

CHARACTERIZING THE STRENGTH OF WOOD TRUSS JOINTS

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ABSTRACT

Probability density functions (normal, lognormal, and three-parameter Weibull) were used to characterize strength data for three different types of metal-plate-connected wood truss joints (web at the bottom chord, tension splice, and heel). Modulus of elasticity (MOE) of the lumber used to fabricate the joints was also characterized. A probability-plot technique, in conjunction with Kolmogorov-Smirnov and chi-square statistics, was used to determine which distribution best fit the data. Lumber MOE was best described by a lognormal distribution. No single distributional form fit the strength data for all three joint types with equal accuracy. Lumber MOE and joint strength were unrelated. Strength data for the web at the bottom chord and heel joints were best described by normal distributions; however, none of the distributions considered fit the data for the tension splice joints. The probability-plot technique provided a better visual inspection of fit than did a density function superimposed over a histogram. Fitted distributions are easy to work with and can be used in reliability analyses to simulate strength values of joints. The results presented here are for particular joint types and plates and should not be extrapolated to other truss joints.

KEYWORDS. Truss joints, Modulus of elasticity, Probability density function, Reliability analysis, Timber engineering, Wood engineering, Wood-connection strength.

INTRODUCTION

Wood has long been recognized as a material with significant variability in its structural properties (e.g., strength and stiffness), which can be characterized as random variables. In recent years, lumber material properties (modulus of elasticity, bending strength, compressive strength, and tensile strength) have been characterized by probability distributions (Galligan et al., 1986). Accurate probabilistic models for joint strength also are needed to accurately predict moments, axial forces, and shear forces for reliability

analyses of trusses. Indeed, it would be difficult to improve or even assess the safety and reliability of a structure without viable information (failure modes; strength and stiffness) on structural connections.

The necessary information for the application of reliability-based design (RBD) to wood structures is currently being developed. In a recent study, Galligan et al. (1986) collected data on material properties of lumber from worldwide literature and statistically analyzed these data using two approaches. In the first approach, the data were fit by simple linear regression of strength on modulus of elasticity (MOE). In the second approach, they were characterized by probability distributions. The authors concluded that no single distributional form fits all mechanical properties with equal accuracy but that the three-parameter Weibull distribution dominates. Moreover, strength distributions were often positively skewed.

Little information is available on probabilistic models for strength and stiffness of truss joints (Gupta, 1990). McLain (1986) has emphasized the need to develop probability distributions for strength of joints, pointing out that resistance of the whole joint is generally less variable than that of the joined members. He also stated that if moisture content is uniform, specific gravity low, installation practice constant, and the test procedure uniform, then strength distributions of joints would be symmetric with a coefficient of variation (CV) less than 15%. Otherwise, the distribution would be skewed and the CV would increase.

This article presents a portion of the results from ongoing research on probabilistic design of a roof system comprised of metal-plate-connected wood trusses. The first part of the research dealt with the experimental investigation of the strength and stiffness of three metal-plate-connected wood truss joint types (Gupta and Gebremedhin, 1990). The second part, which is described in this article, focused on the development of probabilistic models for characterizing the strength of those three joint types. Specifically, the research described here characterized both the strength of the three types of metal-plate-connected wood truss joints and the MOE of the lumber used to fabricate the joints by one of three probability density functions—normal, lognormal, or three-parameter Weibull.

The long-term goal of this research is to predict the reliability of a roof truss system under various loads. Before that can happen, however, the reliability of a single truss, and therefore the reliability of its members and connections, must be determined.

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MATERIALS AND METHODS

Truss joints were designed for a fink truss. The 38-×89-mm (2-×4-in.) Southern Pine No. 2 KD 15 members were connected by 20-gauge punched metal plates supplied by Alpine Engineered Products, Inc. Two 76-×127-mm (3-×5-in.) plates were used for each heel and web at the bottom chord joints, and two 76-×102-mm (3-×4-in.) plates were used for the tension splice joints. All joints were fabricated by commercial truss fabricators. For a complete description of the joint fabrication, testing procedure, data-acquisition system, determination of ultimate strength values, and failure modes, see Gupta and Gebremedhin (1990). Before fabricating the joints, 250 pieces of Southern Pine lumber used in joint fabrication [all 2.44 m (8 ft) long and 10% moisture content] were nondestructively tested for MOE by using static flatwise bending with a concentrated dead load at mid-span (clear span 1.83 m) (Gupta, 1990).

PROCEDURE FOR CHARACTERIZING STRENGTH

Strength data from 60 specimens of each of three joint types found in metal-plate-connected wood trusses were used to determine probability distributions for joint strength. These joints were: 1) web at the bottom chord joint (hereafter referred to as "web joint"); 2) tension splice joint (hereafter referred to as "tension joint"); and 3) heel joint. In addition, 250 MOE values, from lumber used in joint fabrication, were used to develop probability distributions for MOE. Least square regression analyses were used to determine the relationship between lumber MOE and strength. Probability density functions were fitted to joint-strength and MOE values to characterize the data.

CHOOSING PROBABILITY DISTRIBUTIONS

The three probability distributions most widely used to model wood strength properties (Galligan et al., 1986)—normal, two-parameter lognormal, and three-parameter Weibull—were considered in this study. The probability density function, $f_x(x)$, of each distribution is expressed as follows:

Normal density function:

$$f_x(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (1)$$

where μ is mean and σ is standard deviation.

Lognormal density function:

$$f_x(x) = \frac{1}{x \xi \sqrt{2\pi}} e^{-\frac{(\ln x - \lambda)^2}{2\xi^2}} \quad x > 0 \quad (2)$$

where λ is log mean and ξ is log standard deviation.

Three-parameter Weibull density function:

$$f_x(x) = \frac{\eta}{\sigma} \left(\frac{x-\mu}{\sigma}\right)^{\eta-1} e^{-\left(\frac{x-\mu}{\sigma}\right)^\eta} \quad x > \mu \quad (3)$$

where

- μ = location parameter
- σ = scale parameter
- η = shape parameter
- x = random variable (e.g., strength)

The Statistical Analysis System (SAS, 1991a) software package was used to estimate the parameters for the normal and lognormal distributions (using the unbiased estimators). A computer program developed by Simon and Woeste (1980) was used to estimate the parameters for the three-parameter Weibull distribution. This program uses the method of maximum likelihood for parameter estimation.

DETERMINING THE "BEST" DISTRIBUTION

To date, no standard and/or powerful procedure exists for determining the distribution with the "best fit". Traditionally, the Kolmogorov-Smirnov (K-S) and chi-square goodness-of-fit tests have been used for deciding among the normal, lognormal, and Weibull distributions. The use of Anderson-Darling and Shapiro-Wilk goodness-of-fit tests have also been suggested, especially for Weibull distribution (Evans et al., 1989). Galligan et al. (1986) used a two-step procedure for selecting among these three distributions to model lumber strength properties; this procedure always selects a distribution for a particular strength property. Occasionally, there may not be a probability distribution that suitably describes the data, in which case a procedure other than that used by Galligan et al. (1986) needs to be followed to characterize the data.

After the parameters for each distribution were estimated for each data set as previously described, we used the following procedure to select the distribution with the best fit:

- The data and distributions (cumulative frequencies, or probabilities, p) were plotted on specified graph paper ("probability paper") constructed in association with each distribution. The $m/(N+1)$ (the m^{th} value among the N observations, arranged in increasing order) plotting position of each data point was used to plot the cumulative probability.
- The linearity of each probability plot was visually inspected to determine how well the data at the lower and upper tails of the distribution were characterized by that distribution. Emphasis was placed on the distribution's fit in the lower tail because the lower tail of the strength distribution is particularly important in structural reliability analyses.
- The distribution that best fit the probability plot was selected. If none of the distributions fit, the original data were considered not characterized by any of the three distributions and so these data could be used for

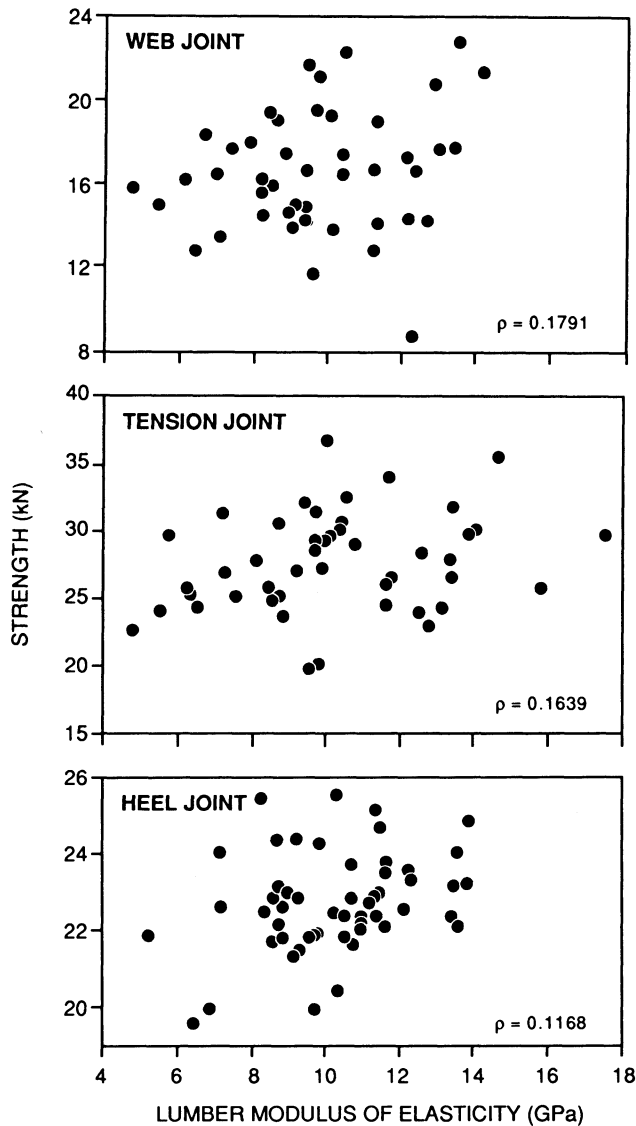


Figure 1—Scattergrams relating modulus of elasticity of the lumber comprising the joint to the strength of the three types of joints studied (ρ = correlation coefficient).

random sampling. If more than one distribution could possibly characterize the data, the K-S and chi-square tests were then used to determine the best-fitting distribution.

Critical values for the normal and lognormal K-S tests were obtained from Law and Kelton (1991), whereas for the Weibull K-S test, critical values were obtained from Evans et al. (1989). The original form of the K-S test is valid only if all of the parameters of the hypothesized distribution are known; i.e., the parameters of the distribution could not have been estimated from the data. However, recent research has allowed the K-S test to be extended to allow for estimation of the parameters in the cases of the normal, lognormal, and Weibull distributions. The chi-square test statistics were calculated using the SAS CAPABILITY procedure (SAS, 1991b). This procedure uses the expected values less than 1 to compute chi-square statistics. The critical values for this test were taken from Law and Kelton (1991).

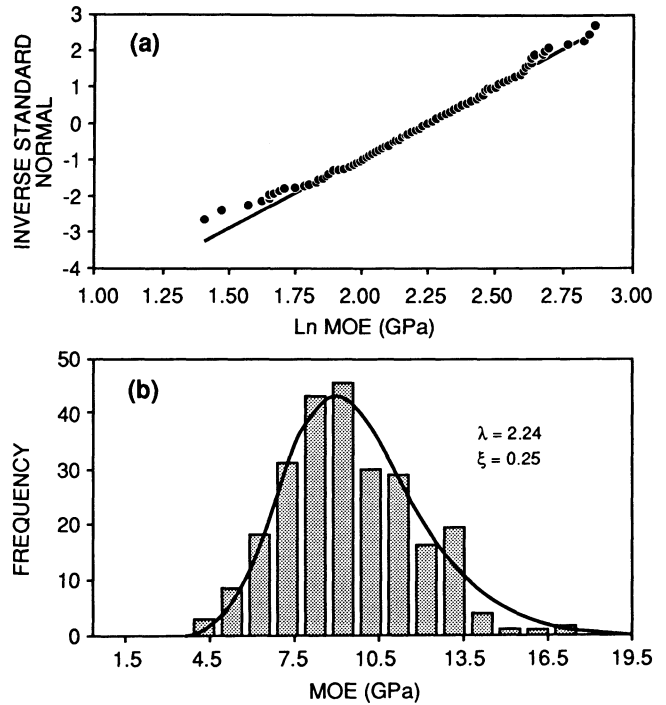


Figure 2—a) Lognormal probability plot; b) Histogram and lognormal density function for modulus of elasticity (MOE) of lumber used to fabricate the joints ($N=250$).

RESULTS

MODULUS OF ELASTICITY

MOE for the 250 lumber specimens ranged from 4.1 GPa (0.6×10^6 psi) to 17.5 GPa (2.5×10^6 psi), with a mean value of 9.7 GPa (1.4×10^6 psi) and a CV of 25%. Visual inspection of the probability plots revealed that the lognormal distribution fit the data better than the normal and three-parameter Weibull distributions (fig. 2a). Distribution parameter estimates are given in Table 1. Chi-square test statistics (Table 2) and visual appraisal of the fitted density function and its corresponding frequency histogram of the actual test data (fig. 2b) supported the choice of distribution. The estimated fifth percentile for the lognormal distribution was 6.1 GPa (0.9×10^6 psi).

MODULUS OF ELASTICITY AND STRENGTH OF JOINT

The MOE of the respective pieces forming the joint was assumed to be identical because a single piece of lumber, cut in half, was used to fabricate each joint. However, as evident from the scattergrams shown in figure 1, lumber MOE seems unrelated to strength, possibly because plate parameters also affect joint strength.

TABLE 1. Distribution parameters for lumber modulus of elasticity (MOE) and joint strength

Property	Distribution	Parameters	
MOE (GPa)	Lognormal	$\lambda = 2.24$	$\xi = 0.25$
Strength (kN)			
Web joint	Normal	$\mu = 16.7$	$\sigma = 2.9$
Tension joint	— Use Original Data for Random Sampling —		
Heel joint	Normal	$\mu = 22.6$	$\sigma = 1.4$

TABLE 2. Chi-square and Kolmogorov-Smirnov (K-S) statistics applied to the three distributions considered for lumber modulus of elasticity (MOE) and joint strength*

Property	Chi square			K-S		
	Normal	Lognormal	Weibull	Normal	Lognormal	Weibull
MOE (N† = 250)	17.60 (14.07, 7)‡	10.96 (14.07, 7)	12.59 (12.59, 6)			
Strength						
Web joint (N = 55)	3.30 (9.49, 4)	7.68 (9.49, 4)	3.63 (7.82, 3)	0.0665 (0.1190)§	0.0562 (0.1190)	0.0744 (0.1098)
Tension joint (N = 52)	19.40 (9.49, 4)	39.02 (9.49, 4)	16.65 (7.82, 3)	0.1105 (0.1223)	0.1540 (0.1223)	0.1063 (0.1128)
Heel joint (N = 56)	3.20 (7.82, 3)	3.88 (7.82, 3)	3.42 (5.99, 2)	0.0893 (0.1180)	0.0972 (0.1180)	0.0895 (0.1089)

* Level of significance for all tests was 0.05.

† N = number of observations.

‡ Numbers in parentheses under chi-square statistics are critical values and degrees of freedom, respectively.

§ Numbers in parentheses under K-S statistics are critical values.

WEB JOINT

Strength values averaged 16.7 kN (CV=17.1%) for the web joints. Almost 95% of the web joints failed in tension: The tension web pulled out due to tooth bending, and the plate remained on the bottom chord and the compression web. Other failure modes included plate failure and plate peeling.

Visual inspection of the probability plots revealed that the normal and three-parameter Weibull distributions both fit the data well enough that K-S and chi-square statistics had to be used. K-S statistics for both distributions were lower than their corresponding critical values (Table 2). Chi-square statistics for both distributions were much lower than the corresponding critical values, although statistics for the normal distribution were relatively lower (Table 2). Thus, the normal distribution was selected. With the exception of one data point, all the data in the probability plot of figure 3a fell on the straight line. Distribution parameter estimates are given in Table 1. Superimposing the fitted density function on the histogram of the actual data showed that the density function fit the data quite well (fig. 3b).

TENSION JOINT

Strength values averaged 27.0 kN (CV=17.6%) for the tension joints. Of the three joint types, CV was greatest for the tension joint, which failed in three different modes: tooth bending and plate peeling (63%), tooth bending and plate peeling combined with wood failure (31%), and plate failure (6%).

Visual inspection of the probability plots revealed that none of the distributions fit the data, although the three-parameter Weibull was relatively better than the other two (fig. 4a); as can be seen in the figure, the data at the lower tail fell far from the straight line. The lack of fit of all three distributions also was verified by the chi-square test (Table 2). Superimposing the fitted density function on the histogram of the actual data showed that the density function does not characterize the data very well (fig. 4b). Because none of the density functions considered were acceptable, the test data (Table 3) could be used for random sampling in reliability analysis. It is also possible to use the test data to specify an empirical distribution from

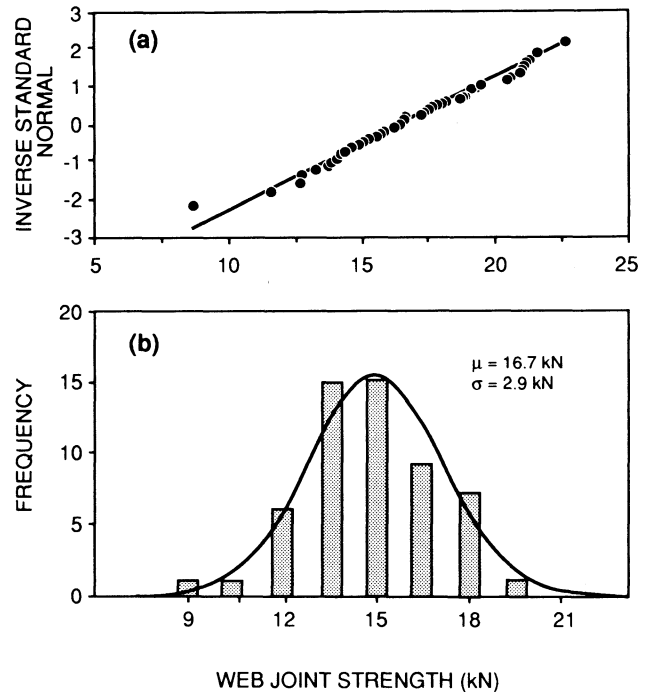


Figure 3-a) Normal probability plot; b) Histogram and normal density function for the web joints (N=55).

which random values can be generated (Law and Kelton, 1991).

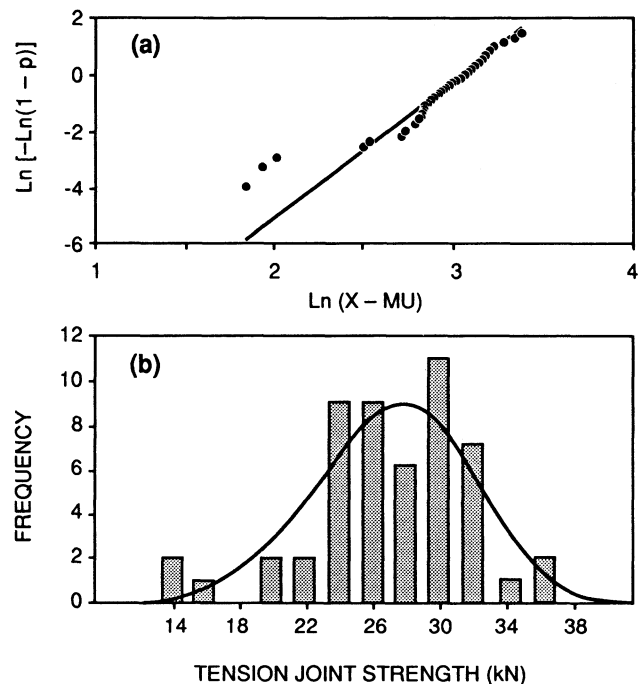


Figure 4-a) Three-parameter Weibull probability plot, in which p=cumulative probability, MU=mean, and x=any observation; b) Histogram and three-parameter Weibull density function, for which location=7.49 kN, scale=21.19 kN, and shape=4.89 kN for the tension joints (N=52).

TABLE 3. Ultimate strength (kN) of tension joints*

No.	Strength	No.	Strength	No.	Strength
1	29.8	19	22.7	36	29.6
2	25.1	20	27.7	37	25.1
3	26.8	21	32.1	38	23.0
4	29.2	22	27.2	39	36.8
5	30.1	23	29.6	40	24.4
6	32.5	24	20.2	41	15.1
7	29.4	25	34.0	42	27.1
8	31.4	26	24.2	43	35.5
9	30.1	27	25.7	44	29.3
10	27.8	28	24.2	45	25.9
11	24.9	29	24.5	46	23.9
12	25.3	30	29.1	47	28.3
13	30.7	31	28.5	48	13.8
14	25.7	32	26.5	49	24.5
15	26.6	33	26.1	50	31.0
16	31.3	34	30.6	51	14.4
17	19.8	35	31.7	52	31.7
18	23.7				

* Mean = 27.0 kN; coefficient of variation = 17.6%.

HEEL JOINT

Strength values averaged 22.7 kN (CV=6.7%) for the heel joints. Of the three joint types, CVs were least for the heel joint because all those joints failed in a similar manner, as a result of tooth bending; they pulled out either from the top or bottom chord.

Visual inspection of the probability plots revealed that the normal distribution best characterized the data (fig. 5a); the plotted data points nearly all fell on the straight line. Distribution parameters are given in Table 1. The chi-square statistic for the normal density function was less than its corresponding critical value (Table 2); therefore, the normal distribution could not be rejected. Superimposing the fitted density function on the histogram

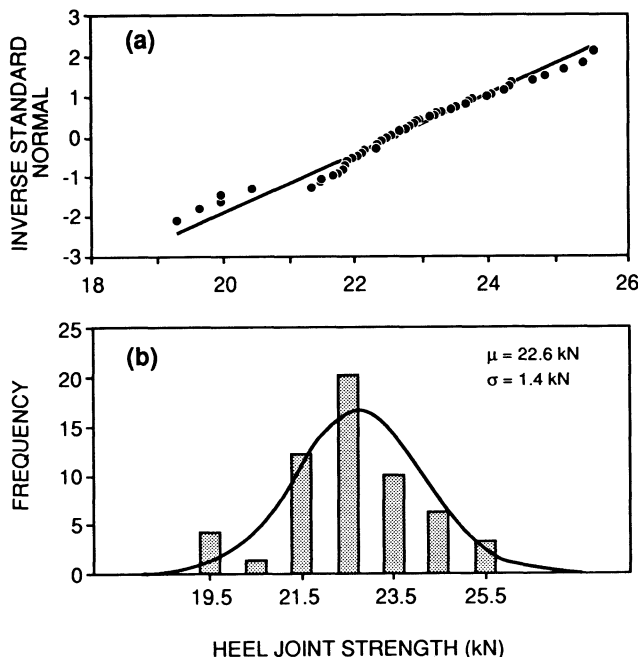


Figure 5—a) Normal probability plot; b) Histogram and normal density function for the heel joints (N=56).

of the actual data showed that the density function fit the data quite well (fig. 5b).

DISCUSSION

No single distributional form fit the strength data of the three types of joints with equal accuracy. The modeling of any random variable (e.g., strength or load) by a probability distribution is always associated with some uncertainty. There is model uncertainty because mathematical modeling can never be exact, statistical uncertainty because distribution parameters are estimated and the estimates depend on the amount of sample data, and physical uncertainty because physical variability of the measured quantities (e.g., strength) may be present. Models do not represent reality, they only approximate it.

The choice of a density function is of great importance because of sensitivity of data at the tails of the distribution, which influences reliability estimates for the structure. For example, if strength of the tension joint was modeled by a Weibull distribution (see fig. 4a), predictions at the lower tail would be poor because this distribution did not fit the lower tail very well. The lowest strength value in the actual data was 13.70 kN. Its cumulative probability based on the ranking described earlier was 0.0189. However, the cumulative probability of 13.70 kN based on the Weibull distribution is only 0.0028. Thus, the actual probability is almost seven times that of the theoretical probability. Therefore, the Weibull distribution is a poor representation of the strength of the tension joint, especially at the lower tail.

The strength data of the web and heel joints were best described by normal distributions. It is sometimes argued that the normal distribution should not be used to model a resistance variable (e.g., joint strength) because it gives a finite probability of negative strength. Usually, this probability is extremely small, and in such cases it may be possible, when sampling from the normal distribution, to discard the negative values. However, doing so may change the distribution of the simulated data. Therefore, the researcher may want to use the lognormal or Weibull distributions.

The MOE data were best described by a lognormal distribution. Several researchers (Simon and Woeste, 1980; Galligan et al., 1986) have used a three-parameter Weibull distribution to characterize MOE; however, a lognormal distribution also has been used (Galligan et al., 1986).

Visually inspecting the linearity, or lack of linearity, of a probability plot is an excellent technique for screening density functions, as compared to visually inspecting the fit of a density function superimposed on a histogram. Note that in the probability plots, the variance of the data points at the tails is larger than that at the center of the distribution. Thus, the relative fit of the data at the tails is often poorer than that at the center even if the correct distribution is chosen (Hahn and Shapiro, 1967).

The lack of wood failure in any of the three joint types suggests that joints are indeed the weakest link in a truss and could limit truss strength. In the way that a chain is no stronger than its weakest link, a wooden truss is no stronger than its weakest joint. The variability in joint strength was less than expected—less than that in MOE of the visually graded lumber forming the joints. Variability

due to fabrication was reduced because during joint fabrication, the plates on both sides of the lumber were centered over the joint and aligned such that one was on top of the other. In commercial truss production, however, these conditions may not exist; therefore, one would expect joint strength in this study to vary less than that in typical commercial trusses.

The fitted distributions arrived at herein may not be the "true underlying distributions," but they closely represent the sample data. In the third part of this ongoing long-term research, we used these distributions to simulate joint strength for input to the truss simulation model (Gupta and Gebremedhin, 1992).

CONCLUSIONS

- Lumber MOE and joint strength apparently are unrelated. MOE was best described by a lognormal distribution. Strength data for the web and heel joints were best described by normal distributions; data for the tension joints could not be characterized by any of the distributions considered.
- Visually inspecting probability plots is an excellent technique for screening density functions, preferable to using K-S and chi-square statistics and the conventional approach of superimposing a density function on the histogram of the data.

Although these results should not be extrapolated to other truss joints because they depend on joint geometry, we found that probability distributions are easy to work with mathematically and can easily be used in reliability analysis.

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