0.0235	0.9391	-0.0215	0.0012	0.6566	0.0669	0.0566	0.4900
6th decile	-0.0006	0.0329	0.6113	-0.0209	-0.0134	0.9605	-0.0180	0.0288	0.6902	0.0749	0.0684	0.6476
7th decile	0.0224	0.0725	0.6760	0.1093	0.1818	0.8595	0.0485	0.1441	0.8113	-0.0362	0.0541	0.6742
8th decile	-0.0276	0.0578	0.9863	0.0752	0.1564	0.8160	0.0447	0.1314	0.8280	-0.1280	-0.0300	2.2038
9th decile	-0.1170	0.0224	1.0872	-0.1291	-0.0442	1.7080	-0.0992	-0.0042	0.9887	-0.0697	0.0189	0.9673
10th decile	0.0524	0.1538	1.3939	0.0771	0.1779	2.0568	0.0247	0.1352	1.3536	-0.0138	0.0553	1.1322

Table 2

Conditional market crash probabilities

The table presents conditional probabilities of market crashes for the equal-weighted all-country index. Risk states are determined by FI, which is the average of $\beta_{j,i,t}$ across all countries at a given point t, and defined as $\mu_{PC1,t}$. $\beta_{j,i,t}$ for each country j and each point in time t is the coefficient of the first component (PC1) and estimated from daily observations from day t-500 through day t-1. The coefficient of PC1 is estimated by regressing country stock returns on ten PCs constructed according to Pukthuanthong and Roll (2009). We restrict the analysis to countries with at least 100 usable observations during any particular period, and use weighted least squares in which the weight placed on daily observation t-x is equal to 0.995^{x-1}. Market crashes are defined as a daily return falling below the percentile listed in the column heading. Table rows present the expected number of crashes, the frequency of crashes, and the percentage of crashes. The final row in each subpanel presents a *Z*-score and associated *p*-value testing that the probability of a crash is equal across risk states. The sample is daily from December 29, 1994 through November 30, 2010. The Appendix lists the countries included in the sample.

	$R_{j,t} \leq P20\%$	$R_{j,t} \leq P10\%$	$R_{j,t} \leq P5\%$	$R_{j,t} \leq P2\%$
$Ex(X \mid \mu_{PC1,t} < P80\%(\mu_{PC1}))$	598.36	299.18	149.59	59.20
$f(X \mid \mu_{PC1,t} < 80\%(\mu_{PC1}))$	498	213	90	27
$f/n(X \mid \mu_{PC1,t} < P80\%(\mu_{PC1}))$	16.63	7.11	3.01	0.90
$Ex(X \mid \mu_{PC1,t} > P80\%(\mu_{PC1}))$	149.64	74.82	37.41	14.80
$f(X \mid \mu_{PC1,t} > P80\%(\mu_{PC1}))$	250	161	97	47
$f/n(X \mid \mu_{PC1,t} > P80\%(\mu_{PC1}))$	33.38	21.50	12.95	6.28
$H_0: d = 0$	9.042	9.145	7.857	5.952
	(0.000)	(0.000)	(0.000)	(0.000)
$Ex(X \mid \mu_{PC1,t} < P90\%(\mu_{PC1}))$	673.28	336.64	168.32	66.61
$f(X \mid \mu_{PC1,t} < 90\%(\mu_{PC1}))$	627	288	128	47
$f/n(X \mid \mu_{PC1,t} < P90\%(\mu_{PC1}))$	18.61	8.55	3.80	1.39
$Ex(X \mid \mu_{PC1,t} > P90\%(\mu_{PC1}))$	74.72	37.36	18.68	7.39
$f(X \mid \mu_{PC1,t} > P90\%(\mu_{PC1}))$	121	86	59	27
$f/n(X \mid \mu_{PC1,t} > P90\%(\mu_{PC1}))$	32.35	22.99	15.78	7.22
$H_0: d = 0$	5.477	6.483	6.260	4.304
	(0.000)	(0.000)	(0.000)	(0.000)
$Ex(X \mid \mu_{PC1,t} < P95\%(\mu_{PC1}))$	710.64	355.32	177.66	70.30
$f(X \mid \mu_{PC1,t} < 95\%(\mu_{PC1}))$	673	319	147	56
$f/n(X \mid \mu_{PC1,t} < P95\%(\mu_{PC1}))$	18.92	8.97	4.13	1.57
$Ex(X \mid \mu_{PC1,t} > P95\%(\mu_{PC1}))$	37.36	18.68	9.34	3.70
$f(X \mid \mu_{PC1,t} > P95\%(\mu_{PC1}))$	75	55	40	18
$f/n(X \mid \mu_{PC1,t} > P95\%(\mu_{PC1}))$	40.11	29.41	21.39	9.63
$H_0: d = 0$	5.815	6.073	5.720	3.716
	(0.000)	(0.000)	(0.000)	(0.000)

Table 2
Cont'd

$Ex(X \mid \mu_{PC1,t} < P98\%(\mu_{PC1}))$	733.22	366.61	183.30	72.54
$f(X \mid \mu_{PC1,t} < 98\%(\mu_{PC1}))$	715	345	163	64
$f/n(X \mid \mu_{PC1,t} < P98\%(\mu_{PC1}))$	19.48	9.40	4.44	1.74
$Ex(X \mid \mu_{PC1,t} > P98\%(\mu_{PC1}))$	14.78	7.39	3.70	1.46
$f(X \mid \mu_{PC1,t} > P98\%(\mu_{PC1}))$	33	29	24	10
$f/n(X \mid \mu_{PC1,t} > P98\%(\mu_{PC1}))$	44.59	39.19	32.43	13.51
$H_0: d = 0$	4.318	5.23	5.13	2.957
	(0.000)	(0.000)	(0.000)	(0.002)

Table 3
Conditional probabilities of joint crashes
The table presents the probability of joint market crashes across cohorts, conditional on market states. We consider equal-weighted returns to the three cohorts within our sample. Countries included in the sample and their cohort assignment are listed in the Appendix. Risk states are indicated in the initial column, and return thresholds that define crashes are indicated in the panel headings. Risk states are determined by FI, which is the average of $\beta_{j,i,t}$ across all countries at a given point *t*, and defined as $\mu_{PC1,t}$. $\beta_{j,i,t}$ for each country *j* and each point in time *t* is the coefficient of the first component (PC1) from Pukthuanthong and Roll (2009). Estimation of $\beta_{j,i,t}$ is described in Table 2. We present the frequency ($f(X)$) and empirical probabilities ($f/n(X)$) of *X* cohort indexes jointly crashing on a given day across risk states, with *X* labeled in the column headings. For the high risk states, we present the expected number of joint occurrences ($Ex(X)$), and the chi-squared statistic and associated *p*-value testing for independence between risk states and crashes. The sample is daily from December 29, 1994 through November 30, 2010.

Panel A: *Crash defined as* $R_{j,t} \leq P20\%$

Risk state	Statistic	$X = 0$	$X = 1$	$X = 2$	$X = 3$
$\mu_{PC1,t} \leq P80\%$	$f(X)$	2038	531	270	156
	$f/n(X)$	68.05	17.73	9.02	5.21
$\mu_{PC1,t} \geq P80\%$	$f(X)$	436	79	76	158
	$Ex(X)$	494.93	122.03	69.22	62.82
	$f/n(X)$	58.21	10.55	10.15	21.09
	χ^2	7.02	15.78	0.66	144.23
		(0.071)	(0.001)	(0.883)	(0.000)
$\mu_{PC1,t} \leq P90\%$	$f(X)$	2253	576	309	232
	$f/n(X)$	66.85	17.09	9.17	6.88
$\mu_{PC1,t} \geq P90\%$	$f(X)$	221	34	37	82
	$Ex(X)$	247.14	60.94	34.56	31.37
	$f/n(X)$	59.09	9.09	9.89	21.93
	χ^2	2.76	11.91	0.17	81.74
		(0.430)	(0.008)	(0.982)	(0.000)
$\mu_{PC1,t} \leq P95\%$	$f(X)$	2378	593	324	262
	$f/n(X)$	66.85	16.67	9.11	7.37
$\mu_{PC1,t} \geq P95\%$	$f(X)$	96	17	22	52
	$Ex(X)$	123.57	30.47	17.28	15.68
	$f/n(X)$	51.34	9.09	11.76	27.81
	χ^2	6.15	5.95	1.29	84.10
		(0.105)	(0.114)	(0.732)	(0.000)
$\mu_{PC1,t} \leq P98\%$	$f(X)$	2442	600	339	289
	$f/n(X)$	66.54	16.35	9.24	7.87
$\mu_{PC1,t} \geq P98\%$	$f(X)$	32	10	7	25
	$Ex(X)$	48.90	12.06	6.84	6.21
	$f/n(X)$	43.24	13.51	9.46	33.78
	χ^2	5.84	0.35	0.00	56.91
		(0.120)	(0.950)	(1.000)	(0.000)

Table 3
Cont'd

Panel B: *Crash defined as* $R_{j,t} \leq P10\%$					
Risk state	Statistic	$X = 0$	$X = 1$	$X = 2$	$X = 3$
$\mu_{PC1,t} \leq P80\%$	$f(X)$	2540	296	107	52
	$f/n(X)$	84.81	9.88	3.57	1.74
$\mu_{PC1,t} \geq P80\%$	$f(X)$	538	66	45	100
	$Ex(X)$	615.76	72.42	30.41	30.41
	$f/n(X)$	71.83	8.81	6.01	13.35
	χ^2	9.82	0.57	7.00	159.27
		(0.020)	(0.903)	(0.072)	(0.000)
$\mu_{PC1,t} \leq P90\%$	$f(X)$	2816	328	127	99
	$f/n(X)$	83.56	9.73	3.77	2.94
$\mu_{PC1,t} \geq P90\%$	$f(X)$	262	34	25	53
	$Ex(X)$	307.47	36.16	15.18	15.18
	$f/n(X)$	70.05	9.09	6.68	14.17
	χ^2	6.72	0.13	6.35	94.18
		(0.081)	(0.988)	(0.096)	(0.000)
$\mu_{PC1,t} \leq P95\%$	$f(X)$	2961	342	138	116
	$f/n(X)$	83.24	9.61	3.88	3.26
$\mu_{PC1,t} \geq P95\%$	$f(X)$	117	20	14	36
	$Ex(X)$	153.74	18.08	7.59	7.59
	$f/n(X)$	62.57	10.70	7.49	19.25
	χ^2	8.78	0.20	5.41	106.30
		(0.032)	(0.978)	(0.144)	(0.000)
$\mu_{PC1,t} \leq P98\%$	$f(X)$	3039	354	145	132
	$f/n(X)$	82.81	9.65	3.95	3.60
$\mu_{pC1,t} \geq P98\%$	$f(X)$	39	8	7	20
	$Ex(X)$	60.84	7.15	3.00	3.00
	$f/n(X)$	52.70	10.81	9.46	27.03
	χ^2	7.84	0.10	5.31	96.15
		(0.049)	(0.992)	(0.150)	(0.000)

Table 3
Cont'd

Panel C: *Crash defined as* $R_{j,t} \leq P5\%$					
Risk state	Statistic	$X = 0$	$X = 1$	$X = 2$	$X = 3$
$\mu_{PC1,t} \leq P80\%$	$f(X)$	2807	119	45	24
	$f/n(X)$	93.72	3.97	1.50	0.80
$\mu_{PC1,t} \geq P80\%$	$f(X)$	606	58	33	52
	$Ex(X)$	682.78	35.41	15.60	15.20
	$f/n(X)$	80.91	7.74	4.41	6.94
	χ^2	8.63	14.41	19.39	89.05
		(0.035)	(0.002)	(0.000)	(0.000)
$\mu_{PC1,t} \leq P90\%$	$f(X)$	3123	143	56	48
	$f/n(X)$	92.67	4.24	1.66	1.42
$\mu_{PC1,t} \geq P90\%$	$f(X)$	290	34	22	28
	$Ex(X)$	340.94	17.68	7.79	7.59
	$f/n(X)$	77.54	9.09	5.88	7.49
	χ^2	7.61	15.06	25.91	54.86
		(0.055)	(0.002)	(0.000)	(0.000)
$\mu_{PC1,t} \leq P95\%$	$f(X)$	3283	152	63	59
	$f/n(X)$	92.30	4.27	1.77	1.66
$\mu_{PC1,t} \geq P95\%$	$f(X)$	130	25	15	17
	$Ex(X)$	170.47	8.84	3.90	3.80
	$f/n(X)$	69.52	13.37	8.02	9.09
	χ^2	9.61	29.54	31.65	45.93
		(0.022)	(0.000)	(0.000)	(0.000)
$\mu_{PC1,t} \leq P98\%$	$f(X)$	3373	163	66	68
	$f/n(X)$	91.91	4.44	1.80	1.85
$\mu_{PC1,t} \geq P98\%$	$f(X)$	40	14	12	8
	$Ex(X)$	67.46	3.50	1.54	1.50
	$f/n(X)$	54.05	18.92	16.22	10.81
	χ^2	11.18	31.52	70.95	28.11
		(0.011)	(0.000)	(0.000)	(0.000)

Table 3
Cont'd

Panel D: *Crash defined as* $R_{j,t} \leq P2\%$					
Risk state	Statistic	$X = 0$	$X = 1$	$X = 2$	$X = 3$
$\mu_{PC1,t} \leq P80\%$	$f(X)$	2933	39	18	5
	$f/n(X)$	97.93	1.30	0.60	0.17
$\mu_{PC1,t} \geq P80\%$	$f(X)$	677	35	14	23
	$Ex(X)$	722.19	14.80	6.40	5.60
	$f/n(X)$	90.39	4.67	1.87	3.07
	χ^2	2.83	27.55	9.02	54.04
		(0.419)	(0.000)	(0.029)	(0.000)
$\mu_{PC1,t} \leq P90\%$	$f(X)$	3277	54	25	14
	$f/n(X)$	97.24	1.60	0.74	0.42
$\mu_{PC1,t} \geq P90\%$	$f(X)$	333	20	7	14
	$Ex(X)$	360.61	7.39	3.20	2.80
	$f/n(X)$	89.04	5.35	1.87	3.74
	χ^2	2.11	21.50	4.53	44.87
		(0.550)	(0.000)	(0.210)	(0.000)
$\mu_{PC1,t} \leq P95\%$	$f(X)$	3452	59	28	18
	$f/n(X)$	97.05	1.66	0.79	0.51
$\mu_{PC1,t} \geq P95\%$	$f(X)$	158	15	4	10
	$Ex(X)$	180.31	3.70	1.60	1.40
	$f/n(X)$	84.49	8.02	2.14	5.35
	χ^2	2.76	34.57	3.61	52.90
		(0.430)	(0.000)	(0.307)	(0.000)
$\mu_{PC1,t} \leq P98\%$	$f(X)$	3554	64	30	22
	$f/n(X)$	96.84	1.74	0.82	0.60
$\mu_{PC1,t} \geq P98\%$	$f(X)$	56	10	2	6
	$Ex(X)$	71.35	1.46	0.63	0.55
	$f/n(X)$	75.68	13.51	2.70	8.11
	χ^2	3.30	49.83	2.96	53.60
		(0.348)	(0.000	(0.398)	(0.000)

Table 4
Logistic regressions within cohort indexes
This table presents logistic regression results in which the occurrence of market crashes is regressed on FI. For each subpanel, the dependent variable takes a value of one if the return to the equal-weighted index given in the column heading falls below the threshold identified in the panel heading. We report coefficient estimates on FI, denoted $\emptyset$, as well as associated *p*-values. FI is the average of $\beta_{j,i,t}$ across all countries within the given cohort at a given point *t*. $\beta_{j,i,t}$ for each country *j* and each point in time *t* is the coefficient of the first component (PC1) and estimated from daily observations from day *t*-500 through day *t*-1. The coefficient of PC1 is estimated by regressing country stock returns on ten PCs constructed according to Pukthuanthong and Roll (2009). We restrict the PC calculation to countries with at least 100 usable observations during any particular period and use weighted least squares in which the weight placed on daily observation *t*-x is equal to 0.995^{x-1}. $OR_{1\sigma}$ and $OR_{2\sigma}$ represent the odds ratio of a crash when FI increases by one and two standard deviations, respectively. Cohort 1, Cohort 2, and Cohort 3 are formed from country indexes with available data beginning prior to 1984, from 1984 through 1993, and post-1993, respectively. The sample is daily from December 29, 1994 through November 30, 2010. The Appendix provides the list of countries included in the sample and their cohort assignment.

	$World\ index$	$Cohort$ 1	$Cohort$ 2	$Cohort$ 3
		$R_{j,t} \leq P20\%$		
$\emptyset$	4.450	2.376	5.240	4.192
	(0.000)	(0.000)	(0.000)	(0.000)
$OR_{1\sigma}$	1.438	1.332	1.483	1.407
$OR_{2\sigma}$	2.069	1.775	2.198	1.980
		$R_{j,t} \leq P10\%$		
$\emptyset$	6.378	3.741	7.520	6.063
	(0.000)	(0.000)	(0.000)	(0.000)
$OR_{1\sigma}$	1.684	1.571	1.760	1.639
$OR_{2\sigma}$	2.835	2.468	3.096	2.687
		$R_{j,t} \leq P5\%$		
$\emptyset$	8.125	4.938	8.416	8.736
	(0.000)	(0.000)	(0.000)	(0.000)
$OR_{1\sigma}$	1.942	1.815	1.882	2.038
$OR_{2\sigma}$	3.771	3.294	3.542	4.153
		$R_{j,t} \leq P2\%$		
$\emptyset$	9.808	6.829	8.568	10.587
	(0.000)	(0.000)	(0.000)	(0.000)
$OR_{1\sigma}$	2.228	2.280	1.904	2.370
$OR_{2\sigma}$	4.964	5.200	3.624	5.617

Table 5

Logistic regressions across cohorts

The table presents logistic regression results in which co-exceedances are regressed on FI. Column headings define return thresholds that determine market crashes. In Panels A through C, the dependent variables are indicator variables that take the value of one for any day *t* in which the number of market crashes across the three cohorts exceeds the specified value, and the value of zero otherwise. For example, in Panel B the dependent variable takes the value of one for any day in which at least two of the three cohort index returns fall below the percentile given in the column heading. Panel D presents ordinal logistic regressions in which the dependent variable is set equal to the number of cohorts experiencing a crash on the given day. Panel E presents generalized logistic results in which the dependent variable is as defined in Panel D. The final three rows of Panel E present statistical tests of equality across coefficients. We use $\emptyset$ to represent the coefficient estimate on FI, and the associated *p*-value is reported below. FI estimation and row headings are defined in Table 4. Finally, the notation $\emptyset_i$ and $OR_{1\sigma.i}$ in Panel E refers to parameter estimates, and associated odds ratios for a one standard deviation increase in FI, for the state in which *i* cohorts jointly crash. Cohort 1, Cohort 2, and Cohort 3 are formed from country indexes with available data beginning prior to 1984, from 1984 through 1993, and post-1993, respectively. The sample is daily from December 29, 1994 through November 30, 2010. The Appendix provides the list of countries included in the sample and their cohort assignment.

	$R_{j,t} \leq P20\%$	$R_{j,t} \leq P10\%$	$R_{j,t} \leq P5\%$	$R_{j,t} \leq P2\%$
Panel A: $Y_t = I_{\sum X_i \geq 1}$				
$\emptyset$	2.192	4.256	6.623	8.159
	(0.000)	(0.000)	(0.000)	(0.000)
$OR_{1\sigma}$	1.196	1.416	1.718	1.947
$OR_{2\sigma}$	1.431	2.004	2.950	3.792
Panel B: $Y_t = I_{\sum X_i \geq 2}$				
$\emptyset$	4.979	7.186	8.525	9.244
	(0.000)	(0.000)	(0.000)	(0.000)
$OR_{1\sigma}$	1.502	1.798	2.006	2.128
$OR_{2\sigma}$	2.255	3.235	4.025	4.527
Panel C: $Y_t = I_{\sum X_i = 3}$				
$\emptyset$	7.569	9.942	10.364	13.543
	(0.000)	(0.000)	(0.000)	(0.000)
$OR_{1\sigma}$	1.856	2.252	2.331	3.023
$OR_{2\sigma}$	3.443	5.073	5.435	9.136

Panel D: $Y_t = \sum X_i$				
$\emptyset$	3.430	4.917	6.828	8.229
	(0.000)	(0.000)	(0.000)	(0.000)
$OR_{1\sigma}$	1.323	1.494	1.747	1.958
$OR_{2\sigma}$	1.751	2.233	3.051	3.835
χ^2	153.291	99.602	18.978	11.804
	(0.000)	(0.000)	(0.000)	(0.003)
Panel E: *Generalized logistic regression*				
$\emptyset_3$	7.441	10.250	10.908	13.873
	(0.000)	(0.000)	(0.000)	(0.000)
$OR_{1\sigma.3}$	1.836	2.310	2.437	3.105
$\emptyset_2$	1.352	3.891	6.812	6.023
	(0.051)	(0.000)	(0.000)	(0.000)
$OR_{1\sigma,2}$	1.117	1.374	1.744	1.635
$\emptyset_1$	-1.846	0.787	4.480	7.081
	(0.006)	(0.260)	(0.000)	(0.000)
$OR_{1\sigma,1}$	0.860	1.066	1.442	1.783
$\emptyset_3 = \emptyset_1$	134.410	93.441	25.568	9.569
	(0.000)	(0.000)	(0.000)	(0.002)
$\emptyset_3 = \emptyset_2$	55.659	33.416	7.937	9.696
	(0.000)	(0.000)	(0.005)	(0.002)
$\emptyset_2 = \emptyset_1$	13.013	8.600	3.400	0.304
	(0.000)	(0.003)	(0.065)	(0.581)

Table 6
Fragility index and right tail joint co-exceedances
The table presents logistic regression results in which positive co-exceedances are regressed on FI. Column headings define return thresholds that determine extreme positive market returns. In Panels A through C, the dependent variable takes the value of one for any day *t* in which the number of market positive exceedances across the three cohorts exceeds the specified value. For example, in Panel B the dependent variable takes the value of one for any day in which at least two of the three cohort index returns fall above the percentile given in the column heading. Panel D presents ordinal logistic regressions in which the dependent variable is set equal to the number of cohorts experiencing positive return exceedances on the given day. Panel E presents generalized logistic results in which the dependent variable is as defined in Panel D. The final three rows of Panel E present statistical tests of equality across coefficients. We use $\emptyset$ to represent the coefficient estimate on FI, and the associated *p*-value is reported below. FI estimation and row headings are defined in Table 4. Finally, the notation $\emptyset_i$ and $OR_{1\sigma.i}$ in Panel E refers to parameter estimates, and associated odds ratios for a one standard deviation increase in FI, for the state in which *i* cohorts jointly increase. Cohort 1, Cohort 2, and Cohort 3 are formed from country indexes with available data beginning prior to 1984, from 1984 through 1993, and post-1993, respectively. The sample is daily from December 29, 1994 through November 30, 2010. The Appendix provides the list of countries included in the sample and their cohort assignment.

	$R_{j,t} \geq P80\%$	$R_{j,t} \geq P90\%$	$R_{j,t} \geq P95\%$	$R_{j,t} \geq P98\%$
Panel A: $Y_t = I_{\sum X_i \geq 1}$				
$\emptyset$	1.656	3.869	5.965	7.270
	(0.000)	(0.000)	(0.000)	(0.000)
$OR_{1\sigma}$	1.145	1.372	1.628	1.811
$OR_{2\sigma}$	1.311	1.881	2.649	3.279
Panel B: $Y_t = I_{\sum X_i \geq 2}$				
$\emptyset$	4.869	7.699	10.250	13.146
	(0.000)	(0.000)	(0.000)	(0.000)
$OR_{1\sigma}$	1.488	1.875	2.310	2.926
$OR_{2\sigma}$	2.215	3.517	5.335	8.562
Panel C: $Y_t = I_{\sum X_i = 3}$				
$\emptyset$	8.145	11.713	15.599	15.048
	(0.000)	(0.000)	(0.000)	(0.000)
$OR_{1\sigma}$	1.945	2.603	3.575	3.418
$OR_{2\sigma}$	3.783	6.776	12.782	11.682
Panel D: $Y_t = \sum X_i$				
$\emptyset$	3.008	4.630	6.403	7.455
	(0.000)	(0.000)	(0.000)	(0.000)
$OR_{1\sigma}$	1.278	1.460	1.687	1.838
$OR_{2\sigma}$	1.634	2.130	2.846	3.379

Table 6
Cont'd

Panel E: *Generalized logistic regression*				
$\emptyset_3$	7.982	12.061	16.079	15.441
	(0.000)	(0.000)	(0.000)	(0.000)
$OR_{1\sigma.3}$	1.919	2.678	3.718	3.529
$\emptyset_2$	1.609	4.480	7.095	12.195
	(0.011)	(0.000)	(0.000)	(0.000)
$OR_{1\sigma,2}$	1.140	1.442	1.785	2.707
$\emptyset_1$	-2.203	0.471	2.714	4.382
	(0.001)	(0.466)	(0.000)	(0.000)
$OR_{1\sigma,1}$	0.835	1.039	1.248	1.430
$\emptyset_3 = \emptyset_1$	156.047	119.892	63.994	16.148
	(0.000)	(0.000)	(0.000)	(0.000)
$\emptyset_3 = \emptyset_2$	61.763	42.281	24.417	1.093
	(0.000)	(0.000)	(0.000)	(0.296)
$\emptyset_2 = \emptyset_1$	21.822	16.592	12.371	15.700
	(0.000)	(0.000)	(0.000)	(0.000)

Table 7
Logistic regressions controlling for volatility
The table presents logistic regression results in which co-exceedances are regressed on FI. The dependent variable is detailed in the initial column, and is based on $\sum X_i$, which represents the number of cohort indexes that experience a crash on day *t*. In the first three rows of each panel, the dependent variable is equal to an indicator variable that takes the value of one an any day *t* in which the number of cohort indexes that crash is greater than or equal to one, greater than or equal to two, or equal to three, respectively. In the final row of each panel the dependent variable equals the number of cohort indexes that crash on day *t*. For each cohort, a crash is defined as a return that falls below the fifth percentile of full-sample returns. The dependent variable in each specification is regressed on FI, calculated through day *t*-1, and a control variable. In Panel A the control variable represents forecasted volatility for day *t* of the all-country index return from a GARCH specification. In Panels B and C the control variable is the cross-sectional average of country index standard deviations, and the standard deviation of the all-country index, respectively, each calculated through day *t*-1 with the same 500-day rolling-window and weighting scheme as FI. In Panel D the control variable is the cross-sectional average adjusted *R*-square from the FI regressions. Table entries represent coefficient estimates, and associated *p*-values, as well as odds ratios for a one standard deviation increase in the given variable. Cohort 1, Cohort 2, and Cohort 3 are formed from country indexes with available data beginning prior to 1984, from 1984 through 1993, and post-1993, respectively. The sample is daily from December 29, 1994 through November 30, 2010. The Appendix provides the list of countries included in the sample and their cohort assignment.

Panel A: *GARCH forecasted volatility*				
Dependent variable	$\emptyset_{FI}$	OR_{FI}	$\emptyset_{\sigma}$	OR_{σ}
$I_{\sum X_i \geq 1}$	5.933	1.623	10.056	1.201
	(0.000)		(0.000)	
$I_{\sum X_i \geq 2}$	7.759	1.885	10.445	1.210
	(0.000)		(0.000)	
$I_{\sum X_i = 3}$	9.428	2.160	12.428	1.254
	(0.000)		(0.000)	
$\sum X_i$	6.107	1.647	11.286	1.228
	(0.000)		(0.000)	

Panel B: *Cross-sectional average standard deviation*				
Dependent variable	$\emptyset_{FI}$	OR_{FI}	$\emptyset_{\sigma}$	OR_{σ}
$I_{\sum X_i \geq 1}$	4.188	1.408	1.427	1.384
	(0.000)		(0.000)	
$I_{\sum X_i \geq 2}$	5.688	1.591	1.599	1.440
	(0.000)		(0.001)	
$I_{\sum X_i = 3}$	9.778	2.222	0.305	1.072
	(0.000)		(0.674)	
$\sum X_i$	4.402	1.433	1.409	1.379
	(0.000)		(0.000)	

Panel C: *World index standard deviation*				
Dependent variable	$\emptyset_{FI}$	OR_{FI}	$\emptyset_{\sigma}$	OR_{σ}
$I_{\sum X_i \geq 1}$	2.950	1.272	1.753	1.442
	(0.007)		(0.000)	
$I_{\sum X_i \geq 2}$	3.467	1.327	2.397	1.650
	(0.024)		(0.000)	
$I_{\sum X_i = 3}$	8.027	1.926	1.066	1.249
	(0.002)		(0.344)	
$\sum X_i$	3.149	1.293	1.749	1.441
	(0.004)		(0.000)	
Panel D: *Cross-sectional average adjusted R-square*				
Dependent variable	$\emptyset_{FI}$	OR_{FI}	$\emptyset_{AR}$	OR_{AR}
$I_{\sum X_i \geq 1}$	5.445	7.657	0.015	1.122
	(0.000)		(0.363)	
$I_{\sum X_i \geq 2}$	5.972	1.629	0.033	1.290
	(0.003)		(0.184)	
$I_{\sum X_i = 3}$	9.307	2.139	0.014	1.112
	(0.003)		(0.721)	
$\sum X_i$	5.701	1.593	0.014	1.116
	(0.000)		(0.384)	

Table 8

Regressions of number of crashes on FI and realized volatility

The table presents Poisson regression results of the number of crashes within calendar periods. The sample is divided into three-month and six-month calendar periods in Panels A and B, respectively. For each period *T*, we calculate FI as the average loading on the first PC across all countries using observations only from period *T*. Similarly, for each period *T*, we calculate the realized volatility of our all-country world index by summing daily squared returns. Both FI and volatility are standardized. We define $I_{X\geq n}$ as a daily indicator variable taking the value of one for any day in which the number of cohort index returns falling below their fifth percentile is greater than or equal to *n*. The value of the dependent variables in the regression models for period *T* are created by summing across all daily observations of the given indicator variable during the period. In this way, the dependent variables represent the number of occurrences of daily co-exceedances during each period *T*. Table entries represent coefficient estimates and associated p-values on lagged FI, FI_{t-1}, and contemporaneous, and lagged volatility, σ_t, and σ_{t-1}, respectively. In each panel, the sample starts with the second period during 1995, such that the first observation in Panel A is the three-month period from April through June, 1995 and the observation *T*=1 in Panel B corresponds to the six-month period from June through December, 1995. Cohort 1, Cohort 2, and Cohort 3 are formed from country indexes with available data beginning prior to 1984, from 1984 through 1993, and post-1993, respectively. The Appendix provides the list of countries included in the sample and their cohort assignment.

Panel A: *Three-month periods*

	FI_{T-1}	σ_T	σ_{T-1}
$Y_T = \sum I_{X\geq 1}$	0.505	-	-
	(0.000)		
	0.327	0.215	-
	(0.000)	(0.000)	
	0.459	-	0.068
	(0.000)		(0.084)
$Y_T = \sum I_{X\geq 2}$	0.689	-	-
	(0.000)		
	0.478	0.232	-
	(0.000)	(0.000)	
	0.681	-	0.011
	(0.000)		(0.839)
$Y_T = \sum I_{X=3}$	0.786	-	-
	(0.000)		
	0.562	0.235	-
	(0.000)	(0.000)	
	0.792	-	-0.008
	(0.000)		(0.909)

Panel B: *Six-month periods*			
	FI_{T-1}	σ_T	σ_{T-1}
$Y_T = \sum I_{X\geq 1}$	0.540	-	-
	(0.000)		
	0.345	0.235	-
	(0.000)	(0.000)	
	0.628	-	-0.115
	(0.000)		(0.029)
$Y_T = \sum I_{X\geq 2}$	0.720	-	-
	(0.000)		
	0.522	0.237	-
	(0.000)	(0.000)	
	0.861	-	-0.178
	(0.000)		(0.010)
$Y_T = \sum I_{X=3}$	0.840	-	-
	(0.000)		
	0.667	0.209	-
	(0.000)	(0.015)	
	1.038	-	-0.246
	(0.000)		(0.007)

Table 9

Logistic regressions for robustness

The table presents logistic regression results. The dependent variable is based on the number of cohort indexes that exhibit a crash on the given day. The benchmark specification is detailed in Panel D of Table 5 with crashes defined based on the fifth percentile of returns. Results in each row are based on the benchmark specification, with the only difference in methodology described in the first column of each row. Cohort 1, Cohort 2, and Cohort 3 are formed from country indexes with available data beginning prior to 1984, from 1984 through 1993, and post-1993, respectively. The sample is daily from December 29, 1994 through November 30, 2010. The Appendix provides the list of countries included in the sample and their cohort assignment.

Alteration	$\emptyset_{FI}$	$OR_{1\sigma}$	$OR_{2\sigma}$
Benchmark case: Table 5, Panel D, crashes defined based on fifth percentile of returns	6.828 (0.000)	1.747	3.051
Sample period: 12/29/1994–12/31/2007	15.284 (0.000)	1.311	1.720
Sample period: 12/01/2000 – 11/30/2010	7.635 (0.000)	2.115	4.471
FI estimation: 500-day rolling-window with equal-weighting scheme for all observations	5.865 (0.000)	1.594	2.541
FI estimation: 60-day rolling-window	6.302 (0.000)	1.822	3.321
FI specified as the difference between FI estimated from 60-day rolling-window and FI estimated from 500-day rolling-window	3.773 (0.000)	1.220	1.488
FI estimation: 60-day rolling-window. Results analyzed only in months April through December	3.958 (0.000)	1.234	1.524
FI specification: 75th percentile of beta	4.096 (0.000)	1.722	2.965
FI specification: Standard deviation of beta	6.382 (0.000)	1.452	2.108
Crash definition: Absolute return below -5%	13.201 (0.000)	2.939	8.639
Only observations not preceded by a crash within any cohort in the previous 10 trading days	5.854 (0.000)	1.401	1.964
Only observations not preceded by a crash within any cohort in the previous 20 trading days	6.823 (0.001)	1.376	1.893
Only observations not preceded by a crash within any cohort in the previous 50 trading days	10.974 (0.017)	1.341	1.798