AN ABSTRACT OF THE DISSERTATION OF

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Title: Designing and Using Multiplayer Tabletop Mathematics Learning Games.

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Thomas P. Dick

Teachers may be attracted to the use of a game in a learning activity under the presumption that students will find the game experience to be more "fun" than typical classroom activities. The use of a game in a learning activity should help students attain important learning outcomes and engage them in mathematical reasoning and sense making. While most of the research attention has been devoted to digital learning games, multiplayer tabletop mathematics learning games afford unique opportunities (and challenges) to engage students in meaningful mathematical activity and discourse.

Engle and Conant's (2002) construct of *productive disciplinary engagement* is used to frame the notion of learner engagement with mathematical ideas during gameplay. The expectancy-value theory of achievement motivation applied to the mathematics learning game context suggested a motivational construct called *subjective gameplay-value* that captures multiple reasons behind a student's willingness to play a learning game, including both enjoyment and learning value.

Two sets of principles are proposed, based on these theoretical frameworks and the game-based learning literature: 1) design principles to guide creation of games that engage students with important mathematical ideas while also motivating them to play, and 2) implementation principles to help teachers make effective use of multiplayer tabletop mathematics learning games as classroom learning activities and to facilitate mathematical discourse.

The function representations card game *Curves Ahead!* was created using the design principles, and then playtested as part of a design experiment to refine the game. Embedded in the game are mathematical tasks requiring students to connect and interpret multiple representations of functions. Early playtests suggested which game features were impacting subjective gameplay-value. After iterative refinements and modifications, the resulting game was playtested with differential calculus students to assess perceptions of its subjective gameplay-value.

The calculus board game *Assembly Lines* was also created using the design principles. Embedded in the game are tasks requiring graphical interpretation of derivatives, antiderivatives, and the Fundamental Theorem of Calculus. Game sessions with calculus students were conducted using the implementation principles. Video recordings of the sessions were analyzed to investigate how calculus students were productively engaging with mathematical ideas. The results suggested that all students were engaged in mathematical reasoning and sense making during gameplay, and most students were making "intellectual progress." The sessions also revealed potentially positive and negative impacts of participant interactions that have implications for teacher facilitation during gameplay.

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Designing and Using Multiplayer Tabletop Mathematics Learning Games

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APPROVED:

Major Professor, representing Mathematics

Head of the Department of Mathematics

Dean of the Graduate School

I understand that my dissertation will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my dissertation to any reader upon request.

Michael Renne, Author

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Designing and Using Multiplayer Tabletop Mathematics Learning Games

CHAPTER 1: INTRODUCTION AND OVERVIEW

Game-based learning is a pedagogical strategy that uses games to help students attain important learning outcomes. Typically, game-based learning activities have included students building games to learn, repurposing existing games, or playing a game that has been purposefully designed around specific learning outcomes (Van Eck, 2006). Games designed around specific mathematics learning outcomes are called *mathematics learning games* or *educational mathematics games*. This dissertation focuses on the design and implementation of multiplayer tabletop mathematics learning games.

Much of the research in game-based learning has been focused on single-player (Harteveld & Bekebrede, 2011) digital (video) learning games (Ke, 2011). Digital games are played on a console, computer, or mobile device. Gee's (2003) influential book, *What Video Games Have to Teach Us About Learning and Literacy*, outlines 36 learning principles that Gee believes are embodied by video games. Devlin used Gee's principles (Devlin, 2011) in an ambitious project to create an exemplar for single-player digital mathematics learning games. That exemplar game is *Wuzzit*™[1] *Trouble*, designed for learning number sense, and investigations of its use have demonstrated the positive impact of video game learning in mathematics (Kiili, Devlin, Perttula, Tuomi, & Lindstedt, 2015; Pope & Mangram, 2015).

Mathematics learning games have potential for helping students attain important learning outcomes and meet mathematical content standards like those of *The Common Core State Standards for Mathematics (CCSSM)* (National Governors Association Center for Best Practices, 2010). However, some mathematical practice standards imply that students need to interact as members of a mathematical learning community. For example, if teachers seek to engage students in opportunities to "construct viable arguments and critique the reason of others" (Mathematical Practices Standard 3, CCSSM), then an activity that encourages student-student interaction and discourse around mathematical reasoning tasks will be particularly attractive.

Multiplayer tabletop games (e.g., card games, board games, etc.) often involve many player-to-player interactions. If game tasks are centered on mathematical sense making and reasoning, then the multiplayer tabletop format could provide special opportunities to support student-student interactions and engagement with

[1] Wuzzit is a registered trademark of BrainQuake. The use of this name does not imply any affiliation with or endorsement by BrainQuake.

important mathematical ideas. In turn, these kinds of games also create opportunities for teachers to orchestrate productive mathematical discourse around students' game experiences.

How can a multiplayer tabletop game be designed to foster students' productive engagement with mathematical ideas? How can a teacher support student engagement during gameplay and facilitate productive mathematical discussions? These are questions of mathematics learning game design and implementation that motivated this dissertation study.

"Gamification" versus "game"

Early research on games and learning emphasized isolating game features and elements that contributed to the "fun" and placing them in an educational setting to increase engagement and motivation. This process became known as *gamification*, and it was perceived negatively by many who are passionate about playing and designing games. Robertson (2010) prefers to call it "pointsification" due to the common practice of introducing points and referring to the result as a game. Most implementations of gamification would not be recognized as games.

So, what exactly is a game? There is no consensus in the literature on a definition for game, but drawing on the important characteristics of games that several authors have identified, the following is presented in the dissertation as a working definition.

> **A *game* is a voluntary play activity in a pretended reality governed by rules, wherein the participant(s) try to achieve one or more goals, and where degrees of success in the attainment of goals are conveyed by a feedback system.**

The four defining characteristics of a game are: *voluntary play activity*, *pretended reality governed by rules*, *adoption of goals*, and *progress conveyed by a feedback system*.

Theoretical framing: productive disciplinary engagement, expectancy-value theory

To theoretically frame the notion of student engagement with mathematical ideas during a learning game, the dissertation uses Engle and Conant's (2002) construct of *productive disciplinary engagement*. The expectancy-value theory of achievement motivation (Eccles & Wigfield, 2002; Wigfield & Eccles, 2000) provides a useful lens with which to frame a student's willingness and motivation to play a mathematics learning game. The application of expectancy-value theory to the context of mathematics learning games resulted in a proposed motivational construct called *subjective gameplay-value.* This construct is attractive because it can be used to capture multiple possible reasons behind a student's willingness to play a mathematics learning game, including both enjoyment and learning value.

Overview of the three-paper model structure of the dissertation

This dissertation uses the "three-paper model." Chapters 2, 3, and 4 are presented as three thematically related, but self-contained papers.

Chapter 2 is the "principles" paper. Two sets of principles for multiplayer tabletop mathematics learning games are proposed that are based on these theoretical frameworks and the game-based learning literature. The first set of principles aims to guide the creation of games that engage students with important mathematical ideas while also motivating them to play. The second set of principles is proposed in order to help teachers effectively use these kinds of games as classroom learning activities and to facilitate mathematical discourse about the game and its embedded mathematics.

This attention to both sets of principles (design and implementation) was motivated by Dick and Burrill's (2016) work on digital interactive mathematics learning technologies. They also discuss both design principles for the creators of interactive digital environments for mathematics learning, as well as implementation principles for teachers to use those kinds of environments to support mathematical sense making and reasoning. (Many of Dick and Burrill's design principles could apply to digital mathematics learning games, since such games can be viewed as a special case of an interactive mathematics learning technology envisioned by Dick and Burrill.)

Chapter 3 is the "design experiment" paper. The design principles of Chapter 2 were used to create the function representations card game *Curves Ahead!* The game was then playtested as part of a design experiment to refine and improve the game. Mathematical tasks that require students to translate between multiple representations of functions in multiple contexts are embedded in the game. Early playtests highlighted game features that were impacting subjective gameplay-value. After iterative refinements to improve *Curves Ahead!*, the final playtest involves an entire class of differential calculus students in playing the game to evaluate its subjective gameplay-value.

Chapter 4 is a "gameplay interaction analysis" paper. The design principles were used to create the calculus board game *Assembly Lines*, and then game sessions with calculus students were conducted using the implementation principles. Mathematical tasks requiring the graphical interpretation of derivatives, antiderivatives, and the Fundamental Theorem of Calculus are embedded in the game. The study uses video recordings of the gameplay sessions to investigate how calculus students are engaging with mathematical ideas, and the extent to which that engagement might be productive.

Chapter 5, the dissertation's final chapter, positions the dissertation's contribution to the mathematics game-based learning literature. Implications and discussion of each of the papers presented in Chapters 2, 3, and 4, follow. Included in the final chapter is a presentation of a design process for developing multiplayer tabletop mathematics learning games. The dissertation concludes with a discussion of directions for future research and mathematics learning game design efforts.

CHAPTER 2: DESIGN AND IMPLEMENTATION PRINCIPLES FOR MULTIPLAYER TABLETOP MATHEMATICS LEARNING GAMES

Introduction

Game-based learning (GBL) is the use of gameplay as a means for attaining desired learning outcomes (Plass, Homer, & Kinzer, 2015). Educators may be attracted to the idea of game-based learning under the presumption that learners will be more "engaged" and will find the game experience "more fun" than typical classroom activities (Ke, Xie, & Xie, 2016; Kebritchi, Hirumi, & Bai, 2010; Plass et al., 2015; Slussareff, Braad, Wilkinson, & Strååt, 2016; Wouters, Van der Spek, & Van Oostendorp, 2009; Wouters, Van Nimwegen, Van Oostendorp, & Van Der Spek, 2013).

In this paper, the focus is on game-based learning in mathematics, and a careful consideration of the nature of students' engagement and interactions when playing a mathematics learning game. When students report enjoyment from playing a mathematics learning game during class time, is that simply an indication that the activity was perceived as a welcome distraction from the usual classroom routine? Was their "engagement" actually a diversion away from learning mathematics? Or, as mathematics educators employing game-based learning would hope, did students genuinely engage with mathematical ideas on cognitive, affective, or sociocultural levels (Plass et al., 2015) while they played a game intended to help them learn important mathematics?

Digital learning games seem to be primarily played by an individual student (Harteveld & Bekebrede, 2011), and most of the research in game-based learning appears to have been devoted to these kind of digital games (Ke, 2011). One particularly ambitions and notable effort in this arena is the design, development, and research on the digital mathematics learning game *Wuzzit™ Trouble* (Kiili, Devlin, Perttula, Tuomi, & Lindstedt, 2015; Pope & Mangram, 2015).

This paper devotes special attention to multiplayer tabletop mathematics learning games and the special opportunities they afford for significant interactions between players. Two sets of principles are presented to support more effective use of game-based learning in mathematics. One is a set of design principles to guide developers of multiplayer tabletop educational mathematics games. The other is a set of implementation principles to assist teachers in effectively using such games as learning activities in the classroom setting.

This attention to both design and implementation principles was motivated by Dick and Burrill's (2016) work on digital interactive mathematics learning environments, in which they also discuss both design principles for the creators of such digital environments as well as implementation principles for teachers to use such environments to support mathematical sense making and reasoning. (Many of the design principles presented here could apply to digital mathematics learning games which, in turn, could be viewed as a special case of the interactive environments Dick and Burrill discuss.)

Underlying both sets of principles, in this paper, is the use of Engle and Conant's (2002) construct of *productive disciplinary engagement* to theoretically frame the notion of learner engagement with mathematical ideas during gameplay. The overarching goal of supporting and facilitating students' productive disciplinary engagement in mathematics shaped the formulation of both the design and the implementation principles.

The design principles are intended to guide the development of educational mathematics games that intrinsically involve students in mathematical tasks in ways that address not only mathematical content standards, but also mathematical practice standards (e.g., *The Common Core State Standards for Mathematics*, by National Governors Association Center for Best Practices, 2010). The implementation principles are intended to provide teachers with guidance that supports many of the effective mathematical teaching practices suggested by the National Council of Teachers of Mathematics (NCTM, 2014).

The implementation principles are intended to provide teachers with guidance consistent with the effective mathematical teaching practices suggested by the National Council of Teachers of Mathematics (NCTM, 2014). These implementation principles explicitly recognize the special role the teacher has in facilitating gameplay as a classroom activity, and the teacher's orchestration of associated discourse with the players before, during, and after the gameplay activity.

Before turning to detailed descriptions of the design and implementation principles, important background and terminology on games and game-based learning are discussed, and the theoretical framework underlying the principles is presented.

Terminology of game-based learning

There is no clear consensus on a definition of *game* (Plass et al., 2015; Salen & Zimmerman, 2004) and any description is unlikely to include all games (Adams, 2010). Salen and Zimmerman (2004) define a game as, "a system in which players engage in an artificial conflict, defined by rules, that results in a quantifiable outcome" (p. 80), but they acknowledge that this definition may not cover all games. Fullerton (2008) defines a game as "a closed, formal system that engages players in structured conflict and resolves its uncertainty in an unequal outcome" (p. 43), but

Fullerton also admits that this description may not fully capture the nature of games. Adams (2010) offers this definition: "A game is a type of play activity, conducted in the context of a pretended reality, in which the participant(s) try to achieve at least one arbitrary, nontrivial goal by acting in accordance with rules" (p. 3), but Adams calls his definition "nonrigorous" and "practical rather than complete" (p. 3).

On the other hand, McGonigal (2011) defines games through "four defining traits: a *goal*, *rules,* a *feedback system,* and *voluntary participation*" (p. 21), and Koster (2013) simply states that "Games are puzzles to solve, just like everything else we encounter in life" (p. 34). If the definitions offered above are not inclusive enough, one could argue that these two definitions are simply *too* inclusive, for they admit experiences that would not be considered games. For example, enrollment in a four-year university would entail a goal, rules, a feedback system, and is voluntary, but would not normally be considered a game (except perhaps by analogy).

For the purposes of the discussion in this paper, the operational definition of *game* proposed here includes the important elements of play activity and pretended reality described by Adams (2010) as well as the four traits emphasized by McGonigal (2011):

> **A *game* is a voluntary play activity in a pretended reality governed by rules, wherein the participant(s) try to achieve one or more goals, and where degrees of success in the attainment of goals are conveyed by a feedback system.**

There are four important characteristics of this definition: *voluntary play activity*, *pretended reality governed by rules*, *adoption of goals*, and *progress conveyed by a feedback system*.

Voluntary play activity

Participants willingly take part in the activities within a game, and the voluntary nature of playing a game differentiates it from "work." The cultural historian and anthropologist Johan Huizinga, in his influential book *Homo Ludens: A Study of the Play Element in Culture* (1938/1950) argues that play activities are inherently voluntary. Caillois (1961), building on Huizinga's work, suggests that forcing someone to play a game undermines the nature of play. McGonigal (2011) writes that voluntary participation ensures that a game "is experienced as *safe* and *pleasurable*" (p. 21, emphasis in original).

Note that the definition of game does not include a reference to "fun." Huizinga (1938/1950) argues that "fun" itself cannot be properly defined, and no attempt is made here to define it. Even in Raph Koster's (2013) book *A Theory of Fun for Game Design*, the term "fun" itself is not defined. Rather, Koster's book is a treatise on the important characteristics of games that attract people to play them, i.e., what motivates voluntary play activity.

Pretended reality governed by rules

The *pretended reality* is the game setting, situated in some play space (think of a gameboard or playing field or virtual setting on a computer screen), and a context within which the players take actions. It is where "the players assign artificial significance to the situations and events in the game" (Adams, 2010, p. 5).

The allowable player actions and their consequences within the pretended reality are governed by a set of rules. In a multiplayer tabletop game, there is an understood agreement by the players to abide by the governing rules, and thus a necessity of a shared knowledge of these rules. In contrast, a digital game enforces its own rules, and there are examples of such games where the players are expected to discover these rules through trial and error.

Adoption of goals

Participants accept or set goals while they play a game. A careful distinction should be made between the learning outcomes that a game designer may have in mind in creating an educational game, and the game goals that arise during play, i.e., those determined by the pretended reality governed by rules. A player could also set personal goals related to these game goals, such as doing better than a particular opponent, or achieving a certain level or number of points (for example, a "personal best" score).

Progress conveyed by a feedback system

The role of the feedback system is to inform the player(s) of their actual, or potential for, progress toward attaining their goals. Some examples of feedback are rewards, awards, points, access, resources, degrees of completion (e.g., progress bars, star-ratings, levels), and adjudication (the determination of whether a player's action was within the rules). In multi-player games, other players can be a source of feedback (e.g., through social standing, competition, or adjudication).

Illustrating operational game characteristics with some well-known games

Key characteristics of the operational definition of game are illustrated here using a few well-known examples, all of which are readily recognized as having a large following of voluntary players, so that aspect is not discussed. The pretended reality governed by rules, the goals, and the feedback system in each example is discussed.

Chess, in one form or another, is more than 1,000 years old. In its present form, the pretended reality is that of two armies on a battlefield (an 8x8 checkered grid in Western Chess) with scripted rules for allowable movement. The goal is to threaten the opponent's king with capture so that any legal move that king might make will cause an immediate capture the next turn (known as checkmate). The feedback system has both qualitative and quantitative aspects: *position* and *material*. Position refers to the placement of the pieces and a consideration of the overall configuration of the gameboard. Players use board position to inform their strategy and take stock of the quality of each other's play because it can be predictive of achieving the game goal (players that control more space have strategic and tactical advantage). Material refers to the game pieces. Capturing the game pieces of the opponent confers a material advantage, which can also give strategic and tactical advantages. Both position and material can inform the player of progress toward their goal of checkmating the opponent's king.

Scrabble®,[2] is a crossword boardgame that has players randomly draw tiles representing letters of the alphabet and place them on a gameboard to form words either horizontally or vertically. The pretended reality is that of 2 to 4 cruciverbalists competing to construct the most valuable words on a 15x15 grid of squares. Each letter has a point value based on its frequency in allowable words of the designated dictionary (e.g., allowable words with the letter "q" are rare in English, so it is worth 10 points), and the gameboard contains squares that can

[2] Scrabble is a registered trademark of Hasbro. The use of this name does not imply any affiliation with or endorsement by Hasbro.

multiply the point value of a letter or the total value of the word formed. The rules govern the placement of the words from an agreed upon dictionary by restricting tile placement to a single horizontal or vertical line on the grid so that a newly placed word must use one or more letters of a previously played word through at least one crossing or at least one adjacency.

The goal is to earn more total points than the opponent(s), and the feedback system includes the point value of a word and the evolving state of the board via the words that have been placed. The point values of all words placed by a single player are summed to compute a total score. The player with the highest total score wins the game, so the point value of a word informs the player as to their potential for attaining the game goal. The state of the board is a form of feedback because it either provides or constrains opportunities for future play. The opportunity (or lack of opportunity) to play a high-value word informs the player of their potential for earning more points and the goal of attaining the highest total score.

Jigsaw puzzles are a special kind of game under this working definition, because all three aspects are interconnected. The goal is to reconstruct an image from a collection of its fragments. The pretended reality is the collection of fragments that interlock uniquely to form the goal image and is governed by the requirement that the entire collection of fragments is to be placed so that they interlock exactly in order to perfectly reconstruct the entire goal image. If two pieces do not interlock exactly, if the entire goal image is not perfectly reconstructed, or if there are unused pieces, then the player knows that they have violated a rule and that they are also not making progress toward their goal. Thus, the rules also form the feedback system.

Super Mario Bros®,[3] is a well-known video game where a player traverses an obstacle course that is sometimes within a maze (known as a platform video game). The pretended reality is that of a hero (usually Mario) chasing the enemy (Bowser) through a kingdom (Mushroom Kingdom). The pretended reality is governed by the rules which permit the player to take certain actions (e.g., running and jumping) and move through the map along pre-defined paths. The goal is to rescue a victim of kidnapping (Princess Toadstool) from the enemy (Bowser). The feedback system includes coins, points, time, "lives," and levels. Coins convert to "lives," which are the number of failures the player can have before having to start over. Time forces a player to complete a level relatively quickly or pay with one of their "lives." The princess is at the end of World 8-4 (there are 8 levels called "worlds," with 4 stages each). The points are, strictly speaking, superfluous, but can be used to compare one player to another. The primary feedback that informs a player of their progress toward the goal is in the completion of worlds and stages, but "lives" and time

[3] Super Mario Bros. is a registered trademark of Nintendo of America, Inc. The use of this name does not imply any affiliation with or endorsement by Nintendo of America, Inc.

inform the player about their potential for completing the stage that they are playing (a sub-goal).

Game terminology relevant to design and implementation principles

With an operational definition of game in hand, other key terms are presented here for the purpose of the discussion of design and implementation principles for educational mathematics games.

A *digital (video) game* is a game that is played on an electronic computation device, such as a console, computer, or mobile device. Digital games for learning have usually been designed to be played by a single individual. In most digital games the software itself enforces the rules of the game as well as provides immediate feedback to the player.

A *tabletop game* is a game that is typically played on, or requires the use of, a flat surface.[4] Examples might include board games or card games. Tabletop games are usually designed to be played by two or more people and require an agreement by the players to abide by the game rules. Feedback in a tabletop game often require player-player interactions and adjudication, and feedback may have a delay (e.g., after every player has completed their turn).

[4] There have been examples of games that blur the distinction between digital and tabletop games.

An *educational game* is a game that is designed for the purpose of aiding in the attainment of specified learning outcomes, and an *educational mathematics game* is an educational game with mathematical learning goals. The terms, *educational math game, math game, mathematics game, mathematics learning game,* and *math learning game* are all taken to be synonymous with educational mathematics game.

A game designed for some purpose other than purely entertainment is called a *serious game*, so educational games are examples of serious games. (A flight simulator or role-playing exercise in a business setting provide some examples of serious games.) In arguing that serious games must be "fun," Ravyse et al. (2017) "take the stance that serious games are successful if they get played (granted that the pedagogic infusion is decent) – fun games get played" (p. 50). In a discussion of games as a participatory activity, Adams (2010) goes further by saying, "for a game to exist, it must be played; otherwise it is simply a theoretical abstraction" (p. 9). Hence, the value of an educational game is judged in part on students' willingness and motivation to voluntarily play the game.

Game mechanics are the structured actions within the pretended reality of the game that can occur between players, between the player(s) and the game, or internally to the game. A player-to-player game mechanic might be an exchange of resources, while a player-to-game mechanic might be the placement of a resource. A game mechanic which might be game-to-player is the allocation of a resource, while a game mechanic internal to the game might be the generation or instantiation of resources. *Game interactions*, more generally, include game mechanics and other interactions of players with each other or the game. For example, game interactions could also include adjudication, an exchange of information, or socializing. The *game world* includes the players, the pretended reality, the rules, the goals, and the feedback system.

To *grok* a game means to thoroughly comprehend every aspect of the game (Koster, 2013). This is different from simply beating the game. Grokking a game suggests a deeper structural understanding of the game. Koster (2013) uses the example of *Tic-Tac-Toe* to illustrate the meaning of *grok*. *Tic-Tac-Toe* can be enjoyable for young children, but there eventually comes a time when children "grok" the game and understand (implicitly or explicitly) that there is always a strategy that forces a draw. At this point, *Tic-Tac-Toe* loses its appeal and is viewed as "boring."

Grokking a game could result in a player's loss of interest in any further voluntary play, especially in an individually played digital game, and that could be viewed as undesirable from an entertainment game design perspective. However, grokking an educational mathematics game could suggest that the player has completely mastered the intended learning outcomes and is able to easily perform all the mathematics tasks that the game has to offer. Such a result might not be viewed by a designer or a teacher as undesirable at all.

A *playtest* refers to a designer-observed gameplay session, along with possible follow-up interviews or surveys of the players, for the purposes of evaluating the structure of the game and/or the perceptions of the players to the game. Such playtests may involve content experts, teachers, or students representing the target audience. The designer uses that feedback to determine the degree to which design goals have been achieved, and then modify accordingly. Playtesting is usually done as part of a cycle in which a game is refined between playtests. Playtesting is no longer necessary when a game has achieved its design goals.

Grounding the design and implementation of mathematics learning games in theory

This paper discusses design principles for the development of educational mathematics games and implementation principles for the deployment of educational mathematics games in a classroom setting. Many of these principles can apply to both individual player digital games and multiplayer tabletop games, but there are some very important distinctions to be made.

Multiplayer tabletop educational games give rise to player-to-player interactions and discourse, and when used in the classroom setting, there can also arise player-to-teacher interactions and discourse. These interactions and discourse can have significant impacts on players' opportunities for learning and for their engagement with mathematical ideas. The special affordances for such interactions offered by multiplayer tabletop educational games also pose special challenges in design and in effective implementation as a classroom activity, and the principles attempt to address those challenges.

The theoretical grounding of the design and implementation of an educational mathematics game should provide an appropriate framing for how the gameplay activity both supports students meeting intended mathematics learning outcomes and enhances students' interest in playing the game. The discussion that follows situates the proposed principles for design and implementation within a framework for supporting productive disciplinary engagement in mathematics as described by Engle and Conant (2002). The voluntary play activity aspect of educational mathematics games is viewed through the lens of expectancy-value theory of achievement motivation described by Wigfield and Eccles (2000), and this is used to frame the discussion of enhancing students' interest in voluntarily playing an educational mathematics game.

There is increasing interest in moving from questions of efficacy in game-based learning, to questions of designing for learning outcomes (Gaydos, 2015; Ke et al., 2016) and implementation in the classroom (Gaydos 2015; Van Eck, 2006; Westera, 2015). Plass et al. (2015) suggest designing learning games around cognitive and affective engagement. Considering students' productive disciplinary engagement in mathematics during game play affords a useful way of evaluating students' cognitive engagement, and expectancy-value theory can provide insights into students' affective engagement.

Fostering productive disciplinary engagement in mathematics

Engle and Conant (2002) pose principles for fostering productive disciplinary engagement, and these overlap with principles of fostering communities of learners (FCL) because Engle and Conant's "principles were developed in the course of analyzing an FCL case" (p. 406). Principles for FCL are rooted in guided discovery (Brown, 1997; Brown & Campione, 1994), which has been argued to be a useful learning theory for game-based learning (Slussareff et al., 2016). The construct of productive disciplinary engagement shapes both the design and implementation principles. It also provides a useful lens for the analysis of in-game communication and discourse, an area of game-based learning research that is undeveloped (Wouters et al., 2009).

Engle and Conant (2002) have used the constructs of engagement, disciplinary engagement, and productive disciplinary engagement in detailed descriptive analyses focused specifically on student discourse (including nonverbal communication). The constructs appear well-suited for characterizing not only the discourse between players, but also other kinds of interactions arising during the play of an educational mathematics game.

For example, Engle and Conant (2002) describe *engagement* as a focused and active participation in the present discourse. The present use of that term will mean, "*a focused and active participation in the present (game) activity*." They describe *disciplinary engagement* as "contact between what students are doing and the issues and practices of a discipline's discourse" (p. 402), while the present use will be "*contact between what students are doing and the issues, practices, or discourse of a discipline*," retaining the quality of the original description while also slightly generalizing it. And Engle and Conant (2002) go on to say that disciplinary engagement is productive for participants if "they make intellectual progress" (p. 403). Similarly, in the present discussion, *productive disciplinary engagement* will mean that the participants are making intellectual progress during or through their disciplinary engagement. In this paper, the discipline is understood to be mathematics. (As such, the name could easily be changed to *productive mathematics engagement*.)

Engle and Conant (2002) posit four principles that foster productive disciplinary engagement: *problematization, authority, accountability,* and *resources*. A description of each principle follows, along with implications for game design.

Problematization

Engle and Conant (2002) say, "the core idea behind *problematizing* content is that teachers should encourage students' questions, proposals, challenges, and other intellectual contributions, rather than expecting that they should simply assimilate facts, procedures, and other 'answers'" (p. 404, emphasis in original). The original conception of problematization provided by Hiebert et al. (1996) is more specific to mathematics and is the basis for Engle and Conant's more general definition: "students should be allowed and encouraged to problematize what they study, to define problems that elicit their curiosities and sense-making skills" (p. 12). Combining these, it can be said that students should be encouraged to define their own tasks and problems, as well as question, propose, and challenge relevant (or possible) mathematical facts, procedures, and solutions.

Hiebert et al. (1996) consider reflective inquiry as conceived by Dewey (1933, cited in Hiebert et al., 1996) to be a chief component of problematization and problem solving. They describe three features of reflective inquiry: *problem identification, searching for a resolution,* and *reaching conclusions*. Problem identification occurs when "the participant sees a quandary or feels a difficulty or doubt that needs to be resolved" (Hiebert et al., 1996, pp. 14 – 15), and the search for a resolution commences once the problem is identified. The search for a solution to the problem requires activity and "overt doing," while the participant "[calls] up and [searches] out related information, [formulates] hypotheses, [interacts] with the problem, and [observes] the result" (Hiebert et al., 1996, p. 15). The participant then reaches some conclusion, even if only temporary or partially refined. "Eventually some conclusion is reached, some resolution is achieved, some hypotheses are refined. The outcome of the process is a new situation, and perhaps a new problem, showing new relationships that are now understood" (Hiebert et al., 1996, p. 15).

Implications for game design: An educational mathematics game that supports problematization and reflective inquiry would encourage and allow players to formulate goals that are essentially self-posed mathematical problems, as well as allow them to devise their own strategies to achieve those goals. Such a game would permit the player to explore some space of possibilities and make their own reasoned conclusions about the efficacy of their problem-solving strategies.

Authority

Engle and Conant (2002) hold that students should be given authority to solve problems by “having an active role, or agency, in defining, addressing, and resolving [those] problems” (p. 404). In addition to agency, students should be positioned as stakeholders, contributors, and potential local experts (Engle & Conant, 2002). To give authority to students means “that the tasks, teachers, and other members of the learning community generally encourage students to be authors and producers of knowledge, with ownership over it, rather than mere consumers of it” (Engle & Conant, 2002, p. 404).

Implications for game design: A mathematics learning game that enables player creativity and agency would be supporting authority. Such a game would provide the player ample opportunity to make mathematically purposeful choices and employ their creativity while problem-solving.

Accountability

Students should not be given complete authority, but their “intellectual work” should be “made accountable to others and to disciplinary norms” (Engle & Conant, 2002, p. 401). This means that the participant’s “intellectual work is responsive to content and practices established by intellectual stakeholders inside and outside their immediate learning environment as well as to relevant disciplinary norms" (Engle & Conant, 2002, p. 405). To Engle and Conant (2002), this accountability is internal to the learning environment, having less emphasis on external standards and assessment.

Implications for game design: While rules are an obvious way that games support accountability, so too would an adjudication process which involves the players. Such an adjudication process would encourage the justification of mathematical reasoning as part of the gameplay.

Resources and support

Engle and Conant (2002) argue that resources and support for learners are for both productive disciplinary engagement and "embodiment of the other principles" (p. 405). They offer some examples that include time for deep investigations of a problem, access to information or disciplinary experts, scaffolding, models and norms of appropriate engagement and activity, and "public forums for student work" (p. 406).

Implications for game implementation: In the context of game-based learning, the description of resources and support suggest a role for the teacher in facilitating productive disciplinary engagement during the gameplay. This facilitation might include providing feedback, assisting with adjudication, clarifying or explaining rules, and management of interaction dynamics. In gameplay, interaction dynamics can take the form of player(s)-with-player(s) and player(s)-with-game.

One example of a player-with-player interaction that may require oversight and careful management is that of an *alpha gamer*. An alpha gamer is one that seeks control over the behaviors and actions of other players, especially during cooperative gameplay. The participant subjected to the alpha gamer's pressures may find that their authority, for example, is being usurped by the alpha gamer. In this case, the teacher may need to provide a balancing interaction that restores the authority of the player(s) affected by the alpha gamer.

An example of a player-with-game interaction that may require intervention is that of an *exploit*. An exploit occurs when the player discovers a way to circumvent the intended routes to achieve the given game goal(s). In an educational mathematics game, this would represent a player achieving a game goal (solving a mathematical problem) without engaging in the relevant mathematics. The availability of an exploit can be viewed as a design flaw, but if the exploit is discovered in the classroom implementation of the game as an activity, then the teacher may impose some additional rule that effectively blocks the exploit.

Expectancy-value theory of achievement motivation

Expectancy-value theory has been recognized as a possible framework for studying motivation in game-based learning (Ke, 2011; Plass et al., 2015; Zusho, Anthony, Hashimoto, & Robertson, 2014), and it provides the language used here to describe and explain the interest learners might have in playing a mathematics learning game. Expectancy-value theory attends to two important (self-directed) questions that speak to voluntary participation for mathematics learning games: "Can I do it?" and "Do I want to do it?" The theory appears well-suited to game-based learning design and implementation and outlines multiple dimensions along which the answers to those questions are answered by the player.

Can I do it? is a question *answered* based on the learner's *expectancy*, which Wigfield and Eccles (2000) describe as the combination of the learner's *ability beliefs* about their competence to do a given task and *outcome expectancies*, the learner's beliefs about the likely outcome if they attempt a task. While these are technically different, Eccles and Wigfield (2002) point out that "in real-world achievement situations they are highly related and empirically indistinguishable" (p. 119). Some have argued that a game can be used to provide a space in which a participant feels safe to explore the boundaries of what they can do (e.g., Gee, 2003; McGonigal, 2011).

Do I want to do it? weighs three dimensions of *achievement values*, or *subjective task-values* (Eccles & Wigfield, 2002; Wigfield & Eccles, 2000) for engagement in an activity against the perceived *cost* of that engagement. The three dimension of *subjective task-values* are *attainment value* (personal importance of succeeding at the task or activity), *intrinsic value* (enjoyment gained from doing the task or activity), and *utility value* (usefulness of the task or activity in attaining goals). The *cost* refers to negative results and trade-offs for performing the task or activity. Costs might include fear of failure or success, anxiety, effort, and lost opportunities to engage in other activities (Eccles & Wigfield, 2002).

As discussed above, reasons that a person might want to play a mathematics learning game include enjoyment, learning value, or beating the game. The dimensions of subjective task-value align well with these reasons. Enjoyment of the game aligns with intrinsic task value, learning value aligns with utility value, and beating the game aligns with attainment value. (In a multiplayer game, the opportunity to socialize or collaborate could increase the subjective task-value on one or more of these dimensions.)

Implications for game design: The voluntary play dimension of a learning game distinguishes it from other learning activities. A learning game designer seeks to address desired learning outcomes through creating a pretended reality with rules, goals, and a feedback system that foster productive disciplinary engagement during gameplay. In addition, the learning game designer should attend to game elements that can increase student interest and motivation to play the learning game, what could be called its *gameplay-value*. Expectancy-value theory suggests that enhancing a player's perception of the *gameplay-value* of a learning game involves more than just increasing its perceived "entertainment" value. The perceived value of the disciplinary engagement activity itself and the expectations of achieving learning outcomes could also contribute to gameplay-value.

Background for design and implementation principles

Dick and Burrill (2016) provide five design principles and five implementation principles for the development and use of dynamic interactive mathematics technologies to support the learning of mathematical concepts. Digital educational mathematics games, being one form of dynamic interactive mathematics technology, can certainly leverage these principles, but a case can also be made for their applicability to tabletop games. Their design principles support conceptual learning in mathematics by ensuring the technology tool allows a student to take mathematically meaningful and deliberate actions that lead to mathematically meaningful consequences, while maintaining faithfulness to the mathematics (factually and as perceived by the student). Dick and Burrill's implementation principles support teachers in their use of mathematics technology tools to engage students in sense-making, reflection, and inquiry.

Dick and Burrill's design principles for dynamic interactive mathematics technologies

A dynamic interactive mathematics technology would be a tool that provides immediate feedback in response to user actions in a mathematical scenario (Dick & Burrill, 2016). The immediacy stems from its dynamic nature, the feedback from its interactivity, and the mathematical scenario from it being a mathematics technology. A multiplayer tabletop mathematics learning game is neither dynamic nor interactive in the sense that Dick and Burrill (2016) use those words, but their principles for technology-based environments offer striking parallels to games. A clear emphasis in their design principles is that the user (player) should be allowed to engage in mathematically meaningful and deliberate (game) actions with mathematically meaningful (game) consequences in a mathematical (game) scenario. This is restated for emphasis in the context of designing mathematical learning games:

> **The design of multiplayer tabletop mathematics learning games should allow students to take mathematically meaningful and deliberate actions that have mathematically and situationally meaningful consequences.**

The five design principles proposed by Dick and Burrill (2016) are listed here:

1. **Action-Consequence Principle:** The learner has an opportunity to take a mathematically meaningful action that leads to an immediately perceptible consequence that is mathematically meaningful.
2. **Purposefulness Principle:** The learner has an opportunity to take actions motivated by mathematical intentions and purpose.
3. **Sandbox Principle:** The learner should be guarded against the possibility of arranging the virtual objects in a way that has no mathematical meaning, as well as from extraneous and irrelevant aspects of the technology (e.g., software-specific syntax).
4. **Mathematical Fidelity Principle:** The virtual environment remains faithful to the mathematics.
5. **Cognitive Fidelity Principle:** The virtual environment remains faithful to the learner perceptions of the mathematics.

The mathematical and cognitive fidelity principles ensure that the technology tool remains faithful to both the mathematics and the learner's perception of the mathematics, respectively. Dick and Burrill (2016) point out that technology is sometimes limited mathematically (e.g., numerical precision), but mathematical errors or ambiguities and sloppiness in language and notation must be avoided. Cognitive fidelity refers to presentations that are mathematically correct but misleading in some way. They provide the example of a screen display of perpendicular lines graphed correctly, but the displayed coordinate system has visually different scales on the horizontal and vertical axes. In this case, these lines could be perceived as not perpendicular.

Design principles for multiplayer tabletop mathematics learning games

The design principles presented in this section are intended to guide the development of multiplayer tabletop educational mathematics games that create opportunities for, and support, productive disciplinary engagement in mathematics as described by Engle and Conant (2002). Table 1 lists the ten design principles for multiplayer tabletop mathematics learning games.

Table 1: The design principles for multiplayer tabletop mathematics learning games.

Mathematical Fidelity Principle	A mathematics learning game should be faithful to the mathematics, being free of errors, ambiguities, and sloppiness.
Cognitive Fidelity Principle	A mathematics learning game should be faithful to the mathematics as perceived by a player.
Embedding Principle	Each mathematical task in a mathematics learning game should be embedded in a way that elicits the formulation of a mathematical problem statement by the player, without the need to overtly indicate the task to the player.
Rules Principle	The rules of a mathematics learning game should be simple, clearly stated, consistent, and perceived as fair by the players.
Adjudication Principle	A mathematics learning game should provide error-free, simple, and fair judgment of player actions.
Reward System Principle	Every mathematical task in a mathematics learning game should have a reward associated with its successful performance and a minimal cost associated with its unsuccessful performance.
Discovery & Reflection Principle	Feedback provided by a mathematics learning game should stimulate discovery and reflection, and it should not be provided through overt telling.
Variety Principle	A mathematics learning game should provide many opportunities for its players to learn through engagement with important mathematical ideas that contribute to the attainment of the intended learning outcomes.
The Virtuous Cycle Principle	A mathematics learning game should give a player meaningful control to make consequential choices that brings their creativity to bear. Success should yield more (or different kinds of) meaningful control.
Flow Principle	A mathematics learning game should immerse each player in a flow experience that sustains the player's engagement in game-based mathematical activities throughout the duration of the game.

The first seven design principles for mathematics learning games relates to one of three defining aspects of game: *goals, pretended reality with rules,* and *feedback system*. The last three design principles support the defining aspect of *voluntary participation* by enhancing the subjective value of the gameplay (henceforth, "*subjective gameplay-value*").

Faithfulness to the mathematics: Mathematical and Cognitive Fidelity

A mathematics learning game has game goals that are aligned with mathematics learning outcomes. The fidelity principles support productive disciplinary engagement by helping to ensure that the game acts as a reliable *resource* for mathematics learning and engaging with mathematical ideas.

> ***Mathematical Fidelity Principle:*** **A mathematics learning game should be faithful to the mathematics, being free of errors, ambiguities, and sloppiness.**

Dick and Burrill (2016) make the case for faithfulness to the mathematics as an essential design principle for dynamic interactive mathematics technologies, and that rationale extends naturally to educational mathematics games, both digital and tabletop. Inaccurate or misleading representations of mathematical facts, rules, processes, structures, or concepts presented within a mathematics learning game could obviously have undesirable effects on a player's mathematical learning.

> ***Cognitive Fidelity Principle:*** **A mathematics learning game should be faithful to the mathematics as perceived by a player.**

Bos (2009) refers to cognitive fidelity as meaning “the math object makes sense and fits into a memorable schema befitting the concept” (p. 111) and adds, “cognitive fidelity enables one to make connections by seeing developing patterns” (p. 112). Adherence to cognitive fidelity can be more nuanced than simply ensuring mathematical correctness. For example, the individual mathematical tasks arising repeatedly in a learning game could be presented correctly mathematically, but suggest a false pattern or a solution strategy that works locally (within in the game) but does not generalize adequately.

Situating mathematical activity within the game: the embedding principle

> ***Embedding Principle:*** **Each mathematical task in a mathematics learning game should be embedded in a way that elicits the formulation of a mathematical problem statement by the player, without the need to overtly indicate the task to the player.**

A math learning game should embed (or situate) the mathematics within the game by using the game’s pretended reality and mechanics so that important actions in the game require the use of mathematical reasoning. Ke et al. (2016) argue that learning “should be necessitated as a component of core game actions and/or rules" (p. 1198). This notion has variously been called "intrinsic integration" (Habgood & Ainsworth, 2011), "gameplay-situated learning content" (Ke et al., 2016), and "situated action" (Lameras, Arnab, Dunwell, Stewart, Clarke, & Petridis, 2017). The underlying idea in all cases is that the game should present situations that are "natural" to the game world, but also cause the player to use mathematical reasoning in a meaningful way. In writing about mathematics learning games, Devlin (2011) says,

> "the math should arise naturally… It should not stand out as something that does not fit, a mere hurdle to overcome; rather, it should seem perfectly natural to the player… in order to complete the task at hand." (p. 139)

In their description of intrinsic integration, Habgood and Ainsworth (2011) say that

> Intrinsically integrated games embody the learning material within the structure of the gaming world and the player's interactions with it, providing an external representation of the learning content that is explored through the core mechanics of the gameplay. (p. 173)

Nicholson (2011) extends this notion of intrinsic integration and then applies it to tabletop learning games. He argues that "to create these games, the first design constraint is: 'Do not ask questions.' Instead, the design goal is to locate the game within the content" (p. 62). To Nicholson, the presence of mathematics "questions" makes the mathematics tasks "mere hurdles", as Devlin (2011) might say. This should not be construed as, "remove the questions" by employing grammatic reformulations that avoid question marks. Rather, the intent is to elicit the formulation of the mathematical tasks and problem statements from the player through their gameplay. The game should not directly ask questions or pose problem statements, nor should the game overtly tell the player the mathematical tasks to perform. The embedding or situating of mathematics in a learning game supports productive disciplinary engagement through problematization and the first stage of reflective inquiry described by Hiebert et al. (1996).

Successful embedding of the mathematics learning content within a game's pretended reality relies on the game mechanics mapping to mathematical concepts or tasks. Bright et al. (1985) suggest using Bloom's taxonomy (Bloom, Englehart, Furst, Hill, & Krathwohl, 1956) for the cognitive domain to classify the complexity of mathematical tasks in a math learning game, and Kapp (2013) provides an alignment of learning objectives classified by Bloom's revised taxonomy for the cognitive domain with game activities/mechanics (adapted in Table 2).

Table 2: Adapted from Kapp (2013)

Cognitive Process	**Associated Verbs**	**Sample Game Activities**
Remember	Define, Duplicate, List, Memorize, Recall, Repeat, Reorganize	Matching, Collecting
Understand	Classify, Identify, Locate, Recognize, Report, Select, Interpret, Exemplify, Summarize, Infer, Compare, Explain	Puzzle solving, exploring
Apply	Demonstrate, Dramatize, Employ, Illustrate, Operate, Schedule, Sketch, Solve, Use, Execute, Implement	Role playing
Analyze	Compare, Contrast, Differentiate, Discriminate, Distinguish, Examine, Experiment, Question, Organize, Attribute	Allocating resources
Evaluate	Appraise, Argue, Defend, Judge, Select, Support, Value, Evaluate, Critiquing, Checking	Strategy
Create	Assemble, Construct, Create, Design, Develop, Formulate, Write, Generate, Plan, Produce	Building, Building your own game

Keep the game simple and fair: the rules principle

> ***<u>Rules Principle:</u>*** **The rules of a mathematics learning game should be simple, clearly stated, consistent, and perceived as fair by the players.**

Unlike a digital learning game, in which rules can be delivered through a tutorial or discovered through interacting with the environment (Adams, 2010; Fullerton, 2008), a tabletop game requires each player to know, understand, and agree to the rules before play begins. To enable knowledge and understanding of the rules, they should be stated as clearly and simply as possible (Adams, 2010; Fullerton, 2008). Ambiguity and "gray areas" should be avoided, and the rules should be internally consistent (Adams, 2010). Rules that are complicated or unclear may hinder the players from making meaningful gameplay choices and the players may perceive less control (Fullerton, 2008). Mathematics learning games have specific mathematics learning outcomes. Complicated game rules that are not necessary to enforce disciplinary norms can add unnecessary cognitive demand on the players that detract from more productive disciplinary engagement in the mathematics.

Players need to perceive the rules as fair (Adams, 2010; Fullerton, 2008). Adams (2010) points out that perceptions of fairness are dependent on individual players and cultural norms, but he adds, "for all players to enjoy a game, they must all be in general agreement about what constitutes fair play" (p. 12).

Rules also serve to govern the pretended reality by providing constraints that motivate creative strategies (McGonigal, 2011) and imparting meaning to the objects and activities in the pretended reality (Adams, 2010). Rules in a tabletop math learning game should provide players with the necessary constraints so that interactions with the mathematical objects that do not have mathematical meaning are disallowed or judged as erroneous play. The constraints should also motivate creative problem-solving strategies and help to enforce the appropriate mathematical meaning of the objects and actions in the game.

Producing a rule set with the above characteristics will enable players to be responsive to disciplinary norms and each other's performance on mathematical tasks. Supporting accountability in this way will be enhanced if the math learning game adheres to the embedding principle. If the mathematical content is embedded, then the rules will either have mathematical meaning or the rules will help players enforce mathematics norms and rules during gameplay.

Feedback principles: adjudication, reward system, and discovery and reflection

Feedback provides information to the players that conveys actual and potential progress toward their goals. In turn, this information supports productive disciplinary engagement in mathematics by enabling the game to act as a resource for its player-learners. Whatever choices are made for the feedback forms (e.g., points, badges, levels, etc.) are not of concern here. Instead, the design principles seek to create a feedback system that is situated and used within the game to support productive disciplinary engagement in mathematics.

<u>Adjudication Principle:</u> **A mathematics learning game should provide error-free, simple, and fair judgment of player actions.**

Correct play in a digital learning game would have the advantage of immediate enforcement provided by the machine. Tabletop learning games, however, require the players to adjudicate play for (and between) themselves (usually with a delay). Like rules, this process should be clear, simple, and fair. The feedback to the player should come as quickly as is reasonable. In the case of a tabletop game, this might mean the end of a turn or the end of a round. To provide the teacher flexibility in how they use their time to support learners during gameplay, most tabletop math learning game should provide a means for the players to adjudicate without the need for the teacher. However, some game designs could be played by the entire class or in large groups, which may admit teacher adjudication as the primary means of judging correct play.

Advantages of incorporating a challenge-defense mechanic: A multiplayer tabletop math learning game has a unique affordance in that a player can be put in a position to act as a contributor of knowledge and an enforcer of mathematics as a discipline during the adjudication process. These types of games can have a *challenge-defense mechanic* which incentivizes players to challenge incorrect plays or defend themselves from erroneous challenges made by other players. A successful challenge or defense could result in an associated reward and/or a loss to the players involved. The incentive embeds the challenge-defense related actions within the pretended reality so that players do not perceive it as a mathematical task that exists outside the gameplay. Recommendation for the incentive to be present for *both* players in the exchange is to demotivate players from posing baseless challenges and to keep the game moving.

The ideal challenge-defense mechanic has players engaging in constructing mathematical arguments and responding to the reasoning of other players (MP3. mathematical practice standard 3 from The Common Core State Standards by National Governors Association Center for Best Practices, 2010). In a discussion of representations, patterns, and communication in mathematics learning, Goldin (2002) posits, "mathematical power consists not only in being able to detect, construct, invent, understand, or manipulate patterns, but in being able to communicate these patterns to others" (p. 213). The challenge-defense mechanic is a unique affordance of multiplayer tabletop math learning games and can support productive disciplinary engagement through authority and accountability. This mechanic can help learners become "authors", "contributors", and "owners" of their mathematics knowledge, and it can enable a learner's responsiveness to mathematics content and practices.

> ***Reward System Principle:*** **Every mathematical task in a mathematics learning game should have a reward associated with its successful performance and a minimal cost associated with its unsuccessful performance.**

Incorporating a system of balanced rewards and costs into a game is an essential game design practice (Adams, 2010; Fullerton, 2008; Koster, 2013; Salen & Zimmerman, 2004). Koster (2013) says that the minimum amount of cost would be opportunity cost, which is better than no cost at all. Fullerton says that only a small amount of cost is necessary, and that cost provides meaning to the choices that a player makes.

There may be rewards and costs in addition to those related to performance of mathematical tasks. To provide more depth to the gameplay, most of the rewards should have an impact on future play or help a player reach a goal (Fullerton, 2008). Adams (2010) argues that all costs need to have a counterbalancing reward, or there is no reason to take the risk. To maintain a general sense of fairness, rewards and costs should match the difficulty (Adams, 2010) of the mathematics tasks.

McGonigal (2011) points out that roughly 80% of gameplay time is spent failing, and that players enjoy the gameplay with the in-game failures if they perceive a game as fair and the goals as attainable. McGonigal argues that enjoyable failures in game are often accompanied by what she calls, *positive failure feedback*. Such feedback is perceived to be interesting, is provided immediately, causes positive feelings (sometimes even a smile), and provides a player with a stronger sense of agency and optimism. It is this kind of risk, a cost accompanied by interesting and enjoyable feedback, that seems wholly appropriate for a mathematics learning game.

A feedback system which associates rewards and costs with performance of the mathematics tasks in the game can provide a resource to the learner by being a source of information that conveys learning progress. Such a feedback system can also support productive disciplinary engagement in mathematics through accountability, by positioning a participant's gameplay as "intellectual work [that is] responsive to content and practices" (Engle & Conant, 2002, p. 405) within the learning environment (the game) and the discipline. Mathematics tasks that are without associated rewards or costs will not connect a player's actions in the game to the discipline in such a way as to make the player "responsive" to mathematics. Instead, those mathematical tasks may be perceived as separate from the gameplay.

> ***Discovery & Reflection Principle:*** **Feedback provided by a mathematics learning game should stimulate discovery and reflection, and it should not be provided through overt telling.**

Feedback should be meaningful (Dick & Burrill, 2016; Fullerton, 2008), and it should *stimulate* reflection and discovery without overt telling (Gee, 2003). In digital games, messages from the game providing mathematical instruction to the player may reduce the quality of the play experience (Wouters, et al., 2013), and players have been found to generally ignore feedback in the form of post-play summary or messages providing educational content that appear between levels (Ke et al., 2016). These findings suggest that overt mathematical instruction as part of the feedback mechanism in a tabletop math learning game may be viewed as disruptive to the game and could possibly be ignored by players.

A common way that games stimulate reflection is through meaningful consequences of failing the game task (for example, trying and failing with a cost can lead a player to update their strategy). If the player wants to achieve the game goal(s), they will reflect on failures (Gee, 2003). To leverage this style of feedback to support mathematics learning, the mathematics tasks should be embedded so that failure of the game task constitutes unsuccessful performance of the mathematics task.

Games can encourage players to make their own discoveries through experimentation (Gee, 2003). A common way that games encourage discovery is through providing players the opportunity to try various strategies. In a tabletop mathematics learning game, this could involve giving players time to explore and try different strategies before proposing that their solution be judged. This is different from a player being allowed to take back a "move." For example, if there are game pieces whose movement or placement have mathematical meaning, then players could be permitted (and encouraged) to try different movements and placements without being judged and without those trials being considered their "move" or their turn. This kind of exploration could help a learner see their mathematical ideas manifested in the play space. After deciding a final position, the player could inform the others that they are ready to have their "move" judged. If players advancing in skill come to view that kind of behavior as unfair, they may adopt stricter rules amongst themselves (Adams, 2010).

Supporting voluntary participation and enhancing subjective gameplay-value: variety, the virtuous cycle, and flow

The notion of subjective task-value provided by the expectancy-value theory of achievement motivation is a useful way of describing possible reasons for voluntary participation in a mathematics learning game. The components of subjective task-value are attainment value, intrinsic value, and utility value, weighed against the cost of performing the task (Eccles & Wigfield, 2002; Wigfield & Eccles, 2000). For example, a student might play a math learning game voluntarily because they want to overcome all its challenges (attainment), they think it is enjoyable (intrinsic value), or they view it as a useful resource for learning mathematics (utility). In the context of learning games, this could be called its *subjective* *gameplay-value*. Note that the evaluation of a game's subjective gameplay-value would necessarily involve actual playtesting of the game with a sample of the intended audience of players.

Expectancy-value theory also provides a framing of a student's beliefs about their abilities or the likely outcome of engaging in an activity (Eccles & Wigfield, 2002; Wigfield & Eccles, 2000), which can impact a person's desire to play a game (Koster, 2013). The discussion of each design principle in this section uses the language of expectancy-value theory to consider what implications can be drawn from game design literature for enhancing the subjective gameplay-value of a mathematics learning game.

> ***Variety Principle:*** **A mathematics learning game should provide many opportunities for its players to learn through engagement with important mathematical ideas that contribute to the attainment of the intended learning outcomes.**

A game should continue to offer its players opportunities to learn new things (Koster, 2013), and should avoid *stagnation*, which occurs when nothing new seems to happen for too long (Fullerton, 2008). At first glance, this might seem to suggest that a designer should include as many mathematical tasks as possible within a single game. However, simply giving a player many of the same tasks may lead to *overlearning*, which is an excessive amount of repetition that does not provide more learning gains (Rohrer & Taylor, 2006). While the inclusion of many types of mathematics tasks might provide a player with many opportunities to learn, if too many of those tasks do not contribute to the attainment of the intended learning outcomes, then a player might make better use of their time by engaging in other learning strategies. Variety is more than numerosity. A mathematics learning game requires a balance in its variety of tasks (quantity, type, and sequencing) in a way that facilitates the attainment of the intended learning outcomes. Some possible ways to provide meaningful variety are discussed below.

Incorporating a collection of mathematics tasks that could be related to each other within the context of the game, but that would not seem obviously related to students, could potentially delay the impression of repetitive tasks. It might only cause a delay because a player who begins to see the relationship between the tasks will begin to see them as repetitive. However, the observation of those relationships between tasks might be a desirable learning outcome. One such example could be a tabletop math learning game that includes mathematical tasks involving multiple representations (e.g., situational, symbolic, or visual), which might be particularly attractive in that it is also aligned with the "Representations" process standard recommended by the National Council of Teachers of Mathematics (NCTM, 2000).

Fullerton (2008) argues that the right balance of surprise and anticipation will make a game more enjoyable. Chance events can be used to increase the perceived variety in a tabletop math learning game by introducing elements of surprise. Random events can be triggered by player actions and interactions, or by the game world itself. For example, if a game requires players to take turns using game pieces to co-construct the graph of a function, then one player's decision can create new opportunities and challenges for another player through the randomness of that decision. If the game board contains locations that trigger random events when occupied, the game world would be creating new opportunities and challenges for the player(s). Many of these random events should have an impact on the use of mathematical knowledge or the engagement with mathematical ideas. For example, an event could change (or give) a mathematical constraint that a player must interpret and apply appropriately to carry out their intended actions. If such an event favors a player, it could relax a constraint or give them an additional reward if they successfully perform the embedded mathematics task.

Variety can enhance subjective gameplay-value through providing more challenges (attainment value), piquing curiosity through novelty (intrinsic value), or providing meaningful learning opportunities that contribute to the achievement of the learning outcomes (utility value). By enhancing the subjective gameplay-value of an educational mathematics game, this kind of meaningful variety supports productive disciplinary engagement by giving learners increased agency over whether and to what degree they will engage with the game as a learning activity.

<u>The Virtuous Cycle Principle:</u> **A mathematics learning game should give a player meaningful control to make consequential choices that brings their creativity to bear.**

Success should yield more (or different kinds of) meaningful control. Games should give players control, choices, and opportunities for creativity (Adams, 2010; Caillois, 1961; Fullerton, 2008; Koster, 2013; Salen & Zimmerman, 2004). While some authors have drawn distinctions between these three notions (e.g., Fullerton, 2008; Salen & Zimmerman, 2004), there seems to be an underlying interdependency between the three concepts. Control without choice or creativity would be manifested in a game like *slot machines*, which may not be desirable as a learning activity (except, perhaps, if the learning activity involves probability). Choice without control seems to be nonsensical, if not impossible, and choice without creativity can become tedious (Fullerton, 2008). Lastly, creativity without control or choice is simply imagination.

Players are motivated by meaningful control (Garris, Ahlers, & Driskell, 2002) and want agency (Fullerton, 2008; Ke et al., 2016). Enabling players to use their creativity to solve problems can make a game "fun" (Squire, 2011) and offering multiple ways to achieve success engenders creative thinking (Fullerton, 2008). A game should give its players meaningful control which leads to consequential choices which allow players to bring their creativity to bear. After a player makes their choice and takes an action, the player should again have some meaningful influence.

The virtuous cycle can help a learner feel more competent because it rewards them with continued agency and empowerment in performing mathematics tasks. If the learner perceives some agency in their problem solving within the game, then that could mean they perceive those (and possibly similar) mathematical tasks as being attainable by them. The virtuous cycle empowers a learner by giving them agency and authority and can improve their expectancy beliefs.

> ***Flow Principle:*** **A mathematics learning game should immerse each player in a flow experience that sustains the player's engagement in game-based mathematical activities throughout the duration of the game.**

The psychological experience of flow is a subjective state which occurs when an individual is so thoroughly focused or immersed in an activity that their sense of time and space becomes distorted, the activity is perceived as being inherently rewarding, and the individual perceives a sense of agency in that they become confident that they can handle whatever challenges may arise in the activity (Csikszentmihalyi, 1991; Nakamura, & Csikszentmihalyi, 2009). Quoting from Nakamura and Csikszentmihalyi (2009), the necessary conditions for a flow experience are:

- perceived challenges, or opportunities for action, that stretch but do not overmatch existing skills;
- clear proximal goals and immediate feedback about the progress being made. (p. 195)

Keller and Landhäußer (2012) argue that the conditions believed to be necessary for a flow experience can be reduced to the first condition (i.e., the perceived challenges match the perceived skills). This reduction is helpful for designers because it narrows the focus to perceived difficulty in the game. In a mathematics learning game, a player could perceive difficulty in the mathematics or in the game (e.g., the mechanics or rules). The remainder of the discussion establishes the need for a math learning game to position itself to enable flow, followed by offering some ways that designers can understand player feedback that might suggest "the game is too difficult."

Van Eck (2006) argues that "good games promote flow" (p. 11), and Garris et al. (2002) say "the concept of flow provides one perspective on the feelings of enjoyment and engagement that can be experienced" (p. 452) by players. In addition to contributing to "enjoyment," a flow experience in a learning game can facilitate learning as well. For example, Hamari et al. (2016) found a positive association between "conditions for flow (i.e., challenge and skill)" (p. 176) and learning, and Plass et al. (2015) argue that in-game flow experiences are a form of "optimal engagement, that is, engagement optimized to facilitate learning" (p. 262). Ke et al. (2016) says that a learning game can cause a player to be "so engaged and absorbed in the problem-solving activity that he/she loses the sense of effort and repetition, and gains powerful satisfaction from solving the game challenge" (p. 1183).

Describing how to induce flow in the context of gameplay, Garris et al. (2002) say that "flow derives from activities that are optimally challenging" (p. 452), and Ravyse et al. (2017) state, "challenge should constantly be on the fringes of player ability" (p. 51). Ravyse et al. later add,

> as gameplay progresses, player abilities go up and that challenges should always be on the edge of player ability. This suggests that game tasks should become gradually more difficult in order for a player's cycle of mastery to be continuously challenged. (p. 53)

It follows, from these descriptions, that designing a math learning game to induce flow is not perfectly attainable due to the dependence on individual player perspectives. Instead, a mathematics learning game should position itself to enable flow, attempting to satisfy the necessary condition. It can be useful to consider ways of understanding in-game experiences of difficulty.

As many of the descriptions point out, the game's challenges need to be aligned with learner-player skill. If the game is too easy, then they will become bored, and if the game is too hard, then they will become anxious (Fullerton, 2008; Koster, 2013; Salen & Zimmerman, 2004). Some ways of doing this in a multiplayer tabletop math learning game might be providing different rule sets for variations of the game, including game mechanics that make the game harder as the game progresses, or including game mechanics and constraints that both depend on and inform player interactions with each other and with the game. While *Scrabble*® is not a math learning game, it exemplifies the last two examples. The game becomes more challenging as the game progresses because resources deplete (tiles are played) and space availability diminishes (board space fills), and both of those aspects depend on and inform player interactions with the game.

One aspect of perceived in-game difficulty is cognitive load. Cognitive load theory assumes that working memory can only process a few pieces of current information (Muller, 2008; Pollock, Chandler, & Sweller, 2002). Cognitive overload occurs when the cognitive demand exceeds cognitive capacity of one's working memory (Mayer & Moreno, 2003). There are three types of cognitive load: *intrinsic*, *germane*, and *extraneous* (Leppink, Paas, Van der Vleuten, Van Gog, & Van Merriënboer, 2013; Zhang, Ayres, & Chan, 2011). Intrinsic cognitive load is the cognitive demand inherent to a learning task. In a mathematics learning game, intrinsic cognitive load would be the difficulty of the mathematics tasks with which a player engages. Germane cognitive load is the cognitive demand associated with organizing information and relationships between concepts. In a mathematics learning game, germane cognitive load would relate to the player's mental organization of mathematical objects, their properties, and the relational connections among them. Extraneous cognitive load is the cognitive demand that is not necessary to learn a topic. A mathematics learning game will have a certain amount of extraneous cognitive load that is unavoidable (e.g., the non-mathematical aspects of the pretended reality and its governing rules), and there is some evidence to suggest that extraneous cognitive load can be tolerated in a learning game (Zhang et al., 2011). However, due to limited cognitive resources, germane cognitive load is generally viewed positively and extraneous cognitive load negatively (Leppink et al., 2013).

Designers should minimize the extraneous cognitive load that would cause the player to perceive the game itself as too difficult. Examples of extraneous cognitive load that could interrupt the flow experience might include playing cards with font that is too small to read or an ordering of gameplay that seems chaotic or frantic. Designers should also be mindful of the intrinsic cognitive load of the mathematics tasks embedded within the game. If those mathematics tasks are too challenging, then designers should attempt to mix in easier tasks or break the challenging tasks into smaller constituent tasks that do not make the game "too easy" or violate the variety principle. Teachers should be aware of the germane and intrinsic cognitive loads at the level of each individual student (see the *timing principle* for implementation of multiplayer tabletop mathematics learning games).

Multiplayer games run the risk of one player dominating the other players in a way that makes the other players feel that achieving success is impossible. Koster (2013) refers to this as *the mastery problem*, and he says that it "must be dealt with" (p. 124). A perception that success is impossible can degrade a student's ability beliefs and outcome expectancy (Wigfield & Eccles, 2000), making it less likely that the student will want to engage in the gameplay. These lowered expectancies could be related to the embedded mathematics content.

Some game design authors approach one potential underlying cause of the mastery problem through an engineering lens (e.g., Fullerton, 2008; Salen & Zimmerman, 2004). Fullerton (2008) describes a *reinforcing mechanism* as something that causes instability in a game by amplifying the effects of player choices and actions. A reinforcing mechanism often favors a successful player and compounds the effects of their successes early in the game to rapidly create insurmountable challenges for their opponent later in the game (Salen & Zimmerman, 2004). Fullerton describes a *balancing mechanism* as something that brings about stability or equilibrium in a game by damping the effects of player choices and actions. A balancing mechanism often temporarily favors an unsuccessful player by making it more difficult for the more successful player to win near the end of the game (Salen & Zimmerman, 2004). A designer of a multiplayer tabletop math learning game should avoid game structures that could widen the goal attainment gap between a successful player and an unsuccessful player in the early part of the game, making it impossible for the unsuccessful player to close that gap in the later part of the game.

Stress caused by the game or player interactions could interrupt a flow experience and cause a player to want to stop playing. On the other hand, tension could be beneficial in a game, if that tension does not lead to stress. Definitions of *tension* include "inner striving, unrest, or imbalance" and "a state of latent hostility or opposition" (Merriam-Webster Dictionary online). Both kinds of tension can arise in a game (Salen & Zimmerman, 2004). Generally, tension is a desirable component of most forms of play (Huizinga, 1950), and Caillois (1961) argues that play originates with tension related to uncertainty. Salen and Zimmerman (2004) argue that games are "artificial conflict" (p. 80), even when a game is single-player or cooperative. Games produce tension because the player aims to overcome a challenge while it is uncertain that they will succeed (Adams, 2010). Players enter the pretended reality voluntarily (Huizinga, 1950; McGonigal, 2011) and as a result of the absurd means by which challenges manifest (Salen & Zimmerman, 2004).

In regard to tension in a game, Suits (2014) points out that outside the pretended reality, the goals of most games would easily be achieved by means not present in the game. Suits uses the example of transporting a small ball into a hole hundreds of yards away. This task is quite easy if one carries the ball and places it in the hole. However, golfers are more than happy to attempt this feat with a couple of sticks, only allowing themselves to strike the ball with those sticks. Golf is an extreme example to make the point that games incorporate "unnecessary" stressors. Suits (2014) continues,

> In anything but a game the gratuitous introduction of unnecessary obstacles to the achievement of an end is regarded as a decidedly irrational thing to do, whereas in a game it appears to be an absolutely essential thing to do. (p. 41)

Put another way, people often play games with a goal of engaging with mildly tense challenges in order to overcome the otherwise unnecessary obstacles (McGonigal, 2011).

Game features and elements that are associated with inner striving, (mild) unrest, and (mild) imbalance can be called, *positive tension*. For example, the tension that may arise as a result of nearness to goal completion or anticipating achievement is expected to increase motivation to play due to an association with mastery and inner striving (Chou, 2015). Game features and elements that are associated with stress or a latent state of hostility can be called, *negative tension*. For example, the tension that may arise out of loss avoidance or a fear of failure is expected to decrease motivation to play due to an association with anxiety (Chou, 2015). Chou (2015) characterizes this dichotomy with the notions of "white hat" and "black hat," which correspond to the present uses of *positive* and *negative*, respectively. Chou argues that neither is inherently "good" or "bad" in a game. He argues that a small amount of loss avoidance or fear of failure can be a good thing, if it does not cause one of the two extremes: quitting or addiction.

Making the distinction between positive tension and negative tension can be useful for designers during playtesting. If players report that certain game features are causing loss avoidance, fear of failure, or other feelings associated with stress, then the subsequent design choices can be framed around balancing or replacing the negative tension with positive tension, rather than simply eliminating the tension.

Understanding individual gameplay experiences are important for designers to consider in the design of a mathematics learning game that aims to adhere to the flow principle. Playtesting will be a key part of designers coming to that understanding for any game (Fullerton, 2008). The discussion above can be useful for framing the feedback that playtesters provide designers. If players for whom the game is designed commonly express a perception that the game is too difficult, designers can begin to investigate the degree to which a high cognitive load (and what kind), a reinforcing mechanism, or too much negative tension might be related to the perceived difficulty. If the game is perceived as being too easy by students, then designers of mathematics learning games can use the above discussion to explore ways of making the game more challenging.

Designing a math learning game to adhere to the flow principle is likely to have a positive impact on a learner's expectancies (ability belief and expectation for outcomes) because the game is more likely to maintain a balance between the challenges and their skill level. In addition, a mathematics learning game that induces a flow experience can support productive disciplinary engagement through an immersive experience that causes a learner to want to play the game, giving them a sense of agency over how they reach the learning outcomes.

Implementation principles

The implementation principles presented in this section are intended to assist teachers in effectively using multiplayer tabletop mathematics learning games as learning activities in the classroom setting to create opportunities for, and support of, productive disciplinary engagement in mathematics as described by Engle and Conant (2002).

In addition to their design principles, Dick and Burrill (2016) offer implementation principles for teachers to make the best use of dynamic interactive mathematics technologies in the classroom, while engaging students in sense-making, reflection, and inquiry. Each of their implementation principles are applicable to educational mathematics games and strongly influenced the formulation of the tabletop game implementation principles discussed here. The five implementation principles proposed by Dick and Burrill (2016) are:

1. **Mathematical Content Principle:** The activity should have mathematical content that aligns with the curricular goals.
2. **Curricular Timing Principle:** The activity should come at an appropriate time in the curricular sequence and when students are developmentally ready.
3. **Questioning Principle:** The environment should allow for mathematical inquiry and questions that challenge a student's mathematical thinking.
4. **Reflection Principle:** The activity should foster a student's mathematical sense-making and reasoning.

5. **Mathematical Practice Principle:** The activity should engage students in mathematical practices.

Table 3 lists the five implementation principles for multiplayer tabletop mathematics learning games.

Table 3: The implementation principles for multiplayer tabletop mathematics learning games.

Timing Principle	Teachers should use a math learning game at a time appropriate to student development and curricular goals.
Planning Principle	Teachers should plan the implementation of a math learning game in terms of what the learners will need in order to successfully play the game and attain the learning outcomes.
Briefing Principle	Teachers should prepare students for a math learning game by establishing behavioral norms and explaining the game and its relevance.
Managing Gameplay Principle	Teachers should monitor a math learning game activity and its player interactions. The teacher should clarify rules and assist with adjudication as needed and facilitate the mathematical discourse when asked for help.
Debrief Principle	The teacher should follow a math learning game activity with a moderated debriefing session to help students make connections between the game and the learning outcomes.

The first three implementation principles all describe what preparation a teacher should make in advance of a game being deployed as an activity in the classroom. The fourth principle provides guidance on what should be done during gameplay, and the fifth principle addresses teacher-led discussion after the mathematics learning game activity.

Timing the use of a mathematics learning game

> ***Timing Principle:*** **Teachers should use a math learning game at a time appropriate to student development and curricular goals.**

The National Council of Teachers of Mathematics (NCTM) recommends tasks that promote mathematical reasoning and problem solving (NCTM, 2014), and a game that does not serve important curricular goals could be a waste of valuable class time. In addition, the mathematics tasks in a learning game should build on a student's prior knowledge (Stein & Smith, 1998) and should engage students in productive struggle (NCTM, 2014). A mathematics learning game with content that is too advanced for a student could lead to lower ability beliefs, frustration, or both. If a game involves mathematics tasks that are more complex than "remembering" in Bloom's revised taxonomy (Krathwohl, 2002), then it is best if implementation occurs near the time students learn those topics (Bright, Harvey, & Wheeler, 1985). In terms of productive disciplinary engagement, timing the use of mathematics learning game to match a student's readiness helps to enable the student to be an author of their knowledge.

Planning for the use of a mathematics learning game

> ***Planning Principle:*** **Teachers should plan the implementation of a math learning game in terms of what the learners will need in order to successfully play the game and attain the learning outcomes.**

With an eye toward managing gameplay and the debrief session following gameplay, a teacher should fully understand the game, be fully aware of the mathematical ideas that are embedded within the game, and be cognizant of the mathematical practices that are enabled through the game. It is best if a teacher plays the game as part of their planning, following the advice of Stein et al. (2008) when they say,

> Anticipation requires that teachers, at a minimum, actually do the mathematical tasks that they are planning to ask their students to do. However, rather than finding a single strategy to solve a problem, teachers need to devise and work through as many different solution strategies as they can. Moreover, if they put themselves in the position of their students while doing the task, they can anticipate some of the strategies that students with different degrees of mathematical sophistication are likely to produce and consider ways that students might misinterpret problems or get confused along the way. (p. 323)

Playing the game can also help a teacher identify the kinds of resources students may need during the gameplay. For example, if students have not seen the mathematical language or notation in the game, a short reference sheet that is simple and easy to understand could facilitate gameplay. Such a reference sheet could help keep a student "in the game" by providing them with necessary tools to complete the in-game mathematics tasks, rather than having to repeatedly stop play to ask the teacher for "translations" or to locate the relevant material in a book (which itself could be a laborious process external to the gameplay). Teachers should be careful not to include resources that effectively negates the learning by providing students with “the answers.” Rather, the resources should engage students in mathematical sense making and reasoning.

Preparing students for a mathematics learning game

> ***Briefing Principle:*** **Teachers should prepare students for a math learning game by establishing behavioral norms and explaining the game and its relevance.**

The establishment of norms and standards of behavior before students begin to play the game can reduce the likelihood that students might use the game environment to insult, intimidate, or bully their peers. Such negative behavior could arise due to an intense interest in “winning the game” or otherwise showing superiority.

A walkthrough of the rules and how to play a mathematics learning game can reduce the impact of the extraneous cognitive load introduced by the game, especially if the game is complex or the mathematics is new to the students. Some students could benefit further from a practice round of play to avoid feelings of being overwhelmed by the game in addition to the mathematics. The walkthrough and practice round could serve as a form of scaffolding in the (game) activity, consistent with advice from Stein and Smith (1998).

Teachers should make students aware of the mathematical relevance of the game, and prime students for mathematical connections to be made. This can serve to focus the students' learning through the establishment of mathematics goals and learning outcomes (NCTM, 2014). If the game uses mathematical language or notation that is unfamiliar to the students, the teacher should inform the students, and the teacher should explain or point out any in-game references and resources to help the students effectively "translate" the unfamiliar terms and symbols.

If briefing students on the game and its relevance requires a lot of time, teachers could break up the briefing or the gameplay to occur across multiple sessions in order to allow ample time for students to become fully engaged with the game and its embedded mathematics. This is consistent with advice from Stein and Smith (1998) that advocates for providing students with enough time to explore mathematical tasks or ideas that have a high level of cognitive demand.

Monitoring and managing the game activity

> ***Managing Gameplay Principle:*** **Teachers should monitor a math learning game activity and its player interactions. The teacher should clarify rules and assist with adjudication as needed and facilitate the mathematical discourse when asked for help.**

Actively monitoring the game activity by circulating around the room serves two purposes. One purpose is to help a teacher "identify the mathematical learning potential of particular strategies or representations used by the students" (Stein, Engle, Smith, & Hughes, 2008, p. 326), that could be leveraged during the debrief session following the game activity. The other purpose of monitoring gameplay is to manage and respond to player interactions. Monitoring can help a teacher identify, and then respond to, any instance of students not following the established behavioral norms, not engaging in the intended mathematics learning activity, needing assistance with understanding rules, or getting lost or "stuck" in the game.

If students ask for assistance, teachers should help in a way that facilitates meaningful mathematical discourse among the students playing the game (NCTM, 2014). There may be times that students become "stuck" trying to perform the embedded mathematics tasks, or get confused by the relationships between performing the embedded mathematics tasks and the rules of the game. In such instances, teachers can point out something that a student said that could be useful, remind the players of the relevant in-game resource(s) that could be useful, indicate the presence of an achievable goal or implementable strategy, or indicate the achievable goal or implementable strategy. However, teachers should be wary of interrupting any productive disciplinary engagement with mathematical ideas that might occur during gameplay. Interruptions could stop the game flow (see *flow principle* above) and should only be done as a last resort to keep students from going too far afield of the intended learning outcomes or from repeated errors in adjudication.

In monitoring gameplay, teachers may find that students have been matched up in gameplay groups in unproductive ways. For example, a game group could be socializing, but not collaborating on the gameplay or its relevant mathematics. A game group could also have widely varying skill levels amongst its players that disrupt the game flow, whether that variation in skill occurs relative to the game task performance or the mathematics task performance. Less skilled players/students could begin to feel overmatched by skilled players/students, which could negatively impact their ability beliefs. Skilled players/students could disengage or feel "bored" while they wait for less skilled players/students to increase in skill, which could potentially lead to disruptive behavior. If a game group formation gives rise to these kinds of mismatches between players, then the teacher should consider modifying the game groups to make the game activity more productive and engaging for all the students.

Teacher-led discussion following gameplay

> ***Debrief Principle:*** **The teacher should follow a math learning game activity with a moderated debriefing session to help students make connections between the game and the learning outcomes.**

Teachers should leverage the game activity to ask good questions that lead to meaningful insights (as in Dick & Burrill, 2016; NCTM, 2014). Teachers should also help students make connections between the gameplay and the mathematics learning outcomes (as in NCTM, 2014; Stein & Smith, 1998). Student contributions to the debrief discussion could be in the form of describing gameplay moments or strategies that reveal, to their classmates, insights into the mathematical ideas at play. For some mathematics learning games, steering the debrief discussion toward "best-play strategies" could be revelatory in terms of mathematics sense making and reasoning. Students' ideas could be selected, sequenced, and then connected to each other, the game, and the relevant mathematics concepts (as in Stein et al., 2008).

Discussion

Multiplayer tabletop mathematics learning games afford unique challenges and opportunities to productively engage students in mathematics as a discipline. Such games give rise to player-player and player-teacher interactions that can help students make deep connections while engaging in meaningful mathematics discourse and mathematical sense making and reasoning. This paper proposes a set of design principles that are intended to support productive disciplinary engagement (Engle & Conant, 2002) in mathematics, while helping developers to craft mathematics learning games which students perceive as having gameplay-value. At the same time, teachers play an important role in using a game activity to support productive disciplinary engagement in mathematics, and a set of implementation principles is provided in order to help teachers realize the potential of a game-based learning activity. These two sets of principles are positioned for future game-based learning research into questions of designing for learning outcomes (Gaydos, 2015; Ke et al., 2016) and implementation in the classroom (Gaydos 2015; Van Eck, 2006; Westera, 2015).

An educational mathematics game could be evaluated on two dimensions: 1) its subjective gameplay-value, and 2) its impact on learning. Expectancy-value theory helps to frame subjective gameplay-value in a way that can inform design. Playtesting can potentially reveal game features and elements that enhance (or reduce) gameplay-value. Designers can use the language of expectancy-value theory to determine if changes in game-play value are related to changes in ability beliefs, attainment value, intrinsic value, or utility value. Those findings could, in turn, be weighed against the game's potential for supporting productive disciplinary engagement in order to make design choices for subsequent versions of the game that "balance the learning with the fun." A direction for future research might be to investigate how a multiplayer tabletop mathematics learning game could be designed in accordance with the proposed design principles, then cycle through playtests and design modifications while attending to subjective gameplay-value and opportunities for productive disciplinary engagement, to see how (or even, if) a "balance" can be struck.

Assessment of a mathematics learning game for its impact on learning should include productive disciplinary engagement. Engle and Conant (2002) offer some evidentiary indicators of productive disciplinary engagement that could be leveraged in the assessment of a multiplayer tabletop mathematics learning game's implementation (e.g., students/players making coordinated contributions toward the achievement of game and mathematics goals). A direction for future research could be to use the two sets of principles to design and implement a multiplayer tabletop mathematics learning game and assess whether and to what degree students might productively engage with mathematical ideas during gameplay.

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CHAPTER 3: MATHEMATICS LEARNING GAME DEVELOPMENT AS DESIGN EXPERIMENT: THE CASE OF THE FUNCTION REPRESENTATIONS CARD GAME CURVES AHEAD!

Introduction

A mathematics learning game is a game that is designed to help students attain specified learning outcomes. Such games may be attractive to mathematics teachers who hope to increase "engagement" or make the classroom environment more "fun." Some argue that learning games must balance the desired learning outcomes with "fun," in order to avoid both a failure to help students achieve learning outcomes and student perceptions that the game is simply a repackaging of schoolwork (e.g., Garris, Ahlers, & Driskell, 2002; Habgood & Ainsworth, 2011; Nicholson, 2011; Van Eck, 2006; Weitze, 2014). To stress their view that achieving this balance is nontrivial, Garris et al. (2002) say,

> the instructional games that we wish to design are not merely games in which learning is a by-product of play but games that are devoted to learning. The challenge is to adapt game features for instructional purposes… that sustains self-directed interest, without squeezing out what is enjoyable about games in the first place. If we succeed, we will be able to develop games that instruct and instruction that engages the student. If we fail, we end up with games that are dull and instructional programs that do not teach. (p. 459)

The use of games in the classroom to help students attain learning outcomes is known as *game-based learning*. Most of the recent research appears to be in the area of *digital game-based learning* (Ke, 2011), which is the use of a computer, console, or mobile game as a learning activity. In addition, digital learning games seem to be primarily played by an individual student (Harteveld & Bekebrede, 2011). In his influential book, *What Video Games Have to Teach Us About Learning and Literacy*, James Paul Gee describes 36 learning principles that are embodied by video games (Gee, 2003). Applying Gee's insights and then extending them into the area of mathematics education, Devlin (2011) argues that digital games can be an ideal learning environment for mathematics.

While there have been some efforts in designing multiplayer digital learning games (Ke, 2011), efforts to leverage player-to-player interactions have often come up short of expectations (Westera, 2015). In most instances, it seems that "open conversations are likely to be absent and that information is kept secret rather than being shared and discussed" (Wester, 2015, p. 10). On the other hand, multiplayer tabletop mathematics learning games, which include board games, card games, and other kinds of games that are generally played on a flat surface, afford unique opportunities in player-to-player (and player-to-teacher) interactions that can be leveraged into engagement with important mathematics ideas. Multiplayer tabletop games can require that players adopt and enforce a set of rules while they play, which can then be leveraged to engage players in the exchange of mathematical ideas through crafting and critiquing mathematical reasoning and arguments (Renne, 2019, Chapter 2), a recommended standard for mathematical practice (*The*

Common Core State Standards for Mathematics, by National Governors Association Center for Best Practices, 2010).

Renne (2019, Chapter 2) proposes a set of design principles and design considerations to guide the development of multiplayer tabletop mathematics learning games. The design principles and design considerations are framed by the construct of productive disciplinary engagement (Engle & Conant, 2002), which provides a useful language for describing student engagement with mathematical ideas. Slightly extending Engle and Conant's (2002) construct, while remaining consistent with its original use and intent, Renne (2019, Chapter 2) defines disciplinary engagement in mathematics as the focused and active participation in an activity that engages students in the issues, practices, or discourse of mathematics. Engle and Conant (2002) say that disciplinary engagement is productive to the extent that students are making intellectual progress during that engagement.

An educational mathematics game can be evaluated on two dimensions: 1) its impact on student learning, and 2) its subjective gameplay-value. Productive disciplinary engagement in mathematics is one way to frame potential impacts on student learning, and expectancy-value theory of achievement motivation described by Wigfield and Eccles (2000) provides a useful language to understand student willingness and motivation to play a mathematics learning game. Expectancy-value theory of achievement motivation focuses on a student's perceptions of their likelihood for success (expectancies) in an activity, and whether a student wants to engage in an activity (task-value). Through this lens, subjective gameplay-value can depend on a student's expectancies for success as it relates to the gameplay and the embedded mathematics tasks, as well as a student's subjective value of the gameplay as a task in and of itself (though it might constitute many mathematics tasks).

This paper presents a detailed account of the development of the function representations card game *Curves Ahead!,* using Renne's design guidelines and this view of subjective gameplay-value to steer an iterative design process. The iterative refinement and subsequent testing of the game are treated as a design experiment.

The function representations card game *Curves Ahead!*

The card game *Curves Ahead!* is designed for precalculus and calculus students. The game requires players to match a given *graphical* representation of a mathematical model to one or more cards that describe a representation that is *tabular, symbolic, verbal,* or *physical*. Engagement with multiple representations is a process standard recommended for K-12 mathematics by the National Council of Teachers of Mathematics (NCTM, 2000), multiple representations of functions is a centrally important topic in calculus (Dick & Edwards, 2008), and translations between function representations is an important ability in mathematics learning (Gagatsis & Shiakalli, 2004; Janvier, 1987).

The mathematical content that is targeted by *Curves Ahead!* focuses on the signs of the first and second derivative of a function. The intended learning outcomes include fluency and automaticity in the identification of whether a function is increasing or decreasing and whether its graph is concave up or concave down, given descriptions of the function across several different representations. The motivation behind choosing this content is the repeated finding that students struggle to conceive the relationships represented by a function, especially across intervals.

For example, calculus students have misconceptions in relating the graph of a function with its analytic properties, with common struggles in coordinating the signs of the first and second derivative across intervals (Baker, Cooley, & Trigueros, 2000). Students are slow to develop the understanding that functions relate two variables that change together, or covary (Carlson, Jacobs, Coe, Larsen, & Hsu, 2002), and students struggle to interpret graphical representations of rate of change (Monk, 1992; Monk & Nemirovsky, 1994).

The design challenge for *Curves Ahead!* was to

1. embed (or situate) the selected mathematics content within the game,
2. through tasks that required matching,

3. in order to engage students in the process of translating function representations, and
4. in a way that would engender student perceptions of gameplay-value
5. while maintaining opportunities for productive disciplinary engagement in mathematics.

Terminology related to games

Definitions of key terms are presented here for the purpose of the discussion of game design.

Renne (2019, Chapter 2) provides the following definition of *game*:

> **A *game* is a voluntary play activity in a pretended reality governed by rules, wherein the participant(s) try to achieve one or more goals, and where degrees of success in the attainment of goals are conveyed by a feedback system.**

The four aspects of a game in this definition are *voluntary participation, pretended reality governed by rules, adoption of goals,* and *progress conveyed by a feedback system*. A pretended reality is a game's setting, where "the players assign artificial significance to the situations and events in the game" (Adams, 2010, p. 5). Player goals can be internal to the game (e.g., successful completion of all challenges) and external to the game (e.g., playing with a friend). A learning outcome can be a goal that is adopted by the player, teacher, and designer of the game. A feedback system can convey actual or potential progress to a player (Renne, 2019, Chapter 2). Feedback of actual progress is information that conveys to a player "how far they have come" in relation to a game goal (i.e., proximity to a goal), while feedback of potential progress is information that conveys to a player "how far they can go" in relation to a game goal (i.e., potential for achievement).

An *educational game* is a game that is designed to assist a student in the attainment of specified learning outcomes, and an *educational mathematics game* is an educational game with mathematics learning outcomes. The terms, *educational math game, math game, mathematics game, mathematics learning game,* and *math learning game* are all taken to be synonymous with educational mathematics game.

Renne (2019, Chapter 2) defines a *game mechanic* to be:

> **A structured action within the pretended reality of a game that can occur between players, between the player(s) and the game, or internally to the game.**

Renne provides the examples of resource exchange as an example of a player-to-player mechanic, resource placement as an example of a player-to-game mechanic, resource allocation as an example of a game-to-player mechanic, and resource generation as an example of a mechanic internal to a game.

A *playtest* occurs when a version of a game is played by a representative sample of likely players in order to test the degree to which it adheres to design guidelines or attains design goals.

Iterative refinements of a multiplayer tabletop mathematics learning game

Game design relies on an iterative process (Adams, 2010; Fullerton, 2008; Salen & Zimmerman, 2004) that involves playtesting. Due to some dependencies on player perceptions, playtests are necessary to thoroughly assess whether a multiplayer tabletop mathematics learning game conforms to Renne's (2019, Chapter 2) design guidelines. For example, player perceptions of difficulty, fairness, tension, and cognitive load are particularly important to consider, and evaluating the clarity and simplicity of the rules and adjudication process (checking for valid actions) also benefits from player input. Feedback from players is necessary to understand how specific game features might positively or negatively impact a player's perception of gameplay-value. However, modifications made to a learning game to increase the subjective gameplay-value must be weighed against the impact those changes may have on opportunities for productive disciplinary engagement in mathematics.

The development and refinement of the function representations card game *Curves Ahead!* included a method that utilized playtests to systematically obtain player feedback to aid in the design process. That method was employed to iteratively check for conformity to Renne's (2019, Chapter 2) design guidelines and to participant perceptions of gameplay-value. As such, the game design process for

Curves Ahead! appears to fit closely to the structure of design experiments, which are described in detail in the next section.

Design experiments

Iteration and refinement in design is necessary, "because designers never get the solution right the first time" (Rogers, Sharp, & Preece, 2011, p. 329). Each cycle of an iterative design process yields more insights than the last, which helps to refine a design so that it converges to a desired state or solution (Norman, 2013; Rogers et al., 2011). Each cycle informs the next through an examination of the proximity to the desired state. Norman (2013) proposes that designers engage in all four of the following activities during each cycle of iteration.

1. *Observation*: identify the design requirements, goals, and the target audience.
2. *Idea generation (ideation)*: propose mechanisms and solutions to achieve the design goals.
3. *Prototyping*: convert ideas to informative, usable, or testable models.
4. *Testing*: mimic authentic use of the prototype with small samples of the intended users as though it is the end product.

Testing leads back into observation as the designers observe the degree to which "the new design meets the needs and abilities of those who will use it" (Norman, 2013, p. 229). Norman (2013) adds that an iterative design process is nonlinear, and designers can (and sometimes should) go back and revisit past activities. He situates iterations of the four activities within both phases of a two-phase model of design that begins with identifying the right problem and concludes with identifying the right solution.

As in design science, use of design experiments in educational research strives to achieve usability of findings outside of the lab or research setting (Brown, 1992). To achieve this goal, teachers become part of the research team (Collins, 1992; Gorard, Roberts, & Taylor, 2004), as do students (Brown, 1992). Design experiments in educational settings involve the participation of researchers, teachers, and students alike.

Design experiments are an iterative process that allow for alterations to both the underlying conjectures and the learning intervention *during* the experiment (Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003; Gorard et al., 2004; Middleton, Gorard, Taylor, & Bannan-Ritland, 2008). The iterations allow for attention to why an intervention may not have the intended effects and how the intervention might be modified to get nearer to the intended effects (Brown, 1992).

Design experiments might be useful for investigations in game-based learning and may provide a coherent research paradigm (Squire, 2003). However, design experiments make multiple changes simultaneously among interwoven components of the environment that are difficult to isolate (Brown, 1992). The sources of variation within a design experiment lead to questions about the generalizability of findings for learning interventions (Desforges, 2000; Middleton et al., 2008; Shavelson, Phillips, Towne, & Feuer, 2003; Squire, 2003).

In engineering, design studies iterate through successive refinements of a tool or product, aiming for a pragmatic solution to a given problem (Middleton et al., 2008). Successful design of a tool or product requires that the tool has utility (it solves the stated problem) and that people have buy-in (Norman, 2013). Learning games can be viewed as tools[5] that are designed to engage a learner (buy-in) while providing educational content (utility). The most effective use of a design experiment in game-based learning research may be in the design *process*, so that a game under development is more likely to achieve its goals.

Middleton et al. (2008) propose a research model for education and learning interventions that has seven phases:

Phase 1. *Grounded models*: Identify the research problem and grounded theoretical model.
Phase 2. *Development of artifact*: Propose an intervention based on the theory.
Phase 3. *Feasibility study*: Assess the theory and design, and estimate the effects.
(if necessary, return to phase 1)
Phase 4. *Prototypes and trials*: Pilot one or more small-scale interventions.
Phase 5. *Field study*: Use the intervention in situ, modify as necessary, and document carefully.

[5] Middleton et al. (2008) argue that learning interventions can be viewed as tools.

Phase 6. *Definitive test*: Scale up to an experiment in the traditional sense.
(if necessary, return to phase 4)

Phase 7. *Dissemination and impact*: Share the results with the field and practitioners.

The first two phases and the last two phases form the classic research model, and the design experiment is the middle phases 3, 4, and 5 (Middleton et al., 2008). Like the model proposed by Norman (2013), this model encourages iteration to both properly articulate the problem, and to find a suitable solution.

The few differences that exist between the models of Norman (2013) and Middleton et al. (2008) may be due to context. Commercial product development can benefit from keeping the details of the process proprietary, sharing only the input (the crafting of the problem) and the output (the proposed solution). However, educational research has an added requirement of conveying a careful record of the experiment, its modifications, and other environmental conditions, to enable the research community to carefully examine the data from multiple perspectives (Cobb et al., 2003).

Both kinds of iterative design processes outlined above necessitate an analysis of how well the proposed solution is performing given the problem in consideration. The examination of progress in light of the goal informs the modifications for the next cycle and requires a clear way of making a judgement

(Cobb et al., 2003; Norman, 2013). The intention behind the modifications is to make progress toward solving the problem.

For the design of learning games, Vanden Abeele et al. (2011) recommend an iterative process that includes significant involvement from players and a broad range of specialists that can contribute to the game design. They break the design process into three significant phases: *concept design, game design,* and *game development*. Most of the iterative process occurs during the last phase which includes the prototypes and playtesting, and each phase includes the participation of players and outside experts.

In this paper, the developmental stages, phases, steps, and activities of the design experiment will be called *design activities*, and significant modifications or redesigns will be called *design pivots*. Each stage of the design experiment has a different focus but follow a similar template to the one recommended by Vanden Abeele et al. (2011). Table 4 provides an outline of the design activities that are present in each stage of the design experiment.

The table is presented in an order that becomes less meaningful as the number of iterations increases. This “ordering” was only loosely followed in the design experiment. The indications of layering are included because game design involves an entanglement and interdependence between its features (Renne, 2019, Chapter 2). Importantly, each design activity may lead to a design pivot in any other design activity, but the bulk of design pivots occur as a result of playtesting.

Table 4: Design activities for refining multiplayer tabletop mathematics learning games in a design experiment.

Design Activity	**Description**	**Layering of Activities**
Identify (or alter) possible learning goals	Select educational content and tasks that support desired learning goals. Start broadly and then possibly narrow the selection during other design activities.	Consider game mechanics that might match the educational content. Some consideration of other features may be beneficial.
Develop (or modify) a game concept	Look to common game mechanics while adhering to the situated content principle for the design of math games.	Some consideration of other features may be beneficial.
Produce (or modify) a proto game[6]	Create a playable mockup that exhibits the core mechanics with a handful of relevant and embedded educational tasks. There is little need to attend to features that do not directly support the core mechanics.	Consider game structure that will incentivize engagement with the core game mechanics.
Playtest the proto game	The designers test the fundamental game interactions.	Be ready to make a design pivot in any design activity.
Develop (or modify) a game prototype	Create a version of the game for playtesting that adheres to some design principles. The prototype should include most or all the features of the game that are believed to make the game a success.	Consider possible play testers: the design team, outside experts, or the learners.
Playtest	Play testers play the game to estimate effectiveness of design (proximity to the design goals). Achieve this through subjective experience surveys, interviews, or pre/post-tests, for example.	

Renne's design principles for multiplayer tabletop mathematics learning games

Renne (2019, Chapter 2) proposes 10 design principles in the creation of tabletop mathematics learning games. Those will be reviewed briefly here.

[6] Not to be confused with an unrelated game-development tool of the same name.

- **Mathematical Fidelity Principle:** The game should remain faithful to the mathematics, and be free of mathematical errors, ambiguities, and sloppiness.

- **Cognitive Fidelity Principle:** The game should remain faithful to the mathematics as perceived by the player. (For example, the mathematics presented in the context of the game could be correct but suggest to the player a conclusion that does not hold more generally.)

- **Embedding Principle:** The game should embed the mathematical content so that the game elicits the formulation of the mathematical tasks and problem statements from the player through their gameplay. The game should not directly or overtly pose mathematical tasks or problem statements to the players.

- **Rules Principle:** The rules of the game should be simple to understand, clearly stated without ambiguity, consistent, and perceived by players as fair. Adams (2010) argues that fairness is subjective to each player, but that it is extremely important that players generally judge a game to be fair.

- **Adjudication Principle:** The game should adjudicate play fairly, correctly, and simply. Ideally, a tabletop math learning game would provide a means for the players to adjudicate the gameplay themselves so that a teacher has more logistical flexibility (e.g., more time to facilitate meaningful mathematical discourse).

  Multiplayer tabletop mathematics learning games afford the opportunity to involve players in the adjudication process through a challenge-defense mechanic that incentivizes players to challenge each other on mathematical grounds and reply with mathematical justifications for their play.

- **Reward System Principle:** Every mathematical task should have a potential reward for its successful performance, and a minimal cost for unsuccessful performance. The rewards should not themselves be opportunities for more reward, but rewards in and of themselves.

Rewards should impact future play and all rewards and costs should be commensurate with the difficulty of the task.

- **Discovery & Reflection Principle:** The feedback provided by the game should stimulate discovery and reflection on the part of the player. Players may ignore overt instruction and feedback that is too much like a textbook or disrupts how the game flows.

- **Variety Principle:** The game should provide as many opportunities to learn as possible. *Stagnation* occurs when the game seems to be the same for too long (Fullerton, 2008). In a multiplayer tabletop math learning game, the use of random events and player choices, actions, and interactions can enhance the variety and avoid stagnation. The game should avoid unnecessary repetition in task types, and the game should leverage multiple representations to give players the impression that there are many different tasks.

- **The Virtuous Cycle Principle:** The game should provide the player with control, choice, and creativity in a cycle. Players are given some influence, which leads to meaningful choices, which brings their creativity to bear, and produces more (or different) control.

- **Flow Principle:** The game should immerse the players in a flow experience that sustains their engagement with embedded mathematical activities throughout the duration of the game.

  Flow is a subjective experience which occurs when an individual is deeply immersed in an activity and their sense of time and space becomes distorted, they perceive the activity as intrinsically rewarding, and they become confident that they can overcome any challenge presented in the activity (Csikszentmihalyi, 1991; Nakamura, & Csikszentmihalyi, 2009). Keller and Landhäußer (2012) argue that the conditions believed to be necessary for a flow experience can be reduced to perceived challenges matching an individual's perceived skills.

  Renne (2019, Chapter 2) describes three factors that can contribute to player-perceived difficulty of a game: cognitive load, tension, and stability of game interactions.

Cognitive load is of three types: *extraneous, intrinsic,* and *germane* (Leppink, Paas, Van der Vleuten, Van Gog, & Van Merriënboer, 2013; Zhang, Ayres, & Chan, 2011). Extraneous load represents the cognitive effort that is strictly unnecessary to learn the material. Intrinsic load is the cognitive effort that is inherent to learning the material. Germane load is the cognitive effort for organizing patterns and making connections. In a learning environment, germane cognitive load is usually desirable, while extraneous cognitive load is generally undesirable (Leppink et al., 2013).

The game should balance these three kinds of cognitive load. Removing all extraneous cognitive load is unrealistic because it is a game (Renne, 2019, Chapter 2), and it might even be well tolerated (Zhang et al., 2011).

Renne (2019, Chapter 2) describes two types of tension that might arise during the play of an educational mathematics game: *positive tension* and *negative tension*. Examples of positive tension include that tension which might arise out of nearness to goal completion or the anticipation of achievement. Examples of negative tension include that tension which might arise out of a drive for loss avoidance or a fear of failure. Rather than eliminating tension, a mathematics learning game should strive to balance it so that players do not become stressed.

Fullerton (2008) describes a *reinforcing mechanism* as one that allows the game to amplify effects, usually in favor of the successful player, and may lead to instability. A *balancing mechanism* is one that mitigates a reinforcing mechanism and restores stability. A stable system for interactions in a multiplayer tabletop mathematics learning game is one that does not allow one player to compound their successes in a way that makes it impossible for other players to catch up.

While most of these components may be perceived by players as part of "difficulty," they are made distinct by Renne (2019, Chapter 2) to

> provide a means of analysis for designers of mathematics learning games that refines the notion of game difficulty and the flow experience.

Renne (2019, Chapter 2) proposes that the last three principles contribute to a player's willingness and motivation to play the game. To provide a language for understanding player views of their willingness or motivation to play a mathematics learning game, Renne (2019, Chapter 2) uses the framework provided by the expectancy-value theory of achievement motivation. Expectancies relate to a learner's beliefs about their abilities and likely outcomes if they engage in a particular activity (Wigfield & Eccles, 2000). Task-values are subjective, and have four components: *attainment value, intrinsic value, utility value,* and *cost* (Wigfield & Eccles, 2000). A slight modification of the original use in Wigfield and Eccles (2000) is helpful for the context of math learning games. Renne (2019, Chapter 2) points out that attainment value could be to overcome all the game's challenges ("beating the game"), intrinsic value could be "enjoyment" of the game, and utility value could be the perceived effectiveness in the game's ability to help that player learn the mathematics. Together, a player's expectancies for success and their value of a game as a task in and of itself, comprise that player's *subjective gameplay-value*.

Research questions

The present study attempts to investigate how a multiplayer tabletop mathematics learning game can be developed and refined to meet the design challenge of embedding multiple representations of functions that are categorized by the signs of their first and second derivative within that game, through representation translation tasks that require matching, in a way that engenders student perceptions of gameplay-value while maintaining opportunities for productive disciplinary engagement in mathematics.

> *Question 1:* How can a multiplayer tabletop mathematics learning game be effectively structured to embed gameplay tasks that require students to interpret graphical function behavior across a wide variety of representations and contexts?
>
> *Question 2:* How can a design experiment methodology be leveraged to inform game modifications that maintain or increase gameplay opportunities for productive disciplinary engagement while also enhancing the subjective gameplay-value to players?
>
> *Question 3:* Do participant perceptions of the gameplay-value, based on a final playtest of the refined game, support the modifications made to the game during the design process?

Methodology

This study used a design experiment to iteratively refine the card game *Curves Ahead!*. Each iteration step in the design experiment included a playtest, and each playtest had a different set of participants, treatment, instrumentation, procedure, and analysis of the data. Results will be reported and explained between playtests and may include design activities that are different from the playtest.

Curves Ahead! before the first playtest

The card game, *Curves Ahead!*, is designed for precalculus and calculus students[7] using the design principles from Renne (2019, Chapter 2) and outlined above. The game requires players to match a given *graphical* representation of a mathematical model to one or more cards that describe a representation that is *tabular, symbolic, verbal,* or *physical*. The players (or teams) take turns and try to collect as many points as possible by making correct matches. Players have assets in the form of tokens that give them a temporary strategic advantage over their opponents. The game also includes a mechanic that incentivizes players to catch erroneous play through a challenge-defense mechanic. This mechanic awards players that successfully challenge erroneous play or successfully defend against erroneous challenges.

Game concept

The core mechanic in this game is matching. *Players have a hand of playing cards, each with a representation of a mathematical model or function that may be in the form of a table of coordinate pairs, symbols or an equation, a story or context, or a physical representation (see*

Figure 1). Each card describes a function for which the signs of its first and second derivative do not change over its domain. The graph of each function, then, will be one of the four shapes in

[7] The original design was for calculus students, but the design experiment revealed a wider applicability.

Figure 2, which is on the card to be matched (called the *target card* or *curve card*).

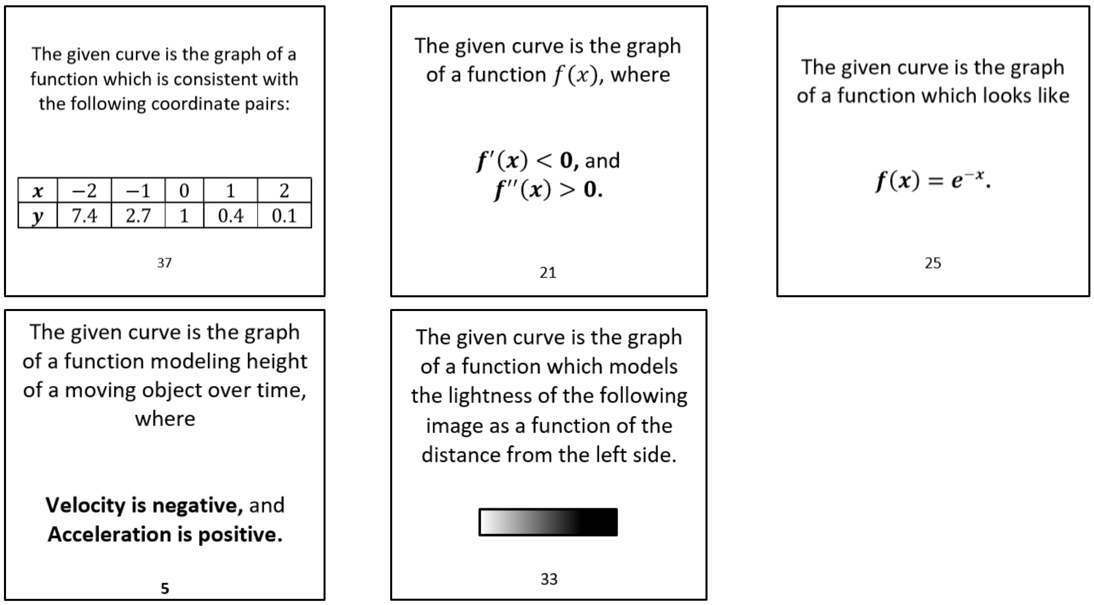


Figure 1: Sampling of playing cards that shows the kinds of function representations in the game Curves Ahead!

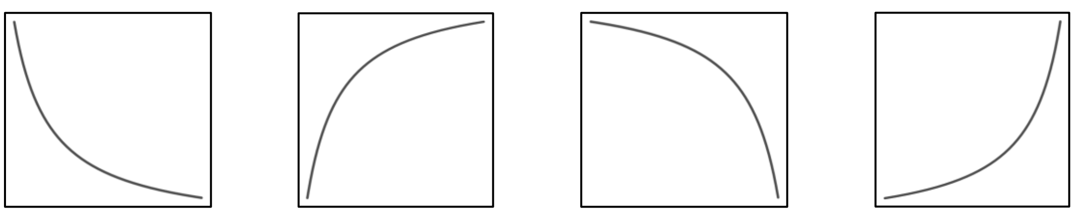


Figure 2: The four orientations of the curve card.

The initial game concept had a deck of target cards and a deck of playing cards. The idea at that time was for the players to draw a target card and match their playing card to it, then repeat by drawing a new target card. However, any one of the four shapes can be rotated in 90° increments to obtain any other, so that the game only needs one target card. This was the first *design pivot*. It was a significant observation because it simplified the game concept considerably and improved the flow of the gameplay. Flow is a kind of immersion or intense focus that might cause distortion of an individual's sense of time and place (Csikszentmihalyi, 1991). Improving the flow is expected to contribute positively to the subjective gameplay-value (Renne, 2019, Chapter 2).

Proto game

A deck of 48 square playing cards, 1 square curve card, and rules were printed as the proto game. The rules of the proto game dictated that players must be seated so that each of them would see a unique orientation of the curve card that was placed face up in the center (Figure 3). Seven cards would be dealt to each player, and each turn would deplete each player's hand by the one card they played.

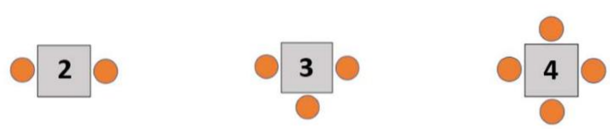


Figure 3: The proto game required different seating arrangements for different numbers of players.

All players that matched the curve card would earn a point during that turn (represented by collecting the card). Playing cards had card numbers that served two roles. One role was to look up the matching graph in an answer key. The second role was an ordering of the cards that depended on the difficulty of the mathematical task. The matching card with the highest card number earned that player the control of the curve card orientation for the next turn.

Cards were to be played in turn, face up. The played cards would then be judged for correctness, with points awarded for catching erroneous matches. Judgements were to be done by players in order of least total points to highest so that players that had fewer points would have an opportunity to catch up. The game ended when all card hands were entirely depleted, and the player with the most points was declared the winner.

Early trials of the proto game

The first trial playtests were conducted by the creator of the game and a content expert. There was an early realization that this version of the game had induced cognitive overload, violating one of the design principles (Chapter 2).

Each player was seated so they would see a different orientation of the target card, which would force each player to play a match to a different graph. The challenge-defense mechanic that awarded points for finding player errors meant that players would have to judge each of their opponents from a different perspective (see Figure 4). It seemed likely that this arrangement would require most non-experts to walk around the table and check each card with an alternative orientation, and each judgement was expected to be a difficult task for a typical player. This was also thought to be disruptive to the game flow and the flow experience.

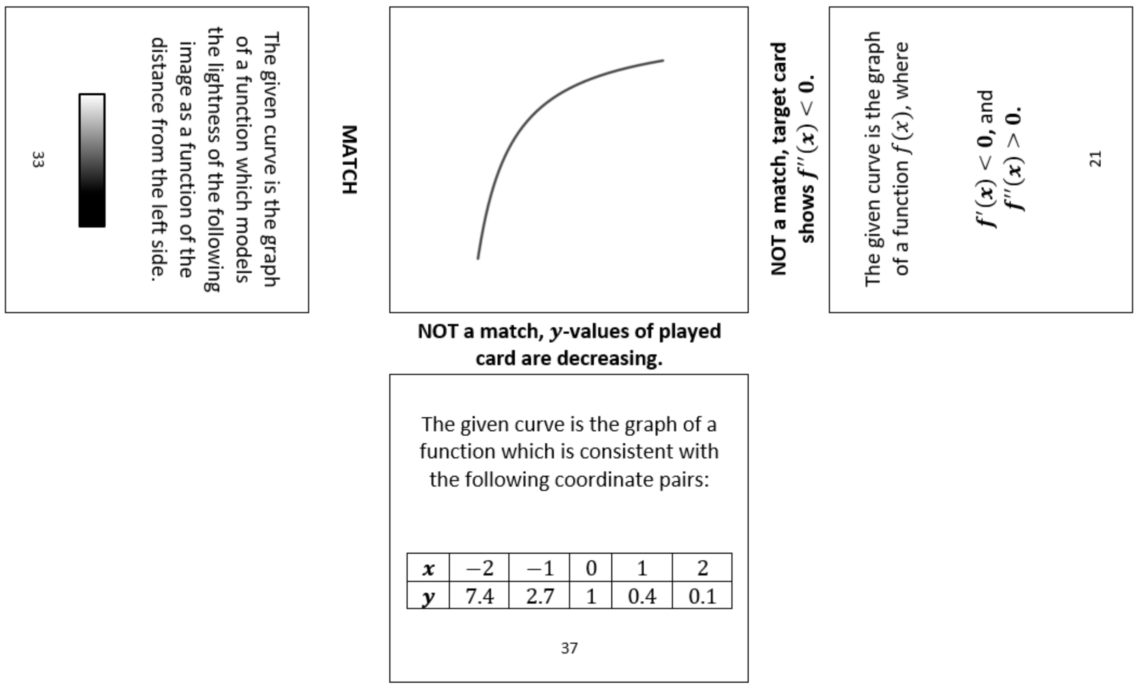


Figure 4: The proto game had each player matching a different curve card orientation.

To reduce the intrinsic cognitive load, the rules were changed so that the curve card would be placed standing up for all players to see the same orientation. This *pivot* was expected to reduce the complexity of judging opponent plays and maintain game flow, and therefore increase subjective gameplay-value (Renne, 2019, Chapter 2). With the curve card standing up, players would not be required to sit in a rigid formation, but the card would need to be placed so everyone could see it.

There were two additional *pivots* as a result of this proto game playtesting that serve as an example of the interdependence of game features. One change was a response to the ease with which a player could memorize the matches based on card numbers, which is a failure in adjudication (see Chapter 2). To combat this exploit, a decision was made to incorporate four-digit card identifiers that were randomly generated. However, randomizing the card identifiers meant that the card numbers would no longer reflect difficulty. To address this consequence, the next change was to explore the possibility of assigning point values to each playing card that depended on mathematical difficulty. This change is consistent with the view that in-game achievements can generate further engagement with a game (Chou, 2015). However, this was a difficult choice because of the worry that this might seem to players as arbitrary or unfair.

As it pertains to maintaining flow, another realization was that playing cards in turn allowed for off-task diversions while a player waited for the preceding player(s) to make their selection. Subsequent play could depend on previous play, so there would be little motivation to engage with one's hand while waiting for preceding players to make their choices. Such diversions reduce game flow, and as a result, subjective gameplay-value decreases (Renne, 2019, Chapter 2).

A *pivot* was made so that players would make and play their selection at the same time. To allow for differences in processing time, players would play their card face down so others would not be able to base their decision on present selections of their opponents. This has its significance in the fact that the highest valued matching card would win the round. If a player saw the values of the cards of their opponents, that would provide them an exploit. Players, being likely to observe such exploits as a means of in-game survival (Koster, 2013), would then refuse to be the first to play their cards. Such a refusal would cause a stall in gameplay, which would reduce flow and then subjective gameplay-value.

Note the example of an entanglement in the game mechanics. Playing the cards in turn means that seeing other player choices informs strategy as play proceeds to manage the depleting resource (cards in hand). This has the potential to enrich the gameplay experience by adding depth of possibility to one's gameplay strategy. However, increasing intrinsic and germane cognitive load and reducing extraneous cognitive load by playing cards at the same time, means that a feature before the pivot becomes an exploit after the pivot.

Game prototype for the first playtest

The game prototype featured tokens that would give players temporary strategic advantage against their opponents. This choice was in the service of increasing subjective gameplay-value by introducing a virtuous cycle (Renne, 2019, Chapter 2) that was missing from the early trial game. The tokens also facilitated the orchestration of mild tension (Renne, 2019, Chapter 2) that did not rest solely on the educational task of matching function representations. There were four circular tokens:

- The *shield* token allowed the player to play a mismatching card so that it could not be challenged. The player would discard their played card without earning a point that turn.
- The *wild* token had an image of the curve that is on the curve card. This token allowed the player to play a card that did not match the curve card but guarantee a match by rotating the token to the orientation that matches the playing card. Error could be challenged.
- The *'+2'* token increased the point value of the played card by two points.
- The *rotate* token allowed the player to rotate the curve card after all players had played their intended matches. None of the effected players could be challenged, but they also could not match the card after it had been rotated. If multiple rotate tokens were played simultaneously, the player with the lowest total score earned was able to rotate the curve card while all others lost their token as a wasted play.

The tokens were to be played face down with the player cards and revealed at the same time as the playing cards. The tokens could only be played once per game and were expected to introduce positive tension and reduce negative tension. Incorporating features that were more in the service of the game than in the service of the learning goals was an intentional choice with the hope of increasing subjective gameplay-value. However, there was some concern that the tokens may have substantially increased the extraneous cognitive load within the game.

Playtest I

The first playtest had participants provide real-time feedback and discussion during the gameplay. The primary focus of the playtest was to examine game features in accordance with Renne's design principles (Renne, 2019, Chapter 2), with particular attention to subjective gameplay-value.

Participants

The first playtest of the prototype game was conducted with six players with a strong mathematical background and an interest in teaching mathematics. The small number of players during this activity is in keeping with the recommendations from Middleton et al. (2008) and Norman (2013). An important advantage of playtesting with these individuals was that they could play the game without spending considerable cognitive effort to attend to the mathematics. This would allow for discussions of the qualities of the game in terms of subjective gameplay-value.

Treatment

An explanation of design principles from Renne (2019, Chapter 2) was provided to participants to frame the discussion during gameplay. The rules of the game were explained by the creator before play began, and participants played the card game *Curves Ahead!*. The specific version of *Curves Ahead!* that they played is summarized below:

- The curve card (or target card) would be standing with the same orientation for all players during a turn.
- Players would have 7 cards in their hand and play one each turn.
- Cards would be played face down.
- If a player wanted to use one of the four tokens (described above), they would do so by playing it face down with their card.
- After everyone played their card, all cards and tokens would be played face up.
- All effects from tokens would be resolved before judging cards played.
- Players would collect cards they won through correct matches or through the challenge-defense mechanic.
- Control of the orientation of the curve card for the next turn was granted to the player that won the most points during the present turn.
- The game would end when all card hands were depleted.
- The winner was the player with the most points.

Instruments

This playtest used a moderated group discussion during gameplay, to capture feedback that surfaced naturally within the context of the game. This allowed real-time attention to how an event or feature in the game impacted the subjective gameplay-value. Some of the players also had experience teaching mathematics, which enabled strands of commentary that hypothesized how a typical student might engage with the game.

Questions that were asked of the participants during the discussion were in response to player statements or conversation. Since the conversation could not be predicted in advance, question types were devised, rather than a list of specific questions. This strategy is consistent with an unstructured interview that can be useful in gaining insight into the perceptions of participant experiences, and the question types form an agenda for the interview (Zhang & Wildemuth, 2009).

Question types served two similar and overlapping purposes: comparing the actual gameplay experience to the predicted gameplay experience and gaining potential insight into impacts the gameplay experience might have on subjective gameplay-value. Most questions either requested a suggestion for an alternative to the implicated design feature, or an elaboration, specification, or clarification of what was said.

Curves Ahead! was designed with the intention of conforming to the design principles put forth by Renne (2019, Chapter 2), but assessing conformance to some of those design principles requires input from players (e.g., the perceived fairness of the rules or perceptions of flow). For example, if a participant were to indicate that the rules were unfair, then a question from the moderator might be, “can you elaborate on what you think is unfair?” A follow up to the answer to that might be, “what would you propose as a potential alternative to make it more equitable?”

One desirable quality of an educational math game is that learners will be interested in playing (Renne, 2019, Chapter 2). To investigate how the gameplay experience might impact subjective gameplay-value for the game *Curves Ahead!*, statements from participants that spoke to the design principles related to subjective gameplay-value, and flow in particular, were given special attention. Statements regarding flow (e.g., cognitive load or difficulty) were given the most attention due to its perceived importance in the design of learning games (Renne, 2019, Chapter 2). For example, if a participant were to indicate that the game is too hard, then a question from the moderator might be, “what do you think is making the game particularly difficult?” or “what do you think could make the game easier?” If a participant were to indicate that the mathematics is too hard, then a question from the moderator might be, “how do you think the game could provide scaffolding for the mathematical tasks?” The term, “scaffolding,” would have been familiar to these participants. With other participants, that question might be split into multiple parts that might begin with, “what could the game provide in order to help you get started on that mathematical task?”

Adhering to the advice of Gaydos (2015), the focus of the questions was the possible deficiencies in the design of *Curves Ahead!* According to Gaydos, this "is the first step for improving it" (Gaydos, 2015, p. 481).

Procedure

The creator of the game explained the design principles around which the game was created in order to frame the in-game discussion. This explanation lasted 45 minutes. While participants were aware of the design principles, the principles were not explicitly discussed during the in-game discussion. Following the exposition, participants were informed that one aim of their playtest was to improve the quality of the game. They were explicitly encouraged to freely provide their opinions as to deficiencies that they perceived in the game design and how improvements could be made.

The rules of the game were thoroughly explained before the gameplay. The explanation of the rules included an explanation of the core mechanic with some example cards, and an explanation of the tokens with their effects (without offering possible strategies of playing with them).

The participants played in pairs, so that the game was played by three teams of two players each. This allowed observation of meaningful discourse between players that might not exist if they played alone. It was hypothesized that the players would discuss game strategy and mathematics in an integrated way to coordinate their play as team members. This was expected because the players had a strong background in calculus, which reduced the likelihood that mathematical discussions would occur naturally and spontaneously if they played alone.

During gameplay, participants openly discussed the game and the mathematics with each other and the moderators. The discussion, which bore resemblance to an unstructured interview with an agenda, was moderated by the creator of the game and a mathematics education researcher that specializes in the use of technology tools in mathematics classrooms. While the presence of the creator of the game may have negatively affected the transparency of the dialogue, the discussion was able to produce actionable feedback to enhance subjective gameplay-value (evidenced below). There was great care taken to avoid defending the design of the game or explaining design choices to encourage transparency in the dialogue. Defense of the game design could lead to a (sub)conscious submission to the designer who may be perceived as an authority in the discussion.

The agenda in moderating the discussion consisted of capturing the feedback from participants that spoke to the design principles broadly, or to design principles related to subjective gameplay-value specifically. Specific areas of focus included cognitive load and flow. Notes were taken to record the feedback, but notes were not verbatim. Rather, notes described the feedback provided by the participants as well as answers to the questions that were asked of the participants. Immediately following the game, the moderators discussed their individual views of the discussion and gameplay. More details were added to the notes after the debrief.

Data analysis

Each feedback item was analyzed as to its implications regarding the game's conformity to the design principles in Renne (2019, Chapter 2), with a focus on subjective gameplay-value. Data that addressed the design principles of *flow*, *the virtuous cycle*, or *variety* were given the highest priority, in that order.

For example, if feedback indicated that the game was too hard or the cognitive load was too high, then there is a possibility that flow needed to be improved in order to increase subjective gameplay-value. If the feedback indicated a sense of powerlessness in the face of changing conditions, then players needed to be given more control, choice, or creativity to enhance the virtuous cycle, and hence increase subjective gameplay-value. If the feedback indicated that the game failed to offer new opportunities for growth or learning (i.e., the game stagnated), then variety needed to be increased in order to increase subjective gameplay-value.

Results of the first playtest and subsequent design pivots

Table 5 summarizes the feedback provided by the participants during their playtest of *Curves Ahead!*, along with design implications and the affected category of design principles in Chapter 2. Some of the feedback is categorized as general design if it was unrelated to the design principles in Chapter 2 *and* it was related to general design practices (for examples, see Norman, 2013). The table is loosely ordered thematically by the type of feedback, but overlap does exist. The first three items are generally touching on game appeal, items 4 thru 9 are organizational in nature, items 10 thru 12 are addressing tension and stability, and items 13 thru 20 regard the complexity introduced by the tokens.

Some trends that stand out in the player feedback include the concern over extraneous cognitive load and tension, as well as a desire to ensure adequacy of balancing mechanisms. The first feedback item is particularly noteworthy and was monitored carefully during subsequent playtests. That participant expressed a fear of failure and stress arising from loss avoidance, so this was coded with the design implication of reducing negative tension. The hope was that other design pivots might reduce the negative tension that player-learners might feel.

Table 5: Descriptions of the feedback from the first playtest of Curves Ahead!

	Description of Feedback	**Design Implications**	**Affected Category**

1	Feeling too much pressure.	Reduce negative tension.	Subjective Gameplay-Value
2	Many of the cards did not require calculus knowledge.	Broader educational reach.	General Design
3	Inverse functions should be included in the game.	Improve variety.	Subjective Gameplay-Value
4	The answer key should be picture-based.	Simplify adjudication.	Feedback System
5	Make the playing cards rectangular, like a standard deck, to make them easier to hold and read.	Reduce extraneous cognitive load. Improve sensory design.[8]	Subjective Gameplay-Value General Design
6	Create a discard pile to organize play.	Reduce extraneous cognitive load.	Subjective Gameplay-Value General Design
7	The order of play is too chaotic.	Reduce extraneous cognitive load. Simplify the rules. Simplify adjudication.	Subjective Gameplay-Value Rules Feedback System
8	There is too much going on in the game.	Reduce extraneous cognitive load.	Subjective Gameplay-Value
9	There are too many strategies that are unrelated to the mathematics.	Reduce extraneous cognitive load. Increase germane cognitive load.	Subjective Gameplay-Value
10	Replenish the hand after each turn to replace the played card.	Provide a balancing mechanism. Reduce negative tension.	Subjective Gameplay-Value
11	The player should be allowed to swap unwanted cards with undealt cards.	Provide a balancing mechanism. Increase positive tension. Reduce negative tension.	Subjective Gameplay-Value
12	Limit the time that players have for selecting a card and token to play.	Provide a reinforcing mechanism. Increase tension. Maintain flow.	Subjective Gameplay-Value

Table 5: Continued

13	Multiple rotate tokens played in the same turn should cancel each other.	Improve fairness in the rules. Reduce negative tension.	Rules Subjective Gameplay-Value
14	The rules for the rotate token are too complicated.	Reduce extraneous cognitive load. Simplify the rules.	Subjective Gameplay-Value Rules
15	Four tokens each, played in only seven turns, makes for too many	Reduce extraneous load. Simplify the rules.	Subjective Gameplay-Value

[8] *Sensory design* is the design of a product around the physical senses of its users.

	tokens. Maybe only two or three each.	Possibly reduce positive tension.	Rules Subjective Gameplay-Value
16	It should be allowable for a player to play multiple tokens in one turn.	Increase positive tension. Increase extraneous cognitive load.	Subjective Gameplay-Value
17	The tokens can be removed.	Reduce extraneous cognitive load. Reduce tension. Reduce the virtuous cycle.	Subjective Gameplay-Value
18	To reduce tokens, the game should only give a token to the player with the lowest total score after each turn.	Provide a balancing mechanism. Possibly reduce positive tension.	Subjective Gameplay-Value
19	The rotate token should only be given to the player with the least total points at some designated point in the game.	Provide a balancing mechanism. Reduce positive tension.	Subjective Gameplay-Value
20	There should be a bluff token. One which does nothing, but since it is played face down, it can impact play.	Increase positive tension. Add to the virtuous cycle.	Subjective Gameplay-Value

Feedback items 2, 3, and 4 were accepted for future versions of the game to capitalize on the design implications indicated in the table, but they would not be present until the third playtest.

Feedback items 5, 10, 13, and 20 were tentatively accepted for future versions of the game with an aim to increase subjective gameplay-value, pending what the second playtesting group indicated.

There was no action taken for feedback items 6 and 12. Both suggestions were such that players could spontaneously organize in accordance with those desires without intervention from the designer. The institution of time limits was recommended in all play tests by some of the players, but not the majority. This recommendation was ignored in all playtests on the grounds that players can decide this for themselves during play, and that it could lead to bottom-feeding or bullying. Bottom-feeding in a game occurs when a player takes advantage of one or more opponents that are not as well-equipped or skilled in the gameplay. Bottom-feeders exploit the gap in skills and resources that may exist between them and their opponents. An educational math game that supports bottom-feeding could lead to anxiety for those subjected to it. Bottom-feeding may enhance subjective gameplay-value for some players, but it would be at the expense of other players. Design choices that support bottom-feeding are undesirable for an educational math game. The choice to ignore this feedback was a nontrivial decision because players may disengage while waiting for their opponents to make choices, possibly reducing subjective gameplay-value.

Feedback item 7, unordered play causing chaos, could have led to a potential design pivot and was to be monitored in subsequent playtests. This version of the game had no order of play for making matches or playing tokens, one order of play for the challenge-defense mechanic, and yet another order of play for tiebreakers to determine control of the curve card. No suggestion was provided by the participants as to what could improve this. It was possible that this feedback item was related to the reinforcing mechanism that gave control of the curve card orientation for the next turn to the player with the most points in the present turn.

The first few versions of the game had no order of play for making matches, one order of play for the challenge-defense mechanic, and another order of play for tiebreakers to determine which player would control the curve card. Players appeared to prefer an order for making matches and playing tokens, and no order for the challenge-defense mechanic. Distribution of the control of the curve card addresses these preferences.

The design implications for feedback items 8, 11, 18, and 19 were thought to be addressable through other design pivots. It seemed that the cognitive load reductions in other areas might alleviate the feeling expressed in item 8. There was a possibility that replenishment of the player hand might obviate the need for swapping unwanted cards (item 11). Improving the mechanics built around the tokens in other ways was expected to address items 18 and 19.

Feedback item 9 was potentially of serious concern. Game strategies being unrelated to the mathematics could lead to cognitive overload and disengagement. However, the feeling could have been a result of player expectations as (future) educators of mathematics students. It could be that the players expected the game to feel more demanding mathematically or more focused on the mathematics. The decision was made to wait and see how calculus students felt about gameplay strategies during subsequent playtests.

It was decided that a temporary pivot based on feedback items 14 and 15 would be made for the next playtest to see how calculus students might perceive the experience. That is, to see if calculus students agreed that doing so would increasing subjective gameplay-value. This pivot was made by removing the rotate token from play during the second playtest.

Feedback items 16 and 17 were deferred with a wait-and-see approach. Allowing players to play multiple tokens in one turn (item 16) only adds to the extraneous cognitive load, which was a recurring concern in the feedback. Multiple tokens as an idea was also something players could spontaneously decide as a kind of "house rule."

Removal of the tokens (item 17) would reduce the extraneous cognitive load as desired, but the reduction would not warrant contravention of two important design principles. Following the recommendation would remove most of the tension in the game and dramatically inhibit the virtuous cycle of control, choice, and creativity. The decision was made that removing the tokens would be too costly in terms of subjective gameplay-value.

Playtest II

The second playtest had participants provide real-time feedback and discussion during the gameplay and a brief discussion following play. The primary focus of the playtest was to examine game features in accordance with Renne's design principles (Renne, 2019, Chapter 2), with particular attention paid to subjective gameplay-value.

Participants

The second playtest was conducted with four differential calculus students at a four-year university in the Pacific Northwest. The advantage of playtesting with differential calculus students is in their being members of the population of learners that the game is designed to serve.

Treatment

The rules of the game were explained by the creator before play began, and participants played the card game *Curves Ahead!*.

Two aspects of the game were changed from the first playtest and enacted for this playtest, to provide scaffolding that might address the concerns that the cognitive load was too high. This was done by removing the cards that required calculus for the first game and removal of the rotate token. The specific version of *Curves Ahead!* that they played is summarized below:

- The curve card (or target card) would be standing with the same orientation for all players during a turn.
- Players would have 7 cards in their hand and play one each turn.
- Cards would be played face down.
- If a player wanted to use one of the *three* tokens, they would do so by playing it face down with their card.
- After everyone played their card, all cards and tokens would be played face up.
- All effects from tokens would be resolved before judging cards played.
- Players would collect cards they won through correct matches or through the challenge-defense mechanic.
- Control of the orientation of the curve card for the next turn was granted to the player that won the most points during the present turn.
- The game would end when all card hands were depleted.
- The winner was the player with the most points.

Instruments

There were two moderated group discussions that took place during this playtest. One discussion was during gameplay, and the other followed gameplay. Since the participants were differential calculus students, their background knowledge was not strong enough to engage in both the mathematical tasks and a thorough discussion related to the game itself during gameplay. In order to facilitate their focus on the mathematical tasks that occurred during the game, much of the moderated group discussion during the game attended to adjudication of play and to clarify rules. Immediately following the gameplay, another moderated group discussion took place which focused on the game design.

For both discussions, question types were devised, instead of a list of specific questions. This strategy accommodated the unpredictable nature of the conversations and is consistent with an unstructured interview that has an agenda (Zhang & Wildemuth, 2009). Questions that were asked of the participants during the first discussion were only in response to player statements or conversation, and they were kept to a minimum to avoid disrupting the gameplay.

Question types served three similar and overlapping purposes: following up on specific feedback items from the first playtest, comparing the actual gameplay experience to the predicted gameplay experience and gaining potential insight into impacts the gameplay experience might have on subjective gameplay-value. If a question was intended to follow up on feedback from the first playtest, then it would have wording like, "What do you think about X?" or "Some players have suggested X, what do you think?" Here, "X" is a stand-in for prior feedback or a potential design change that arose out of prior feedback. If a question was not related to prior feedback, then the question either requested a suggestion for an alternative to the implicated design feature, or an elaboration, specification, or clarification of what was said, in a manner consistent with the questions asked during the first playtest.

Like the first playtest, the questions in the discussions in this playtest focused on the possible deficiencies in the design of *Curves Ahead!*

Procedure

The game took place in week 9 (of 10) during the fall quarter and was played for a total of 50 minutes. The creator of the game explained the rules of the game and included an explanation of the core mechanic with some example cards, and an explanation of the tokens with their effects (without offering possible strategies of playing with them). The rotate token was not described to them and they were unaware of its existence.

The participants played in pairs, so that the game was played by two teams of two players each. This allowed further scaffolding so that players could cooperate on the mathematical tasks. The players had a weak background in calculus and the pairing was intended to reduce the cognitive load during play.

Participants were informed that one aim of their playtest was to improve the quality of the game. They were explicitly encouraged to freely provide their opinions as to deficiencies that they perceived in the game design and how improvements could be made. Participants were not aware of the design principles and the principles were not explicitly discussed during the moderated discussions.

The discussions were moderated by the creator of the game. As in the first playtest, the presence of the creator of the game may have negatively affected the transparency of the dialogue. However, the discussion was able to generate useful feedback to increase subjective gameplay-value (evidenced below).

Most of the participants' in-game discussion was related to the mathematics, rather than gameplay. During play, most of the moderator involvement was to assist with adjudication and rules. After their first game concluded, the participants were asked if they would play again, but with the inclusion of the cards which required calculus knowledge. They were interested to see more and responded positively to the request. After only two rounds of play (the second not fully completed), the players asked to stop because it was too hard. The postgame discussion commenced at that point. Most questions from the moderator occurred during the postgame discussion.

The postgame moderated group discussion was directed at the game design, in the manner of an unstructured interview with an agenda as discussed above. However, there was one student that asked for explanation of a key mathematical concept that arose in the game and was relevant to her differential calculus coursework. A concept she claimed to find confusing for most of the term (the relationships between position, velocity, and acceleration). The other players took initiative in attempting to explain the concept to her, followed by a wrap-up explanation from the moderator.

To follow up on feedback from the first playtest and to gain insight into the potential impact of changes that were made between playtests, players were made aware of the rotate token and the scaffolding choice which separated the calculus cards from the main deck of cards. These points were only made during the postgame discussion.

The postgame discussion would cycle through a question from the moderator, followed by each participant having an opportunity to respond to the moderator and each other. Questions from the moderator either probed prior commentary in the manner outlined in the agenda or followed up on feedback items from the first playtest.

Notes were taken during the discussions in the same manner as the first playtest.

Data analysis

Data analysis following the second playtest was the same as that which followed the first playtest, with the addition of follow up feedback items from the first playtest.

Results of the second playtest and subsequent design pivots

Table 6 summarizes the feedback provided by the participants during the second playtest of *Curves Ahead!*, along with design implications and the affected category of design principles in Chapter 2. Some of the feedback is categorized as general design if it was unrelated to the design principles in Chapter 2 *and* it was related to general design practices (for examples, see Norman, 2013). The table is ordered so that items discussed as follow up and prompted by the moderator appear first, prepended by an "F". The feedback descriptions are numbered in continuation of the feedback from the first playtest. Recurring feedback across both playtests that was spontaneously offered by participants in the second playtest are not prepended with an "F," but given a new number and emphasized.

Table 6: Descriptions of the feedback from the second playtest of Curves Ahead!

	Description of Feedback	**Design Implications**	**Affected Category**
F-5	Make the playing cards rectangular, like a standard deck.	Reduce extraneous cognitive load. Improve sensory design.	Subjective Gameplay-Value General Design

F-7	The order of play is too chaotic.	Reduce extraneous cognitive load. Simplify the rules. Simplify adjudication.	Subjective Gameplay-Value Rules Feedback System
F-10	Replenish the hand after each turn to replace the played card.	Provide a balancing mechanism. Reduce negative tension.	Subjective Gameplay-Value
F-11	The player should be allowed to swap unwanted cards with undealt cards.	Provide a balancing mechanism. Increase positive tension. Reduce negative tension.	Subjective Gameplay-Value
F-13	Multiple rotate tokens played in the same turn should cancel each other.	Improve fairness in the rules. Reduce negative tension.	Rules Subjective Gameplay-Value
F-15	There were a good number of tokens at three each.	Reduce extraneous load. Simplify the rules. Possibly reduce positive tension.	Subjective Gameplay-Value Rules Subjective Gameplay-Value
F-17	The tokens should **not** be removed.	Increase extraneous cognitive load. Increase tension. Increase the virtuous cycle.	Subjective Gameplay-Value
F-19	The rotate token can be given to the player with the least total points at some designated point in the game.	Provide a balancing mechanism. Reduce positive tension.	Subjective Gameplay-Value
F-20	There should be a bluff token. One which does nothing, but since it is played face down, it can impact play.	Increase positive tension. Add to the virtuous cycle.	Subjective Gameplay-Value
21	*Limit the time that players have for selecting a card and token to play.*	Provide a reinforcing mechanism.	Subjective Gameplay-Value
22	It takes a few turns to realize how to play the game.	Incorporate scaffolding in the rules.	Rules
23	Player personality impacted motivation.	Alter game concept.	Implementation Principles
24	Point values on the non-calculus cards are appropriate.	Rewards appropriately tied to performance of educational task.	Feedback System
25	Point values on the calculus cards are too low.	Rewards not appropriately tied to performance of educational task.	Feedback System
26	The calculus cards are extremely difficult.	High germane cognitive load.	Subjective Gameplay-Value

During the game, one of the few comments that were made regarding the game design was provided by the pair of students that were performing better than the other (i.e., "winning"). They spontaneously declared that turns should have a time limit, which indicates a risk of cognitive disengagement for players if there is a significant difference in skill within the game.

The second group of playtesters mostly agreed with the first group on each of the follow-up items. Unlike the first playtest group however, the second group was disappointed that the rotate token was removed and insisted that tokens should remain in the game.

Follow-up items 5, 10, 13, and 20 were supported by the feedback in the second playtest, leading to design pivots for the third playtest in order to increase subjective gameplay-value. Item 7 continued to lack a clearly beneficial solution, and the design implications for item 11 was again considered to be addressable through another design pivot.

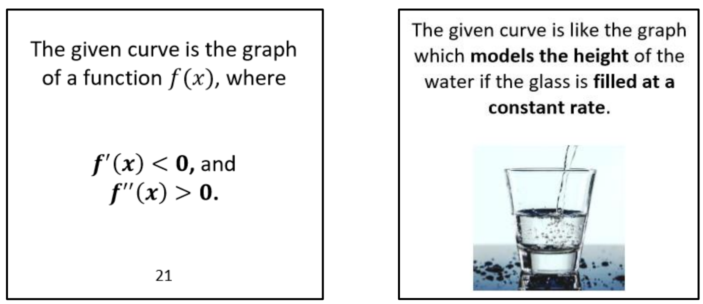
Follow-up items 15, 17, and 19 produced another pivot in that players would be directed to select only three tokens with which to play. Follow-up item 17 indicated that players wanted access to the rotate token. Follow-up item 19 was considered by the playtesters to be suboptimal to everyone having the rotate token, but better than no one having it. The pivot to having players select three tokens of their choice at the beginning of the game would balance the desire of players to have access to the rotate token, while maintaining the number of tokens in play at 3 per player. This change was made with the intention of increasing subjective gameplay-value.

Feedback item 21 was rejected for the same reason as item 12 in the first playtest. It is listed as a separately numbered item because it was spontaneously offered by the playtesters.

Feedback item 22 was noteworthy in two respects: it reflects an early concern from the first playtest regarding the difficulty of the game, and the participants were at a level for which the game was designed. Rather than changing the design of the game, it was decided to change the explanation of the game during implementation.

Feedback item 23 implicates implementation guidelines more than it does game design changes. Player personality is a component of any game, whether cooperative or competitive. The only pivot which would fully insulate players would be to make the game for one player. The decision was to reject this as it was not informative for the intended design and use of the game as a multiplayer game.

Feedback items 24, 25, and 26 were highly informative to the game design. The game at that point had the symbolic calculus playing cards (as in Figure 5A) valued at 2 points. Players were reporting that these cards were extremely difficult for them and that they should be worth more points. The cards worth the maximum of 5 points were the story problems with related rates that could be matched without calculus (as in Figure 5B). This feedback resulted in an important design pivot in order to enhance the support offered by the game (see Chapter 2), as well as an intended improvement in subjective gameplay-value (item 26).



(A) (B)

Figure 5: (A) Symbolic calculus cards valued at 2 points, and (B) Story problems with related rates at 5 points.

The playing card point values were initially chosen to reflect the intrinsic cognitive load. However, the processing required to acquire the schema (i.e., the germane cognitive load) involved in matching the symbolic card seemed to be substantially higher than was anticipated. It was decided to increase the point value of the symbolic calculus playing cards by one point. This choice was motivated by trying to balance the player experience of difficulty with encouraging them to perceive that the task *should* be easier than many of the others. Upon acquisition or automation of an appropriate schema, the symbolic cards will *feel* easier to players and a maximum point value may not seem fair to them.

In contrast, the players reported that the related rates tasks were also very hard, but they felt accomplished when they made a match. They felt that the reward for those cards should be the maximum. It seems that they were at least subconsciously aware of the intrinsic cognitive load associated with matching the related rates cards. It was decided to keep the point values for the related rates cards at the maximum of 5 points.

Refinements in preparation for the final playtest

There had been several changes made to the game before the third and final playtest. The playing cards were professionally printed, and the shape of the playing cards had been changed from square to rectangular (similar to regular poker-sized playing cards). Four-digit randomized identifiers had been assigned to the cards for reference for judging correct matches. The deck was also split into two parts: 48 playing cards did not use any calculus terminology or notation, and 24 cards did use calculus language and symbols. These card counts reflected an overall increase in the types of mathematical tasks represented on the cards. Inverse function tasks (8), related rates tasks (8), and duplicates of some easy cards (8) were added to the deck.

Point values had been changed as discussed above, and the inverse function tasks had different point values depending on whether they required calculus. The inverse function tasks that did not require calculus were valued at 3 points and the inverse function tasks that did require calculus were valued at 5 points.

The collection and play of tokens had also been changed. A bluff token was added to the game. The bluff token is a token that is played and has no action. Since the tokens are played face down, playing a bluff token might alter the choices of other players as they contemplate the possibility that the token may have some effect. The effect of playing the rotate token was also simplified so that two or more rotate tokens played on the same turn simply canceled each other. The game rules were also changed so that players would secretly select three of the five possible tokens at the beginning of the game and discard the others.

The rulebook formally recognized the way players were treating challenges during playtesting. The initial design had players make challenges in order of least total points to greatest. This was done to introduce a balancing mechanism. However, during playtesting, players spontaneously reverted to a chaotic, "first-come, first-serve." The rulebook was changed so that the former design was the recommendation, rather than a prescription.

The rulebook also included an answer key that was organized visually. Each of the four orientations of the curve card was pictured and occupied a quadrant of the rulebook page. Under each picture was a list of all card identifiers that match that orientation. Players were instructed to look for the picture that matched the curve card, and then look up the identifier number of the card they were checking.

During previous playtesting, the likelihood of being able to match an arbitrary orientation of the curve card became quite low as the player's hand became depleted. A balancing mechanism of replenishing the hand during play was also adopted. Instead of dealing seven cards and depleting the hand until there were none, the players would deal 5 cards and replenish the one they played by drawing a new playing card between turns. Each player would have 5 cards to choose from each turn. Replenishment significantly reduced the likelihood of not having any correct match during a turn.

Playtest III

The third and final playtest was conducted in a differential calculus class as an in-class activity, with a follow up survey to obtain feedback. The aim of the third playtest was to ascertain whether design pivots that arose out of prior playtests might increase the subjective gameplay-value of *Curves Ahead!* Data were collected seeking indications of each of the following:

- improvements in the design of *Curves Ahead!* resulting from design pivots made in response to recurring feedback across the first two playtests,
- subjective gameplay-value (either they would play it again or they would recommend it to others), and
- ability beliefs and utility values in order to provide a means of interpreting the gameplay-value indicators.

Participants

The formal playtest was conducted with 29 participants from a differential calculus class that was taught by one of the game designers at a two-year community college in the Pacific Northwest. The game was played as the daily in-class activity for that class period. The students were grouped so that four tables had six players (3 teams of 2) and one table had five players (2 teams of 2 and an individual). Three experienced calculus teachers, including the instructor of the class, circulated around the room and observed the participants in the five games. The instructor of the class was also the primary developer of the game.

Treatment

The rules of the game were explained by the instructor (and game developer) before play began, and participants proceeded to play the card game *Curves Ahead!*. The specific version of *Curves Ahead!* that they played is summarized below:

- The curve card (or target card) would be standing with the same orientation for all players during a turn.
- Players would have 5 cards in their hand, play one each turn, and draw a replacement at the end of the turn.
- Cards would be played face down.
- Players selected 3 of the 5 tokens before the game began.
- If a player wanted to use 1 of the 3 tokens, they would do so by playing it face down with their card.
- After everyone played their card, all cards and tokens would be played face up.
- All effects from tokens would be resolved before judging cards played.
- Players would collect cards they won through correct matches or through the challenge-defense mechanic.
- Control of the orientation of the curve card for the next turn was granted to the player that won the most points during the present turn.
- The game would end after 7 rounds.
- The winner was the player with the most points.

Instruments

A questionnaire (Appendix D) was administered immediately following the gameplay to determine participant perceptions of gameplay-value. The questions are provided and explained below. The order of the questions was a combination of thematic ordering and formatting considerations to optimize the use of space on the printed survey.

1. *How difficult was it to learn how to play the game (independent of the mathematics)?*
   Response choices: Too easy / Just right / Too hard

   This question attempted to follow up on prior concerns that indicated the game might be too hard to learn for a typical student. It could also be used to assess extraneous cognitive load (Leppink, Paas, Van der Vleuten, Van Gog, & Van Merriënboer, 2013). In the context of a game, it is desirable to have the game difficulty between too easy and too hard (Fullerton, 2008; Salen & Zimmerman, 2004).

2. *How difficult was the math?*
   Response choices: Too easy / Just right / Too hard

   This question attempted to determine player perceptions of mathematical difficulty and could be used to assess intrinsic cognitive load (Leppink et al., 2013). In the context of a game, it is desirable to have the game difficulty between too easy and too hard (Fullerton, 2008; Salen & Zimmerman, 2004).

3. *Did the experience feel like a game or more like a dressed-up math activity?*
   Response choices: Felt like a game / Felt like a dressed-up math activity / Unsure

   This question attempted to address a common concern in game-based learning literature (e.g., Habgood & Ainsworth, 2011; Lee et al., 2014; Lee & Doh, 2012; Smith & Mann, 2002; Weitze, 2014). Though responses to this question would be subjective in nature, positive results ("Felt like a game") would be a possible indicator of success within the domain of game-based learning research more generally.

4. *Do you feel like you learned mathematics as a result of the experience?*
   Response choices: Not at all / A little / Some / A lot / Unsure

This question attempted to find evidence of utility value, with positive responses indicating possible usefulness of the activity in attaining goals (Wigfield & Eccles, 2000). This question could also be used to assess germane cognitive load (Leppink et al., 2013).

5. *Do you feel that the experience strengthened existing mathematical understanding?*
   Response choices: Not at all / A little / Some / A lot / Unsure

   This question attempted to find evidence of utility value, with positive responses indicating possible usefulness of the activity in attaining goals (Wigfield & Eccles, 2000). This question could also be used to assess germane cognitive load (Leppink et al., 2013).

6. *To learn the mathematics presented in the game, would you rather play this game, work on a typical worksheet activity, attend lecture, or do something else entirely?*
   Response choices: This was best / Typical activity is best / Lecture is best / Other (what?)

   This question attempted to find evidence of utility value, with positive responses ("This was best") indicating possible usefulness of the activity in attaining goals (Wigfield & Eccles, 2000).

7. *How has your confidence regarding the mathematical material changed?*
   Response choices: Lower confidence / No change / Higher confidence

   This question attempted to find evidence of changes in ability beliefs. Positive answers would indicate possible improvements in the participant's view of their mathematical ability (Wigfield & Eccles, 2000) that they might attribute to the game.

   No item was included on the questionnaire to assess participant perceptions of whether success is likely or not. While ability beliefs and expectancies of success are technically different, Eccles and Wigfield (2002) point out that "in real-world achievement situations they are highly related and empirically indistinguishable" (p. 119).

8. *Ideally, how many times should this game be played in a term?*
   Response choices: 0 / 1 / 2 / 3 / 4 or more

   This question attempted to assess the degree to which a participant valued the gameplay by evaluating interest.

9. *Ideally, where would you rather play this game?*
   Response choices: In class / Outside of class / Both / Unsure

   This question attempted to find evidence for enhanced subjective gameplay-value (Renne, 2019, Chapter 2), but the final print of the survey mistakenly omitted the response choice of "I would rather not play the game." As a result, participants may interpret this question in light of a perceived expectation that the game must be played *somewhere*. For example, they may interpret the question as akin to, "If you were to play this game somewhere, where would it be?"

   Question 9 was removed from the analysis due to this mistake.

10. *How likely would you be to request that we play this game in class again?*
    Response choices: Not at all likely / Somewhat likely / Highly likely / Unsure

    This question attempted to assess the degree to which a participant valued the gameplay by evaluating interest.

11. *How likely would you be to recommend that others play this game in future classes?*
    Response choices: Not at all likely / Somewhat likely / Highly likely / Unsure

    This question attempted to assess the degree to which a participant valued the gameplay by evaluating their likelihood to recommend the game.

12. *How did you feel about the game overall?*
    Response choices: It was no fun at all. / It was a little fun. / It was fun. / It was so much fun that I'd play this game in my free time with my friends. / It was only fun when compared to my typical experiences in a math class. / Other (please specify):

    This question attempted to assess the degree to which a participant valued the gameplay by evaluating perceived enjoyment.

    Like question 3, the response to this question would be entirely subjective in nature, but positive results would be a possible indicator of success within the domain of game-based learning research more generally.

13. *What could make the experience feel more like a game, and/or make it more fun?*
    Response choices: Open response

    This question attempted to gain insight into responses to questions 3 and 12. It was attempted to supplant the group discussion that was not feasible as a result of the group size.

Procedure

As in the previous playtests, the rules were explained before play, with examples. Unlike the first two playtests, the players were advised to deal six cards, and play the first round as practice. They did not replenish the card that was played during this practice round. This was intended to scaffold the game to reduce the number of turns it took players to learn the mechanics and how to play. This was also the first playtest where adjudication would involve the answer key. No adjudication was provided by the teachers. The questionnaire was administered immediately following the gameplay.

Data analysis

The data were analyzed with subgroupings of survey items that were intended to assess player perceptions of cognitive load, interest in playing the game, utility value, and one survey item was intended to assess a player's self-perceived change in confidence. The subgroupings are as follows:

- Questions 1, 2, 4, and 5 were combined to assess cognitive load, which would reveal something about flow (Renne, 2019, Chapter 2), and hence gameplay-value.
- Questions 3, 8, 10, 11, and 12 were combined to evaluate whether participants would be interested in playing the game again (or recommending it to others).
- Questions 4, 5, and 6 were combined to assess utility value.
- Question 7 was used to assess changes in ability beliefs that the participant might attribute to the game.

Question 9 was omitted from the analysis, due to the printing error mentioned above.

Results of the third playtest and subsequent design pivots

Most groups were only able to complete one game, and one group played approximately half of a second game.

Completed questionnaires were obtained from 26 of the players (two of the students were below the age requirement and one chose not to take part in the questionnaire). The questionnaire results are reported in the tables below (Table 7 and Table 8). The first table reports the response counts for the first 12 items.

Table 7: Responses to questionnaire following the third playtest of Curves Ahead!

	Question	**Responses (count)**
1	How difficult was it to learn how to play the game (independent of the mathematics)?	Too easy (2); Just right (24); Too hard (0)
2	How difficult was the math?	Too easy (1); Just right (24); Too hard (1)
3	Did the experience feel like a game or more like a dressed-up math activity?	Felt like a game (21); Felt like a dressed-up math activity (4); Unsure (1)
4	Do you feel like you learned mathematics as a result of the experience?	Not at all (1); A little (7); Some (15); A lot (3); Unsure (0)
5	Do you feel that the experience strengthened existing mathematical understanding?	Not at all (0); A little (3); Some (14); A lot (8); Unsure (1)
6	To learn the mathematics presented in the game, would you rather play this game, work on a typical worksheet activity, attend lecture, or do something else entirely?	This was best (14); Typical activity is best (6); Lecture is best (1); Other (what?) (4 combos of other three)
7	How has your confidence regarding the mathematical material changed?	Lower confidence (0); No change (8); Higher confidence (18)
8	Ideally, how many times should this game be played in a term?	0 (0); 1 (3); 2 (8); 3 (11); 4 or more (4)
10	How likely would you be to request that we play this game in class again?	Not at all likely (0); Somewhat likely (16); Highly likely (9); Unsure (1)
11	How likely would you be to recommend that others play this game in future classes?	Not at all likely (0); Somewhat likely (4); Highly likely (21); Unsure (1)
12	How did you feel about the game overall?	It was no fun at all (0); It was a little fun (2); It was fun (16); It was so much fun that I'd play this game in my free time with my friends. (3); It was only fun when compared to my typical experiences in a math class. (5);

The results from the first subgrouping (questions 1, 2, 4, and 5) suggest that the cognitive load of the game may be well tolerated. However, it is unclear whether or to what degree each design pivot might explain this finding. For example, rules were simplified, the playing card shape was changed to a rectangle and professionally printed (making it easier to hold and read the card), and there was an extra "practice round" at the beginning of the game (reducing the number of turns to learn the game).

The results from the second subgrouping (questions 3, 8, 10, 11, and 12) suggest a modest degree of interest in playing the game. The responses to question 8 suggest that the participants may want to play multiple times. The responses to questions 3, 10, and 12 appear to be less enthusiastic than their responses to question 11. Taken as a subgrouping, it appears that participants were more likely to recommend the game to others than to ask to play again. The results may also suggest that the participants may only want to play the game on one occasion (e.g., one class period).

The results from the third subgrouping (questions 4, 5, and 6) suggest that participants see a modest degree of utility value for attaining their learning goals (a component of subjective task-value). Given that 23 of the participants said that the ideal number of times to play was 2 or more (item 8) and that most only played the game once, it seems that this task-value is related to playing again, rather than having just played it moments before. This supports the interpretation of the results from the second subgrouping.

Table 8 summarizes and discusses the responses to item 13, which attempted to gain insight into possible reasons for participant responses to items 3 and 12. Most participants provided no more than one response. The discussion for each response is included in the right-hand column.

Table 8: Responses to item 13.

Response (count)	**Discussion**
Timer or time limit (8)	This was a surprisingly widespread recommendation, including during gameplay. This is a request for more tension and a reinforcing mechanism. This was rejected due to the possibility for negative ramifications.
Increase competition (4)	Players wanted more competitive interactions. This seems to be a request for more tension.
Class-level prizes (3)	Players wanted extra credit or other boosts to their course grade as a result of stellar performance in the game. This has implementation implications.
Spread out control of the curve card (2)	This was only expressed by two players during playtesting, but it was something we considered in the creation of the game.
More story problems (1)	This is a request for more variety and more difficulty. This may result in more content.
More tokens (1)	This request indicates that tokens need more attention.
A plot (1)	It is not clear what the player meant by this comment. It may be that they are asking for a storyline or narrative. Doing so would significantly alter the game concept.
Math refresher (1)	This has implementation implications.
Put it on the internet (1)	This player may be indicating their personal preference for digital games over tabletop games.

The player feedback indicating that control of the curve card orientation should be spread around was interesting in that it was rare, but something that was thoroughly considered during the design. The game to this point, had no order of play for making matches or playing tokens, one order of play for the challenge-defense mechanic, and yet another order of play for tiebreakers to determine which player would control the curve card. Players appeared to prefer an order for making matches and playing tokens, and no order for the challenge-defense mechanic.

Distribution of the control of the curve card can address the first of those two player preferences. A pivot was made to rotate control of the curve card so that each turn has a new person controlling the orientation, with the intention of enhancing subjective gameplay-value through a balancing mechanism. The new rule also indicates that players take turns to play their matching card and token. This change has a significant balancing effect, but it also reduces tension, so the potential effect is currently unclear. The rulebook now states both variants with a recommendation for rotating control among the players each turn.

The tokens have shown themselves to be an important part of increasing the potential for the perceived gameplay-value of *Curves Ahead!*. However, coordinating the tokens with the game concept in a balanced way has proven difficult. During this playtest, the players expressed doubts to each other about whether they could optimally choose tokens for the game. The combination of observations during all playtests led to a further alteration. The latest change to the game has removed the *'+2'* token and gives players all four tokens, with the intention of simplifying the use of the tokens. The reasoning behind this choice is that the simplicity may enhance flow through a reduction in cognitive load or angst associated with token selection, which would increase subjective gameplay-value.

Conclusion

The present study intended to answer three questions:

1. How can a multiplayer tabletop mathematics learning game be effectively structured to embed gameplay tasks that require students to interpret graphical function behavior across a wide variety of representations and contexts?
2. How can a design experiment methodology be leveraged to inform game modifications that maintain or increase gameplay opportunities for productive disciplinary engagement while also enhancing the subjective gameplay-value to players?
3. Do participant perceptions of the gameplay-value, based on a final playtest of the refined game, support the modifications made to the game during the design process?

Concluding remarks for question 1

Curves Ahead! is designed so that the mathematical task of translating between function representations is embedded within the game task of matching. A player has a hand of cards which has various functions represented in words, tables, symbols, or situations, and they must select one that matches a given graph. There are 72 playing cards, with a wide variety of function representations and contexts. The point value of each playing card corresponds to either intrinsic cognitive load or germane cognitive load, and players are motivated to make matches (i.e., translate between function representations) by the game goal of maximizing point earnings. Incorporating a challenge-defense mechanics further motivated student engagement with representation translation tasks by incentivizing them to judge each other's plays. The challenge-defense mechanic also provided the students with opportunities for meaningful mathematical discourse while they made and defended challenges.

Concluding remarks for question 2

The iterative natures of game design and design experiments in educational research suggest that a design experiment could be used to iteratively refine the mathematics learning game *Curves Ahead!* Six design activities were presented, that could potentially be used to guide the iterative design process for the game: identify (or alter) possible learning goals, develop (or modify) a game concept, produce (or modify) a proto game, playtest the proto game, develop (or modify) a game prototype, and playtest. These design activities may occur in cycles or in a nonlinear order. Indeed, the design of *Curves Ahead!* followed the trajectory:

(i) identify learning goals
(ii) develop a game concept

(iii) produce a proto game
(iv) playtest the proto game
(v) develop a game prototype
(vi) playtest 1
(vii) modify the game prototype (design pivots)
(viii) playtest 2
(ix) modify the game concept (replenish hand) and prototype (multiple design pivots)
(x) playtest 3
(xi) modify a portion of the game concept (control of curve card) – stop.

The first playtest sought feedback from stakeholders in the community of mathematics educators. The second playtest sought feedback from a small subset of the target audience. The third playtest sought feedback from an entire calculus class. While the playtests could have been conducted with different participants in different ways, this combination and sequencing appeared to be helpful for the design of *Curves Ahead!* The design of *Curves Ahead!* was relatively straightforward, as there were no indications during the playtests that any of the first three activities would need to be revisited.

Concluding remarks for question 3

Each playtest appeared to incrementally improve the game with respect to subjective gameplay-value. The results from the third playtest suggest that overall views of subjective gameplay-value are moderately favorable for *Curves Ahead!* To understand which aspects of the game contributed to this, the three design principles of variety, the virtuous cycle, and flow will be examined in relation to implicated game features. This is consistent with the view that these principles support subjective gameplay-value in a math learning game (Renne, 2019, Chapter 2).

The first and second playtests revealed that the tokens had a significant impact on subjective gameplay-value, and the second playtest suggested that the rotate token was particularly important. The tokens increased both forms of tension but seemed to have more positive tension than negative tension. In particular, the rotate token is likely to induce both forms of tension for the one that plays it, because it is played upside down. It could be that another player plays a rotate token, causing a loss for both (negative tension). Or, it could be that no one else plays a rotate token, potentially causing a significant gain for the one that plays it (positive tension). Removal of the rotate token seemed to lead to disappointment, suggesting that it has more positive tension than negative.

The tokens also enhanced the virtuous cycle through their strategic use. For example, one player observed that if they played the wild token when someone else played the rotate token, the wild token would shield them from the negative effects of the rotate token. Another strategy used was using the rotate token while also having control of the curve card. That player oriented the curve card to one of the three orientations that they did not want, then played their card with a rotate token.

The other players matched the initial orientation, only to discover that the player in control of the curve card played the rotate token to specify its orientation (again) after everyone had played their card. This appeared to garner emotional reactions in response to the unexpected (creativity), which suggests passionate involvement.

While the tokens appeared to enhance the virtuous cycle and tension, they appeared to have made the game more complex. That complexity increases the extraneous cognitive load, which could potentially reduce flow. However, Zhang et al. (2011) provide an example that illustrates that players of learning games may tolerate some forms of extraneous cognitive load if the game absorbs their attention. The third playtest revealed that players generally tolerated the cognitive load in the game, but it is not clear whether the tokens were specifically tolerated in the same way as the game overall.

The replenishment of the hand at the end of each turn seemed to increase subjective gameplay-value by giving more players more opportunities for control, choice, and creativity. The replenishment also introduced an important balancing mechanism, and is expected to have reduced overall negative tension, by increasing the likelihood of having a matching play all or most turns.

While each playtest group indicated a desire for more variety, the third playtest had only 1 out of 26 participants indicate that desire. It seems then, that the increase in variety from the second to the third playtest may have had a positive influence on subjective gameplay-value.

Although it was not tested, it is hypothesized that distributing control of the curve card to each player throughout the game may reduce both forms of tension and have a positive balancing effect. The expectation is that this change will provide a net increase to subjective gameplay-value.

The first two playtests focused on deficiencies, in alignment with the advice from Gaydos (2015). According to Gaydos (2015), sharing those deficiencies can be an important part of game-based learning research. The game elements in *Curves Ahead!* that were found to reduce subjective gameplay-value are listed here:

- **Removal of the tokens:** discussed above.
- **Perceived unfairness in the use of tokens:** the original rules for the rotate token resolved simultaneous play of a rotate token by giving the "power" to the player with the least total points earned (a balancing mechanism), and all other played rotate tokens would be "wasted." Players in the first playtest suggested that it would be preferable that all the played rotate tokens be "wasted" by canceling each other, or that the rotate token should only be given to one player in the game (the one with the lowest total score at some specified time).
- **Square cards:** players found these uncomfortable to hold and due to the rotational symmetry of a square, players found them cumbersome to orient before reading. Alternatively, a rectangular card that is the same size as a standard poker deck, will be more comfortable in the hand, and would either be right-side up or upside down, but never

sideways.

- **Chaotic order of play:** the first few versions of the game had no order of play for making matches or playing tokens, one order of play for the challenge-defense mechanic, and yet another order of play for tiebreakers to determine which player would control the curve card. Players appeared to prefer an order for making matches and playing tokens, and no order for the challenge-defense mechanic. Distribution of the control of the curve card can address the first of these preferences.

- **Depletion of the hand of cards:** as the game developed, with control of the curve card going to the player that "won" the round, depletion of the hand meant that players became less likely to have a matching play and were unable to regain control of the curve card. This reduced subjective gameplay-value. Instead, reducing the size of the hand from 7 to 5, and replenishing the hand, significantly increased the likelihood of a matching play throughout the game.

- **No time limit on plays:** each playtest had participants requesting a time limit for others to make a play. Those participants were typically those that were "winning" in terms of points earned, which suggests that they wanted a reinforcing mechanism. These participants also seemed to want to maintain their immersion in the game, instead of disengaging while waiting. However, institution of a time limit is expected to introduce a form of negative tension for the "losing" players and might have an overall reduction in subjective gameplay-value. The negative tension could also cause a player to disengage from mathematical reasoning and sense making related to the embedded mathematics. Although this seems to be a no-win situation, it may be possible to address this through implementation (e.g., matching players on skill in *Curves Ahead!*).

- **Mismatch between the difficulty and the reward:** the participants in the second playtest (calculus students) indicated that the cards requiring calculus knowledge were too difficult, and they wanted to stop playing. Elaboration on that subject revealed that the players felt that the points were not commensurate with the difficulty. The point

> values were changed to reflect the higher germane cognitive load, but also balance the low intrinsic cognitive load. The third playtest gave no indication that there continued to be a mismatch between the reward and the difficulty.

Implications for design

The present study revealed significant interdependencies among game elements in the game *Curves Ahead!* A recurring challenge in addressing participant feedback was the creation of a new deficiency, like trying to remove a bubble in wallpaper. Improvements in one area sometimes reduced conformity to the design principles in other areas (e.g., distribution of control of the curve card). And in at least one case (time limits), it seems impossible to adequately address the player experiences through design because of an inherent conflict in player values. This gives rise to an important design question: "What should be done in response to conflicting design demands?"

One way to address this question in the theory would be research into possible hierarchies for the design principles that might act as meta-principles that resolves such conflicts. In the absence of such a hierarchy, clearly defined values and goals could be established before beginning the design process. If design conflicts arise, or it seems impossible to meet the demands of all participants, the choice that seems most likely to support those values or advance those goals should be made. In *Curves Ahead!*, conflicting design demands were reconciled by giving precedence to supporting productive disciplinary engagement.

Implications for implementation

The present study also revealed the relatedness between the design of *Curves Ahead!* and its use in the class, giving rise to questions of implementation strategies. From the third playtest, it appears possible that a different way of introducing the players to the game *Curves Ahead!* may have had some positive impact on their perception of the game's difficulty. This was not a design choice, but an implementation choice. Furthermore, the request for time limits seems to have been associated with success in the game. One way of addressing the desires of these players would be in implementation. For example, the teacher could match players based on success in playing the game, or those players that make faster decisions. This would maintain the cognitive engagement of those players that want things to happen faster, while also allowing other players to spend more time deliberating.

Coda

The present study attempted to investigate the design and refinement of the function representations card game *Curves Ahead!* A design experiment with playtests were used to gather player feedback to determine the degree to which certain features in *Curves Ahead!* might maintain or increase opportunities for productive disciplinary engagement while enhancing player perceptions of gameplay-value. That feedback informed subsequent design pivots in the refinement process of *Curves Ahead!*, which culminated in a version of the game for which students in a differential calculus class perceived gameplay-value.

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CHAPTER 4: USING GAME-BASED LEARNING TO FOSTER PRODUCTIVE DISCIPLINARY ENGAGEMENT

Introduction

Teachers may be attracted to mathematics learning games with the idea that students will be more "engaged" and will find the experience "more fun" than the usual classroom activity. But is this engagement in playing games really an effective use of class time? That is, can playing a mathematics learning game genuinely support students' learning of important mathematical content, or provide students with substantive experience with important mathematical practices?

The use of a game to help students attain learning outcomes is known at *game-based learning*. Renne (2019, Chapter 2) defines a *game* as a voluntary play activity in a pretended reality governed by rules, wherein the participant(s) try to achieve one or more goals, and where degrees of success in the attainment of goals are conveyed by a feedback system. A *learning game* (or educational game) is a game with specified learning outcomes, and a *mathematics learning game* (or educational mathematics game) is a learning game in which the learning outcomes are in the domain of mathematics. A *digital game* (or video game) is a game that is played on a computer, console, or mobile device, while a *tabletop game* is one that is usually played on a flat surface (e.g., card games and board games).

Much of the design and research attention in game-based learning to date has been given to digital learning games, and these games are usually intended for a single player (Ke, 2011). Gee's (2003) influential book describes learning principles that are embodied by *digital* games. Devlin has used Gee's principles (Devlin, 2011) in an ambitious (and expansive) project aimed at creating an exemplar in the game *Wuzzit™ Trouble,* to illustrate the power of video game learning in mathematics (Kiili, Devlin, Perttula, Tuomi, & Lindstedt, 2015; Pope & Mangram, 2015).

While such games may have potential for helping students meet mathematical content standards like those of the Common Core, mathematical practice standards imply interactions between the members of a mathematical learning community (of which both students and the teacher are participants). For example, teachers that provide students the opportunity to "construct viable arguments and critique the reason of others" (Mathematical Practices Standard 3, *The Common Core State Standards for Mathematics*, by National Governors Association Center for Best Practices, 2010), are likely to be orchestrating an activity that encourages and supports students in that practice.

Multiplayer tabletop games often involve many player-to-player interactions. (Indeed, one of the attractions to playing such games may well be the setting they create for socializing among friends.) If designed appropriately, could a mathematics learning game in a multiplayer tabletop format offer special affordances to support students' productive engagement with mathematical ideas? That is, can a properly designed multiplayer tabletop mathematics learning game engage students in actions and interactions centered on mathematical sense making and reasoning, and encourage productive mathematical discourse?

Renne (2019, Chapter 2) has proposed both design principles (for creative development by game designers) and implementation principles (for effective deployment by classroom teachers) for multiplayer tabletop educational mathematics games. The design principles situate the mathematics content within the pretended reality of the game, convey progress to the player(s), and position the game so that players have interest in playing. The design principles were devised to foster productive disciplinary engagement as proposed by Engle and Conant (2002), in the discipline of mathematics. The implementation principles are intended for educators to make effective use of a game as a classroom activity, with special attention paid to supporting mathematical sense making during play in a way that fosters productive disciplinary engagement. Both sets of principles attend to interaction dynamics that may arise during multiplayer games in a classroom setting.

Productive disciplinary engagement is a construct that arose out of analyzing a class project in a fostering community of learners classroom (Engle & Conant, 2002). *Engagement* and its two modifiers, *productive* and *disciplinary*, need unpacking. By engagement, Engle and Conant mean "focused and active participation in the present discourse" (Renne, 2019, Chapter 2). By disciplinary engagement, they mean, "contact between what students are doing and the issues and practices of a discipline's discourse" (Engle & Conant, p. 402). And they take productivity to be "intellectual progress" (p. 403). Connecting productive disciplinary engagement to a mathematics learning game, Renne (2019, Chapter 2) views productive disciplinary engagement in mathematics as intellectual progress during or through a focused and active participation in the activity, while maintaining contact between what students are doing and the issues, practices, or discourse in mathematics.

The study reported in this paper is an investigation of the effectiveness of a recently developed multiplayer tabletop calculus game called *Assembly Lines* to support productive disciplinary engagement in mathematics. The game was developed using Renne's design principles (Renne, 2019, Chapter 2), and a playtest involving student participants was conducted with Renne's implementation principles in mind. The game *Assembly Lines* involves players, either collaboratively or competitively, building piece by piece the graph of a piecewise linear function f. This graphically presented function f, in turn, must at times be interpreted by the players as the *derivative* of another function F, which gives players practice with a mathematical task which has been shown to cause students some difficulty (Orhun, 2012). The mechanics of the gameplay (in-game structured actions) of *Assembly*

Lines involve creating the graph of f to achieve certain characteristics or satisfy properties or constraints of F, f, and f' that are stated using the language and notation of calculus.

The students' engagement with, and discourse about mathematical ideas during the gameplay was of primary interest in this study, with special attention to the interactions of students with each other. To evaluate the productivity (or "intellectual progress") during gameplay, the students' calculus content learning was also considered. To frame the question of student engagement with mathematical ideas, the construct of productive disciplinary engagement is used (Engle & Conant, 2002). This framework provides not only a useful language for describing students' mathematical engagement, but also evidentiary indicators of that engagement.

Engle and Conant (2002) point out that evidence of productive disciplinary engagement will depend on the discipline and that "expressions of engagement are both culturally relative and subject to interpretation" (p. 402). However, they suggest some indicators of engagement that might be applicable somewhat broadly to students in the U.S and multiple disciplines. Quoting Engle and Conant (2002):

> it [seems] appropriate to infer greater engagement to the extent that: (a) More students in the group sought to make, and made, substantive contributions to the topic under discussion; (b) students' contributions were more often made in coordination with each other, rather than independently of each other…; (c) few students were involved in unrelated "off-task" activities; (d)

> students were attending to each other as assessed by alignment of eye gaze and body positioning…; (e) students often expressed passionate involvement by making emotional displays…; and (f) students spontaneously got reengaged in the topic and continued being engaged in it over a long period of time. (p. 402)

A multiplayer tabletop game, such as *Assembly Lines*, can give rise to player-player interactions and player-teacher interactions that present opportunities for engagement with mathematical ideas. The list of indicators provided by Engle and Conant (2002) is a helpful place to start when assessing the possible effectiveness of a math learning game to engage players with mathematical ideas.

The four guiding principles for fostering productive disciplinary engagement

Engle and Conant (2002) offer four guiding principles that they argue will foster productive disciplinary engagement: *problematization, authority, accountability,* and *resources*.

- **Problematization:** "The core idea behind *problematizing* content is that teachers should encourage students' questions, proposals, challenges, and other intellectual contributions, rather than expecting that they should simply assimilate facts, procedures, and other 'answers'" (Engle & Conant, 2002, p. 404, emphasis in original).

- **Authority:** "The tasks, teachers, and other members of the learning community generally encourage students to be authors and producers of knowledge, with ownership over it, rather than mere consumers of it" (Engle & Conant, 2002, p. 404).

- **Accountability:** "Students' intellectual work is made accountable to others and to disciplinary norms" (Engle & Conant, 2002, p. 405).

- **Resources:** "Students are provided with sufficient resources to do all of the above" (Engle & Conant, 2002, p. 401).

Renne's design principles for multiplayer tabletop mathematics learning games

Renne (2019, Chapter 2) proposes 10 design principles for the creation of tabletop math learning games to support productive disciplinary engagement in mathematics. Those will be discussed briefly.

- **Mathematical Fidelity Principle:** The game should remain faithful to the mathematics, and be free of mathematical errors, ambiguities, and sloppiness. This design principle positions the game as a reliable source of mathematical information, making it a resource.

- **Cognitive Fidelity Principle:** The game should remain faithful to the mathematics as perceived by the player. While the mathematics could be correct, the game could give a player a false impression of some pattern or strategy that is local to the game and does not generalize to other mathematics contexts. Like mathematical fidelity, this design principle positions the game to be perceived by the learner as a reliable resource.

- **Embedding Principle:** The game should embed the mathematical content so that the game elicits the formulation of the mathematical tasks and problem statements from the player through their gameplay. The game should not directly or overtly give the mathematical tasks or problem statements to the players. This principle will assist learners in problematization.

- **Rules Principle:** The rules of the game should be simple, clear, consistent, and fair. This design principle supports accountability.

- **Adjudication Principle:** The game should adjudicate play fairly, correctly, and simply. This aspect of adjudication makes the game a valuable resource, as it can provide learners feedback on their learning progress.

  Ideally, a tabletop math learning game would enable the players to adjudicate the gameplay themselves, which in turn would enable a teacher to have logistical flexibility (e.g., more time to facilitate discourse). This aspect of adjudication would partly support authority, by giving players some "ownership" over the feedback process, and it could free the teacher up as a resource.

- **Reward System Principle:** All mathematics tasks should have rewards and costs tied to successful (or unsuccessful) performance of those tasks. All rewards and costs should reflect the difficulty of the task. Costs should be minimal and can include loss of opportunities. These rewards and costs are a kind of feedback, which makes them a resource. Rewards and costs reinforce successful performance and steer learners away from unsuccessful performance.

- **Discovery & Reflection Principle:** The in-game feedback should stimulate discovery and reflection on the part of the player. Such feedback is both a resource and positions the learner to be an author of their own knowledge.

- **Variety Principle:** The game should provide many opportunities to learn. A wide variety of learning opportunities is more likely to enhance subjective gameplay-value, which, in turn, promotes practice. This design principle supports the authority of the student.

- **The Virtuous Cycle Principle:** The game should provide the player with control, which leads to meaningful choices, which in turn, motivates creativity. This design principle supports the authority of the student.

- **Flow Principle:** The game should immerse players in a flow experience that sustains engagement with in-game mathematical activities for the entire game.

  Flow is a psychological experience which occurs when an individual becomes fully absorbed in an activity and their sense of time and space becomes distorted (Csikszentmihalyi, 1991; Nakamura, & Csikszentmihalyi, 2009).

Renne's implementation principles for multiplayer tabletop mathematics learning games

Drawing on work done by Dick and Burrill (2016), Stein et al. (2008), Stein and Smith (1998), and the National Council of Teachers of Mathematics (NCTM, 2014) *Effective Mathematics Teaching Practices*, Renne (2019, Chapter 2) proposes 5 implementation principles for the use of multiplayer tabletop math learning games that are intended to support productive disciplinary engagement in mathematics. Those will be discussed briefly.

- **Timing Principle:** Educators should use a game when the students are ready and timed to align with the curriculum. Games that require learners to engage in mathematics tasks that are more complex than simply recall, are best done around the time of the instruction on the subject (Bright, Harvey, & Wheeler, 1985). Timing the implementation of a game to a learner's readiness will support their ability to become authors of their knowledge.

- **Planning Principle:** Educators should plan for the implementation in terms of what the learners will need in order to successfully play the game and attain the learning goals. Planning will provide learners with adequate resources to learn during the gameplay. For example, a reference sheet that is simple and easy to understand could facilitate completion of game tasks, without giving “the answers”.

- **Briefing Principle:** The teacher should explain the rules of the game and how to play. If a game or the mathematics is complex, then it might benefit the learners to play a practice round.

  The teacher should explain the mathematics involved to set learners up for connections to be made. Longer explanations of the mathematics may require the briefing and game activity to be spread out over multiple class periods. Additionally, complex games with new mathematics could cause the learners to feel overwhelmed before they even begin to play the game.

  If applicable, the teacher should explain any mathematical language or notation that is unfamiliar to the students and in the game.

  If applicable, the teacher should briefly explain how to use any external resources while playing the game.

- **Managing Gameplay Principle:** The teacher should monitor gameplay and player-player interactions, clarify rules, assist with adjudication as needed, and facilitate the mathematical discourse when asked for help. Interrupting gameplay might disrupt the flow of the game, and the flow experience. Interruptions should only be done as a last resort to guide learners that have strayed too far from the intended mathematics or are repeatedly adjudicating incorrectly.

  Occasionally, learners may need to be matched up by the teacher to make the most effective use of the gameplay in terms of learning and potential for enjoyment. This matching may be along prior performance on coursework, or it may be along prior gameplay success or preferences.

  If players get stuck with the mathematics tasks or how those tasks relate to the rules of the game, teachers can help by giving an indication of the potential utility of something that was said in the players’ conversation, bringing the players’ attention to relevant in-game resources that might be helpful, indicating that a move or strategy is possible, or indicating a move or strategy that is possible, all while avoiding giving the players "the answer".

- **Debrief Principle:** The teacher should follow gameplay with a debriefing session to help students make connections. The debriefing session can include discussion of gameplay moments or strategies, or mathematical ideas related to the game. The debrief should serve to help students make connections between the game and the mathematics. It is possible to discuss student ideas that relate to mathematical ideas or related gameplay. The debrief session could be a good time to select student ideas for discussion, sequencing those ideas, and then connecting those ideas to each other and the mathematics learning goals, as in Stein et al. (2008).

The game, *Assembly Lines*

The boardgame *Assembly Lines* is designed for integral calculus students using the design principles found in Renne (2019, Chapter 2) and outlined above. The game can be played by 1 or 2 players, cooperatively, competitively, or in teams. The game requires players to build the piecewise linear graph of a function f, which is understood to be the derivative of another function F, using line segments. Points are scored by satisfying given local and global constraints to the functions f and F and their derivatives. Players try to earn as many points as possible while constraints are randomly generated, and resources (available line segments) dwindle. The game also includes a challenge-defense mechanic for competitive play that incentivizes player participation in the adjudication process.

The mathematical tasks in *Assembly Lines* involve building a piecewise linear graph of a function f to satisfy "local" or "global" constraints on one of the three functions: F (whose derivative $F' = f$), the function f itself, or f' (the derivative of f). Examples of local constraints include a given value of f or f' at some point, the value of the definite integral of f over the interval spanned by a single linear piece of its graph, or a given location for a local extremum on the graph of any anti-derivative of f. Examples of global constraints include the value of the definite integral of f over its entire (bounded) domain, a total number of local extrema on the graph of any anti-derivative of f, or a total number of inflection points on the graph of any anti-derivative of f.

The learning outcomes include proficiency in identification of features and characteristics of a function given the graph of its derivative function and facility with the fundamental theorem of calculus. The selection of the learning outcomes was inspired by research showing that calculus students struggle to build graphs when given constraints to satisfy (Baker, Cooley, & Trigueros, 2000), and research suggesting that students struggle to interpret the graph of the derivative function (e.g., Monk, 1992; Monk & Nemirovsky, 1994; Orhun, 2012). The requirement that players interpret different representations of the same mathematical object also affords opportunities for engagement in "procedures with connections" (Stein & Smith, 1998, p. 10).

The game setting and mechanics were motivated by an important type of summative assessment question that has appeared on the released free response section of the Advanced Placement®,[9] (AP®) calculus exams almost every year for the last 20 years. The AP® exam question usually depicts a piece-wise function, f, such that all but perhaps one piece of the graph is linear, and the one non-linear piece is the arc of a circle or parabola. The task commonly identifies a related function, g, that is defined as a definite integral function with f as an integrand and upper limit variable. Such a definition of g implies that its derivative is f by the Fundamental Theorem of Calculus. The different parts of the question typically include evaluating g, g', or g'' at specific points, identifying local or global extrema, identifying points of concavity, and justifying those choices. Student performance on this type of task suggests that it is indeed challenging for calculus students, and average scores on this exercise have hovered around 3.5 out of 9 possible points, with hundreds of thousands of students taking the AP® calculus AB exam each year.

The core mechanics in *Assembly Lines* are building (the graph), collecting (cards/points), and managing dwindling resources (line segment pieces). The game has the following hardware:

- A Cartesian grid in the form of a rectangle with evenly spaced peg holes to form a 1:1 ratio in the scaling of the axes. The portion of the

[9] Advanced Placement and AP are registered trademarks of the College Board. The use of these names does not imply any affiliation with or endorsement by the College Board.

Cartesian plane corresponds to $[-10,10] \times [-7,7] \subset \mathbb{R} \times \mathbb{R}$.

- Flat line segment pieces, whose ends have circular holes and are designed to span peg holes spaced two horizontal units apart on the grid. Each line segment has a length that will create a segment with one of these pre-defined slopes: $0, \pm\frac{1}{2}, \pm1, \pm\frac{3}{2}, \pm2, \pm\frac{5}{2}, \pm3$, depending on whether the line segment slants up or down from the left to right. There are two line segment pieces available for each of these slopes.

- Pegs that hold the line segments in place on the grid.

  (The rectangular grid board, the line segment pieces, and the pegs were all created using a 3D printing device.)

- A set of playing cards present the local or global constraint statements that players try to satisfy as the game progresses.

Assembly Lines has several variants to scale difficulty, but all of them share the same basic actions: collect constraint cards that are satisfied by building a continuous piece-wise linear derivative function, f, using the available line segment pieces so that each line segment spans an interval of length 2 in the horizontal axis. The game includes the fundamental assumption that $F' = f$ is continuous.

Each constraint card presents a single constraint that is either local to the line segment being played or global to the entire graph that has been built up to that moment. Players may satisfy more than one constraint in a turn, which allows for players to collect more than one constraint card per turn. To allow players to plan future moves, there are always three or more constraint cards visible during play. The game ends when the graph is built for the entire domain $[-10,10]$.

Constraints require different interpretations and uses of the fundamental theorem of calculus. Some constraints give the value of a definite integral, while others present the same task by giving the net change in F over the same interval. Some constraints require evaluation of f or f', while others present the same tasks by having players evaluate F' or F'', respectively. There are also tasks that constrain local extrema on the graph of F and some cards that constrain inflection points on the graph of F.

There are two types of game cards in *Assembly Lines*. One type of game card involves satisfying mathematical constraints by strategic placement of a single line segment. In the game, these are called *segment cards*, and they correspond to satisfying local constraints. The other types of game cards involve a search across the domain of the function being built to determine if a constraint has already been satisfied by previous play or if strategic planning might lead to satisfying that constraint in the future. These cards are called *key point cards*, and they usually correspond to satisfying global constraints. Except in rare, usually serendipitous cases, these cards require analysis that spans multiple line segments. The key point cards are intrinsically more difficult than the segment cards, so the segment cards are worth 1 point and the key point cards are worth 3 points.

Figure 6 illustrates the game during play (at the beginning of the 8th turn of 10). There are 4 stacks of constraint cards directly below the graph (from the perspective of the players). The view in this figure is with the positive y-axis to the right and the positive x-axis downward. From top to bottom of the figure, the first three stacks are the segment cards, and the fourth is a key point card. The remaining stacks of cards around the board include cards out of play (face down) or cards that were earned (face up). At the bottom of the figure is a reference sheet and the game pieces.

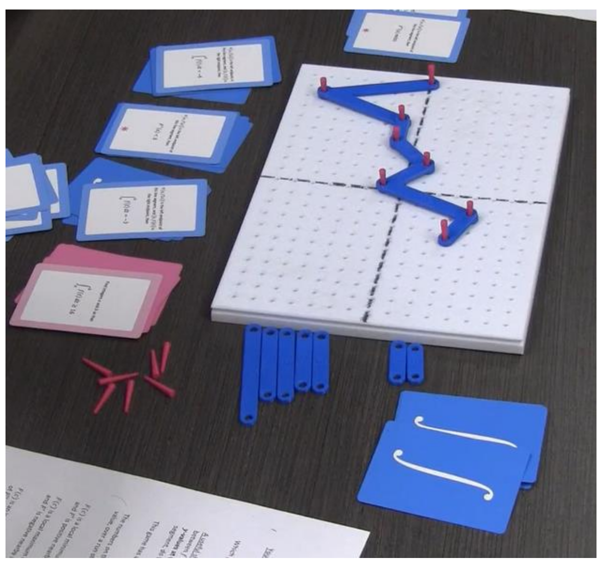

Figure 6: View of a game of Assembly Lines in progress, from the side. The positive x-axis is pointing downward, positive y-axis is pointing rightward.

Figure 7 is an enlarged view of the cards in play in the above figure, with the key point card in the bottom right of the figure below.

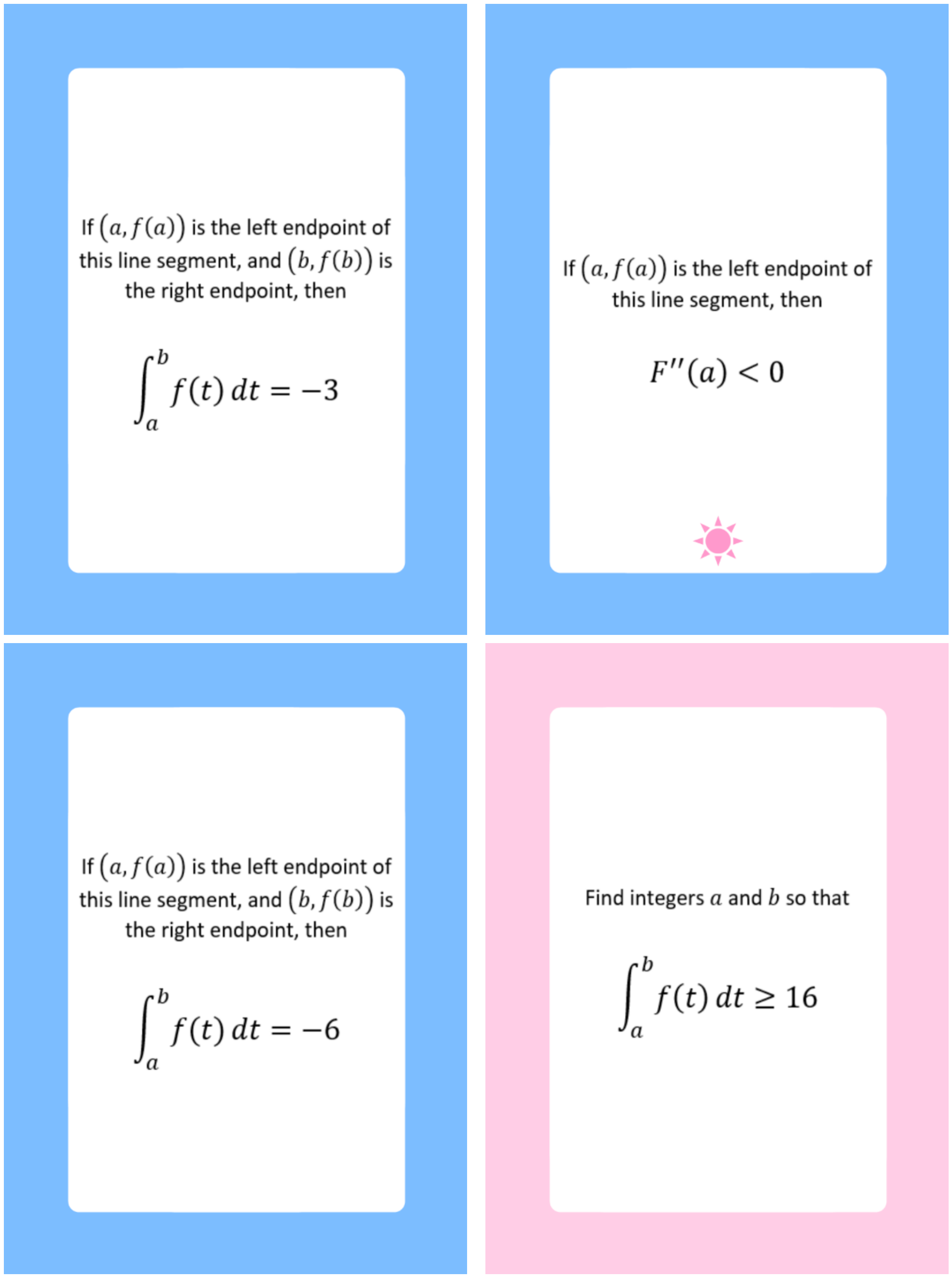

Figure 7: An enlarged view of the cards in play from the game shown in Figure 6.

Research questions

There are two research questions in this study. The first research question considers the potential for a game-based learning activity to foster engagement with mathematical ideas. The second research question considers the degree to which that disciplinary engagement might be productive.

> *Question 1*: How can a game-based learning activity foster engagement with mathematical ideas?
>
> > *Sub-question 1.1*: How can player contributions toward performing the mathematical tasks support engagement with mathematical ideas?
> > *Sub-question 1.2*: To what extent will players stay "on task" relative to the mathematics?
> > *Sub-question 1.3*: To what extent will players express "passionate involvement" during a math learning game?
>
> *Question 2*: To the extent that learners are engaging with mathematical ideas, how can a game-based learning activity foster productivity, or "intellectual progress"?
>
> > *Sub-question 2.1*: How does performance transfer to mathematical tasks outside of the game?
> > *Sub-question 2.2*: How does student dependence on in-game mathematical resources change over time?
> > *Sub*-question 2.3: To what extent do players become more efficient in performing mathematical tasks during gameplay?

Methodology

Participants were administered a background questionnaire followed by a 15-minute pretest using an AP® exam question of the type that motivated the design of the game Assembly Lines. The participants then played *Assembly Lines* in pairs for approximately 55 minutes. All gameplay was video recorded. After the gameplaying session, players were administered a 15-minute posttest, again using a different AP® exam question of the same type. Participants were made aware before the pretest that the posttest would be administered following the gameplay.

The background questionnaire collected demographic information, prior knowledge of integral calculus, gameplay habits, and goal orientations. Goal orientation items were included to explore possible relationships between goal orientations and the performance measures that might suggest hypotheses for future studies. The pretest and posttest were used to evaluate changes in performance that might be attributed to the gameplay. Videos were used to collect and analyze discourse data and participant body language to find possible evidence for engagement with each other, the game, and mathematical ideas.

Participants

Volunteer participants were recruited from calculus courses at a university in the Pacific Northwest. Twelve (12) students volunteered to participate in the study, all of whom had the pre-requisite knowledge to play the game (differential calculus and the fundamental theorem of calculus). Of those 12, nine (9) of the participants were enrolled in a single-variable integral calculus class, two (2) had been enrolled in the integral calculus class the previous term (quarter system), and one participant was enrolled in the integral calculus 3 years prior. Four of the 9 participants that were currently enrolled in the integral calculus class reported having learned about integrals prior to their current class. None of the participants had taken the Advanced Placement® (AP®) exam prior to the study.

Video recordings were made of all games, and two videos contained audio of the discourse. Even the videos without audio contributed data to the player engagement and interaction analysis, for they showed body language and the physical actions of the players. The body language can be evidence of engagement with each other and the game, and the gameplay can show evidence of possible "intellectual progress" (Engle & Conant, 2002).

Age and gender demographic data were obtained by participant responses to a background questionnaire. There were 6 female participants and 6 male participants, and ages ranged from 19 to 48 years, with 9 participants (4 female and 5 male) between 19 and 22 years of age.

The questionnaire also included an item to indicate the participant's average time spent playing games of any kind. Five (5) of the participants averaged less than 30 minutes of gameplay per day and 7 of the participants averaged 30 minutes or more per day.

Instruments

Data were collected using the following instruments:

1. **Background questionnaire:**
   Questionnaire items (Appendix E) included

   a. Demographic information (gender and age),
   b. Prior knowledge of integral calculus and the recency of coursework in integral calculus,
   c. Gameplay habits (frequency and duration), and
   d. Goal orientations[10] (collected to explore possible relationships between a player's goal orientations and their performance measures, which might inform hypotheses for future studies).

2. **Pretest and posttest performance measures:**
   Two AP® free-response questions (Appendix F) were used as pretest and posttest performance measures. These questions were of the type that had motivated the creation of the *Assembly Lines* game, and hence, closely reflected the mathematics content addressed by *Assembly Lines*. This provided an opportunity to look for transfer from the game through performance gains from the pretest to the posttest. The AP® free-response questions chosen were from the examination years 2004 and 2012. The AP® exam performance data available suggested that these problems were of roughly equal difficulty, and the tasks and graphs were highly similar.

Procedure

The study took place over two evenings of one week in mid-April of Spring quarter. The timing of the game in relation to the curriculum was shortly after

[10] The game goal orientation items are based on items developed for digital games by Quick and Atkinson (2014), which in turn, were based on items developed for learning goal orientations by Elliot et al. (2011).

students learned about the fundamental theorem of calculus in the integral calculus course. The volunteers were also recruited from calculus courses that would ensure students had the necessary mathematical background to play the game. These choices were made in order to follow the Timing Principle for the implementation of math learning games (Renne, 2019, Chapter 2).

There were two research gameplay sessions, involving two groups each of 6 participants playing in pairs. Pairings in each session were randomly assigned.

For purposes of the discussion, the participants were assigned pseudonyms. The six game pairs are labeled 1A, 1B, 1C, 2A, 2B, and 2C, respectively. The numerical prefix indicates the session number, while the alpha suffix designates the game pair within that session. Table 9 provides the pairings of the participants, with their gender indicated by (M/F), and sources of data available for each pair.

Table 9: Game pair identifiers and corresponding sources of data.

Identifier	**Pseudonyms**	**Questionnaire**	**Pre/Posttest**	**Video**	**Audio**
1A	Alice (F) & Chad (M)	✓	✓	✓	
1B	Bernard (M) & Helena (F)	✓	✓	✓	
1C	Beau (M) & Eleanor (F)	✓	✓	✓	
2A	Joseph (M) & Turner (M)	✓	✓	✓	✓
2B	Kayla (F) & Mei (F)	✓	✓	✓	✓
2C	Sabastian (M) & Zinnia (F)	✓	✓	✓	

The participants were administered the background questionnaire for 15 minutes, immediately followed by the pretest question for 15 minutes. One player from each game pair was given the 2004 AP® exam question as a pretest, and the other player was given the 2012 AP® exam question.

Following the pretest, the game was introduced to the players and the rules were explained. The mathematics topic of the fundamental theorem of calculus and the key relationship that $F' = f$ were pointed out, but not explained. There was no need to give an exposition of the mathematics topic because of the adherence to the Timing Principle. These steps were consistent with the recommendations of the Pre-brief Principle for the implementation of math learning games (Renne, 2019, Chapter 2).

Players were provided with a reference sheet (Figure 8, Appendix G), along with a brief explanation of its use and usefulness. While some of the language on the reference sheet is non-standard (e.g., "net area"), the language was commonly used in the classes at that university and the students would have been aware of its intended meaning. The reference sheet was intended to assist players in deciding game moves to make, and to help players decide if the result of a game move correctly satisfied a given mathematical constraint posed on one of the playing cards (i.e., adjudication). The creation and provision of the reference sheet are in keeping with the Planning Principle for the implementation of math learning games

(Renne, 2019, Chapter 2) and consistent with "[making] an effort to actively envision how students might mathematically approach the instructional tasks(s) that they will be asked to work on" (Stein, Engle, Smith, & Hughes, 2008, p. 322).

Assembly Lines!
Reference Sheet

You are graphing f, which is the derivative of F. That is, $F' = f$.

Which gives a useful relationship: $F(b) - F(a) = \int_a^b f(t)\,dt$

A useful shortcut: This game has been specially designed so that the **net area** between f and the horizontal axis **under a single line segment** is the **sum of the y-values at the endpoints of that line segment**. Caution: For more than one segment, do this shortcut for *each* line segment, then add the results.

This game has also been designed to allow the use of geometry to calculate areas.

The numbers *on* the line segments are the **change in y** from the last played y-value, over a run of 2. (ALWAYS a run of 2.)

$F(c)$ is a local minimum if $F'(c) = 0$ and F' is negative nearby and to the left of c, and F' is positive nearby and to the right of c (i.e., $F'(c^-) < 0$ and $F'(c^+) > 0$).

$F(c)$ is a local maximum if $F'(c) = 0$ and F' is positive nearby and to the left of c, and F' is negative nearby and to the right of c (i.e., $F'(c^-) > 0$ and $F'(c^+) < 0$).

$F(c)$ is an inflection point if F'' nearby and to the left of c has the **opposite sign** of F'' nearby and to the right of c (i.e., $F''(c^-)$ and $F''(c^+)$ have opposite sign).

For $F''(c)$ to exist, it must be that F' is smooth (no corners) near c. Or, for this game, we can say, $F''(c^-) = F''(c^+)$

Figure 8: The reference sheet given to participants playing Assembly Lines.

Each pair of participants played during a session that lasted approximately 55 minutes. All gameplay was video recorded. The games played by Mei/Kayla and Joseph/Turner included audio.

Players were given the choice of playing cooperatively or competitively, and all players chose to play cooperatively (which was somewhat unexpected). To facilitate the learning of the game rules, the first game was played with the *segment cards* only. Players were then given a choice whether to incorporate the *key point cards* in their second game. Four of the six game pairs elected to incorporate the key point cards in their second game, and two did not.

During the game, two facilitators circulated around the room to check in on players and monitor gameplay. The two facilitators answered any questions relating to the game or the relevant mathematical ideas (without providing solutions to game tasks) and helped with judging the correctness of game moves (i.e., adjudication). When answering math questions, the facilitators took great care in doing so by encouraging collaboration between game partners, asking questions of the players, and pushing players to explain what they were thinking. That is, the facilitators were careful to support the learners in their authority and accountability. To help players problematize, the facilitators would ask players to rephrase the mathematical task posed on the playing card under consideration.

The mathematical tasks represented by satisfying a segment or key point card constraint were considered quite difficult by the players. Occasionally, the difficulty of the task resulted in "player paralysis," where the players were unable to progress, and the gameplay stalled. To assist players in moving forward, the facilitators would have the players explain their thinking about the goal they were trying to achieve before employing one of the following "hint" strategies (in order of precedence, with the first two being the most common).

- Giving an indication of something the players said that was useful (or correct) for moving forward or a redirection through questions that were chosen (at that moment) to steer participants in the right direction.
- Directing the players' attention to the relevant portion of the reference sheet.
- Giving an indication that a card could be satisfied.
- Giving indication of a specific card that could be satisfied.

The first hint type was given if the conversation indicated that one of the players had a substantive contribution. The second hint type was given if the conversation revealed that both players were confused or lost. The third hint type was given if players were still exhibiting play paralysis after the first two hint types. If the first three hint types did not alleviate the play paralysis, the fourth hint type was given.

The implementation choices made for how the facilitators would engage the participants during gameplay were consistent with the Managing Gameplay Principle (Renne, 2019, Chapter 2).

After gameplay, 15-minute posttests were administered. If the participant was given the 2004 AP® exam question as the pretest, then they were given the 2012 AP® exam question as the posttest, and vice versa.

During the research study gameplay of *Assembly Lines*, the facilitators followed the same implementation principles that Renne (2019, Chapter 2) suggests for teachers' deployment of a mathematics learning game in a classroom setting, but with one very important difference: there was no moderated debrief session (Debrief Principle) following the gameplay. In that sense, the research gameplay should not be viewed as an example of "best practice" in classroom implementation of a mathematics learning game, for a teacher moderated debrief discussion would be a critically important opportunity for the teacher to facilitate students' learning from the game. The choice made to administer a posttest immediately following gameplay was deliberate, for it afforded an opportunity to investigate performance gains that could be attributed to the gameplay itself.

The students' work on the AP® exam questions were graded independently by two people (one with several years of experience as a grader of AP® exams), each using the same standards that were used for the original grading of the AP® exam questions (Appendix F). Any differences in the scoring between the two graders were discussed and resolved to produce a single common grade score. Performance gains/losses were recorded.

Analysis of player engagement

The analysis of player engagement sought to answer the research questions, stated again here:

1. How can a game-based learning activity foster engagement with mathematical ideas?
    1.1. How can player contributions toward performing the mathematical tasks support engagement with mathematical ideas?
    1.2. To what extent will players stay "on task" relative to the mathematics?
    1.3. To what extent will players express "passionate involvement" during a math learning game?

2. To the extent that learners are engaging with mathematical ideas, how can a game-based learning activity foster productivity, or "intellectual progress"?
    2.1. How does performance transfer to mathematical tasks outside of the game?
    2.2. How does student dependence on in-game mathematical resources change over time?
    2.3. To what extent do players become more efficient in performing mathematical tasks during gameplay?

The player engagement analysis was conducted using video and audio recordings to look for evidence of productive disciplinary engagement in mathematics.

For the first research question, the following indicators adapted from Engle and Conant (2002), were considered evidence of engagement with mathematical ideas during gameplay.

(a) By the end of the game, both players are attempting to make substantive contributions toward the performance of the mathematical tasks presented by the game.

This was assessed by determining whether both players were physically engaging with the game at the same time and both players offering suggestions to achieve goals. Evidence of physical engagement with the game included body position (e.g., facing the game board), hand gestures around or toward playing cards, the board, or the reference sheet, and physical interactions with the game pieces in a way that suggested an attempt to satisfy the mathematical constraints posed on the playing cards (as opposed to fidgeting or other irrelevant physical interactions).

(b) Player contributions are coordinated with each other, rather than independent of each other.

Independent play in this context was assessed by observing whether players were alternating turns (since all games were played cooperatively), one player was ignoring the other, and players were working toward different goals.

(c) Players are attending to each other's learning needs and contributions toward successful completion of the mathematics tasks or achievement of game goals.

This was assessed by body language and speech directed to their partner. Body language indicating the affirmative included players facing one another, and speech indicating the affirmative was their responsiveness to the questions and contributions of one another.

This was also assessed by observing whether one player was dominating the gameplay or discourse. If one of the players was consistently too assertive or consistently too passive, that conversation was coded as having a dominant participant.

(d) Players exhibit a high degree of engagement in gameplay.

This was assessed by measuring the portion of time that participants were on task (game or mathematics).

(e) Players express passionate involvement by making emotional displays or statements (e.g., “We’re on a roll!”, “Dang!”, or high-fives).

Indicators (a), (b), and (c) were used to answer sub-question 1.1. Indicator (d) was used to answer sub-question 1.2. Indicator (e) was used to answer sub-question 1.3.

To answer the second research question, evidence of productivity or “intellectual progress” was assessed during or following gameplay through the following indicators adapted from Engle and Conant (2002):

(f) Players exhibit performance gains from a pretest to a posttest.

This was assessed using the AP® exam questions described above and included in Appendix F. Gains were associated with intellectual progress.

(g) Players show a reduction in the reliance on resources and are more independent as the game progresses.

This was assessed through analyzing the change in frequency with which the players relied on the reference materials and the facilitators from one game to the next. Productivity was associated with a reduction in the frequency of using in-game resources.

(h) Players play more efficiently as the game progresses, by having greater success in less time.

This was assessed through analyzing changes in points earned per turn, time of play per turn, and points earned per minute.

Gains in points earned per turn would suggest that the player is satisfying more mathematical constraints with each play, but this could be influenced by the combination of cards that come up through random events. A simultaneous reduction in time of play per turn could support the inference that gains in points earned per turn can be partially attributed to mathematical progress. However, by itself, a decrease in time of play per turn could be attributed to learning the game mechanics or could be achieved by simply building a graph as quickly as possible and ignoring the constraint cards.

A gain in points per minute could be attributable to a gain in points per turn or a reduction in time of play per turn. For example, it is possible to see a gain in points per minute if there is a serendipitous sequence of cards that present themselves while the players take slightly longer to play, which may not suggest mathematical progress as much as it suggests convenience. Including this measurement in combination with the other two measurements could reveal efficiency of gameplay and could suggest an efficient use of time. Points in the game indicate performance of challenging integral calculus tasks, and more points per minute could suggest that participants are either performing more (or harder) tasks in the same amount of time.

Indicators (f), (g), and (h) were used to answer sub-questions 2.1, 2.2, and 2.3, respectively.

Results

In the discussion of these results, all names are pseudonyms, and the two facilitators are labeled with “F1” and “F2”, respectively. To give a sense for inflection, intonation, and emotion, italics are used to indicate emphasis in the speech, periods to indicate full stops in speech, and ellipses to indicate short pauses in speech. Commas required in grammar are omitted if the player did not pause, and statements that terminate with rising inflection are completed with a question mark. Long pauses are indicated with a “[p].” Unclear recordings of the audio are indicated with a “[-?-].”

When the player is referring to or otherwise indicating a line segment piece by its number label, a subscript p is used, as in “4_p.” The placement of a line segment with positive slope is indicated by a superscript arrow pointing upward and to the right, as in “$4_p^{\nearrow}$.” An arrow pointing downward and to the right in the superscript indicates that the piece was placed to have a negative slope, as in “$4_p^{\searrow}$.”

Indications of a segment card being discussed, collected, or read are given by “C” with a subscript of 1, 2, or 3 to indicate the specific card. Recall that each segment card corresponds to a mathematical constraint of the graph that is being built, and there are three face up at the beginning of each turn. And “RS” will indicate the reference sheet.

Players struggled with distinguishing between f and F, both in terms of the mathematics, and conversing about them. Sometimes, they would just say "eff" without indicating which one they intended. If the participant gave clear indication using modifiers, as in, "little eff" or "big eff," then the quote uses the corresponding mathematical notation. In all other cases, the letter will be spelled out, as in, "eff."

Were both players attempting to make substantive contributions?

Engle & Conant (2002) argue that attempts by all participants to make substantive contributions can be evidence of productive disciplinary engagement. Note that in the context of this game study, many of the players were strangers to each other, so a risk averse or shy player may have been less inclined to contribute meaningfully at the beginning of the game. By the end of the game, however, both players should be attempting to make substantive contributions.

By the middle of the first game, both players were attempting to achieve the mathematical goals set forth by the game. This was evidenced even in the videos without audio by players simultaneously pointing at and making use of the game materials. Sometimes, both players would point at the board and make gestures that mimicked potential play of a line segment for their present or future turns. Other times, one player would place a line segment piece and begin gesturing and the other player might respond with gestures and/or changing the line segment.

During the fifth turn of their second game, Mei and Kayla were trying to figure out how to satisfy two constraints simultaneously to collect all three cards (the second constraint appeared on two cards). The language on the cards read:

1. If the right endpoint of this line segment is at $x = b$, then $F'(b) \geq 0$. (This was C_1.)

2. If $(a, f(a))$ is the left endpoint of this line segment, and $(b, f(b))$ is the right endpoint, then F attains a local minimum between a and b. (This was both C_2 and C_3.)

In the game, the segment cards refer to the location of the left and right endpoints of the line segment as $x = a$ and $x = b$, respectively. Since all line segments are played over an interval of length 2 units, such that $b - a = 2$, references to $a + 1$ are, in fact, references to the x-coordinate of the midpoint of the line segment to be played. Recall that players are building, from left to right, the graph of a continuous piecewise linear f, such that $f = F'$. The state of the graph at the beginning of that turn was equivalent to Figure 9.

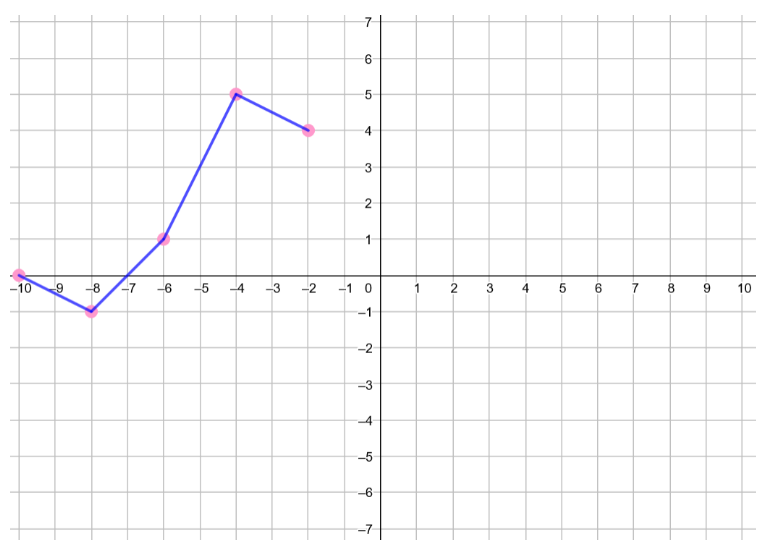

Figure 9: The state of Kayla & Mei's graph during their fifth turn.

The conversation captured in Table 10 is an example of both Kayla and Mei making attempts to contribute substantively toward the attainment of game goals and completion of mathematics tasks.

Table 10: A conversation showing substantive contributions from both players.

Name:	**Quote**	**Notes**
Mei:	So we won't be able to do this unless we use (silently counting)… 6_p. We'd have to use 6_p and we still wouldn't get it this turn though.	This is a reference to obtaining the local minimum on the graph of F, and they are discussing the possible play of $6_p^{\searrow}$.
Kayla:	Hold on.	This indicates her desire to have a voice.
Mei:	Well, we can use it to get closer to that (pointing to C_3). We just wouldn't be able to do this one yet (pointing to C_1).	
Kayla:	Which one?	She appeared to be consulting the RS.
Mei:	If we did 6_p that brings us negative (pointing at the board). So you'd have to do that and then do another one to get over the x-axis and then we would get these two (pointing at C_2 and C_3).	
Kayla:	But this one. We wouldn't accomplish this one (pointing to C_1).	
Mei:	But we would if we did one more.	
Kayla:	But I don't think we can add one just to like add one.	This is a reference to playing a piece without satisfying a constraint.
Mei:	I don't know if we can… I think you're right.	
Kayla:	What if we did like (placing $4_p^{\searrow}$ to intercept the horizontal axis)?	
Mei:	But I think this one has to go under. Yeah. For these, you'd have to go under. So…	It appears that she is still thinking about the second constraint.
Kayla:	(Placing $3_p^{\searrow}$) We have to plan for two turns ahead.	Offering a compromise between their desire to optimize play and avoid breaking any rules.
Mei:	Yeah. That will work for this one.	A reference to C_1.

The game allows for playing a line segment piece without satisfying a constraint, but Kayla and Mei were unaware of that fact. In an apparent effort to balance their desire to maximize point totals with rule-following, they chose to satisfy the first constraint and uncover the next card. They both engaged with the game physically, contributed to the choice for that turn, and contributed to planning for future turns.

Mei was attempting to position them to collect all three cards in their next turn, while Kayla was unsure of whether that would be allowed. Kayla offered a counter proposal that involved satisfying one card this turn and preparing for the other two cards in the next turn. Mei agreed and the play moved to the next turn. During the discussion, Mei repeatedly gestured toward the board or cards, while Kayla experimented with different game pieces.

One game, involving Turner and Joseph, saw a slight disengagement near the end of the first game. There was a widening gap in skill as the game progressed, and Joseph began to allow Turner to make all the decisions. While Joseph did continue to ask relevant mathematical questions for the duration of gameplay, he did not appear to persist in trying to understand. This seems to have been the only game pair that exhibited this phenomenon.

Were player contributions coordinated?

Another possible avenue for showing productive disciplinary engagement involves the coordination of contributions between participants (Engle & Conant, 2002). One way to assess this is to examine its negation, independent play. Since all players chose to play cooperatively, the design of the game would not have players alternating play. Only one game pair (Alice/Chad) exhibited this behavior, but in the second game, they began to coordinate the placement of the pieces rather than take turns. Despite their turn-taking, this pair did appear to discuss the choice before the piece was played. While it is impossible to be certain since the video did not have audio, both players would gesture toward the cards, board, and reference sheet before each piece was played.

Another indicator of independent play would be that one player ignores the contributions of the other. This occurred in only one of the six games. One player was attempting to make all the final decisions and generally ignored the input of his game partner. When a player engages in this type of behavior, they are called an *alpha gamer*. Despite continued efforts from both facilitators, this independent behavior only abated slightly. Interventions from the facilitators included explicit instructions that he should ask his game partner questions before asking the facilitators and repeated affirmations of the contributions from his game partner.

Around the sixth or seventh turn of the first game, the alpha gamer's partner, Zinnia, began to display overt and sustained contributions to the discussion. Despite not having audio, it was possible to estimate the moment when she began making sustained contributions to the discussion by analyzing the audio of a nearby game. That audio did not provide enough clarity to ascertain details of their discussion, but it was clear who was speaking. Roughly the first 35 minutes of their game was dominated by Sabastian telling Zinnia what he thought, and then engaging with a facilitator for feedback, despite Zinnia's continued attempts to contribute. The video footage of their game is consistent with this estimate in that she began to demonstrate more physical engagement with the board game, game pieces, and cards during the sixth turn. The video also shows that the alpha gamer placed all but two pieces (one in each game), while all game pieces were within reach of both players. Additionally, one of the facilitators noted that the alpha gaming persisted until the end. Alpha gaming has several implications for implementation of game-based learning activities, which will be discussed below.

The last indication of independent play that occurred during this game study was that of the two players working toward different goals (this does not include a "divide-and-conquer" approach, which would be a coordination of contributions). The only games that showed players working toward different goals in an exclusionary way were those two games played by Sabastian (alpha gamer) and Zinnia. Zinnia generally went along with trying to satisfy whichever constraints Sabastian chose, but in two instances, they were working toward different goals (the two pieces that she placed).

The video does not reveal any insight into the two instances in which Sabastian and Zinnia did not coordinate, but one of the facilitators noted that the second instance (the third turn of the second game) may have been her attempt to remove all doubt that she was correct about her advice from the previous turn (the second turn). Zinnia recognized that they had satisfied a global constraint that involved finding an integer endpoint to an interval so that $\int_{-10}^{b} f(x)\,dx \leq -18$. Indeed, after their second turn, the graph of f was such that $\int_{-10}^{-6} f(x)\,dx = -18$. She then tried to collect the card, while Sabastian doubted her correctness.[11] During his deliberation of the matter, she simply played the next line segment piece entirely below the horizontal axis and declared b to be the right endpoint of her piece (at -4). Thus, the inequality was satisfied beyond doubt.

Nearly all other gameplay had players coordinating their contributions. This was exemplified by players continually asking each other which card or cards to do next, checking in with the other player to see what they thought about a matter, or asking permission before proceeding.

[11] Cynicism can be useful in eliciting strong mathematical justifications. However, rather than cynicism, the alpha gamer's repeated failure to admit the contributions and reasoning of his game partner was more like repudiation.

Were players attending to each other?

Engle & Conant (2002) suggest an assessment of body language and speech directed to group participants to determine whether participants are attending to one another. In the context of a multiplayer tabletop mathematics learning game being played cooperatively, attending to one another might include learning needs, contributions toward successful completion of mathematics tasks, and contributions toward achievement of game goals.

During the game, players were seated beside each other and facing the game board. This seating arrangement naturally enforced a minimum amount of body language being directed toward each other. Indeed, it was only brief moments that players would turn away from each other.

In five of the six games, there was not one player that dominated the gameplay or the discourse. Except for the alpha gamer mentioned previously, speech was generally directed toward one another, and players were invested in what their partner had to say (both in terms of learning and in terms of gameplay). Facilitators usually positioned themselves so that the players would still be facing each other by standing between and behind the players, or in front of the players. Such positioning encouraged players to maintain inclusiveness while receiving help from the facilitators.

The video for Turner and Joseph showed that the pair were attending to each other for the first 9 turns of the first game. A typical example occurred during Turner and Joseph's first game, before Joseph slightly disengaged. Joseph and Turner were working to understand the graphical features of f that would indicate the presence of local extrema on the graph of F. In this case, Joseph was particularly perplexed by the fact that the graph of $f = F'$ would be *decreasing* as it crossed the horizontal axis, for there to be a local maximum on the graph of F. The conversation shown in Table 11 took place immediately following a lengthy conversation with one of the facilitators.

Table 11: An example conversation showing players attending toward one another.

Name:	**Quote**
F1:	See if you can satisfy this card (pointing to C_1) and another one, because you can. You can satisfy two cards.
Turner:	(pointing at C_1 and C_2) These two. But not this one (pointing at C_3). I think. (F1 leaves) [p] Do… do you see why that is?
Joseph:	No.
Turner:	Ok. Um. Well… (grabbing the RS) appealing to the authority of this white sheet of paper… um, because I didn't know this before, uh, apparently, um. The, the anti-derivative does indeed have a local maximum if, um, f is equal to 0 – so this function is equal to 0 (pointing to graph) at wherever. Uh… and is positive on the left side of c (pointing at the board near the horizontal axis) and negative on the right side of c (pointing at the board)
Joseph:	Right. So. This has always confused me. (and the conversation continued, with Turner explaining mathematical concepts to Joseph)

After a detailed conversation involving both players and the facilitator, Turner appeared to have a profound breakthrough in his mathematical understanding of the graphical relationship between $f = F'$ and F (evidenced by subsequent efficiencies in his play). Immediately following the facilitator's departure, Turner gave a long pause. It is not clear whether he was deliberating or giving Joseph an opportunity to deliberate. However, it is clear from what Turner asked Joseph, that he wanted to give Joseph the opportunity to understand the relevant mathematics before moving on. Turner was attending to the Joseph's learning needs, and Joseph was attending to Turner's contributions toward his achievement of learning outcomes.

During the last turn of the first game, Joseph slightly disengaged. Turner was careful to incorporate Joseph after mathematical breakthroughs but became slightly impatient near the end. It is unclear which occurred first, the disengagement or the impatience. However, it became clear that they were not attending to each other as thoughtfully as before that turn. Joseph continued to ask questions, and Turner continued to provide answers and justifications for his game moves, but Joseph would reply with comments like, "I guess… ok. I'm happy to say we got it."

What portion of the time was on task?

Engle & Conant (2002) approach the question of staying on task through its negation. Namely, by assessing whether participants avoid off-task activities. In the context of the gameplay, an example of an off-task activity might be the player's engaging in discourse unrelated to the game or the mathematics (socializing).

However, as players of a game become comfortable with the setting, unrelated discourse may arise as a natural filler for the time between games or during brief reorganization of the game materials (between-play downtime). Social discourse that occurred during the between-play downtime was not measured in this study, but it did occur. Socializing during the game was extremely rare. The videos with audio showed only one moment that lasted approximately 20 seconds, and one facilitator noted that players were generally on task throughout the gameplay.

Another example might be for players to look at their mobile phone during play. The videos did not show the players' eyes, so it is not possible to know where players were looking with certainty. However, the videos showed that player body positions were generally angled slightly toward each other while facing the game board. In addition, there were only three brief instances caught on camera in which the participants looked at their mobile phone, each lasting no more than 5 seconds. In one recorded instance, a participant appeared to ignore, or did not notice, the incoming notification on her phone. All other mobile phone interactions caught on camera were during between-play downtime.

As discussed above, Joseph exhibited a slight disengagement from the game while playing with Turner. However, they both stayed on task for the duration of gameplay. This was evidenced by Joseph's continued mathematical questions as Turner made plays.

Alice and Chad finished their second game approximately 7 minutes earlier than everyone else and decided not to play again. However, they chose to spend some of that extra time reflecting on the game and their choices. In this way, they were partially on task.

The video angles were chosen so that the game board, the cards, and the play choices could be determined from the video footage. As such, the cameras were unable to have a wide enough angle to see the entire player and their eye gaze. However, the discussion above regarding the general lack of evidence for off-task activities suggests a high degree of engagement in gameplay. The portion of the time that was on task handily exceeded 95%, if between-play downtime is excluded from consideration. For all game pairs, the between-play downtime was less than 5 minutes.

Were there emotional displays or statements?

Engle & Conant (2002) suggest that emotional displays and statements show passionate involvement and can be evidence of productive disciplinary engagement. In this game study, both videos that contained audio showed moments of emotion, and one of the videos without audio also showed some moments of emotion. In the latter case, Chad would occasionally rub his hands together at the end of a turn, as if eagerly awaiting the next card(s), and for much of their second game, Chad's partner Alice would roll her chair rapidly back and forth, away and then toward the desk, as one might do in nervous excitement. Some emotional highs and lows were found in the audio, where Turner once excitedly exclaimed, "Wow!" and another time, "Cool!", while Kayla once reacted with, "doooope,[12]" and in a moment of frustration, Mei slapped the desk. Excerpts are collected from Kayla and Mei's games in Table 12.

Table 12: Examples of passionate involvement indicated by emotional statements.

Name:	Quote	Notes
Kayla:	Doooope! We got momentum. Let's work on that. (both laugh)	This occurred immediately following successfully interpreting and applying the RS to create an inflection point and then explain it to F1.
	After flipping a new card:	
Mei:	Can we do this one though?	
Kayla:	Yeah. Oh my god (both laugh). It keeps getting worse. It's like it brings up your confidence and knocks [-?-] down (while chuckling).	This was a response to turning over a new card that they perceived as difficult to achieve given their remaining line segment pieces.
	When experimenting with a line segment piece:	
Kayla:	So that would be a little bit more than 2, huh?	A reference to the area under the curve.
Mei:	I think it's a little bit, well, if there was a line here this would be one box, that's 1. And this is a little less. Oh wait. Is it? A little more I guess.	
Kayla:	Yeah. It's a little bit more than 2. Dang!	This exclamation was quite loud.
	Then, after much discussion and satisfying a different, but challenging constraint:	
Mei:	We did it! (laughs)	
Kayla:	Yes! *Yes!* (louder) (Mei laughs again)	

[12] A slang term with highly positive connotations when said in this way.

Table 12: Continued

F2:	Plus, if you guys would've put the 0_p, it would've been $1 + 1$.	This clarified their doubts regarding "a little more" in the previous excerpt.
Kayla:	Oh! So we *could've* done that.	
Mei:	For this? (pointing to the card) We *were* right.	
Kayla:	Yeah. Oh my gosh! Ok, that's just like a bonus point then. A personal bonus point.	
Mei:	Yeah.	
Kayla:	I need it.	
F2:	A bonus point for your self-esteems.	
Kayla:	Yeah. I needed this. (Mei laughs). Can we start over now?	This is a request to play again.
Mei:	(Counting cards earned) We got like 12 points.	
Kayla:	Alright. I'm starting to get the hang of this now. I feel a little bit better now.	

How did performance change from pretest to posttest?

The pretest and posttest questions were the relevant AP® exam questions from the years 2004 and 2012 (Appendix F). One player from each game pair was given the 2004 exam question as the pretest question and the 2012 exam question as the posttest question. The other player in each game pair was administered the same two questions, but with the pretest/posttest order reversed. Players were given 15 minutes to complete each of the pretest and posttest questions. The students' work on these AP® exam questions were graded independently by two people (one with several years of experience as a grader of AP® examinations), each using the same standards that were used for the original grading of the AP® exam questions (Appendix F). Any differences in the application of the standards between the graders were resolved to produce consistent grading.

The mean score on the pretests was 1.25 and the mean score on the posttests was 1.67. Overall, the players appeared to show modest learning gains on the immediate posttest (Table 13).

Table 13: Change in scores from pretest to posttest.

	Mean Score
Pretest	1.25
Posttest	1.67
Increase/(Decrease)	0.42

Zinnia, the partner of the alpha gamer, Sabastian, did not seem obviously affected during gameplay but showed the largest decrease in performance when compared to all other players. The 2004 pretest score for Zinnia was a 4 out of 9, and her 2012 posttest score was a 0 out of 9. She was the top performer of all pretests and dropped to the lowest possible score on the posttest.

Removing the test scores of the apparently affected player reveals more impactful results. The mean scores change from 1.00 in the pretests to 1.82 in the posttests (Table 14). While modest in absolute terms, the change reflects a near doubling in scores in relative terms.

Table 14: Change in scores from pretest to posttest excluding Zinnia.

Trimmed	Mean Score
Pretest	1.00
Posttest	1.82
Increase/(Decrease)	0.82

An analysis of pre-posttest performance measures grouped by enrollment may suggest that participants that were currently enrolled in the single-variable integral calculus course exhibited greater gains (Table 15). While the sample size is small, these data plausibly suggest that curricular timing may be related to student motivation to transfer the mathematics outside the game.

Table 15: Change in scores from pretest to posttest, split by course enrollment.

	Currently Enrolled Mean Score N = 9	Not Currently Enrolled Mean Score N = 3
Pretest	1.11	1.67
Posttest	2.00	0.67
Increase/(Decrease)	0.89	(1.00)

The AP® exam questions included a piecewise function whose pieces were either linear or semi-circular. The game only included tasks involving line segments, which means that the sub-tasks of the AP® exam questions that involve the semi-circular piece of the given graph would be transfer tasks. Only one participant (Chad) earned a point on the transfer task, suggesting that no appreciable gains were made on the transfer tasks.

Did players become more independent of the in-game resources?

A possible indicator of mathematical progress might be a decrease in reliance on resources and facilitation as the participant becomes more independent. The game *Assembly Lines* was designed so that all game moves correspond to a meaningful mathematical move, and if the player is attempting to earn a card by satisfying its mathematical constraint, then that game move is purposeful as it relates to the mathematics. In addition, the reference sheet is a mathematical reference. A reduction in the use of the reference sheet and receiving help from the facilitators, while simultaneously continuing to perform tasks successfully, could suggest increasing understanding of the corresponding mathematics. Such a reduction could be explained by learning how to play the game, but learning how to play *Assembly Lines* successfully, will, at least in part, correspond to learning some mathematics.

All six game pairs began a second game. Only the game pairs Beau/Eleanor and Joseph/Turner did not use the key point cards in their second game. The game pairs Beau/Turner, Joseph/Turner, and Sabastian/Zinnia only completed 4 turns (40%) of their second game, and game pair Kayla/Mei only completed 9 turns (90%) of their second game. To account for this difference between game pairs, all the measurements are divided by the number of turns to provide a uniform basis for comparison.

Kayla and Mei correctly determined that playing their 10th and final piece was superfluous and would not give them any more points. They chose to focus on the key point card at that time, to see if they could figure it out without continuing to build their graph. They were able to determine how their graph had satisfied it just before time ran out for the session, so they were unable to work on another key point card which could have motivated playing a 10th piece.

Of the six games whose data are presented below, only game pairs Joseph/Turner and Kayla/Mei had audio. Since the videos did not show eye gaze, body language was used. The videos that contained audio suggest that almost all uses of the reference sheet were accompanied by physical evidence that could be seen in the videos (like pointing, bring it closer, holding it up, etc.). Similarly, almost all instances of facilitation were accompanied by physical evidence which showed the facilitator in action.

Players would commonly use the reference sheet, then interact with the board or each other, and sometimes repeat this process in a single turn. If both players interacted with their own reference sheet at the same time, that was counted as two lookups. If both players appeared to be sharing a reference sheet, it was counted as one lookup because most videos did not have audio and it was not always possible to be certain that both players were making use of it. If a player began interacting with the game pieces, game board, game cards, or their game partner after using the reference sheet, and then shifted their attention back to the reference sheet, then the second lookup would be counted. If the player appeared to be glancing back and forth to compare the reference sheet to the game pieces, game board, or game cards, then those were not counted as separate instances of using the reference sheet. If a player engaged with a facilitator and were instructed to consult the reference sheet (indicated by the facilitator pointing at the reference sheet), then that would be counted as a lookup and help. Table 16 shows the changes in reference sheet lookups from game one to game two.

Table 16: Changes in reference sheet lookups from game one to game two.

Game Pair	Game 1 RS Lookups per turn	Game 2 RS Lookups per turn	Change in RS Lookups per turn	Percent Change in RS Lookups per turn
Alice/Chad	1.7	0.1	– 1.6	– 94%
Bernard/Helena	0.8	0.0	– 0.8	– 100%
Beau/Eleanor*	1.8	0.5	– 1.3	– 72%
Joseph/Turner*	3.1	0.2	– 2.9	– 92%
Kayla/Mei	3.8	0.4	– 3.4	– 88%
Sabastian/Zinnia	2.1	1.0	– 1.1	– 52%
Mean	*2.2*	*0.4*	*– 1.8*	*– 83%*

*These game pairs did not play with key point cards (global constraint cards) in their second game.

Any engagement with the facilitator was counted as receiving help, since nearly all interactions with the facilitators were questions regarding rules or mathematics. The duration of an instance of facilitation is not represented in the data below. Table 17 shows the changes in the amount of help received from facilitators from game one to game two.

Table 17: Changes in receiving help from facilitators from game one to game two.

Game Pair	Game 1 F1/F2 Help per turn	Game 2 F1/F2 Help per turn	Change in F1/F2 Help per turn	Percent Change in F1/F2 Help per turn
Alice/Chad	0.5	0.3	– 0.2	– 40%
Bernard/Helena	0.6	0.2	– 0.4	– 67%
Beau/Eleanor*	0.9	0.3	– 0.6	– 72%
Joseph/Turner*	0.7	0.5	– 0.2	– 29%
Kayla/Mei	1.0	0.7	– 0.3	– 33%
Sabastian/Zinnia	1.6	1.0	– 0.6	– 38%
Mean	*0.9*	*0.5*	*– 0.4*	*– 46%*

*These game pairs did not play with key point cards (global constraint cards) in their second game.

The data show that game pairs averaged 2.2 reference sheet lookups per turn in their first game, and they averaged 0.4 reference sheets lookups per turn in their second game. That is, the game pairs changed from roughly two lookups per turn in the first game, to one lookup every two turns in their second game. That is an 83% reduction in the frequency from one game to the next. The primary use of the reference sheets was to interpret and understand the mathematical concepts put forth by the constraint cards and connect that to the graph they were building. This result suggests that players were learning the mathematical connections between the cards and the graph.

Similarly, all game pairs showed a reduction in the frequency with which they needed assistance from one of the facilitators, dropping from an average of 0.9 instances of help per turn in the first game, to 0.5 instances of help per turn in the second game. That is, the game pairs averaged nearly one instance of help per turn in the first game, and only one instance of help every two turns in their second game. That is nearly a halving (46%) in the frequency in the instances of facilitation from one game to the next.

The game pair Sabastian/Zinnia stands out in some remarkable ways. Their first game showed them engaging with a facilitator nearly as frequently as they engaged with the reference sheet. Their second game showed equality in the frequencies. They showed the smallest reduction in use of the reference sheet per turn. This was the pair with the alpha gamer. One of the facilitators noted that this game pair absorbed more of their time during the session than any other group, which is not reflected in the counts. Much of that time was spent attempting to steer the alpha gamer (Sabastian) toward coordination with their game partner (Zinnia).

A comparison of the two sessions shows that the first session exhibited more impressive improvements in these frequencies than did the second session. This may suggest that certain interaction dynamics that require considerable attention from a facilitator could reduce the access other students have to support. A reduction in support or access to resources could hinder productive disciplinary engagement (Engle & Conant, 2002). This result may have implications for a teacher that uses game-based learning activities in the classroom.

Did players become more efficient with their gameplay?

In the case of *Assembly Lines*, an increase in the efficiency of gameplay may be partially attributable to progress toward the mathematics learning goals. *Assembly Lines* was designed so that the game moves had mathematical meaning, and that players attempting to earn points through the collection of constraint cards would be acting purposefully. That is, if a player made a move with the intention of satisfying a constraint card, they were doing so with mathematical ideas in mind. Improvements in gameplay efficiency could mean a better understanding of the game, of the mathematics, or both. Three measurements were used in order to gain insights into possible reasons for changes in gameplay efficiency: *changes in points earned per turn (Δppt), changes in time of play per turn (Δmpt),* and *changes in points earned per minute (Δppm)*. As discussed above, all three measurements are needed to adequately support attributing improvements in gameplay efficiency to mathematics learning.

All players chose to play cooperatively, and 4 of the 6 game pairs elected to incorporate the key point cards in their second game. It might be expected that game pairs which chose to incorporate key point cards would score more points than game pairs that did not. However, the difficulty of the key point cards meant that game pairs might take longer to play the second game, ask for more help, or both. The results discussed in the previous section suggest that they did not ask for more help. Game pairs that played with the key point cards in their second game appeared to seek facilitation with comparable frequency to those game pairs that did not use the key point cards, and they showed similar reductions in that frequency from game 1 to game 2.

Data showing measures of efficiency in games one and two are in are Table 18, and the data showing changes in the measures of efficiency from game one to game two are in Table 19.

Table 18: Game 1 and Game 2 measures of efficiency.

	Game 1			Game 2		
Game Pair	**Points per turn (ppt)**	**Min. per turn (mpt)**	**Points per Min.(ppm)**	**Points per turn (ppt)**	**Min. per turn (mpt)**	**Points per Min.(ppm)**
Alice/Chad	1.10	2.66	0.41	2.90	2.02	1.44
Bernard/Helena	1.20	2.28	0.53	1.20	3.03	0.40
Beau/Eleanor*	1.30	4.86	0.27	1.50	1.88	0.80
Joseph/Turner*	1.60	5.73	0.28	1.25	1.17	1.07
Kayla/Mei	1.20	4.48	0.27	1.56	1.69	0.92
Sabastian/Zinnia	1.00	4.25	0.24	1.25	4.76	0.26
Mean	*1.23*	*4.04*	*0.33*	*1.61*	*2.42*	*0.81*

*These game pairs did not play with key point cards (global constraint cards) in their second game.

Table 19: Changes in measures of efficiency from game one to game two.

	Change			Percent Change		
Game Pair	**Δppt**	**Δmpt**	**Δppm**	**Δppt**	**Δmpt**	**Δppm**
Alice/Chad	1.80	– 0.64	1.02	164%	– 24%	247%
Bernard/Helena	0.00	0.75	– 0.13	0%	33%	– 25%
Beau/Eleanor*	0.20	– 2.99	0.53	15%	– 61%	199%
Joseph/Turner*	– 0.35	– 4.56	0.79	– 22%	– 80%	282%
Kayla/Mei	0.36	– 2.79	0.65	30%	– 62%	243%
Sabastian/Zinnia	0.25	0.51	0.03	25%	12%	12%
Mean	*0.38*	*– 1.62*	*0.48*	*35%*	*– 30%*	*160%*

*These game pairs did not play with key point cards (global constraint cards) in their second game.

The data suggest that incorporation of the key point cards may have slightly disadvantaged gameplay efficiency gains. However, Bernard and Helena noted to the facilitators that they drew an unfavorable key point card that had a considerable impact on the progression of the game. And, as noted before, Sabastian and Zinnia was the game pair with the alpha gamer, which could partially explain their underwhelming gain in efficiency. Kayla and Mei did not attend to the key point cards until their last turn, which may have contributed to their considerable gains in time per turn (it was consistent with the two pairs that did not incorporate key point cards). In general, it appears that incorporation of the key point cards did appear to yield more points and consume more time, with a slightly greater effect in the consumption of time. The results from an earlier section suggest that the difficulty in obtaining key point cards did not lead to greater use of in-game resources. This suggests that players were handling greater challenges with less help.

Overall, the data seem to suggest considerable gains in gameplay efficiency that may be partially attributable to the learning of mathematics. The overall changes indicate a 35% gain in points per turn (ppt) and a 160% gain in points per minute (ppm), with a simultaneous 30% reduction in time per turn (mpt). The overall gain in ppt suggests players were satisfying more constraints per turn, or more challenging constraints per turn (e.g., the key point cards). The overall reduction in mpt suggests players were playing faster, which could be attributed to increased understanding of the game rules or the mathematics. Given the reductions in the frequency in using resources, the reduction in mpt seems to be related to both. The overall gain in ppm suggests players were satisfying more constraints per minute, or more challenging constraints per minute.

Discussion of interactions between players during gameplay

The previous section discussed the evidence for productive disciplinary engagement in mathematics indicated across both research gameplay sessions. In this section, the discussion turns to lessons that can eb learned from the progression of interactions between players in each game pair.

Game pair 1A (Alice & Chad): Alice and Chad played considerably faster than everyone else and earned 5 key point cards. Despite being slightly more than double the next highest score, their overall gain in efficiency was comparable to the gains in efficiency of the two game pairs that did not incorporate the key point cards. This was likely due to their efforts in completing more challenging mathematical tasks. In addition, each player improved 2 points out of 9 from the pretest to the posttest.

The video for Alice and Chad did not have audio, but their body language generally supported the idea that they were working together, on task, and engaged with the mathematics throughout the game. Alice's mobile phone visibly alerted her to notifications during play, and she appeared to ignore it. In addition, each player exhibited attenuated excitement during the second game, as exhibited by Alice rolling her chair back and forth and Chad rubbing his hands together while new cards were being drawn.

Taken together, the evidence seems to suggest that Alice and Chad were productively engaged in the discipline of mathematics while playing *Assembly Lines*.

Game pair 1B (Bernard & Helena): Bernard and Helena showed a decrease in gameplay efficiency, but they complained of having an unfavorable draw of cards in their second game. The unfavorable draw of an extremely difficult key point card appears to have consumed a considerable amount of their time. They did not appear to use their reference sheet during their second game, and they only asked for facilitation twice. This suggests that it may be the bad draw that bogged them down, rather than their continuing to struggle with mathematical concepts. Indeed, Bernard improved by 1 point from the pretest to the posttest, while Helena improved 2 points.

The video for Bernard and Helena did not have audio, but their body language generally supported the idea that they were working together, on task, and engaged with the mathematics throughout the game. There were no noticeable emotional displays from this game pair.

Taken together, the evidence seems to suggest that Bernard and Helena were productively engaged in mathematics.

Game pair 1C (Beau & Eleanor): Beau and Eleanor only completed 4 turns (of 10) in their second game, and they did not incorporate key point cards. They showed a reduction in the frequency with which they utilized in-game resources, but it is unclear whether their gains in efficiency could be attributed to attainment of mathematics learning goals. They showed a 15% gain in points per turn, which could have been due to favorable draws. They also showed a considerable reduction in time per turn (61%), but this could easily be attributed to having learned the rules of the game. Additionally, their having only played 4 turns of the second game makes it uncertain whether the gains in efficiency would have been sustained. Optimistically, the reduction in their use of resources seems to suggest slight improvements in mathematical understanding, but that conclusion is not corroborated by the other evidence.

The video for Beau and Eleanor did not have audio, but their body language generally supported the idea that they were working together, on task, and engaged with the mathematics throughout the game. There were no noticeable emotional displays from this game pair, but they did incorporate social norms that exist in other card games. This game pair cut the deck after shuffling and before dealing the segment cards for their second game. They also used a hand-sign that is sometimes used at a Blackjack table to indicate that they want to "stand." In this case, they used the hand-sign to indicate that they were ready to adjudicate the play.

Beau improved 2 points from the pretest to the posttest, while Eleanor showed a decrease of 1 point. The results for this pair of players were not entirely clear, but it does appear that both were engaged in mathematics. The evidence suggests that only Beau showed "intellectual progress" (Engle & Conant, 2002), and Eleanor may not have done so. Without the audio, there is scant evidence to counter the suggestive nature of the test performances for Eleanor.

Game pair 2A (Joseph & Turner): Joseph and Turner only completed 4 turns of their second game and did not incorporate key point cards during their second game. It is difficult to determine the degree to which this game *pair* was exhibiting productive disciplinary engagement in mathematics. The improvements from game 1 to game 2 are attributable to the attainment of mathematics learning and game goals by one of the two players (Turner). The less skilled player (Joseph) disengaged near the end of their first game, and then began to play more passively. Joseph only reengaged sporadically during the second game. The last turn of the first game, and all four turns of the second game were decided solely by Turner.

The video for Joseph and Turner did have audio. It showed that the pair was engaged in mathematics as a discipline for the first 9 turns of the first game, and that Joseph partially disengaged at that time. Turner was careful to include Joseph in the action after mathematical breakthroughs but became slightly impatient near the end. They began to ask about the time and sighed a few times (apparently out of frustration). However, Turner stayed after the session to ask when he might see the game commercially available. This seems to suggest that Turner valued the game, but not playing with Joseph.

Neither player changed from pretest to posttest, but the other evidence available seems to suggest that Turner exhibited productive disciplinary engagement, while Joseph showed a lack of productivity.

Game pair 2B (Kayla & Mei): Kayla and Mei only completed 9 turns of their second game, but made that decision based on an attempt to optimize gameplay. During their 10th and final turn, they recognized that there was no available play that would satisfy the available constraints on the segment cards. At that time, they decided to incorporate and attend to the key point cards. The card they drew had been satisfied by an earlier play and they identified that fact right before the gameplay session concluded. Thus, they were not in a situation that would motivate play of the 10th piece. Most of their efficiency gains were in the form of time savings, which may reflect that ending sequence of events.

The video for Kayla and Mei, which included audio, showed that they needed help with adjudicating every turn of the first game and most turns from their second game. It was not until near the end of their second game when their independence from the in-game resources began to emerge. As they progressed toward that independence, the nature of their mathematical discourse began to show increasing understanding of the mathematics. They did not seem to improve their use of formal mathematical language, but they did appear to make deeper connections.

For example, one late card that they drew contained the notation, $F'(b)$, which they immediately identified as f. When reading the new card, Mei said, "(quietly) eff prime of b, (loudly) which is just eff!" One of the significant hurdles for the players of *Assembly Lines* was non-rhetorical recognition that $F' = f$, which is consistent with other research (Orhun, 2012). Most players would repeat that fact throughout the gameplay, but early on, it typically would occur only after a reminder from the facilitators to revisit that part of the reference sheet. In this case, there was no facilitator, there was no reference sheet lookup, and almost no time transpired between the two parts of that statement. This suggests that they recognized the most essential mathematical connection between objects in the game in a way that was non-rhetorical.

Kayla and Mei proved to be exemplars in terms of attempting to make substantive contributions, coordinating contributions, attending to one another, staying on task, and engaging emotionally. However, neither pair seemed to improve from the pretest to the posttest. Mei dropped in performance 1 point, and Kayla performed the same in both tests.

Overall, the evidence for this pair seems to suggest that they productively engaged in mathematics, despite lackluster test performances. Kayla had expressed fatigue during the gameplay by pointing out that she had "been at school since like 5AM," which was more than 14 hours before the posttest. It is possible that fatigue played a part in the lack of improvement in posttest performance.

Game pair 2C (Sabastian & Zinnia), the case of the alpha gamer: Sabastian improved 2 points from pretest to posttest, while Zinnia dropped 4 points (changing from being the top performer on the pretest to a score of zero on the posttest). The pair showed a modest reduction in the utilization of in-game resources when counting instances, but the duration of time for each instance of facilitation may have cost the other game pairs access to valuable in-game resources. Overall, each player of the pair may have been engaged in the mathematics in an uncoordinated way, but that is not reflective of disciplinary engagement as envisioned by Engle and Conant (2002). Thus, it seems that Sabastian and Zinnia did not exhibit productive disciplinary engagement in mathematics.

In summary, it appears that 5 of the 6 participants from the first gameplay session engaged productively with mathematics concepts and tasks, while 4 of the 6 participants from the second gameplay session showed mathematical engagement that was slightly productive.

The presence of the alpha gamer during the second gameplay session may have contributed to the results being less convincing for participants of that session. Not only did the alpha gamer negatively impact the learning of his game partner, but he consumed facilitator time that may have been beneficial to other players. The evidence suggests that all players engaged with the mathematical concepts and tasks, and a majority of players were productive.

Conclusions and implications for implementation of a game-based learning activity

This study investigated how a game-based learning activity could foster engagement with mathematical ideas and how that game activity could make that engagement productive. During gameplay it was found that participants engaged with mathematical ideas in numerous ways. The players collaborated in attempting to make substantive contributions toward completion of mathematical tasks, they attended to one another as players and leaners, they remained on task for nearly the entire gameplay session, and they showed moments of passionate involvement.

Additionally, several participants showed performance gains from a pretest to a posttest, game pairs showed greater independence in terms of gameplay and performance of mathematics tasks as the games progressed, and they showed gains in efficiency by playing faster, and earning more points each turn. Since the earning of points maps exactly to performance of mathematics tasks, the gain in points per minute indicates that participants were either completing more mathematics tasks in the same time, or harder mathematics tasks.

The alpha gamer (Sabastian) in the second session appeared to have an outsized impact on the learning environment and other players by consuming resources (facilitator time). His repeated efforts to avoid collaboration consumed the valuable resource of content expert facilitation, and he and his partner showed below average reductions in use of in-game resources overall. Sabastian was the only participant from that session to show performance gains from the pretest to the posttest, and his partner showed the largest decrease in posttest performance, dropping from the top performer in the pretest to the lowest possible score in the posttest. The second session's group did not show the performance gains that were shown in the first session's group, and the second session's group showed less productivity, overall, in their engagement with mathematical ideas. It is possible that one alpha gamer may have negatively impacted the learning environment for all players. Research into possible effects of an alpha gamer in a game-based learning activity on the learning environment and other learners could inform teachers on effective strategies for working with such a student.

One possibility available to the facilitators was to rearrange the players, which was not done. It is an open question of whether that would have improved the situation. Another question that arises relates to the player with whom the alpha gamer should be paired. Who would be a good match in that they would not be adversely affected by the alpha gaming behavior? What should be done if there is no one with whom to pair the alpha gamer? Could an alpha gamer situation be averted by establishing norms of behavior during a game-based learning activity? Answers to such questions could inform use of games in the classroom.

During the first session, one game pair had an unfavorable draw of cards that slowed them down and kept them from earning points. As the gameplay session ended, they voiced their frustration with that outcome. In some game-based learning activities, this kind of frustration could hinder learning for some students, since they might internalize the unfavorable draw as reflecting something negative about their mathematical abilities. In this case, it did not appear to have significant negative effects on the pair's learning as both players showed performance gains from pretest to posttest.

It is possible that design choices could be made to reduce the likelihood of unfavorable draws, but the event highlights an interesting dilemma for the facilitator (or teacher). One possibility was that the facilitators could have over-ridden the rules to "cheat" by discarding one of the cards causing the most trouble, in order to make the game move more quickly for those players (give them more opportunities to do mathematical tasks) and relieve their frustration (make the gameplay experience more enjoyable). However, this action could have led to undesirable consequences in that players might take liberties in removing from play those cards they think are too hard. This could include players from other games that saw the event transpire, and then possibly spread throughout the classroom. Investigations into the conditions under which such dilemmas arise might be a fruitful area of inquiry for game-based learning, in order to inform the design and implementation of multiplayer tabletop mathematics learning games.

All of this seems to point to the important role that a teacher might play during the deployment of a game-based learning activity in the classroom, for all students to realize the learning benefits. Establishing, managing, and enforcing appropriate behavioral norms for player-to-player and player-to-teacher interactions during a game-based learning activity seem critically important. As noted earlier, the implementation of the game *Assembly Lines* during the research sessions followed the first four of Renne's implementation principles (Renne, 2019, Chapter 2), but deliberately did not include a teacher-moderated debrief. It is possible that a debrief session could have led to greater performance gains and deeper mathematical connections. These results may lend support to the commonly held belief that a debriefing session following a game-based learning activity is essential (e.g., Garris, Ahlers, & Driskell, 2002; Nicholson, 2012; Westera, 2015; Wouters, Van Nimwegen, Van Oostendorp, & Van Der Spek, 2013).

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CHAPTER 5: DISCUSSION

Much of the work in the arena of game-based learning in mathematics has focused on digital games, usually intended to be played by a single individual. In his book *Mathematics Education for a New Era: Video Games as a Medium for Learning* (2011), Devlin describes effective design practices for digital mathematics learning games, which contributed to the design of what is, perhaps, the most notable effort to date in video game design for mathematics learning, *Wuzzit™ Trouble*. Devlin's work was heavily influenced by the 36 learning principles embodied in video games proposed by James Paul Gee (2003).

Video learning games offer the important affordances of automatic adjudication (enforcement of game rules) and feedback (progress toward game goals) to the player. Indeed, many of the design challenges in creating a successful digital learning game concern how best to provide this adjudication and feedback in ways that both enhance the player's mathematics learning and sustain the individual's interest and engagement in playing the game.

In contrast, the format of a multiplayer tabletop game (non-digital, such as a board game or a card game) lacks these features of automatic adjudication and feedback. Such games must rely on either the players themselves (or another person acting as a referee) to adjudicate play within the rules as well as to evaluate each player's progress toward game goals. However, multiplayer tabletop games can also involve many player-to-player interactions that are absent in the single player video game scenario. These player-to-player interactions afford valuable opportunities to promote productive mathematical discourse around the players' engagement with mathematical ideas arising in the game. Taking full advantage of those opportunities suggests not only different challenges for multiplayer tabletop game designers, but also has implications for the teacher's role in supporting student engagement in mathematical activity and discourse.

The work described in this dissertation on the design and use of multiplayer tabletop mathematics learning games is intended to be an effort that is complementary to the work on single-player digital mathematics learning games done by Devlin and others. Both the different limitations and the unique affordances presented by the multiplayer tabletop game format pose special challenges for the design of mathematics learning games of this type. Moreover, recognizing the special role the teacher could play in facilitating student discourse around mathematical ideas, there is an additional need for guidance for effective implementation of such a game as a learning activity. Faced with similar concerns, Dick and Burrill (2016) offered principles for both design and implementation of dynamic interactive mathematics learning technologies. Their approach motivated our presentation of both a set of design principles to guide the creation of multiplayer tabletop mathematics learning games, and a set of implementation principles to aid teachers effectively leverage such games as classroom learning activities.

The three-paper dissertation model structure appeared to be particularly appropriate for this work. Chapter 2 is a “principles” paper that provides a theoretical framing for, and describes in detail, both a set of design principles and a set of implementation principles for multiplayer tabletop mathematics learning games. Chapter 3 is a “design experiment” paper that documents the use of the design principles in the creation and iterative refinement of the function representations card game *Curves Ahead!* and reports on an investigation of its perceived gameplay-value. Chapter 4 is a “gameplay interaction analysis” paper that reports on an analysis of video-recorded game sessions involving the calculus board game *Assembly Lines* and considers the degree to which the players were productively engaged in mathematical activity. These sessions were conducted following the proposed implementation principles, and the analysis of player interactions suggested implications for teachers facilitating gameplay as a learning activity.

Implications of the design and implementation principles paper (Chapter 2)

The construct of productive disciplinary engagement (Engle & Conant, 2002) provided a very useful theoretical framing for both the design principles and the implementation principles for multiplayer tabletop mathematics learning games. Teachers are generally attracted to the idea of game-based learning by the allure of increased "engagement" of students, but exactly what that notion means to the teacher may be left vague. Engle and Conant's framework puts a clear and sharp focus on the kind of student engagement to aim for in a learning game: *productive disciplinary engagement*. While this framework has primarily been used in the analysis of student discourse (e.g., Engle & Conant, 2002), it appears to be entirely relevant to the consideration of a broader set of player interactions while playing a learning game. Indeed, this dissertation would suggest that:

> *creating sustained opportunities for, and facilitation of, productive disciplinary engagement in mathematics should be the central aim of both the development and deployment of a mathematics learning game.*

Engle and Conant's framework also provides useful evidentiary indicators of productive disciplinary engagement that can be readily applied in analyzing gameplay interactions.

Importance of situating the mathematics

The design principle that most distinguishes a mathematics learning game from gamification (the superficial introduction of game elements to an essentially non-game learning activity) is the

> *Embedding Principle: Each mathematical task in a math learning game should be embedded in a way that elicits the formulation of*

> *a mathematical problem statement by the player, without the need to overtly indicate the task to the player.*

Essential to adhering to this principle is establishing a meaningful mapping of the basic game mechanics to mathematical tasks involving sense making and reasoning, and not complicated computations.

Leveraging a challenge-defense mechanism for productive mathematical discourse

In a tabletop game, the players themselves will often be involved in adjudicating player actions. This adjudication process could include a game mechanic where one player can formally challenge another player's action as invalid, and the challenged player can formally respond with a defense of the validity of that action. If the game mechanics have situated mathematical meaning for the player actions, then one player's challenge may be viewed as essentially a critique of the mathematical reasoning of another player, whose defense is a counter argument. This player-to-player discourse exchange involves both participants in the mathematical practice of critiquing the reasoning of another.

Subjective gameplay-value – a useful construct for student motivation

The success of a mathematics learning game will be judged not only on the productive mathematical engagement of its players, but also on the motivation or interest students have to play the game. The expectancy-value theory of achievement motivation was a useful lens through which player interest could be considered, which yielded the construct of *subjective gameplay-value*. This construct takes into account multiple reasons behind a player's interest in a mathematics learning game, including valuing the game for sheer enjoyment or for the expected learning outcomes.

In short, the set of design principles presented in Chapter 2 is intended to help designers create a multiplayer tabletop mathematics learning game in a way that fosters productive disciplinary engagement, situates mathematics within the pretended reality of a game, conveys progress toward goals to the player(s), and positions the game so that players have interest in playing.

The postgame debrief as an essential teacher-led discussion

The set of implementation principles was proposed to help teachers make effective use of multiplayer tabletop games as mathematics learning activities in the classroom. The guidance positions the teacher as a facilitator of productive disciplinary engagement in mathematics and of meaningful mathematical discourse before, during, and after the game. The implementation principles are influenced by, and deliberately invite comparison to, Stein et al.'s (2008) five practices for orchestrating productive mathematics discussions.

Special attention to the postgame debrief discussion is warranted. Even if game designers have done a spectacular job of situating mathematical activity into the game's structure and students are productively engaged in mathematics throughout the gameplay, the debrief discussion gives the teacher a critical opportunity to make sure that important mathematical ideas arising during the gameplay are surfaced for all students.

Discussion of the function representations card game *Curves Ahead!* (Chapter 3)

The function representations card game *Curves Ahead!* was created using the design principles proposed in Chapter 2. Further refinements and improvements were made by using playtests in a design experiment to determine which features were impacting productive disciplinary engagement and/or subjective gameplay value. The first playtest allowed members of the community of mathematics educators to provide input for important design changes. The second playtest took feedback from a small number of calculus students to gain further insight into the potential of *Curves Ahead!* to support productive engagement with mathematical ideas and maintain player interest for the duration of the game. The results from the first two playtests allowed for an advanced prototype to be tested with an entire class of differential calculus students. The results from the third and final playtest support the design decisions that were made following the first two playtests and suggested that students perceived *Curves Ahead!* as offering them learning value.

The results were encouraging enough to suggest that the proposed design principles employed using a design experiment methodology could be effectively used to iteratively develop a multiplayer tabletop mathematics learning game that supports productive disciplinary engagement in mathematics having subjective gameplay-value for learners.

Discussion of the calculus board game *Assembly Lines* (Chapter 4)

The calculus board game *Assembly Lines* was also created using the design principles proposed in Chapter 2. This game includes the embedded mathematics tasks of graphically interpreting derivatives, antiderivatives, and the Fundamental Theorem of Calculus, and can be played cooperatively or competitively, and single-player or multiplayer. *Assembly Lines* was playtested in video-recorded gameplay sessions that followed most of the implementation principles proposed in Chapter 2 (there was not moderated debrief session following the gameplay). Using the construct of productive disciplinary engagement and its attendant evidentiary indicators suggested by Engle and Conant (2002), the videos were carefully analyzed for the verbal and nonverbal communication that students used during the gameplay to determine whether the students were productively engaging with mathematical ideas. Performance measures were obtained by using a pretest and posttest to determine if any learning might be attributable to the gameplay session.

While performance gains were slight, all participants demonstrated that they were engaged in mathematical reasoning and sense making for all but a few moments of the gameplay. In addition, most participants showed "intellectual progress" by becoming more independent of the reference sheet and facilitators, as well as performing more (or harder) mathematics tasks in the same amount of game time.

The results from Chapter 4 were also promising in that they suggested the design and implementation principles proposed in Chapter 2 could be effectively utilized to create and deploy a multiplayer tabletop calculus game. Performance gains from a pretest to a posttest were modest, but the study also deliberately chose not to include a moderated postgame debrief. It is quite possible that a teacher-led discussion following gameplay could have significantly helped students make mathematical connections that would contribute to greater performance gains. It may well be that the facilitated debriefing session is of critical importance in the successful implementation of a game-based learning activity.

The impact of the behavior of an alpha gamer may have had a particularly negative impact on his partner's performance, and possibly even other players in his gameplay session. The importance of attention to interpersonal dynamics between students working cooperatively is not new, but the phenomenon of alpha gamer behavior may warrant special consideration in the implementation of game-based learning activities. For example, it is conceivable that the behavior demonstrated by the alpha gamer might not have manifested itself so dramatically if the students were working together in a cooperative, but non-game, learning activity.

A design process for multiplayer tabletop mathematics learning games

Devlin (2011) suggests that while learning principles like those proposed by Gee (2003) for learning from video games can be helpful for design, there is no "how-to" manual for developing a successful video mathematics learning game. That said, Devlin does provide detailed descriptions of how to apply Gee's learning principles in developing a digital mathematics learning game.

The case of multiplayer tabletop mathematics learning game design is similar. It is hoped that the design principles proposed here will be found helpful to other learning game creators, but neither is there a "how-to" manual for this kind of mathematics learning game. The following description of a general design process is offered as an accompaniment to the design principles presented in Chapter 2, and to provide more depth to the process described in Chapter 3.

Game-based learning researchers suggest that designers begin with the desired learning outcomes and formulate the essential mathematical tasks with which players are to engage (e.g., Stanford, Wiburg, Chamberlin, Trujillo, & Parra, 2016; Weitze, 2014). Next, using the *embedding principle* and *rules principle*, embed those tasks within the pretended reality and form rules, respectively. Then, devise a feedback system that will convey actual or potential progress to the player (e.g., an adjudication process and a points system). Lastly, consider those features of the mathematics learning game that are intended to enhance the subjective gameplay-value.

Table 20 lists each game aspect, their related design principles, and how the design principles support productive disciplinary engagement in mathematics.

Table 20: Alignment of design principles with game aspects and productive disciplinary engagement.

Game Aspect	**Design Principle**	**Supported Principles of Productive Disciplinary Engagement**
Goals	Mathematical Fidelity Principle Cognitive Fidelity Principle	Resources Resources
Pretended Reality with Rules	Embedding Principle Rules Principle	Problematization Accountability
Feedback System	Adjudication Principle Rewards & Risks Principle Discovery & Reflection Principle	Resources, Authority, & Accountability Resources & Accountability Resources & Authority
Enhancing the Subjective Gameplay-Value	Variety Principle The Virtuous Cycle Principle Flow (or Immersion) Principle	Authority Authority Authority

Game design typically relies on an iterative process (Adams, 2010; Fullerton, 2008; Salen & Zimmerman, 2004). Early prototypes of a game can be playtested after the first two “steps” and into the beginning stages of the third (Vanden Abeele et al., 2011). That is, the *goals*, the *pretended reality with rules*, and some of the *feedback system*, are all that is necessary to begin playtesting.

Directions for future design and research: digital versus tabletop mathematics learning games

Muller (2008) argues that there is no reason to believe that any one type of multimedia is better for learning. Perhaps, each type of learning game can offer a different learning experience that supports different learning goals.

Single-player video and multiplayer tabletop games present different limitations and affordances. Much of the game-based learning design research and work to date has focused on digital learning games, and this dissertation work was presented in an effort to complement that work with attention to a game format that has special affordances for player-to-player interactions.

Adjudication in a digital game is fast, automatic, and (usually) error-free. However, adjudication in a digital game is usually passive in that players do not generally participate in the judgements. A digital game is unlikely to admit player participation during adjudication in a way that facilitates productive disciplinary engagement or meaningful mathematical discourse.

This suggests some lines for future design and research. Given similar basic game mechanics, how is the mathematical engagement of the player qualitatively different between the two game formats? For example, the function-representation-translation-via-card-matching mechanic of *Curves Ahead!* or the graph building mechanic under constraint(s) in *Assembly Lines* both lend themselves to the creation of digital game versions of those games. These new games would have the advantage of automatic adjudication and feedback, but without the player-to-player interactions. An investigation comparing the experiences of learners in both formats, where the essential game mechanics are identical, would be a natural line of inquiry.

If each of the two formats are found to provide distinct advantages, then this leads to interesting design challenges. For example, are there ways of combining the affordances offered by both formats by creating a multiplayer video game that allows significant opportunities for player-to-player interactions and discourse in ways that do not disrupt the flow of the game?

Finally, there has been little attention paid to implementation principles for single-player digital learning games. There is no real occasion for the teacher to moderate discourse during truly individual gameplay, but an implementation where students work in pairs or small cooperative groups to play a digital learning game as an “individual” might offer an opportunity for productive interactions between players. Even in the case of truly individual gameplay, would implementation of a teacher-facilitated whole-class pregame brief and postgame debrief session before and after individual gameplay yield significantly positive results in meeting the learning outcomes envisioned by the game designers?

Simply "turning students loose" to play either a digital or tabletop learning game with the expectation that any learning is simply a product of the game is likely squandering precious opportunities of the teacher to leverage the game as a focus of productive mathematical discourse. Evaluating the "(learning-)value added" by discussion before, during, and following gameplay could serve to illustrate its importance and reveal impactful implementation strategies.

Concluding remarks and a look to the future

While it is possible that different types of multimedia do not have inherent advantages in terms of learning effects (Muller, 2008), the present discussion has made a case that different learning game formats do indeed offer distinct opportunities that suggest the need for different design principles and implementation guidelines to optimize their effectiveness. It seems that both digital mathematics learning games and tabletop mathematics learning games have genuine promise as vehicles for engaging students in productive mathematical activity, and the teacher has an important role in realizing that promise.

The interest in game-based learning is following a growth curve that has yet to reach its inflection point. Game-based learning research has much to offer in advancing our collective wisdom regarding design of mathematics learning games and the effective implementation of these games in the classroom. It is hoped that the work presented here has contributed in some way to that collective wisdom.

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Appendices

Appendix A: Glossary

ability beliefs (in *expectancy-value theory*)*:* one's beliefs about one's competence in performing a task

accountability (in *productive disciplinary engagement*)*:* responsiveness to content, practices, and disciplinary norms established in a community of learners

alpha gamer: a player that seeks to control the behaviors and actions of other players, especially during cooperative gameplay

attainment value (in *expectancy-value theory*)*:* personal importance of succeeding at a task or activity

authority (in *productive disciplinary engagement*): to be able to define, address, and resolve problems for oneself

cost (in *expectancy-value theory*)*:* negative results and trade-offs from performing a task or activity

digital game: a game played on an electronic computation device, such as a console, computer, or mobile device

educational (mathematics) game: a game designed to support attainment of specified learning outcomes (in mathematics)

expectancy-value theory: a theory of achievement motivation that attempts to explain an individual's willingness and motivation to perform a task using the individual's beliefs about the potential for success and its personal value

exploit: a way to circumvent the intended routes to achieve given game goal(s)

flow: a subjective psychological state which occurs when an individual is so thoroughly focused or immersed in an activity that their sense of time and space becomes distorted, the activity is perceived as being inherently rewarding, and the individual perceives a sense of agency in that they become confident that they can handle whatever challenges may arise in the activity

game: a voluntary play activity in a pretended reality governed by rules, wherein participant(s) try to achieve one or more goals, and where degrees of success in the attainment of goals are conveyed by a feedback system *game-based learning:* a pedagogical strategy that uses gameplay as a means for attaining desired learning outcomes

game interaction: a player-player or player-game interaction (incl. *game mechanic*)

game mechanic: a structured action in the pretended reality of a game that can occur between players, between the player(s) and the game, or internally to the game

game world: the players, the pretended reality, the rules, the goals, and the feedback system of a game

grok: to thoroughly comprehend every aspect of a game, suggesting a deeper structural understanding of the game

intrinsic value (in *expectancy-value theory*)*:* personal enjoyment gained from doing a task or activity

learning game: see *educational game*

outcome expectancy (in *expectancy-value theory*)*:* one's beliefs as to the likely outcome of performing a task

playtest: a designer-observed gameplay session, along with possible follow-up interviews or surveys of the players, for the purposes of evaluating the structure of the game and/or the perceptions of the players to the game

problematize (in *productive disciplinary engagement*)*:* to define a problem that elicits one's curiosity and sense-making skills

productive disciplinary engagement: intellectual progress during or through a focused and active participation in an activity, while maintaining contact between what students are doing and the issues, practices, or discourse in a discipline

serious game: a game designed for some purpose other than purely entertainment

subjective gameplay-value: perceived value of playing a game, explained through applying *expectancy-value theory* to a *game-based learning* activity

tabletop game: a game typically played on, or requiring the use of, a flat surface

utility value (in *expectancy-value theory*)*:* usefulness of a task or activity in attaining personal goals

video game: see *digital game*

Appendix B: Gee's 36 learning principles

These principles are quoted from Gee (2003):

1. **Active, Critical Learning Principle**
   All aspects of the learning environment (including the ways in which the semiotic domain is designed and presented) are set up to encourage active and critical, not passive, learning.
2. **Design Principle**
   Learning about and coming to appreciate design and design principles is core to the learning experience.
3. **Semiotic Principle**
   Learning about and coming to appreciate interrelations within and across multiple sign systems (images, words, actions, symbols, artifacts, etc.) as a complex system is core to the learning experience.
4. **Semiotic Domain Principle**
   Learning involves mastering, at some level, semiotic domains, and being able to participate, at some level, in the affinity group or groups connected to them.
5. **Metalevel Thinking about Semiotic Domains Principle**
   Learning involves active and critical thinking about the relationships of the semiotic domain being learned to other semiotic domains.
6. **"Psychosocial Moratorium" Principle**
   Learners can take risks in a space where real-world consequences are lowered.
7. **Committed Learning Principle**
   Learners participate in an extended engagement (lots of effort and practice) as extensions of their real-world identities in relation to a virtual identity to which they feel some commitment and a virtual world that they find compelling.
8. **Identity Principle**
   Learning involves taking on and playing with identities in such a way that the learner has real choices (in developing the virtual identity) and ample opportunity to meditate on the relationship between new identities and old ones. There is a tripartite play of identities as learners relate, and reflect

on, their multiple real-world identities, a virtual identity, and a projective identity.

9. **Self-Knowledge Principle**
The virtual world is constructed in such a way that learners learn not only about the domain but about themselves and their current and potential capacities.

10. **Amplification of Input Principle**
For a little input, learners get a lot of output.

11. **Achievement Principle**
For learners of all levels of skill there are intrinsic rewards from the beginning, customized to each learner's level, effort, and growing mastery and signaling the learner's ongoing achievements.

12. **Practice Principle**
Learners get lots and lots of practice in a context where the practice is not boring (i.e., in a virtual world that is compelling to learners on their own terms and where the learners experience ongoing success). They spend lots of time on task.

13. **Ongoing Learning Principle**
The distinction between learner and master is vague, since learners, thanks to the operation of the "regime of competence" principle listed next, must, at higher and higher levels, undo their routinized master to adapt to new or changed conditions. There are cycles of new learning, automatization, undoing automatization, and new reorganized automatization.

14. **"Regime of Competence" Principle**
The learner gets ample opportunity to operate within, but at the outer edge of, his or her resources, so that at those points things are felt as challenging but not "undoable."

15. **Probing Principle**
Learning is a cycle of probing the world (doing something); reflecting in and on this action and, on this basis, forming a hypothesis; reprobing the world to test this hypothesis; and then accepting or rethinking the hypothesis.

16. **Multiple Routes Principle**
There are multiple ways to make progress or move ahead.

This allows learners to make choices, rely on their own strengths and styles of learning and problem solving, while also exploring alternative styles.

17. **Situated Meaning Principle**
The meanings of signs (words, actions, objects, artifacts, symbols, text, etc.) are situated in embodied experience. Meanings are not general or decontextualized. Whatever generality meanings come to have is discovered bottom up via embodied experiences.

18. **Text Principle**
Texts are not understood purely verbally (i.e., only in terms of the definitions of the words in the text and their text-internal relationships to each other) but are understood in terms of embodied experiences. Learners move back and forth between texts and embodied experiences. More purely verbal understanding (reading texts apart from embodied action) comes only when learners have had enough embodied experience in the domain and ample experiences with similar texts.

19. **Intertextual Principle**
The learner understands texts as a family ("genre") of related texts and understands any one such text in relation to others in the family, but only after having achieved embodied understandings of some texts. Understanding a group of texts as a family (genre) of texts is a large part of what helps the learner make sense of such texts.

20. **Multimodal Principle**
Meaning and knowledge are built up through various modalities (images, texts, symbols, interactions, abstract design, sound, etc.), not just words.

21. **"Material Intelligence" Principle**
Thinking, problem solving, and knowledge are "stored" in material objects and the environment. This frees learners to engage their minds with other things while combining the results of their own thinking with the knowledge stored in material objects and the environment to achieve yet more powerful effects.

22. **Intuitive Knowledge Principle**
Intuitive or tacit knowledge built up in repeated practice and experience, often in association with an affinity group, counts a great deal and is honored. Not just verbal and conscious knowledge is rewarded.

23. **Subset Principle**
Learning even at its start takes place in a (simplified) subset of the real domain.

24. **Incremental Principle**
Learning situations are ordered in the early stages so that earlier cases lead to generalizations that are fruitful for later cases. When learners face more complex cases later, the learning space (the number and type of guesses the learner can make) is constrained by the sorts of fruitful patterns or generalizations the learner has found earlier.

25. **Concentrated Sample Principle**
The learner sees, especially early on, many more instances of fundamental signs and actions then would be the case in a less controlled sample. Fundamental signs and actions are concentrated in the early stages so that learners get to practice them often and learn them well.

26. **Bottom-up Basic Skills Principle**
Basic skills are not learned in isolation or out of context; rather, what counts as a basic skill is discovered bottom up by engaging in more and more of the game/domain or game/domains like it. Basic skills are genre elements of a given type of game/domain.

27. **Explicit Information On-Demand and Just-in-Time Principle**
The learner is given explicit information both on-demand and just-in-time, when the learner needs it or just at the point where the information can best be understood and used in practice.

28. **Discovery Principle**
Overt telling is kept to a well-thought-out minimum, allowing ample opportunity for the learner to experiment and make discoveries.

29. **<u>Transfer Principle</u>**
Learners are given ample opportunity to practice, and support for, transferring what they have learned earlier to later problems, including problems that require adapting and transforming that earlier learning.

30. **<u>Cultural Models about the World Principle</u>**
Learning is set up in such a way that learners come to think consciously and reflectively about some of their cultural models regarding the world, without denigration of their identities, abilities, or social affiliations, and juxtapose them to new models that may conflict with or otherwise relate to them in various ways.

31. **<u>Cultural Models about Learning Principle</u>**
Learning is set up in such a way that learners come to think consciously and reflectively about their cultural models of learning and themselves as learners, without denigration of their identities, abilities, or social affiliations, and juxtapose them to new models of learning and themselves as learners.

32. **<u>Cultural Models about Semiotic Domains Principle</u>**
Learning is set up in such a way that learners come to think consciously and reflectively about their cultural models about a particular semiotic domain they are learning, without denigration of their identities, abilities, or social affiliations, and juxtapose them to new models about this domain.

33. **<u>Distributed Principle</u>**
Meaning/knowledge is distributed across the learner, objects, tools, symbols, technologies, and the environment.

34. **<u>Dispersed Principle</u>**
Meaning/knowledge is dispersed in the sense that the learner shares it with others outside the domain/game, some of whom the learner may rarely or never see face-to-face.

35. **<u>Affinity Group Principle</u>**
Learners constitute an “affinity group,” that is, a group that is bonded primarily through shared endeavors, goals, and practices and not shared race, gender, nation, ethnicity, or culture.

36. **<u>Insider Principle</u>**
The learner is an "insider," "teacher," and "producer" (not just a "consumer") able to customize the learning experience and domain/game from the beginning and throughout the experience.

(p. 207-212)

Appendix C: A summary of the design and implementation principles for multiplayer tabletop mathematics learning games

Design principles for multiplayer tabletop mathematics learning games:

- **Mathematical Fidelity Principle:** A mathematics learning game should be faithful to the mathematics, being free of errors, ambiguities, and sloppiness.

- **Cognitive Fidelity Principle:** A mathematics learning game should be faithful to the mathematics as perceived by a player.

- **Embedding Principle:** Each mathematical task in a mathematics learning game should be embedded in a way that elicits the formulation of a mathematical problem statement by the player, without the need to overtly indicate the task to the player.

- **Rules Principle:** The rules of a mathematics learning game should be simple, clearly stated, consistent, and perceived as fair by the players.

- **Adjudication Principle:** A mathematics learning game should provide error-free, simple, and fair judgment of player actions.

- **Reward System Principle:** Every mathematical task in a mathematics learning game should have a reward associated with its successful performance and a minimal cost associated with its unsuccessful performance.

- **Discovery & Reflection Principle:** Feedback provided by a mathematics learning game should stimulate discovery and reflection, and it should not be provided through overt telling.

- **Variety Principle:** A mathematics learning game should provide many opportunities for its players to learn through engagement with important mathematical ideas that contribute to the attainment of the intended learning outcomes.

- **The Virtuous Cycle Principle:** A mathematics learning game should give a player meaningful control to make consequential choices that brings their creativity to bear. Success should yield more (or different kinds of) meaningful control.

- **Flow Principle:** A mathematics learning game should immerse each player in a flow experience that sustains the player's engagement in game-based mathematical activities throughout the duration of the game.

Implementation principles for multiplayer tabletop mathematics learning games:

- **Timing Principle:** Teachers should use a math learning game at a time appropriate to student development and curricular goals.

- **Planning Principle:** Teachers should plan the implementation of a math learning game in terms of what the learners will need in order to successfully play the game and attain the learning outcomes.

- **Briefing Principle:** Teachers should prepare students for a math learning game by establishing behavioral norms and explaining the game and its relevance.

- **Managing Gameplay Principle:** Teachers should monitor a math learning game activity and its player interactions. The teacher should clarify rules and assist with adjudication as needed and facilitate the mathematical discourse when asked for help.

- **Debrief Principle:** The teacher should follow a math learning game activity with a moderated debriefing session to help students make connections between the game and the learning outcomes.

Appendix D: Questionnaire for third playtest of *Curves Ahead!*

Note that the formatting here is slightly different from the original.

Please answer each question to the best of your ability. To improve the quality of educational games, your honest opinion is of utmost importance. This is anonymous and will not be viewed until after the term is completed.

Circle the answer that fits your experience the best.

1. How difficult was it to learn how to play the game (independent of the mathematics)?

   Too easy   Just right   Too hard

2. How difficult was the math?

   Too easy   Just right   Too hard

3. Did the experience feel like a game or more like a dressed-up math activity?

   Felt like a game   Felt like a dressed-up math activity   Unsure

4. Do you feel like you learned mathematics as a result of the experience?

   Not at all   A little   Some   A lot   Unsure

5. Do you feel that the experience strengthened existing mathematical understanding?

   Not at all   A little   Some   A lot   Unsure

6. To learn the mathematics presented in the game, would you rather play this game, work on a typical worksheet activity, attend lecture, or do something else entirely?

   This was best   Typical activity is best

   Lecture is best   Other (what?)

7. How has your confidence regarding the mathematical material changed?

   Lower confidence   No change   Higher confidence

8. Ideally, how many times should this game be played in a term?

   0   1   2   3   4 or more

9. Ideally, where would you rather play this game?

   In class   Outside of class   Both   Unsure

10. How likely would you be to request that we play this game in class again?

    Not at all likely   Somewhat likely   Highly likely   Unsure

11. How likely would you be to recommend that others play this game in future classes?

    Not at all likely   Somewhat likely   Highly likely   Unsure

12. How did you feel about the game overall?

    It was no fun at all.   It was a little fun.   It was fun.

    It was so much fun that I'd play this game in my free time with my friends.

    It was only fun when compared to my typical experiences in a math class.

    Other (please specify):

13. What could make the experience feel more like a game, and/or make it more fun?

Appendix E: Background questionnaire for gameplay sessions of *Assembly Lines*

Participant Survey:
Participant Information:
Instructions: Please answer the following questions to the best of your ability.

1. What is your name?
2. What is your gender?
3. What is your age?
4. Have you ever taken the Calculus AP® exam (not for practice)?
    a. If so, in what year?
    b. Was the exam AB or BC?
5. Have you ever learned about anti-derivatives, integrals, and the Fundamental Theorem of Calculus before your math class this term?
    a. If so, how long has it been since you did this kind of mathematics?
6. How often do you play games? (number of times per week and/or duration)

Game Goal Orientation

Instructions: The following statements represent types of goals that you may or may not have when playing games. Circle a number to indicate how true each statement is of you. All of your responses will be kept anonymous and confidential. There are no right or wrong responses, so please be open and honest.

1. To beat the game
2. To win on a challenging difficulty level
3. To overcome many challenges
4. Avoid being defeated by the game
5. Avoid losing on a challenging difficulty level
6. Avoid failing challenges
7. To play better than I have in the past
8. To play well relative to how I have in the past
9. To play better than I typically do
10. Avoid playing worse than I normally do
11. Avoid playing poorly compared to my typical performance
12. Avoid playing worse than I have in the past
13. To outperform other players
14. To play well compared to other players
15. To do better than other players
16. Avoid underperforming relative to other players
17. Avoid playing poorly compared to other players
18. Avoid doing worse than other players

Goal Orientation for the Math Class

<u>Instructions:</u> The following statements represent <u>types of goals</u> that you may or may not have for your math class. Circle a number to indicate how true each statement is of you. All of your responses will be kept anonymous and confidential. There are no right or wrong responses, so <u>please be open and honest</u>.

1. To get a lot of questions right on the exams in my math class.
2. To know the right answers to the questions on the exams in my math class.
3. To answer a lot of questions correctly on the exams in my math class.
4. To avoid incorrect answers on the exams in my math class.
5. To avoid getting a lot of questions wrong on the exams in my math class.
6. To avoid missing a lot of questions on the exams in my math class.
7. To perform better on the exams in my math class than I have done in the past on these types of exams.
8. To do well on the exams in my math class relative to how well I have done in the past on such exams.
9. To do better on the exams in my math class than I typically do in this type of situation.
10. To avoid doing worse on the exams in my math class than I normally do on these types of exams.
11. To avoid performing poorly on the exams in my math class compared to my typical level of performance.
12. To avoid doing worse on the exams in my math class than I have done on prior exams of this type.
13. To outperform other students on the exams in my math class.
14. To do well compared to others in my math class on the exams.
15. To do better than my classmates on the exams in my math class.
16. To avoid doing worse than other students on the exams in my math class.
17. To avoid doing poorly in comparison to others on the exams in my math class.
18. To avoid performing poorly relative to my fellow students on the exams in my math class.

Note that all goal orientation items included the following scale for responses:

1	2	3	4	5	6	7
not true of me		slightly true of me	moderately true of me	very true of me		extremely true of me

Appendix F: AP® Exam Questions and Grading Standards

AP® CALCULUS AB
2004 SCORING GUIDELINES

Question 5

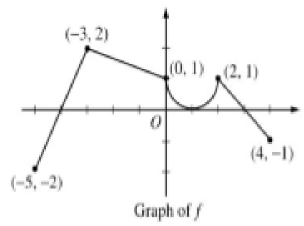
The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.



Graph of f

(a) $g(0) = \int_{-3}^{0} f(t)\,dt = \frac{1}{2}(3)(2+1) = \frac{9}{2}$ $g'(0) = f(0) = 1$	$2 : \begin{cases} 1 : g(0) \\ 1 : g'(0) \end{cases}$
(b) g has a relative maximum at $x = 3$. This is the only x-value where $g' = f$ changes from positive to negative.	$2 : \begin{cases} 1 : x = 3 \\ 1 : \text{justification} \end{cases}$
(c) The only x-value where f changes from negative to positive is $x = -4$. The other candidates for the location of the absolute minimum value are the endpoints. $g(-5) = 0$ $g(-4) = \int_{-3}^{-4} f(t)\,dt = -1$ $g(4) = \frac{9}{2} + \left(2 - \frac{\pi}{2}\right) = \frac{13-\pi}{2}$ So the absolute minimum value of g is -1.	$3 : \begin{cases} 1 : \text{identifies } x = -4 \text{ as a candidate} \\ 1 : g(-4) = -1 \\ 1 : \text{justification and answer} \end{cases}$
(d) $x = -3, 1, 2$	2 : correct values ⟨−1⟩ each missing or extra value

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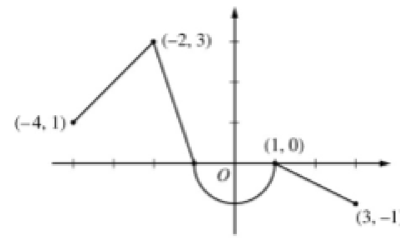
Figure 10: 2004 AP® exam question 5 and its grading standards.

AP® CALCULUS AB
2012 SCORING GUIDELINES

Question 3

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.



Graph of f

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

(a) $g(2) = \int_1^2 f(t)\,dt = -\frac{1}{2}(1)\left(\frac{1}{2}\right) = -\frac{1}{4}$ $g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^{1} f(t)\,dt$ $= -\left(\frac{3}{2} - \frac{\pi}{2}\right) = \frac{\pi}{2} - \frac{3}{2}$	$2: \begin{cases} 1: g(2) \\ 1: g(-2) \end{cases}$
(b) $g'(x) = f(x) \Rightarrow g'(-3) = f(-3) = 2$ $g''(x) = f'(x) \Rightarrow g''(-3) = f'(-3) = 1$	$2: \begin{cases} 1: g'(-3) \\ 1: g''(-3) \end{cases}$
(c) The graph of g has a horizontal tangent line where $g'(x) = f(x) = 0$. This occurs at $x = -1$ and $x = 1$. $g'(x)$ changes sign from positive to negative at $x = -1$. Therefore, g has a relative maximum at $x = -1$. $g'(x)$ does not change sign at $x = 1$. Therefore, g has neither a relative maximum nor a relative minimum at $x = 1$.	$3: \begin{cases} 1: \text{considers } g'(x) = 0 \\ 1: x = -1 \text{ and } x = 1 \\ 1: \text{answers with justifications} \end{cases}$
(d) The graph of g has a point of inflection at each of $x = -2$, $x = 0$, and $x = 1$ because $g''(x) = f'(x)$ changes sign at each of these values.	$2: \begin{cases} 1: \text{answer} \\ 1: \text{explanation} \end{cases}$

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Figure 11: 2012 AP® exam question 3 and its grading standards.

Appendix G: *Assembly Lines* Reference Sheet

You are graphing f, which is the derivative of F. That is, $F' = f.$

Which gives a useful relationship: $F(b) - F(a) = \int_a^b f(t)\,dt$

A useful shortcut: This game has been specially designed so that the **net area** between f and the horizontal axis **under a single line segment** is the **sum of the y-values at the endpoints of that line segment**. Caution: For more than one segment, do this shortcut for *each* line segment, then add the results.

This game has also been designed to allow the use of geometry to calculate areas.

The numbers *on* the line segments are the **change in y** from the last played y-value, over a run of 2. (ALWAYS a run of 2.)

$F(c)$ is a local minimum if $F'(c) = 0$ and F' is negative nearby and to the left of c, and F' is positive nearby and to the right of c (i.e., $F'(c^-) < 0$ and $F'(c^+) > 0$).

$F(c)$ is a local maximum if $F'(c) = 0$ and F' is positive nearby and to the left of c, and F' is negative nearby and to the right of c (i.e., $F'(c^-) > 0$ and $F'(c^+) < 0$).

$F(c)$ is an inflection point if F'' nearby and to the left of c has the **opposite sign** of F'' nearby and to the right of c (i.e., $F''(c^-)$ and $F''(c^+)$ have opposite sign).

For $F''(c)$ to exist, it must be that F' is smooth (no corners) near c. Or, for this game, we can say, $F''(c^-) = F''(c^+)$