AN ABSTRACT OF THE DISSERTATION OF

Prashant Tiwari for the degree of Doctor of Philosophy in Industrial Engineering presented on May 16, 2022.

Title: Analysis and Comparison of Fixed-Size Lot and Fixed-Time Lot Batch Production Systems

Abstract approved: ____

David S. Kim

In high volume automated discrete item batch production systems, the batches or lots are typically fixed quantity (i.e., size), with setups incurred between the different production lots. Processing times for the fixed-size lots are relatively constant when the workstation is operating however, random workstation disruptions cause variability in the lot completion time making the operations of interrelated activities such as material handling and setup crews less efficient. In this dissertation, the long-run average cost performance of fixed-size lot batch production systems is compared to batch production systems where the lot size is defined as a fixed time. In the "fixed-time" lot batch production system, there is ideally no variability in the time length to produce a lot, but the production output in this fixed time length may vary. In this comparison, the batch production systems considered are workstations operating under a continuous review (Q, r) inventory system. The comparison was conducted assuming unmet demands are

lost (lost-sales policy), and also when unmet demand can be backordered (backordering policy).

One objective of this research was to identify the factors having the largest effect on the long-run average cost differences between fixed-size lot and fixed-time lot systems. Because of the system complexity due to the inclusion of multiple real-world factors, a designed experiment is employed to compare the fixed-sized and fixed-time lot systems using discrete event simulation. For every treatment combination tested the batch sizes and reorder point levels (quantities or time) were optimized, so that differences between systems cannot be attributed to poor batch size and re-order point selection. The experimental results show that for the lost sales policy the factors: interarrival time between demands, and the coefficient of variation of the demand probability distribution have the largest impact on the long-run average cost difference between a fixed-size lot and fixed-time lot batch production systems. For the backordering policy the factors: workstation stand-alone availability, failure and repair frequency, and capacity utilization have the largest impacts

Another research objective was to identify functional relationships between the input factors and the output. A feedforward backpropagation neural network with the connection weight approach was applied to the experimental results database to search for relationships between various input factors and the categorical outcomes 1) a fixed-size lot production system has significantly lower cost performance than a fixed-time lot system, 2) a fixed-time lot production system has significantly lower cost performance than a fixed-size lot system, and 3) the cost performance of two systems is not significantly different. The results show that for the lost sales policy the factors: demand coefficient of variation, and stand-alone availability, have the largest relative importance in predicting the outcomes. For the backordering policy the factors: demand coefficient of variation, stand-alone availability, and inventory holding cost have the largest relative importance in predicting the outcomes. In general, at higher stand-alone availability levels and lower demand coefficient of variation the production time to produce a fixed-size lot is low enough that the system can operate in a "just-in-time" manner and a fixed-size lot production system will result in lower costs than a fixed-time lot system. However, as the stand-alone availability reduces and demand coefficient of variation increases, the fixed-time lot system results in significantly lower costs than the fixed-size lot system. The insights developed from this research can be utilized by the decision makers to select which batch production system should be utilized such that the long-run average cost can be minimized.

Analysis and Comparison of Fixed-Size Lot and Fixed-Time Lot Batch Production Systems

by
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1 Introduction

An automated workstation can represent a single machine, or a set of machines (e.g., sheet metal stamping line) that transforms incoming units of material into a product. The performance of the workstation is affected by random disruptions such as machine breakdown, maintenance issues, and tool wear. The performance measures of the workstation can include – time to produce a job, variability in the time to produce a job, the number of jobs produced over a fixed time period, and variability in the number of jobs produced. These performance measures affect the ability to meet a customer's service level requirements. The nonfulfillment of a service level requirement may lead to a penalty from the customer, or the customer switching their supplier. Current performance level requirements of manufacturing enterprises reflected in phrases such as "world-class manufacturing", "lean and sustainable manufacturing", and "Just-In-Time production" requires manufacturers to adopt a production control system to meet the customer's desired service level in the most cost efficient manner.

The decision on the adoption of different production control systems can depend on the time horizon under consideration, and how long the effects of the decision will last. *Strategic decisions* focus on long term decisions that involve some degree of uncertainty. For example, what will the customer demand trajectory look like in the next five years or how many new products can be launched in the next few years? *Tactical decisions* are medium term focusing on allocating resources to meet the strategic goals. *Operational decisions* are short term decision that focus on effective utilization of resources to meet short term goals such as daily/weekly fulfilment of customer orders. This research provides insights to decision makers to assist them in adopting a production control system such that the customer's desired service level is met in the most cost efficient manner.

In many high volume automated discrete item batch production systems, such as automated sheet metal stamping facilities, the production of products occur in a fixed-size lots, with workstation setup required after completion of a lot. A fixed-sized lot is defined by the number of discrete units of a particular product to produce. Due to their dynamic and stochastic nature, how the workstation is managed has a significant impact on long-run cost performance. For example, if workstation disruptions occur at random time points, and last for a random time lengths then it is impossible to predict the production completion time of a fixed-size lot. The inability to predict the lot completion time also affects the planning and management of several interrelated activities such as production planning and scheduling, setup crew scheduling, and scheduling the delivery of final product to the customer.

In this research a "fixed-time lot" batch production system is introduced. In this production system the lot is defined by a fixed time T rather than a fixed number of items. The motivation for fixed-time lot production control is that the planning and management of interrelated activities will be less affected by workstation disruptions, and should lead to more efficient overall operations. With fixed-time lot size production control system, the workstation may still experience random failures and random repairs. However, once the fixed time length (T) has elapsed the workstation stops producing, and the jobs produced up to time T are treated as the production quantity for the lot time T. With this production control system, there is ideally no variability in the time length to produce a lot, the lot completion time is always known in advance. However, there is a variability in the uptime i.e., it is impossible to predict the exact uptime in each lot of fixed time length. This is exactly the opposite of fixed-lot production where the batch quantity is fixed, but the total time to produce the lot varies.

In the research literature there are a large number of studies conducted to examine the performance of fixed-size lot production systems operating under continuous review inventory policy. The primary focus of prior research has been optimizing the lot size and reorder level such that the total cost of the system is minimized. With respect to fixed-time lot production system, Kletter (1996) is the only prior research discovered that compared fixed-size lot and fixed-time lot batch production system. Kletter (1996) derived the density function of production output for a fixed-time lot. They also derived the density function, expected time length, and variance of time to produce a fixed-size lot. These results were used to examine and compare the performance of several production and inventory policies. However, they did not include the production system costs as a measure to compare the two production systems, and the results they presented may not be generalizable and may be applicable only to the specific manufacturing system studied. Therefore, a comparison of the two batch production systems in a more general manner is conducted with a goal of that the results can be used by production managers and decision makers.

For the unreliable production environments considered, *tactical or operational decisions* need to be made on the adoption of a production system such that the customer's desired service level is met in the most cost efficient manner. The goal of this dissertation is to investigate and understand the contribution of several input factors that affect the cost performance of a production system, and provide deeper understanding and useful insights that can be used by the decision makers to decide when either of the batch production systems should be utilized.

1.1 Research Approach

In this study the performance of fixed-size lot and fixed-time lot batch production systems is compared such that from a customer's perspective both production control systems perform identically (Figure 1.1). In Chapter 2, an automated unreliable workstation is considered, and discrete event simulation is used to compare fixed-lot and fixed-time production control systems through a factorial experiment. The system parameters that define the factorial space included 1) *stand-alone availability*, 2) *failure and repair frequency*, 3) *service level*, 4) *capacity utilization*, 5) *demand interarrival time*, 6) *coefficient of variation of demand*, 7) *ordering cost*, 8) *inventory holding cost*, and 9) *backordering cost*. System parameters and system related quantities such as demand size, demand variability, production lot size, reorder level, on-hand inventory, are converted into the equivalent time units of production expressed in processing minutes or processing hours. The inventory holding cost and backordering cost on per job basis were scaled such that they are charged on per processing minute basis. This conversion expands the inference space of the results obtained since after conversions of system parameters to processing minutes, many more production systems fall within the factorial space examined.

The workstation examined adopts a continuous review (Q, r) inventory system operating under either a lost sales, or backordering policy. Simulation was used to estimate production system performance, and an optimization search procedure was used to optimize the lot sizes and reorder levels of the fixed-size lot and fixed-time lot batch production systems each time any system parameter was changed. The optimization search procedure used was a hybrid of greedy search and random walk algorithms. This hybrid algorithm is designed to reduce the chance that a solution found is a local minima.

The Python programming language and discrete event simulation library SimPy is used to develop the simulation model. The expectation and variance formulas of the time length to produce a fixed-size lot, and the uptime in a fixed-time lot provided by Kim and Alden (1997), Keltter (1996) were utilized to validate simulation models. After conducting 1,536 different experiments with both fixed-lot and fixed time systems, ANOVA was used to identify the factors and interactions that have the largest effect on the long run average cost difference between the two batch production control systems.

In Chapter 3, a feedforward backpropagation neural network is utilized to discover functional relationship between inputs factors and predefined categorical outputs by analyzing the experimental database and results obtained from Chapter 2. Neural networks process multiple inputs in parallel and capture the causal relationships between input factors and outputs. The Python programming language is used to develop the feedforward backpropagation neural network with connection weight approach. This approach is found to be one of the most effective approaches in determining the relative importance of the input factors to predict the outcomes (Olden et al., 2004).

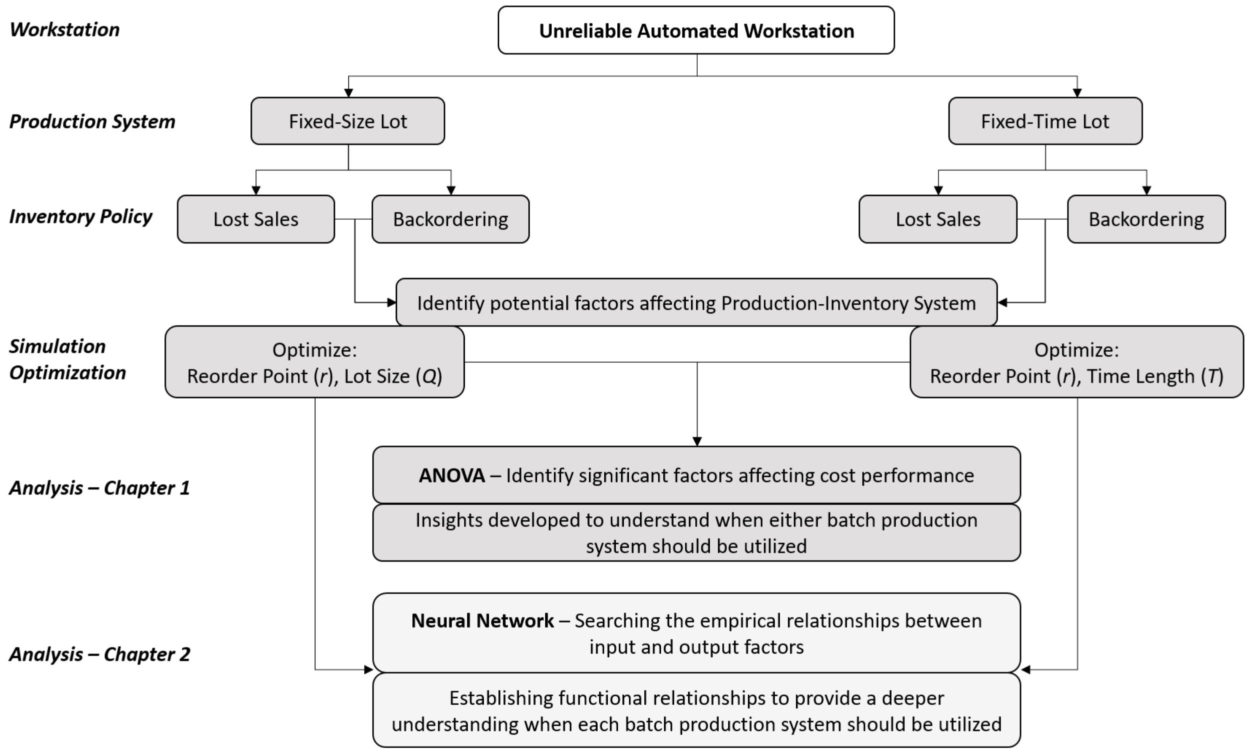

Figure 1.1 Overview of Research Approach

1.2 Research Contribution

This research compares the performance of fixed-size lot and fixed-time lot production control systems and provide general insights for lost sales and backordering policies by 1) analyzing the long run average cost difference of the two production systems and identifying the input factors and their interactions that are significant, and 2) establishing strong predictive functional relationships between the input factors and outputs.

The experimental results show that for a lost sales policy the *interarrival time between demands*, and the *coefficient of variation* of the demand probability distribution have the largest impact on the long-run average cost difference between fixed-size lot and fixed-time lot batch production control systems. For a given demand interarrival length, a higher *demand variability*, and lower workstation *stand-alone availability* creates more variable production, which favors a fixed-time lot batch production system. For the batch production systems operating under a backordering policy the experimental results show that the workstation *stand-alone availability*, *failure and repair frequency*, and *capacity utilization* have the largest impact on the long-run average cost difference between fixed-size lot and fixed-time lot batch production systems. If the workstation operates under low *capacity utilization* (workstation is underutilized) and a high *stand-alone availability* the fixed lot batch production system had lower cost. However, the fixed-time lot batch production system was preferred when the workstation operates with high *capacity utilization* and lower *stand-alone availability*.

The feedforward backpropagation neural network with connection weight approach results show that for a lost sales policy the *coefficient of variation* of the demand probability distribution and the *stand-alone availability* of the workstation have the highest relative importance in predicting the three categorical outcomes, 1) a fixed-size lot system has lower average cost than a fixed-time lot system, 2) a fixed-time lot system has lower average cost than a fixed-size lot system, and 3) the cost performance of the two systems is not different. For the batch production systems operating under the backordering policy the feedforward backpropagation neural network results show that the demand *coefficient of variation*, *stand-alone availability* and *inventory holding cost* have the highest relative importance in predicting the outcomes.

In a real production environment, the general insights from lost sales and backordering policy results, and the short and medium term changes in *demand variability*, workstation *stand-alone availability* can be utilized for making *tactical or operational decisions*. For example, the higher demand *CV* drives larger optimized *Q* values for both fixed-size lot and fixed-time lot systems. However, higher Q values in a fixed lot system combined with lower workstation *stand-alone availability* leads to higher variability in the time to complete the fixed lot. It was discovered that this fact causes many lot completion times to be late or to occur after a demand arrival. To protect against the variability and meet the customer's desired service level, the fixed lot system has to maintain a relatively higher on-hand inventory level, which makes the average inventory holding cost significantly higher for fixed-size lot system than fixed-time lot system. At a high stand-alone availability such as 98%, the variability in the time length to produce a fixed lot is low enough such that the probability of completing a fixed-size lot before the next demand arrival is close to 1. However, in the fixed-time lot production system the variability in the uptime in a fixed time length drives higher reorder levels leading to a higher average inventory than fixed-size lot system.

The decision makers can utilize the insights and decide the most appropriate production system to meet the customer's desired service level with minimum long-run average cost. For example, the fixed-size lot batch production system should be preferred if the workstation operates under lost sales policy, receives weekly demand of 4500 *processing minutes* with standard deviation of 225 *processing minutes*, 95% workstation stand-alone availability and it expects to meet 95% customer's demand.

1.3 Dissertation Organization

This dissertation is divided into two chapters. In Chapter 2 the performance of fixed-size lot and fixed-time lot production systems is compared by analyzing the factors that affect the long-run average cost difference of the two production systems, and Chapter 3 identifies the relationships between various input factors and output by implementing feedforward backpropagation neural networks by utilizing the experimental database obtained from Chapter 2. The common theme that connects the two chapters is the identification of inputs factors that affect the performance of the two batch production systems.

Chapter 2 begins by introducing the fixed-size lot and fixed-time lot batch production systems. The literature review of fixed-sized lot and fixed-time lot systems is presented in Section 2.2. The workstation, its operating assumptions, and various production-inventory cycles observed in the two batch production systems are described in Section 2.3. The simulation optimization model is presented in Section 2.4. The experimental design and results are discussed in Section 2.5. The result discussion, and conclusions with future work are presented in Section 2.6 and 2.7 respectively.

Chapter 3 begins by briefly introducing the fixed-size lot and fixed-time lot batch production systems. A brief literature review of fixed-sized lot and fixed-time lot systems is presented in Section 3.2. The workstation, and the operating assumptions are described in Section 3.3. The data utilized in this chapter is presented in Section 3.4. The feedforward backpropagation neural network and the connection weight approach is presented in Section 3.5. The results from the neural network are presented in section 3.6. Finally the conclusions and future work are presented in Section 3.7.

2 Performance Comparison of Fixed-Sized Lot and Fixed-Time Lot Batch Production Systems

Abstract:

In high volume automated discrete batch production systems, the batches or lots are typically fixed quantity (i.e., size), with costs/setup incurred between the different production lots. Processing times for the fixed-size lots are relatively constant when the workstation is operating however, random workstation disruptions cause variability in the lot completion time making the operations of interrelated activities such as material handling and setup crews less efficient. In this research, the long-run average cost performance of fixed-size lot batch production systems is compared to batch production systems where the lot size is defined as a fixed time. In the "fixed-time" lot batch production system, there is ideally no variability in the time length to produce a lot, but the production output in this fixed time length may vary. In this comparison, the batch production systems considered are workstations operating under a continuous review (Q, r) inventory system. A designed experiment is employed to compare the fixed-sized and fixed-time lots using discrete event simulation. The objective is to identify the factors having the largest effect on the long-run average cost differences. For every treatment combination tested the batch sizes and reorder point levels (quantities or time) were optimized. The experimental results show that for the lost sales policy the factors interarrival time between demands, and the coefficient of variation of the demand probability distribution, and for the backordering policy the factors workstation stand-alone availability, failure, and repair frequency, and capacity utilization have the largest impact on the long-run average cost difference between a fixed-size lot and fixed-time lot batch production systems. The insights from the lost sales and backordering experimentation can be utilized in an

actual production environment to decide the most appropriate batch production system that can minimize long-run average costs.

Keywords: Productivity and competitiveness, Batch production systems, Fixed-size lot, Fixed-time lot, Experimental design, Simulation optimization

2.1 Introduction

In high volume automated discrete item batch production systems, such as automotive sheet metal stamping, workstation production occurs in fixed-size lots with workstation setups required between lots. A fixed-sized lot is defined by the number of discrete units of a particular product to produce, and a "workstation" can represent a single machine, or a set of machines (e.g., sheet metal stamping line). If the workstation is perfectly reliable and there are no disruptions due to machine breakdown, maintenance issues, and tool wear, etc., the time to produce a fixed-size lot is predictable. However, most often workstation disruptions are likely to occur. These disruptions occur at random time points and last for random time lengths, which causes the time length to produce a fixed-size lot to be variable. This variability may lead to an excessive delay in the production of a lot, which may further delay the production of all the fixed-size lots that follow. Such variability also makes the planning and management of production and interrelated activities more complex, and overall reduces operational efficiency. Examples of such activities are production scheduling, setup crew scheduling, and scheduling the delivery of final product to the customers.

In this research fixed-size lot discrete item batch production systems are compared to systems where the lot size is defined by a fixed time T rather than a fixed number of items. The production of fixed time length batches will be referred to as "fixed-time lot" production. The initial motivation for fixed-time lot production is that the planning and management of interrelated activities will be less affected by the workstation disruptions, and lead to more efficient overall operations. However, as will be seen even without consideration of interrelated activities, the fixed-time lot production approach stands on its own when compared to a fixed-lot production approach with regards to long run average cost.

During a fixed-time lot size of *T* the workstation may experience random failures and random repairs. However, once the fixed time length has elapsed the workstation stops producing, and the units produced up to time *T* are treated as the production quantity for the batch. With this strategy, there is ideally no variability in the time length to produce a lot, but instead variability in the production output. This is exactly the opposite of fixed-lot production where the batch quantity is fixed, but the total time to produce the lot varies.

The comparison of the long-run average cost of utilizing fixed-size lot and fixed-time lot batch production systems was completed so that from the customer perspective the systems perform identically. This was accomplished by enforcing the same service level or backorder constraints in both cases. Both systems were examined under similar scenarios (i.e., experimental treatment combinations) where various combinations of workstation reliability (and thus production system variability), service levels, and customer demand variability were examined. Each system operated under a continuous review *(Q, r)* inventory policy, and for each specific scenario examined both batch sizes and reorder point levels (in units or time) were separately optimized for each system. It is worth noting that fixed-size lot and fixed-time lot production are equivalent if the workstation is perfectly reliable.

The remainder of this paper is organized as follows. Section 2.2 presents a literature review. In Section 2.3 the workstation and operating assumptions are described. Next the simulation optimization model is presented in Section 2.4. Section 2.5 discusses the experimental design and results. Finally a results discussion is presented in Section 2.6, and conclusions and future work are presented in Section 2.7.

2.2 Literature Review

The thesis of Kletter (1996) is the only prior research discovered that directly or indirectly compares the cost performance of fixed-size lot and fixed-time lot batch production systems. Kletter (1996) derived the density function of production output over a fixed time period (fixed-time lot). Different density functions were obtained for the cases when the workstation is initially in a working, or failed state, or at steady state. Using Laplace transforms, a workstation's expected uptime and variance over a fixed time period, and the expected time length and variance of time to produce a fixed-size lot were derived. The performance of several production and inventory policies, including fixed-size lot and fixed-time lot production on a single multi-product workstation requiring setups was examined and compared. However, cost was not used in the comparison. For fixed-time lot production an order-up-to policy was used, and order-up-to levels were adjusted to achieve the same average inventory level as the fixed-size lot system. Relatively lower service levels with fixed-time lot production was observed.

The fixed-lot batch production system operating under a continuous review inventory policy has been studied extensively with much of the focus on optimizing the lot size and reorder point such that the ordering, inventory holding, and backordering costs are minimized. Results have been obtained under various assumptions such as fixed and random demand (Abboud, 2001; Hadley & Whitin, 1963; Johansen & Thorstenson, 1993; Song et al., 2010), deterministic and random lead times (Federgruen & Zheng, 1992; Mohebbi, 2003; Mohebbi & Posner, 2002), unreliable suppliers (D. Gupta, 1996; Mohebbi, 2003; Parlar, 1997), and service level constraints (Fattahi et al., 2015; Moon & Choi, 1994; Sarkar et al., 2015; Tajbakhsh, 2010).

The fixed-lot and fixed-time systems considered in this research utilize workstations that experience random failures and repairs. As a result, the time to produce a fixed lot size, and the number of units produced in a fixed time length are random variables. In addition to Kletter (1996) other prior research has examined these random variables. Kim and Alden (1997) derived the density function, and variance of the time to produce a fixed-size lot. They consider a single workstation with deterministic processing time and exponentially distributed time between failures and exponentially distributed time to repair. Gershwin (1993) obtains a formula for the production output variance over a fixed time period for an unreliable workstation with deterministic processing times. Carrascosa (1995) extended Gershwin's work and obtained the output variance for a single unreliable machine with two failure modes during a time interval.

Other studies that are not directly related to the current research analyzed and calculated the output variability of the production lines with unreliable workstations. These studies considered a single unreliable workstation (Bariş Tan, 1999), *N*-station production line with no buffer (Bariş Tan, 1997), production lines with workstations in series and parallel (Bariş Tan, 1998), and buffered production lines (He et al., 2007; Li & Meerkov, 2000; Barış Tan, 2000).

2.3 System Description

The system considered is a single automated unreliable workstation that produces one product with a fixed processing time *s* (time/job) when the workstation is up. This workstation experiences random operation-time dependent failures (i.e., it fails only when it is operating). The time between failures and times to repair are assumed to be exponentially distributed, which has been shown to be a reasonable assumption (Inman, 1999). During repair, part processing is pre-empted, and the part remains in the workstation. After repair completion, part processing resumes to complete the remaining processing. Workstation production is dictated by a continuous review (*Q, r)* inventory policy, where *Q* is the reorder quantity, and *r* is the reorder level. A comparison between *Q* as a fixed quantity (fixed-lot size), and *Q* as fixed elapsed time (fixed-time lot) will be conducted.

A *Q, r* inventory policy is frequently implemented with either a backordering or a lost sales policy, or a mix of both. With a backordering policy, if a demand cannot be met with on-hand inventory, the excess demand is backordered, and a time-based penalty is incurred on the backordered demand. With a lost sales policy, unmet demand is lost when the on-hand inventory level drops to zero and a penalty is incurred for each lost order instance. The demand is lost in all cases, even if the completion of the in-process lot is imminent. In a production system operating under a *Q, r* inventory policy the unreliable workstation alternates between three states: up (operating), down (repair), and idle (up and not operating) Figure 2.1 illustrates the production systems examined. Figure 2.2 and Figure 2.3 present examples of backordering and lost sales respectively.

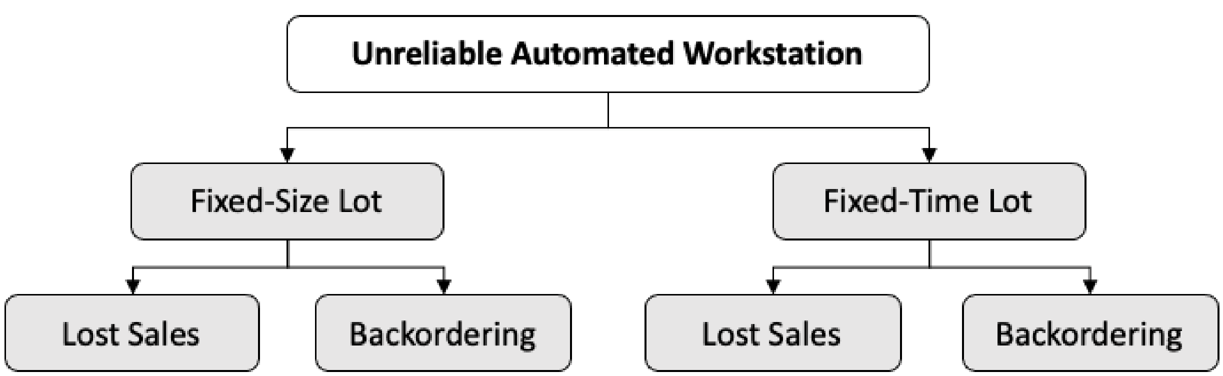

Figure 2.1 Production systems under study

To make the comparison of different workstation and production systems more straightforward, all parameters such as demand sizes, lot size *Q*, reorder-level *r*, on-hand inventory, and backordered inventory are expressed in their equivalent time units of production – processing minutes (p.mins). For example, if the workstation's speed is 60 jobs per hour, then each job is worth 1 processing minute (time/job). To obtain the normalized value (p.mins) of an inventory parameter, the inventory value normally expressed as jobs/units is multiplied by the processing time of the workstation (time/job).

Table 2.1 presents the normalized values of two workstations whose parameters as typically specified imply differences that are not present after normalization.

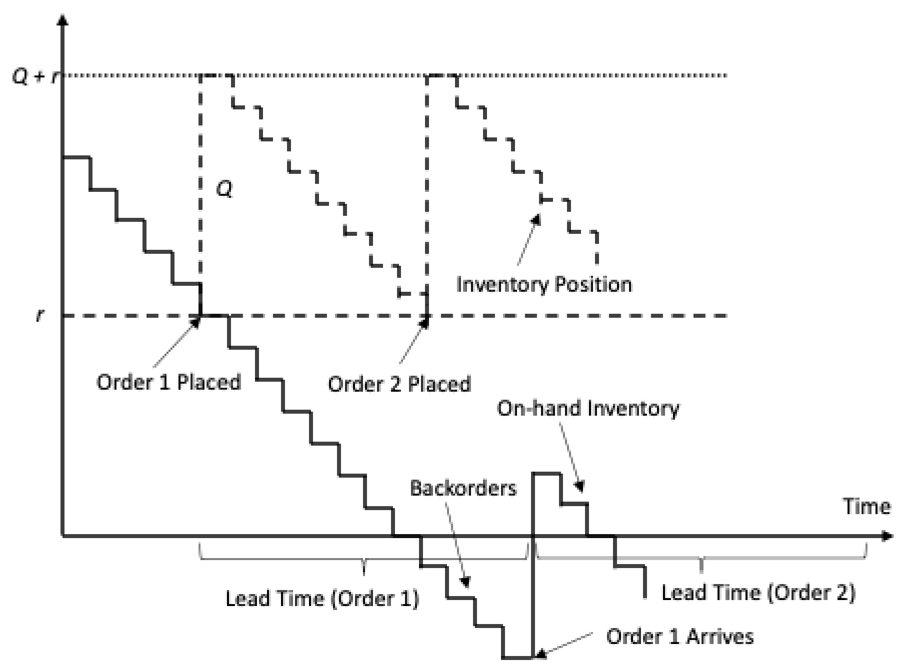

Figure 2.2 Backordering System

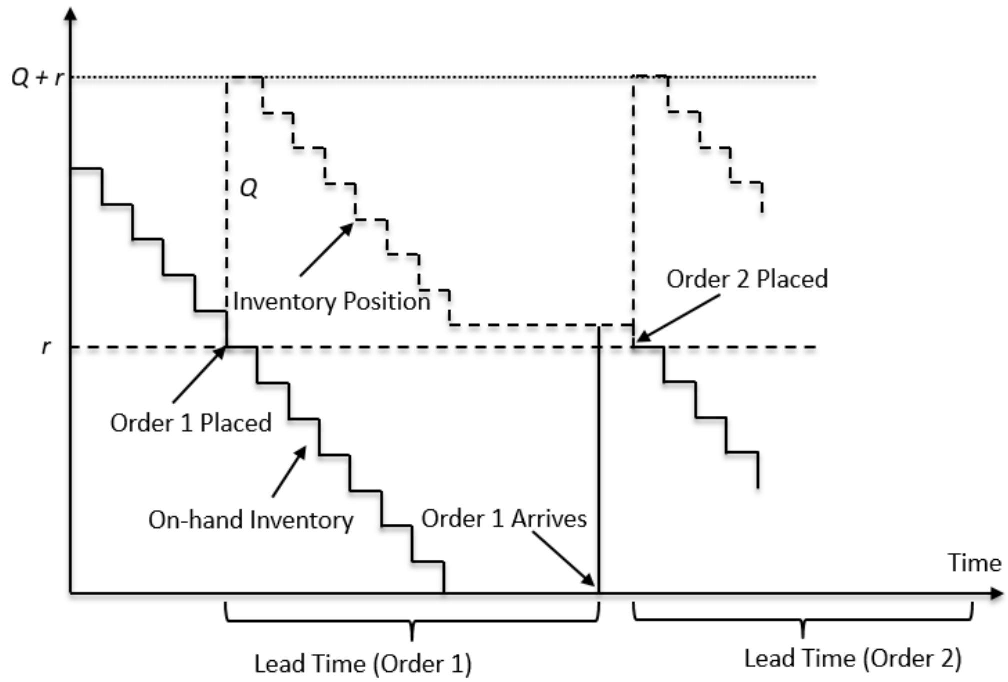

Figure 2.3 Lost Sales System

Table 2.1 Normalization of production system parameters

Parameter	**Workstation 1**	**Equivalent Time Units of Production**	**Workstation 2 Kletter (1996)**	**Equivalent Time Units of Production**
Workstation speed	60 JPH	1 min./job*	495 JPH	0.1212 min./job*
Demand per week	4509 jobs	4509 min.	37200 jobs	4509 min.
Demand standard deviation	901 jobs	901 min.	7440 jobs	901 min.
Lot size	2254 jobs	2254 min.	18600 jobs	2254 min.
Reorder level	1127 jobs	1127 min.	9300 jobs	1127 min.

**Processing rate of the workstation*

2.3.1 Workstation with Fixed-Size Lot Production

While producing, the workstation experiences random failures followed by random repair times (Figure 2.4). If no failures occur during lot production, the minimum time to produce a fixed-size lot Q is realized and equals the processing time s (time/job) multiplied by Q. However, if the workstation experiences failures, the time to produce Q is greater than the minimum time. Kim and Alden (1997) derived the expected time length (Equation 1) and variance of the time length (Equation 2), as well as the density function for the time to produce a fixed-size lot. Equation 2.1, Equation 2.2 are used to validate the fixed-size lot simulation model.

$$E(T_n) = \frac{\lambda n}{S\mu} + nS$$

Equation 2.1 Expected time to produce a fixed-size lot

$$Var(T_n) = \frac{2n\lambda}{S\mu^2}$$

Equation 2.2 Variance of time length to produce a fixed-size lot

where, T_n – time length to produce a fixed-size lot of n jobs; λ - failure rate; μ - repair rate; S - workstation speed (jobs/hour).

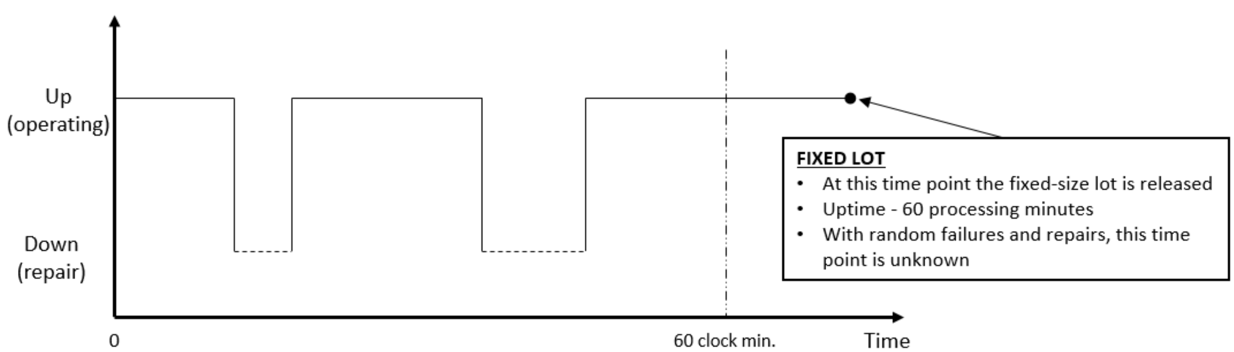

Figure 2.4 Up and down profile of a fixed-size lot

Figure 2.5 illustrates the on-hand inventory path of a workstation that experiences random failures during the production of fixed-size lot Q. The workstation starts production at t_0 and experiences a failure at time t_1. At this time the workstation transitions to a down state and stops production. It stays in the down state until the end of a repair with length $t_2 - t_1$. At t_2 the workstation transitions to an up-state and resumes production. Another failure occurs at t_3 and the workstation transitions to a down state. At time t_4, a random demand of size D arrives. Since inventory level $(IL) < D$, the excess demand is either lost or backordered. At t_5, the repair is completed, and the workstation resumes production until the fixed-size lot Q is produced. At t_6, Q processing minutes are added to the current IL. At this time the IL is checked and a decision is made to either reorder and produce another fixed-size lot, or transition to an idle state and wait until the IL drops to or below r. With random failures and random times to repair, the time length to produce Q is variable.

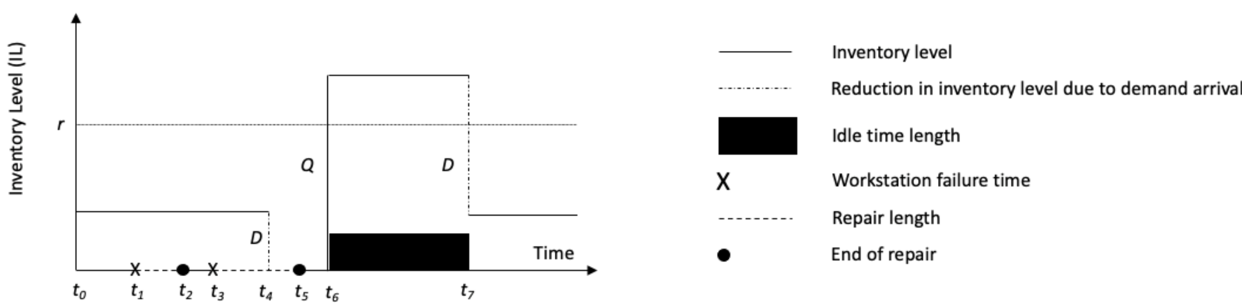

Figure 2.5 Fixed-Size Lot Q, r Policy with Failures and Repairs

2.3.2 Workstation with Fixed-Time Lot Production

In this production system the lot size is defined by a fixed elapsed time T rather than fixed size-lot Q or fixed amount of production time (Figure 2.6). If no failure is observed the production quantity in units (P) in fixed time T is T multiplied by the workstation production rate. However, if the workstation experiences random failures and random times to repair, the production in a fixed time length T will be in the interval [0, P]. Kletter (1996) derived expressions for the mean uptime and variance of uptime of a single unreliable workstation if it is initially up (Equation 2.3, Equation 2.4), or down (Equations 5-6). Equations 3-6 are used to validate the fixed-time lot simulation model.

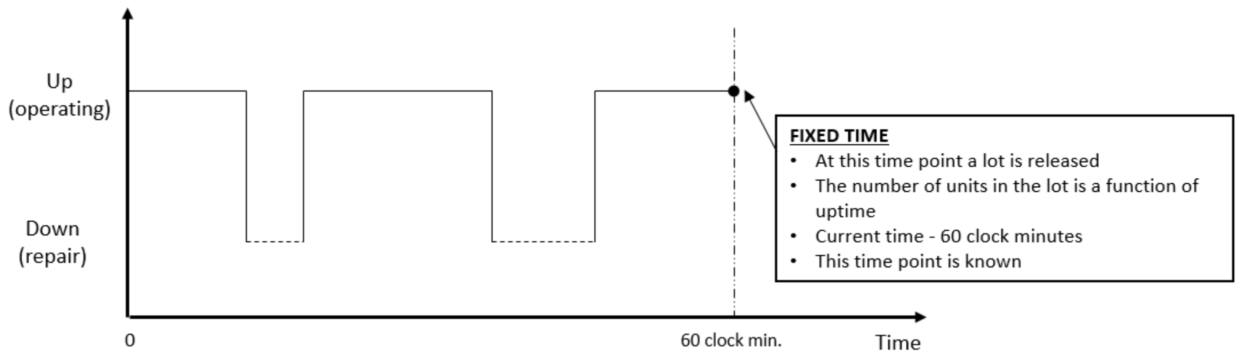

Figure 2.6 Up and down profile of a fixed-time lot

$$E(Uptime_{up}) = \left(\frac{\mu}{\lambda+\mu}\right)T + \frac{\lambda}{(\lambda+\mu)^2}\left(1 - e^{-(\lambda+\mu)T}\right)$$

Equation 2.3 Expected uptime of a single unreliable workstation if initially in 'up' state

$$Var\left(Uptime_{up}\right) = \frac{\lambda^2}{(\lambda+\mu)^4}\left(1 - e^{-2(\lambda+\mu)T}\right) - \frac{4\lambda\mu}{(\lambda+\mu)^4}\left(1 - e^{-(\lambda+\mu)T}\right)$$
$$+ \frac{2\lambda\mu}{(\lambda+\mu)^3}T\left(1 + e^{-(\lambda+\mu)T}\right) - \frac{2\lambda^2}{(\lambda+\mu)^3}Te^{-(\lambda+\mu)T}$$

Equation 2.4 Variance in uptime of a single unreliable workstation if initially in 'up' state

$$E(Uptime_{down}) = \left(\frac{\mu}{\lambda+\mu}\right)T - \frac{\mu}{(\lambda+\mu)^2}\left(1 - e^{-(\lambda+\mu)T}\right)$$

Equation 2.5 Expected uptime of a single unreliable workstation if initially in 'down' state

$$Var(Uptime_{down}) = \frac{\mu^2}{(\lambda+\mu)^4}\left(1 - e^{-2(\lambda+\mu)T}\right) - \frac{4\lambda\mu}{(\lambda+\mu)^4}\left(1 - e^{-(\lambda+\mu)T}\right)$$

$$+\frac{2\lambda\mu}{(\lambda+\mu)^3}T-\frac{2\mu(\lambda-\mu)}{(\lambda+\mu)^3}Te^{-(\lambda+\mu)T}$$

Equation 2.6 Variance in uptime of a single unreliable workstation if initially in 'down' state

Similar to fixed-size lot production, a workstation producing fixed-time lots may also experience several production-inventory cycles. Figure 2.7 illustrates an example of fixed-time lot production system with failures and repairs. In a fixed-time length system $T = T_1 = T_2 = T_3$, and the number of units produced is a function of uptime in T. At the start time t_0 of a production lot, the end time is known.

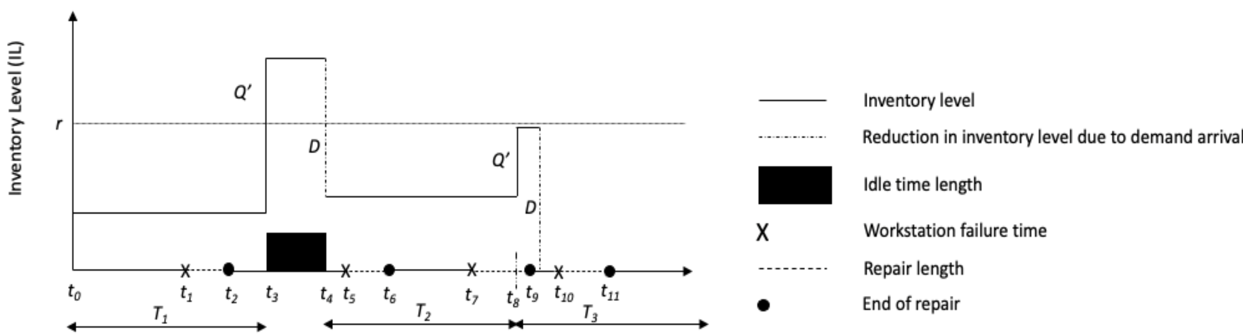

Figure 2.7 Fixed-Time Lot Q, r Policy with Failures and Repairs

During fixed-time length T_1 the workstation experiences a failure at t_1 with a repair length of $t_2 - t_1$. At t_3 the units produced (Q') are released and the on-hand inventory level is updated. The probabilities that a workstation is up (Equation 2.7) or down (Equation 2.8) at the end of time T_1 (t_3), given it is working at t_0 are provided by Ross (2014).

$$P_{up-up}(T) = \frac{\mu}{\lambda+\mu}+\frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)T}$$

Equation 2.7 Probability a workstation is up at the end of time T

$$P_{up-down}(T) = \frac{\lambda}{\lambda + \mu}\left(1 - e^{-(\lambda+\mu)T}\right)$$

Equation 2.8 Probability a workstation is down at time T

Assuming the workstation remains up at t_3, since $IL > r$, the workstation transitions to an idle state and stays idle until IL is less than r. A random demand of size D arrives at t_4. After the demand arrival, IL goes below r and production restarts at t_4. The fixed time length available for production in this cycle is T_2. The units produced through time t_8 (Q') are released. If at the end of T_2 the workstation is in a down state and the repair time length extends into the next production time lot, the repair time that extends beyond T_2 is subtracted from the fixed-time length T_3. Therefore, the maximum available time for production in next period is $T_3 - (t_9 - t_8)$. Figure 2.8 illustrates a cycle where the random repair length ($t_{10} - t_6$) consumes the entire fixed time length T_3. In such cases, there will be no production in the fixed time length.

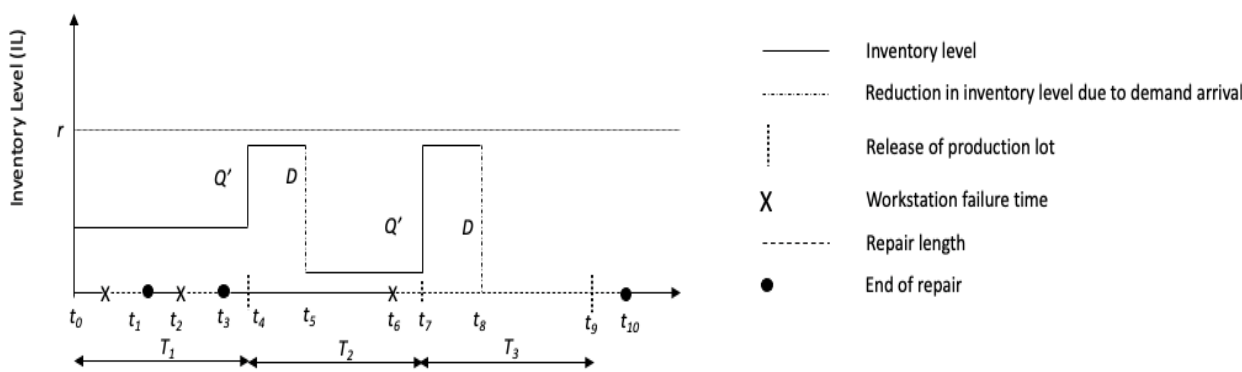

Figure 2.8 Fixed-Time Lot with a Repair Eliminating a Fixed-time Lot

2.4 Simulation Model

The simulation model is divided into three processes (Demand, Inventory, and Production) as shown in Figure 2.9. Table 2.2 shows the input parameters for this simulation model. In the first process, a demand quantity (D) following a specified probability distribution (a lognormal distribution rounded to the nearest integer was utilized) arrives at fixed time intervals (weekly or daily). In the second process, the incoming demand is compared against the on-hand inventory level (*IL*). If $D < IL$, then either the excess demand is backordered, or it is lost. Next, if the $IL \leq r$, an order is placed to produce either a fixed-size lot *Q* or a fixed-time lot *T*. In the third process, the workstation produces either the fixed lot *Q*, or produces for the fixed time *T*. During production the workstation may experience random failures and random repairs. After production of *Q* or *T* the on-hand inventory level, and backorders if any, are updated. When backordering is allowed, the number of outstanding backorders can be greater than one. Outstanding backorders are processed on a first-come-first-serve basis. In a lost sales system, there are no outstanding orders and demand that is not met is assumed to be lost. At the end of the simulation run length, the long-run average cost and service level for the given set of parameters for both fixed-size lot and fixed-time lot production are recorded.

For each treatment combination in the designed experiment, this simulation model uses optimized values for *Q or T* and *r.* These parameter values are determined from the optimization algorithm described in Appendix A, and are set to minimize long-run cost while meeting specified service level constraints.

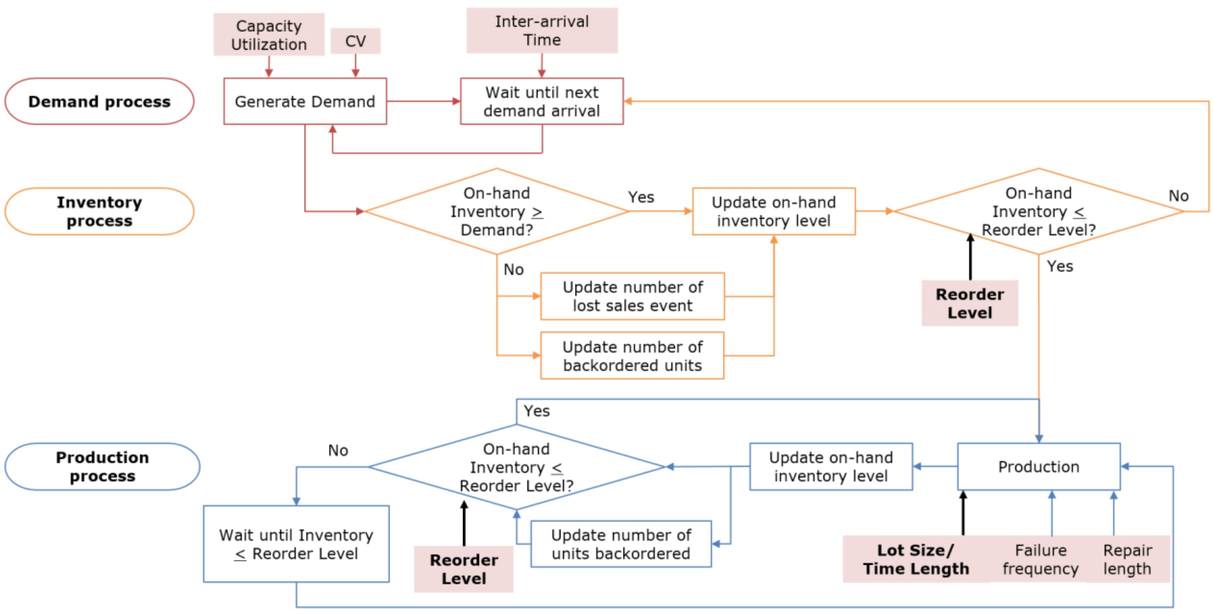

Figure 2.9 Demand, Inventory, and Production Sub-processes

Table 2.2 Simulation Parameters

Sub Process	Input Parameter
Production	Processing rate (minutes/job)
	MTBF (hours)
	MTTR (hours)
	Reorder level (processing minutes) *
	Fixed-size lot (processing minutes) *
	Fixed-time lot (hours)*
Demand	Demand inter-arrival time (hours)
	Capacity utilization
	CV of demand size

* Value from optimization algorithm

2.4.1 Simulation Model Validation

The performance of the simulation models was compared to analytical results where possible. For a fixed-size lot system the analytical results expected time length $E(T_n)$ and variability in the time length $Var(T_n)$ to produce a fixed-size lot (Equation 2.1, Equation 2.2) are compared to simulation model results. As shown in Table 2.3, there are no statistically significant differences between the analytical results and simulation output. For a fixed-time lot system analytical results for the expected uptime *E(U)* and variability in uptime *Var(U)* of a fixed-time lot (Equation 2.3 - Equation 2.6) are compared to simulation model results. In Table 2.4, there are no statistically significant differences between the analytical results and simulation output.

Table 2.3 Validation of fixed-size lot production simulation model

$$Workstation\ speed = 60\ JPH, MTBF = 10\ hrs., MTTR = 2.5\ hrs.$$

Q	**Analytical Results** $E(T_n)$	**Simulation Output** $E(T_n)$	**Absolute Difference**	**Analytical Results** $Var(T_n)$	**Simulation Output** $Var(T_n)$	**Absolute Difference**
2000	41.67	41.66	0.01	41.67	41.79	0.12
3000	62.50	62.52	0.02	62.50	62.87	0.37
4000	83.33	83.33	0.00	83.33	83.89	0.56
5000	104.17	104.14	0.03	104.17	104.53	0.36

Table 2.4 Validation of fixed-time lot production simulation model

$$Workstation\ speed = 60\ JPH, MTBF = 10\ hrs., MTTR = 2.5\ hrs.$$

T **(hrs.)**	**Analytical Results** ***E(U)***	**Simulation Output** ***E(U)***	**Absolute Difference**	**Analytical Results** ***Var(U)***	**Simulation Output** ***Var(U)***	**Absolute Difference**
50	40.40	40.42	0.02	29.60	29.48	0.12
75	60.40	60.41	0.01	45.60	45.70	0.10
100	80.40	80.40	0.00	61.60	61.32	0.28
125	100.40	100.43	0.03	77.60	77.09	0.51

2.5 Experimental Design and Simulation Result Analysis

A designed experiment utilizing the simulation model was to examine the differences in long-run average cost between fixed-size lot and fixed-time lot production systems. When conducting empirical experimental-based research, the inference space for which any results are applicable is an important question. The factors considered in this research, which are known to affect the long-run average cost of the high-volume automated batch production systems (Table 2.5) were carefully selected so the inference space will include many real systems. The factor values utilized were determined from a literature review and discussion with production managers[1], and are intended to be representative of many scenarios that occur in actual high-volume batch production systems. Preliminary experiments were conducted with three levels of each factor to investigate the presence of curvature (non-linearity) in the response. Since no curvature was observed a two-level full factorial design (Table 2.6) was utilized.

Table 2.5 Factors and levels in the experiments

Factor	Level 1		Level 2	
Stand-alone availability	0.80		0.90	
Failure/Repair frequency	Frequent		Infrequent	
Mean time between failures (hours)	10	10	20	20
Mean time to repair (hours)	$1.1\overline{1}$	2.5	$2.2\overline{2}$	5
Service level	0.95		0.98	
Capacity utilization	0.70		0.78	
Demand inter-arrival time (hours)	24 (Daily)		120 (Weekly)	
CV of demand (120 hours)*	0.015		0.1	
CV of demand (24 hours)*	0.03		0.2236	
Ordering cost ($)	500		1000	
Inventory holding cost ($/processing minutes/month)	1		2	
Backordering cost** ($/processing minutes/month)	0.5		2.5	

** Nested in Demand inter-arrival,*

[1] S. Jain, B. Kumar (personal communication, June 2020)

***Factor used only in backordering system*
Month = 480 hours

Table 2.6 Total treatment combinations for each production system

System	***Q, r* Policy**	**Treatment combinations**
Fixed-size lot	Lost sales	$2^8 = 256$
	Backordering	$2^9 = 512$
Fixed-time lot	Lost sales	$2^8 = 256$
	Backordering	$2^9 = 512$

For each treatment combination, 25 replications were conducted. At the beginning of each replication, the inventory level is zero and the workstation is in an up (operating) state. A warm-up period of 6,240 simulated hours (1 year) is run to eliminate any start-up effects, and then statistics collection for 624,000 simulated hours (100 years) is conducted. At the end of 25 replications the long-run average cost and average service level are computed.

2.5.1 Generality of the Factorial Space

The rationale for selecting each factor and their levels is presented next.

Stand-alone availability (SAA): The SAA is the expected fraction of time the machine will be in a working state when not idle. For the stand-alone availability (SAA), the expectation is to have as high a workstation SAA as possible. For manufacturing operations the standard for achieving world-class performance includes a workstation availability of at least 90% (Ahuja & Khamba, 2008). Therefore one SAA level considered is 90%. For the lower SAA level, any SAA lower than 80% is judged to be uncompetitive. At very high SAA levels (> *95%*), the two batch production control systems tend towards very similar performance (they are the same at 100% SAA), so these higher SAA levels are not considered. Moreover, a 10% difference in the two levels of SAA is a relatively large difference that should show any factor effects if they exist. From a realistic standpoint, a SAA below 80% was judged to be unreasonable. Table 2.7 lists prior production related research where workstation SAA were specified, and the values assumed.

Table 2.7 Prior research involving the workstation availability

Workstation Stand-Alone Availability (%)	**Reference**
80	Kletter (1996),
	Gupta and Garg (2012),
88	Panagiotis (2018),
	Nurprihatin et.al. (2019)
90	Ahuja and Khamba (2008),
	Groenevelt et. al. (1992)
91	Mendez and Rodriguez (2017)

Failure/Repair frequency: This factor is nested in the factor SAA. In a nested design the levels of this factor are not identical for different levels of SAA. Prior research (Kim & Alden, 1997; Patti & Watson, 2010) has shown that for the same stand-alone availability, different combinations of downtime frequency (i.e., frequent failures with shorter mean repair times and infrequent failures with longer mean repair times) impact the production system differently. Using the density function for the time distribution to produce *n* jobs (Kim and Alden (1997), and the density function for the distribution of uptime in a fixed-time *T* (Kletter, 1996), the impact of different *MTBF* and *MTTR* combinations with the same SAA is shown in Figure 2.10 and Figure 2.11. Figure 2.10 shows how different *MTBF* and *MTTR* combinations change the production time distribution required to complete 4500 *processing minutes*. Increases in the *MTBF* and *MTTR*, lead to increased production time variance. Figure 2.11 shows uptime (processing minutes) distributions for a fixed time length of 4500 *minutes* for different *MTBF* and *MTTR* combinations with the same SAA. The SAA is a function of the mean time between failure (*MTBF*) and the mean time to repair (*MTTR*), and various combinations of *MTBF* and *MTTR* can be used to obtain the same SAA. In this research frequent failures with shorter repair lengths and infrequent failures with longer repair lengths are considered (Table 2.8).

Table 2.8 Factor levels Failure/Repair Frequency

Failure/Repair	**Frequent failures**		**Infrequent failures**	
MTBF (hours)	10	10	20	20
MTTR (hours)	1.11	2.5	2.22	5

In many high-volume production systems such as automated stamping plants, the number of unplanned stoppages occur regularly. Since the extremes may not be representative of a real systems and since the question revolves around factor effects, *MTBF* values of 10 hours (*frequent failure*) and 20 hours (*infrequent failures*) were selected. A 10 hours *MTBF* is 1 failure per shift on average, and a 20 hour *MTBF* is 1 failure per every three shifts on average. Kim and Alden (1997) show that variability of time length to produce a fixed-size lot increases proportionally to the square of mean repair length. The levels selected should be sufficiently far apart to show any effects from this factor.

<u>*Service level*</u>: The service level is the fraction of demand met with on-hand inventory, also known as the fill rate. The service levels utilized are 0.95 and 0.98. Customers in general desire a high service level, and at higher service levels (> 0.95) a slight service level increase can significantly affect the inventory, ordering, and backordering costs (Bartezzaghi et al., 1999). Therefore the two

service levels utilized, 0.95 and 0.98 are high levels and should show the factor effects if they exist.

Capacity utilization: This is the ratio of average demand and average production in a given period. Based on the levels of SAA used in a design, the levels of capacity utilization (C) can be computed as,

$$C = \frac{Avg.\ demand\ \left(\frac{\text{P.}\,minutes}{day}\right)}{Max.\ production\ \left(\frac{\text{P.}\,minutes}{day}\right) * SAA}$$

The *capacity utilization* is inversely affected by the levels of *SAA*, i.e., for constant average demand and average production over a time period, high *SAA* will yield a lower *capacity utilization*, whereas low *SAA* will yield a higher *capacity utilization*. The question of interest is the impact of capacity utilization at realistic utilization values. Given the multiple sources of variability from both the demand and production sides a higher utilization value in the neighborhood of 0.80 was selected. Similar utilizations were assumed in Groenevelt (1992), Kletter (1996), Taj et. al. (2012).

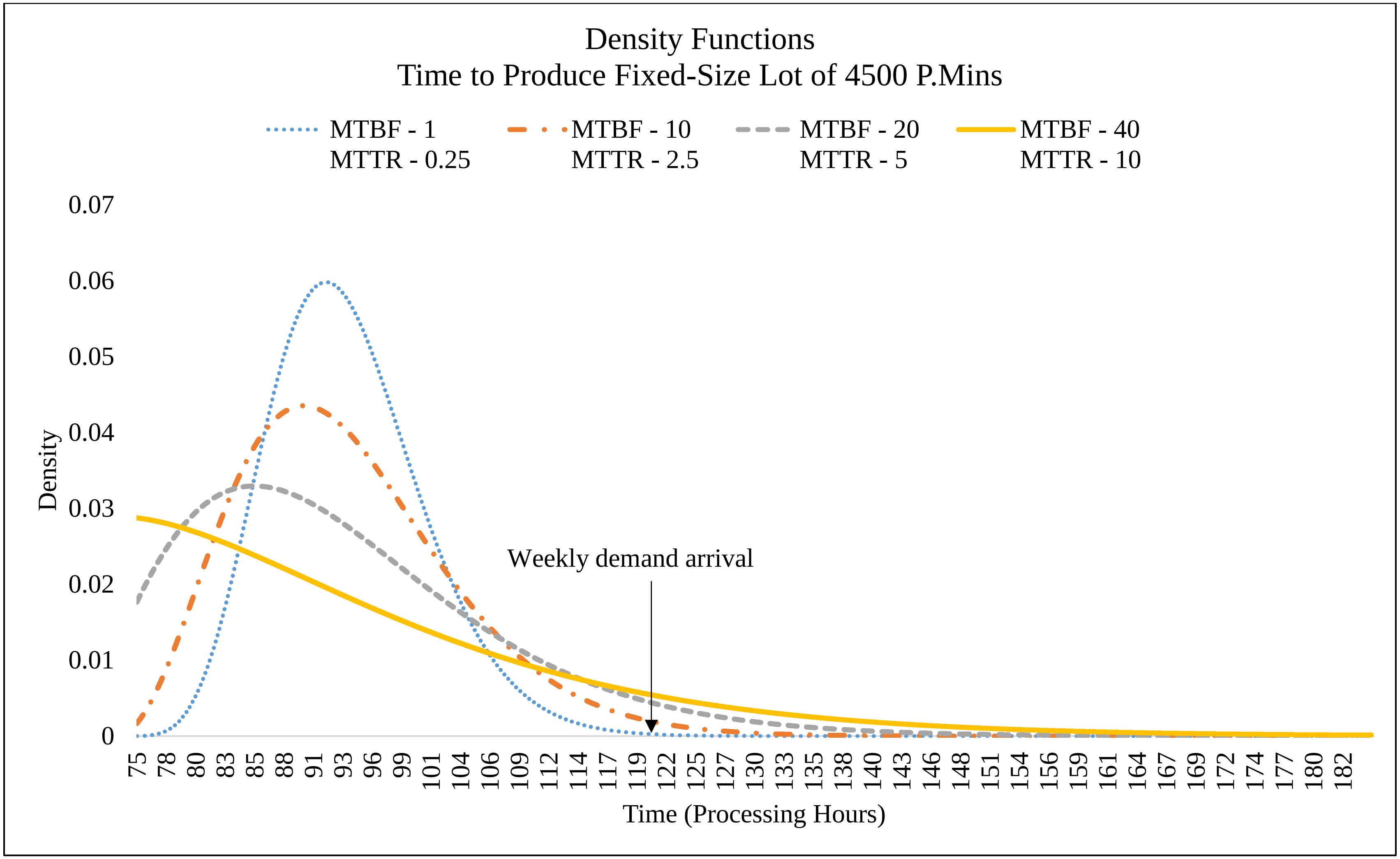

Figure 2.10 Density functions for time to produce job for four combinations of MTBF and MTTR with equal stand-alone availability

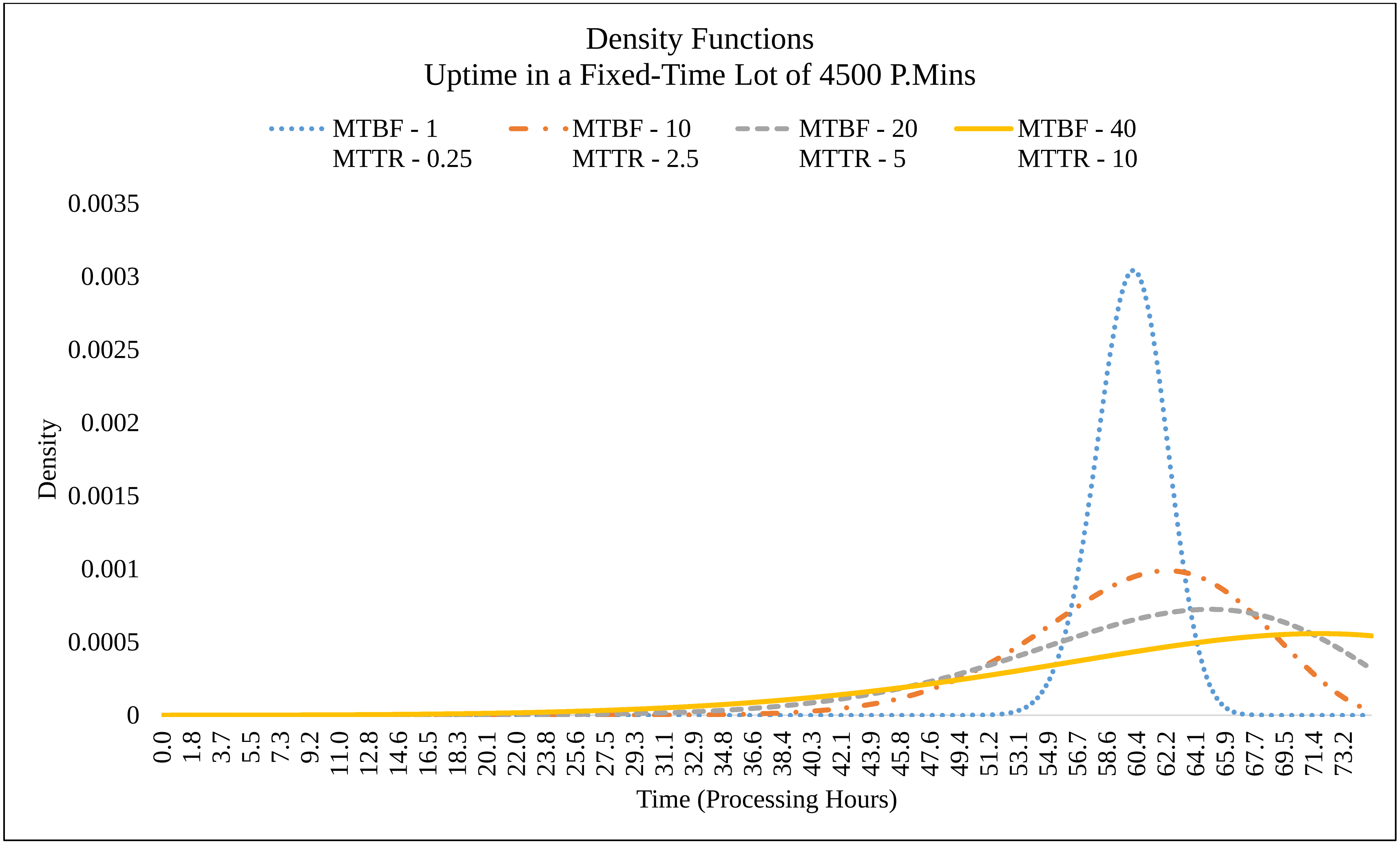

Figure 2.11 Density functions of uptime to produce jobs for four combinations of MTBF and MTTR with equal stand-alone availability

Demand inter-arrival time: Commonly seen in many high-volume batch production system, the inter-arrival times between demands is either daily or weekly. Therefore, demand inter-arrival time of 24 hours (daily) and 120 hours (weekly) are utilized (Table 2.9). The probability distribution for demand sizes for both inter-arrival periods is assumed to be lognormal (rounded to the nearest integer).

Table 2.9 Prior research with various demand inter-arrivals

Demand inter-arrival	**Reference**
Daily	Kalchschmidt (2003)**,**
	Abdulmalek and Rajgopal (2007)**,**
	Ferguson et. al. (2007)**,**
	Sabaghi et. al. (2015)
Weekly	Kletter (1996),
	Kalchschmidt (2003)**,**
	Abdulmalek and Rajgopal (2007)

CV of demand: This factor controls the variability of the demand quantities. Li et al. (2004) shows that CV of demand affects the probability of customer demand satisfaction. This factor is nested in demand inter-arrival time. The CVs at both levels are scaled such that the demand variability per week is same for different demand inter-arrival times (Appendix 2B). For low levels, CVs of 0.015, 0.03 are considered. The standard deviation of demand at the low CV level is assumed to be the square root of the mean demand. For example, if daily demand is 800, then the CV of daily demand is $\frac{\sqrt{800}}{800} \approx 0.03$. At the high level, a CV of 0.1 for weekly demand is assumed, and a CV of 0.2236 for daily demand is used. The daily CV values are scaled such that the demand variability per week remains the same as that of weekly demand. Higher CVs are not considered because for asymmetrical distribution such as the log-normal, higher CVs leads to poor inventory management performance (Tadikamalla, 1984).

Costs: Inventory holding costs and backordering costs are based on the average monthly inventory level. Past studies on inventory systems have considered inventory holding cost on ‘per item per unit time’ basis (Taj et al., 2012; Vander Veen & Jordan, 1989). In this research inventory holding costs are based on *processing minute*s *per month*. The low level for inventory holding cost is assumed to be $1/*processing minutes*/month, and the high level is assumed to be twice the low level at $2/*processing minutes*/month. After conversion to *processing minute*s *per month* the inventory holding costs in (Taj et al., 2012; Vander Veen & Jordan, 1989) fall within this range.

The ordering cost is a fixed cost that is incurred whenever an order of fixed-size lot Q or fixed-time lot T is placed. In inventory systems studied in the literature, the ordering costs considered are significantly higher than the inventory holding costs (Chiu, Wang, et al., 2007; Vander Veen & Jordan, 1989). Chiu et al. (2007) assumed the ordering cost at $450 per order, and Vander Veen & Jordan (1989) assumed it to be $1000. In this study for the low level of ordering cost was increased to $500 per order so that the high level is twice the low level. Therefore, the low level for ordering cost is assumed to be $500/order, and the high level is assumed to be $1000/order.

The backordering cost is incurred on the average excess demand backordered per month. Previous studies on inventory systems have considered backordering cost to be both smaller (Chiu, Ting, et al., 2007) and greater (Wee et al., 2007) than inventory holding cost. The low level of back-ordering cost is assumed to be smaller than the two levels of inventory holding cost at

\$0.5/*processing minutes*/month, and the high level is assumed to be greater than the two levels of inventory holding cost at \$2.5/*processing minutes*/month. Therefore to study the impact of backordering cost on the long run average cost performance, the low level of backordering cost is assumed to be smaller than the two levels of inventory holding cost, and the high level is assumed to be greater than the two levels of inventory holding cost.

2.5.2 Results Analysis

The experimental results implementing lost sales and backordering policies were analyzed separately. Analysis of Variance (ANOVA) was used to identify the factors and interactions that have the largest effect on the long run average cost difference between fixed-time lot and fixed-size lot production systems. The difference in the long run average cost (ΔTC) is computed by subtracting the long-run average cost of fixed-size lot system from the fixed-time lot system. For any treatment combination, if $\Delta TC > 0$ and is statistically significant, then the fixed-size lot is a better system based on cost. If $\Delta TC < 0$ is statistically significant then the fixed-time lot is a better system, and if ΔTC is not significantly different from zero then the systems are not different.

The experiments conducted are 2^8 and 2^9 factorial designs for lost sales and backordering policies respectively. The ANOVA conducted is based on an assumed linear statistical model for the response that includes 8 or 9 main effects, and $\binom{8}{2} - 2$ or $\binom{9}{2} - 2$ two-factor interactions. The factors *failure repair frequency* and *Coefficient of variation (CV) of demand* are nested in factors *stand-alone availability (SAA)* and *demand interarrival time* respectively (see Table 2.5).

2.5.2.1 Results and Analysis - Lost Sales Policy

The subset of the ANOVA results in Table 2.10 show the model terms that are statistically significant and contribute more than 1% to the total sum of squares. The demand process related factors, *CV of demand* and *demand inter-arrival time* have the largest influence on the average long-run cost difference between fixed-lot and fixed-time systems, and account for over 60% of the total sum of squares. The factors *service level*, *stand-alone availability*, and *failure repair frequency* account for about 6%, 5%, and 2% of the total sum of squares respectively.

Table 2.10 Analysis of Variance for the Lost Sales Experiment

Source of Variation	Degrees of Freedom	Sum of Squares	Percent Contribution	Mean Square	F-value	P-value
CV(Demand)	2	2.62E+15	42.71%	1.31E+15	39323.97	0
Demand Inter-arrival	1	1.10E+15	17.87%	1.10E+15	32914.28	0
Service level	1	3.73E+14	6.08%	3.73E+14	11188.54	0
CV(Demand)* Inventory holding cost	2	3.38E+14	5.50%	1.69E+14	5062.63	0
Stand-alone availability (SAA)	1	2.82E+14	4.59%	2.82E+14	8448.58	0
CV(Demand)*Service level	2	2.12E+14	3.44%	1.06E+14	3171.24	0
Inventory holding cost* Demand Inter-arrival	1	1.38E+14	2.24%	1.38E+14	4124.21	0
Failure repair frequency(SAA)	2	1.07E+14	1.75%	5.37E+13	1609.26	0
Demand Inter-arrival* Service level	1	9.54E+13	1.55%	9.54E+13	2858.99	0
Inventory holding cost	1	8.73E+13	1.42%	8.73E+13	2617.59	0

The ANOVA identifies the factors that have the largest impact on *ΔTC,* and a statistically significant (here at a 0.05 level) *ΔTC* determines if fixed-time or fixed-size lot systems is better. The experimental results for significantly different long-run average cost performance at two levels of the most influential factors are summarized in Figure 2.12 and Figure 2.13. Figure 2.12 shows the impact of *demand interarrival time*, *CV of demand*, and *service level* on the performance of fixed-size lot and fixed-time lot production systems as measured by the percentage of treatment combinations where a particular system (fixed-lot or fixed-time) performs better.

Overall, a fixed lot system performs better than a fixed-time lot system with weekly demand arrivals, and vice versa with daily demand arrivals. Weekly demand arrivals increase the day to day demand variability. However, within a specific demand interarrival period a higher demand variability increases the percentage of treatment combinations where a fixed-time lot system performs better. The relationships displayed in Figure 13 can be quantified by fitting a logistic regression model to those treatment combinations where a statistically significant *ΔTC* occurred. For example, for daily demand arrivals and demand CV, and service level as categorical predictors:

$$Y = -0.938 + 2.273 * CV + 1.373 * Service\ Level,$$

$$P(Fixed\ time\ system\ has\ lower\ cost) = \frac{e^Y}{1-e^Y}.$$

Here *CV* and *Service Level* equals 1 when at a high level, and 0 when low. All model coefficients are significant as is the regression model.

Figure 2.13 shows the impact of workstation's *stand-alone availability* or *reliability* (higher stand-alone availability implies higher reliability), *frequency of failures and repair length*, and *service level* on the performance of fixed-size lot and fixed-time lot production systems as measured by the percentage of treatment combinations where a particular system significantly performs better. Overall the fixed lot production system performs better on a workstation with a high *reliability*, and the fixed-time lot system works better with lower *reliability*.

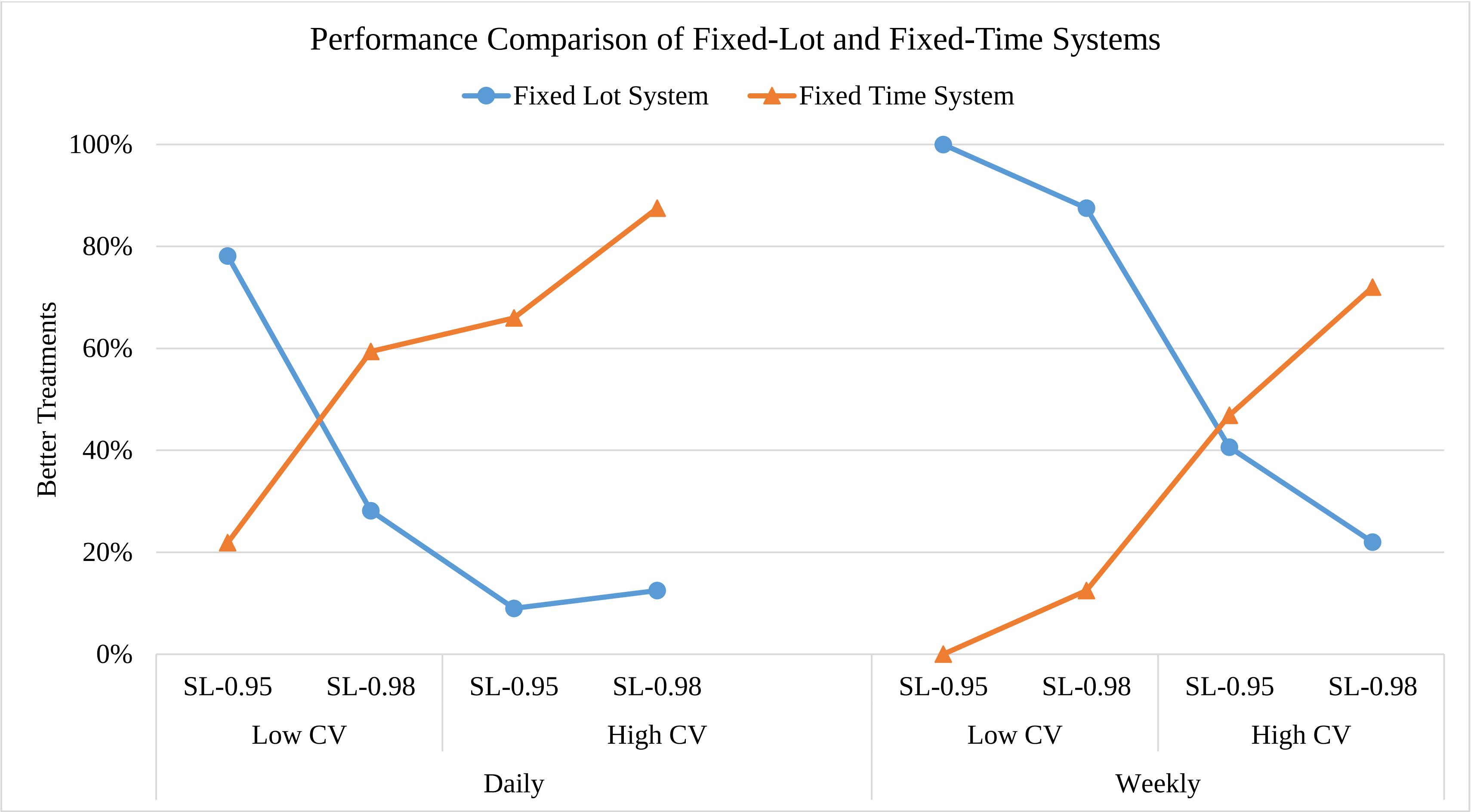

Figure 2.12 Lost Sales Policy - Performance of fixed-size lot and fixed-time lot production systems with different levels of demand interarrival, CV of demand, and service level (SL)

Overall a higher service level pushes results in favor of a fixed-time lot system. In Figure 2.12 and Figure 2.13 moving from a 95% to 98% service level results in a relatively large increase in the percentage of treatment combinations where a fixed-time lot system has lower cost. Other general observations from the experimentation for a lost sales policy are stated next. Recall that Q and r values were optimized for fixed-lot and fixed-time systems for each treatment combination.

- Both high *CV of demand* and lower *stand-alone availability* drive the average number of orders processed in the fixed-time system to be less than the fixed lot system. Fewer orders implies that the average amount produced in the fixed-time lot system is greater than in the fixed lot system, and that lower ordering costs push total cost in favor of a fixed-time lot system
- Low *CV of demand* and higher *stand-alone availability*, drives lower average inventory levels in fixed lot systems and drives costs in favor of fixed lots systems due to lower inventory costs.
- The fixed-time system generally performed better in treatments that had low *stand-alone availability* and high *capacity utilization*, and high *stand-alone availability* and low *capacity utilization* generally favors a fixed-lot systems.

* There was no significant difference in ΔTC for 7% of all treatment combinations.

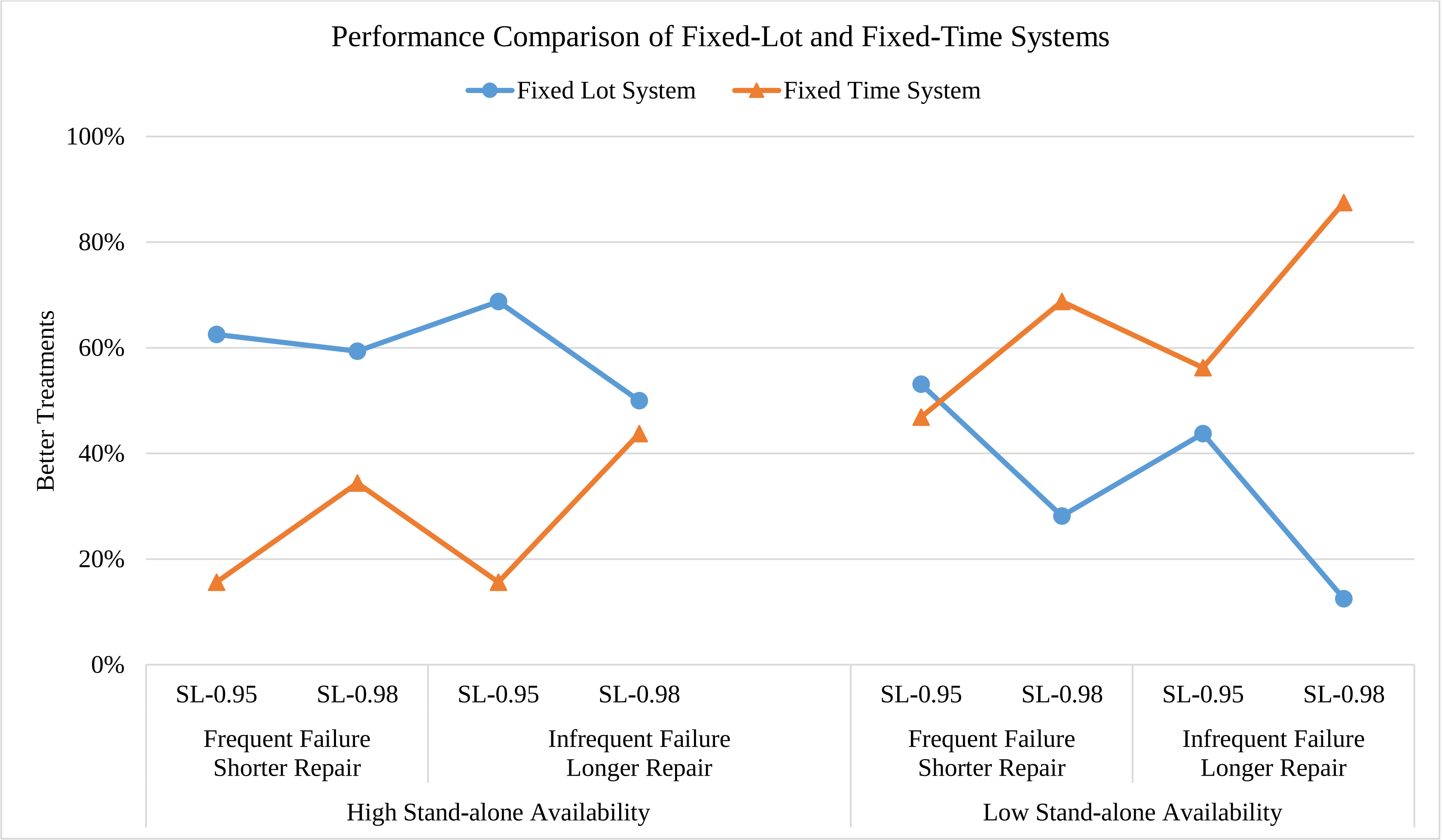

Figure 2.13 Lost Sales - Performance of fixed-size lot and fixed-time lot production systems with different levels of workstation's reliability (SAA), failure repair frequency, and service level (SL)

2.5.2.2 Results and Analysis - Backordering Policy

A subset of the ANOVA results in Table 2.11 show the model terms that are statistically significant and contribute more than 1% to the total sum of squares. The factors *stand-alone availability*, *CV of demand*, *ordering cost*, *service level*, *capacity utilization*, and *failure repair frequency* have the largest influence on the experimental response (ΔTC) accounting for about 37% of the total sum of squares. With a backordering policy the effects are more even and spread out over more factors than with the lost sales policy. The *stand-alone availability* factor is the only factor that stands out relative to other factors listed in Table 2.11. For backordering more of the dominant factors are production side related, rather than demand side related as was seen with a lost sales policy.

Table 2.11 Analysis of Variance for Backordering Experiment

Source of Variation	Degrees of Freedom	Sum of Squares	Percent Contribution	Mean Square	F-value	P-value
Stand-alone Avail.	1	8.63E+12	13.94%	8.63E+12	198.9	0
CV(Demand)	2	3.08E+12	4.97%	1.54E+12	35.44	0
Order Cost	1	2.94E+12	4.75%	2.94E+12	67.75	0
Service Level	1	2.84E+12	4.59%	2.84E+12	65.49	0
Cap Utilization	1	2.65E+12	4.28%	2.65E+12	61.01	0
Failure Repair Frequency(SAA)	2	2.46E+12	3.97%	1.23E+12	28.34	0
Inventory Holding Cost* Demand Interarrival	1	2.04E+12	3.29%	2.04E+12	46.94	0
CV(Demand)*Inventory Holding Cost	2	1.85E+12	2.99%	9.26E+11	21.34	0
Order Cost* Demand Interarrival	1	1.75E+12	2.83%	1.75E+12	40.37	0

Source of Variation	Degrees of Freedom	Sum of Squares	Percent Contribution	Mean Square	F-value	P-value
SAA* Service Level	1	1.69E+12	2.72%	1.69E+12	38.85	0
SAA* Inventory Holding Cost* Order Cost	1	1.03E+12	1.66%	1.03E+12	23.72	0
SAA*Demand Interarrival	1	9.61E+11	1.55%	9.61E+11	22.15	0
Service Level* Failure Repair Frequency (SAA)	2	9.05E+11	1.46%	4.53E+11	10.43	0
SAA* Inventory Holding Cost* Demand Interarrival	1	8.97E+11	1.45%	8.97E+11	20.67	0
Demand Interarrival	1	8.37E+11	1.35%	8.37E+11	19.3	0
SAA* Order Cost	1	6.71E+11	1.08%	6.71E+11	15.47	0

Results for the two levels of the most influential factors for ΔTC, as measured by the percentage of treatment combinations where a particular system (fixed-lot or fixed-time) performs significantly better, are shown in Figure 2.14, Figure 2.15, and Figure 2.16.

Similar to the lost sales policy, the fixed-time system generally performed better in treatments that had low *stand-alone availability* and high *capacity utilization*, however high *stand-alone availability* and low *capacity utilization* generally performed similarly for both systems. With backordering the impact of *interarrival time between demands*, and *CV of demand* is similar to what occurs for a lost sales policy, however a fixed-time system is better for a higher percentage of treatment combinations (Figure 2.15 and Figure 2.16). With daily *interarrival time between demands*, and with both low *CV of demands* and high *CV of demands*, the fixed time systems perform better by reducing the number of orders that are processed and in turn reducing the average ordering cost.

The effects of *capacity utilization* is similar to *service level* where higher *capacity utilization* and a higher required *service level* leads to a higher percentage of treatment combinations where a fixed-time system performs better. Other general observations from the experimentation with a backordering policy are stated next.

- Higher *stand-alone availability* and low *capacity utilization* drives lower average inventory levels for fixed-lot systems and thus pushes more treatment combinations to have lower cost with a fixed-lot system due to lower average inventory cost. However, low *reliability* and low *capacity utilization* or *high capacity utilization*, tends to lower the number of orders processed in fixed-time systems when compared to fixed-lot systems and pushes more treatment combinations to have lower cost with a fixed-time system due to lower ordering costs.
- There was no significant difference in ΔTC for 11% of all treatments. For such treatments, either batch production system, fixed-size lot or fixed-time lot, will give similar cost performance.

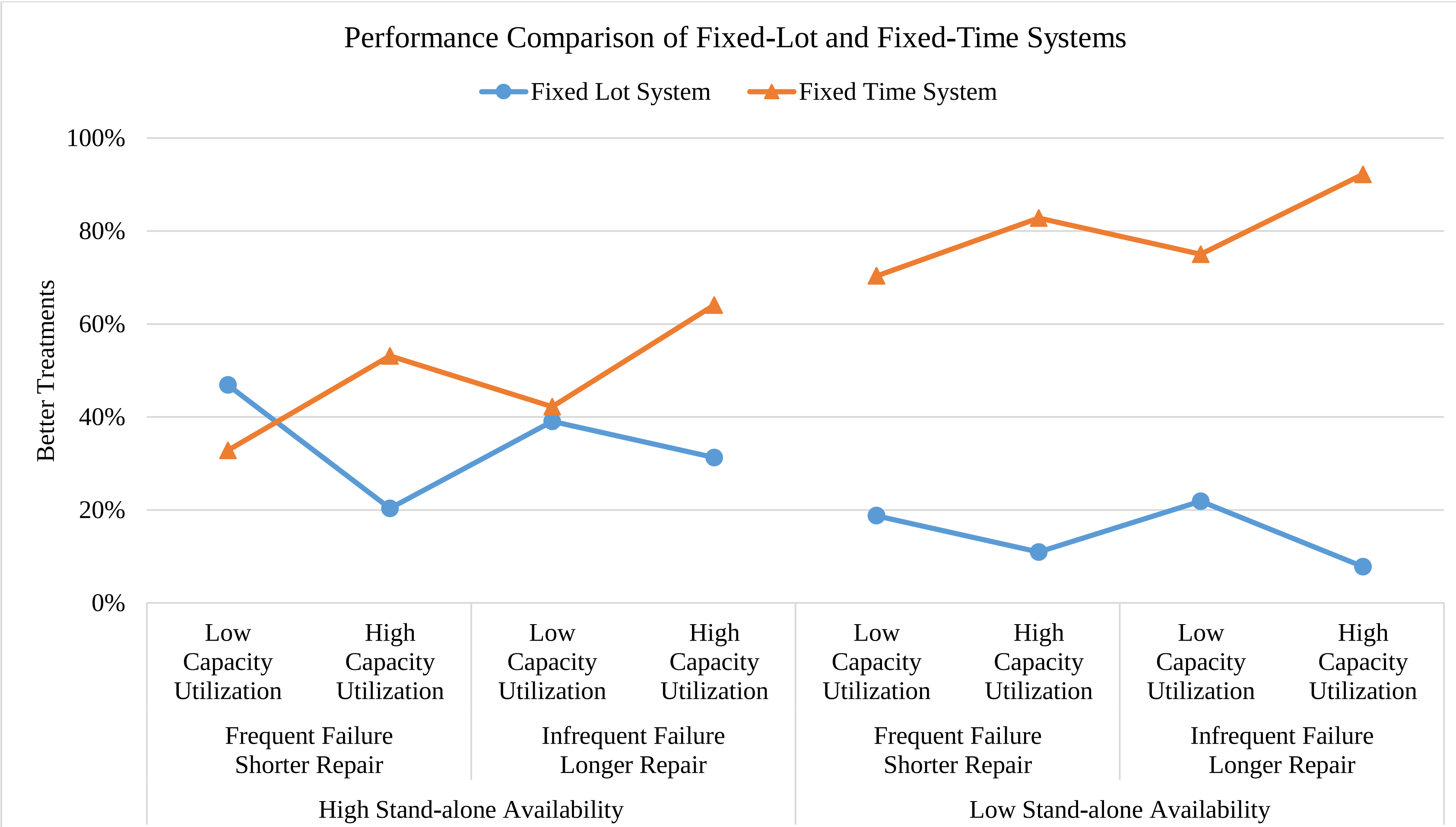

Figure 2.14 Backordering - Performance of fixed-size lot and fixed-time lot production systems with different levels of capacity utilization, failure repair frequency, and workstation reliability

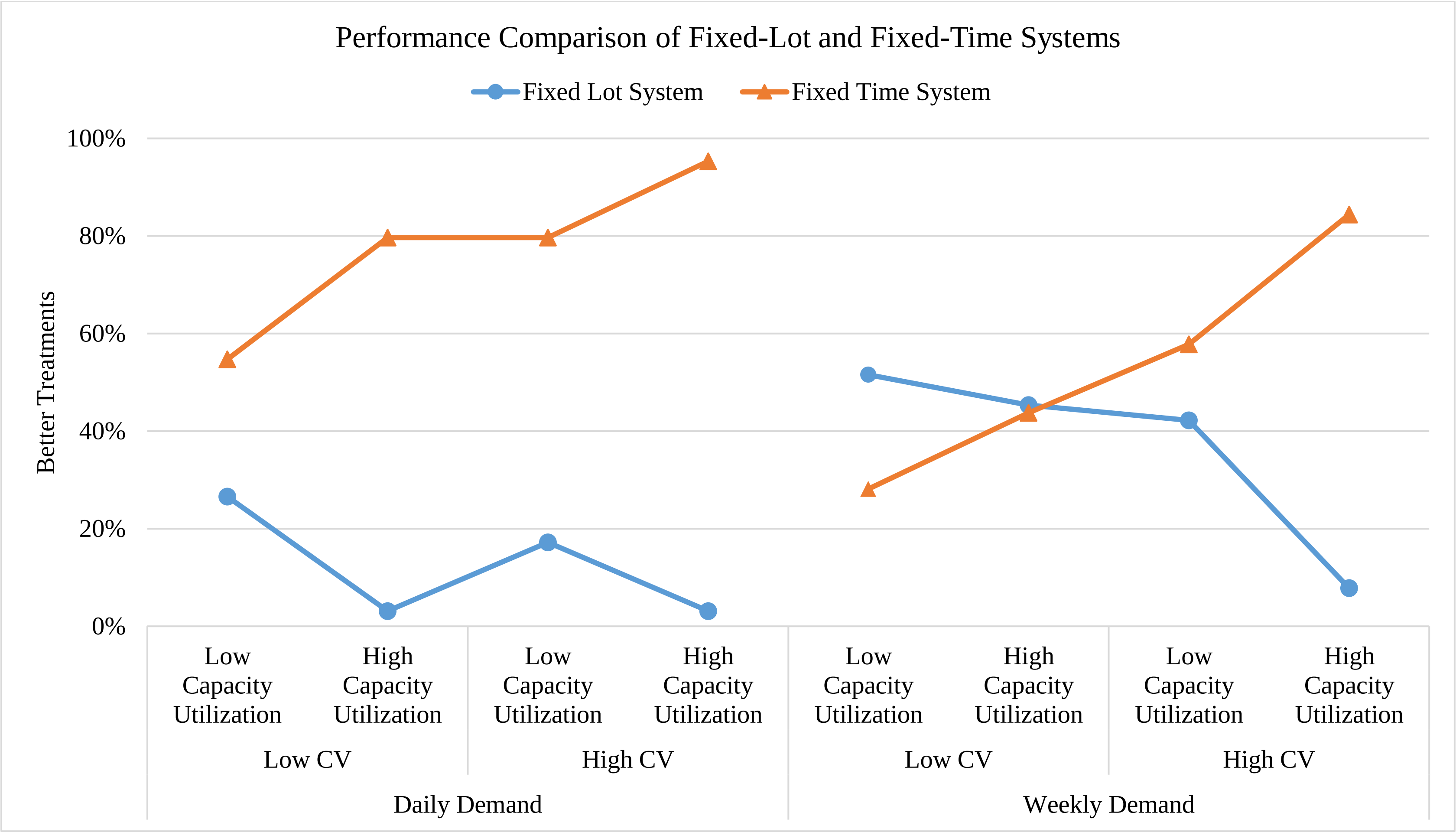

Figure 2.15 Backordering - Performance of fixed-size lot and fixed-time lot production systems with different levels of CV of demand, demand interarrival, and capacity utilization

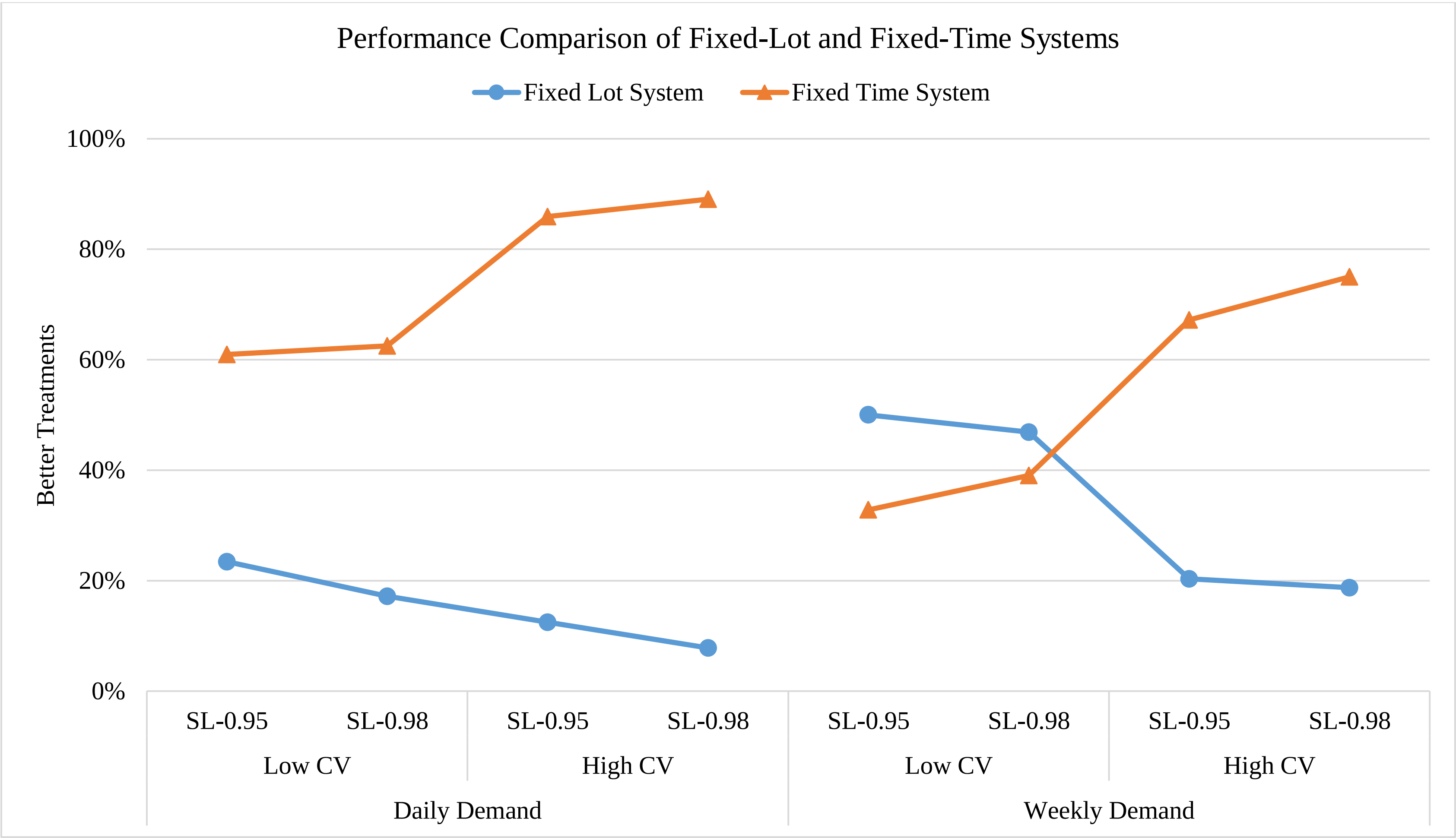

Figure 2.16 Backordering - Performance of fixed-size lot and fixed-time lot production systems with different levels of CV of demand, demand interarrival, and service level (SL)

2.6 Discussion of Results

The experimental results demonstrate that there are no clear rules with respect to particular factors that can be established where one particular strategy (fixed-sized or fixed-time lot sizes) is always better with respect to total long-run costs. Trying to identify such rules is complicated by the fact that the *Q* and *r* parameters are optimized for each test case. However, some explanation/insight for some general results/trends can be offered. The experimental results clearly show that for a lost sales policy the *interarrival time between demands*, and the *coefficient of variation* of the demand probability distribution have by far the largest impact on the long-run average cost difference between fixed-size lot and fixed-time lot batch production systems. For a given demand interarrival length (weekly or daily), a higher *demand variability*, and lower workstation *stand-alone availability* (more variable production) favors a fixed time batch production system. Upon examination of the optimized *Q* values it stands to reason that higher demand *CV* drives larger optimized *Q* values for both fixed lot and fixed time systems. However higher *Q* values in a fixed lot system combined with lower workstation *stand-alone availability* leads to higher variability in the time to complete the fixed lot. This fact causes many lot completion times to be late or to occur after a demand arrival. When used with the "unforgiveness" of a lost sales policy, either the lost sales will be very large or the optimized *Q*, and *r* values will be optimized at a high level to avoid such costs, but this results in higher inventory costs. For the batch production systems operating under a backordering policy the same effects of the *interarrival time between demands*, and the *coefficient of variation* of the demand probability distribution do not occur. In a backordering policy, finishing a lot after a demand arrival is not as severely punished.

For the batch production systems operating under a backordering policy the experimental results show that the workstation *stand-alone availability*, *failure and repair frequency*, and *capacity utilization* have the largest impact on the long-run average cost difference between fixed-size lot and fixed-time lot batch production systems. Unlike the lost sales results, these factors are related to production rather than demand. If the workstation operates under low *capacity utilization* (workstation is underutilized) and a high *stand-alone availability* the fixed lot batch production system had lower cost. However, the fixed time batch production system was preferred when the workstation operates with high *capacity utilization* and lower *stand-alone availability*. In general, the fixed time system can accommodate higher levels of production variability better than a fixed lot system. In the treatment combinations tested, the backordering costs tested were lower than inventory costs at a low level, and higher than inventory costs at a high level. However, the differences between backordering and inventory costs in any treatment was relatively small and seems to have resulted in less significant effects of factors on ΔTC.

The general insights from lost sales and backordering experimentation can be utilized in an actual production environment to decide the most appropriate production system that can minimize the long-run average cost. For example, the fixed-time batch production system is preferred if the workstation operates under backordering policy, receives weekly demand of 4500 units with standard deviation of 450 units, and it expects to meet 98% of demand.

2.6 Conclusion and Future Work

The main objective of this research is to compare fixed-size lot and fixed-time lot batch production systems, and develop useful insights for understanding when either system should be utilized based on minimizing long-run average cost with a constraint on the service level. Expectations at the start of this research was that, in the absence of considering support activities such as set-up crews, the fixed-size lot system will outperform the fixed-time lot system. This expectation was mainly due to the ubiquitous use of fixed-sized lot systems in practice. However, the experimental results demonstrated that this is not the case. Additionally, there are no clear rules that can be established for when a fixed lot strategy is better than a fixed time strategy.

The experimental results show that for lost sales systems the *interarrival time between demands*, and the *coefficient of variation* of the demand probability distribution have (by far) the largest impact on the long-run average cost difference between fixed-size lot and fixed-time lot batch production systems. For the batch production systems operating under a backordering policy the experimental results show that the workstation *stand-alone availability*, *failure and repair frequency*, and *capacity utilization* have the largest impact on the long-run average cost difference. The following are possible directions for extending this research.

1. More complex empirical relationships between the factors examined and the long run average cost differences between fixed lot and fixed time systems can be explored by utilizing other modeling techniques such as artificial neural networks.
2. The performance of the production systems can be examined when a workstation produces multiple products, and requires a setup with a random setup time whenever a new product is produced. This workstation will utilize limited setup crew resources, shared with other workstations, whenever a setup is requested.

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Appendix 2A

SIMULATION OPTIMIZATION MODELS

In this research the simulation model is a substitute for a function that estimates production system performance as a function of specific parameters. The simulation model is used as part of an optimization procedure to optimize specific system parameters. Simulation optimization models (SOM) for the fixed-size lot and fixed-time lot production systems described in section 3 were developed using the programming language Python and discrete event simulation library SimPy. SimPy is a process-based discrete-event simulation framework based on standard Python in which every activity is modeled as a process (Matloff, 2006). The SOM consists of the Optimization Algorithm, and the Simulation Model as shown in Figure 2.17. In SOM, the optimization algorithm provides the input parameters to the simulation model that then provides an estimate of the long-run average cost and service level for those input parameters. The objective of the SOM is to optimize the fixed-size lot Q, fixed-time lot T, and corresponding reorder levels r that minimizes the long-run average cost given that other simulation parameters remain fixed.

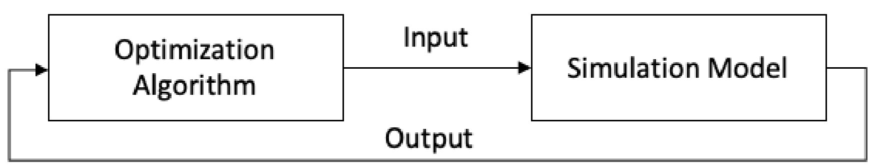

Figure 2.17 Simulation Optimization Model

OPTIMIZATION ALGORITHM

Optimization algorithm provides optimized values of Q, T, and r that minimizes the long-run average cost by utilizing a hybrid greedy and random walk algorithm. This hybrid algorithm is designed to reduce the chance that the solution found is not a local minima. The search boundaries are selected for Q, T, and r by utilizing classical EOQ closed form results (Tajbakhsh, 2010), and outcomes from preliminary simulation runs. From the search boundaries, a random combination of *Q* or *T*, and *r* is generated and treated as candidate solution. The optimization algorithm provides input (candidate solution parameters) to the simulation model, and the simulation model provides an estimate of the long-run average cost and service level for the candidate solution. If customer specified service level is obtained, the candidate solution is accepted as a best candidate solution, otherwise the algorithm keeps generating random combinations and runs the simulation model until the service level is obtained. From the best candidate solution the neighboring solutions are explored for a better solution. To search the neighborhood for a better solution, the best candidate solution is mutated, and the long-run average cost and service level is obtained. This is repeated until a better solution is obtained. If a new candidate solution has a better long-run average cost than the current best candidate solution, it then becomes the best candidate solution. After *n* neighborhood searches with no improvement, a large random jump away from the best candidate solution is made. This ensures the search moves out of the region of a possible local minima and extends the search into other regions. After each random jump, *m* neighborhoods of the randomly generated parameters are searched for a better solution. The random jumps and *m* neighborhood search continues until no better solution is obtained. The simulation optimization ends after a pre-defined number of search iterations are complete. At this time the best solution is reported. The data set in Zheng (1992) is used to validate the optimization algorithm. The proposed algorithm is shown next where the n^{th} iteration represents the iteration up to which no improvement in the *Best Candidate Solution* is observed. *Best Solution* is the combination of decision variables with minimum long-run average total cost meeting a specified service level.

BEGIN

Set termination criteria (total number of iterations)

Set number of iterations (*n*) before making large random move

Set number of iterations (*m*) to search random move's neighborhood

Set desired service level

Set search boundaries for decision variables *Q, T, r*

while *Current service level is less than desired service level*, **do**

Generate a combination of decision variables *Q* or *T*, and *r*

Simulate decision variables

end

Best Candidate Solution = Current Solution

while *Current iteration number is less than total iterations*, **do**

Increment current iteration number

if n^{th} iteration, **then**

if Improvement in *Best Solution*, **then**

Best Candidate Solution = Current Solution

Reset n^{th} iteration

else

Make large move away from *Best Solution*

Simulate random decision variables

Reset m^{th} iteration

if not m^{th} iteration, **then**

Search immediate neighborhood of *random parameters* for *New Solution*

Mutate *random parameters*

Simulate mutated *random parameters*

if *New Solution* is accepted, **then**

Best Candidate Solution = New Solution

end

increment m^{th} iteration

```
                **end**
            **end**
        **else**
            Search immediate neighborhood of Best Candidate Solution for New Solution
            Mutate Best Candidate Solution
            Simulate mutated decision variables
            if New Solution is accepted, then
                Best Candidate Solution = New Solution
                Reset nth iteration
            else
                increment nth iteration
            end
        end
    end
    Report Best Solution
END
```

The optimization algorithm was validated by comparing the optimized Q and r with the numerical results reported by Zheng (1992). As shown in Table 2.12, there are no differences in the numerical values obtained by Zheng, and values obtained by the optimization algorithm developed.

Table 2.12 Validation of optimization algorithm

$Demand\ rate = 50, Lead\ time = 1, holding\ cost = 10, penalty\ cost = 25$

Fixed cost per order	**Zheng's**		**Simulation Optimization**	
	Q*	***r****	***Q****	***r****
1	7	50	7	50
25	23	44	23	44
100	40	38	40	38
1000	120	15	120	15

Appendix 2B

DEMAND VARIABILITY OVER A GIVEN TIME PERIOD

Table 2.13 Computation of demand variability

	Weekly Demand	**Daily Demand**
Jobs per hour	60	
Minutes/job	1	
Demand interarrival time (*processing hours*)	120	24
Expected units per demand arrival	4500	900
Expected demand (processing minutes per arrival)	4500	900
CV of demand	0.1	0.2236
Demand variance (*p.minutes*^{2}/arrival)	202500	40498
Average demand rate (*p.minutes*/hour)	37.5	37.5
Demand variance (*p.minutes*^{2}/hour) *Var[X] = E[X^2]-E[X]^2*	$\frac{4500^2+202500}{120}-37.5^2$ **= 169,031**	$\frac{900^2+40498}{24}-37.5^2$ **= 34,031**

3 Factor Analysis of Fixed-Size Lot and Fixed-Time Lot Batch Production System using Artificial Neural Network

Abstract

High volume discrete item batch production systems such as automotive sheet metal stamping facilities are complex in nature, and effective production control can have a significant impact on cost. Production control in such systems is normally based on fixed-size lots, however recent research has shown that production control using fixed time length lots can be advantageous. In this research, functional relationships between various input factors and three categorical outcomes (representing the lower cost system) are explored using feedforward backpropagation neural networks. Results show that when unmet demand is lost the factors: demand coefficient of variation, and system stand-alone availability have the largest relative predictive importance. When unmet demand can be backordered the factors: demand coefficient of variation, system stand-alone availability, and inventory holding cost have the largest relative predictive importance. Mechanistically, at higher stand-alone availability levels and lower demand coefficient of variation the fixed lot production time variance is low enough that the system can operate in a “just-in-time” manner and results in lower costs than a fixed time lot system. However, as the system stand-alone availability reduces, and demand coefficient of variation increases, the fixed-time lot system results in significantly lower costs than the fixed-size lot system.

Keywords: Batch production systems, Fixed-size lot, Fixed-time lot, Artificial neural network, Connection weight approach

3.1 Introduction

High volume discrete item batch production systems such as automotive sheet metal stamping facility are complex in nature, and their effective management can have a significant impact on cost. Conventionally, the workstation (represented by a single machine, or a set of machines) in such production systems produces a fixed-size lot with workstation setups required between lots. A fixed-sized lot is defined by the number of discrete units of a particular product to produce. Tiwari and Kim (2022) explored an alternate production system where the lot size is defined by a fixed time T rather than a fixed number of items, and called such a time length a "fixed-time lot".

For a perfectly reliable workstation with no disruptions due to machine breakdown, maintenance issues, tool wear, etc., the time to produce a fixed-size lot is predictable, and is the same as a fixed-time lot equal to the fixed-size lot multiplied by the fixed processing time per unit. However, in a real production system, workstation disruptions are likely to occur, and such disruptions occur at random time points and last for random time lengths. This causes the time length to produce a fixed-size lot to be variable. This variability in the time length may lead to an excessive delay in the production of a fixed-size lot, which may further delay the production of all the fixed-size lots that follow. In the fixed-time lot, the workstation disruption introduces no variability in the time length to produce a lot, but does so with respect to production output variability, which is exactly the opposite of fixed-lot production.

Tiwari and Kim (2022) compared the long-run average cost performance of the fixed-size lot and fixed-time lot batch production systems. 1,536 different production systems were simulated based on a two-level full factorial design with the following factors: 1) *stand-alone availability*, 2) *failure and repair frequency*, 3) *service level*, 4) *capacity utilization*, 5) *demand interarrival* *time*, 6) *coefficient of variation of demand*, 7) *ordering cost*, 8) *inventory holding cost*, and 9) *backordering cost*. The simulations estimated production system cost performance, and ANOVA was used to identify the factors and interactions that have the largest effect on the cost difference between the two batch production systems. Based on the results, insights were developed for understanding when either batch production system should be utilized. The use of ANOVA focused on factor effects, and not on functional patterns or relationships that may be established between the various input factors and the output.

Due to its ability to parallel process multiple inputs, and capture the causal relationships between independent and dependent variables in any given data set, the present research utilizes feedforward backpropagation neural network (FFNN), a classification of artificial neural network, to further analyze the experimental database and the results obtained in Tiwari and Kim (2022). This FFNN is suited to discover complex relationships between input factors and outputs from the experimental database such as patterns and relationships between various input factors and their levels. The output of this FFNN is then utilized to provide useful insights and a deeper understanding of when either batch production system should be utilized.

The remainder of this paper is organized as follows. Section 3.2 presents a literature review. In Section 3.3 the workstation and operating assumptions are described. Next the feed forward back propagation neural network is presented in Section 3.4. Section 3.5 discusses the results. Finally, conclusions and future work are presented in Section 3.6.

3.2 Literature Review

The research literature contains extensive studies examining fixed-size lot batch production systems operating under a continuous review inventory policy. The primary focus of this prior research has been optimizing the lot size and reorder point such that the total cost, which includes ordering, inventory, and backordering/lost sales costs, is minimized. Examples of such research in this area includes Federgruen and Zheng (1992), Moon and Choi (1994), Parlar (1997), Mohebbi (2003), Song (2010), Sarkar et al. (2015). With respect to fixed-time lot batch production systems, Kletter (1996) and Tiwari and Kim (2022) are the only prior research discovered that focuses on a comparison of fixed-size lot and fixed-time lot batch production systems. Other research that examines characteristics of production over a fixed time period includes Gershwin (1993), who calculated the variance of output as a function of time, Carrascosa (1995) extended Gershwin's work and analytically derived the variance of output for a single unreliable machine. Tan derived variance of output for a single unreliable workstation (1999), *N*-station production line with no buffer (1997), production lines with workstations in series and parallel (1998). Tan (2000), Li and Meerkov (2000), He et. al. (2007), and Colledani et. al. (2010) analyzed the production output variability for the buffered production lines. A list of expected times length to produce a fixed lot, expected uptime in a fixed time length and their variances are provided in Appendix A.1.

Kletter (1996) derived the density function of production output for a fixed-time lot. They also derived the density function, expected time length, and variance of time to produce a fixed-size lot. These results were used to examine and compare the performance of several production and inventory policies, including fixed-size lot and fixed-time lot production on a single multi-product workstation requiring setups. In addition to Kletter (1996), Kim and Alden (1997) also derived the density function, and variance of the time to produce a fixed-size lot. This lot was produced on a single workstation with deterministic processing times, and exponentially distributed times between failures and repair are times. Several expressions from the literature for the expected output and the variability in output for different production systems is compiled in Appendix 3A.

Tiwari and Kim (2022) compared the fixed-size lot and fixed-time lot batch production systems, and provides insights for when either system is preferred based on minimizing long-run average cost with a constraint on the service level. They consider a single workstation that experiences random failures and repairs, and operates under a continuous review (Q, r) inventory system with either lost sales or backordering of unmet demand. The production system long-run average costs were estimated using simulation. To develop a better understanding of when fixed-time or fixed-lot systems are preferred a two-level full factorial experiment was conducted. The experimental factors were: 1) *stand-alone availability*, 2) *failure and repair frequency*, 3) *service level*, 4) *capacity utilization*, 5) *demand interarrival time*, 6) *coefficient of variation of demand*, 7) *ordering cost*, 8) *inventory holding cost*, and 9) *backordering cost*. 1,536 different batch production systems were simulated, and ANOVA was used to identify the factors and interactions that have the largest effect on the cost difference between the two batch production systems (Table 3.1). The experimental results show that for the lost sales systems the *interarrival time between demands*, and the *coefficient of variation* of the demand probability distribution have the largest impact on the long-run average cost difference between fixed-size lot and fixed-time lot batch production systems. For batch production systems operating under a backordering policy the workstation *stand-alone availability*, *failure and repair frequency,* and *capacity utilization* have the largest impact on the long-run average cost difference between fixed-size lot and fixed-time lot batch production systems. However, the use of ANOVA focused on factor effects, and not on functional patterns or relationships that may be established between the various input factors and the output.

This research utilizes the extensive experimental results obtained from Tiwari and Kim (2022) as inputs to a feed forward back propagation neural network to search for empirical relationships between the input factors and outputs. The objective is to take advantage of their results to establish functional relationships that may provide useful insights and a deeper understanding of when each batch production system should be utilized (Table 3.1).

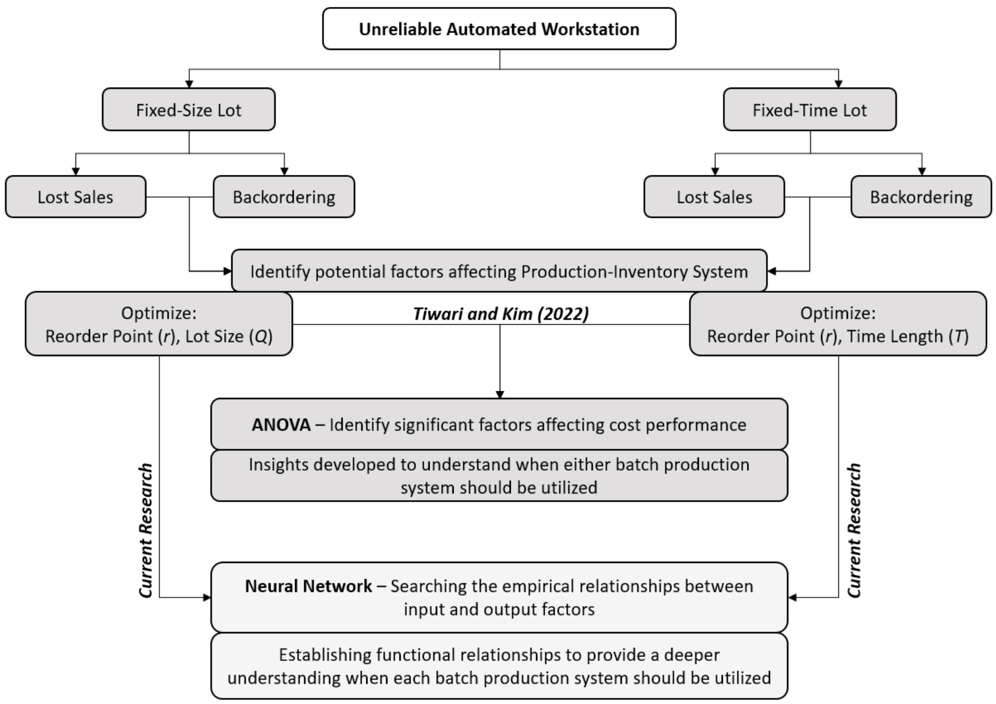

Figure 3.1 Analysis approaches used in Tiwari and Kim (2022) and current research

3.3 System Description

The system considered in this research is the same as the one considered in Tiwari and Kim (2022), which produces batches (fixed-sized lot or fixed-time length) of a single product on an automated unreliable workstation with a fixed processing time per job when the workstation is up. This workstation experiences random failures only when it is operating, with exponentially distributed time between failures and repair times. Workstation production is dictated by a continuous review

(*Q, r)* inventory policy, where *Q* is the reorder quantity, and *r* is the reorder level. For a specified set of parameters *Q* and *r* are optimized to minimize long-run average cost.

3.3.1 Workstation Operating Policy

During the production of a batch (Figure 3.2), if no failures occur then there will be no differences between a fixed-size lot and fixed-time lot batch production systems. In such cases, the time to produce a fixed-size lot *Q* equals the processing time per job multiplied by *Q*, or the number of units (*U*) produced in a fixed time *T* equals *T* divided by processing time per job. However, if the workstation experiences random failures, and random repair times then fixed-size lot and fixed-time lot batch production system will operate and perform differently. The expressions for the expected time length to produce a fixed lot and the variability in the time length to produce this lot on an unreliable workstation (Equation 1, 2) were derived by Kim and Alden (1997).

$$E(T_n) = \frac{\lambda n}{S\mu} + nS \qquad (1)$$

$$Var(T_n) = \frac{2n\lambda}{S\mu^2} \qquad (2)$$

where, T_n – time length to produce a fixed-size lot of *n* jobs; λ - failure rate; μ - repair rate; *S* - workstation speed (*jobs/hour*).

The expressions for the mean uptime and variance of uptime of a single unreliable workstation if it is initially operating (Equations 3, 4), or under repair (Equations 5, 6) were derived by Kletter (1996).

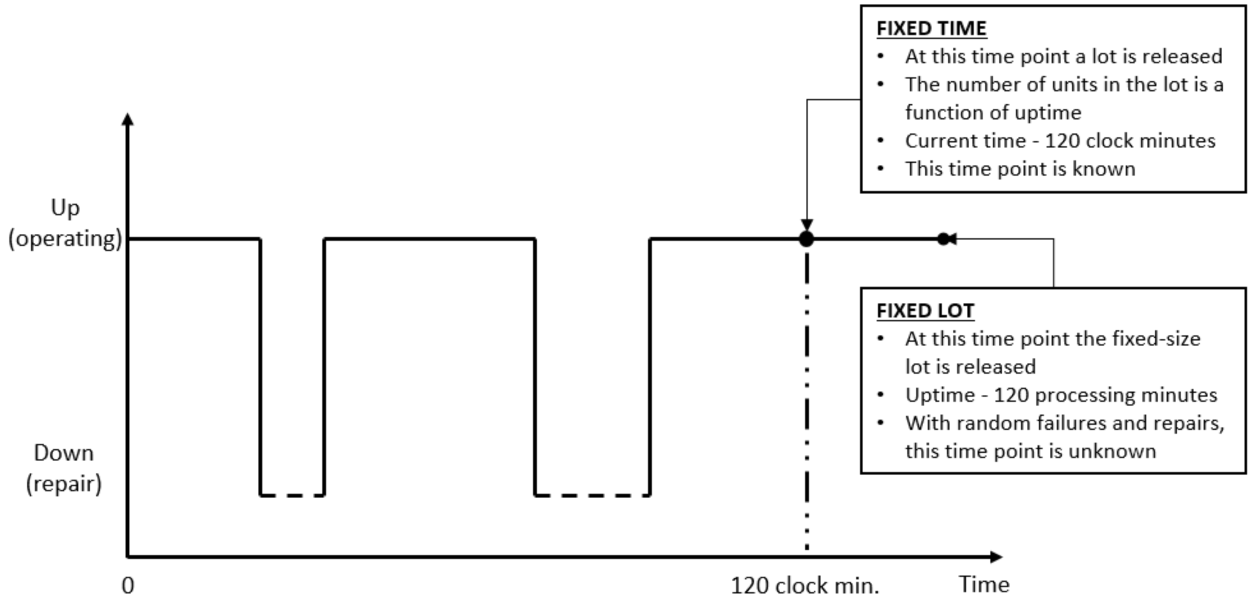

Figure 3.2 Up and down profile of fixed-size lot and fixed-time lot batch production systems

$$E(Uptime_{operating}) = \left(\frac{\mu}{\lambda+\mu}\right)T + \frac{\lambda}{(\lambda+\mu)^2}\left(1 - e^{-(\lambda+\mu)T}\right) \tag{3}$$

$$Var\left(Uptime_{operating}\right) = \frac{\lambda^2}{(\lambda+\mu)^4}\left(1 - e^{-2(\lambda+\mu)T}\right) - \frac{4\lambda\mu}{(\lambda+\mu)^4}\left(1 - e^{-(\lambda+\mu)T}\right)$$
$$+ \frac{2\lambda\mu}{(\lambda+\mu)^3}T\left(1 + e^{-(\lambda+\mu)T}\right) - \frac{2\lambda^2}{(\lambda+\mu)^3}Te^{-(\lambda+\mu)T} \tag{4}$$

$$E(Uptime_{repair}) = \left(\frac{\mu}{\lambda+\mu}\right)T - \frac{\mu}{(\lambda+\mu)^2}\left(1 - e^{-(\lambda+\mu)T}\right) \tag{5}$$

$$Var\left(Uptime_{repair}\right) = \frac{\mu^2}{(\lambda+\mu)^4}\left(1 - e^{-2(\lambda+\mu)T}\right) - \frac{4\lambda\mu}{(\lambda+\mu)^4}\left(1 - e^{-(\lambda+\mu)T}\right)$$
$$+ \frac{2\lambda\mu}{(\lambda+\mu)^3}T - \frac{2\mu(\lambda-\mu)}{(\lambda+\mu)^3}Te^{-(\lambda+\mu)T} \tag{6}$$

where,

T is the fixed-time length;

λ – workstation’s failure rate;

μ – workstation’s repair rate.

Using equations 3-6 an expression for variance of uptime in a fixed time length is derived in Appendix 3B.

3.3.2 Fixed-Size Lot vs Fixed Time Lot Batch Production System

In a fixed-size lot batch production system a fixed-sized lot can be defined as the number of discrete units of a particular product to produce. For a perfectly reliable workstation the time to produce a fixed-size lot is known even before the start of production of a fixed-size lot. However, in reality a workstation experience disruptions at random time points and last for random time lengths, which causes the production time length to produce a fixed-size lot to be variable (Figure 3.2). This variability may lead to an excessive delay in the production of a lot and such variability also makes the planning and management of interrelated activities such as production scheduling, setup crew scheduling more complex.

In a fixed-time lot batch production system the lot size is defined by a fixed time T rather than a fixed number of units. During a fixed-time lot size of T the workstation may experience disruptions at random time points and last for random time lengths. Unlike fixed-size lot batch production system, once the fixed time length has elapsed the workstation stops producing, and the units produced up to time T, a random variable, are treated as the production quantity for the batch (Figure 3.2). In this production system, there is no variability in the time length to produce a lot, but instead variability in the production output, which is exactly the opposite of fixed-lot production where the batch quantity is fixed, but the total time to produce the lot varies. The interested readers are referred to Tiwari and Kim (2022), where the details of several production-inventory cycles are presented.

3.4 Data Description

In the current research, to search for empirical relationships between the input factors and outputs, the experimental database from Tiwari and Kim (2022) is used as input to a feedforward backpropagation neural network (FFNN). This database includes the various combinations of factor levels defined by a 2^k factorial design, and the associated difference in the long-run average cost between optimized fixed-lot and fixed-time systems. Estimated cost differences for 1,536 different combinations were obtained. The factors and their levels are presented in Table 3.1. For more details around the rationale for selecting each factor and their levels, see Tiwari and Kim (2022).

To better understand the contents of this database consider a single factor combination. Discrete event simulation and optimization models were developed to optimize the batch sizes and reorder point levels in both fixed-size lot and fixed-time lot batch production systems. Then the difference in the long-run average cost (ΔAC) is estimated using discrete event simulation by subtracting the long-run average cost of the fixed-size lot from the fixed-time lot production system. If $\Delta AC > 0$ is statistically significant, then the fixed-size lot is considered a better system. If $\Delta AC < 0$ is statistically significant then the fixed-time lot is considered a better system, and if ΔAC is not significantly different from zero then the two systems are not different. Thus the dependent variable used in the FFNN consists of three nominal categorical levels.

Table 3.1 Factors and levels considered in Tiwari and Kim (2022)

Factor	**Level 1**	**Level 2**
Stand-alone availability	0.80	0.90
Failure/Repair frequency	Frequent	Infrequent

Factor	Level 1		Level 2	
Mean processing time between failures (hours)	10	10	20	20
Mean processing time to repair (hours)	1.11	2.5	2.22	5
Service level	0.95		0.98	
Capacity utilization	0.70		0.78	
Demand inter-arrival time (processing hours)	24 (Daily)		120 (Weekly)	
CV of demand (120 processing hours)	0.015		0.1	
CV of demand (24 processing hours)	0.03		0.2236	
Ordering cost ($)	500		1000	
Inventory holding cost ($/p.mins/month)	1		2	
Backordering cost ($/p.mins/month)	0.5		2.5	

3.4.1 Conversion to Processing Minutes

To expand the inference space of the results obtained, various production system parameters will be converted to processing minutes. Parameters such as demand size, demand variability, production lot size, reorder level, on-hand inventory, inventory holding cost, and backordering cost are converted into the equivalent time units of production expressed in processing minutes or processing hours. For example, a demand value that is normally expressed in jobs is multiplied by the processing time of the workstation so if workstation speed is 300 jobs per hour, then each job is worth 0.2 processing minutes (processing time per job). Table 3.2 presents parameters for three different batch production workstations with *original* values, and then expressed in equivalent time units of production. Using this, the varying parametrizations of many batch productions systems in the literature can also be converted to equivalent time units of production.

Example

In this example the production parameters from a real automated high volume sheet metal stamping plant are utilized to compare the long-run average cost performance of system simulated in the *original* parameter units, versus a simulation of the system using parameters converted to their equivalent time units of production (Table 3.3). The parameters demand size, demand standard deviation, lot size, reorder level, and inventory holding cost are converted into the equivalent time units of production by utilizing the processing rate of the workstation (60÷765 minutes per job). For example, the demand of *74,265 (jobs per week) * 60÷765 (minutes per job)* yields an equivalent demand of *5825 (p.mins per week)*.

Both *original* and *converted* systems are then optimized to obtain the fixed-size lot (Q), reorder point (r), and fixed-time length (T). Once optimized, the lot size (Q), reorder point (r), and fixed-time length (T) of the *original* system can be compared to the optimized lot size (*p.mins*), reorder point (*p.mins*), and fixed-time length of the *converted* system (Table 3.3). When the optimized parameters in *original* units are converted, the results are the same as the optimized parameters of the *converted* system. For example, the optimized lot size (Q) in original units is *45229* jobs. When this lot size is converted into equivalent time units of production the result is $45529*60/765 \approx 3571$ *p.mins*, which is very close to the optimized lot size (in *p.mins*) of the *converted* system. Also, the holding costs, ordering costs obtained for the system in *original* units, and those obtained by simulating the *converted* system are not statistically different.

Table 3.2 Conversion of parameters to their equivalent time units of production - processing minutes (p.mins)

Parameter	Workstation 1 Original Specification	Workstation 1 Equivalent Time Units of Production	Workstation 2 Original Specification	Workstation 2 Equivalent Time Units of Production	Workstation 3 Original Specification	Workstation 3 Equivalent Time Units of Production
Workstation speed	495 JPH	0.1212 p.mins/job*	60 JPH	1 p.mins/job*	49.5 JPH	1.212 p.mins/job*
Demand per week	37200 jobs	4509 p.mins	4509 jobs	4509 p.mins	3720 jobs	4509 p.mins
Demand standard deviation	7440 jobs	901 p.mins	901 jobs	901 p.mins	744 jobs	901 p.mins
Inventory holding cost	$0.12/unit/month	$1/p.mins/month	$1/unit/month	$1/p.mins/month	$1.2/unit/month	$1/p.mins/month
Backordering Cost	$0.24/unit/month	$2/p.mins/month	$2/unit/month	$2/p.mins/month	$2.4/unit/month	$2/p.mins/month

**Processing rate of the workstation*

Table 3.3 Cost performance comparison of original and converted parameters of a workstation

	Sheet Metal Stamping	**Equivalent Time Units of Production**
Demand Interarrival	120 hours	120 *processing hours*
Machine Speed	765 JPH	60 JPPH*
Demand Distribution	Log Normal	Log Normal
Demand Size	74265 jobs	5825 *p.mins*
Variability in Demand	8900 jobs	698 *p.mins*
Demand CV	0.120	0.120
Mean Time Between Failures	26.37 hours	26.37 *processing hours*
Mean Time To Repair	2.93 hours	2.93 *processing hours*
SAA	90%	90%
Capacity utilization	90%	90%
Service Level Requirement	95%	95%
Inventory Holding Cost	0.5 ($/unit/month)	6.375 ($/*p.mins*/month)
Ordering Cost ($/Order)	500	500
Optimized Reorder Point (Fixed lot system)	110798 jobs	8690 *p.mins*
Optimized Fixed-Size Lot	45529 jobs	3571 *p.mins*
Fixed Lot Inventory level	33901.47	33894.17
Fixed Lot Ordering Cost	3244.47	3244.59
Optimized Reorder Point (Fixed time system)	80964 jobs	6349 *p.mins*
Optimized Fixed-Time Lot	59.10 hrs.	59.10 *processing hrs.*
Fixed Time Inventory level	31059.91	31053.30
Fixed Time Ordering Cost	3632.87	3633.71

**Jobs per processing hours*

3.4.2 Generality of Factorial Space

When conducting empirical experimental-based research, the inference space for which any results are applicable is an important question. The factors considered in this research, which are known to affect the long-run average cost of the high-volume automated batch production systems (Table 3.1), were carefully selected so the inference space will include many real systems. The factor values utilized were determined from a literature review and discussion with production managers[2], and are intended to be representative of many scenarios that occur in actual high-volume batch production systems.

For the stand-alone availability (SAA), the expectation is to have as high a workstation SAA as possible. For recent manufacturing operations the standard for achieving world-class performance includes a workstation availability of at least 90% (Ahuja & Khamba, 2008). Therefore one SAA level considered is 90%. For the lower SAA level, any SAA lower than 80% is judged to be uncompetitive. At very high SAA levels (> *95%*), the two batch production control systems tend towards very similar performance (they are the same at 100% SAA), so these higher SAA levels are not considered. Table 3.4 lists prior production related research where workstation SAA were specified, and the values assumed.

Table 3.4 Workstation availability

Workstation Stand-Alone Availability (%)	**Reference**
80	Kletter (1996),
	Gupta and Garg (2012),
88	Panagiotis (2018),
	Nurprihatin et.al. (2019)

[2] S. Jain, B. Kumar (personal communication, June 2020)

Workstation Stand-Alone Availability (%)	Reference
90	Ahuja and Khamba (2008), Groenevelt et. al. (1992)
91	Mendez and Rodriguez (2017)

The SAA is a function of the mean time between failure (*MTBF*) and the mean time to repair (*MTTR*), and various combinations of *MTBF* and *MTTR* can be used to obtain the same SAA. In this research frequent failures with shorter repair lengths and infrequent failures with longer repair lengths are considered (Table 3.5).

Table 3.5 SAA with various Failure/Repair Frequency

Failure/Repair Frequency	**MTBF (*processing hours*)**	**MTTR (*processing hours*)**	**SAA** $\left(\frac{MTBF}{MTBF+MTTR}\right)$
Frequent failures with shorter repair length	10	1.11	0.9
	10	2.5	0.8
Infrequent failures with longer repair length	20	2.22	0.9
	20	5	0.8

Since the extremes may not be representative of a real systems and since the question revolves around factor effects, MTBF values of 10 hours and 20 hours were selected. A 10 hour *MTBF* represents one failure per shift on average, and a 20 hour *MTBF* is close to one failure per every three shifts on average.

The factors utilized to model the customer's demand includes the *demand inter-arrival time*, and demand *coefficient of variation*. For several production systems studied in the literature, demand interarrival periods considered are daily and weekly (Table 2.9), and the actual demand in a period is a random variable. The same demand interarrival periods were used here with log normally distributed demands.

Table 3.6 Demand inter-arrival

Demand inter-arrival	**Reference**
Daily	Kalchschmidt (2003)**,** Abdulmalek and Rajgopal (2007)**,** Ferguson et. al. (2007)**,** Sabaghi et. al. (2015)
Weekly	Kletter (1996), Kalchschmidt (2003)**,** Abdulmalek and Rajgopal (2007)

The variability in the demand quantities is controlled by the coefficient of variation (*CV*). Two levels of *CVs* are considered for both daily and weekly demands. These *CVs* are scaled such that the weekly demand variability is the same for both daily and weekly demand interarrival. Due to the asymmetrical nature of log-normal distribution, higher *CVs* are not considered since this leads to unreasonably high demands occurring often enough to lead to very poor inventory performance.

Another factor considered is the *capacity utilization*, which is the ratio of average demand and average production in a given period. The *capacity utilization* is inversely affected by the levels of *SAA*, i.e., for constant average demand and average production over a time period, high *SAA* will yield a lower *capacity utilization*, whereas low *SAA* will yield a higher *capacity utilization*. The question of interest is the impact of capacity utilization at realistic utilization values. Therefore, *capacity utilizations* in the neighborhood of 0.8 are selected. Similar utilizations were assumed in Groenevelt (1992), Kletter (1996), Taj et. al. (2012).

Inventory holding costs and backordering costs are based on the average monthly inventory level. Past studies on inventory systems have considered inventory holding cost on ‘per item per unit time’ basis (Taj et al., 2012; Vander Veen & Jordan, 1989). In this research inventory holding costs are based on *processing minute*s *per month*. The low level for inventory holding cost is assumed to be $1/*processing minutes*/month, and the high level is assumed to be twice the low level at $2/*processing minutes*/month. After conversion to *processing minute*s *per month* the inventory holding costs in (Taj et al., 2012; Vander Veen & Jordan, 1989) fall within this range.

The ordering cost is a fixed cost that is incurred whenever an order of fixed-size lot Q or fixed-time lot T is placed. In inventory systems studied in the literature, the ordering costs considered are significantly higher than the inventory holding costs (Chiu, Wang, et al., 2007; Vander Veen & Jordan, 1989). Chiu et al. (2007) assumed the ordering cost at $450 per order, and Vander Veen & Jordan (1989) assumed it to be $1000. In this study for the low level of ordering cost was increased to $500 per order so that the high level is twice the low level. Therefore, the low level for ordering cost is assumed to be $500/order, and the high level is assumed to be $1000/order.

The backordering cost is incurred on the average excess demand backordered per month. Previous studies on inventory systems have considered backordering cost to be both smaller (Chiu, Ting, et al., 2007) and greater (Wee et al., 2007) than inventory holding cost. The low level of back-ordering cost is assumed to be smaller than the two levels of inventory holding cost at $0.5/*processing minutes*/month, and the high level is assumed to be greater than the two levels of inventory holding cost at $2.5/*processing minutes*/month. Backordering cost is not utilized in the lost sales system, where the unmet demands are lost if on-hand inventory is not available.

3.5 Artificial Neural Network

Functional patterns or relationships between multiple input factors on the performance of fixed-size lot and fixed-time lot production systems can provide insight into system operations, but can be difficult to establish. An artificial neural network (ANN) is one method that is appropriate to deal with such complexities. ANNs process in parallel, multiple inputs and capture the causal relationships between the input factors and the outputs in a given dataset. However, beyond the predictive realm, ANNs can also aid in assessing the contribution of each input factor.

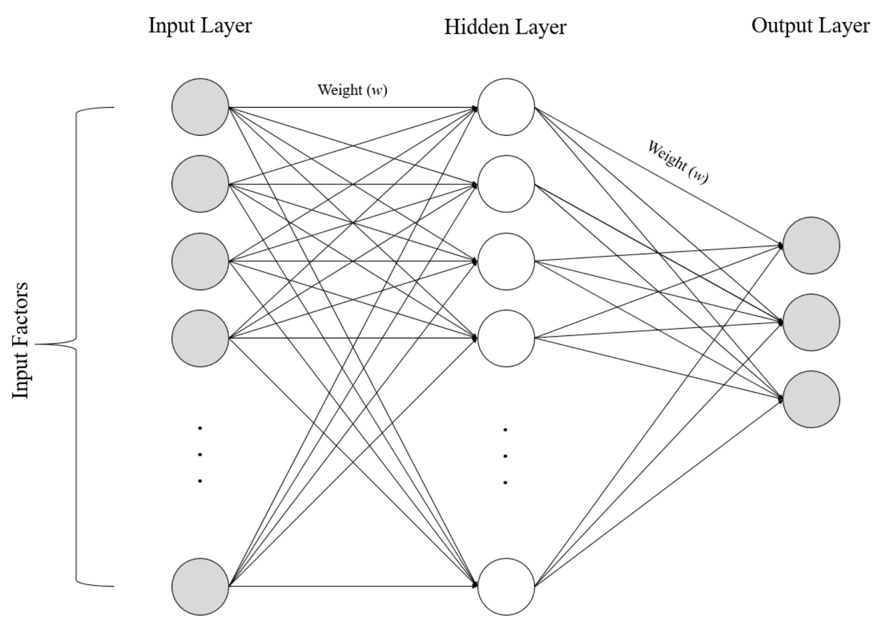

Figure 3.3 Architecture of artificial neural network

Empirical relationships existing in the fixed-time and fixed-lot experimental database are explored using a classical one hidden layer feedforward backpropagation neural network. The schematic diagram of this neural network is shown in Figure 3.3. It consists of three layers: input, hidden, and output layers, with each layer having a sufficient number of neurons. The number of neurons in the input layers depends on the dataset, i.e., the number of input factors or independent variables. The number of hidden layers and their neurons can be determined by trial and error. The number of neurons in the output layers depends on the specific problem. Thus, based on the specific problem, the number of neurons in the input and output layers are fixed. In this research the output layer contains three neurons that represent three categorical outcomes or dependent variables: 1) a fixed-size lot system has lower average cost than a fixed time system, 2) a fixed-time lot system has lower average cost than a fixed-size lot system, and 3) the cost performance of the two systems is not different.

The input data is scaled such that all inputs are in the range [0, 1], and this ensures that all inputs are treated equally. The neurons of a layer are connected to the neurons of the next layer and each connection carries a weight. The neurons of the input layer are multiplied by their respective connection weights to get a single neuron value of the hidden layer. The next step is a mathematical transformation to generate the output between 0 and 1. For this, a more commonly used activation/transfer function such as sigmoid function (logistic function) is utilized. In addition, there is a *bias* component connected to hidden and output layers. This procedure is the same for all of the successive layers (hidden → output), and hence it is called a *feedforward neural network* (Figure 3.4).

$$new\ neuron_j = \sigma(w_1 a_1 + w_2 a_2 + \cdots + w_n a_n + b)$$

Where,

$\sigma = \dfrac{1}{1 + e^{-x}} = sigmoid\ function$

$x = (w_1 a_1 + w_2 a_2 + \cdots + w_n a_n + b)$

w_i is the connection weight between current layer's i^{th} neuron and next layer's j^{th} neuron

a_i is the value of the i^{th} neuron

b is the *bias* component, that is added to make the neuron values more meaningful

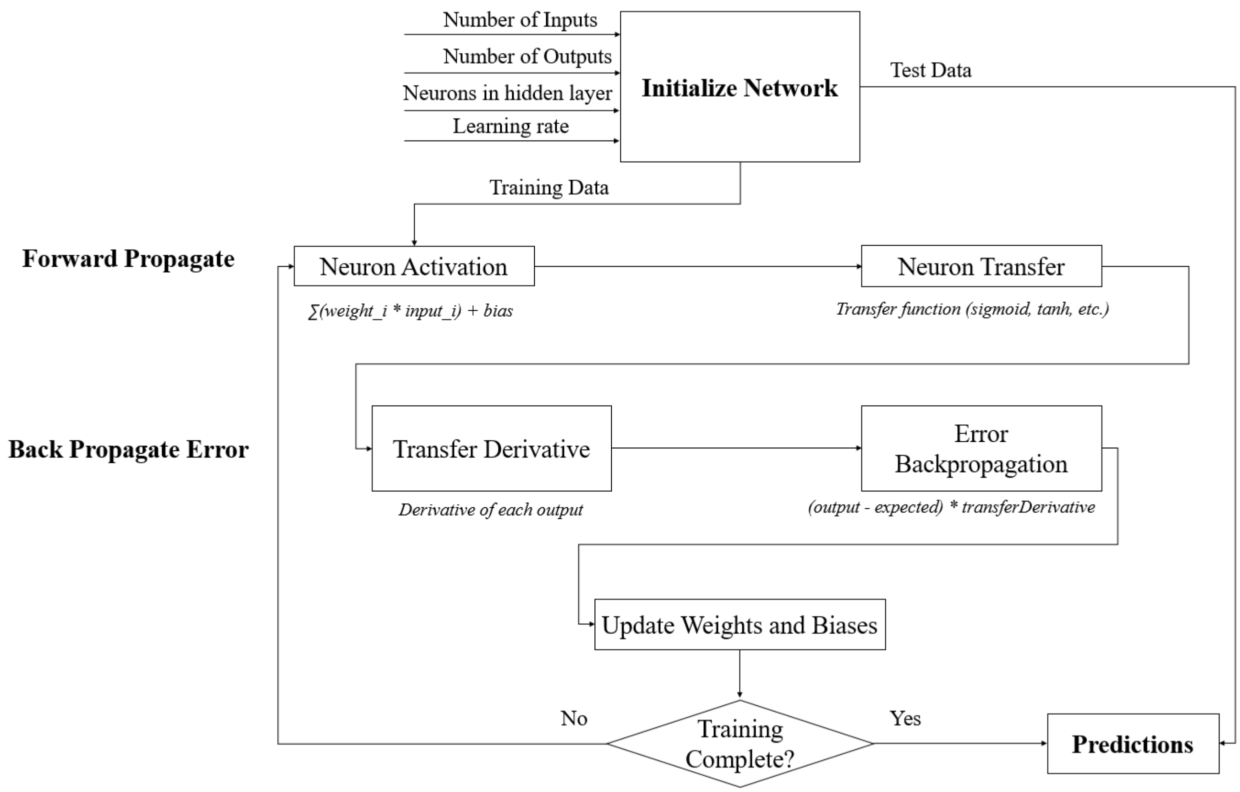

Figure 3.4 Feedforward backpropagation neural network

Backpropagation is a method of adjusting the connection weights of a multilayer feedforward neural network. In backpropagation (Figure 3.4), the connection weights and biases between the input → hidden → output layers are adjusted such that the difference between the neural network output and the targeted output is minimized. Backpropagation along with the *feedforward* process is repeated until the error is minimized to an acceptable value. The final values of the adjusted connection weights are then used to determine the neural network output. The protocol used for implementing FFNN in Python programming language is provided in Appendix 3C.

After the FFNN is trained, a mathematical equation relating the input factors and the outcomes can be written as (Goh et al., 2005):

$$Y = f_{sig}\left\{b_o + \sum_{k=1}^{h}\left[w_k f_{sig}\left(b_{hk} + \sum_{i=1}^{m} w_{ik} X_i\right)\right]\right\}$$

Where,

b_o = bias at output layer, w_k = weight connection between neuron *k* of the hidden layer and the output neuron, b_{hk} = bias at neuron *k* of the hidden layer, w_{ik} = weight connection between input factor *i* and neuron *k* of hidden layer, X_i = the input parameter *i*, normalized in the range (0, 1), f_{sig} sigmoid transfer function.

Using the trained weights and biases, expressions can be written to finally arrive at the outcome with the input parameters (Appendix 3D). The performance of the neural network can be computed using the prediction accuracy obtained over the test dataset.

$$Performance = \left(\frac{Number\ of\ accurately\ predicted\ data\ by\ FFNN\ model}{Total\ number\ of\ data}\right) * 100$$

3.5.1 Connection Weight Approach

After the neural network is trained and its prediction accuracy has reached an acceptable level, the next step is to identify the relative contribution of the input factors to the output. This contribution depends on the magnitude and direction of the inter-neuron connection weights. In a trained neural network the positive connection weights represent excitatory effects on the neurons thereby increasing the value of predicted response whereas, the negative connection weights represent inhibitory effects on the neurons thereby decreasing the value of the predicted response (Olden & Jackson, 2002).

The connection weights based approach has found its application in a wide variety of studies such as ecological, and geological (Das et al., 2011; Das & Basudhar, 2008; Kanungo et al., 2014; Olden et al., 2004, 2006; Olden & Jackson, 2002; Park et al., 2016). However, a literature review suggests that this approach has not been widely utilized in operations research. In a connection weights approach, the calculation includes a product of input-hidden and hidden-output connection weights between each input neuron and output neuron and sums the products across all hidden neurons. The relative importance, if expressed in percentages, of input factors in the neural network can be calculated by dividing the absolute value of each variable contribution by the sum of all absolute variable contributions. (Olden & Jackson, 2002). The relationship between the input and output can be determined in two steps. Positive input-hidden and positive hidden-output weights, or negative input-hidden and negative hidden-output weights, give the direct proportionality of the input factors. The inverse proportionality of the input factors is indicated by the positive input-hidden and negative hidden-output, as well as negative input-hidden and positive hidden-output weights. The input factors with larger connection weights represents a greater effect or relative importance on the prediction of the response than the input factors with smaller connection weights. The magnitude and direction of the relative importance of an input factor can be determined by using the following formula:

$$RelImp_{input} = \sum_{k=1}^{n} \left(ConnectionWeight_{input,hidden(k)} * ConnectionWeight_{hidden,output(k)} \right)$$

where,

$RelImp_{input}$ relative importance of an input factor

n total number of hidden nodes,

k index number of hidden node

Example:

An example of a 2/2/2 neural network architecture is presented to explain the connection weight approach Figure 3.5.

Table 3.7 shows the connection weights between the input and hidden layers that are obtained after training the network. Table 3.8 shows the connection weights between the hidden and output layers. To obtain the relative importance of each input factor, a matrix multiplication of Table 3.7 and Table 3.8 is performed. The output of matrix multiplication is presented in Table 3.9. The input neuron 1 in Table 3.9 positively influences the *Outcome 2*. However, this neuron negatively influences the *Outcome 1*. The advantage of using the connection weight approach is that it takes into account the contrasting influence of each neuron through different hidden neurons resulting in correct estimation of a factor's importance. The final output of the sums of connection weights are presented in Table 3.9. The input neuron *IN1* has large relative importance in predicting *Outcome 1*, as well as in predicting *Outcome 2*.

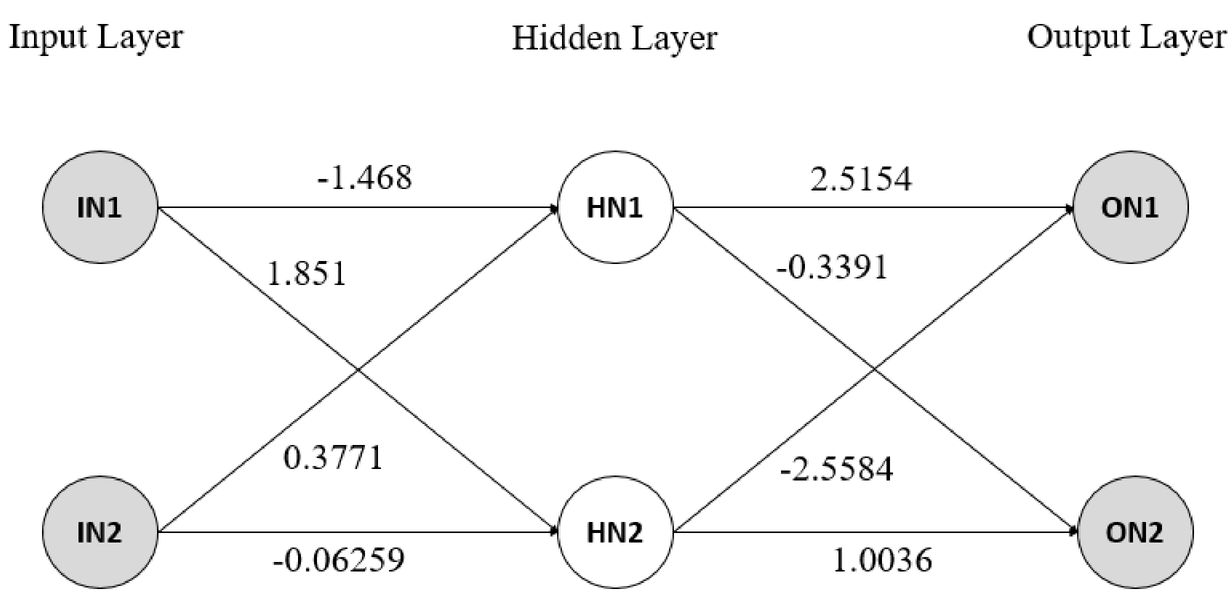

Figure 3.5 Example - Connection Weight Approach

Table 3.7 Input-Hidden connection weights from a trained neural network

	HN1	**HN2**
IN1	-1.468	1.851
IN2	0.3771	-0.06259

X

Table 3.8 Hidden-Output connection weights from a trained neural network

	ON1	**ON2**
HN1	**2.5154**	**-0.3391**
HN2	**-2.5584**	**1.0036**

Table 3.9 Matrix multiplication and addition of Connection Weights

	Outcome 1	**Outcome2**
IN1	-8.4282	2.3555
IN2	1.1086	-0.18093

The advantage of utilizing connection weight approach is that it takes into account the contrasting influence of each neuron that results in the correct estimation of each factor's relative importance. The results and their analysis are presented in the next section.

3.6 Results

The experimental results from Tiwari and Kim (2022) were analyzed to search for empirical relationships between the input factors and outputs. Tiwari and Kim (2022) utilized ANOVA to identify the factors having the largest effect on the long-run average cost performance by analyzing the difference in the long-run average cost performance (ΔAC) of the two batch production system. However, here a feedforward back propagation neural network (FFNN) was applied with three categorical outcomes 1) a fixed-size lot production system has significantly lower cost performance than a fixed-time lot system, 2) a fixed-time lot production system has significantly lower cost performance than a fixed-size lot system, and 3) the cost performance of two systems is not significantly different. A connections weight approach (Olden et al., 2004; Olden & Jackson, 2002) was used to identify the factors that have the highest relative importance/influence on the three categorical outcomes. The FFNN was applied separately to the experimental results for production systems operating under lost sales, and backordering policies.

3.6.1 Lost Sales Policy

A feedforward backpropagation neural network with a 9/9/3 network architecture (i.e., input layers with 9 neurons, one hidden layer with 9 neurons, and output layer with 3 neurons) was constructed. The number of epochs and learning rate parameters were found using trial and error, resulting in a network that was trained for 1000 epochs with a learning rate of 0.1. The performance of the FFNN was evaluated by determining the prediction accuracy of the model on randomly selected testing data. The average classification accuracy of approximately 85% was achieved.

The relative importance/influence of each input factor using the connection weight approach (for a 9/9/3 network architecture) is presented in Figure 3.6. The solid bars in Figure 3.6 are positive valued relative importance that shows the direct proportionality, and the patterned bars are negative valued relative importance that shows inverse proportionality of the input factors. The magnitude and direction of the relative importance of each input factor as per connection weight approach is presented in Appendix 3E. The demand process related factor, *CV of demand* has the largest relative importance for all the three outcomes. There is a stark contrast of the influence of *CV of demand* to predict *Outcome 1* and *Outcome 2*. It can be seen from Figure 3.6 that *CV of demand* is inversely proportional to *Outcome 1*, and directly proportional to *Outcome 2*. Thus it can be inferred that with a shift in *CV of demand* level from low → high the fixed-time lot production system has significantly lower cost performance than a fixed-size lot system.

The production process related factor, *stand-alone availability (SAA)* has the second overall largest relative importance/influence on the prediction of the outcomes. Thus it can be inferred from Figure 3.6 that with a shift in *SAA* level from low → high the fixed-size lot production system has significantly lower cost performance than a fixed-time lot system.

To better understand the mechanisms for how *CV of demand* and *SAA* influence whether fixed-time or fixed-size lot production systems have lower cost, several levels of both factors were identified and simulated. The lot size (Q), reorder point (r), and fixed time length (T) for each treatment combination was optimized. Because other factors have relatively lower importance/influence in predicting the outcomes, the levels of these factors were fixed as shown in Table 3.10. The results of the long-run average cost performance obtained using the optimized parameters at different levels of *CV of demand* and *SAA* for the two production systems are summarized in Figure 3.7. Figure 3.7 shows the impact of the two factors on cost performance, and when a particular system performs better.

At 100% *SAA*, fixed-size lot and fixed-time lot batch production systems are the same. However, with a reduction in the SAA, differences in the cost performance of the two systems starts to appear. For low levels of *CV of demand* and higher *SAA* levels a fixed-size lot system performs better than fixed-time lot system. Whereas, for lower *SAA* levels a fixed-time lot system performs better than fixed-size lot system. Tiwari and Kim (2022) observed a similar pattern. At higher *CV of demand* levels, and higher *SAA* levels the cost performance of the two batch production systems is not significantly different.

Table 3.10 Lost Sales Policy - Levels of CV of Demand and SAA to develop insights

Factors	Level					
Demand	Weekly (120 *p.hrs*)					
Service Level	95%					
Inventory Holding Cost	$1/*p.mins*/month					
Ordering Cost	$500/Order					
CV of Demand	0.015	0.05	0.1	0.2		
SAA (%)	98	95	90	85	80	75

In Figure 3.7, using *boxes*, the *CV of demand* and *SAA* combinations for which a production system's performance is better than the other is shown. A fixed lot production system is preferred when operating at high *SAA* and with a low *CV of demand*. Independent of the *CV of demand*, once the *SAA* drops below 85% there is a steep increase the in the long-run average cost performance of the fixed lot system as indicated by the "elbows" in each graph. The onset of this more rapid cost increase can be attributed to a threshold reached by the fixed lot production time variability caused by increased average repair lengths (that increase as the *SAA* decreases).

If the system operates at a low *SAA* (< 80%), then for any *CV of demand*, a fixed-time lot production system has significantly lower cost. The following two examples provide insights into the mechanisms driving cost differences between fixed-size lot and fixed-time lot production systems under different *CV of demand* and *SAA* combinations.

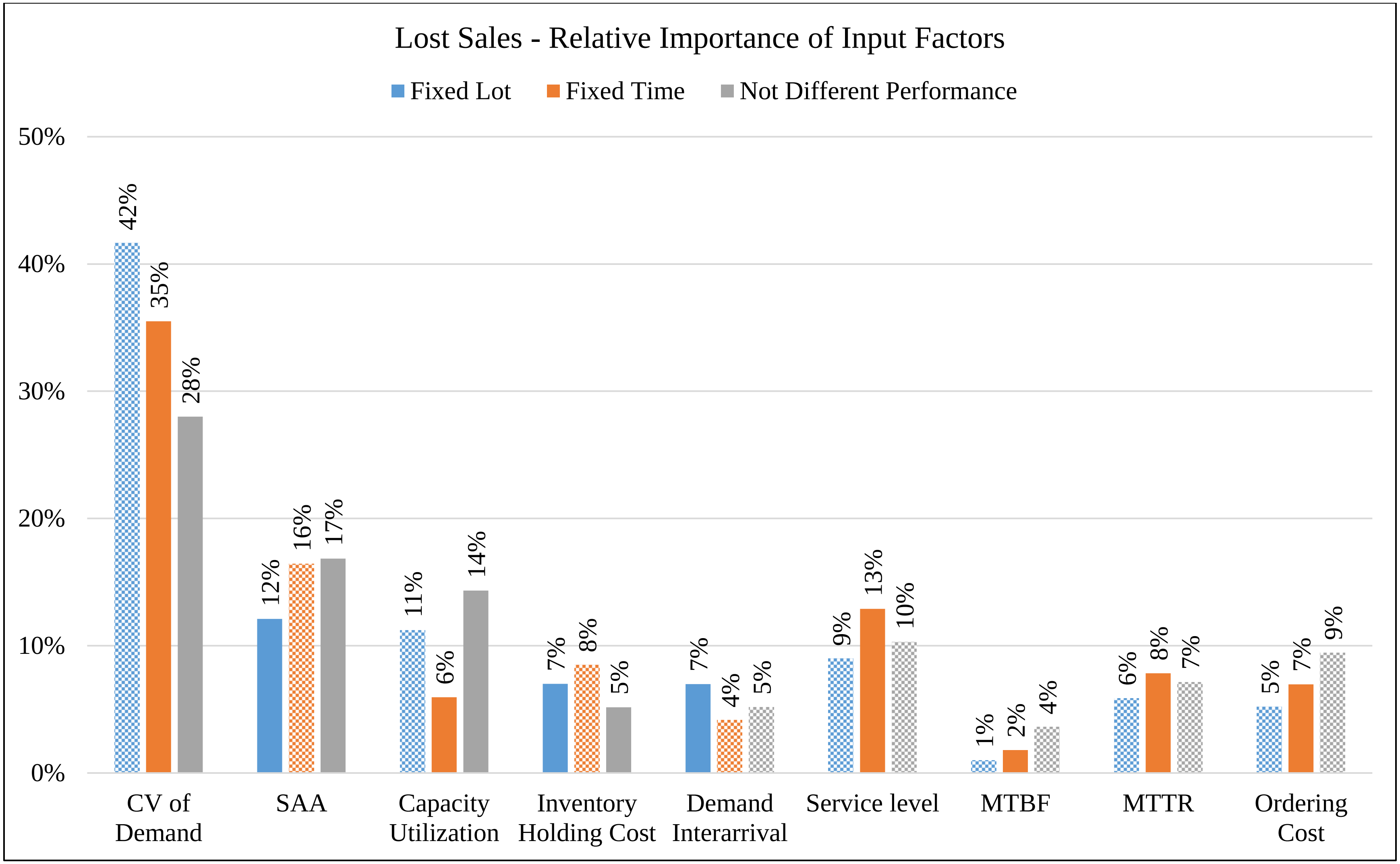

Figure 3.6 Lost Sales Policy - Relative importance for all the input factors from the FFNN

Example 1:

Figure 3.8 and Figure 3.9 show mixed mass/density functions for the time length to produce a fixed-size lot, and uptime in a fixed-time lot, respectively for a workstation operating at 98% *SAA* (*MTBF =30 hrs., MTTR = 0.61224 hrs.).* Density functions for several *CV of demand* levels, and weekly demand arrivals (120 *processing hours*) are shown, where different *CV of demand* levels change the optimal fixed lot and fixed time values.

Figure 3.8 (fixed lot size time production lengths) shows that moving from 0.015 to 0.2 *CV of demand* results in an increase in the optimal lot size (*p.mins*). The increase in the lot size is to buffer against the increasing demand variability. However, at 98% *SAA* the variability in the time length to produce a fixed lot size is low enough such that the probability of completing the fixed-size lot before the next demand arrival is close to 1. This in turn drives lower reorder levels, and leads to lower average inventory levels.

In Figure 3.9 (fixed-time lot uptime) moving from 0.015 to 0.2 *CV of demand* results in an increase in the fixed time length (*p.hours*). Like the fixed lot system, the increases in the optimal fixed time lengths buffer against demand variability increases. At 98% *SAA* the variability in the uptime in a fixed time length is low, but this variability drives higher reorder levels leading to a higher average inventory level to meet the service level requirement.

In general, at higher *SAA* levels when the production time to produce a fixed lot is low enough that the system can operate in a “just-in-time” manner a fixed-size lot production system will result in lower costs than a fixed-time lot system.

Example 2:

Figure 3.10 and Figure 3.11 show mixed mass/density functions for the time length to produce a fixed-size lot, and uptime in a fixed-time lot, respectively for a workstation operating at 85% *SAA* (*MTBF =30 hrs., MTTR = 5.29411 hrs.).* The mixed mass/density functions are shown for several *CV of demand* levels, and weekly demand arrivals (120 *processing hours*). Increasing *CV of demand* results in an increased lot size to buffer against demand variability. The increased lot sizes increase the variability in the time length to produce a fixed lot as shown in (Kim & Alden, 1997), where *n* is the fixed lot size and *S* is the workstation speed.

$$Var(T_{p.mins}) = \frac{2n(MTTR)^2}{S * MTBF}$$

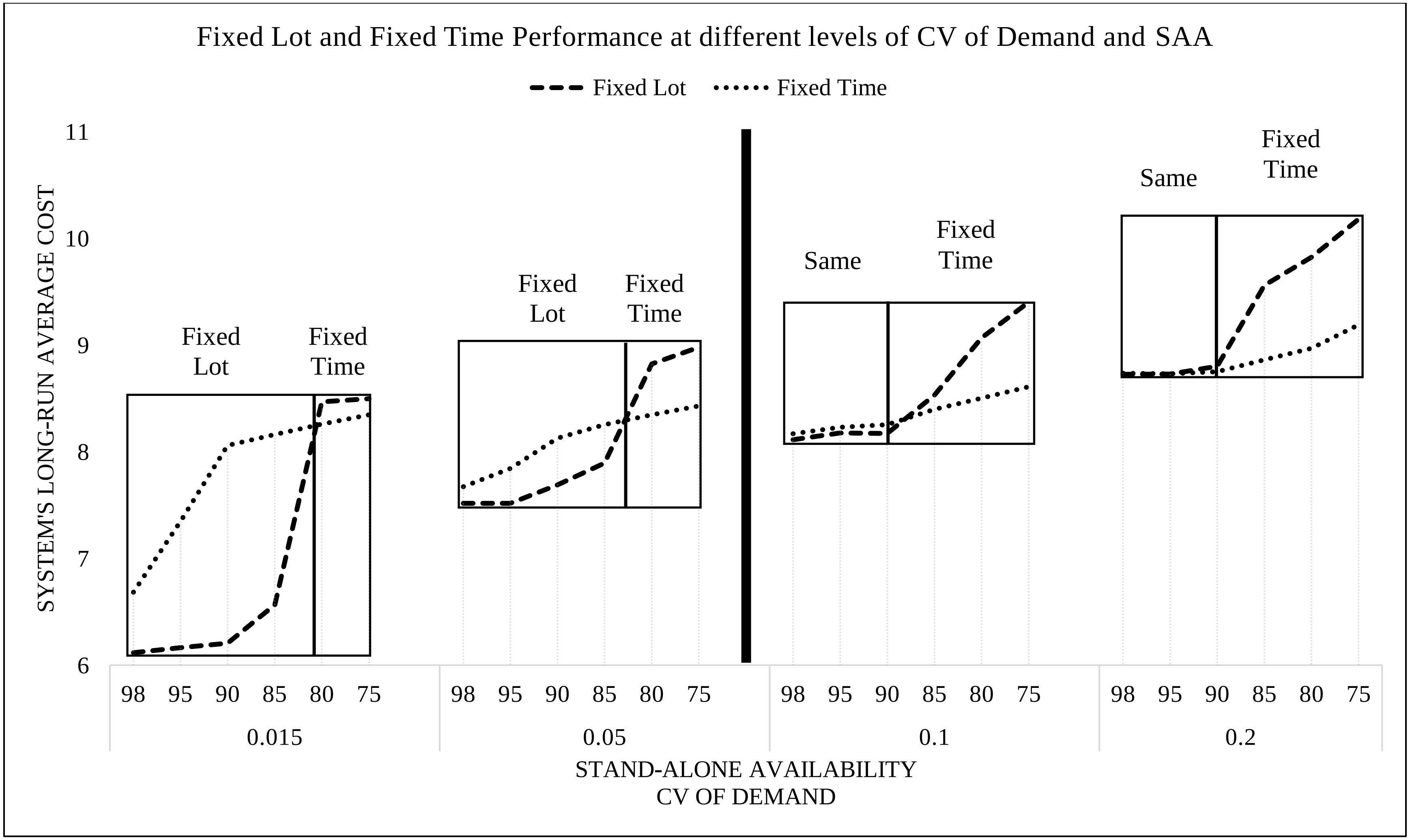

Figure 3.7 Lost Sales - Comparison of long-run average cost performance of fixed-size lot and fixed-time lot production systems at differnet levels of CV of demand and SAA

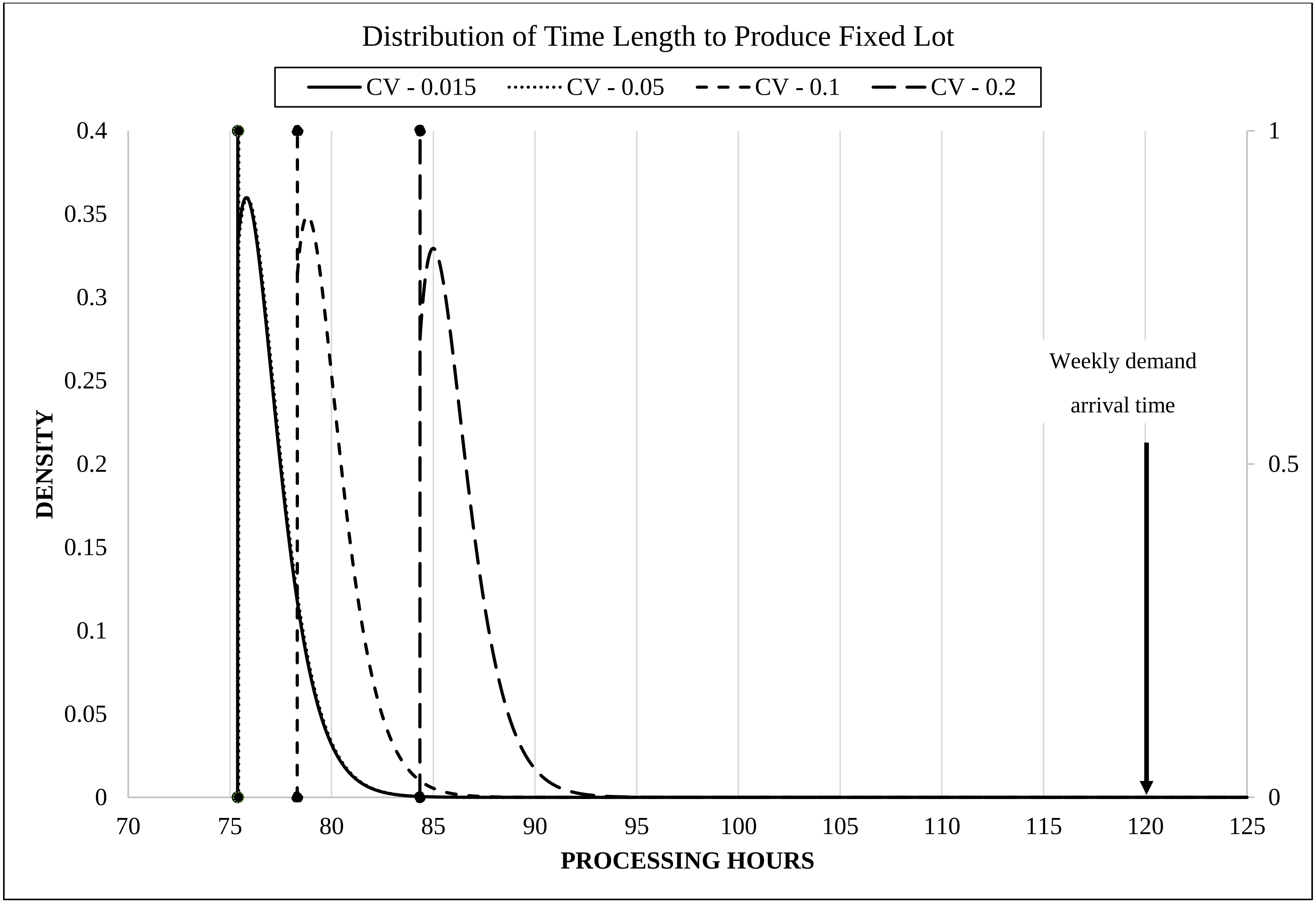

Figure 3.8 Density Functions - Time to Produce a Fixed Lot with 98% Workstation SAA

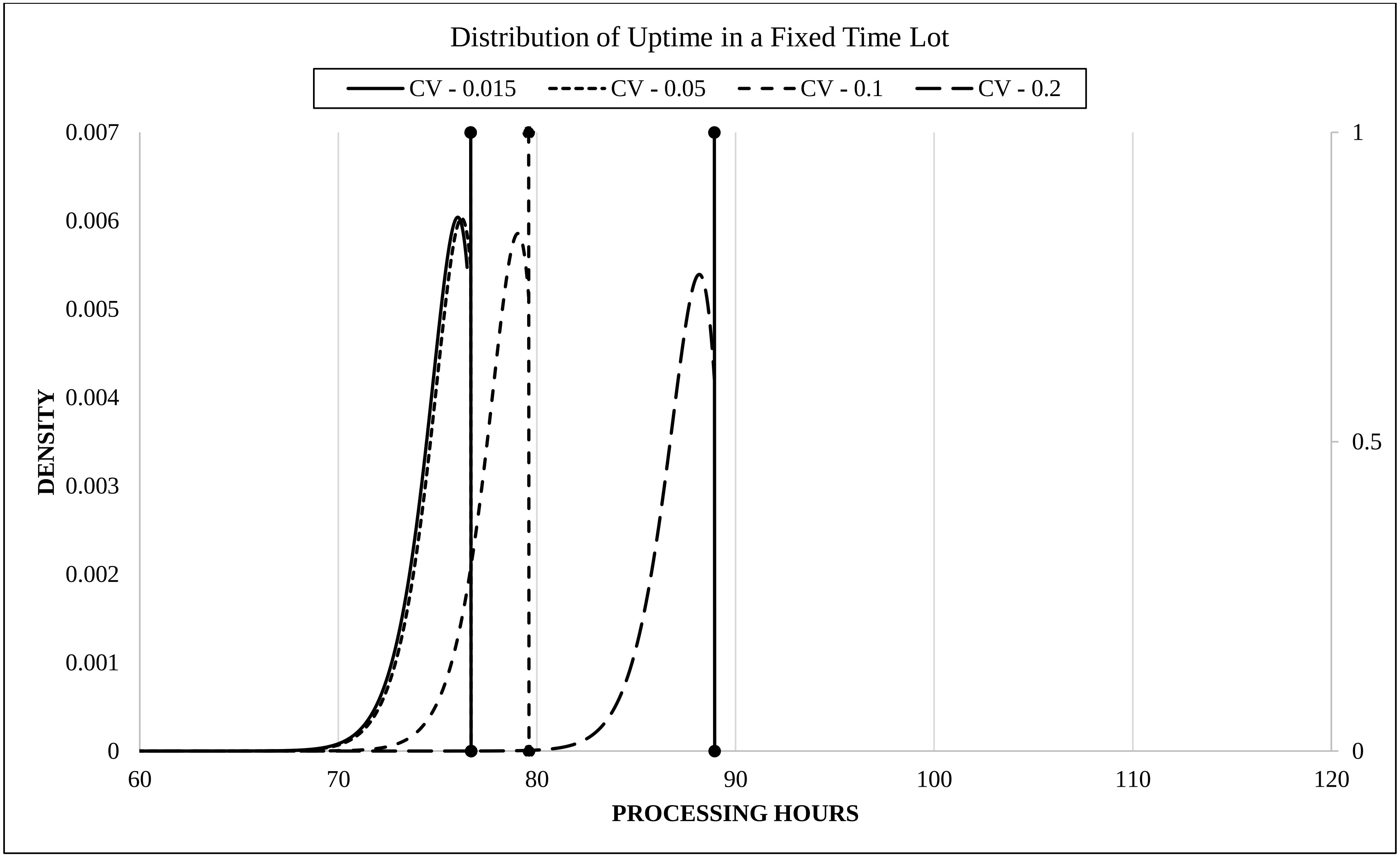

Figure 3.9 Density Functions - Uptime in a Fixed-Time Lot with 98% Workstation SAA

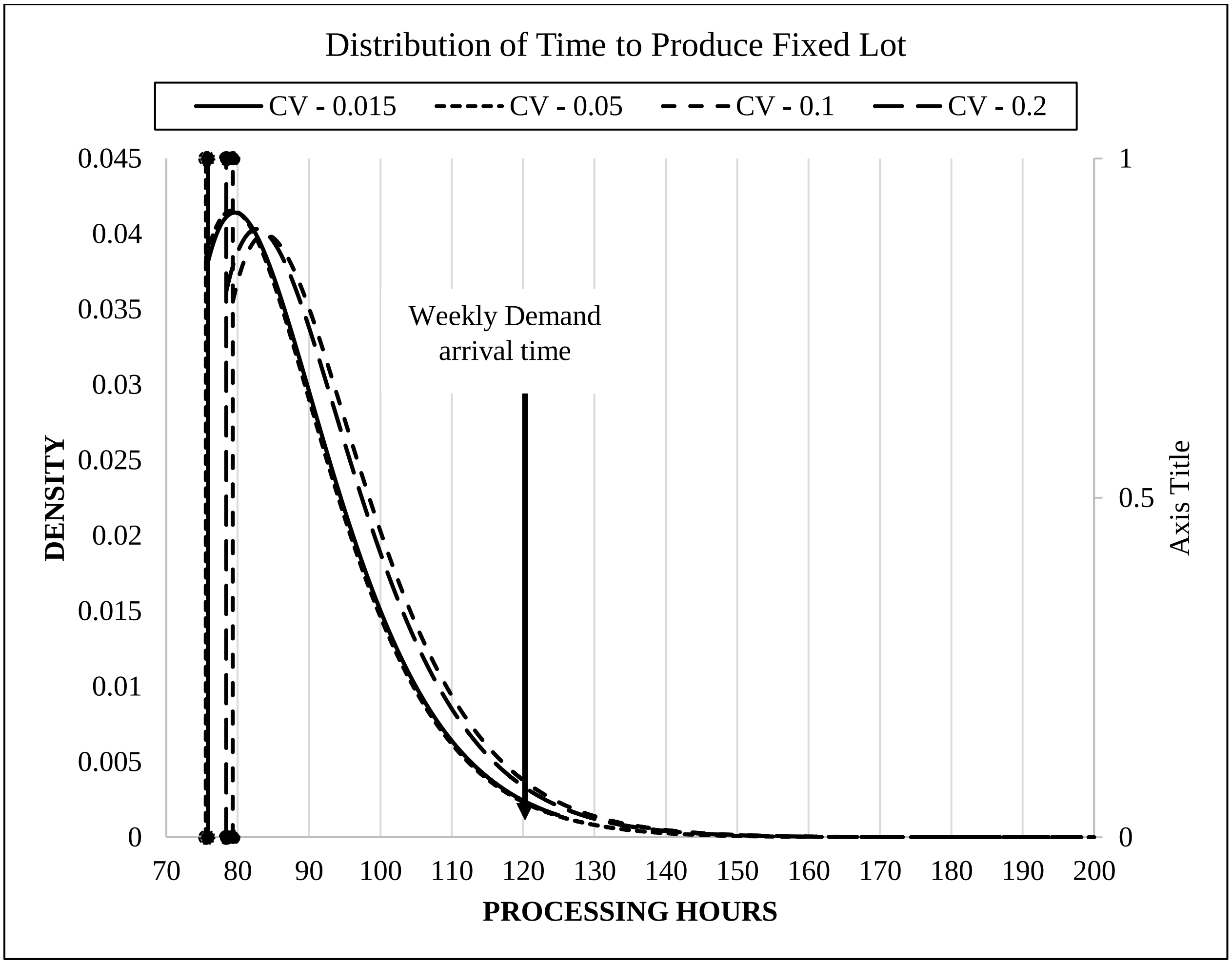

Figure 3.10 Density Functions - Time to Produce a Fixed Lot with 85% Workstation SAA

The variance in the time length to produce a fixed lot increases proportionally to lot size and by the square of the mean time to repair. From Figure 3.10 it is evident that due to large variability in the lot completion times, many lot completion times are late or occur after the demand arrival of 120 *processing hours*. Therefore, to satisfy the service level requirement, relatively large on-hand inventory needs to be maintained that drives higher inventory holding cost, and pushes the overall cost performance of a fixed lot system size lot to be greater than fixed-time lot system.

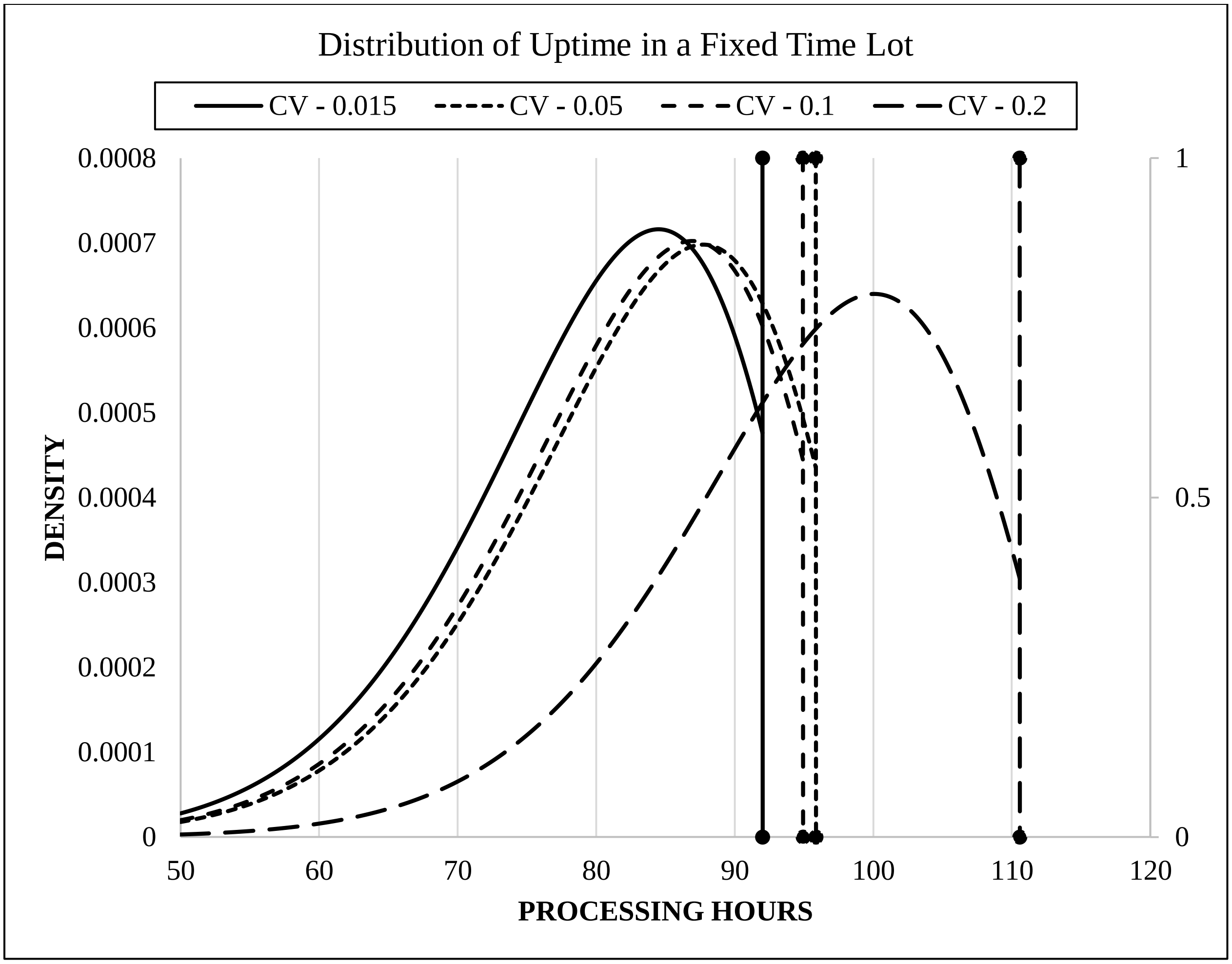

Figure 3.11 Density Functions - Uptime in a Fixed Time Lot with 85% Workstation SAA

Figure 3.11 shows similar behavior as seen in Figure 3.9 in the uptime distribution as the *CV of demand* increases. However, due to the lower *SAA* and larger *MTTR* the variability in the uptime in a fixed-time lot is relatively high. However, the probability of producing no or a very small amount is close to zero. This allows the fixed-time lot system to maintain a lower average inventory than a fixed lot system since the effect of production variability on the service level is not as severe. The lower average inventory levels leads to lower overall costs for a fixed-time lot system. In general, at lower *SAA* levels a fixed-time lot production system will be lower cost except at very low *CV of demand* levels.

3.6.2 Backordering Policy

For the backordering policy a 10/10/3 network architecture was constructed. The network was trained for 1000 epochs with a learning rate of 0.1. The FFNN prediction accuracy using the randomly selected testing data was approximately 82%.

The connection weight approach was utilized to obtain the relative importance/influence of each input factor as shown in Figure 3.12. The relative importance of the input factors as per connection weight approach is presented in Appendix 3E. Similar to the lost sales policy, for a backordering policy the demand process related factor, *CV of demand* has the largest relative importance/influence, and the production process related factor, *stand-alone availability (SAA)* has the second largest relative importance/influence , and for the fixed size-lot system inventory holding cost has high relative importance. To understand the relative importance in predicting the outcomes the several additional levels of the *CV of demand*, *SAA* were identified and simulated (Table 3.11). Because the other factors have lower relative contribution in predicting the outcomes, the levels of these factors were fixed. The lot size (*Q*), reorder point (*r*), and fixed time length (*T*) were optimized whenever an input factor changed.

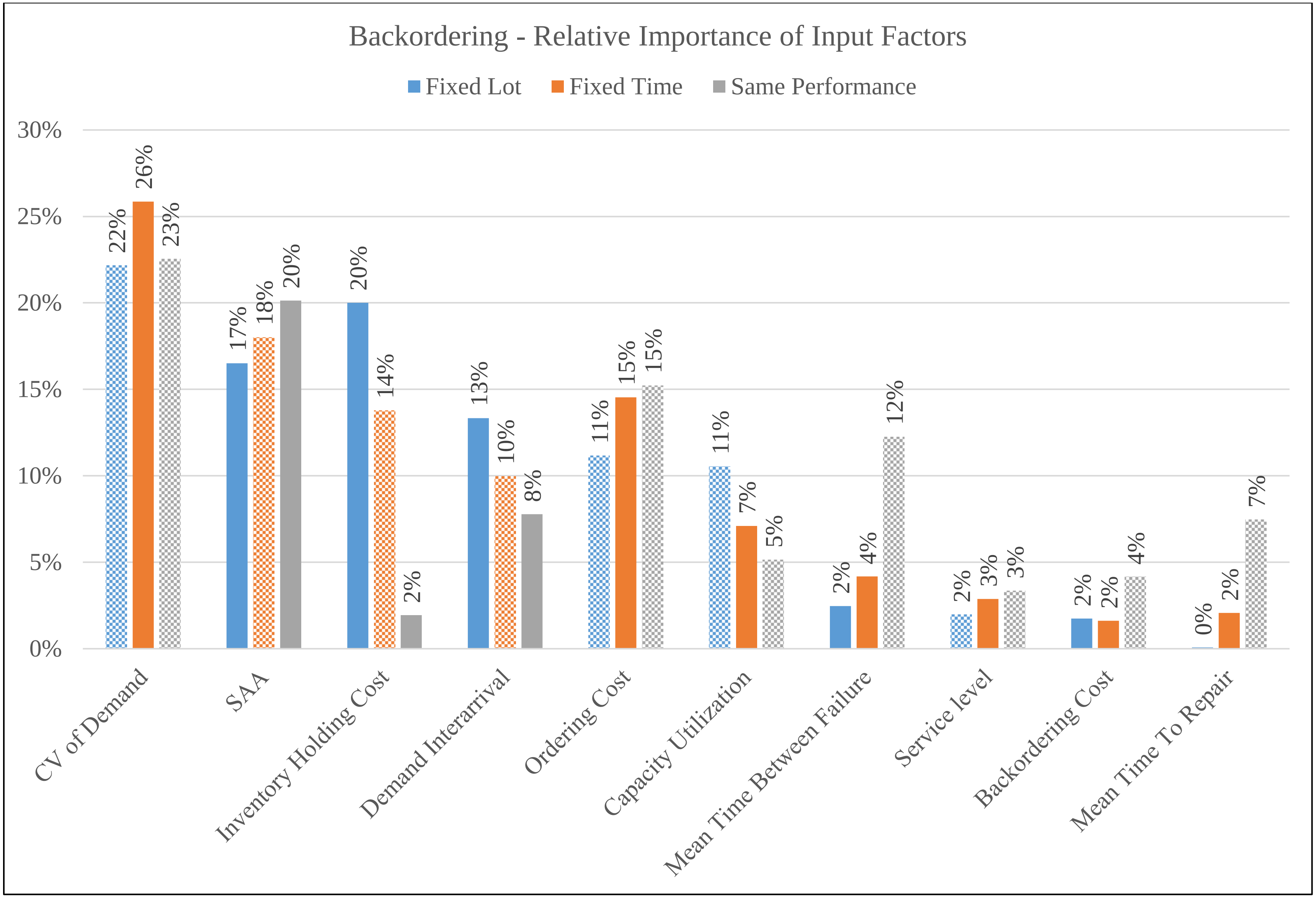

Figure 3.12 Backordering Policy - Relative importance for all input factors from the FFNN

Table 3.11 Backordering Policy - Levels of Inventory Holding Cost and SAA to develop insights

Input Factors			**Levels**		
Service Level			95%		
CV of Demand	0.015	0.05		0.1	0.2
Inventory Holding Cost		$1/*p.mins.*/month		$2/*p.mins.*/month	
Backordering Cost			$0.5/*p.mins.*/month		
Ordering Cost			$500/Order		
Demand interarrival			120 *processing hours*		
SAA (%)	98	95	90	80	75

In Figure 3.13 and Figure 3.14, using *boxes*, the combinations of *CV of Demand*, and *SAA* for which a particular production system's performance is better than the other or if the performances are not different, is summarized. At 100% *SAA*, and any level of *CV of demand*, the cost performance of the two batch production systems is not different. With the reduction in the SAA the difference in the cost performance of the two systems starts to appear. In Figure 3.13, for low *CV of demand* levels and high *SAA* levels there is no difference in the performance of the two batch production systems. For low *SAA* levels a fixed-time lot system performs better than fixed-size lot. At higher *CV of demand* and higher *SAA* levels the cost performance of the two batch production systems is not different. The backordering system here follows the similar mechanisms followed by lost sales as demonstrated in example 1 and 2.

In Figure 3.14, with *inventory holding cost* at high level, high *SAA* levels and low *CV of demand* levels, the performance of the two production systems is not different. However, with reduction in *SAA* with low *CV of demand*, the performance of fixed-size lot production system is better. With increase in *CV of demand*, even at higher *SAA* levels, fixed time is observed to be perform better. Similar to lost sales once the *SAA* drops below 90% there is a steep increase the in the long-run average cost performance of the fixed lot system. At higher *inventory holding cost* level both production control systems produces smaller frequent lots. However, with increasing *CV of demand*, reducing *SAA*, and to meet the service level requirements the fixed-size lot production control system results in an increased reorder level, increased lot size and relatively larger on-hand inventory to buffer against the demand and production variability. This drives higher inventory holding cost in fixed-size lot system, and pushes the overall cost performance of a fixed-size lot system to be greater than fixed-time lot system.

To better understand why the *demand interarrival* has higher relative importance, several levels of SAA and *demand interarrival* were identified and simulated (Table 3.11, Table 3.12). With reductions in the *SAA* the difference in the cost performance of the two systems is evident. For all *demand inter-arrivals* levels, and higher *SAA* levels the cost performance of fixed-size lot and fixed-time lot systems is not significantly different. As the *SAA* reduces, the fixed-time lot system results in significantly lower costs than the fixed-size lot system.

Table 3.12 Backordering Policy - Levels of Demand Interarrival and SAA to develop insights

<table>
<tr><th>Input Factors</th><th colspan="6">Levels</th></tr>
<tr><td>Service Level</td><td colspan="6">95%</td></tr>
<tr><td>Inventory Holding Cost</td><td colspan="6">$1/p.mins./month</td></tr>
<tr><td>Backordering Cost</td><td colspan="6">$0.5/p.mins./month</td></tr>
<tr><td>Ordering Cost</td><td colspan="6">$500/Order</td></tr>
<tr><td>Demand interarrival (p.hrs.)*</td><td colspan="2">24</td><td>48</td><td colspan="2">96</td><td>120</td></tr>
<tr><td>SAA (%)</td><td>98</td><td>95</td><td>90</td><td>85</td><td>80</td><td>75</td></tr>
</table>

* *Weekly demand variability was equal for all demand arrivals*

In Figure 3.15, using *boxes*, the combinations of *Demand Inter-arrival* and *SAA* for which a particular production system's performance is better than the other or if performances are not different, is shown. At high SAA level, the workstation can adopt either lot production system. Similar to lost sales once the *SAA* drops below 85% there is a steep increase the in the long-run average cost performance of the fixed-size lot system. This increase is attributed to the variability in production time lengths driven primarily by increased average repair lengths. If the workstation operates at a lower *SAA*, then a fixed-time lot production system will have significantly lower costs than a fixed lot system. The mechanisms leading to cost differences between fixed-size lot and fixed-time lot systems with backordering are similar to that for lost sales demonstrated in examples 1 and 2.

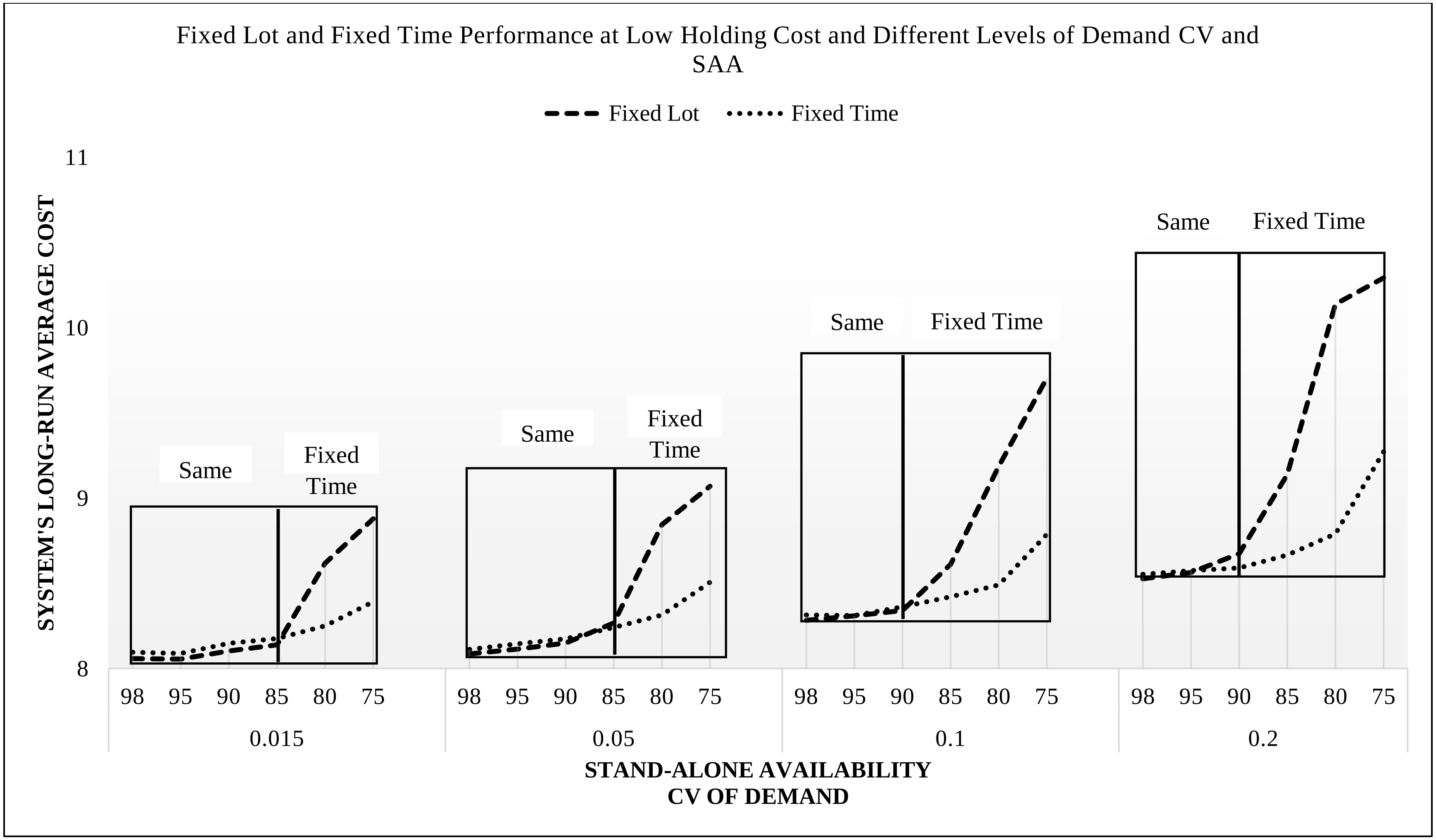

Figure 3.13 Backordering - Comparison of long-run average cost performance of fixed-size lot and fixed-time lot production systems at different levels of CV of demand, SAA, and low level of inventory holding cost

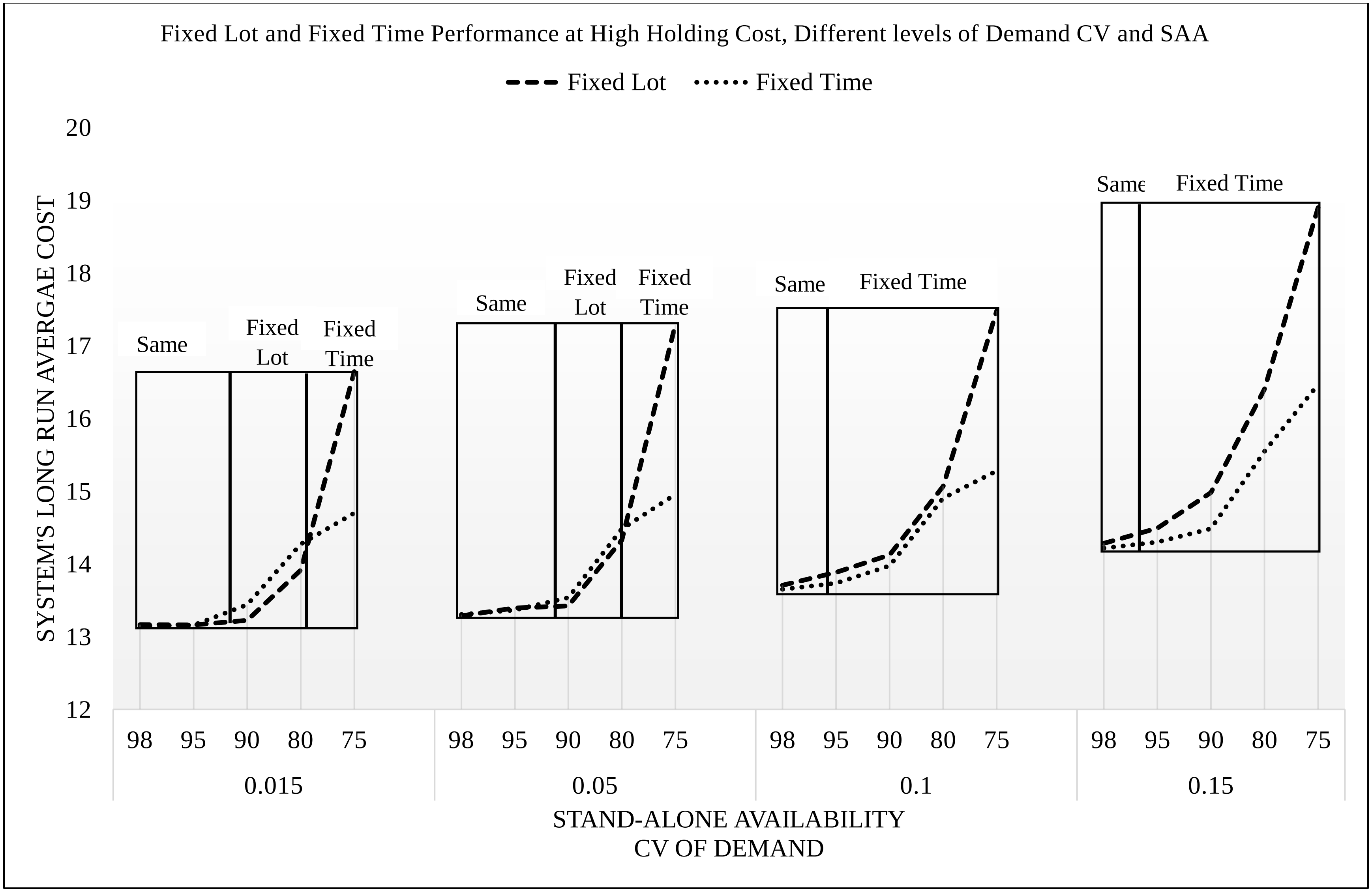

Figure 3.14 Backordering – Fixed-Size Lot and Fixed-Time Lot Performance at different levels of CV of Demand, SAA, and high level of inventory holding cost

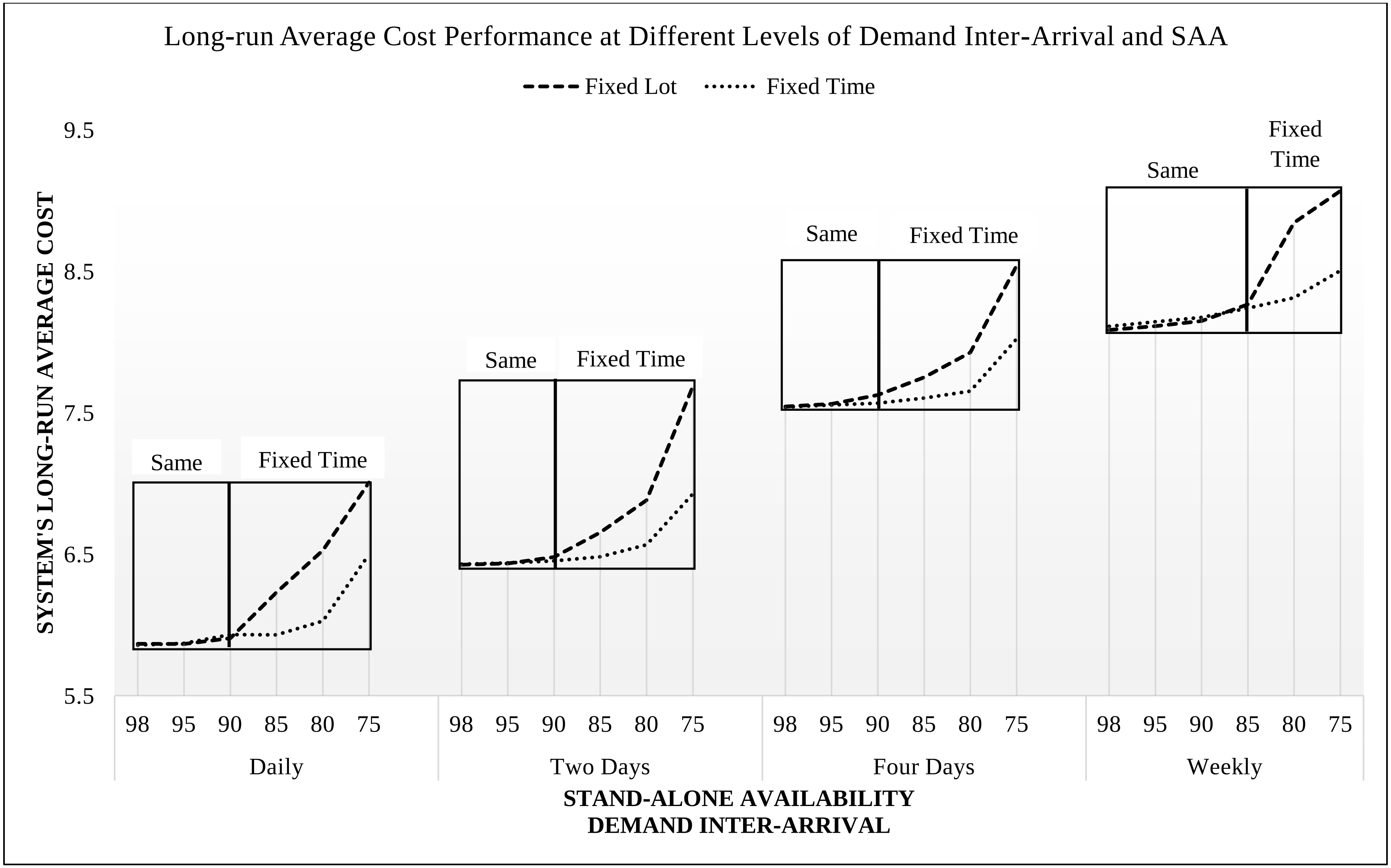

Figure 3.15 Backordering - Comparison of long-run average cost performance of fixed-size lot and fixed-time lot production systems at different levels of Demand Inter-arrival and SAA

3.6.3 Managerial Insights

The results obtained in this research are applicable to the systems analyzed here and can be extended to many other production systems that fall within the same experimental space (after conversion into the equivalent time units of production), and can serve as an aid to production managers.

The results presented clearly demonstrate the impact of *stand-alone availability,* and demand *coefficient of variation* on the performance of the two production control systems. If the workstation follows a (Q, r) inventory policy with high *stand-alone availability* then fixed-size lot production time variability is low enough so that production can operate in a "just-in-time" manner. Before each demand arrival, enough *processing minutes* from the completion of the fixed-size lot are added to on-hand inventory so that lost sales or backorders are minimal. However, with low workstation *stand-alone availability* and high *demand variability* many lots are completed after the demand arrival causing high lost sales or backorders.

Consider an example of a workstation operating under a fixed-size lot production control system where material handling resources are normally scheduled to move completed lots. If lot completion is frequently late so that expedited movement of partially completed lots occurs to avoid lost sales, then such a "boots on the ground symptom" suggests that a fixed-time lot production control system may be more cost effective in the short term. In general, when fixed lot production time is highly variable due to lower *stand-alone availabilities*, a fixed-time lot production system will often yield a lower long run average cost (Figure 3.7, Figure 3.13, and Figure 3.14). However, if and when the workstation *stand-alone availability* is improved, then a move back to fixed-size lot production control should be considered.

The fact remains that the use of fixed-time lot production control in commercial manufacturing environments is rare. The results obtained indicate that when the production environment has particular characteristics, fixed-time lot production control may be more cost effective. Some potential reasons a fixed-time lot production control system has not found widespread implementation in commercial manufacturing environments are listed:

1. The operations at many if not most high volume production systems, including stamping plants, have evolved over time to relatively lower period to period demand variability, and high workstation stand-alone availability. This combination of factors favors the use of fixed lot systems.
2. The impact of using a fixed time system on materials, material handling, and inventory management. Production and warehouse managers are now faced with uncertainty regarding the required quantity of *raw material*, the amount of material handling resources needed, and the storage space required for the raw material and the final product inventory.
    a. In stamping facilities, it is common to store the stamped parts in trolleys for their movement. Such trolleys are specifically designed to hold a fixed number of stamped parts and are expensive and require space to store when not in use. Thus the efficient use of such containers is important.
3. Fixed-time lot system may not be suitable for products whose raw material have short shelf life, especially for the products that are once out of storage and can't be returned such as product mix used to manufacture tires, paint containers for painting vehicle's body, raw material of pharmaceuticals, and certain chemicals that have to be stored under inert environment.

4. To a lesser degree, the scarcity of literature and case studies on a fixed-time lot production system lowers the awareness of its use as a\an alternative production control system.

On the other hand, there are other aspects of production operations that would operate more efficiently with a fixed-time lot system. Workstation setups/changeovers is one such example. Thus, in a more comprehensive comparison of fixed-size lot and fixed-time lot systems, the boundaries of the study should be expanded to include more supporting, upstream, and downstream processes in the whole production facility.

3.7 Conclusion and Future Work

This research examined experimental results comparing the performance of fixed-size lot and fixed-time lot production control systems to search for functional relationships between the input factors and the categorical outputs that indicate when a fixed-size lot or fixed-time lot batch production system should be utilized. A feedforward backpropagation neural network (FFNN) is applied to the experimental database from Tiwari and Kim (2022). The inference space for the FFNN results includes other production system that falls directly within the same experimental space, and many other system after the conversion of production system parameters to the equivalent time units of production.

The FFNN with connection weight approach is used to search for a model to predict three categorical outcomes, 1) a fixed-size lot system has significantly lower average cost than a fixed time system, 2) a fixed-time lot system has significantly lower average cost than a fixed-size lot system, and 3) the cost performance of the two systems is not significantly different. The magnitude and direction (positive and negative) of the connections weights is utilized to identify the relative importance of the input factors in predicting the outcome. The FFNN results show that for the lost sales policy the factors: demand coefficient of variation and workstation stand-alone availability have the highest relative importance in predicting the outcomes. For the batch production systems operating under the backordering policy the factors: demand interarrival time, stand-alone availability, and inventory holding cost have the highest relative importance in predicting the outcomes.

To better understand the mechanisms for how these factors influence the performance of the fixed-size lot and fixed-time lot production systems, additional levels of these factors were identified, and optimized (batch sizes and reorder levels in both fixed-size lot and fixed-time lot batch production systems were optimized) systems were simulated. In general for the lost sales policy, higher *stand-alone availability* levels and a lower demand *coefficient of variation* results in lower production time variability and more predictable demand so that the system can operate in a “just-in-time” manner with fixed-size lot production. This results in lower costs than a fixed-time lot system. Because this just-in-time operation breaks down as the *stand-alone availability* reduces and demand *coefficient of variation* increases, the fixed-time lot system results in significantly lower costs than the fixed-size lot system.

For the backordering policy the demand *coefficient of variation*, *stand-alone availability*, and *inventory holding cost* were observed to have the highest relative importance. In general, at lower *inventory holding cost* level, as the demand *coefficient of variation* increases, and *stand-alone availability* decreases, the fixed-time lot results in significantly lower costs that fixed lot system. The mechanics behind this is similar to lost sales. At higher *inventory holding cost* level both the fixed-size lot and fixed-time lot production control systems produces smaller frequent lots. However, with increasing *CV of demand*, and reducing *SAA* the fixed-size lot results in an increased reorder level, increased lot size and relatively larger on-hand inventory to buffer against the demand and production variability. The increased lot size increase the variability in the time length to produce a fixed lot due to this many lot completion times are late or occur after the demand arrival and the increased on-hand inventory pushes the cost performance of fixed-size lot production control system to be greater than fixed-time lot production control system.

3.7.1 Future work

In this research a single product is considered with a workstation that requires no changeover. Many plants consist of multiple workstations that require changeovers between the production of different products. The findings from this research can be extended to examine the performance of the two production control systems in a larger production environment where multiple workstations produce multiple product types, and each product has their own demand variabilities. Each workstation requires a changeover of random time length whenever a new product is produced. Changeovers are performed by setup crew resources shared among the workstations. Given the more predictable nature of fixed-time lot production, fewer setup resources may be required to meet the performance requirements of a fixed-time lot system.

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Appendix 3A

EXPECTATION AND VARIANCE FOR VARIOUS PRODUCTION LINES

Unless otherwise specified in the table, the following notations are used.

i	station number
p_i	Probability that station i may break down at the end of the cycle
r_i	Probability that down station i is repaired at the end of a cycle
λ	Failure rate
μ	Repair rate
t, T	Time length

Research	Production Line and Station Details	Expected Output	Variability in the Output
Carrascosa (1995)	Amount of material produced in t time steps on machine with two failure modes	Ct where, $C = \frac{1}{1+\sum_{i=1,2}\frac{p_i}{r_i}}$	$\left(C\,\frac{r_1^2 p_2(2-p_2-r_2)+r_2^2 p_1(2-p_1-r_1)-2p_1p_2r_1r_2}{(p_2r_1+r_2p_1+r_1r_2)^3}\right)t - 2C\,\frac{(2+b-p_1-p_2-r_1-r_2)((r_2-r_1)^2-(p_1+p_2-b)^2)}{b(-b+p_1+p_2+r_1+r_2)^3}(1-\beta_1^t) - 2C\,\frac{(-2+b-p_1-p_2-r_1-r_2)((r_2-r_1)^2-(p_1+p_2-b)^2)}{b(b+p_1+p_2+r_1+r_2)^3}(1-\beta_2^t)$ $b^2=(r_2-r_1)^2+(p_1+p_2)^2+2(r_2-r_1)(p_2-p_1)$ $\beta_1 = 1-\frac{p_1+p_2+r_2+r_1-b}{2}$ $\beta_2 = 1-\frac{p_1+p_2+r_2+r_1+b}{2}$

Research	Production Line and Station Details	Expected Output	Variability in the Output
	Output of a machine with a single failure mode in an interval of length t	$\frac{r}{r+p}t$	$\frac{rp}{(r+p)^2}\left(\frac{2}{r+p}-1\right)t - \frac{2rp}{(r+p)^4}(1-r-p)(1-(1-r-p)^t)$
Kletter (1996)	Uptime in length T of a single station initially in operating state	$\left(\frac{\mu}{\lambda+\mu}\right)T + \frac{\lambda}{(\lambda+\mu)^2}\left(1 - e^{-(\lambda+\mu)T}\right)$	$\frac{\lambda^2}{(\lambda+\mu)^4}\left(1-e^{-2(\lambda+\mu)T}\right) - \frac{4\lambda\mu}{(\lambda+\mu)^4}\left(1-e^{-(\lambda+\mu)T}\right) + \frac{2\lambda\mu}{(\lambda+\mu)^3}T\left(1+e^{-(\lambda+\mu)T}\right) - \frac{2\lambda^2}{(\lambda+\mu)^3}Te^{-(\lambda+\mu)T}$
	Uptime in length T of a single station initially in repair state	$\left(\frac{\mu}{\lambda+\mu}\right)T - \frac{\mu}{(\lambda+\mu)^2}\left(1 - e^{-(\lambda+\mu)T}\right)$	$\frac{\mu^2}{(\lambda+\mu)^4}\left(1-e^{-2(\lambda+\mu)T}\right) - \frac{4\lambda\mu}{(\lambda+\mu)^4}\left(1-e^{-(\lambda+\mu)T}\right) + \frac{2\lambda\mu}{(\lambda+\mu)^3}T - \frac{2\mu(\lambda-\mu)}{(\lambda+\mu)^3}Te^{-(\lambda+\mu)T}$
	Time length to produce fixed number of parts	$\frac{n\lambda}{S\mu}$	$\frac{2n\lambda}{S\mu^2}$ *n is the number of parts to be produced* *S is the production rate*
Kim and Alden (1997)	Time length to produce fixed number of parts	$\frac{n\lambda}{S\mu}+\frac{n}{S}$	$\frac{2n\lambda}{S\mu^2}$ *n is the number of parts to be produced*

Research	Production Line and Station Details	Expected Output	Variability in the Output
			S is the production rate
Tan (1998)	N identical stations with no buffer	$\frac{\mu^N}{(\lambda+\mu)^N}$	$\frac{2\mu^{2N}}{(\lambda+\mu)^{2N+1}} \sum_{k=1}^{N} \binom{N}{k} \frac{\left(\frac{\lambda}{\mu}\right)^k}{k}$
	M parallel stations with no buffer	$\frac{(\lambda+\mu)^M - \lambda^M}{(\lambda+\mu)^M}$	$\frac{2(\lambda^M[(\lambda+\mu)^M - \lambda^M])}{(\lambda+\mu)^{2M+1}} \sum_{m=1}^{M} \binom{M}{m} \frac{(-1)^{m+1}}{m}$
	M parallel with N identical stations with no buffer	$\frac{\mu^N}{(\lambda+\mu)^N} \frac{(\lambda+\mu)^M - \lambda^M}{(\lambda+\mu)^M}$	$\left(\frac{2(\lambda^M \mu^{2N}[(\lambda+\mu)^M - \lambda^M])}{(\lambda+\mu)^{2(M+N)+1}}\right) \times \left(\sum_{m=1}^{M} \binom{M}{m} \frac{(-1)^{m+1}}{m} + \sum_{k=1}^{N} \binom{N}{k} ((\lambda+\mu)^M - \lambda^M) \left(\frac{\lambda}{\mu}\right)^k / \lambda^M k + \sum_{k=1}^{N} \sum_{m=1}^{M} \binom{M}{m} \binom{N}{k} \frac{(-1)^{m+1} \left(\frac{\lambda}{\mu}\right)^k}{m+k}\right)$

Research	Production Line and Station Details	Expected Output	Variability in the Output
Tan (1998)	N-stations in series with no intermediate buffer	$\frac{\mu^N}{(\lambda+\mu)^N}$	$\left[\frac{\mu}{(\lambda+\mu)}\right]^N \sum_{n=1}^{N}\sum_{m=0}^{n}\binom{N}{n}\binom{n}{m}\left[\frac{\mu}{(\lambda+\mu)}\right]^{N-n} \times \left[\frac{\lambda\left(\sqrt{s}-2\mu p+\lambda\right)}{2\sqrt{s}(\lambda+\mu)}\right]^{n-m} \times \left[\frac{\lambda\left(\sqrt{s}+2\mu p-\lambda\right)}{2\sqrt{s}(\lambda+\mu)}\right]^{m}\left(\frac{1}{\alpha n-(\beta-\alpha)m}\right)$ $s=(\lambda+2\mu)^2-4\mu p(3\lambda+2\mu-\mu p)$ $\alpha=\lambda+2\mu(1+p)+\sqrt{s}$ $\beta=\lambda+2\mu(1+p)-\sqrt{s}$ *p is the probability that repair job, after receiving service, stays in the same stage for another service*
Tan (1999)	Transfer line with two heterogeneous stations with no buffer	-	$\frac{r_1r_2}{(p_1+r_1)^2(p_2+r_2)^2}\left[p_1r_2\frac{(2-p_1-r_1)}{p_1+r_1} + p_2r_1\frac{(2-p_2-r_2)}{p_2+r_2} + p_1p_2\frac{\left(2-(p_1+r_1)-(p_2+r_2)+(p_2+r_2)(p_2+r_2)\right)}{p_1+r_1+p_2+r_2-(p_2+r_2)(p_2+r_2)}\right]$
	Homogeneous transfer line with N	-	$\frac{r^{2N}}{(p+r)^{2N}}\sum_{j=1}^{N}\binom{N}{j}\frac{\left[1+(1-p-r)^j\right]}{\left[1-(1-p-r)^j\right]}\left(\frac{p}{r}\right)^j$

Research	Production Line and Station Details	Expected Output	Variability in the Output
	identical stations with no buffer		
Tan (1999)	Output of a single machine in a given time interval [*0*, *t*)	$\frac{r}{r+p}t+\frac{\lambda p-(1-\lambda)r}{(p+r)^2}(1-(1-p-r)^t)$ λ = 0, machine initially down λ = 1, machine initially up	$\frac{pr(2-p-r)}{(p+r)^3}t+\left[\frac{2\lambda^2}{(p+r)^2}+\lambda\frac{(p-r)(1-2t)-2}{(p+r)^2}+\frac{r(r^2+2p^2t-2pr-2r^2t-3p^2+4p}{(p+r)^2}\right](1-p-r)^t+\left[\frac{-\lambda^2}{(p+r)^2}+\lambda\frac{2r}{(p+r)^2}-\frac{r^2}{(p+r)^4}\right](1-p-r)^{2t}+\left[-\frac{\lambda^2}{(p+r)^2}-\lambda\frac{p^2-2p-r^2}{(p+r)^2}+r\frac{(-r^2+2pr+3p^2-4p)}{(p+r)^4}\right]$
Assaf et al. (2014)	Output of single machine in a given time interval [*1*, *t*]	-	$\sigma_Y^2\left[\frac{t-t\rho^2-2\rho+2\rho^{t+1}}{(1-\rho)^2}\right]$ $Y\ is\ a\ random\ variable\ of\ stationary\ output\ process$ $\rho\ is\ the\ autocorrelation\ function$

Research	**Production Line and Station Details**	**Expected Output**	**Variability in the Output**
	Output of Bernoulli machine	-	$(1-p)p.t$
	Single machine with multiple failure modes	-	$te(1-3e)+2t\pi\mu_{diag}PZ\mu+2\pi\mu_{diag}(P^{t+1}-P)Z^2\mu$ $e\ is\ the\ mean\ production\ rate\ \left(\frac{r}{r+p}\right)$ $P\ is\ the\ probability\ transition\ matrix$ $\mu_{diag}\ is\ a\ diagonal\ matrix\ with\ rewards\ in\ the\ diagonal$ $Z\ is\ the\ fundamental\ matrix\ given\ by\ (I-P+A)^{-1}$
Rismanchian and Lee (2018)	Output of single station in time length T	$\frac{\pi\begin{bmatrix}\left(\frac{c_2}{2}-a_1c_1\right)s_0e^{T.s_1}+\left(c_1^2-\frac{c_2}{2}\right)s_1e^{T.s_0} \\ -s_1c_1^2+s_0a_1c_1(1+s_1T)-\frac{c_2}{2}(s_0-s_1)\end{bmatrix}}{c_1^2s_1s_0}$ $a_1, a_2, a_3\ and\ b_1, b_2, b_3$ are the first three raw moments of distribution function $F_U(t)$ and $F_D(t)$ respectively $c_1=\ a_1+b_1$ $c_2=\ a_2+b_2+2a_1b_1$ $c_3=\ a_3+3a_1b_2+3a_2b_1+b_3$	Asymptotic variance rate $\frac{2\pi^2a_1(a_1c_1s_0-c_1^2s_1-\frac{c_2}{2}(s_0-s_1))}{c_1^3s_1s_0}$ $s_0=-\frac{6(c_2-2c_1^2)c_1}{2c_1c_3-3c_2^2}$ $s_1=-\frac{6(c_2-2a_1c_1)c_1}{2c_3c_1-3c_2^2+6c_2c_1a_1-6a_2c_1^2}$

Appendix 3B

VARIANCE OF UPTIME IN A FIXED TIME LENGTH

Using the expected uptime and variance equations provided by Kletter (1996), expression for variance in uptime in a fixed time length is derived.

$$Var(X) = E[Var(X|Y)] + Var[E(X|Y)]$$

Therefore, the variance of uptime in a fixed time length can be computed as

$$Var(Uptime) = \left[Var\left(Uptime_{up}\right) * SAA + Var(Uptime_{down}) * (1 - SAA)\right]$$
$$+ Var[E(Uptime|up, or\ down)]$$

$$Var[E(Uptime|up, or\ down)]$$
$$= \left[E\left(Uptime_{up}\right)^2 * SAA + E(Uptime_{down})^2 * (1 - SAA)\right]$$
$$- \left[E\left(Uptime_{up}\right) * SAA + E(Uptime_{down}) * (1 - SAA)\right]^2$$

$$= \frac{\mu^2\lambda - 2\mu^2\lambda * \exp^{-(\lambda+\mu)T} + \mu^2\lambda\, exp^{-2(\lambda+\mu)T} + \mu\lambda^2 - 2\mu\lambda^2 exp^{-(\lambda+\mu)T} + \mu\lambda^2 exp^{-2(\lambda+\mu)T}}{(\lambda+\mu)^5}$$

$$\left[Var\left(Uptime_{up}\right) * SAA + Var(Uptime_{down}) * (1 - SAA)\right] =$$

$$\frac{\left(\lambda^2\mu\left(1 - exp^{-2(\lambda+\mu)T}\right) - 4\lambda\mu^2\left(1 - exp^{-(\lambda+\mu)T}\right) + \left(2\lambda\mu T\left(1 + exp^{-(\lambda+\mu)T}\right) - \left(2\lambda^2 T exp^{-(\lambda+\mu)T}\right)\right) * (\lambda\mu + \mu^2)\right)}{(\lambda+\mu)^5}$$

$$+$$

$$\frac{\left(\mu^2\lambda\left(1-exp^{-2(\lambda+\mu)T}\right)-4\lambda\mu^2\left(1-exp^{-(\lambda+\mu)T}\right)+\left(2\lambda\mu T+2\left(\mu T(\lambda-\mu)exp^{-(\lambda+\mu)T}\right)\right)*(\lambda\mu+\lambda^2)\right)}{(\lambda+\mu)^5}$$

Combining$\left[Var\left(Uptime_{up}\right)*SAA+Var(Uptime_{down})*(1-SAA)\right]$,

$Var[E(Uptime|up, or\ down)]$ and simplifying

$$Var(Uptime)=\frac{2\mu\lambda\left[(2\mu\lambda T+\mu^2T+\lambda^2T)-(\lambda+\mu)\left(1-exp^{-(\lambda+\mu)T}\right)\right]}{(\lambda+\mu)^5}$$

$$Var(Throughput)=\left(\frac{2\mu\lambda\left[(2\mu\lambda T+\mu^2T+\lambda^2T)-(\lambda+\mu)\left(1-exp^{-(\lambda+\mu)T}\right)\right]}{(\lambda+\mu)^5}\right)S^2$$

where, *S* is the processing speed of the workstation The variance in uptime expression divided by T approaches $2\lambda\mu/(\lambda+\mu)^2$ as T approaches infinity, and approaches zero as T approaches zero.

Appendix 3C

Initialize:

1. Set number of folds for the *k-fold* cross validation
2. Set learning rate
3. Set number of epochs
4. Set number of hidden layers, and number of neurons in each layer
5. Scale all inputs between [0,1)
6. Select a transfer/activation function
7. Assign random weights between [0,1] to all connections between input → hidden and hidden → output neurons
8. Assign a random bias between [0,1] to each neuron in hidden and output layers

Feedforward Propagation:

9. Calculate the weighted sum of inputs

$$w = \sum(weight_i * input_i) + bias$$

10. Use transfer function to compute the output of the neuron.

$$neuron_j = \sigma(W * neuron_{j-1} + b)$$

11. Repeat *a.* and *b.* for hidden → output layer

Backpropagation:

12. Use the derivative of sigmoid transfer function to calculate the slope of output value of neuron

$$sigmoid\ function\ derivative = neuronValue_{output} * \left(1 - neuronValue_{output}\right)$$

13. Calculate the error of each neuron in output layer

$$error_{output \to hidden} = (neuronValue_{output} - neuronValue_{expected}) * sigmoid\ function\ derivative$$

14. Calculate the error value of neurons in hidden layer

$$error_{hidden \to input} = \left(weight_j * error_k\right) * derivative\ of\ activation\ function$$

where

$weight_j$ – weight connecting neuron *j* to current neuron

$error_k$ – error from neuron *k* of output layer

15. Update Weights and Bias

$$weight_{updated} = weight_{existing} - learningRate * error * input$$

$$bias_{updated} = bias_{existing} - learningRate * error$$

16. Repeat steps *9a*. through *9c.*for fixed number of epochs

Predictions on Testing Data:

17. Utilize updated weights and biases from feedforward backpropagation
18. For each row of test data set, compute neuron values of hidden and output layers
19. Select index value in output layer with largest probability as prediction

Appendix 3D

The mathematical equation relating the input factors and the output can be written as (Goh et al., 2005),

$$Y = f_{sig}\left\{b_o + \sum_{k=1}^{h}\left[w_k f_{sig}\left(b_{hk} + \sum_{i=1}^{m} w_{ik} X_i\right)\right]\right\}$$

Where,

b_o bias at output layer

w_k weight connection between neuron *k* of the hidden layer and the output neuron

b_{hk} bias at neuron *k* of the hidden layer

w_{ik} weight connection between input factor *i* and neuron *k* of hidden layer

X_i input parameter *i*, normalized in the range (0, 1)

f_{sig} transfer/activation function

Using the values of connection weights and biases are obtained from the trained neural network, the following expression can be written

$$A_i = h_i + \sum_{j=1}^{n} w_j N_j$$

Where,

h_i bias of the *j*th hidden layer neuron

w_j connection weight between the *j*th input layer neuron and *i*th hidden layer neuron

N_j *i*th normalized input parameter

n number of input neurons

$$B_i = \frac{w_k}{[1 + \exp(-A_i)]}$$

where,

w_k weight connection between neuron *k* of the hidden layer and the output neuron

$$C_1 = b_o + \sum B_i$$

where,

b_o bias at output layer

$$Y = \frac{1}{[1 + \exp(-C_1)]}$$

The *Y* value obtained is in the range of [0, 1]. The above expressions and the connection weight values should be applied in the range of the dataset for which the neural network was trained.

Appendix 3E

Lost Sales – Relative importance of the input factors using connection weight approach

Input Parameters	Outcome 1	Outcome 2	Outcome 3
CV of Demand	-236.05 (1)	208.46 (1)	76.51 (1)
SAA	68.62 (2)	-96.44 (2)	46.07 (2)
Capacity Utilization	-63.66 (3)	34.98 (7)	39.16 (3)
Inventory Holding Cost	39.70 (5)	-49.84 (4)	14.14 (7)
Demand Interarrival	39.55 (6)	-24.42 (8)	-14.13 (8)
Service level	-50.97 (4)	75.72 (3)	-28.12 (4)
Mean Time Between Failure	-5.62 (9)	10.55 (9)	-9.86 (9)
Mean Time To Repair	-33.30 (7)	46.06 (5)	-19.50 (6)
Ordering Cost	-29.43 (8)	40.90 (6)	-25.83 (5)

Backordering - Relative importance of the input factors using connection weight approach

Input Parameters	Outcome 1	Outcome 2	Outcome 3
Demand Interarrival	90.22 (4)	-84.72 (5)	27.97 (5)
SAA	111.70 (3)	-153.05 (2)	72.33 (2)
Inventory Holding Cost	135.37 (2)	-117.01 (3)	6.97 (10)
Backordering Cost	11.81 (9)	13.75 (10)	-14.97 (8)

Input Parameters	Outcome 1	Outcome 2	Outcome 3
Mean Time Between Failure	16.72 (7)	35.50 (7)	-44.04 (4)
Service level	-13.37 (8)	24.52 (8)	-12.07 (9)
Ordering Cost	-75.53 (5)	123.52 (4)	-54.71 (3)
Mean Time To Repair	-0.53 (10)	17.58 (9)	-26.81 (6)
CV of Demand	-149.97 (1)	219.80 (1)	-80.98 (1)
Capacity Utilization	-71.32 (6)	60.36 (6)	-18.50 (7)

4 Conclusion and Future Work

The main objective of this research is to develop useful insights for understanding when a fixed-size lot or fixed-time lot production control systems should be utilized based on minimizing long-run average cost with a constraint on the service level. The batch production systems considered are workstations operating under a continuous review (Q, r) inventory system where the order quantities Q, and reorder levels r are optimized to minimize the long-run average costs. When the workstations are perfectly reliable, the fixed-size lot and fixed-time lot production control systems converge since production times are assumed constant, and thus have the same long-run average cost. However, the workstation considered are unreliable and additionally the demand processes are variable.

Due to the availability of an extensive body of knowledge and ubiquitous use of the fixed-sized lot systems in practice, the expectation at the beginning of this research was that, in the absence of considering the support activities such as setup-crews, the fixed-size lot production control system will outperform the fixed-time lot system. However, the results obtained in Chapter 2 demonstrated that this is not the case. It was observed that there are several factors that can drive the long run average cost performance of the fixed-time lot production control system to be superior to the fixed-size lot production control system. This observation motivated the work in Chapter 3, where the functional relationships between the input factors and the output were explored.

Because of the complexity of the systems considered, the work done in Chapter 2 was experimental. The experimental objectives were to identify the factors that have the largest effect on the long-run average cost differences between the fixed-size lot and fixed-time lot production control systems. Discrete event simulation models were created for both fixed-size lot and fixed-time lot production control systems. Multiple factors that are known to drive the cost of such systems were identified. 2^8 and 2^9 experiments were conducted for the lost sales and backordering policies of (Q, r) inventory system of the fixed-size lot and fixed-time lot production systems. This required 1,536 different systems to be simulated. Simulation optimization models were developed to optimize the batch sizes and reorder point levels of each system simulated. System parameters such as demand sizes, lot sizes Q, reorder levels r, on-hand inventory, and backordered inventory are expressed in their equivalent time units of production. This conversion expands the inference space of the results obtained in this research to many other production systems that falls with the same experimental space. The simulation models were validated by comparing the simulation results with available analytical results for the special cases of the systems considered.

The experimental results from Chapter 2 show that for a lost sales systems the *interarrival time between demands*, and the *coefficient of variation* of the demand probability distribution have by far the largest impact on the long-run average cost difference between fixed-size lot and fixed-time lot batch production systems. If the workstation operates under lower *demand variability*, and lower workstation *stand-alone availability* (more variable production), the fixed-time lot batch production system performed better than the fixed lot batch production system. However, if the *coefficient of variation* of the demand probability distribution is low, and *demand interarrival time* is large the fixed lot batch production system had lower cost. For the batch production systems operating under a backordering policy the experimental results show that the workstation *stand-alone availability*, *failure and repair frequency*, and *capacity utilization* have the largest impact on the long-run average cost difference between fixed-size lot and fixed-time lot batch production systems. If the workstation operates under low *capacity utilization* (workstation is underutilized) and a high *stand-alone availability* the fixed lot batch production system had lower cost. However, the fixed-time lot batch production system was preferred when the workstation operates with high *capacity utilization* and lower *stand-alone availability.*

The use of ANOVA in Chapter 2 focused on factor effects, and not on functional patterns or relationships that may be established between the various input factors and the output. In Chapter 3, a feedforward backpropagation neural network is utilized to further analyze the experimental database obtained from Chapter 2 and capture the relationships between the input factors and the output. The neural network considered consists of three layers: input, hidden, and output layers with input and hidden layer having a sufficient number of neurons. The output layer contains three neurons that represent three categorical outcomes: 1) a fixed-size lot system has lower average cost than a fixed-time lot system, 2) a fixed-time lot system has lower average cost than a fixed-size lot system, and 3) the cost performance of the two systems is not different. A connection weight approach was utilized to identify the relative importance of the input factors in predicting the outcomes. The input factors with large connection weights represent a higher relative importance on the prediction of the response than factors with small connection weights.

The results show that for lost sales policy the factors: demand coefficient of variation and stand-alone availability of the workstation have the highest relative importance in predicting the outcomes. To better understand the mechanisms for how these factors influence the performance of the fixed-size lot and fixed-time lot production systems, several levels of these factors were identified, simulated, and summarized. In general, for lost sales policy at higher stand-alone availability levels and lower demand coefficient of variation the production time to produce a fixed lot is low enough that the system can operate in a "just-in-time" manner a fixed-size lot production system will result in lower costs than a fixed-time lot system. However, as the *stand-alone availability* reduces and demand *coefficient of variation* increases, the fixed-time lot system results in significantly lower costs than the fixed-size lot system. For the batch production systems operating under the backordering policy the factors: demand coefficient of variation, stand-alone availability, and inventory holding cost have the highest relative importance in predicting the outcomes. The mechanism leading to the cost performance of fixed-size lot and fixed-time lot systems with backordering are similar to that of lost sales. In general, at the higher inventory holding cost, the lots produced are smaller and frequent, however with increased *CV of demand* and reduced *SAA*, the fixed-size lot production control system results in relatively larger on-hand inventory to buffer against the demand and production variability. This drives higher total cost for the fixed-size lot results than fixed-time lot production control system.

From the results presented in this research a deeper understanding and insights are gained on the performance trends of the fixed-size lot and fixed-time lot production control systems. In this research a single product with a workstation that requires no changeover between production of different products is considered. In the future the findings from this research can be extended to compare the performance of the two production systems in a larger real production environment with multiple workstations where the workstation produces multiple products, with their individual demand variabilities. Whenever a new product is produced, each workstation requires a changeover of random time length that are performed by set up crew resources, shared with other workstations. Given the predictable nature of fixed-time lot production, the setup crew resources and other supporting activities maybe efficiently scheduled to meet the performance requirements. Thus the boundaries of the future study should be expanded to include more supporting, upstream and downstream processes in the whole production facility.

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