Examining the Relationship between Intersection Density and the Priorities of those Traveling the Intersection

By

Daniel Wisnewski

A THESIS

submitted to

Oregon State University
Honors College

in partial fulfillment of
the requirements for the
degree of

Honors Baccalaureate of Science in Industrial Engineering
(Honors Scholar)

Presented March 1, 2021
Commencement June 2021

AN ABSTRACT OF THE THESIS OF

Daniel Alexander Wisnewski for the degree of Honors Baccalaureate of Science in Industrial Engineering presented on March 1, 2021

Title: Examining the Relationship between Intersection Density and the Priorities of those Traveling the Intersection

Abstract approved: ____

Javier Calvo-Amodio

An intersection without a traffic light could be a potentially dangerous place to travel through depending on the mindsets of the of the travelers within it. When travelers are traversing an intersection, they generally have one of two different mindsets that influence the actions they do. One mindset they could have is being focused on saving time as they travel the intersection, causing them to take actions that speed them through while the other is being focused on one's own safety and taking actions that follow the rules of the intersection to keep themselves and others around them out of accidents. Which mindset a traveler has as they travel through the intersection depends largely on how full the intersection is with other cars and pedestrians. This thesis creates an agent-based model to simulate an intersection found on Monroe Ave. on the Oregon State University campus. This model is then used in a series of tests that fills the model with various numbers of agents to test how the intersection runs under various intersection densities. Data is recorded from each test and fitted onto line graphs which are then compared to each other to look for the approximate density where the mindset of a traveler should switch from being focused on saving time to being focused on personal safety. A conclusion was reached after testing the model under five different density levels and comparing the line graphs from the data gathered from those tests to look for the ideal behavior that the target density would have exhibited.

Key Words: intersection, agent-based modeling system, intersection simulation
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I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

Daniel Alexander Wisnewski, Author

ACKNOWLEDGEMENTS

Thank you to Dr. Calvo for holding weekly meetings for us to meet regularly and for guiding me in the directions I needed to research and complete this Thesis.

Thank you to Neal Wilkinson for meeting with me twice a week and helping me organize my thoughts onto paper so that my thesis was not conveyed in a jumbled mess.

Thank you to Dr. Vergara for being on my thesis defense committee and for asking questions that helped me revise my thesis for the better.

Thank you to Malaia Jacobsen for attending my thesis defense practice and for providing advice that helped my presentation flow better.

Thank you to my parents who supported me and loved me throughout my academic career.

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Chapter 1. Introduction

1.1 Background

Within the United States, it is a common occurrence when pedestrians must cross at an intersection with no lights to govern car traffic. Sometimes, pedestrians may be so fixated on getting to their next destination that they do not focus on their current circumstances. Other times, they may think a lot about where they currently are and follow the rules of safety when crossing the road. How much a pedestrian or a driver pays attention to the rules of the road as they navigate an intersection can vary depending on several different factors, but one thing that is common among all of them is that they are not operating in a vacuum. Each pedestrian and driver navigating an intersection must be mindful of the other people that are crossing the intersection at the same time they are. The more people in the intersection at the same time, the more mindful of safety each individual needs to be to navigate without an accident. The point where the greatest shift in mindset occurs is dependent on an unknown number of pedestrians and vehicles that needs to be investigated.

1.2 Problem Statement

Many travelers at Oregon State University navigate a certain intersection on Monroe Avenue that is at the top of the campus. A lot of these travelers navigate it every day and all of them share the same goal of not crashing as they traverse the intersection. However, there exists a variety of factors like weather or time of day that can cause those entities to have different priorities and may affect their mindset as they cross the intersection. Ultimately, these factors can cause the entity in question to have their priorities set on one of two variables as they cross. Do they prioritize acting in a safe manner as they cross, or do they value shorter times when crossing in a way that they may lessen their own safety to shorten their traversal time? One of the most important factors that may influence their priorities is the density of the intersection at the moment an entity arrives to it. Therefore, just how dense does the intersection have to be to get an entity's priorities to shift from prioritizing their traversal time to prioritizing their safety?

1.3 Research Question

When first researching the topic for this paper, the question to answer was "When were travelers safe enough to focus on saving time when crossing the intersection and when did they need to pay more attention to the rules to stay safe?" The main factor that was investigated to answer this question was density of the intersection, and just how full of other travelers it had to be to invoke these changes in mindsets. As the procedure for the study developed, it was decided upon that a model would be created to simulate the Monroe intersection based on intersection density. The research question evolved into "At what density would the intersection be at so that travelers within them switch from a time-saving oriented mindset to a safety-focused mindset?" This way, the model could use intersection density specifically to solve the question rather than focus on naming specific time frames or a general set of intersection conditions.

1.4 General Hypotheses

To answer the research question, an objective way of measuring density needed to be created for this study. This way, the answer could be given in a quantifiable form without any subjective descriptors that could be interpreted in different ways. The study will go into more detail about what these terms means later, but the hypothesis for this study is that the model will have a model density of around 40% with an agent composition of 8 parts cars and 16 parts pedestrians when it is at the model density where travelers will switch their mindsets. This translates to the intersection being about 40% full of travelers, when they decide to switch their mindsets. The composition of the travelers that are in the intersection will be roughly 33% cars and 67% pedestrians.

1.5 Research Purpose

Currently there are numerous studies that study how various intersection factors affect the people who travel through them. However, there are far fewer studies that look at specific tipping points where the behavior of the people drastically in response to one of the intersection factors changing. The purpose of this study is to provide the framework for studying one of these tipping points so that other factors can be observed and researched in different studies in a similar manner. Aside from providing an example for other types of research, finding an answer to the research question in this study will help in identifying scenarios where intersection travelers should focus staying safe over saving time.

1.6 Research Objective

The objectives of this research are as follows:

1. Find the tipping point in terms of intersection density where a traveler's mindset switches from prioritizing saving time to focusing on personal safety.
2. Create a model that can be adapted to other types of intersections to find their tipping points in switching mindsets.

1.7 Limitations and Assumptions of the Model

Below is a list of the limitations of the model that was created as well as some assumptions the model is working under:

- The individual agents in the model have no free will and are bound by the code they are programmed with. They cannot dynamically change their behavior like a real human would.
- The model is taking place in a neutral environment where the only factor affecting agent behavior is intersection density. This is unlike real life where there are a variety of other factors that can affect agent behavior.
- The only entity types that are programmed into the model are cars and pedestrians. There are other types of entities that could cross the intersection in real life like buses or bicycles.
- Each test is run at a static density level where the density of the intersection is kept at a constant level. It cannot change densities mid test like the intersection in real life could.

1.8 Relevance of this Study

As long as roads are used to serve as transportation routes from one location to another, intersections between two or more roads will be formed. It is impractical and costly to set up traffic lights at every single intersection that is formed in the world, so there will be a ton of intersections that will be without an external regulator. Finding the density of the Monroe intersection where traveler's mindsets switch priorities will produce a procedure that can be used to find the densities of other similar intersections. If roads are still used and intersections are formed, this study will produce results that will still be relevant. The results will be useful in facilitating research into intersection behavior and the conditions that cause them.

Chapter 2. Literature Review

2.1 Literature Review Introduction

The use of cars is an everyday occurrence in the lives of many people, and thousands of people drive through the same intersection on any given day. Likewise, pedestrians walking to their destination is another common occurrence and leads them to travel though the same intersections that cars travel through at often the same or near the same times. With the interaction between these two occurring regularly at intersections, the likelihood of an accident occurring at an intersection is high. Within the US alone, there were 33,654 fatal crashes that occurred in 2018 (keeping in mind that this includes all United States car crashes) (Administration, 2020). Using already published papers about these intersection interactions, it is possible to form inferences on how pedestrians and cars behave under different environmental, circumstantial, or personal characteristics. These inferences could be implemented into plans to lessen the number of crashes that occur each year. The following sections provide an overview of the most relevant interactions within intersections.

2.2 Intersections and pedestrian vs. car density tradeoff

Before the 2020 COVID-19 pandemic, it was possible to observe patterns of behavior at intersections directly. It was possible to observe pedestrians and cars becoming more cautious or more brazen based on the increasing or decreasing number of cars and pedestrians travelling through the intersection. This pattern of behavior seemed to indicate that a less dense intersection lets crossing pedestrians and cars prioritize crossing the intersection as quickly as possible while a denser intersection encourages entities to prioritize their own safety. Of the published studies that focus on street intersections, many are focused on the interactions between pedestrians and cars while others looked to see if the layout of the intersection itself had any effect on entity behavior. However, the nature of these interactions can be explained by the rules of a bartering system.

2.2.1 Studies of Pedestrian vs. Car behavior

One common subtopic for published studies about intersection behavior to focus on is about how specific physical and circumstantial characteristics influence the behavior of crossing pedestrians and cars. Some studies focus on driver behavior specifically and try to form algorithms that can predict crash risks based on the pathing of the cars traveling through an intersection. Studies like this one examine driver behavior and try to form inferences about how driver pathing relates to their behavior (Lefevre, 2012). Other studies focus on how both the physical characteristics of the pedestrians and the environmental conditions they are in affect their behavior. One study in particular try to conclude how these factors affect the tendency of pedestrians to either press or ignore a walk button at a signalized intersection (Bradbury, 2012). Every human being is different in terms of circumstance and physical characteristics, which may influence their actions in different ways. By making the connections between what characteristic a pedestrian or driver has or what situation they seem to be in, it becomes easier to predict their behavior.

2.2.2 Studies of intersections

Though a less common subtopic than characteristic/circumstantial influences, some published studies focused on intersections themselves to try and find their influence on behavior. These studies tend to focus on either the physical layout of the intersection themselves or look at the differences in driving customs that intersections located in different nations may have. For example, a driving custom that cars in Japan have is that they drive on the left side of the road, which contrasts with the US's custom of cars traveling on the right side. The difference between the cars' travel path results in different turning radii for the cars when they attempt to turn in the same direction (i.e. both cars turning left from their respective travel lanes). These differences influence the intersections to be built slightly different and alter the behavior of car and pedestrian interactions by changing the general area where pedestrians and vehicles commonly intersect (can potentially occupy the same space) (Park, 2015).

Different layouts for intersections have been studied as well, with many studies observing how these intersection layouts affect the people who travel through them. One study that created a model of an unsignalized T-intersection used it to and predict accident probability based on vehicle approach speed on the priority road of the T-intersection (road making up top of the T). This probability assessment was then used to draw some conclusions about vehicular behavior and what drivers should logically do when turning onto the "stem" of the T-intersection (Spek, 2005). What layout an intersection has and what customs the traveling pedestrians and cars follow will alter what behavior they exhibit as they navigate.

2.2.3 Bartering system psychology

Within every car and pedestrian in an intersection, there exists two possible priorities that each could be focused on and are related to crossing the intersection. These two priorities are: 1) the person in question should travel through the intersection as quickly as possible, and 2) the person in question should travel through the intersection as safely as possible. This decision between prioritizing time vs. safety can be better understood if viewed from a bartering system perspective.

A common strategy within a bartering system involves two or more parties trading resources between each other and each party trying to establish a deal which benefits themselves. The resources being traded can vary wildly, but there are several factors that can make resources more desirable to specific parties involved. Factors for raising or lowering desirability include scarcity of the resource, usefulness to party, acquisition difficulty, and novelty among other potential factors. There are no set guidelines that dictate how the flow of a bartering deal goes, but the factors determining resource desirability remain constant (Chun, 2003).

Connecting the concept of the priority shift to the dynamics of a bartering system can yield better understanding on how the dynamics and interactions in any given intersection work. Doing this can provide an initial set of guidelines to manage traffic flow in an intersection. The two types of priority, time and safety, act as resources to trade. The desirability towards each "resource" for each person in the intersection dynamically changes in response to the density of the intersection (though other personal factors affect desirability as well). Pedestrians and cars that see less crowded intersection will lead them to believe that they can get away with traveling through the intersection quicker, showing an increase in desirability toward time. By contrast, more crowded intersections may cause people in the intersection to care about their own safety more as they attempt to cross, which shows an increase in desirability towards safety. This constant shift in priorities leads to a dynamically changing bartering model between the resources of time and safety occurring within the mind of each person at the intersection.

2.2.4 Gap in Literature

The studies brought up in the past subsections highlighted how intersection behavior has already been explored, while the bartering system terminology was meant to be a lens to examine shifting priorities. Whether it is the circumstances surrounding the person or the intersection layout itself, both seem to have an effect on the behavior of the people traveling through the intersection. The behavior exhibited by the people affects whether they prioritize time or safety when crossing the intersection, which could be more easily understood when viewed through the lens of a bartering system. Published studies focus on identifying what factors causes these shifts in behavior, but there is a deficit of research focusing on how changing the intensity of these factors could affect overall response.

The focus of intersection studies often lay in investigating what factors cause certain changes in behavior. Whether these factors are environmental, locational, circumstantial, or characteristic, the goal of studies is usually to identify correlational relationships between what factor causes what change. What published intersection studies (see sections 2.2.1 – 2.2.3) do not pay as much attention to is the relationship between factor intensity and the subsequent effects. This leaves a gap where more studies can focus on how the intensity of a factor determines how heavily weighted the effect of that factor is. For this study, the density of an intersection and how it influenced priority behavior shifts was examined.

2.3 Examining Intersections Using Agent-Based Modelling

The COVID-19 pandemic caused the U.S. to go into a nationwide lockdown, causing the Monroe intersection to exist in a perpetually near-empty state. To study the intersection when it was behaving normally, it was decided to create a model that emulated the intersection. To create the model, a free agent-based modelling software called NetLogo was selected. NetLogo offers the ability to customize different entities (like cars and pedestrians) as well as code them to run in specific areas (allowing intersections to be made based on these paths), both of which made for a good fit for the objectives of this research.

The NetLogo model made for this study is based on a four-leg intersection with three of the legs acting as two-way streets while the fourth leg is a one-way street (legs in this instance are the different roads branching out from the intersection). A customizable number of pedestrians and cars can be spawned from any of the four legs and are programmed to mimic the behavior of their real-world counterparts, while ensuring that the total number of pedestrians and cars never changes. This way, multiple runs of the model at different static density levels can be performed to observe behavior at these different levels.

All cars and pedestrians (also referred to as entities) within the model are assigned a random value of a variable called "patience," representing its namesake within each entity. This patience value is used to help determine how long an entity waits at a turn point and is comprised of two parts. Part of the patience value is determined randomly every time an entity resets its position, while the other part of the patience value is based on a moving average of previous wait times. This way, the patience value mirrors the way actual patience of an entity's real-world counterpart works (being affected by past events). These two parts of the patience value allows the patience of each entity to be partly randomized and partly affected by previous events already happening in the intersection, a parallel to how patience works in the real world (see Figure 1).

When arriving at a turn point, each entity waits a set amount of wait time based on the value of patience that entity has. While stopped at the turn point, the entity will check several different spaces to see if another entity is on them. If another entity is present on those squares, more wait time is added to the total amount that the entity must wait. This procedure mirrors how cars and pedestrians will end up waiting more time if they see an incoming pedestrian or car in their path of travel (see Figure 1).

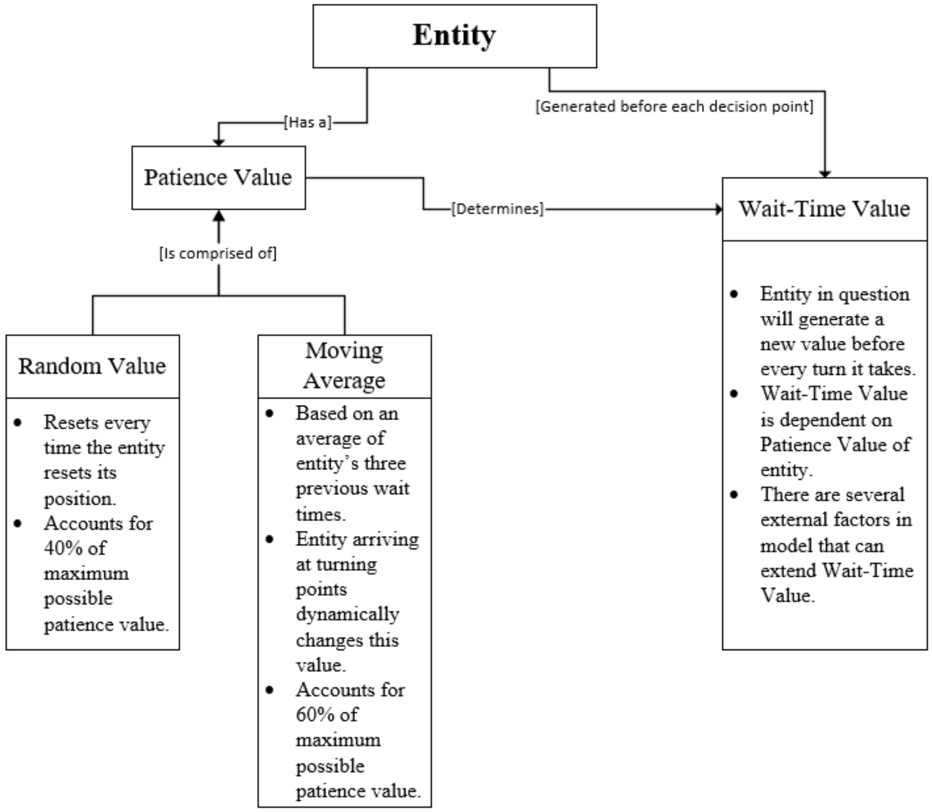

Figure 1. Diagram between Entities, and their Patience and Wait-Time Values.

On every patch (geometric position in model) comprising the center of the intersection and the crosswalks are checks to see if two or more entities are occupying the same patch. If more than one entity is in the same patch, then the model will mark on a graph that a collision has occurred. Through the various graphs on the dashboard, the model can keep track of how many collisions occur and at what times (see Figure 2). These collisions represent when a pedestrian or a driver should logically shift their priority from saving time to being safer, to avoid the collisions depicted in the model. Through various runs at different intersection densities, the relationship between the intersection density and when the shift in priorities should take place will become easier to understand.

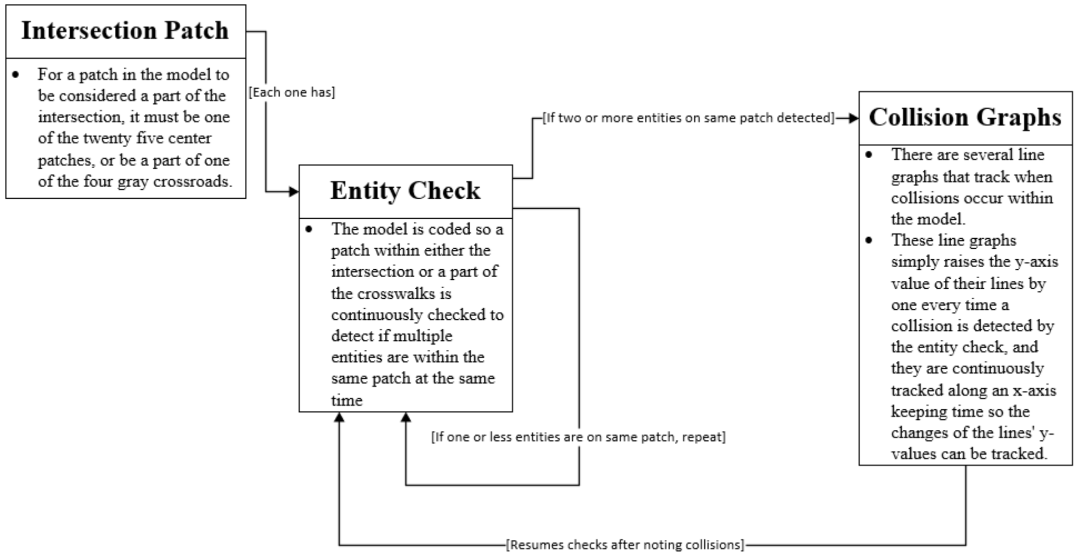

Figure 2. Diagram showing logic behind the intersection patches and collision tracking.

The model is coded so that the patience values, wait-times, and inter-arrival times for each entity type are recorded onto line graphs (see Figures 3 & 4). The number of collisions that occurred as well as when a collision has occurred are also recorded onto line graphs. These line graphs are separated by entity type and recorded information, so that there are three sections of twelve line-graphs, with each graph in each section recording a certain type of data for a certain entity type. All these line graphs are able to be exported in a CVS format so that they can be analyzed in Microsoft Excel. This way, analyses and conclusions about the data from various intersection density trials can be made.

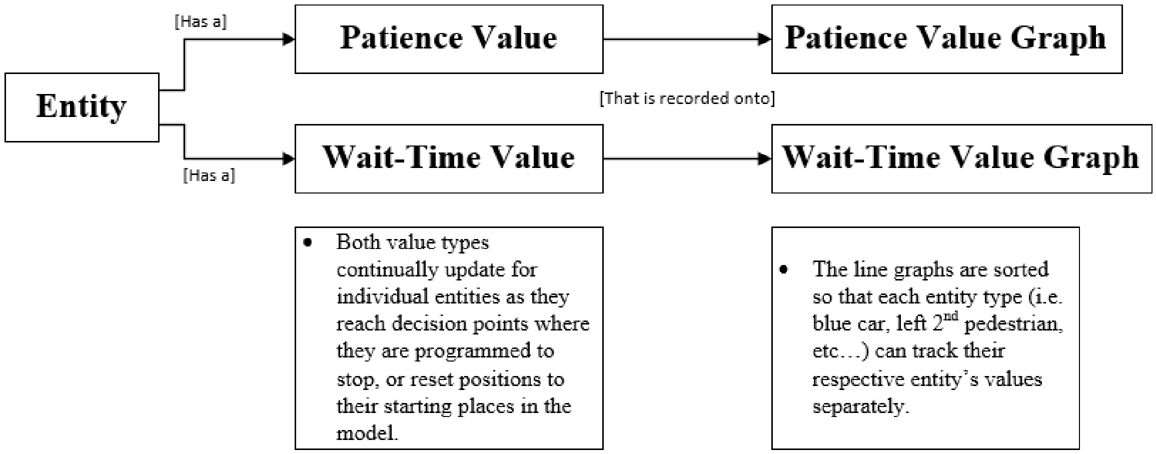

Figure 3. Diagram showing logic behind the patience and wait-time line graphs and how they record information.

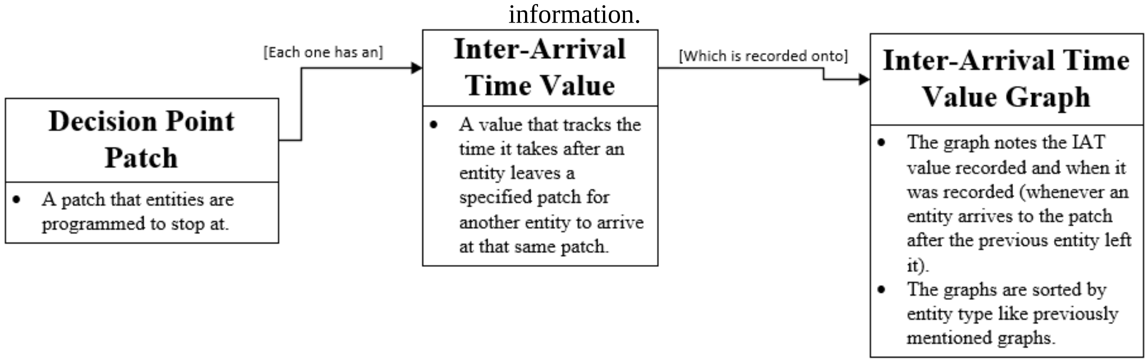

Figure 4. Diagram showing logic behind the Inter-Arrival Time Value line graphs and how they record information.

2.3.1 Agent-Based Modeling over Discrete Event Simulation

A discrete event simulation is a different method of simulating the behavior and performance of a real-life process than an agent-based modeling system. Discrete event simulations depict the system they are simulating as a series of events and assumes there is no change in the system between the events. Each entity is considered independent and coded with a series of parameters that influences what choices they make whenever the series of events has a split path. The information of each entity can be changed as they move through the system and the simulation can account for resources (Allen, 2015).

Despite all these features, an agent-based modeling system is still the better choice to model the Monroe intersection with.

While a discrete event simulation (DES) can model the intersection as a series of events, it is much better at modeling a single starting point that can branch out in several different directions. There are four streets in the Monroe intersection and each street is a possible starting point for a DES, meaning four different starting points that can branch out into even more paths. The DES also does not consider of how the individual entities affect each other, with them assuming the entities are independent. An agent-based modeling system does not have these problems, focusing on the behavior of the agents themselves instead of depicting the intersection as a series of events. That interaction between agents is especially important to this study since it is focusing on intersection density and how the number or lack of agents affects behavior of arriving agents. An agent-based modeling system is superior in this area than a discrete event simulator, making it the preferred system to model the intersection with.

Chapter 3. Methodology

3.1 Introducing Methodology

This study revolves around the observation of the interactions between cars and pedestrians. For data collection, a procedure would normally be made where the natural behavior of the target intersection would be observed under various conditions by researchers, and observations would be recorded for later analysis. While it would be ideal for changes in intersection density to be recorded as naturally occurring patterns, this study is being conducted at a time where the COVID-19 pandemic has caused most of the U.S. to undergo quarantine procedures and has lessened the amount of traffic on the streets nationwide. As a result, direct observation would only result in data about the intersection in a perpetually abnormal state, with no way to raise the traffic levels to regular amounts. Instead, modelling the intersection and collecting data from the model would be the next best way to collect data.

Of the many computer simulation types that exist, the one that is appropriate for this study is one of an agent-based system. An agent-based modelling system uses a collection of autonomous decision-making entities called agents to operate based on a set of programmed rules (Bonabeau, 2002). It is possible to use general knowledge about intersection behavior to program agents in the model to act like their real-world counterparts. Data collection could then be taken by observing the interactions between the agents in the model.

To create the intersection model and collect data, NetLogo software was used. NetLogo is a free agent-based modelling simulation software that had several features needed for this study. In addition to the basic functions needed to program agent behavior, NetLogo also contains several features that allow patch customization, both functionally and aesthetically. This allowed for the intersection to be represented visually in the model and for the agents to acting upon it to bear a more similar representation to their real-world counterparts. The aesthetic likeness of the model and the agents acting upon it allow making observations to be easier, and the fact that it is a model that is being observed where the number of agents can be adjusted means that the NetLogo model is the best alternative for data collection during the COVID-19 pandemic.

3.1.1 Reasoning behind the Model

Even if the COVID-19 was not occurring and data collection was being done through direct observation of the intersection, this study is taking an inductive approach to researching the topic. Inductive reasoning is research approach that starts with a premise based on observations and regularities in experience. After establishing the premise to study, researchers then collect data and identify patterns and relationships within the data. These patterns and relationships are then used to generate theories from which conclusions can be made (Inductive).

Whether direct observations were being done or the model was used, both methods used the inductive approach to research. Learning a bit about general intersection behavior through research and prior experiences made it easier to program the agents in the model to reflect reality. Any inference derived from the behavior of the model was made after data has been collected. The process above aligns with the general process of inductive reasoning.

3.2 NetLogo Model in Detail

To create a model that could reflect the behavior of an intersection, NetLogo was chosen due to the numerous functions that could make this possible. The customization features of the patches were extensive and met the aesthetic and functional needs of the model. Pedestrian and car agent programmability allowed the entities to better reflect their real-world counterparts' behavior in the model. NetLogo's method of tracking different data types also aided in the study conducted. Finally, the ability for different agent types to be programmed in different ways was especially helpful in making the model reflect reality. All these points made NetLogo an excellent choice in software to use to model the intersection.

3.2.1 Patches of the NetLogo Model

The model is made up of patches that are roughly 19 x 19 pixels each and are put together to form a coordinate grid. The center patch (also known as the origin) is labelled with the coordinates (0,0), with the other patches labelled about the origin. The size of the model is a 25 x 25 patch square with coordinates of the x and y axis ranging from -12 to 12. Each entity can fit in a single patch and their location is internally tracked by the software. The entities' current location is based on the relative position of the entity to the center of each individual patch (i.e.: location (3.3, -1) would be tracked as (3, -1) by the model). Through these coordinates, the entities' can be programmed based on their location on the model.

The patches are also programmed to change color to correspond to different intersection features upon setting up the model. The various shades of dark gray represent the roads of the intersection, and the patches have been programmed to vary the shades of gray to make each individual patch stand out. The yellow patches represent the dividing lines between road lanes, green represents sidewalks, and the light gray patches surrounding the center part represent the crosswalks. The white patch in the center does not correspond to any real-life feature of the intersection and is instead used to mark the origin of the coordinate grid.

3.2.2 Behavior of the Entities

Before the COVID-19 pandemic caused the U.S. to go into lockdown in March 2020, observations of the Monroe intersection were recorded starting in January 2020. These observations recorded the general actions of both cars and pedestrians, noted the conditions that the intersection was in, and any abnormal events that occurred during the observation period. The behavior of the model agents is based on these observations, but only to the extent that the general behavior of the cars and pedestrian was informed by. It was during the observation period that scope of this study was vague and still in the exploratory phase, noting down the possible conditions that caused the intersection to behave in the way that it did. The observation records were useful in establishing what behavior the agents in the model would have, but that is the extent of the influence of those records on the agent's behavior.

The agents representing cars and pedestrians are programmed to behave in similar ways despite how different they behave in real life. Once an entity has reached one of the edges of the model, they are immediately inserted at the start of their travel path in the model and made to travel it again. Both cars and pedestrians are programmed to maintain their speed in a cardinal direction unless they reach one of the stop-points or turn-points on the model. A stop-point refers to the patch just before the crosswalk on the road or sidewalk where pedestrians and cars halt their movements before crossing. A turn-point refers to the patches where the entity has the chance to turn and move in a different cardinal direction. For cars, their turn-points are in different places within the center of the intersection, while the stop-points on the sidewalks double as turn-points for the pedestrians.

Coded into the behaviors of the entities are several lines trigger certain actions when the entity moves over a specific patch. A lot of these actions are "preparatory" work, where patience values are reset, or flags are changed so that future behaviors are executed only once. All these preparatory behaviors have no visible effect on the entities as they are enacted. Other actions that visibly affect the entities usually involve them stopping, starting movement, or changing directions. All the entities follow the same basic process regarding their behavior when traveling.

First, the entities will continue traveling in the same direction until they reach the stop-point before the crosswalk. For cars, this means the patch right before the crosswalk while pedestrians stop at the corners of the sidewalks they are traveling on. Once they are stopped, they will wait however long their wait-time value is before each they decide on what direction they want to travel. Both entities types decide on what direction they travel in through random chance, with some directions given a higher priority than others based on previous observations. Once an entity reaches one of the edges of the model, their position is reset to their starting point in the model as described earlier in this section.

3.2.3 Values of the Individual Agents

Every agent in the model has a separate patience value and a wait-time value assigned to them. Each entities' patience value is influenced by a combination of a value assigned to them after resetting positions and an average of the three previous wait-times that entity has experienced. The part of the patience value that is assigned after resetting positions is random and is on a scale from 0 to 40. The part of the patience value determined by the average of the previous three wait-times will continuously change as the oldest wait-time value is replaced by a new value at every stop-point. The total range a patience value can be is between 0 and 100, with higher values representing that entity having more patience and vice versa for lower values.

Most of a wait-time value is largely dependent on that entity's patience value, though there are two other parts that contribute to the wait-time value. Two other components towards an entity's wait-time include a base value that all entities are expected to have (except in cases where the patience of the entity is low), and additive value based on whether a collision occurred recently or not (Figure 5). If an entity's patience value is less than 50 (half of the max possible), then the base value is not added to that entity's total wait time value. This mirrors how a person with low amounts of patience will not be willing to wait a lot at wait points while people with more patience are more willing to wait. There is also a special pedestrian-only procedure where pedestrians who are at wait-point will add time to their total wait-time value if a car agent moves to any patch on either side of the crosswalk that the pedestrian is waiting at. This is supposed to emulate how pedestrians will generally look both ways before crossing a street and will wait for cars to pass by before moving.

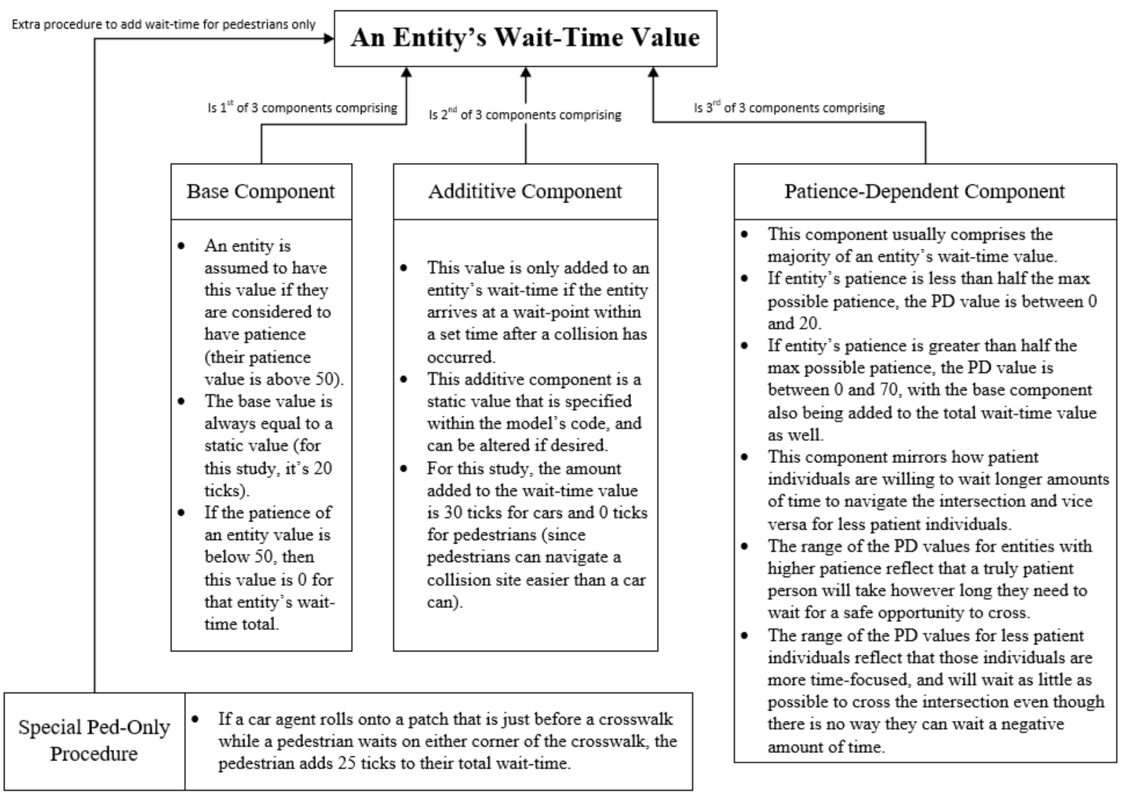

Figure 5. A diagram that explains the logic behind the different components of an entity's wait-time value.

3.2.4 Differences between Car and Pedestrian Entity Programming

The basic programming structure in car agents and pedestrian agents concerning behavior are similar with a few key differences between them. One of these differences was that cars were programmed to accelerate and decelerate in response to the cars in front of them while pedestrians continued moving at a constant speed. The cars were also programmed to avoid moving into patches occupied by other entities, slowing down, or stopping to do so. This behavior is meant to symbolize the differences of how easy/hard it is for pedestrians/cars to move in the intersection. Pedestrians have no trouble changing speeds and can easily navigate around oncoming pedestrians while cars must stop for similar situations.

Pedestrians are also programmed to look out for cars before they cross the intersection. Whenever a pedestrian is waiting at a stop-point, they continuously check the patches in the road that are on both sides of the crosswalk during the duration of their wait-time. Should a car pass over one of those patches, the waiting pedestrians will add more time to their wait-time value. Cars are not programmed to add any additional wait-time to their original value but are instead programmed to immediately stop if they detect an entity in the patch they are about to travel into. This behavior of theirs is a part of the data collection process.

3.3 Data Collection Process

For this study, the model tracks four types of data internally and shows them externally on different line graphs on the model interface. Patience and wait-time data are collected directly from the model. Inter-arrival time data is not directly tracked by the model but is instead derived after collecting data tracking the presence of an entity on certain patches. Collision occurrences of the car agents are tracked both cumulatively and in set intervals of time. Data related to patience, wait-time, and inter-arrival values are further separated by entity type so that the set of data related to a specific entity type can be easily found. This way, it could lead to easier data analysis.

3.3.1 Patience and Wait-Time Values

The patience and wait-time values are entity specific (each entity has one) and are tracked in similar ways. In the middle of the roads and sidewalks for each entity there are patches that are programmed to assign the random portion of the entity's patience value and the past wait-time dependent value as an entity travels over them. The patch that also assigns the patience component based on past wait-time values also records the new total patience value for the entity and displays it on the associated line graph. The wait-time value is recorded in a similar way, but the location of where it is recorded differs. It is recorded when the entity reaches a wait point, and before that value counts down. This way, any last-minute adjustments to the entity's wait-time will be recorded.

3.3.2 Inter-Arrival Values

The model also tracks the interarrival time between entities arriving to the intersection, but in a different manner to the previously two mentioned values. For each agent type, a patch was selected and programmed to let the model know if an entity had passed over it. Each chosen patch was in front of the area that entities spawned from after resetting their position, and essentially let the model know when a new entity arrived at the intersection. Internally, the model recorded each time an entity passed over the patch, appearing as a binary set of 1's and 0's to represent cars on and off the patch respectively. Discerning the actual interarrival time values is done in the analysis portion of the process.

3.3.3 Collision Occurrences

The final type of data that the model tracks is collision occurrences. The patches forming the center of the intersection as well as the crosswalks are coded to watch for any car that slows below a specified speed. Whenever a car agent slows or stops on these patches due to avoiding other entities, a collision occurrence is recorded by the model. These occurrences are recorded both internally by the model as well as displayed on a line graph set aside for them. The collisions are recorded both cumulatively and over set intervals where the count is set to 0 after a certain amount of time has passed

3.4 Data Analysis

Most of the analysis process for data includes taking the data from the model and "preparing" it before looking for any patterns or trends. First, the data is exported and separated on different Microsoft Excel workbooks (the data for inter-arrival time needs a little extra preparation compared to the other two data types). After the preparation, the data is fitted to several distributions to check the validity of the model in simulating the intersection at specified entity numbers. Once the validity has been checked, line graphs of the data are constructed and compared to a line graph of the cumulative total number of collisions. This process is repeated for however many tests are run.

3.4.1 Exportation and Preparation

Once exported in a CSV format, the test data is opened in Microsoft Excel and saved as a Microsoft Excel file. The exported file will have a single work page where all the data corresponding to line graphs is titled and separated from each other in a series of columns. All the data that corresponds to patience values are separated by entity type and transferred to work pages on a blank workbook. The same process is repeated for the wait-time values, though they are saved in a separate workbook than the patience values. The inter-arrival time data is similarly separated into a third workbook but requires a bit more preparation to analyze.

After the data has been separated, a new work page is opened next to one of the existing work pages (indicating the data from the existing page going onto the new page). On the original page, the y-axis data is filtered so that only 1's show in the column, which means that a car or pedestrian was on the patch at that time. Once filtered, the columns with the x and y-axis data are copied onto the new worksheet. A third column on the new worksheet is then filled with cells calculating the differences between each y-value. Due to the speed at which the entities can move, the model may catch an entity twice on the checking patch, which manifests as a bunch of extra 1's in the third column. Because of this, the third column is filtered again so that there are no 1's left, and the resulting column holds the inter-arrival time data.

3.4.2 Checking Validity of Model Through Distribution Fits

The next step in the analysis process is to fit the patience, wait-time, and inter-arrival data to different distributions to ascertain the validity of the model. To do this, the data to be fitted is first pasted onto Notepad text documents (a separate file for each entity type/ data type combo). Once in text documents, the input analyzer function of the ARENA software is used to fit the data to specified data distributions. The square error value on the distribution summary measures how accurate these distributions fit the data, with values under 0.05 being ideal, though values under 0.1 indicate the fit is good. How well the distributions fit the data indicates how well the model emulates the intersection under the chosen number of agents.

All three data types are fit to different distributions, but all data from single data type is fitted to a single distribution fit for the sake of consistency. The data pertaining to inter-arrival time was fitted to an exponential curve after it was filtered twice on Excel. Patience value data was fitted to a normal distribution and was consistently the best fit for that data type among the other data types and their fits. Wait-time data often had a gap in the center of the graphs when displayed in the input analyzer so triangular distributions were used to fit that data type. After each entity/data combination has been fitted, a screenshot of the summary is saved.

3.4.3 Searching for Patterns and Trends

Once all the combinations have been captured using screenshots, line graphs of each data type were constructed along with a line graph of the cumulative total of collisions that occurred during the test. For the inter-arrival time data, the filtered data is used to make the line graph. It is through these line graphs that any conclusions are to be made. The line graphs of the data types will be compared to each other and the graph of the cumulative collisions to see if any trends or patterns are present. After noting down any observations that are present, a new test with varying amounts of agents is to be conducted and the data analysis process repeats again.

Chapter 4. Results

4.1 Introduction and Results Setup

The data gleaned from the model is visually presented on a series of line graphs that can allow conclusions to be made by simply observing and comparing those graphs. Once the tests were completed, the resulting data was sorted by data type and presented on line graphs that would allow the full scale of each test to be easily visualized. The created line graphs were then sorted onto Excel work pages in a way that allows observers to see how each data type changed for each entity type over the course of increasing agent density. Observers would also be able to note how each data type changes over the course of tests of increasing densities and draw conclusions from those observations. With this setup, it will be relatively simple to figure out the approximate point where the priorities of people traveling through the intersection should switch from being time focused to safety focused.

4.1.1 Ticks and their relation to the Tests

To understand how the line graphs for the test data are created, the NetLogo specific time units known as “Ticks” needs to be explained. A tick in NetLogo represents the model updating itself one time and the agents performing a programmed action during that update. In this model, an agent with a speed value of one will move one patch in the direction they are facing or turn itself if it arrives at a turn point during every tick. The speed value of the agents represents how far that agent will move during one update (with a value of 1 equaling 1 patch, a value of 0.5 equaling half a patch, and so on). The model keeps track of the number of times its updates using a tick counter, this counter is used as the basis for any time-related functions of the model.

Since the rate at which the model can update is adjustable by users, there is no set conversion between real-world time units and the ticks used in the model. The actual conversion of ticks to real-world time is not important, but relative lengths of time are. An event or incident occurring for smaller or larger amounts of ticks translates to that event happening for shorter or longer lengths of time. For ease of understanding, it could be thought that a tick equals one second of real-world time (though there is a big source of error with this way of thinking that will be described later in this chapter). Each test ran for roughly 5000 ticks, after which the model was stopped and the data that was recorded by the model was saved and exported to Excel files.

4.1.2 Setup of the Excel Line Graphs

For this study, a test was conducted five times at different density levels, with each repetition representing the intersection at different levels of entity density. The first repetition had one of each agent type on the model, the second repetition had two of each type, and this pattern repeated until the fifth repetition had five agents of each entity type on the model. After a repetition was completed, the associated data was exported to an Excel file, and further sorted to different work pages by entity type. The files containing the sorted data for a repetition consisted of three Excel files, with one file containing all the Inter-arrival time data, one containing all the wait-time data, and one containing all the patience data. Each file contained work pages for each of the agent types, with all the data concerning an agent type relegated to its corresponding work page.

Once all the data for the five repetitions were sorted, line graphs for the patience, wait-time, and inter-arrival data were made. For the wait-time and patience line graphs, both had their respective values as the y-axis and the time in ticks as the x-axis. Observers can easily see how the values change as they are assigned when looking at the y-coordinates for these two graphs. Since the test was conducted for about 5000 ticks for each repetition, the x-axes for all the graphs are relatively similar to each other and will not cause too many problems when compared to each other. The inter-arrival time graphs, on the other hand, are a bit different than the graphs for the other two data types.

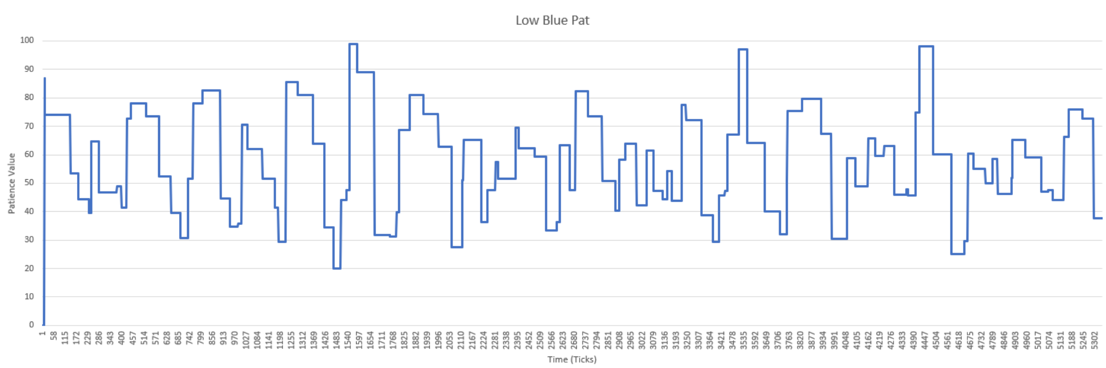

Figure 6. The patience graph of the blue car agents during the low-density repetition.

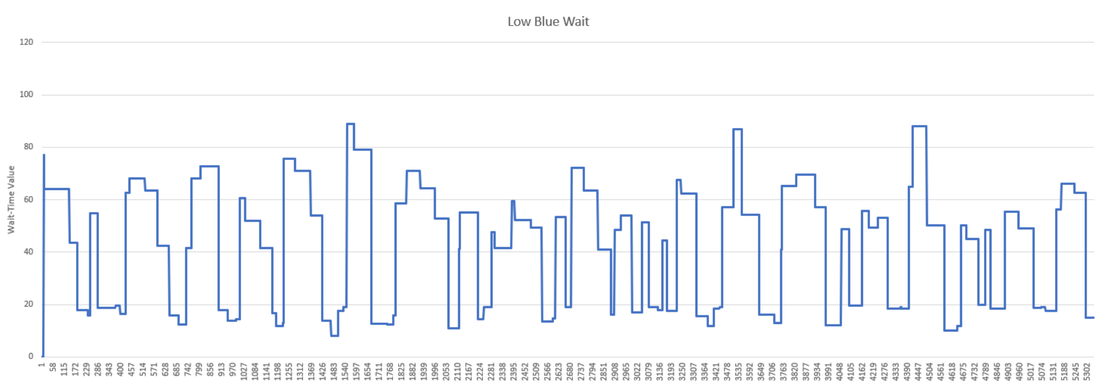

Figure 7. The wait-time graph of the blue car agents during the low-density repetition.

Because the inter-arrival time graphs were constructed using filtered data, their x-axes instead track the number of values that have occurred in their respective tests. The scale of the x-axis between tests varies wildly because of this and need to be kept in mind when making comparisons between inter-arrival graphs. The y-axis of the inter-arrival graphs (as well as the graphs of the other two data types) is consistent throughout the Excel files (i.e., the y-axis for all of the graphs constructed in the inter-arrival Excel file range from 0 to 200). This will make comparisons between graphs using the y-axis less complicated and easier to understand. Once graphs for data from all the tests have been created, the graphs were then copied and further sorted into data comparison Excel files.

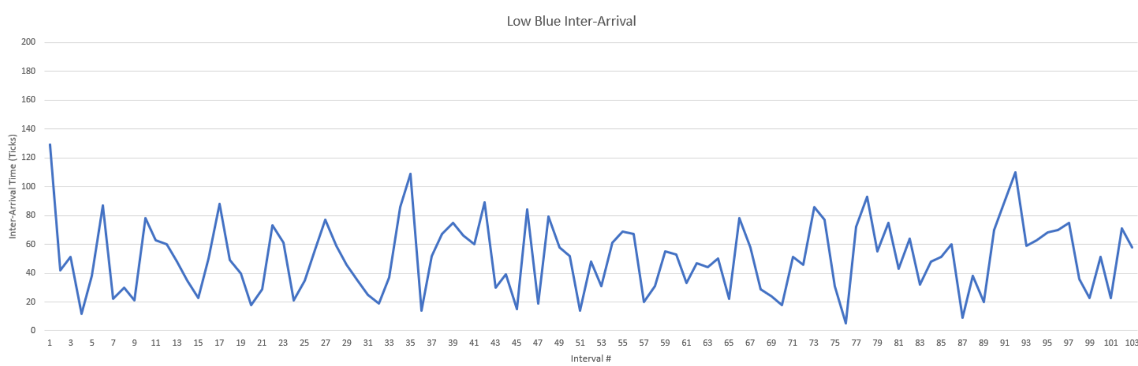

Figure 8. The inter-arrival graph of the blue car agents during the low-density test.

4.1.3 Setup of the Graph Comparison Files

The data comparison files consist of three Excel files labeled for each of the three data types, and each file is separated into twelve work pages labeled for the twelve entity types in the model. For the wait-time file, the line graphs from all the repetitions that were constructed with wait-time data were copied, pasted, and separated by entity type into different work pages. The graphs were size-adjusted and lined up next to each other in intensity-ascending order (graphs from left to right were from repetitions with very-low density to repetitions with very-dense intersections). Below each wait-time graph was another line graph of the total collision amount corresponding to the repetition of the graph above it (i.e., total collisions for the mid repetition under the wait-time graph for the mid repetition , and so on). This resulted in ten line-graphs per page, lined up in 5 x 2 arrays. The files for patience and inter-arrival time graphs were set-up in the exact same manner as the wait-time file.

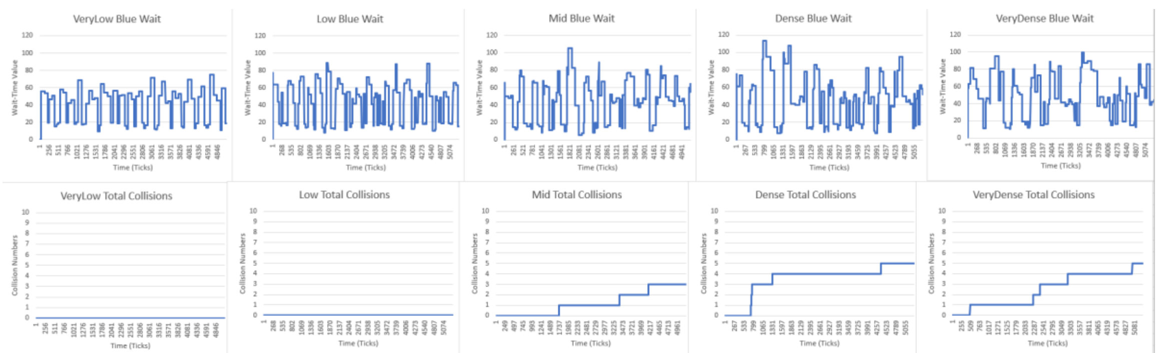

Figure 9. The wait-time comparison file of the blue car agents.

4.2 General Observations from the Results of the Tests

There are several noteworthy aspects about the graphs for all three data types because of both how the data types were tracked on the graph and how the line graphs were created. The different method by which the inter-arrival graphs were created results in those line graphs to be the most different of the three. Conversely, the similar way in which the wait-time and patience graphs were built left them very similar to each other. By observing each of the line graphs on the comparison files, the changes that each graph undergoes as the density of the repetition increases becomes more apparent. This allows conclusions about how density affects the various data types to be made based off those observations.

4.2.1 Inter-Arrival Line Graphs

Looking at the line graphs for both the car and pedestrian agent types reveals some interesting differences between the two. For car agent types, low-density repetitions show that the average range of the inter-arrival (IA) values is on the low end of the y-axis (roughly between 40 and 100 ticks) with the values consistently remaining in that range. As the repetitions increase in density, the average range of the IA values also increases along with there being more instances of outlier values (points outside of the average range). The changes in the values also becomes more erratic as the density increases, with there being periods where the IA values were consistently lower or higher than the average IA value. It is also interesting to note that the total number of IA values for car agents does not linearly increase as the density of repetition increases, with the lowest number of values among the car agents being in the very low-density repetition and the highest number being in the low-density repetition.

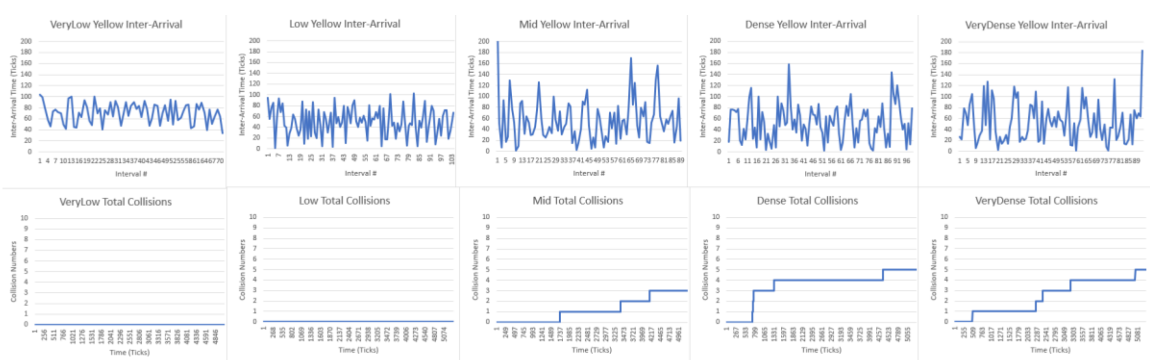

Figure 10. The inter-arrival comparison file for the yellow car agents. This set of inter-arrival graphs clearly demonstrates the decreasing consistency trend mentioned in the above paragraph.

The line graphs for pedestrian agent types show very different behavior than the graphs for car agent types. For repetitions with the lowest density, the range of the IA values are at their largest (generally between 60 and 190 ticks). This range also shortens as the density of the test increases and lowers to the bottom of the y-axis, with graphs from the very-dense tests having ranges of 0 to 60 ticks (on average). Increasing density also both increases the consistency of the IA values staying in the average range and decrease the frequency of outliers appearing. Also, unlike the car agents, the number of IA values for the pedestrian agents increase linearly as the density of the tests increases.

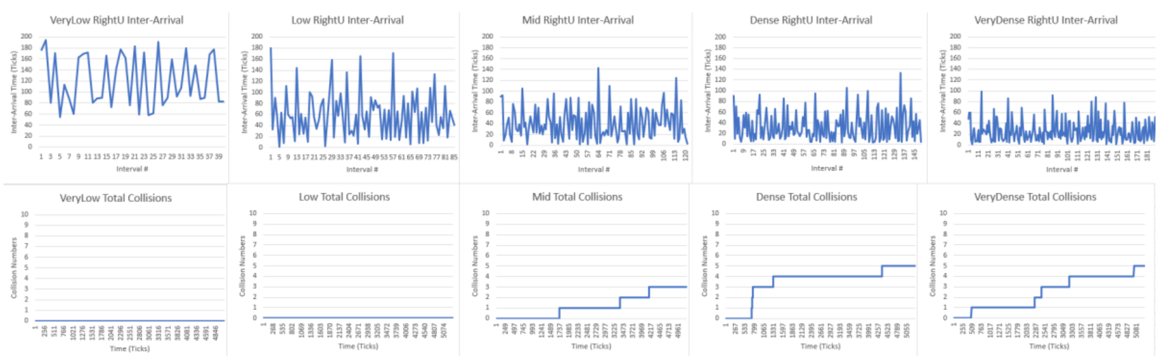

Figure 11. The inter-arrival comparison file for the upper right pedestrian agents. This set of inter-arrival graphs clearly demonstrates the increasing consistency trend mentioned in the above paragraph.

4.2.2 Similarities and Differences between Patience and Wait-Time Graphs

Due to how one of the components for the wait-time value of an entity is their patience value, the general shape between the graphs of the two data types are similar. As an example (see below), the wait and patience graphs of the very-low density repetitions for the LeftD pedestrians are similar in shape with the main difference between the range of the values. This pattern of similar shapes between the two graph types continues as the density of the repetitions increases, only breaking slightly on the graphs of the very-dense repetitions. There are some differences between the two graphs involving specific values (the part of the graph at time 2332 being one example), but the general shape of the graph remains similar between the two. These similarities and differences between the wait-time and patience graphs can be explained by the fact that the wait-time value of an entity is partially dependent on their patience value. The wait-time's dependency on the patience value explains the similarities in the graph's general shape while the differences could be chalked up to the other components of the wait-time (see Chapter 3, Figure 5).

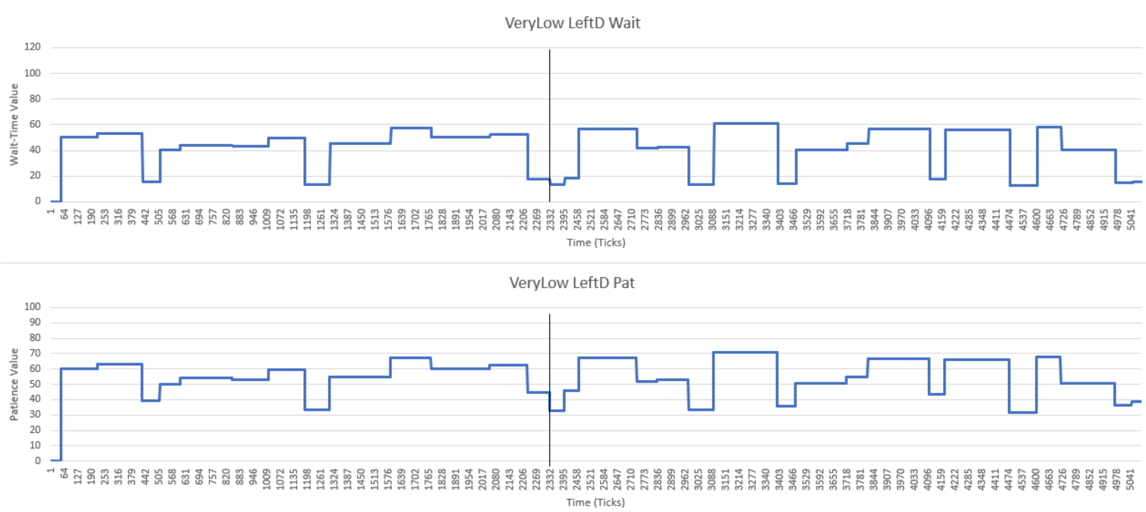

Figure 12. The wait-time and patience graphs for the down left pedestrian agents (vertical line inserted at tick 2332).

4.2.3 Observations of the Patience and Wait-Time Graphs Across Tests

In addition to the shape similarities, there exists a few other trends that the wait-time graphs and the patience graphs share. One trend is that the total number of values being assigned to entities increases as density increases (indicated by how often the graph changes y-values). Another trend is that the range of both graph types increases as the density of the repetition increases. This is more apparent with the wait-time graphs due to the larger y-axis the wait-time graphs use compared to the patience graphs. Both the car agents and pedestrian agents have the trends in both types of graphs so there does not seem to be a significant difference between them.

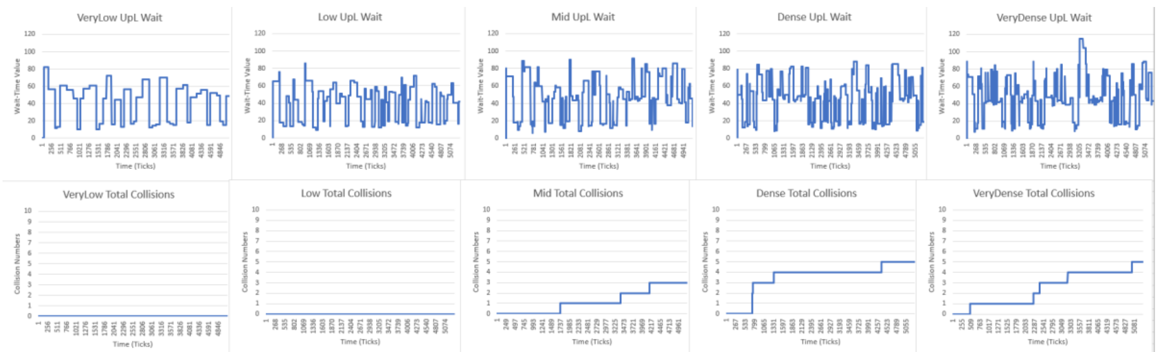

Figure 13. The upper left pedestrian wait-time graphs that demonstrates the mentioned trends shared by both wait-time and patience graphs. The total number of values being assigned along with the range increases as the density of the test increases.

4.3 Analysis of the Observations

Initially, the question that this study aimed to answer was “When are travelers safe enough to try to save time, and when does saving time put people at risk when navigating an intersection?” For the purposes of this research paper, this question later became “At what level of density should travelers in the intersection switch from a time-saving focused mindset to one that prioritizes following the rules for their own safety?” An answer to this question was found by interpreting both the observations made in the previous subsections as well as the obtained results of the graphs themselves. These interpretations allowed for answer to be found that was not only plausible, but empirically sound. The following paragraphs detail the necessary steps taken to arrive at this conclusion.

4.3.1 Classification of the Answer Terms

One thing that needs to be explained before delving into the analysis is how the density of the intersection will be classified for the sake of giving an understandable answer. The density of the intersection will be explained in terms of "model density", with the individual agent types being explained through the concept of "agent composition." Model density refers to the how many agents were used in the test out the total number of models that could be used (it is usually expressed as a percent value). For instance, the maximum possible number of agents that could exist on the model at one time is five agents each from the twelve agent types, resulting in a maximum of 60 agents on the model at the same time. The very-low density repetition uses only one agent from each of the twelve agent types, resulting in twelve agents in the model at the same time, or having a model density of 20%.

Definition of Model Density and Agent Composition

Model Density

- Model Density refers to the ratio of agents currently on the model intersection out of the total possible agents that the model can hold.
- This value is expressed as a percent.
- Model Density represents how full an intersection is at a given moment compared to how full an intersection could possibly be.
- A model density of 20% means that 12 of the possible 60 agents are on the model in a given test. This correlates to an intersection in real life holding enough pedestrians and cars to be 20% full.
- In reality, how many entities an intersection can hold varies, so percentages are used as a universal identifier for when the switch in priorities should occur across many different intersections.

Agent Composition

- Agent Composition refers to the exact amount of car agents and pedestrian agents that makes up the current model density at a given time.
- This value is expressed in parts, referring to x parts cars and y parts pedestrians.
- When x and y are added together, it should come up with a number that when divided by the total possible amount of agents that can fit, will give the model density.
- A model density of 20% could have an agent composition of 4 parts cars and 8 parts pedestrians. 4 + 8 = 12 parts total, and when 12 is divided by the total amount of agents possible (60), it equals 0.2 which is the model density expressed in decimal form.
- Agent composition in real life represents the exact makeup of pedestrians and cars that are in an intersection at the same time.
- For example, a model density of 20% could be made in a variety of ways (4 cars and 8 pedestrians, 6 cars and 6 pedestrians, etc…)

Figure 14. A more in-depth definition of model density and agent composition.

The agent composition of the very-low density repetition is four parts car and eight parts pedestrians, referring to the four car agents and the eight pedestrian agents that were currently on the model. The cars can have a maximum of 20 parts (5 agents of each of the 4 car agent types) and pedestrian can have a max of 40 parts (5 agents of each of the 8 agent types). The model density is used to refer to the overall density of the intersection while the agent composition distinguishes the exact make-up of the model density. The very-low density repetition has a model density of 20% with an agent composition of cars to pedestrians to be 4-8, and each repetition of increasing density raises the model density by 20% and the agent parts by 12 parts per repetition (4 cars and 8 pedestrians). This pattern should end with the very-dense repetition having a model density of 100% and the agent composition being 20 parts cars and 40 parts pedestrians.

Another term that is used in describing graph behavior a lot is the term “erratic.” The previous subsections mention the range of the values a lot, and how the graph usually has wider ranges as the intersection density increases. A graph is said to show erratic behavior if the range established in the graphs of a less dense repetition widens as density increases. The higher the difference is between two graphs, the more erratic the behavior of the agents becomes. The next couple of subsections look for behavior of graphs that borders on being erratic, meaning that the difference in ranges is there, but not too great.

4.3.2 Inter-Arrival Behavior Analysis

To start, looking at the inter-arrival graphs reveals that car agents and pedestrian agents seem to behave in an opposite fashion to each other, and the reason for this lies in their programming. The cars are programmed to move faster than pedestrians normally, meaning that their inter-arrival values would be both lower and more consistent should the car agents move without interference. However, the cars are also programmed to slow and stop in response to other agents in front of them, especially other car agents. A denser intersection means that there are more cars that would get in each other’s way, causing them to stop more frequently and widen the range at which the inter-arrival values generate. This means that the IA value range of the car agents to be wider and the IA values being less consistent at higher intersection densities.

Pedestrians, on the other hand, are not programmed to wait for other pedestrians and instead move forward at a more consistent speed regardless of whatever is in front of them (unless reaching a waiting point). This means that they can ignore the effects that increased intersection density has on the progress of car agents and always move at their default speed. The IA value range for the pedestrians reflects this behavior through the IA graphs of increasing intersection densities. At the very-low density repetitions, the IA value range is wider than those on the car agents graphs due to the lower speeds that pedestrians travel compared to cars. As the density of the intersection tests increases, the IA value range both narrows and relocates to the lower half of the graph due to the increased frequency at which pedestrians arrive to the intersection.

If a repetition was done at the model density where intersection travelers should switch their mindsets from saving time to being safe, then the inter-arrival graphs from that repetition should exhibit the following behavior. Graphs containing data about the IA values for car agents should contain a consistent but relatively wide range of IA values (around 80 to 90 ticks wide) whose average should be on the lower half of the graph's y-axis. The reason for this is because a consistent range in the IA graphs translates to predictability in the frequency of cars entering the intersection in real life. A predictable frequency in turn allows the other travelers already in the intersection to be able to adjust their actions if necessary and lessens the chance for a collision to occur. The target model density should (when tested) produce IA graphs for the car agents where this consistent range is starting to widen, and outlier values are starting to appear. This signals that the intersection is starting to become more unpredictable and a more safety-focused mindset is needed to avoid collisions.

As for graphs containing data about pedestrian agents, the IA value range should also be consistent (though it can be narrower than the car agent range, with around 60 ticks wide being the ideal range) and the average IA value should also be on the lower half of the graph's y-axis. Unlike the graphs for the car agents, the range should avoid the lowest values on the y-axis and instead aim to encompass the center of the lower half of the graphs. The reason why both car and pedestrian agents should have their ranges be on the lower half of the graphs is because the values there are signifies that the cars are starting to enter the intersection more frequently in real life. This shift in frequency is a good indicator that the intersection is starting to fill up more and that a safety-focused mindset will be needed to avoid collisions. As for the reason why ranges for the pedestrian agents need to avoid the bottom of their inter-arrival graphs is because those bottom values indicate that a continuous or near-continuous flow of pedestrians are entering the intersection, especially with the narrower range in IA values that the pedestrian agents have compared to the car agents. The switch in mindset needs to take place before a continuous pedestrian flow form.

4.3.3 Patience and Wait-Time Behavior Analysis

The similarity in the shapes between an entity's patience graph and wait-time graph is because an entity's wait-time is largely dependent on its patience value (see Chapter 3, Figure 5). Since the patience value of an agent helps determine the wait-time, it can be said that the patience value has an indirect effect on the actions of its corresponding agent.

By contrast, the wait-time value of has a direct effect on an agent's actions by causing it to physically stop and wait at the various turn points. It is because of this indirect vs. direct effect of the two graphs that only the wait-time graph is used to make any observations about how the agents in the target model density should behave. The patience graphs act as an indicator of the behavior of their corresponding wait-time graphs, while the wait-time graphs themselves serve as indicators of the agent's behavior in the model (and thereby providing more information when observed).

At the target model density where travelers should make the switch in mindsets, the corresponding wait-time graph should have a consistent wait-time range that is bordering on becoming erratic. Like the IA ranges in the inter-arrival time graphs, a consistent range of values in the wait-time graph translates to predictability of the intersection in real life. The predictability in this case refers to travelers being able to guess how long a car or pedestrian waits at their current location (at a corner) before crossing the intersection. The range of the wait-time graphs should be slightly different for car and pedestrian agents (cars should be around 70 ticks wide and pedestrians should be around 50 ticks, both values based on observations of the wait-time graphs for each agent across the different tests). The graphs should be bordering on an erratic range since the target model density will be the intersection density where predictability starts to go down and collision chances start to rise.

Also like the inter-arrival graphs, the average wait-time value of the range should be located on the lower half of the graph's y-axis, as well as avoid encompassing the bottom values of the wait-time graphs. The values on the y-axis represent how long the agents wait at the various wait points in the model before resuming their journey, reflecting how long pedestrians and cars wait at the corners of the intersection. Having the agents wait less time at the wait points indicates that the intersection "flows" more smoothly, with there being less complications that cause the agents to wait longer than necessary. However, the wait-time values at the very bottom of the wait-time graph are wait-times where the agents would barely pause at the wait-points or not even stop for them and continue through the intersection without stopping. This translates in real life to cars and pedestrians crossing through the intersection without stopping and checking to see if anybody is already crossing. Therefore, having a range with an average wait-time that is close to the 45th percentile is ideal since the average values of all the wait-time graphs is 44.856 and the switch point would likely be around the average.

4.3.4 Answer to the Question

The previous sub-sections (4.3.2 - 4.3.3) outlined the behavior of the inter-arrival and wait-time graphs for the target model density if a repetition was taken and the data was arranged onto line graphs. On the target's inter-arrival graphs, the graphs for both cars and pedestrian agents should have a level of consistency to them that borders on becoming erratic. The range of the IA values for the car agents should be wider than those of the pedestrian agents, while the range of the pedestrian agents should avoid the values at the bottom of the line graph. For the target's wait-time graphs, the consistency of both agent type's graphs should be bordering on erratic (like the inter-arrival graphs) as well as have the average wait-time value around or lower than the 45th percentile. None of the densities that have been tested exhibit the mentioned behavior in their graphs, but the approximate model density can be estimated by looking at the existing graphs and deducing the density where this behavior could be exhibited.

When looking at the various inter-arrival and wait-time graphs, it seems like the two graphs that are the closest to the target behavior are the ones for the low-density repetition (40% model density) and the mid-density repetition (60% model density). Therefore, it seems that the model density that best exhibits the target behavior should be at around a model density of 50% with a composition of 8 parts cars and 22 parts pedestrians (totaling 30 parts out a possible 60 for the model). Since the graphs for the 40% low-density repetition and the 60% mid-density repetition are on either side of the behavior of the target density, it makes sense that a density of 50% would exhibit the target behaviors. The agent composition of the chosen density is based on the results of the inter-arrival graphs for both low-density and mid-density tests.

Looking at the inter-arrival graphs for the car agents, the IA values tend to be lower when the density of the test is lower. The graphs for the low-density tests already shows the IA values for the car agents starting to become inconsistent. Having the same amount of car agents between the low and target model densities seems like the best way to preserve that “bordering-on-erratic” behavior already on the low-density test graphs. Conversely, pedestrian IA values tend to be more consistent as the density of the test increases, which means that having more pedestrians in the agent comp than in the low-density test would bring the behavior of the graphs closer to the target behavior. This agent composition will not have any particular effect on the wait-time graphs since both the car and pedestrian agents share the same general trends in terms of consistency over increasing density. The 50% model density will cause the wait-time graphs to exhibit the target behavior while the inter-arrival graphs will be influenced by the specific agent composition.

4.3.5 Validity of the Model at each Density Level

After the repetitions were completed and all the data had been sorted, the data associated with each entity for each repetition was copied onto a text document to be fitted in the ARENA input analyzer. The purpose of this was to figure out how well the model simulated the intersection at different densities by fitting the data to different distributions and seeing how well those distributions aligned with the data. An exponential distribution was chosen to be fitted onto the inter-arrival time data, normal distributions were chosen for the patience data, and triangular distributions were chosen for the wait-time data. These three fits were chosen because they had the lowest square error values (for each data type set) aside from beta and Weibull distribution fits. Once the data was fitted, a screenshot of the fit was saved in another file and used to record further data about the square error values and p-values for the chi-square test and the Kolmogorov-Smirnov test.

Entity Type	Data Type - Distribution Fit	VeryLow Test			Low Test			Mid Test			Dense Test			VeryDense Test			ChSq = Chi Square Test p-value
		ChSq	KS	SE	ChSq	KS	SE	ChSq	KS	SE	ChSq	KS	SE	ChSq	KS	SE	KS = Kolmogorov-Smirnov Test p-value
Blue Car	Inter-Arrival - Exponential	<0.005	N.A.	0.0211	<0.005	<0.01	0.0572	<0.005	0.0307	0.0299	0.0311	0.06	0.0139	0.0422	>0.15	0.0171	SE = Square Error
	Patience - Normal	<0.005	<0.01	0.0137	<0.005	<0.01	0.0118	<0.005	<0.01	0.0118	<0.005	<0.01	0.0107	<0.005	<0.01	0.0109	
	Wait Time - Triangular	<0.005	<0.01	0.0377	<0.005	<0.01	0.038	<0.005	<0.01	0.0296	<0.005	<0.01	0.0279	<0.005	<0.01	0.0236	
Red Car	Inter-Arrival - Exponential	<0.005	N.A.	0.0144	<0.005	<0.01	0.0314	<0.005	<0.01	0.0499	<0.005	<0.01	0.0446	<0.005	<0.01	0.0453	
	Patience - Normal	<0.005	<0.01	0.0108	<0.005	<0.01	0.0097	<0.005	<0.01	0.0203	<0.005	<0.01	0.012	<0.005	<0.01	0.0093	
	Wait Time - Triangular	<0.005	<0.01	0.0448	<0.005	<0.01	0.0321	<0.005	<0.01	0.036	<0.005	<0.01	0.0345	<0.005	<0.01	0.04	
Yellow Car	Inter-Arrival - Exponential	<0.005	N.A.	0.0186	<0.005	<0.01	0.0414	<0.005	0.0288	0.0339	<0.005	<0.01	0.0403	0.0344	0.0907	0.0185	
	Patience - Normal	<0.005	<0.01	0.016	<0.005	<0.01	0.0112	<0.005	<0.01	0.0083	<0.005	<0.01	0.0154	<0.005	<0.01	0.0163	
	Wait Time - Triangular	<0.005	<0.01	0.0255	<0.005	<0.01	0.0375	<0.005	<0.01	0.0229	<0.005	<0.01	0.0339	<0.005	<0.01	0.0452	
White Car	Inter-Arrival - Exponential	<0.005	N.A.	0.014	<0.005	<0.01	0.0359	<0.005	<0.01	0.0557	<0.005	<0.01	0.035	<0.005	0.0122	0.0168	
	Patience - Normal	<0.005	<0.01	0.0078	<0.005	<0.01	0.0149	<0.005	<0.01	0.0143	<0.005	<0.01	0.0132	<0.005	<0.01	0.0145	
	Wait Time - Triangular	<0.005	<0.01	0.0213	<0.005	<0.01	0.0381	<0.005	<0.01	0.0326	<0.005	<0.01	0.0245	<0.005	<0.01	0.0288	
LeftU Ped	Inter-Arrival - Exponential	<0.005	0.094	0.0829	<0.005	<0.01	0.0378	0.0701	>0.15	0.0102	0.331	>0.15	0.0065	0.0282	N.A.	0.0076	
	Patience - Normal	<0.005	<0.01	0.018	<0.005	<0.01	0.0123	<0.005	<0.01	0.0125	<0.005	<0.01	0.009	<0.005	<0.01	0.0111	
	Wait Time - Triangular	<0.005	<0.01	0.0297	<0.005	<0.01	0.0334	<0.005	<0.01	0.0389	<0.005	<0.01	0.0294	<0.005	<0.01	0.0292	
LeftD Ped	Inter-Arrival - Exponential	<0.005	>0.15	0.0594	0.027	0.145	0.0127	0.0465	>0.15	0.0102	0.399	>0.15	0.0047	<0.005	>0.15	0.0045	
	Patience - Normal	<0.005	<0.01	0.0567	<0.005	<0.01	0.0192	<0.005	<0.01	0.033	<0.005	<0.01	0.0093	<0.005	<0.01	0.0091	
	Wait Time - Triangular	<0.005	<0.01	0.0615	<0.005	<0.01	0.0334	<0.005	<0.01	0.0343	<0.005	<0.01	0.0303	<0.005	<0.01	0.0277	
UpL Ped	Inter-Arrival - Exponential	<0.005	0.0191	0.1152	<0.005	<0.01	0.0501	0.126	0.0498	0.007	0.0375	>0.15	0.0103	0.326	>0.15	0.0033	
	Patience - Normal	<0.005	<0.01	0.0258	<0.005	<0.01	0.0154	<0.005	<0.01	0.0111	<0.005	<0.01	0.0082	<0.005	<0.01	0.0125	
	Wait Time - Triangular	<0.005	<0.01	0.0642	<0.005	<0.01	0.0436	<0.005	<0.01	0.0321	<0.005	<0.01	0.0378	<0.005	<0.01	0.0286	
UpR Ped	Inter-Arrival - Exponential	0.0053	0.1	0.0393	<0.005	<0.01	0.0427	0.0677	>0.15	0.0069	0.136	>0.15	0.0069	0.59	>0.15	0.0023	
	Patience - Normal	<0.005	<0.01	0.019	<0.005	<0.01	0.0127	<0.005	<0.01	0.0109	<0.005	<0.01	0.0082	<0.005	<0.01	0.0079	
	Wait Time - Triangular	<0.005	<0.01	0.0339	<0.005	<0.01	0.0331	<0.005	<0.01	0.0258	<0.005	<0.01	0.0328	<0.005	<0.01	0.0224	
RightU Ped	Inter-Arrival - Exponential	0.0096	>0.15	0.0474	0.0148	<0.01	0.0167	0.159	>0.15	0.0093	0.21	>0.15	0.0035	<0.005	N.A.	0.0084	
	Patience - Normal	<0.005	<0.01	0.0314	<0.005	<0.01	0.0167	<0.005	<0.01	0.0136	<0.005	<0.01	0.0129	<0.005	<0.01	0.0075	
	Wait Time - Triangular	<0.005	<0.01	0.0354	<0.005	<0.01	0.0351	<0.005	<0.01	0.03	<0.005	<0.01	0.0355	<0.005	<0.01	0.02866	
RightD Ped	Inter-Arrival - Exponential	<0.005	0.0427	0.0598	0.0164	0.0477	0.0253	0.416	>0.15	0.0046	0.413	>0.15	0.0032	0.0852	0.066	0.012	
	Patience - Normal	<0.005	<0.01	0.0306	<0.005	<0.01	0.0214	<0.005	<0.01	0.0206	<0.005	<0.01	0.0128	<0.005	<0.01	0.0103	
	Wait Time - Triangular	<0.005	<0.01	0.0532	<0.005	<0.01	0.0389	<0.005	<0.01	0.0347	<0.005	<0.01	0.0277	<0.005	<0.01	0.0363	
DownL Ped	Inter-Arrival - Exponential	<0.005	0.0574	0.0953	<0.005	<0.01	0.0391	<0.005	<0.01	0.034	0.244	>0.15	0.0041	0.509	N.A.	0.0056	
	Patience - Normal	<0.005	<0.01	0.0309	<0.005	<0.01	0.0151	<0.005	<0.01	0.0144	<0.005	<0.01	0.0093	<0.005	<0.01	0.0074	
	Wait Time - Triangular	<0.005	<0.01	0.0402	<0.005	<0.01	0.0394	<0.005	<0.01	0.0359	<0.005	<0.01	0.0337	<0.005	<0.01	0.0289	
DownR Ped	Inter-Arrival - Exponential	<0.005	0.0478	0.0907	<0.005	0.0104	0.0331	0.0084	0.088	0.0166	0.331	>0.15	0.0055	0.183	>0.15	0.0057	
	Patience - Normal	<0.005	<0.01	0.0278	<0.005	<0.01	0.0216	<0.005	<0.01	0.0116	<0.005	<0.01	0.0134	<0.005	<0.01	0.0177	
	Wait Time - Triangular	<0.005	<0.01	0.0599	<0.005	<0.01	0.0482	<0.005	<0.01	0.0324	<0.005	<0.01	0.0371	<0.005	<0.01	0.042	

Table 1. A table detailing the two p-values and the square error value for the fit of each data type (Larger Version provided in Appendix E).

The lower the square error (SQE) values and the p-values are, the less chance there is that the corresponding data was outside of what they should have been normally and the more likely the model was accurate. The chart above splits each entity types' data into inter-arrival, patience, and wait-time fits, and gives the SQE and p-values for each repetition. Some of the values are highlighted in orange to signify that the SQE or p-value in question is above 0.1 (or not-available due to an error) and may be an indicator that the data for the corresponding set was an irregularity. The less orange values a repetition has, the more accurate the model has simulated the intersection under the corresponding density. The orange values currently on the distribution table shows some interesting trends.

All the orange highlighted values are Inter-Arrival values, which hints that the data for the inter-arrival values may be a bit inaccurate. That may be true since the data had to be filtered twice to get rid of extraneous data points. The speed at which the cars and pedestrian agents were moving at caused them to linger on patches over the course of 2 to 3 ticks. This caused them to be counted twice on the patch that checks for new arrivals to the intersection and cause extra data values of 1 tick to be included in the data when inter-arrival times were calculated. The second filtering allowed those extra 1's to be removed, but it also reduced the total number of data points entered into the distribution.

Other than some of the inter-arrival values, most other values (especially the patience and wait-time values) were under 0.1. This argues that for patience and wait-time values, the model did a good job in gathering accurate data, while the inter-arrival methods were a bit hit-or-miss. The dense and the very dense repetitions especially had a lot of orange values, meaning that the data those two repetitions collected were a bit questionable. However, that fact remains that all (save one) the square error values were under 0.1 which means that the fits for each data set are accurate to some extent. That means that this data has some validity and may be considered reliable (albeit with some reservations as to the extent of its accuracy).

The distributions that were chosen for each data set were chosen because they were simple to understand and provided an easier way to analyze the data. The problem that these fits were trying to solve was simple, and that was to determine the validity of the model under the various densities. The triangular, exponential, and normal distribution fits were both simpler to understand and fulfilled the simple goal that the data set fitting was meant for. Using a Weibull or Beta distribution would have resulted in needing more calculations to determine the validity of the model and needlessly complicating the validity verification. The three mentioned fits were both simpler to use and had error values that were low enough to indicate that the fits worked.

4.3.6 Complex Behaviors Emerging from Simple Rules

The coding of the agents caused them to perform basic actions like stopping, accelerating, and turning. These actions are in accordance with the simple rules that are laid out by the model’s code, and each rule is not complex by themselves. However, the series of agents acting at the same time causes complex behaviors to arise from those simple rules. These complex behaviors can range from causing the pedestrian agents to act in a more “aggressive” manner when grouped up to having the car and pedestrian agents take turns in crossing the intersection. These complex behaviors, though not explicitly coded in model, are the constructs of the simple rules outlined by the code.

However, the complex behaviors are not necessarily planned nor are they intentional. Often, the analysis portion of studies with a model like this one includes identifying what complex behaviors arise from the code that was programmed into the model. For a model like this one, what complex behaviors emerged was the result of both the rules that the coding dictated as well as what conditions the model was under. In this model, it was intersection density that caused the agents to exhibit the more aggressive behavior that was akin to a mob mentality (even though no such behavior was explicitly coded). It was these complex behaviors that ultimately caused the graphs to turn out the way they did.

4.4 Limitations and Sources of Error

A major limitation of the model itself is that it can only simulate static density situations and cannot change the overall density mid test. Reality does not work under the assumption that the density of an intersection stays constant, so the model cannot simulate reality to that extent. That is why the series of tests are necessary, so that data between different densities can be compared. Another limitation of the model is the lack of agents' dynamic behavior. Each agent can only follow their programming rigidly and cannot dynamically change their actions like a human in real life could. The agents sticking rigidly to their programming may cause them to get into incidents that a real human would recognize and avoid by altering their approach.

One potential source of error (as seen in the previous section) was the data gathering and preparation methods for inter-arrival time data. The greatest number of p-values over 0.1 are from inter-arrival data and suggests that the method of collecting the data needs to be improved to get more accurate data. The filtering of the data is necessary because without doing that, extra points that are technically not a part of the data get mixed into the data set. Since filtering out extra points leaves less data points overall to analyze, one possible way of improving the gathering method is to extend the time at which the model is running, therefore increasing the number of inter-arrival points to analyze.

One problem with interpreting the data has to do with the conversion of ticks to real-world time units and how there is no set conversion. If average wait-time of the target behavior were to be converted 1 to 1 with seconds, then the ideal wait-time would essentially be 25 to 30 seconds for each pedestrian and car before the switch in mindsets were to take place. This is an unrealistic scenario since realistically, both pedestrians and cars would wait less time than that if they were still in a time-saving focused mindset. Prior observations of the intersection before the model were made saw that the cars would usually wait only 5 to 10 seconds while they were still in a time-focused mindset. The target wait-time should be waited out when the cars and pedestrians are still in this mindset and waiting only a little longer than the mentioned times. Since the 1 to 1 conversion of ticks to seconds provides unrealistic answers, relative lengths should instead be used in interpreting the model and the real-life intersections.

The programming behind the model could potentially be the biggest source of error. The way data is collected, or the way agents behave, there are many ways that to code the kind of behavior that is currently in the model. There could be quirks in the code that cause unforeseen errors, and there could exist ways to code those behaviors without those quirks. Without re-coding the model and redoing the tests, there is no way to know what the best method to code in the behaviors are. Any future study should take note of the behaviors of the agents in this model and see if the ways to code them could be improved upon in any way.

Chapter 5. Discussion and Conclusion

5.1 Discussion of the Claims

The answer that was given to the question "What is the level of density in an intersection where travelers should switch their mindsets from being time-focused to safety-focused?" was given in terms of model density and agent composition. These two terms are specific to the model itself, but they can be translated to real world concepts that help answer the question realistically. Model density can translate intersection density, which means how full of pedestrians and vehicles an intersection is compared to the maximum number of entities that can be in the intersection at the same time. Intersection density is used rather than concrete numbers so that the answer given is not limited to any one intersection. Agent density can translate to the exact makeup of the entities currently in the intersection (exactly like how agent composition is with the model, only referring to actual people rather than agents). Even if two intersections have the same intersection density, the compositions of both could be completely different from each other (at 100% density, one could be filled with a mix of cars and pedestrians while the other is completely full of pedestrians.

The answer given in the previous claims that the point where travelers would change their mindsets from being focused on saving time to being focused on navigating safely was at 50% model density with an agent composition of 8 parts cars and 22 parts pedestrians. Translated, this would mean that travelers would change their mindsets when the intersection they are about to travel through is at half its total capacity and the entities within it consists around 27% cars and 73% pedestrians. Travelers may not necessarily change their mindsets at this density level since there are several extraneous factors (like traveler disposition or weather) that could affect their decision. However, it is at a 50% intersection density level where travelers SHOULD change their mindsets if they want to navigate the intersection without incident. The exact makeup of this density level is also a bit flexible but should follow the trend of consisting of mostly pedestrians with a few cars in the makeup.

5.1.1 Results over Multiple Intersection Types

With the way that the answer was given through concepts that translate to intersection density and the makeup of that density, these results could apply to other intersection types as well. The Monroe Street intersection used to reach this answer is a four-street intersection with three two-way streets and one one-way street leading to the intersection center. It is not unreasonable to assume that other travelers in other intersections like the Monroe intersection behave in a similar manner. This means that the answer from this study could apply to these intersections if they are not too different than the one used for this study. Even if the density for these other intersections is different, then the answer given in this study can act as a starting point to research what these other target densities are.

However, this type of study will not provide target densities for intersections that are regulated by traffic lights due to how there is an external force that regulates the intersection. The intersection that the model was based off did not have a traffic light or any external traffic regulator other than static stop signs and existing intersection rules. It was all up to the individual travelers to follow the rules and make decisions based about the intersection. Having a traffic light that actively told when cars could and could not cross the intersection took a lot of the decision making out of the hands of the individuals and left it to an objective, impartial device. Having pedestrian lights in an intersection does the same thing as the traffic lights, but for pedestrians instead.

The point of estimating the density in this study was to find the density of the intersection where most travelers would likely decide when they want to switch to a more safety-oriented mindset. Having external regulators that tells every traveler when they should or should not act established objective periods of time that dictates the behaviors of others. A switch in mindset for the travelers will not matter as much since the only things that a traveler has to do to safely cross the intersection is obey the traffic lights, which heavily encourages a safety-focused mindset. The only times a driver will cross the intersection with a time focused mindset is if they ignore the red lights at the intersection, arrive at the intersection as green lights pop up, or speed up to cross the intersection when the light is yellow. Pedestrians will be even less likely to disobey traffic laws since if they get into an accident, they are often the ones that come out of it worst off.

5.2 Possible Routes for Future Research

Using this study as a foundation, there are many other research routes that can investigate a variety of other topics related to research. One route is rebuilding the model so that it depicts other intersection types can allow investigations of the target density of other intersection types with different street configurations. Another route is coding the model in a way that can investigate the effects of weather on the switch in mindset (though this requires a lot of observation of car and pedestrian behavior under various types of weather). There is also the possibility of adding other types of other agent types to the model like bikes, trucks, or buses to see how those affect the density point. Basically, there are many ways to improve upon the study to research various aspects of the target density point.

5.2.1 Suggestions for Improving on This Study's Procedure

There are several ways in which the study for this procedure could be altered or improved upon to get a better result or more accurate result than what was already found. For one thing, the way that agents enter and exit the intersection could be reworked to add a bit of dynamic elements to the number of agents in the model at once. The way the model works now is that the users set the number of agents that they want on the model and when run, the model keeps the agent amount constant. Instead of having the agents simply reset their positions when they travel, it could be possible to instead erase the agent after they finish and recreate them at their starting point after a bit of time passes. This could let the user instead set agent limits and have the model never generate a number of agents that is over the limit at the same time. Doing this would allow a degree of realism to the model and depending on how the rate of agents spawning is coded, allow for study on how other external aspects affect the target density point.

Other than agent numbers, a lot of improvement could center around tweaking certain aspects of the code for a more accurate model overall. The speeds of the various agents could be increased to avoid the problem with the inter-arrival patch checker counting the same agent twice. The wait-time assignment could be readjusted so that the assignment and record of the wait-time could happen closer together. The collision checker could be reworked so that it always checking the agents rather than just the agents in the center of the intersection. The are many small and numerous ways to alter the code so that it can be improved for a more accurate experience.

5.3 Conclusion

According to the research done by this study, there exists a tipping point when travelers in an intersection (without traffic lights) should switch their mindsets from one that prioritizes saving time when to one that prioritizes following the rules to stay safe. This tipping point is based on how full an intersection is of travelers at the time a new traveler enters or is about to enter an that intersection. When the intersection in question is at a density of 50% and is comprised of a 27% to 73% ratio of cars to pedestrians, travelers should become more focused of following the rules for their own safety. Though this density and ratio may not be the same in other intersections than the one studied in this paper, it could be close depending on how similar that intersection is to the Monroe Intersection used in this study. Keeping in mind that each intersection not regulated by a traffic light has a similar turning point will help travelers in general keep safe should they attempt to cross a similar intersection when it is at the tipping point.

Chapter 6. References

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Appendix A: Model Screenshots

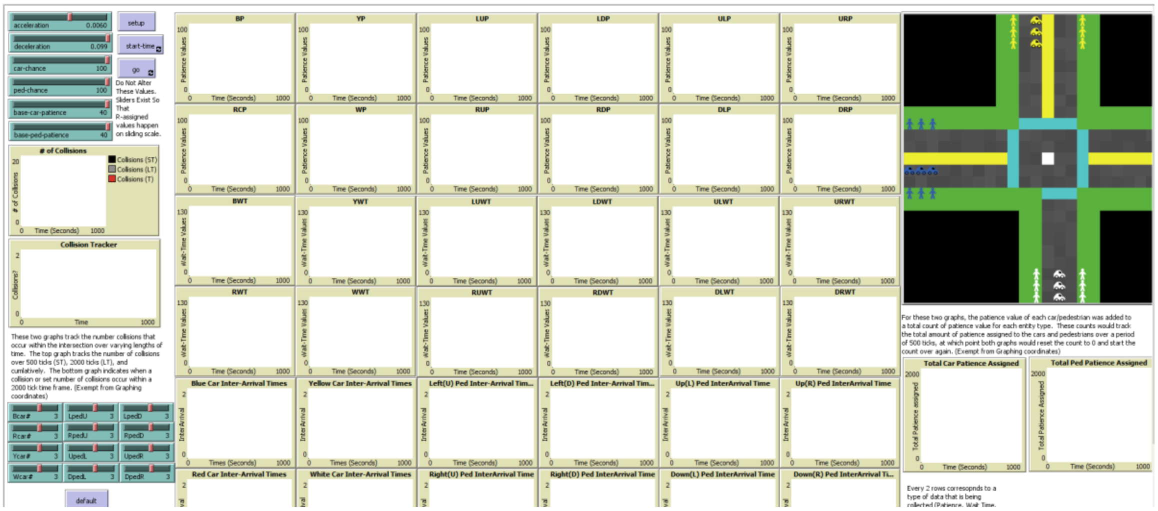


Figure 15. The interface of the model that contained both the control panel for adjusting agent numbers and the various line graphs created to display the data.

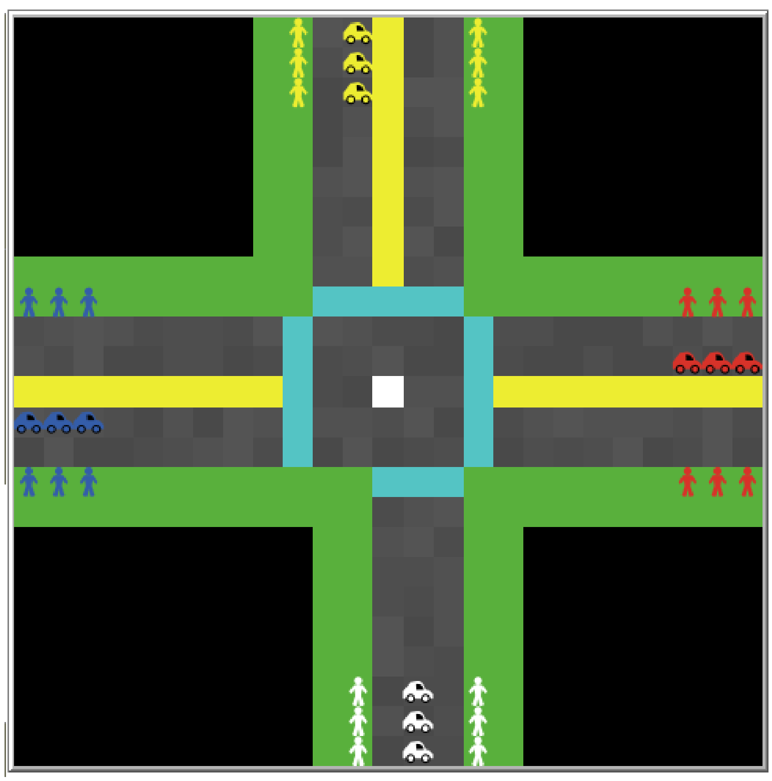
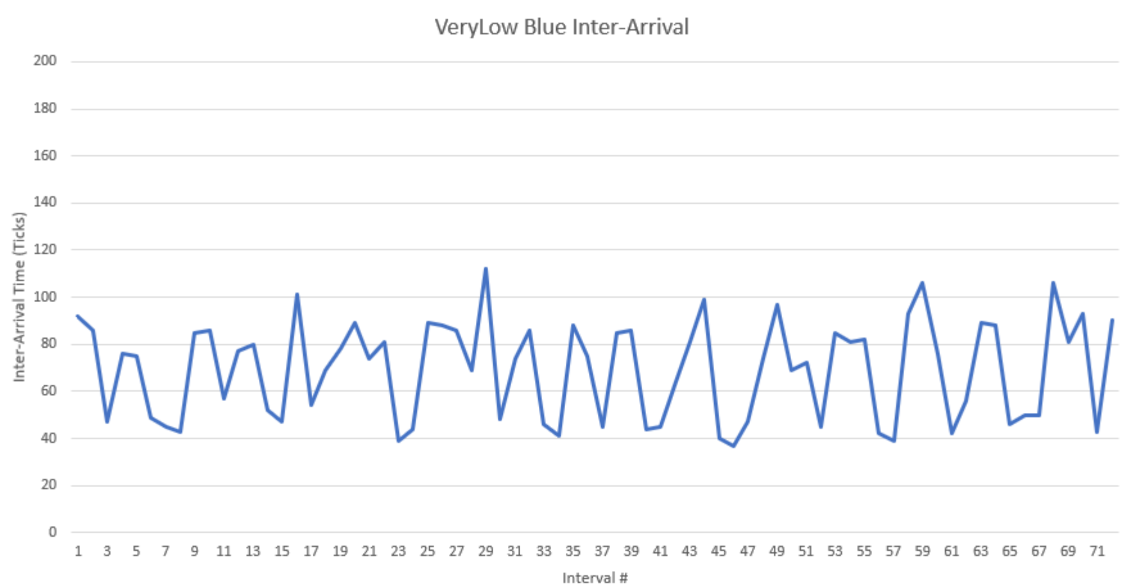


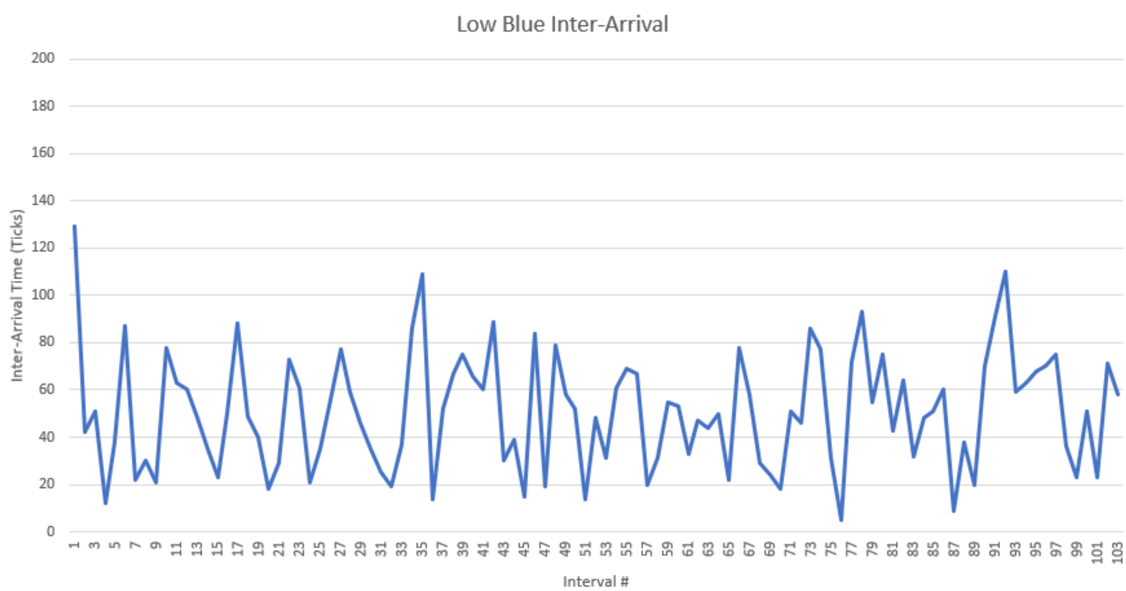
Figure 16. The intersection model that is currently has the number of agents equivalent to that for the mid-density test.

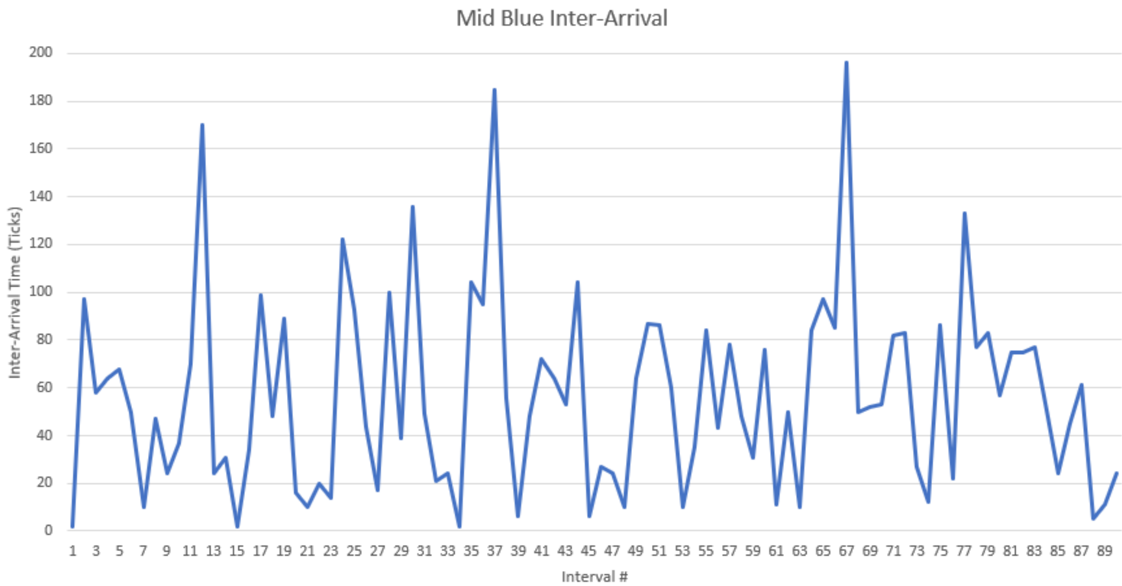
Appendix B: Inter-Arrival Time Graphs

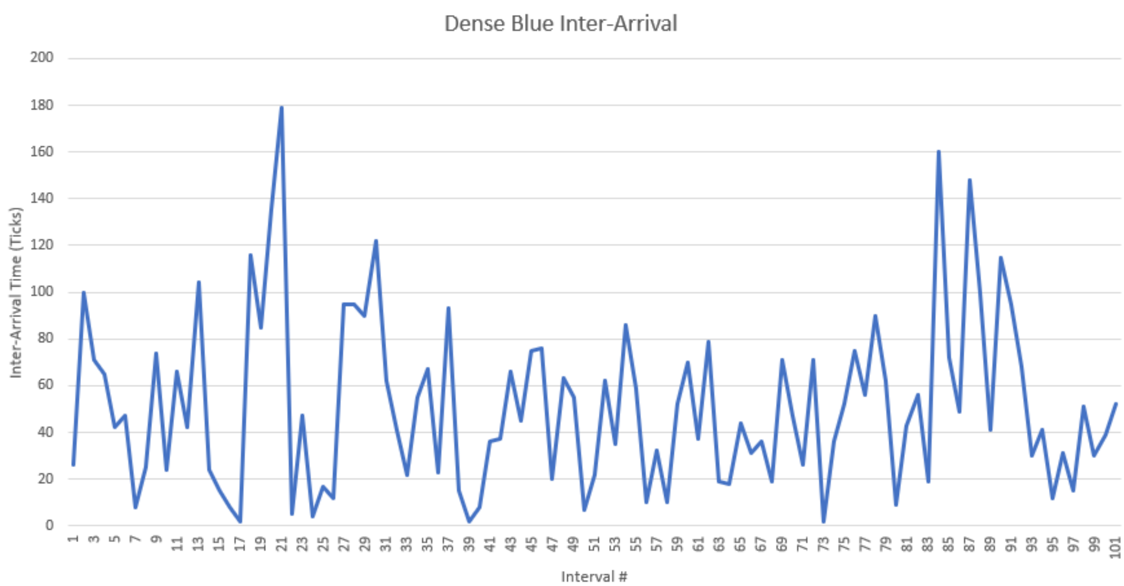
Figures 17 presents all the inter-arrival time graphs of the blue car agents, Figure 18 presents all the inter-arrival time graphs of the red car agents, and the pattern repeats until all the inter-arrival time graphs are presented. Each graph is labeled with what test the data originates from.

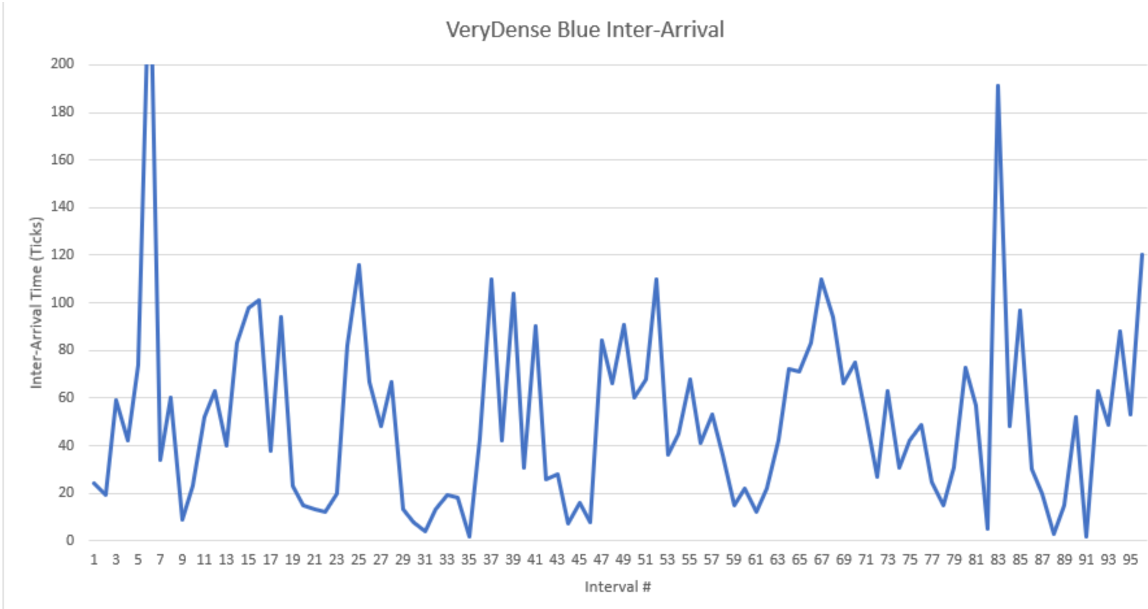
Figure 17. Blue Car Inter-Arrival Time Graphs





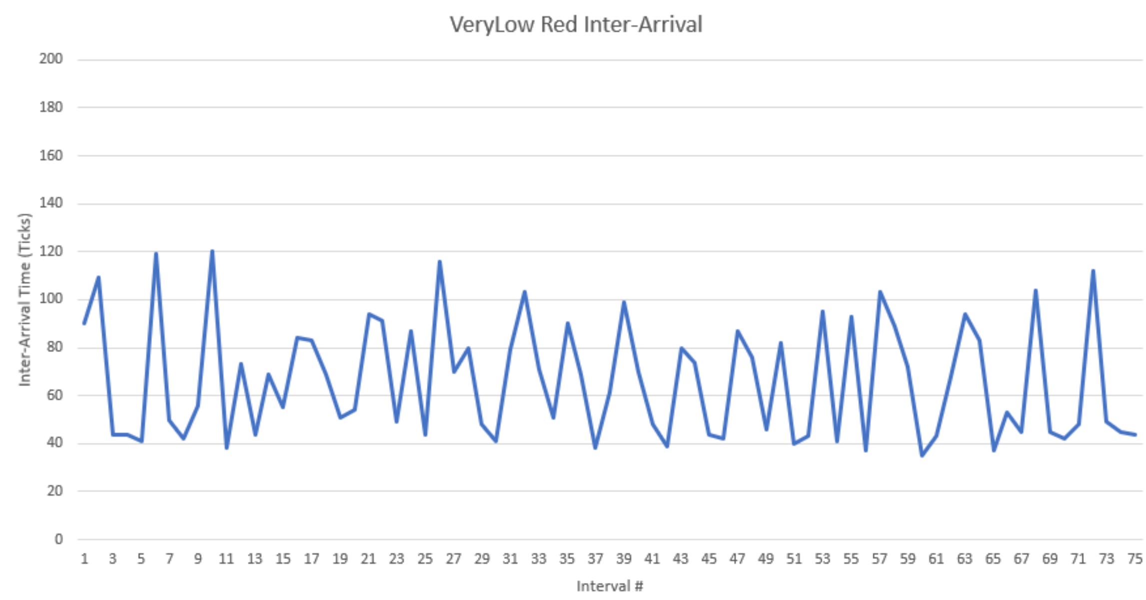

Mid Blue Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #

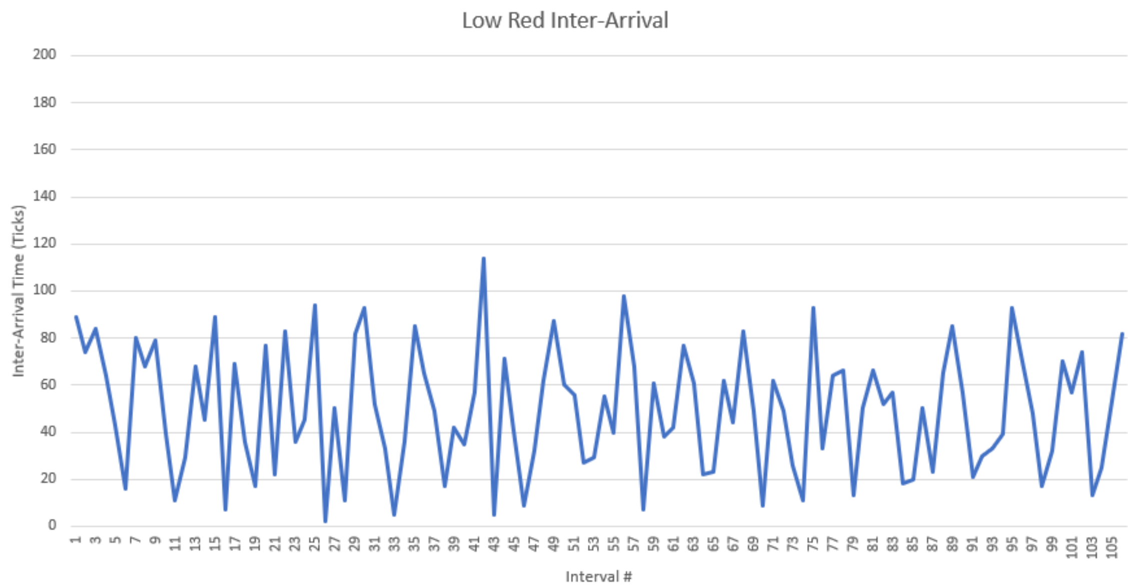

Dense Blue Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #

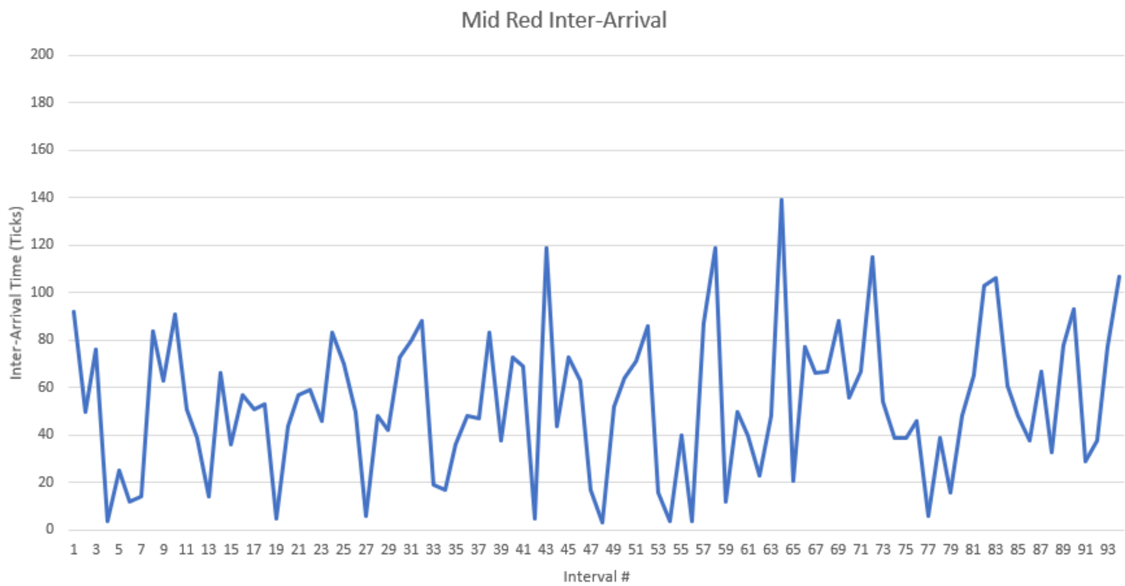


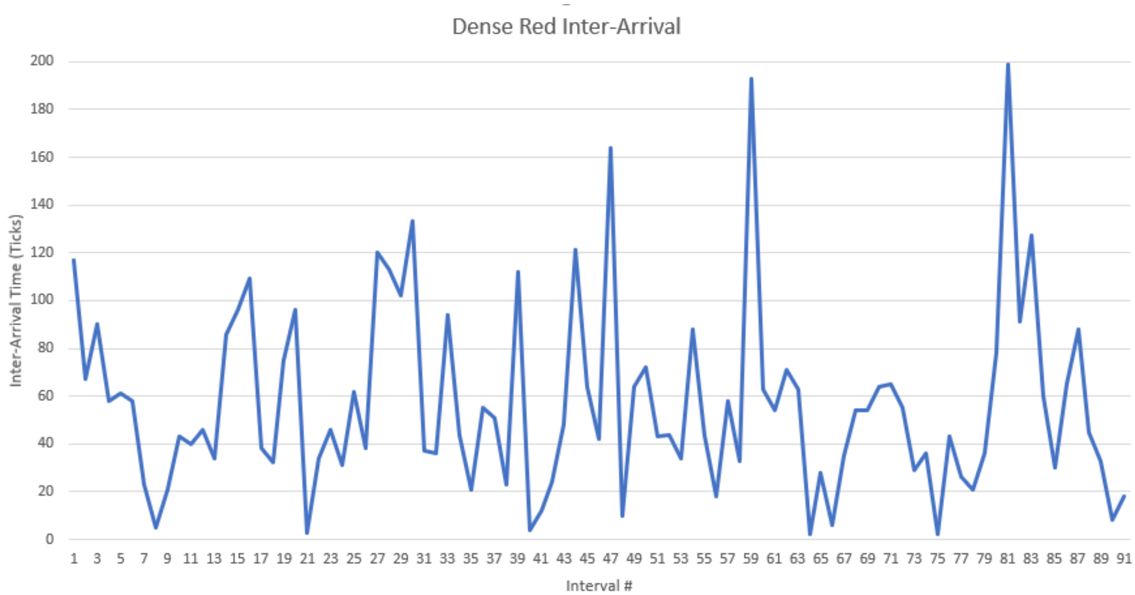
VeryDense Blue Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #
200
180
160
140
120
100
80
60
40
20
0
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71 73 75 77 79 81 83 85 87 89 91 93 95

Figure 18. Red Car Inter-Arrival Time Graphs






Mid Red Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #


Dense Red Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #

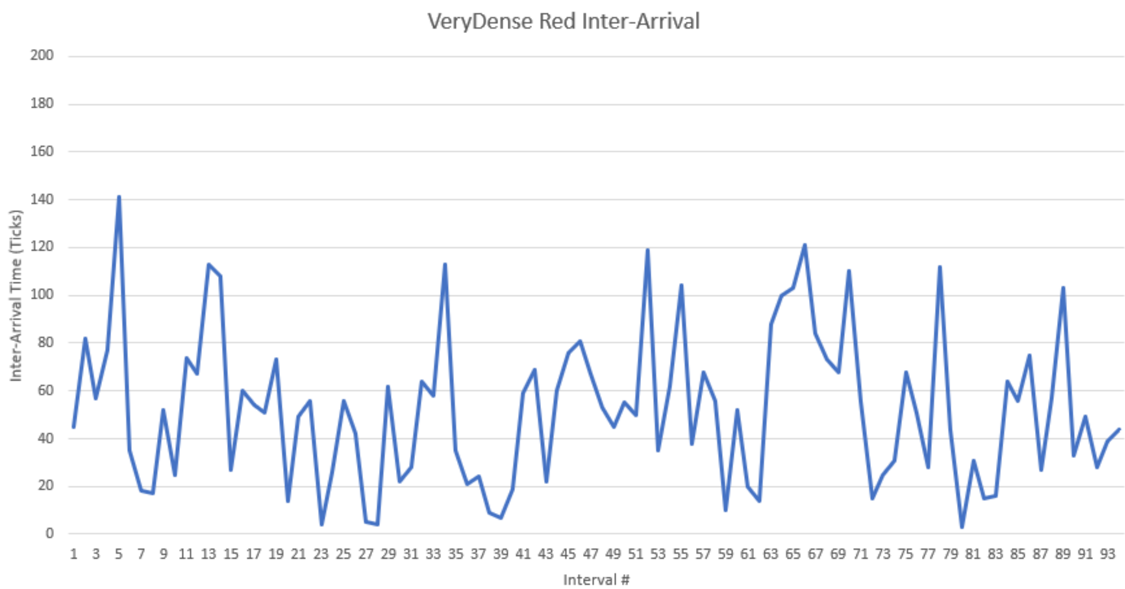
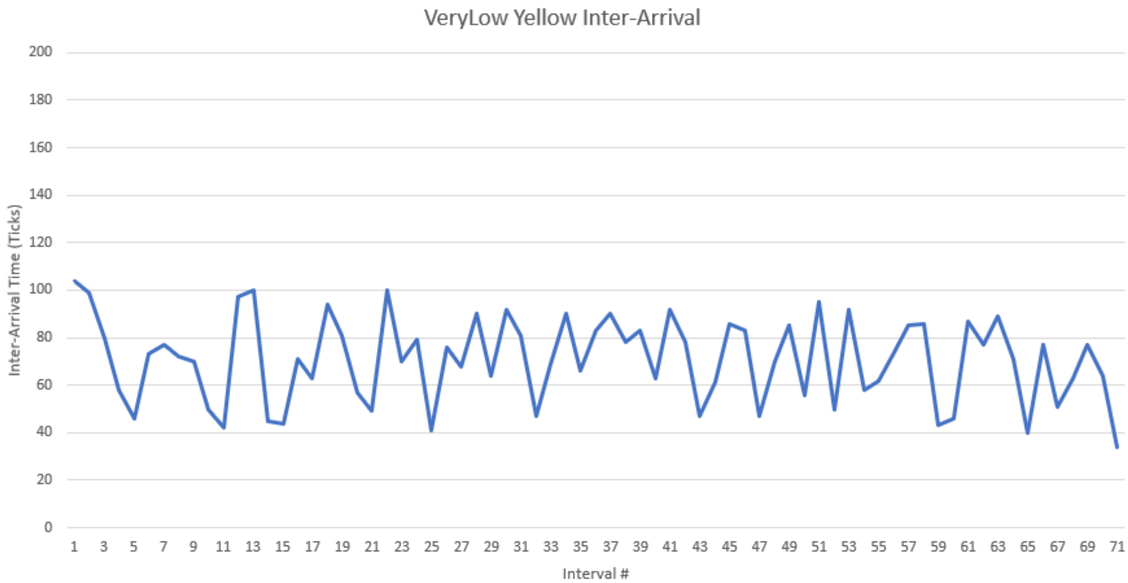
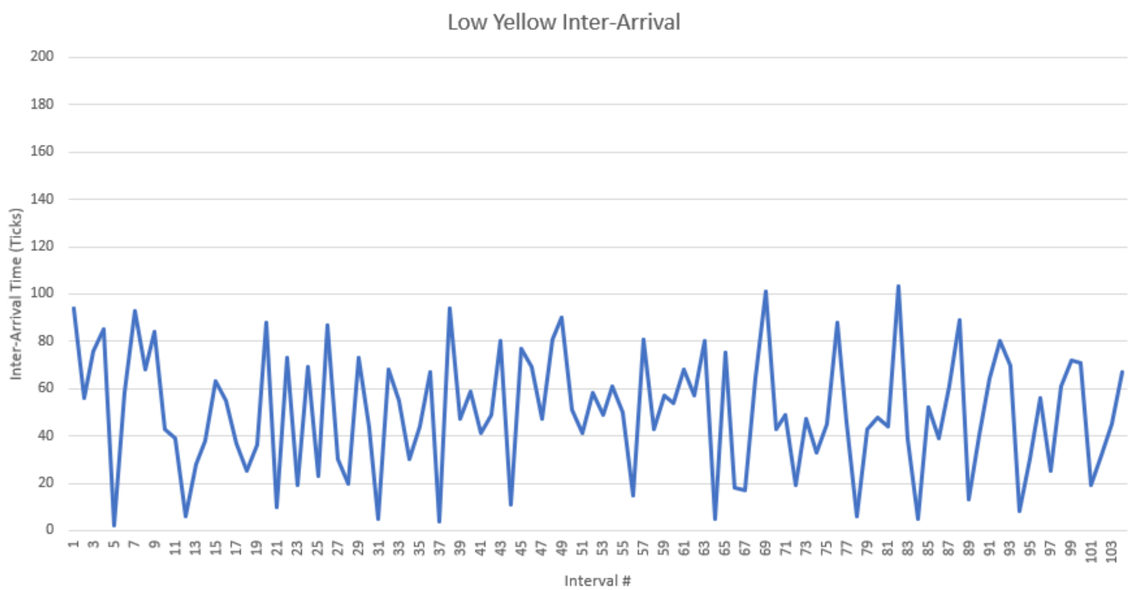
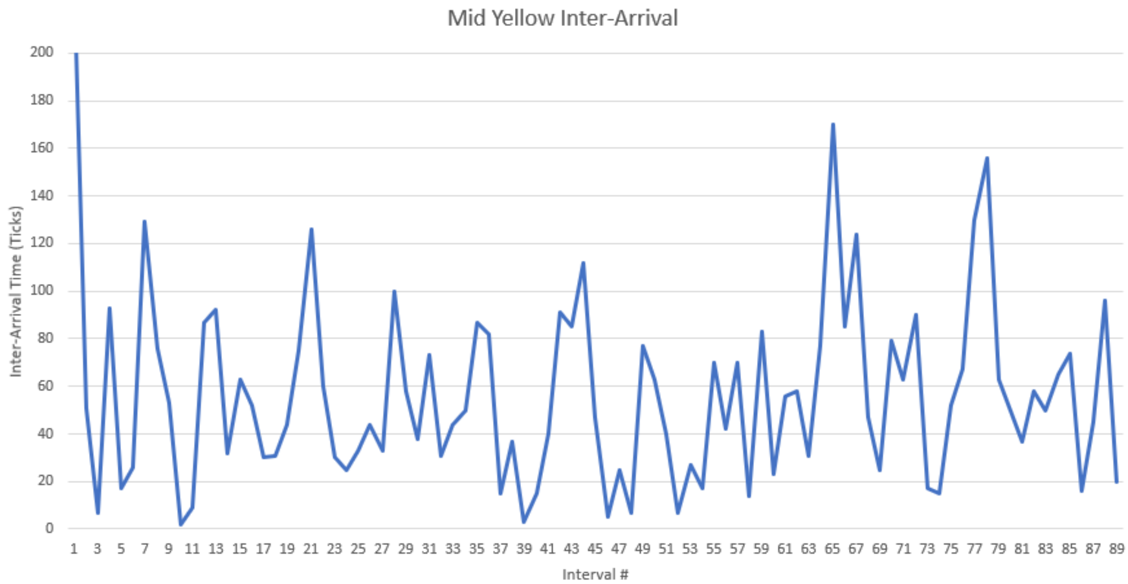
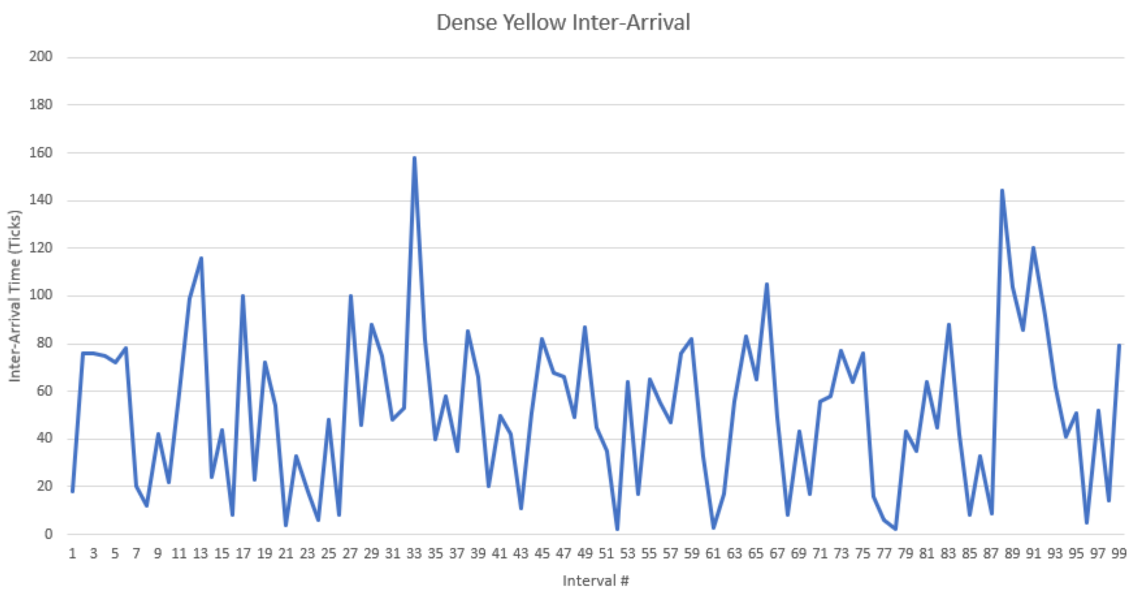

VeryDense Red Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #
200
180
160
140
120
100
80
60
40
20
0
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71 73 75 77 79 81 83 85 87 89 91 93

Figure 19. Yellow Car Inter-Arrival Time Graphs






Mid Yellow Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #


Dense Yellow Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #

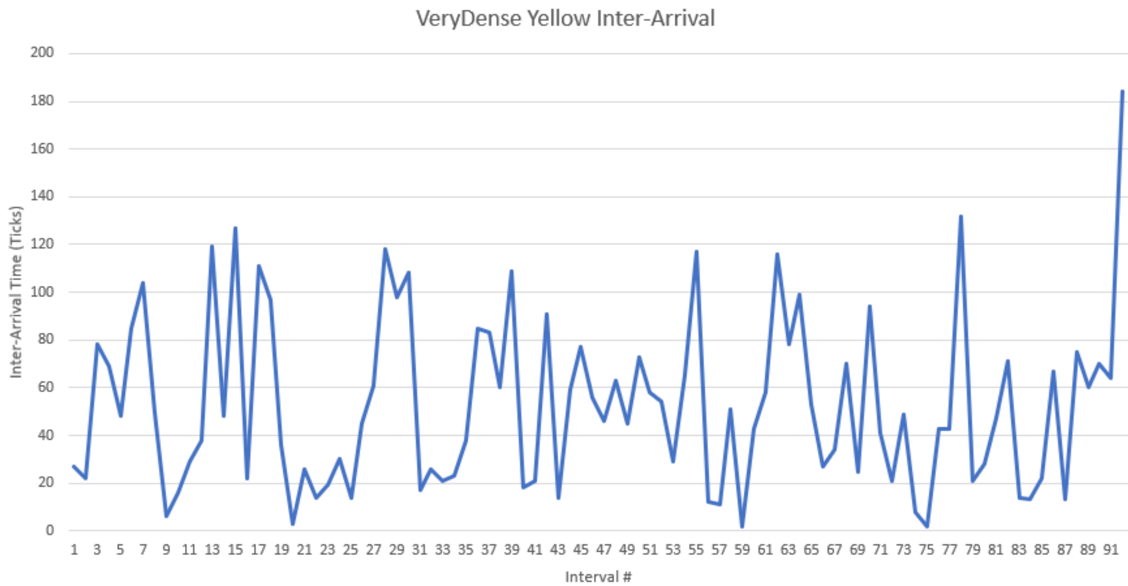
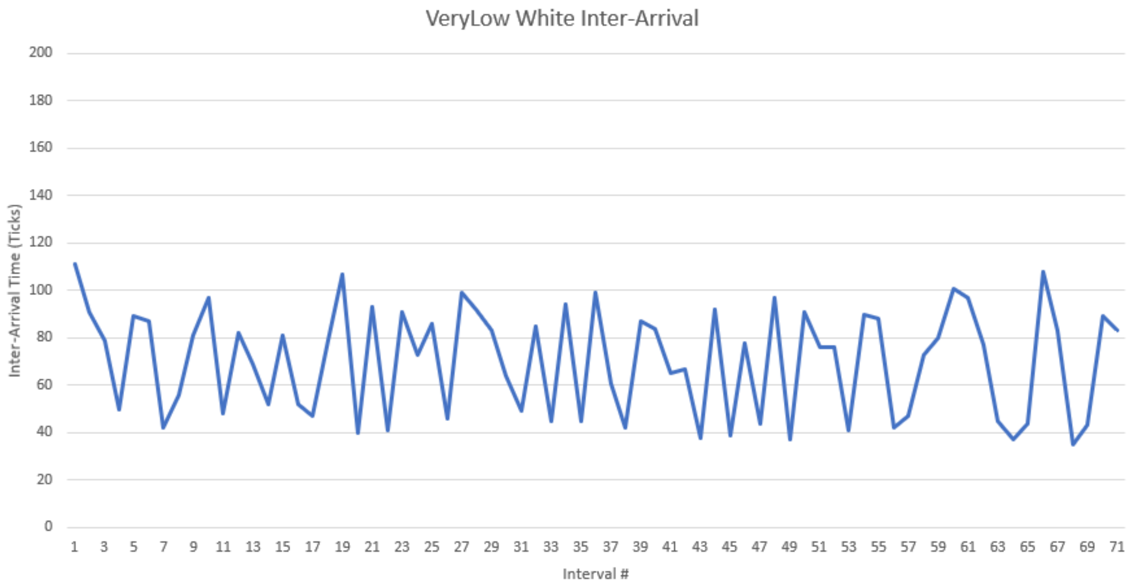
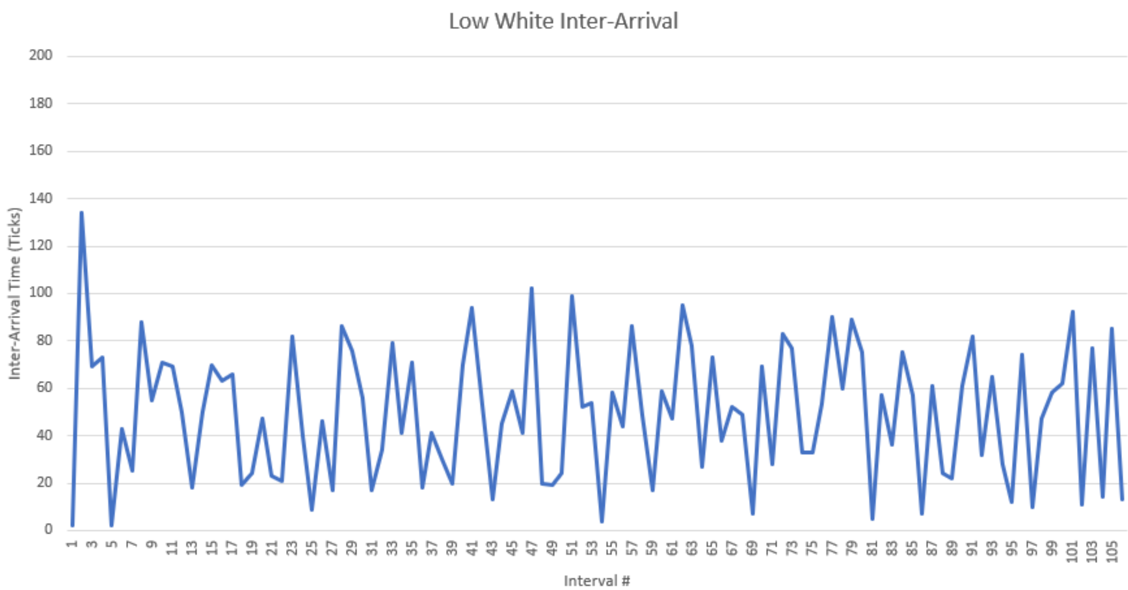
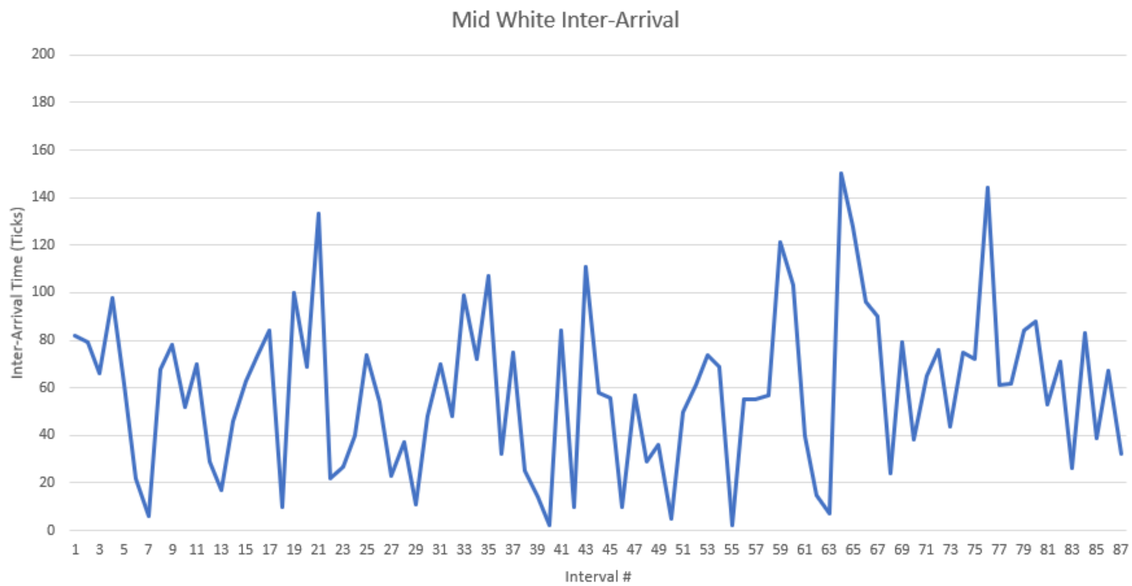
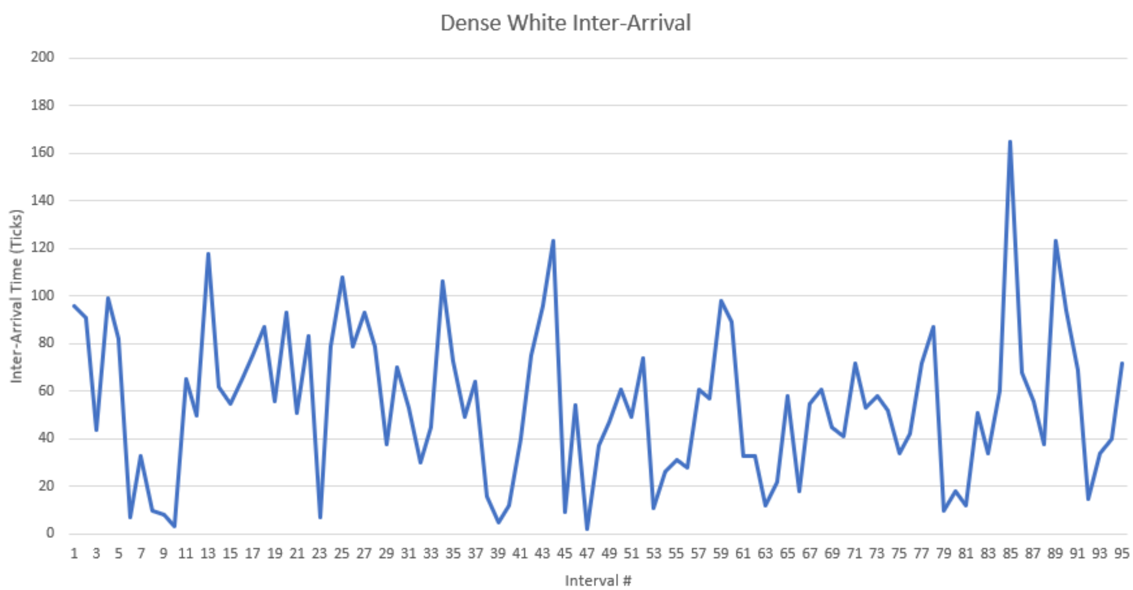

VeryDense Yellow Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #
0
20
40
60
80
100
120
140
160
180
200

Figure 20. White Car Inter-Arrival Time Graphs






Mid White Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #


Dense White Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #

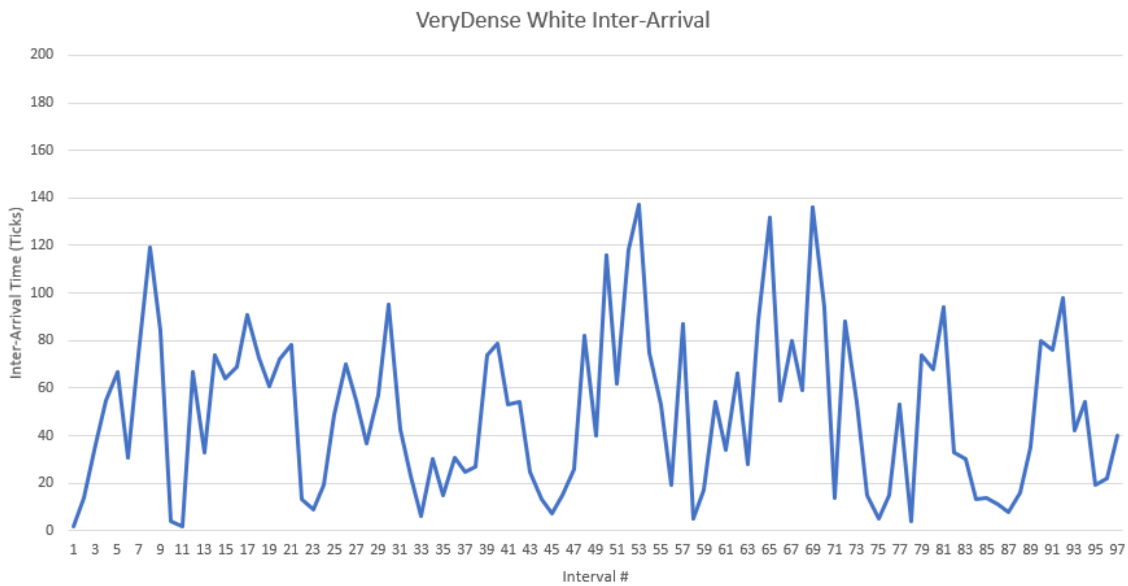
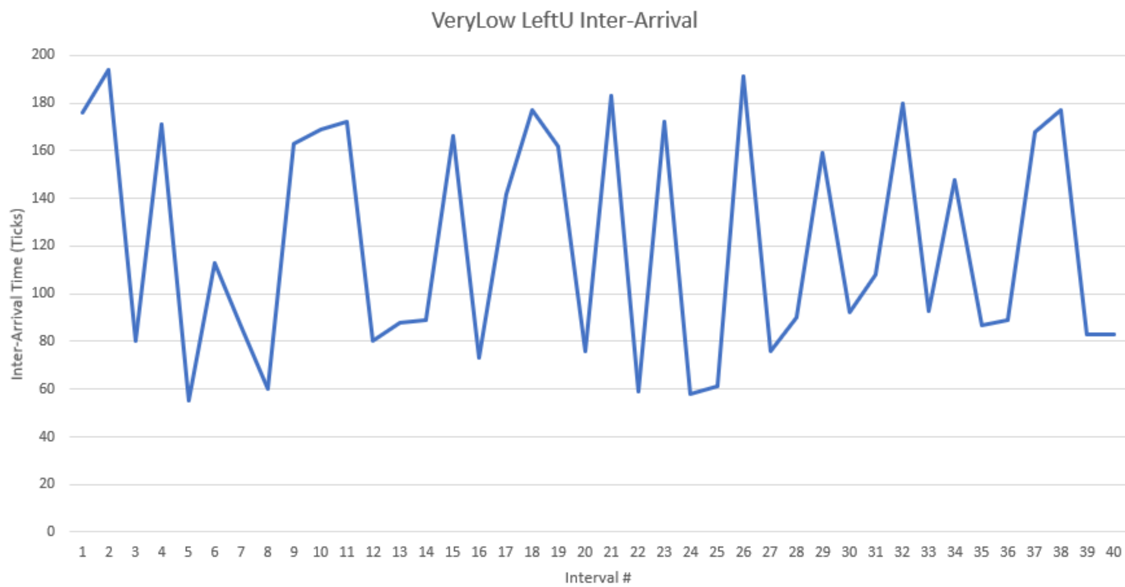
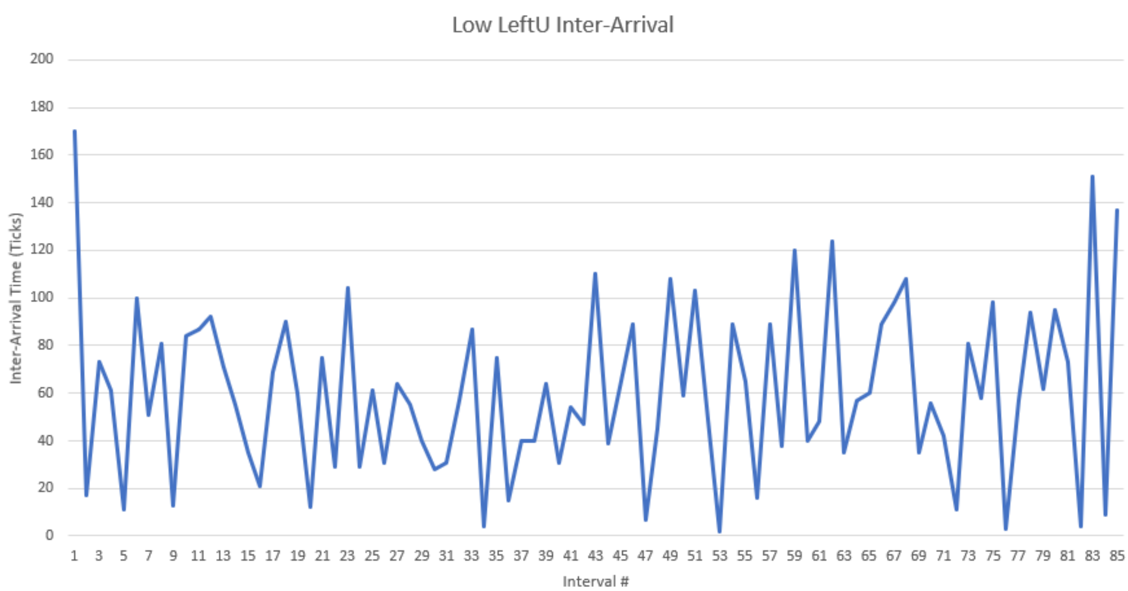
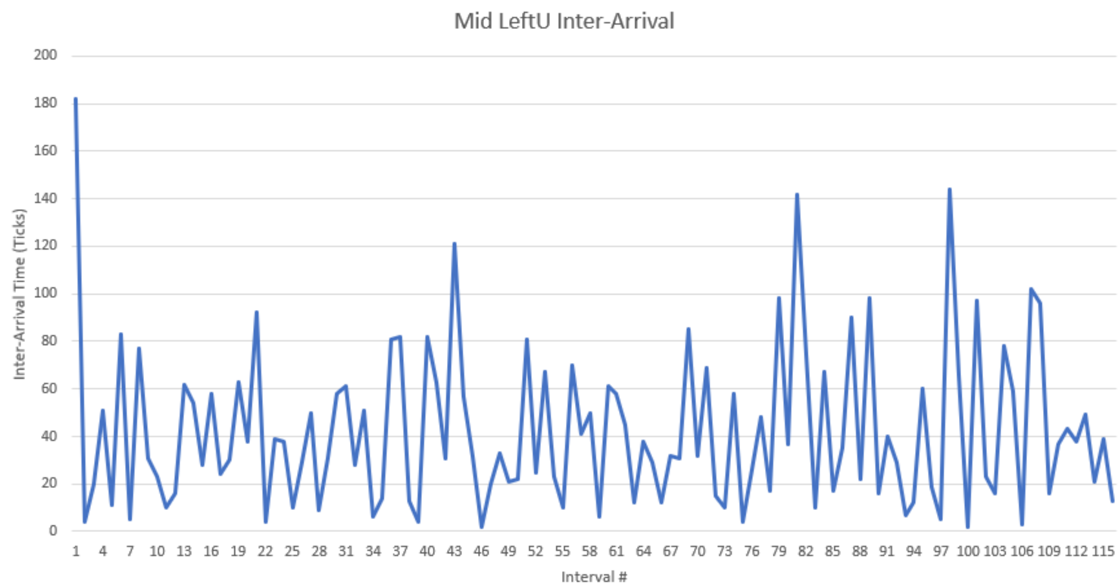
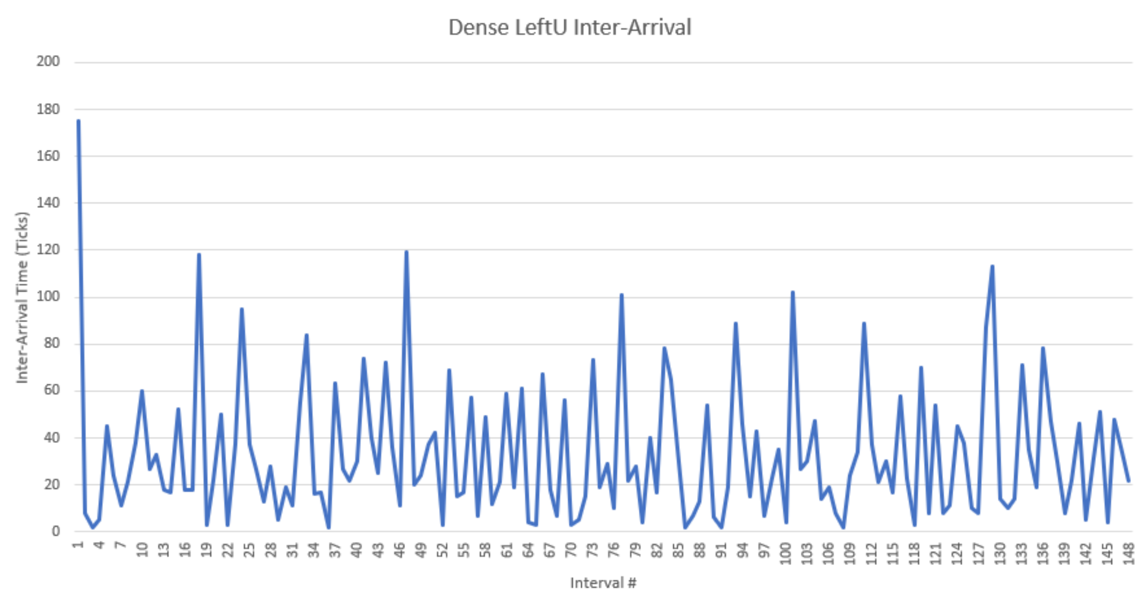

VeryDense White Inter-Arrival
Inter-Arrival Time (Ticks)
200
180
160
140
120
100
80
60
40
20
0
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71 73 75 77 79 81 83 85 87 89 91 93 95 97
Interval #

Figure 21. Left (Up) Pedestrian Inter-Arrival Time Graphs






Mid LeftU Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #


Dense LeftU Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #

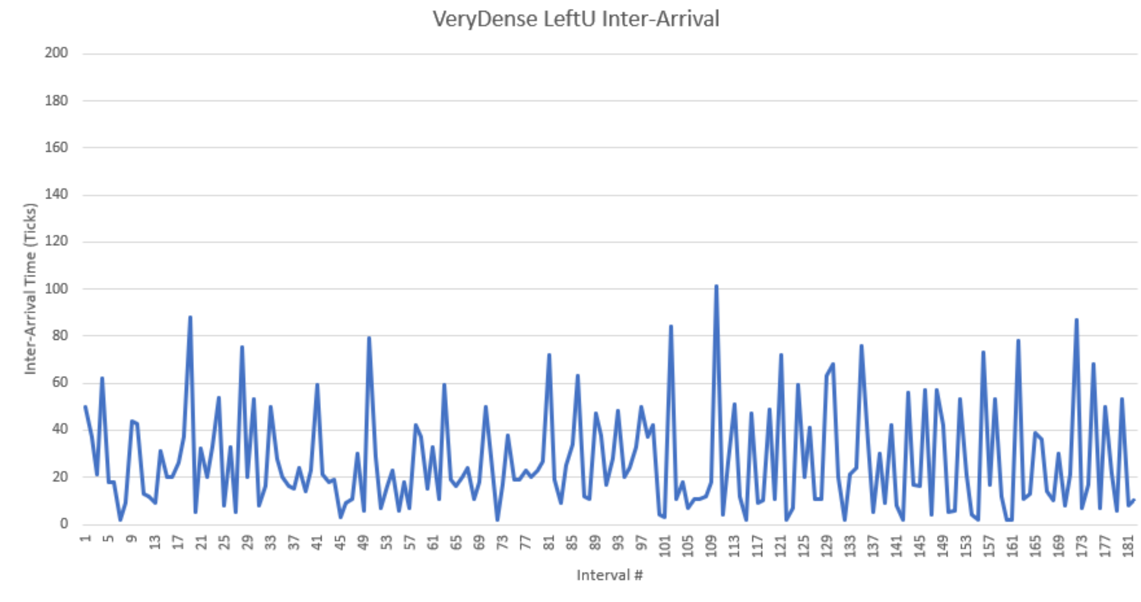
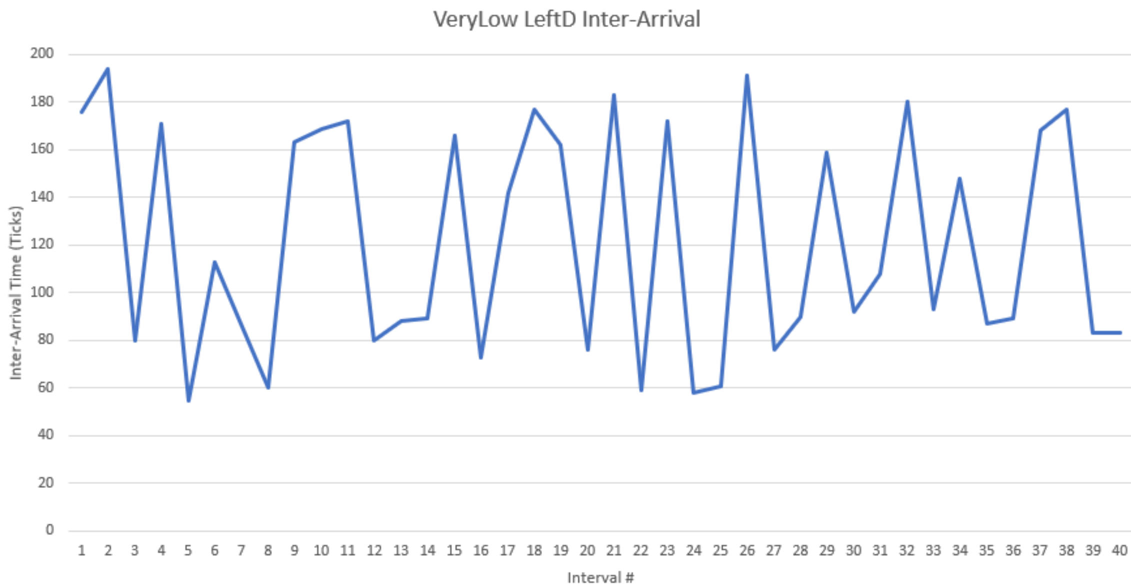
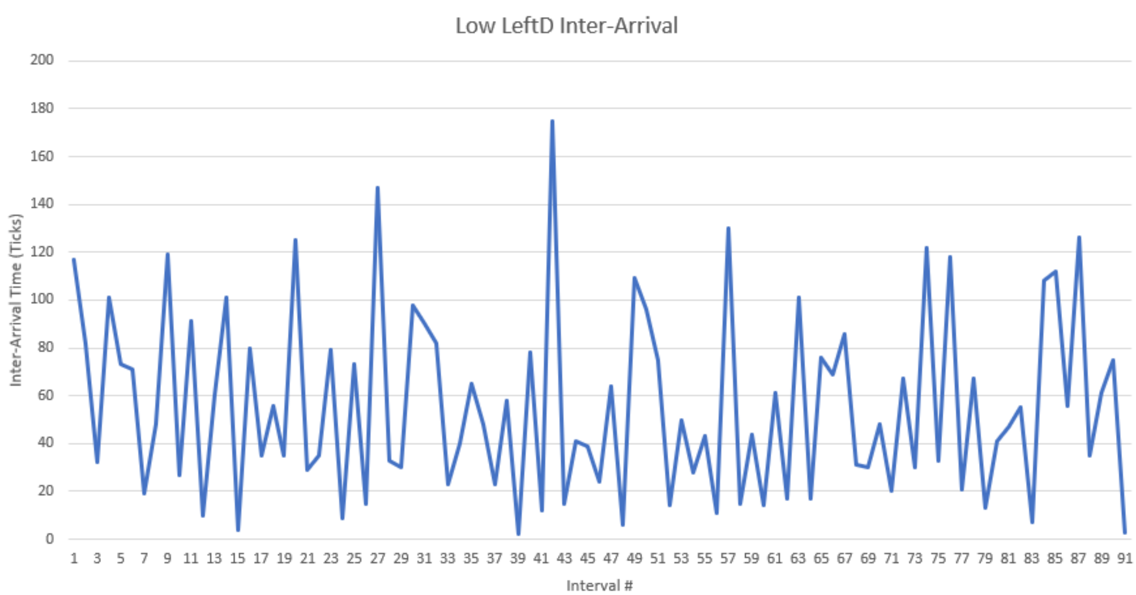
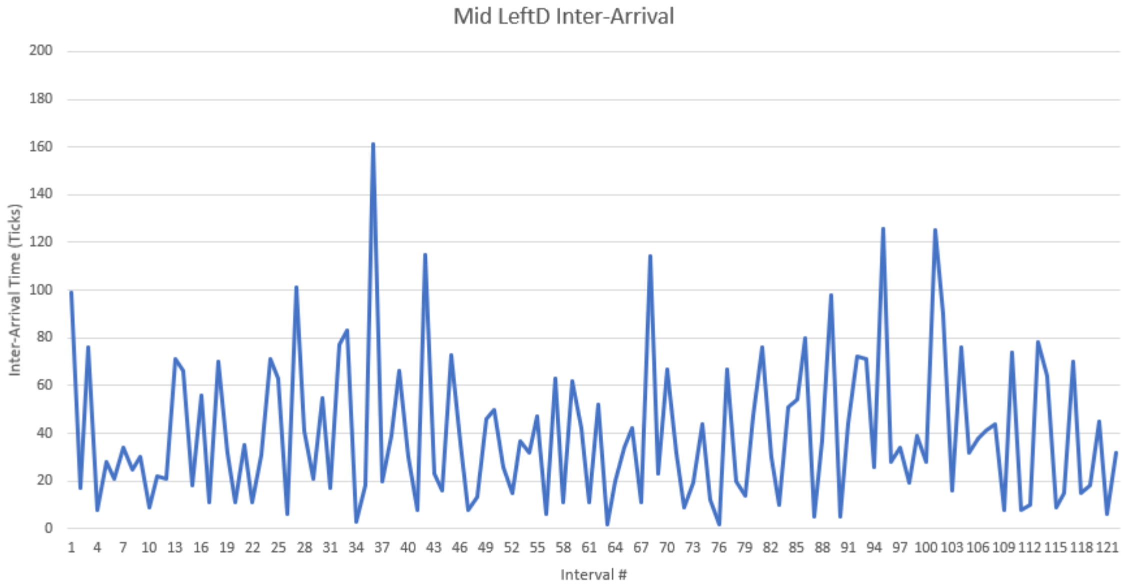
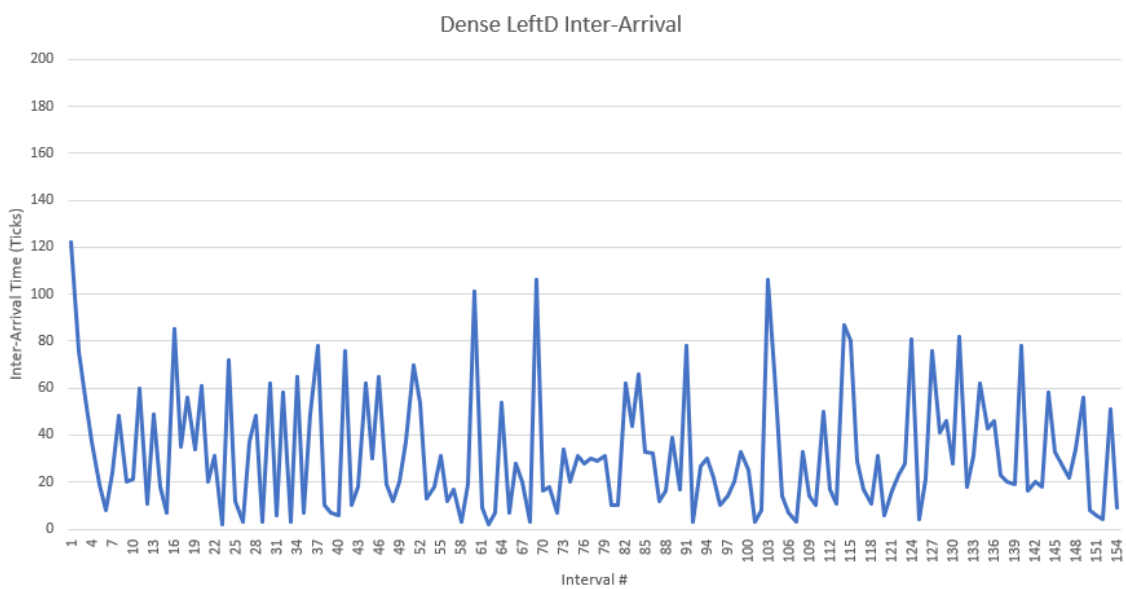

VeryDense LeftU Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #
0
20
40
60
80
100
120
140
160
180
200

Figure 22. Left (Down) Pedestrian Inter-Arrival Time Graphs






Mid LeftD Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #
200
180
160
140
120
100
80
60
40
20
0


Dense LeftD Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #
200
180
160
140
120
100
80
60
40
20
0

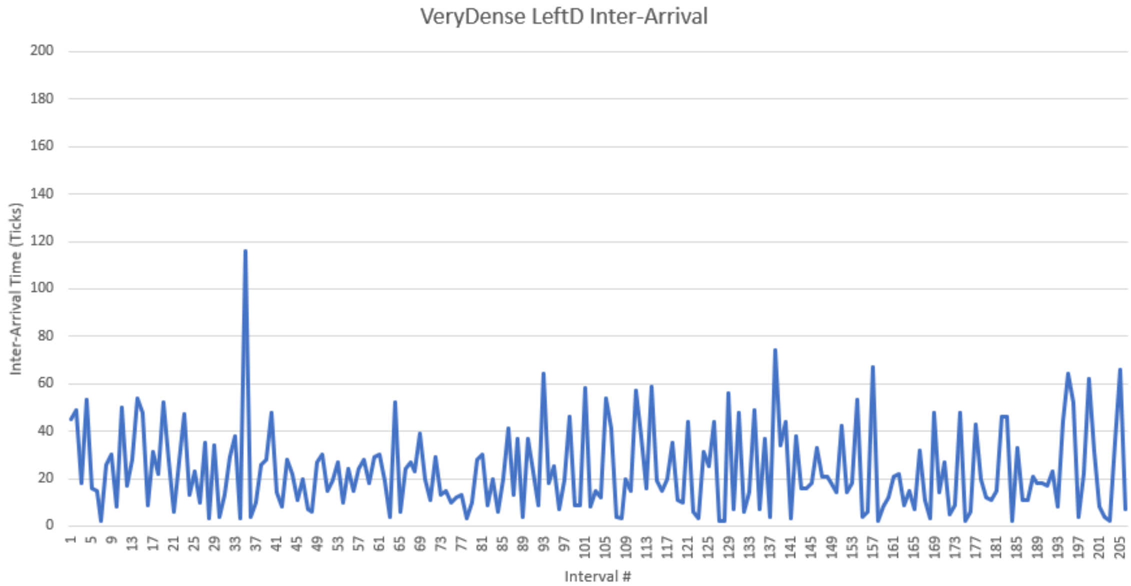
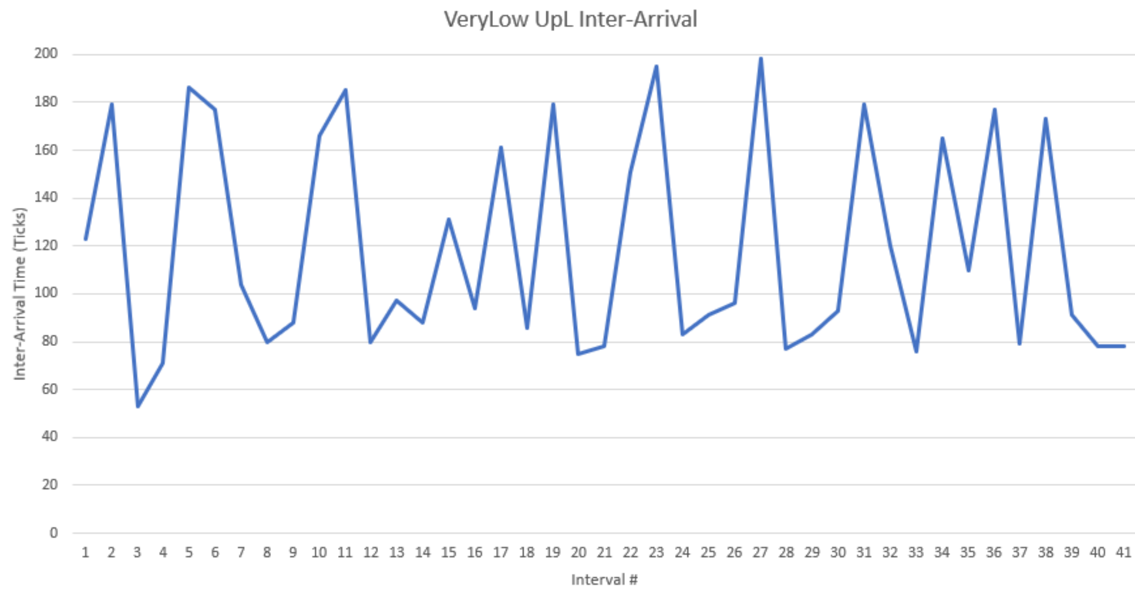
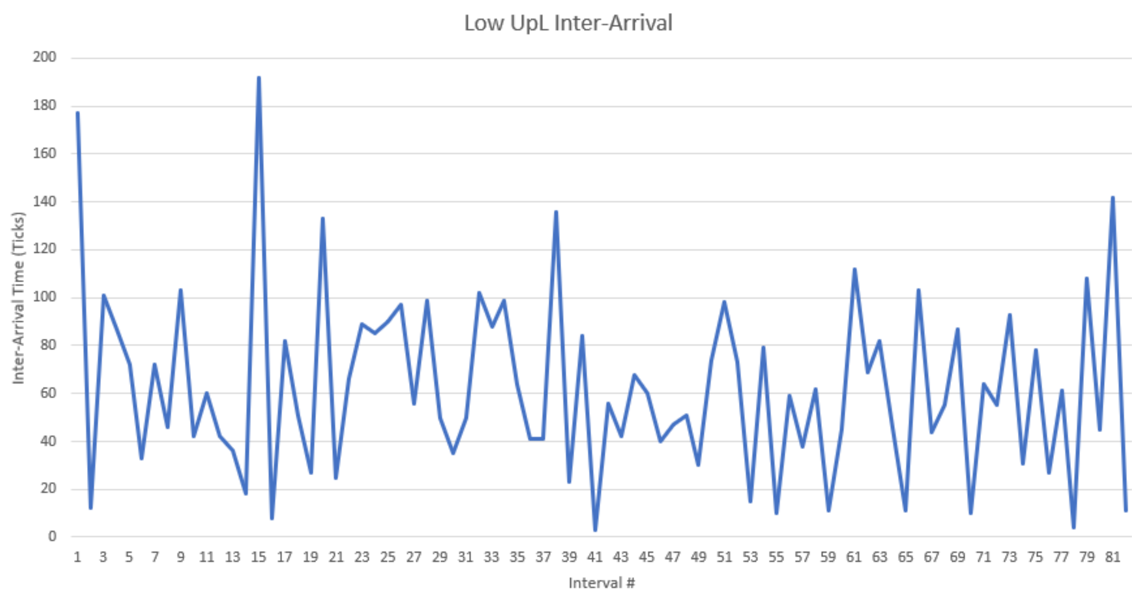
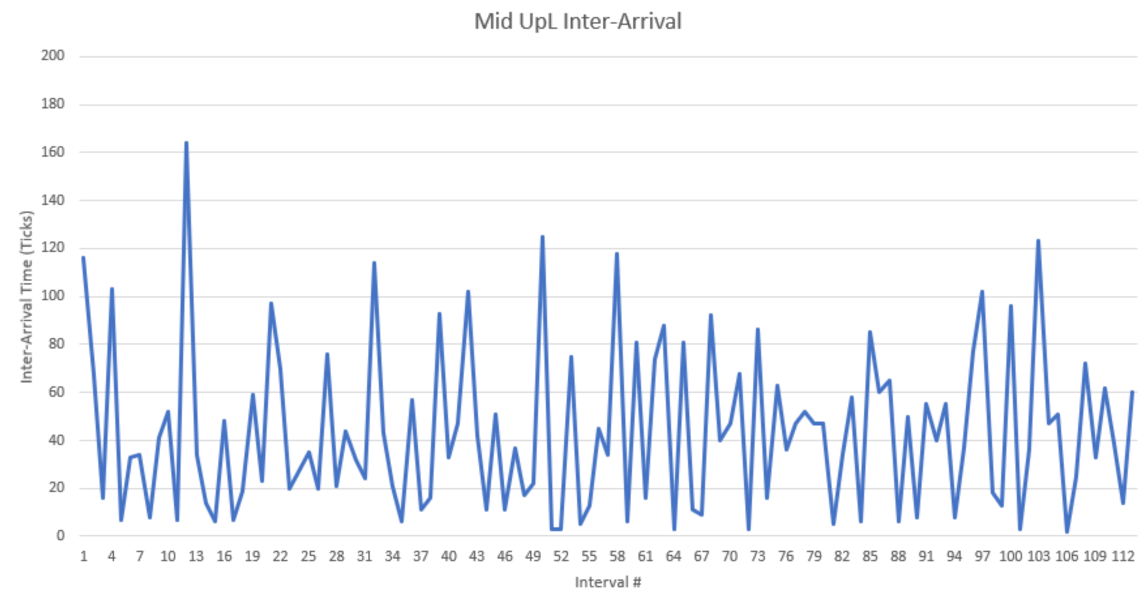
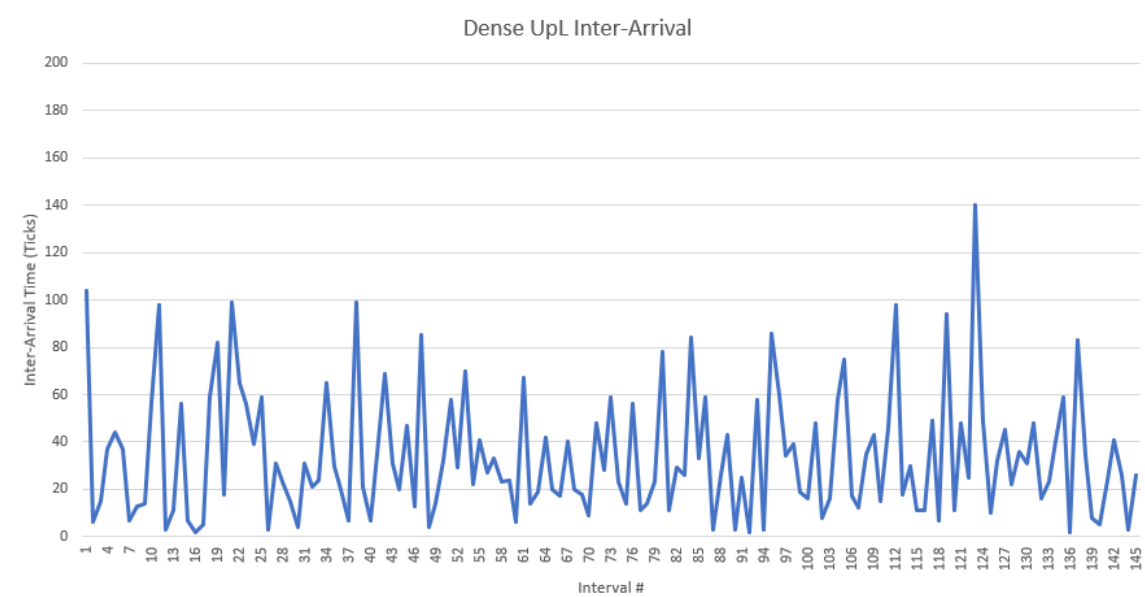

VeryDense LeftD Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #
0
20
40
60
80
100
120
140
160
180
200

Figure 23. Up (Left) Pedestrian Inter-Arrival Time Graphs






Mid UpL Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #
0
20
40
60
80
100
120
140
160
180
200
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79 82 85 88 91 94 97 100 103 106 109 112


Dense UpL Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #
0
20
40
60
80
100
120
140
160
180
200
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79 82 85 88 91 94 97 100 103 106 109 112 115 118 121 124 127 130 133 136 139 142 145

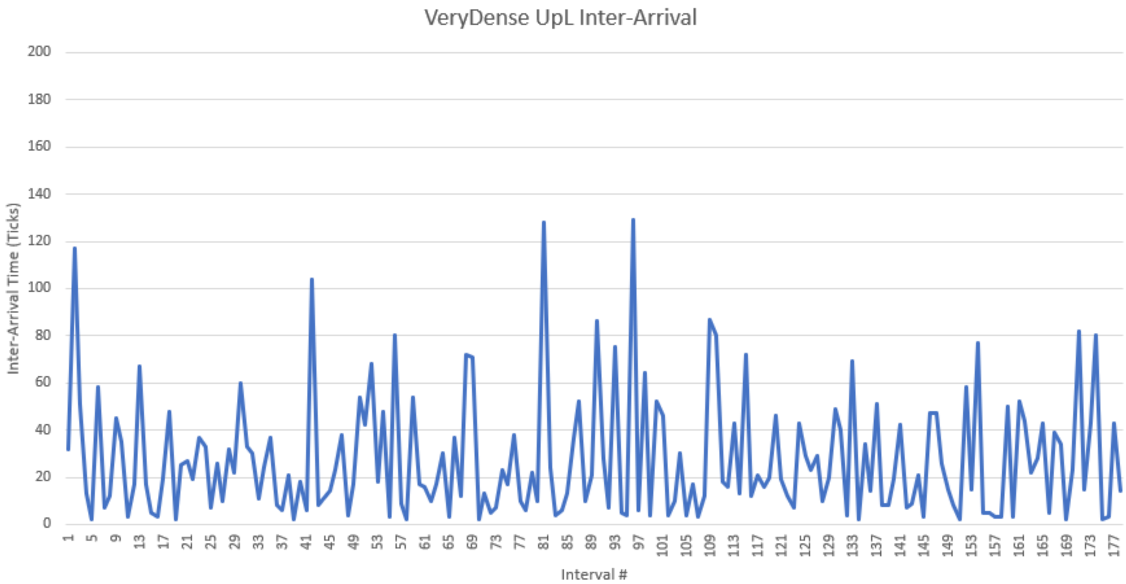
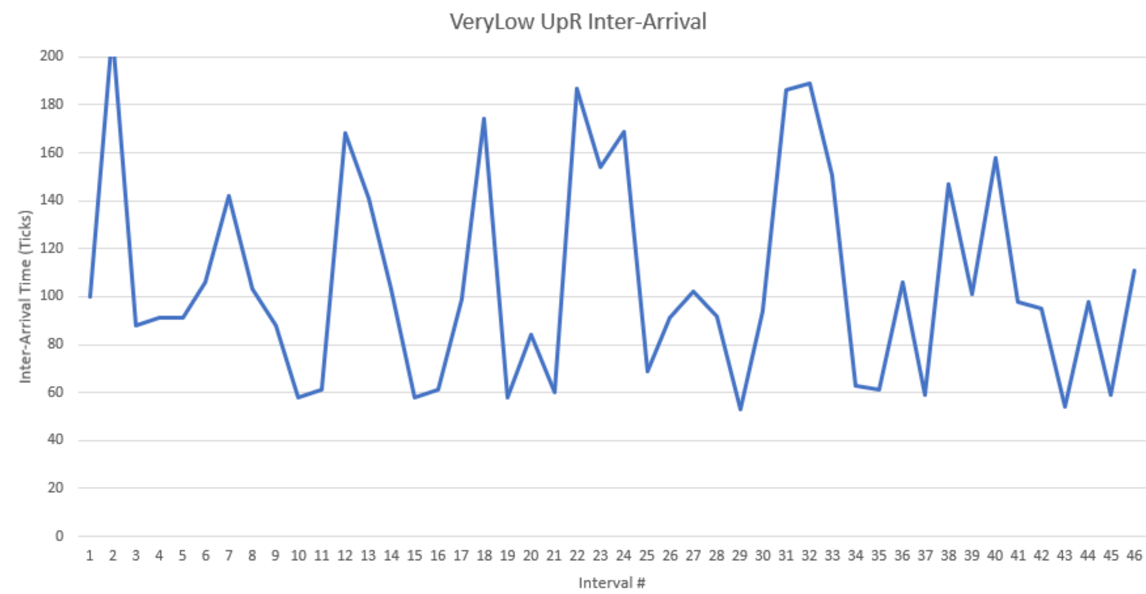
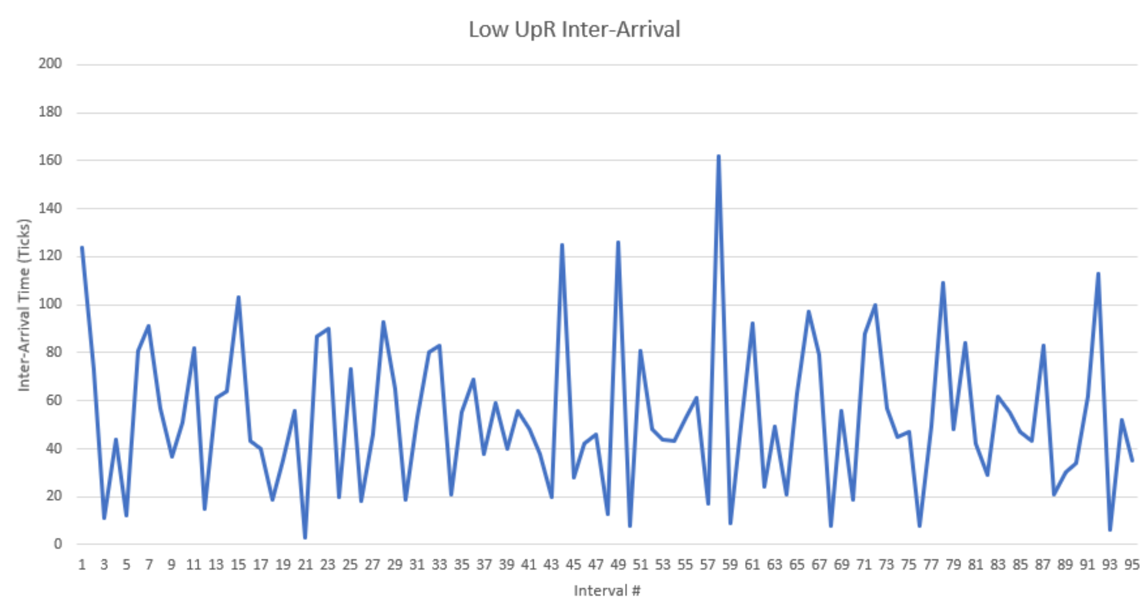
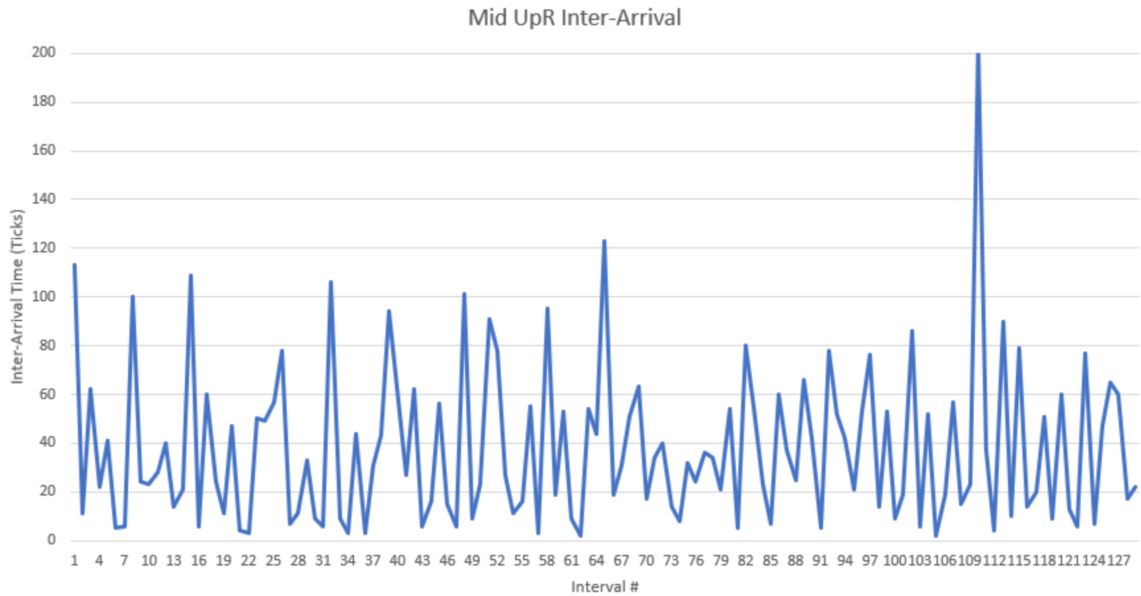
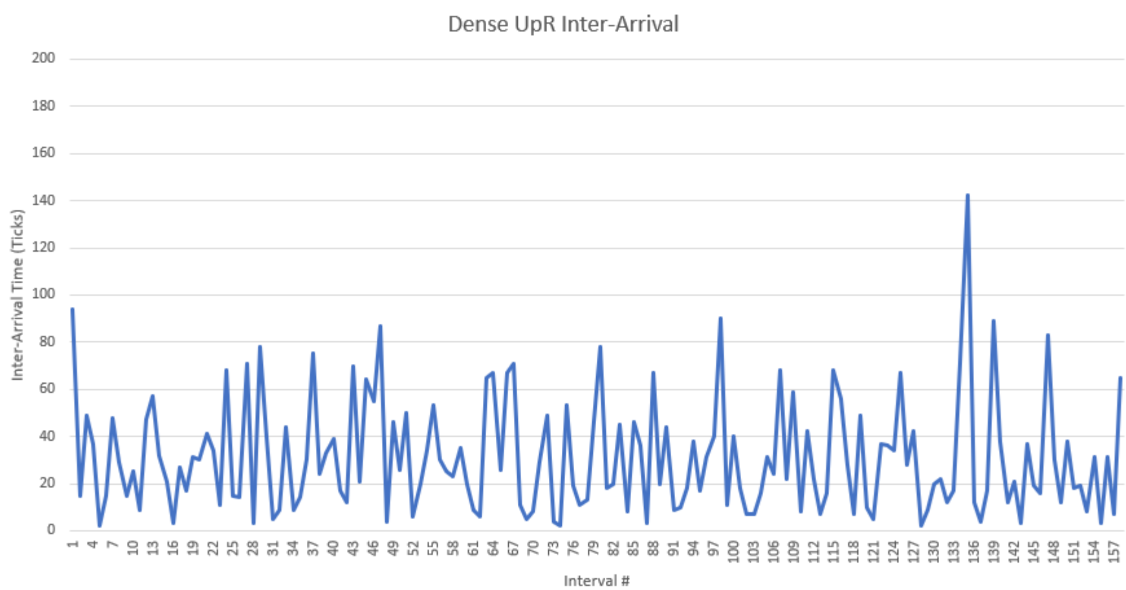

VeryDense UpL Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #

Figure 24. Up (Right) Pedestrian Inter-Arrival Time Graphs






Mid UpR Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #
200
180
160
140
120
100
80
60
40
20
0


Dense UpR Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #
200
180
160
140
120
100
80
60
40
20
0

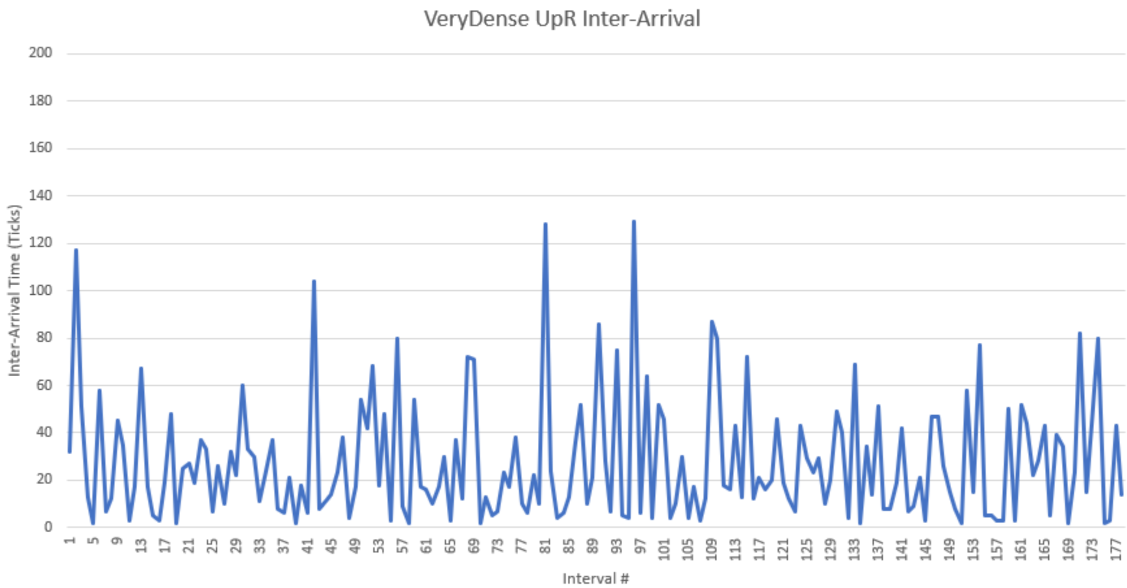
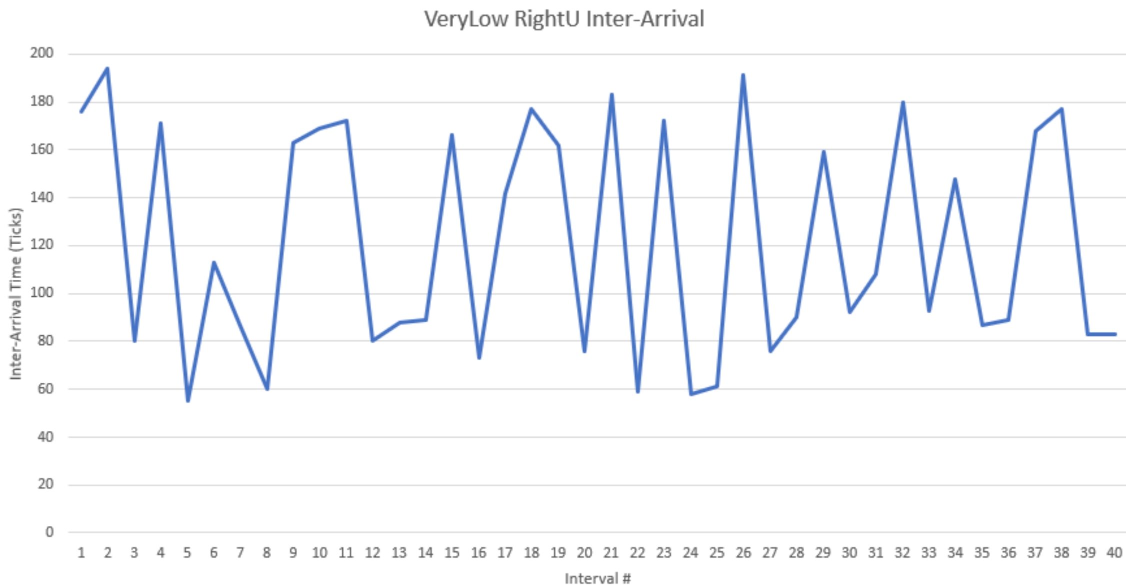
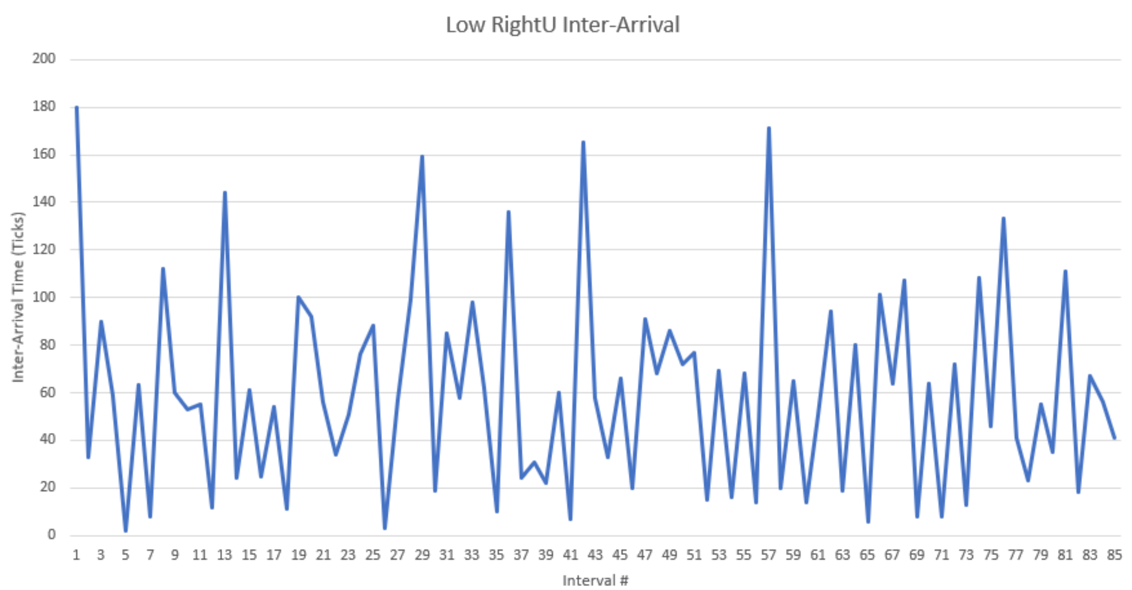
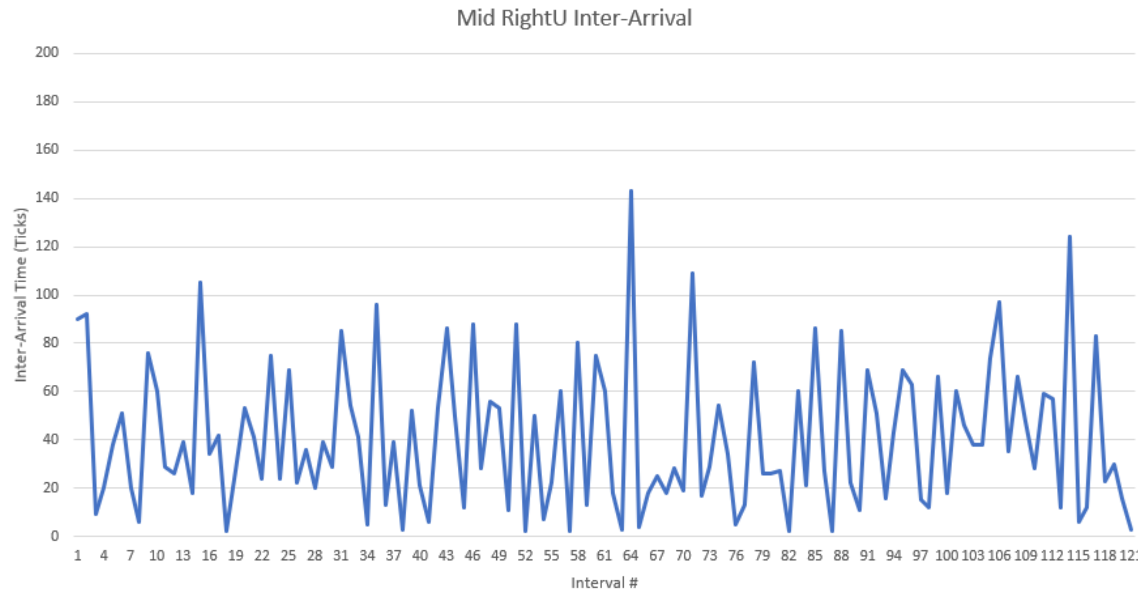
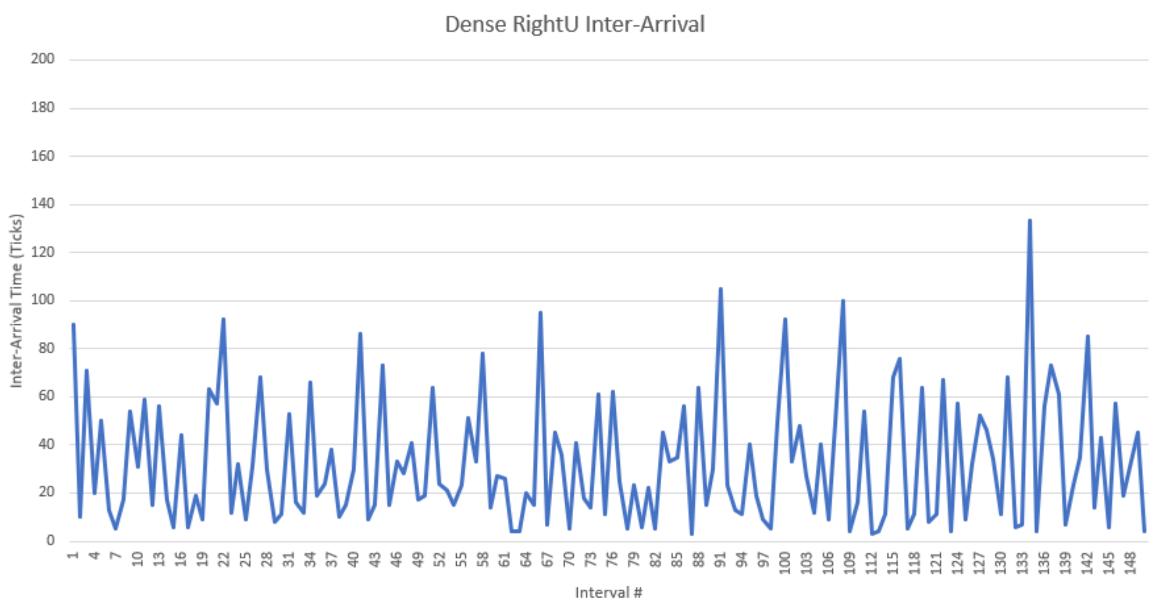

VeryDense UpR Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #
0
20
40
60
80
100
120
140
160
180
200

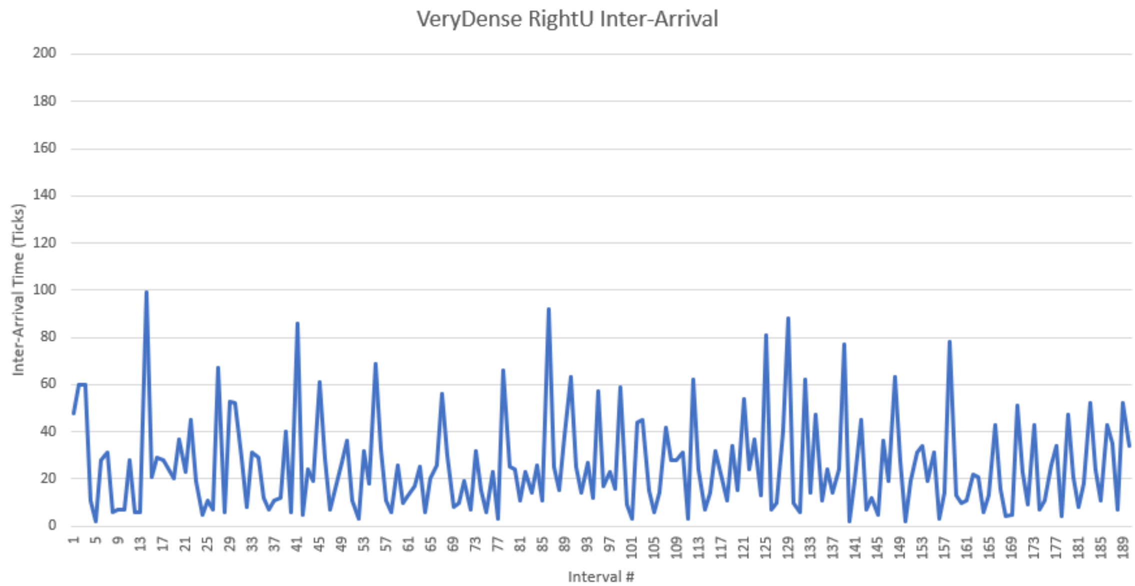
Figure 25. Right (Up) Pedestrian Inter-Arrival Time Graphs





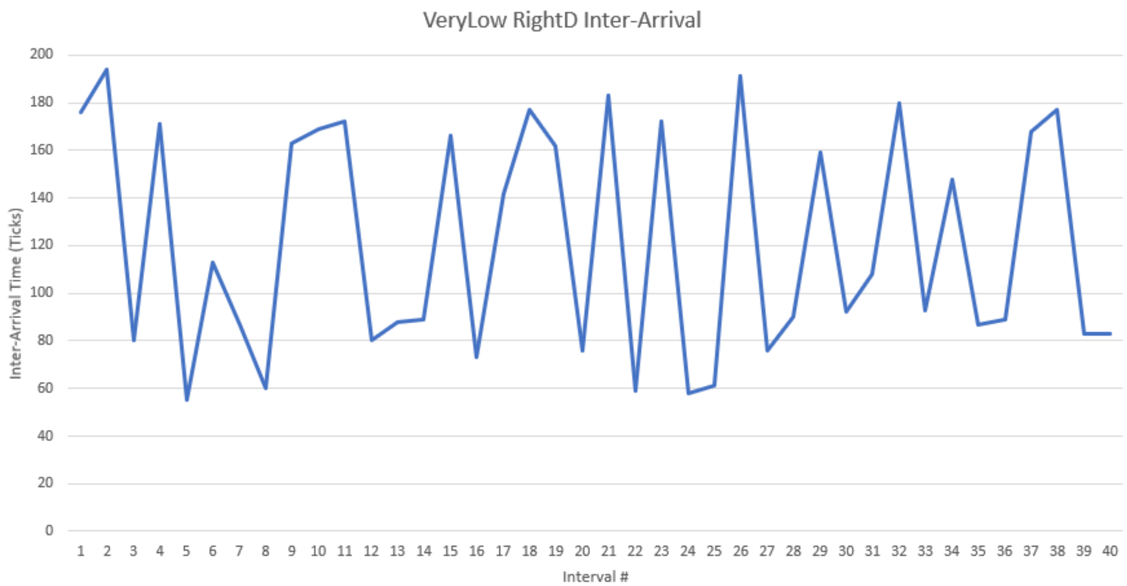

Mid RightU Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #
0
20
40
60
80
100
120
140
160
180
200

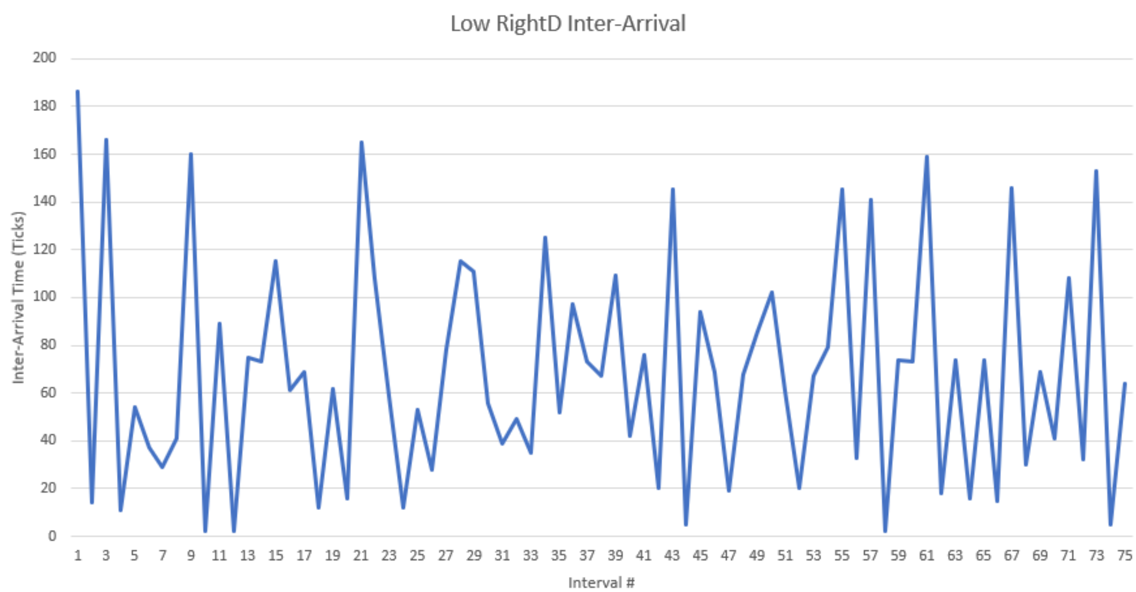

Dense RightU Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #
0
20
40
60
80
100
120
140
160
180
200

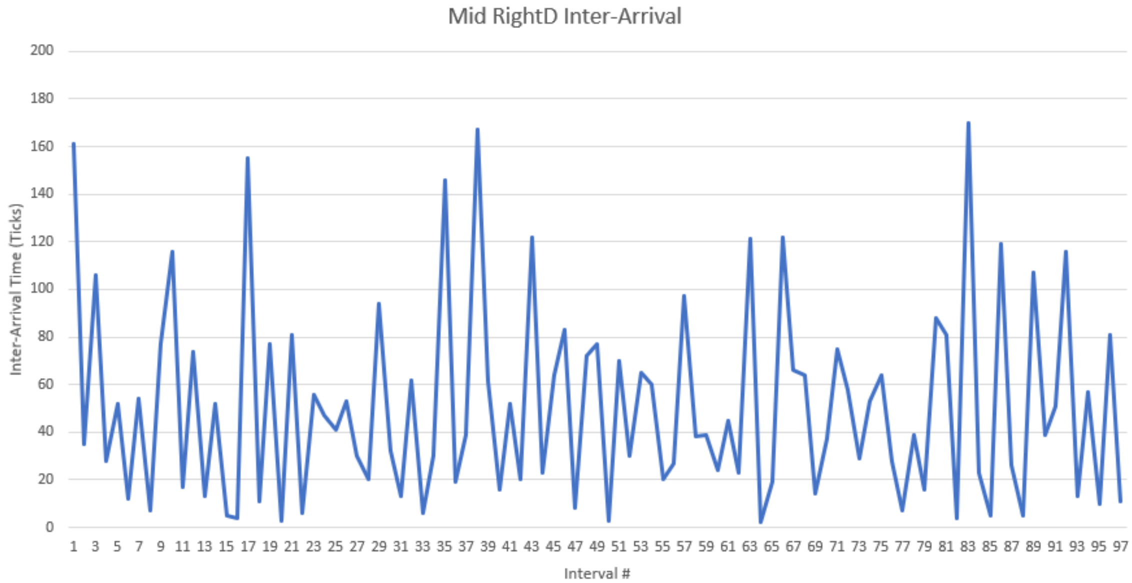


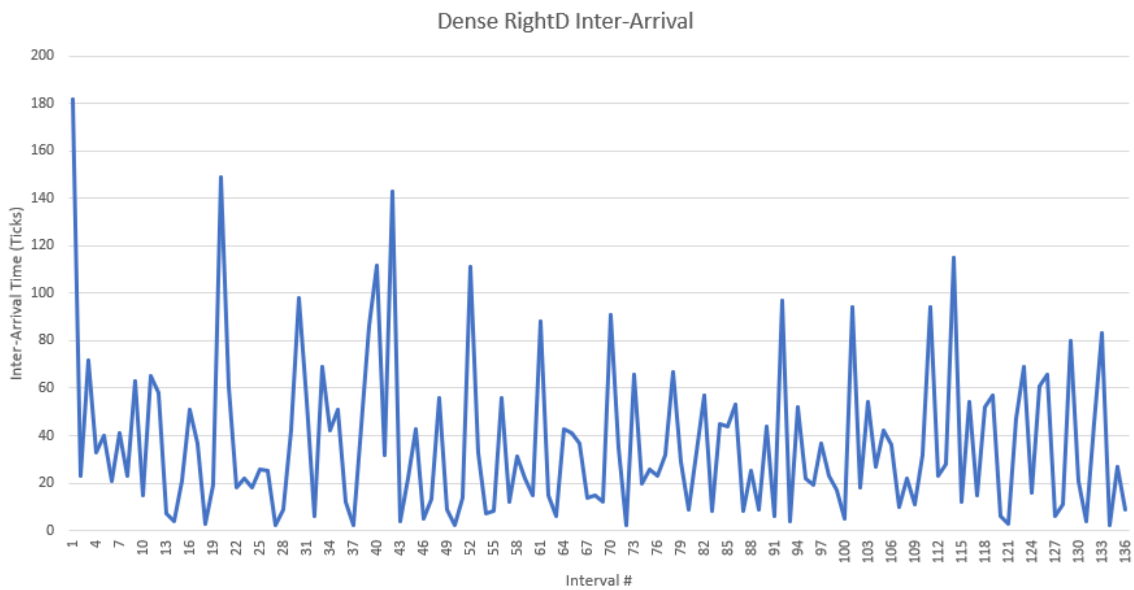
VeryDense RightU Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #

Figure 26. Right (Down) Pedestrian Inter-Arrival Time Graphs






Mid RightD Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #


Dense RightD Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #

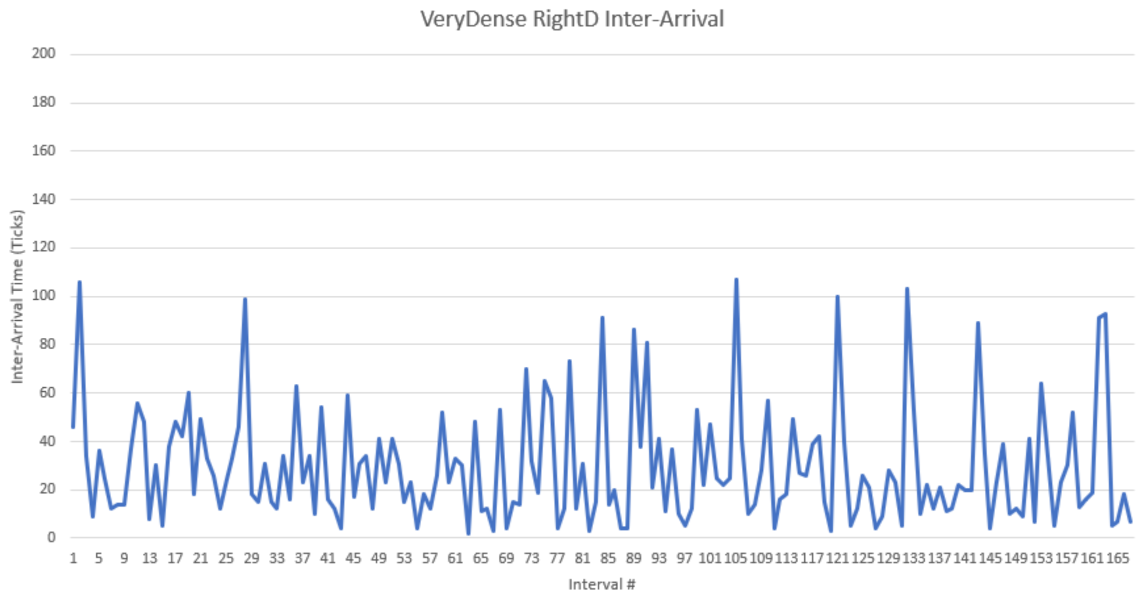
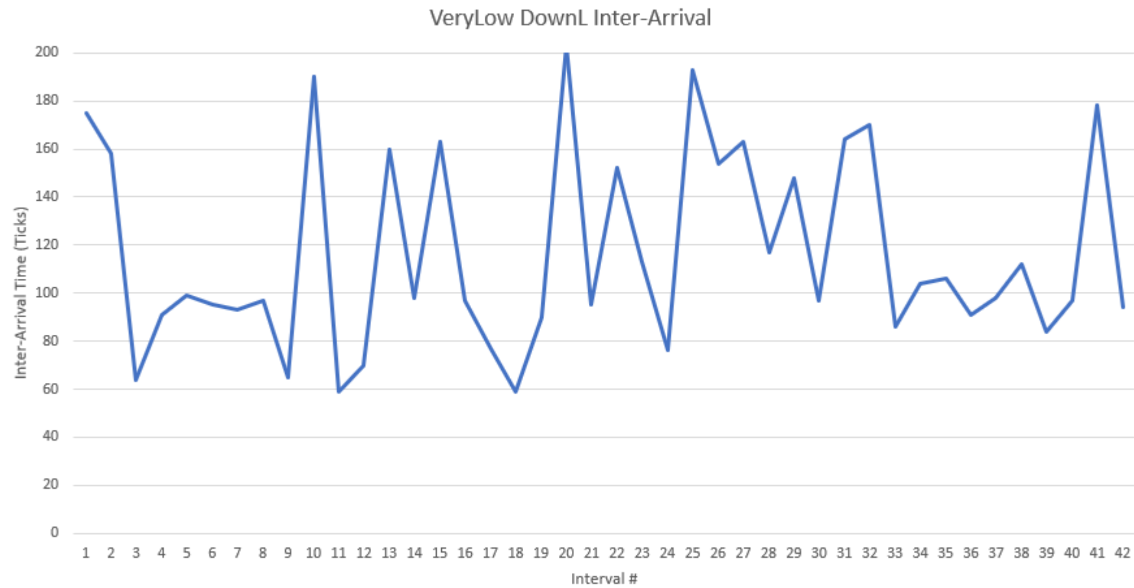
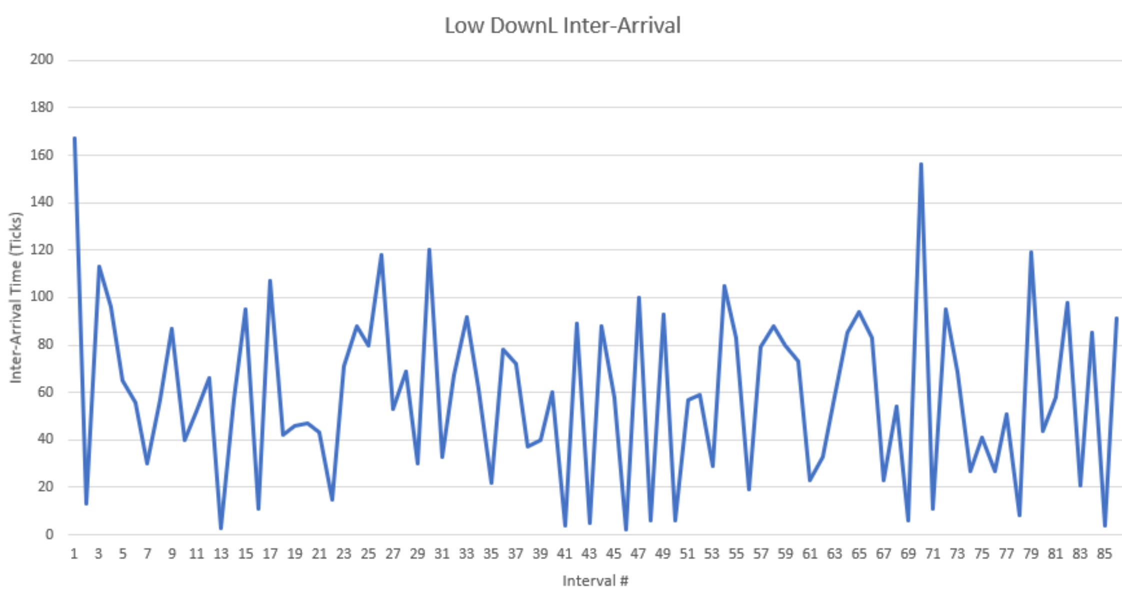
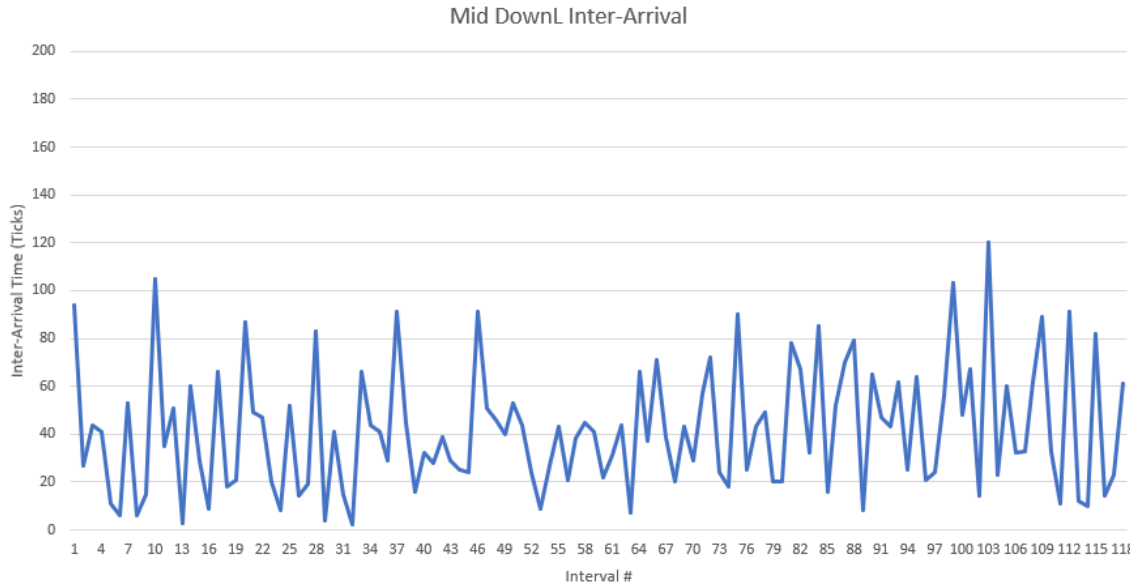
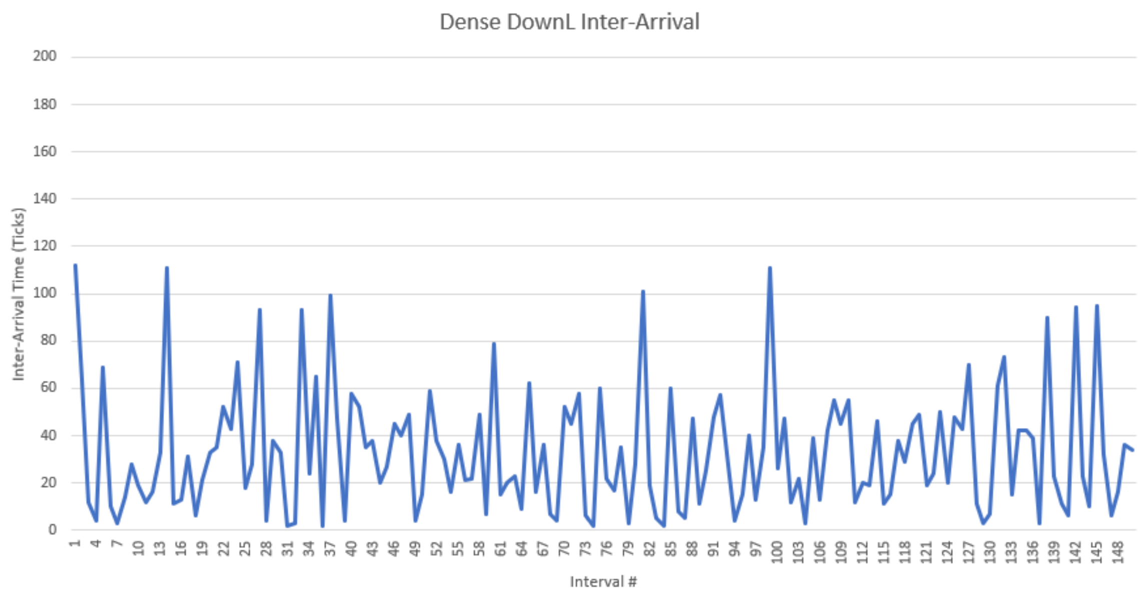

VeryDense RightD Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #
0
20
40
60
80
100
120
140
160
180
200

Figure 27. Down (Left) Pedestrian Inter-Arrival Time Graphs






Mid DownL Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #


Dense DownL Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #

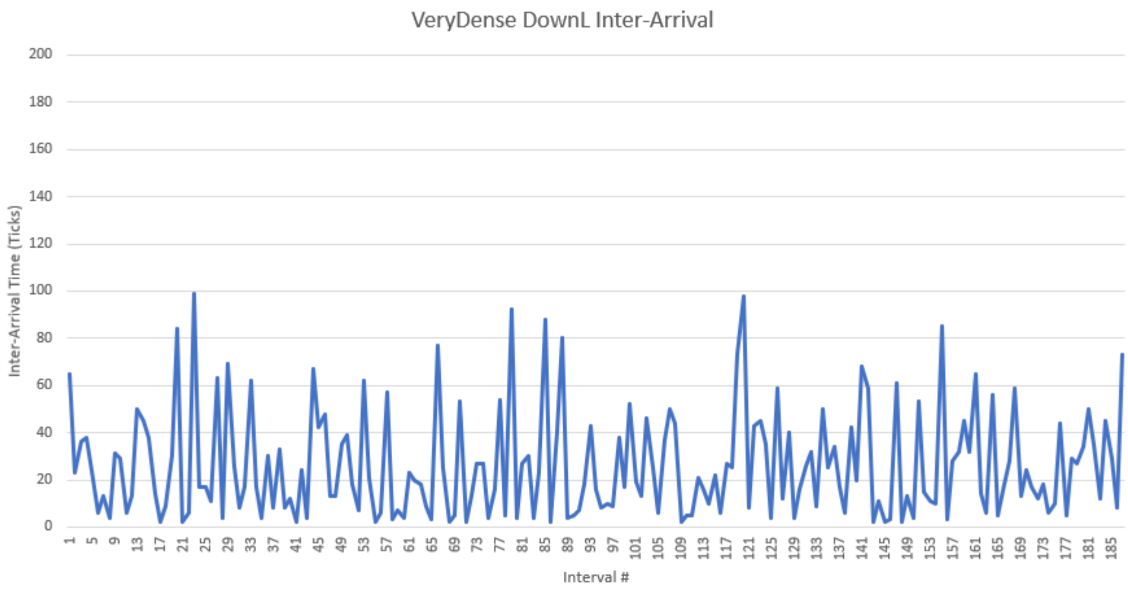
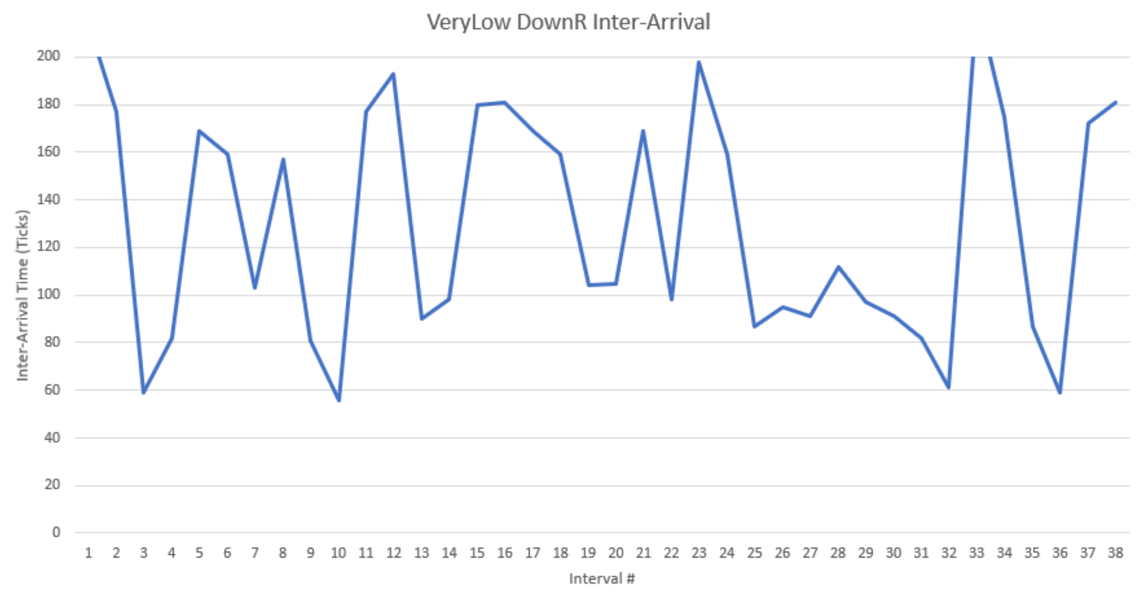
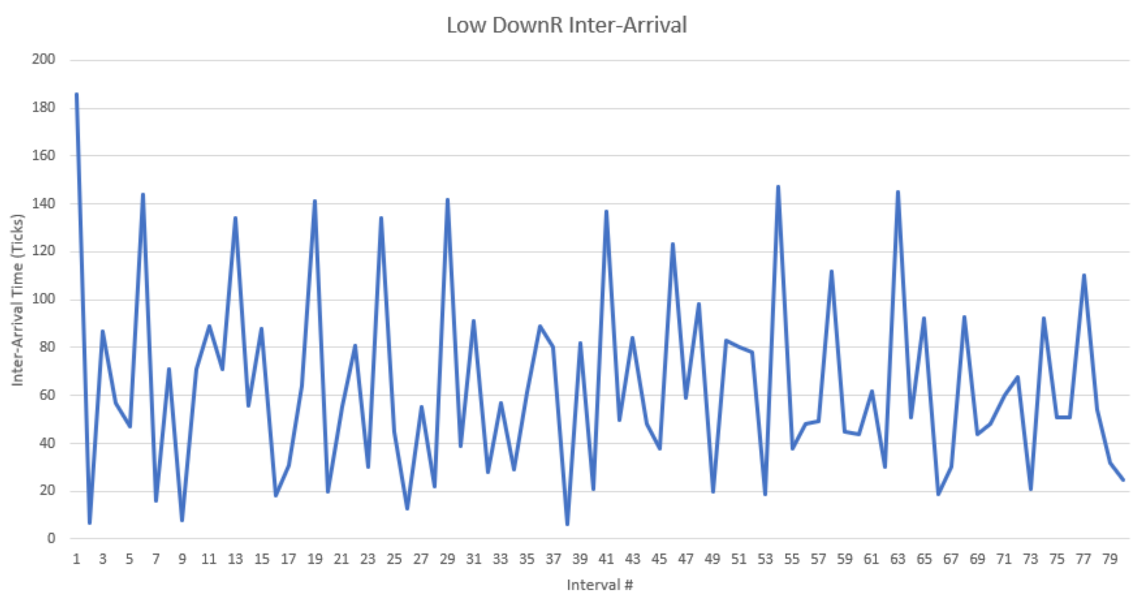
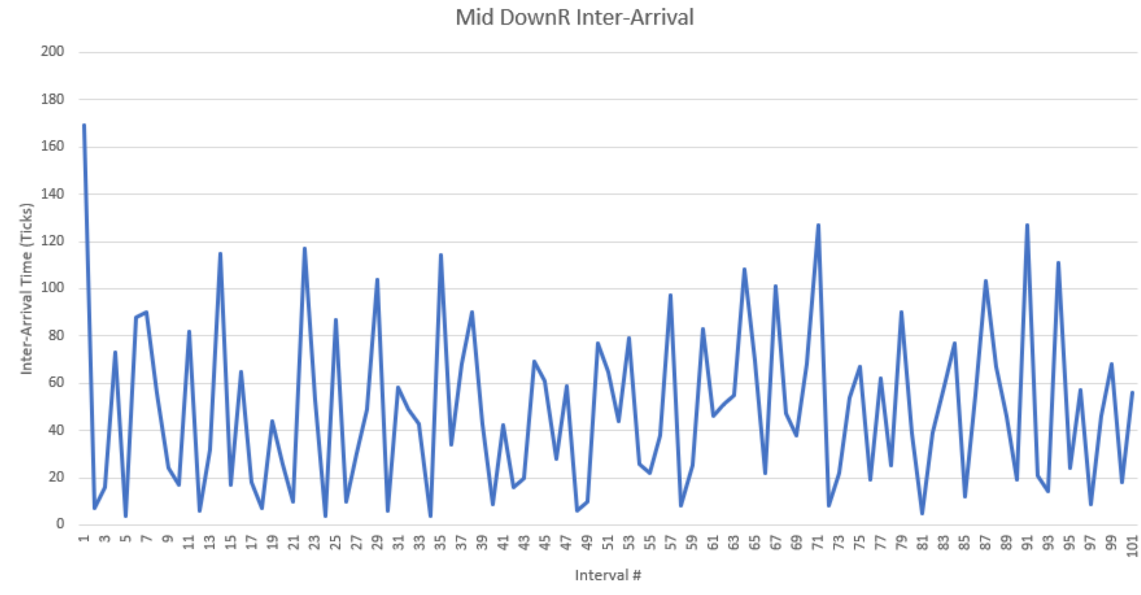
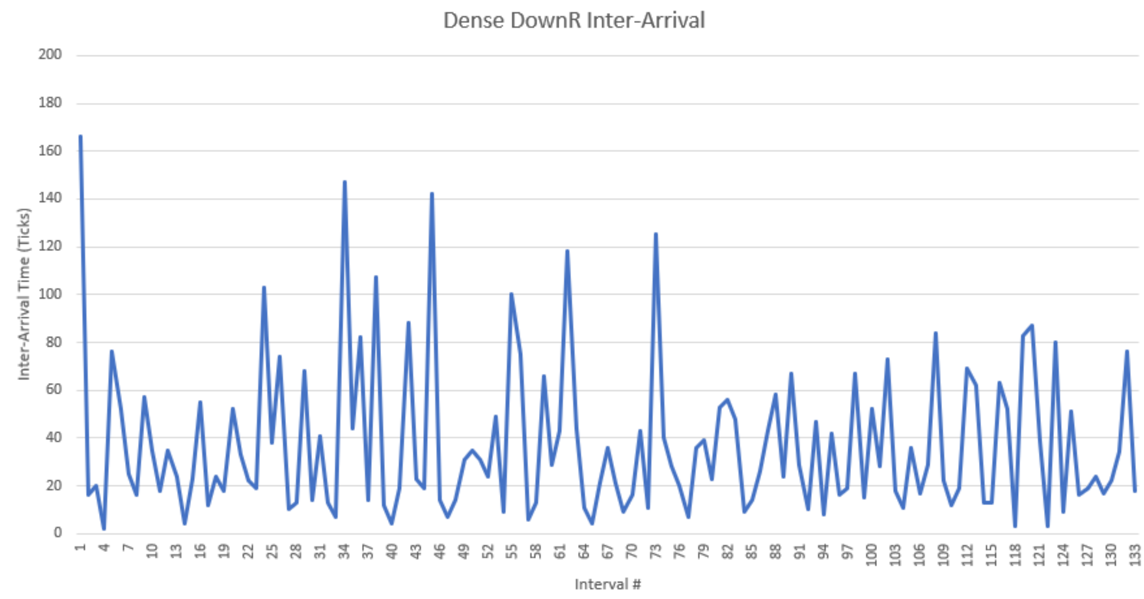

VeryDense DownL Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #

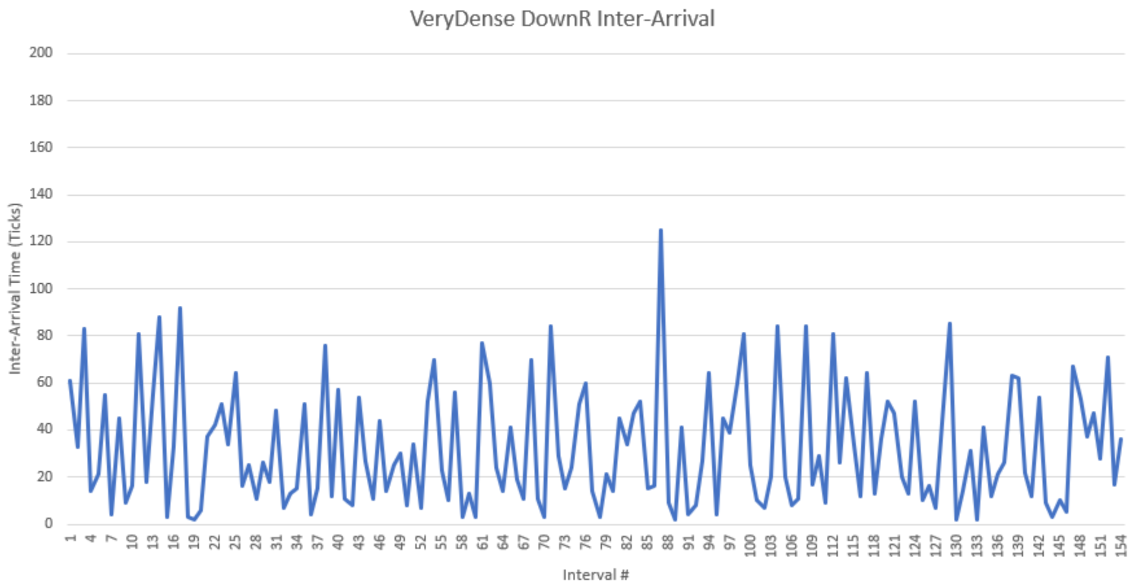
Figure 28. Down (Right) Pedestrian Inter-Arrival Time Graphs






Mid DownR Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #

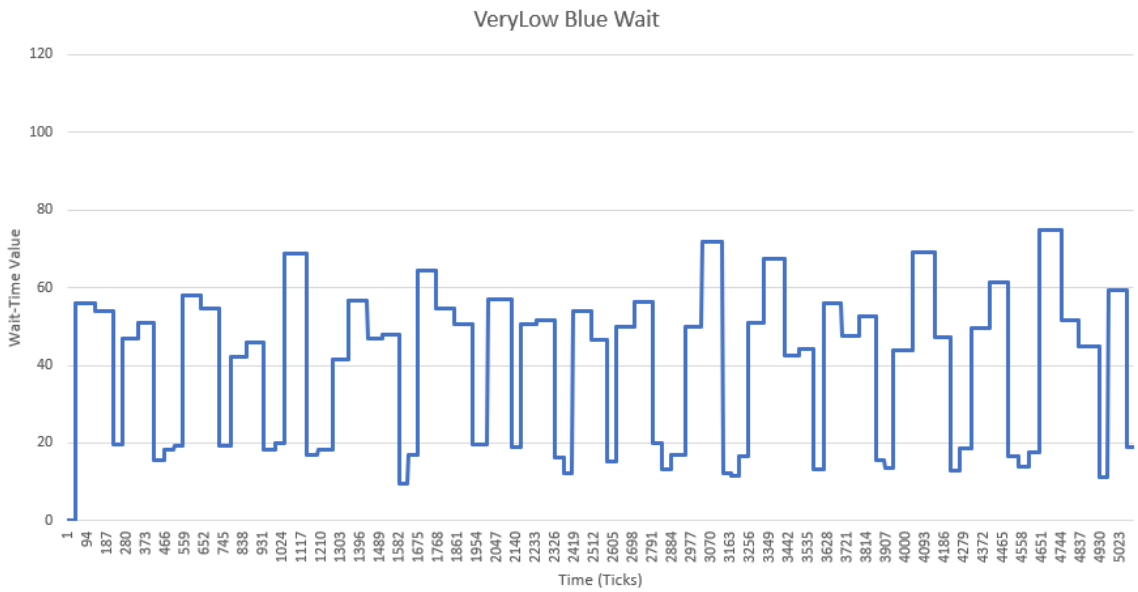

Dense DownR Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #

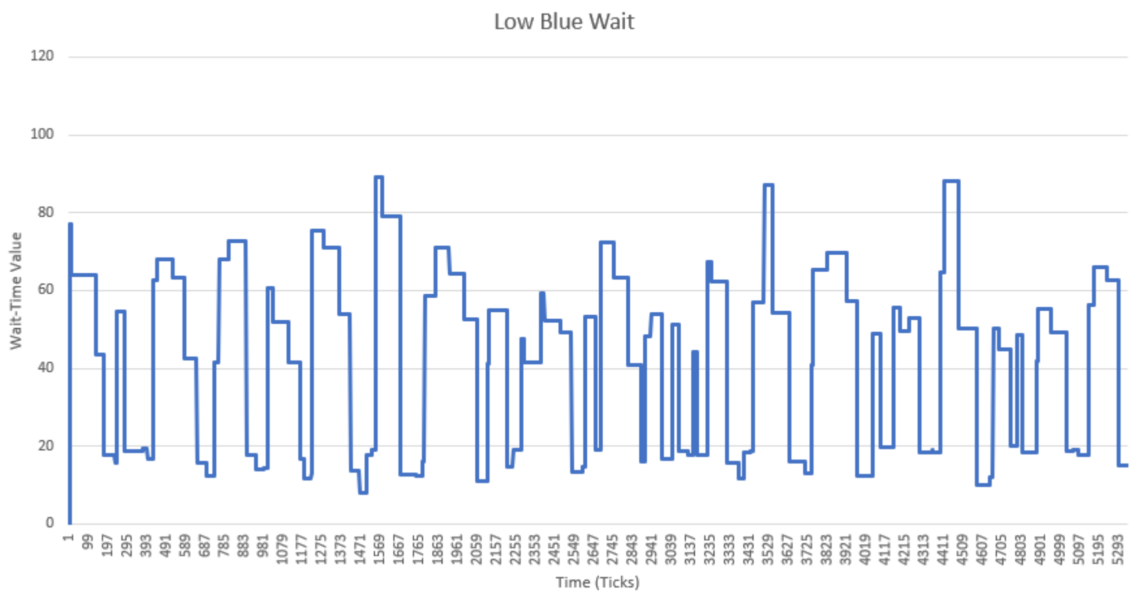

VeryDense DownR Inter-Arrival
Inter-Arrival Time (Ticks)
Interval #
0
20
40
60
80
100
120
140
160
180
200

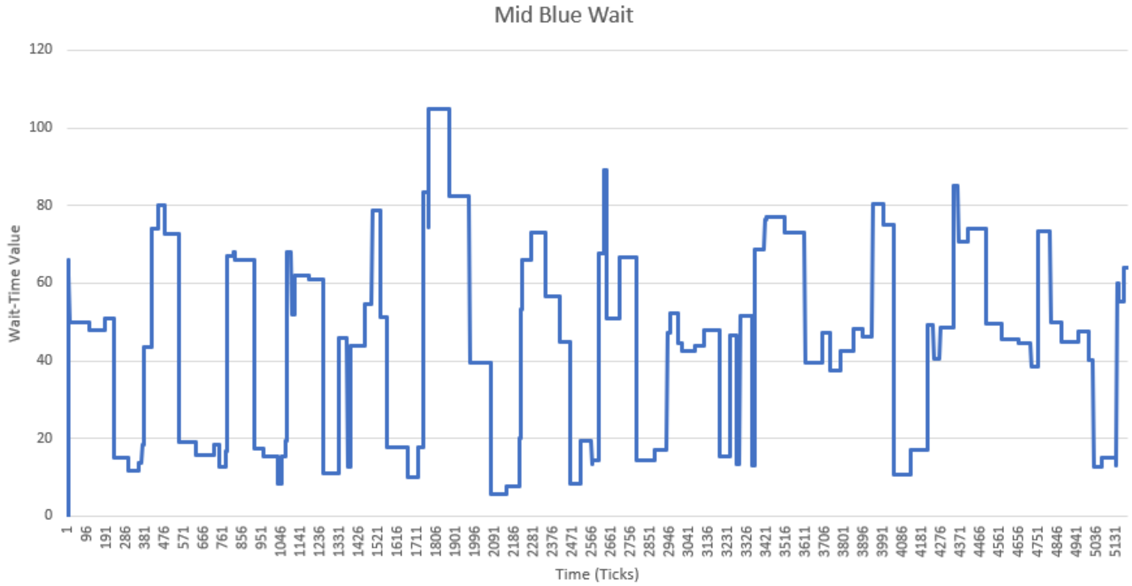
Appendix C: Wait-Time Graphs

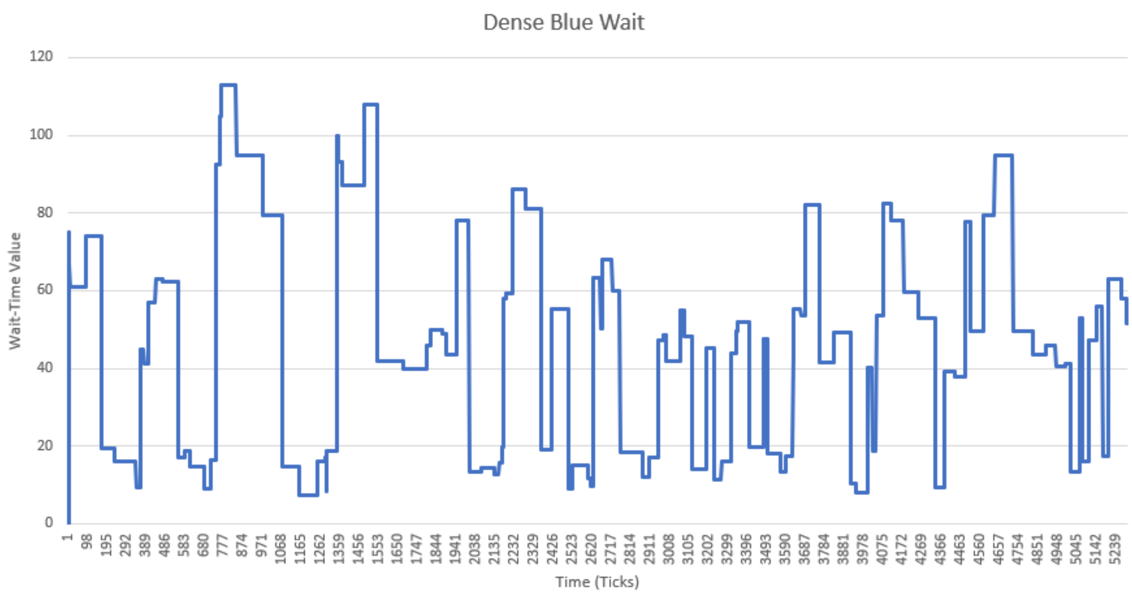
Figures 29 presents all the wait-time graphs of the blue car agents, Figure 30 presents all the wait-time graphs of the red car agents, and the pattern repeats until all the wait-time graphs are presented. Each graph is labeled with what test the data originates from.

Figure 29. Blue Car Wait-Time Graphs






Mid Blue Wait
120
100
80
60
40
20
0
Wait-Time Value
Time (Ticks)


Dense Blue Wait
120
100
80
60
40
20
0
Wait-Time Value
Time (Ticks)

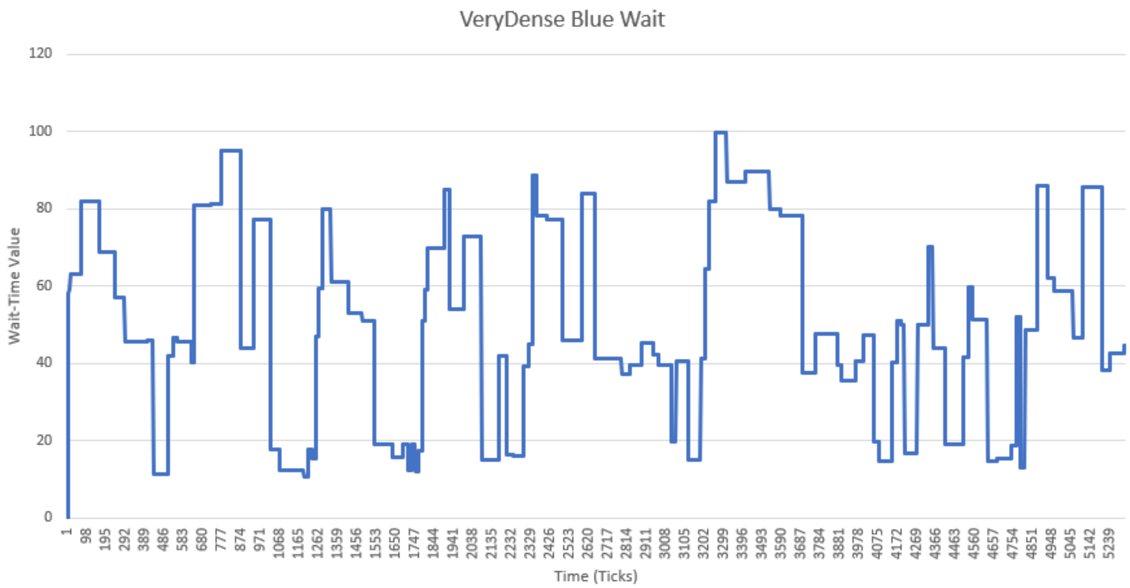
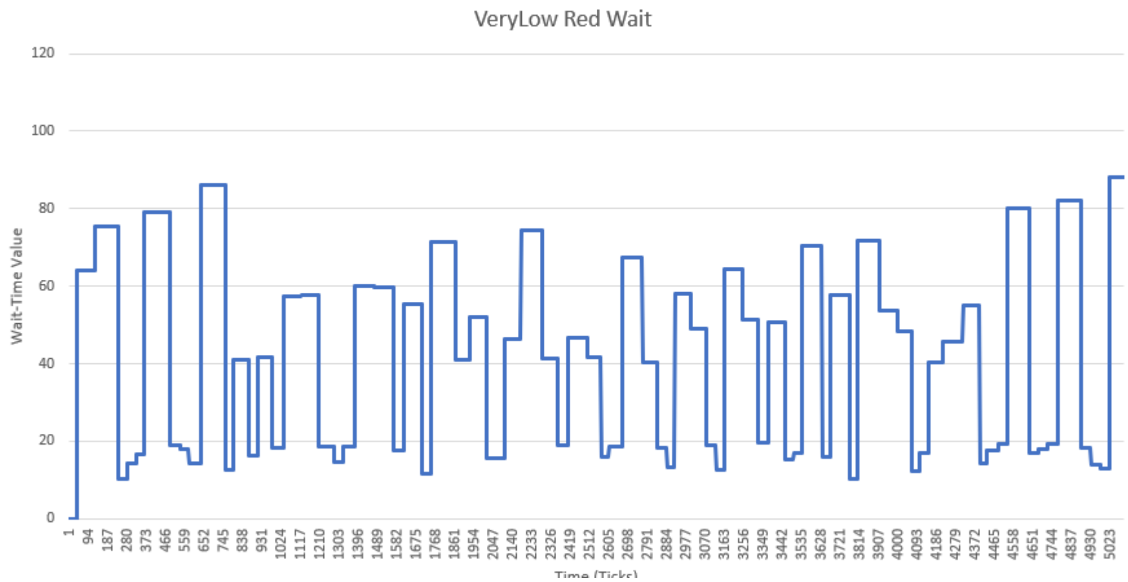
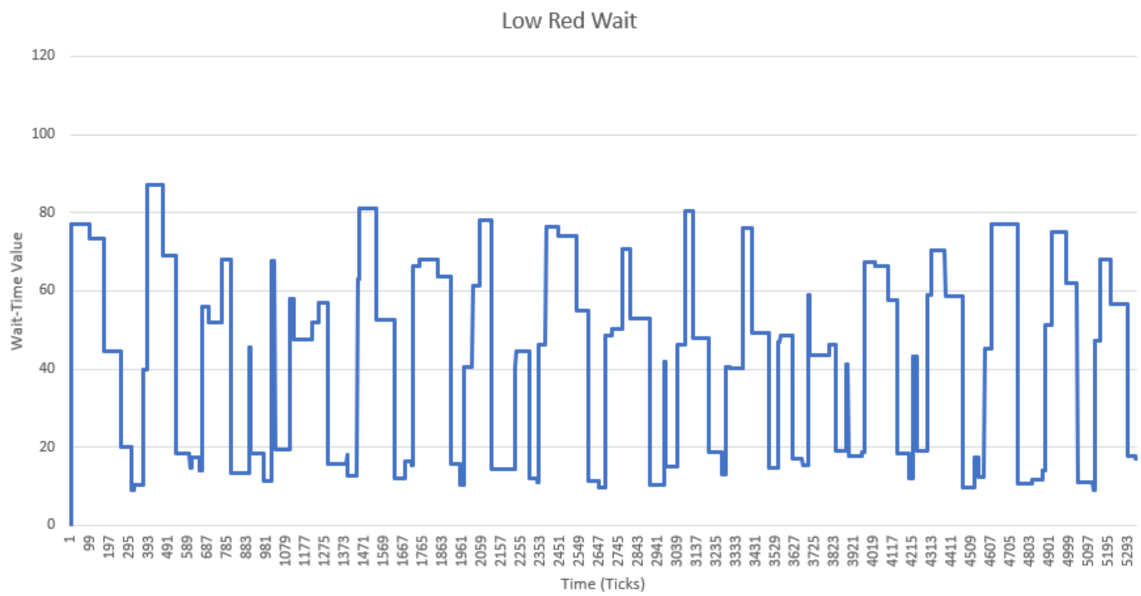
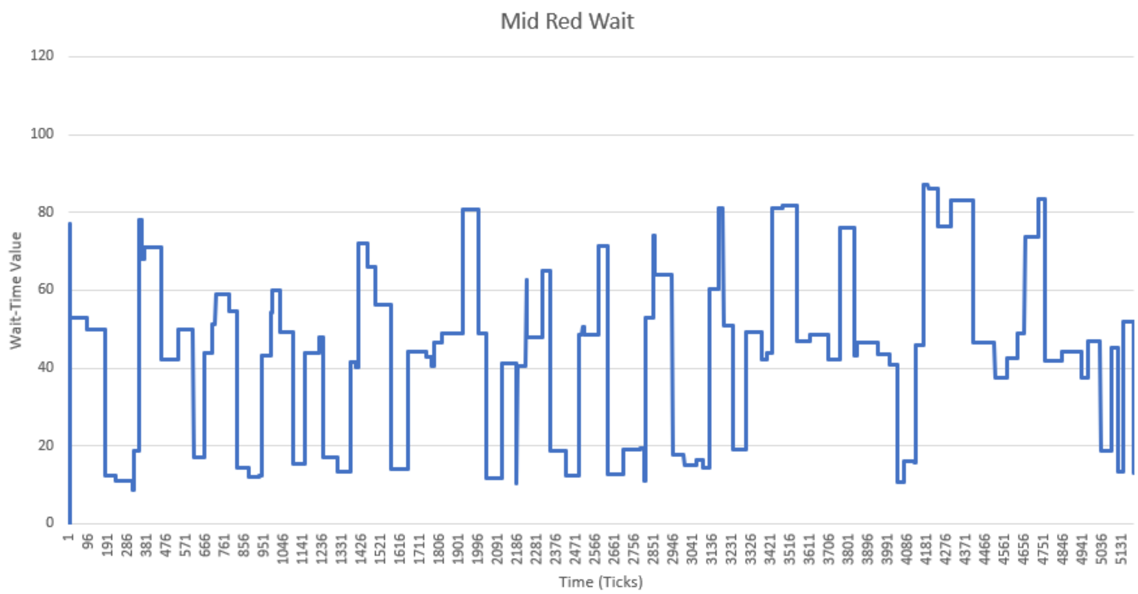
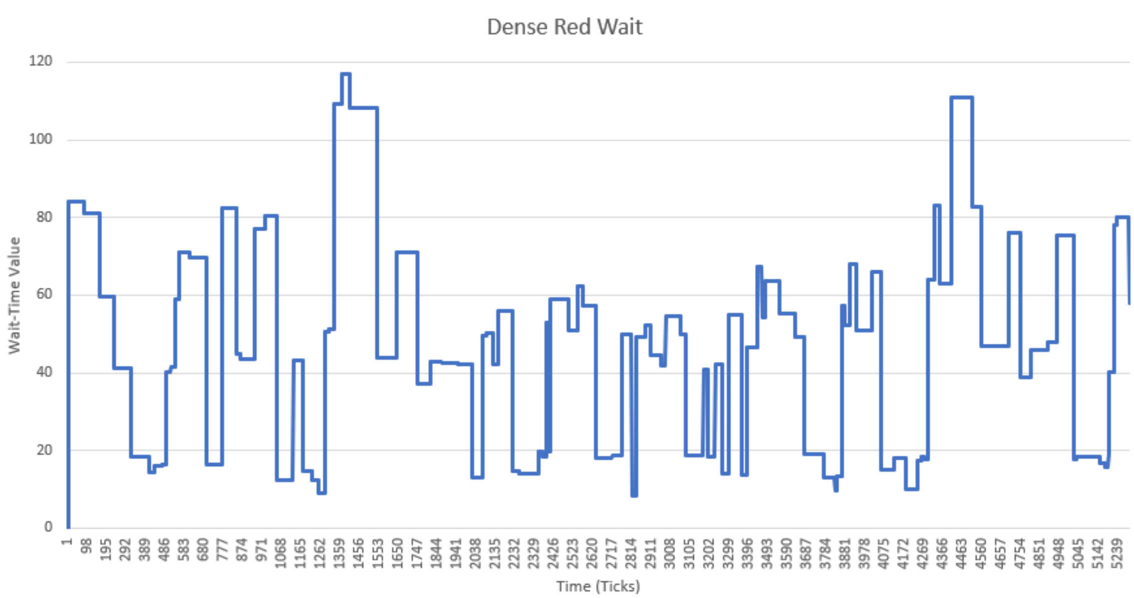

VeryDense Blue Wait
Wait-Time Value
Time (Ticks)
120
100
80
60
40
20
0

Figure 30. Red Car Wait-Time Graphs






Mid Red Wait
Wait-Time Value
120
100
80
60
40
20
0
1 96 191 286 381 476 571 666 761 856 951 1046 1141 1236 1331 1426 1521 1616 1711 1806 1901 1996 2091 2186 2281 2376 2471 2566 2661 2756 2851 2946 3041 3136 3231 3326 3421 3516 3611 3706 3801 3896 3991 4086 4181 4276 4371 4466 4561 4656 4751 4846 4941 5036 5131
Time (Ticks)


Dense Red Wait
Wait-Time Value
120
100
80
60
40
20
0
1 98 195 292 389 486 583 680 777 874 971 1068 1165 1262 1359 1456 1553 1650 1747 1844 1941 2038 2135 2232 2329 2426 2523 2620 2717 2814 2911 3008 3105 3202 3299 3396 3493 3590 3687 3784 3881 3978 4075 4172 4269 4366 4463 4560 4657 4754 4851 4948 5045 5142 5239
Time (Ticks)

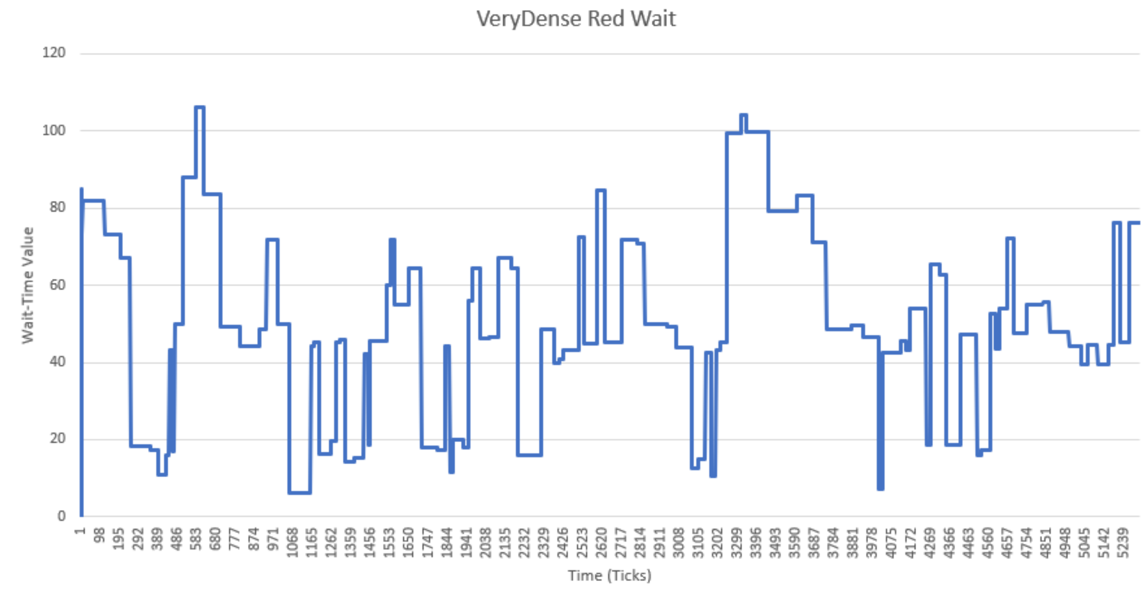
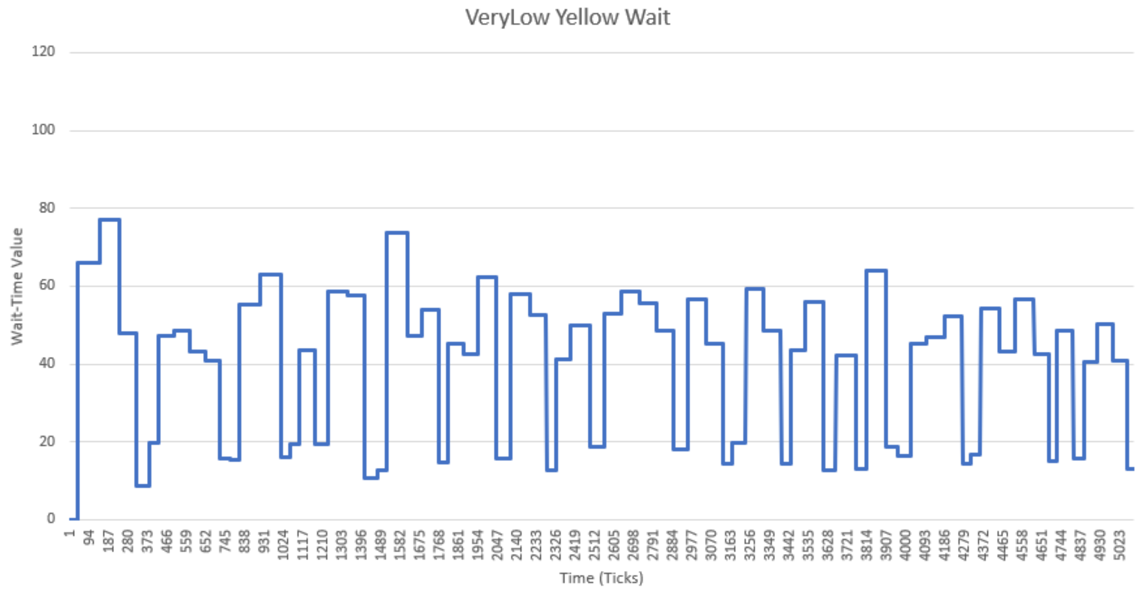
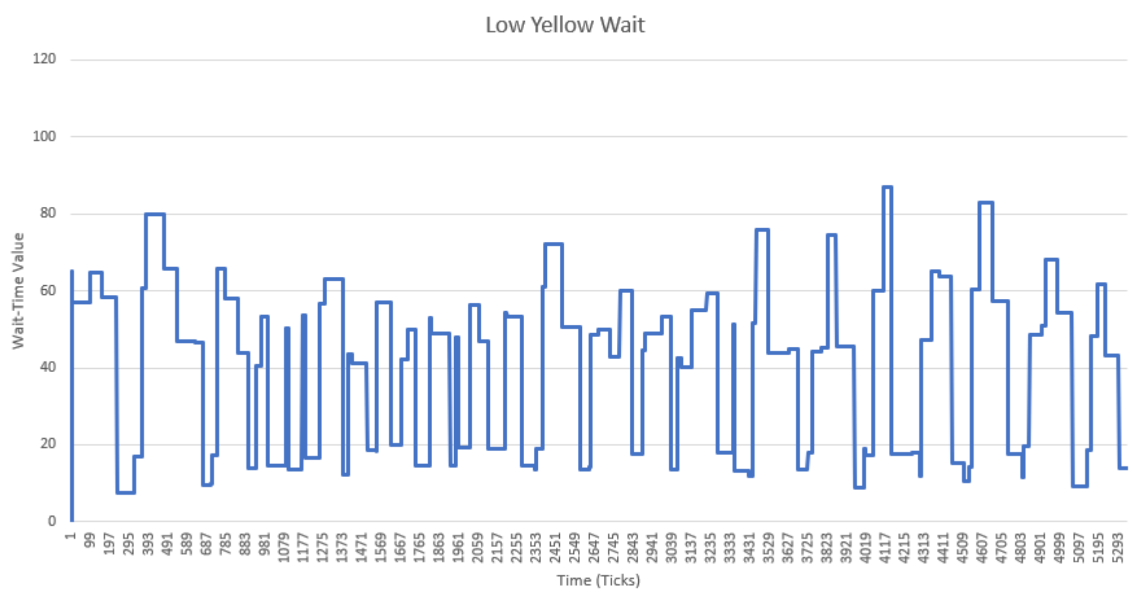
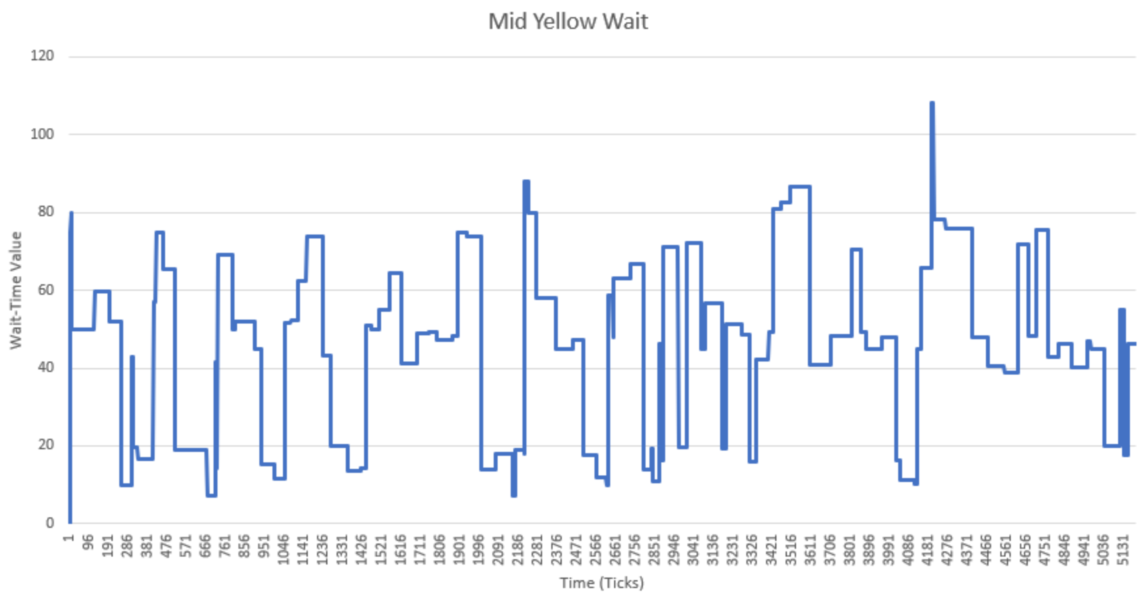
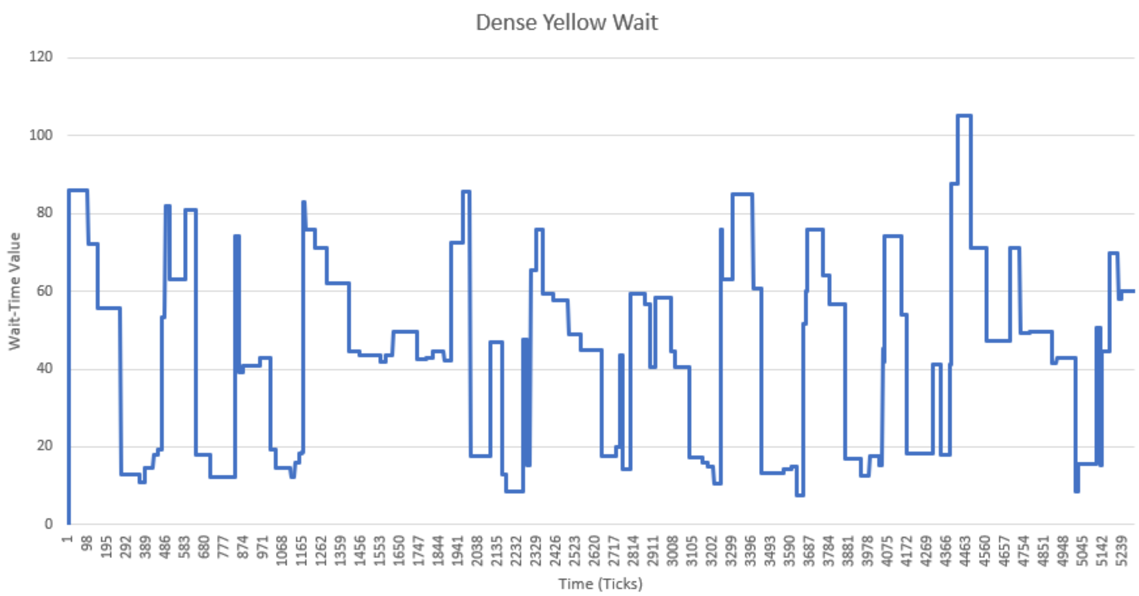

VeryDense Red Wait
Wait-Time Value
Time (Ticks)
120
100
80
60
40
20
0
1
98
195
292
389
486
583
680
777
874
971
1068
1165
1262
1359
1456
1553
1650
1747
1844
1941
2038
2135
2232
2329
2426
2523
2620
2717
2814
2911
3008
3105
3202
3299
3396
3493
3590
3687
3784
3881
3978
4075
4172
4269
4366
4463
4560
4657
4754
4851
4948
5045
5142
5239

Figure 31. Yellow Car Wait-Time Graphs






Mid Yellow Wait
Wait-Time Value
Time (Ticks)


Dense Yellow Wait
Wait-Time Value
Time (Ticks)

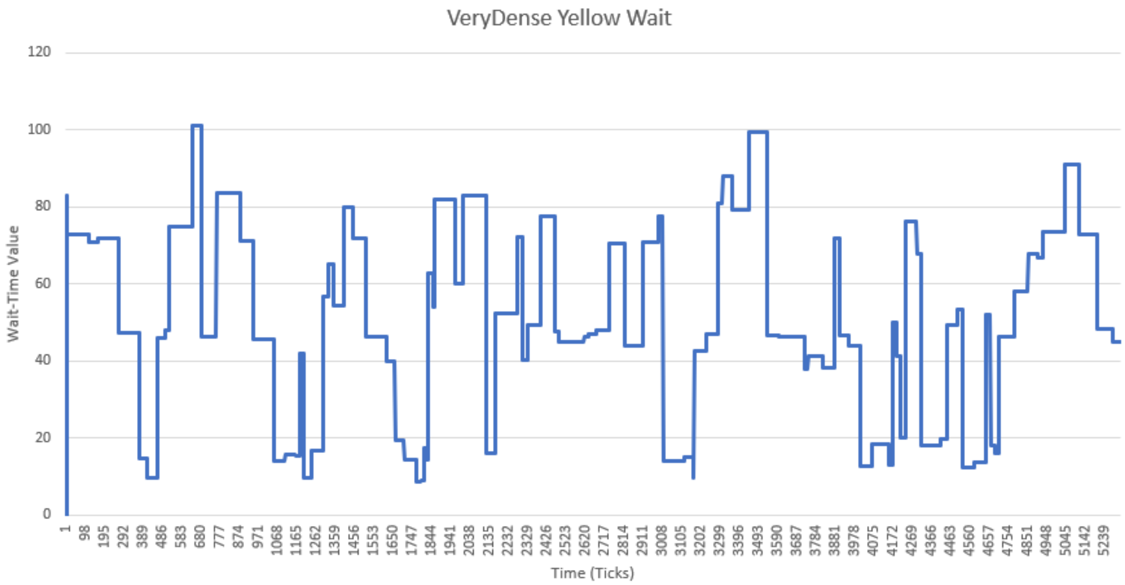
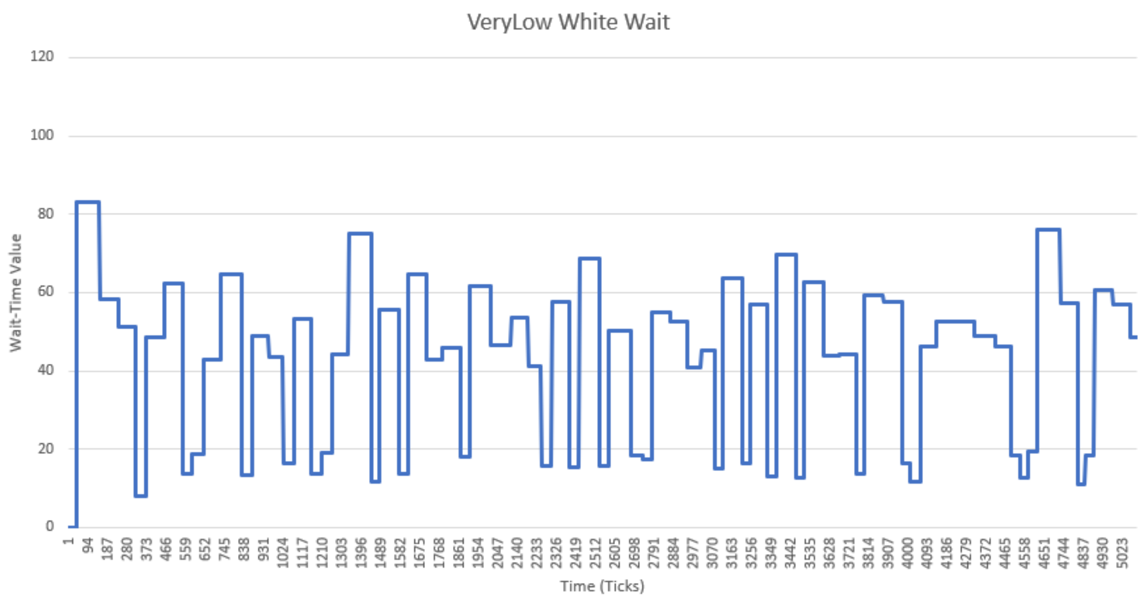
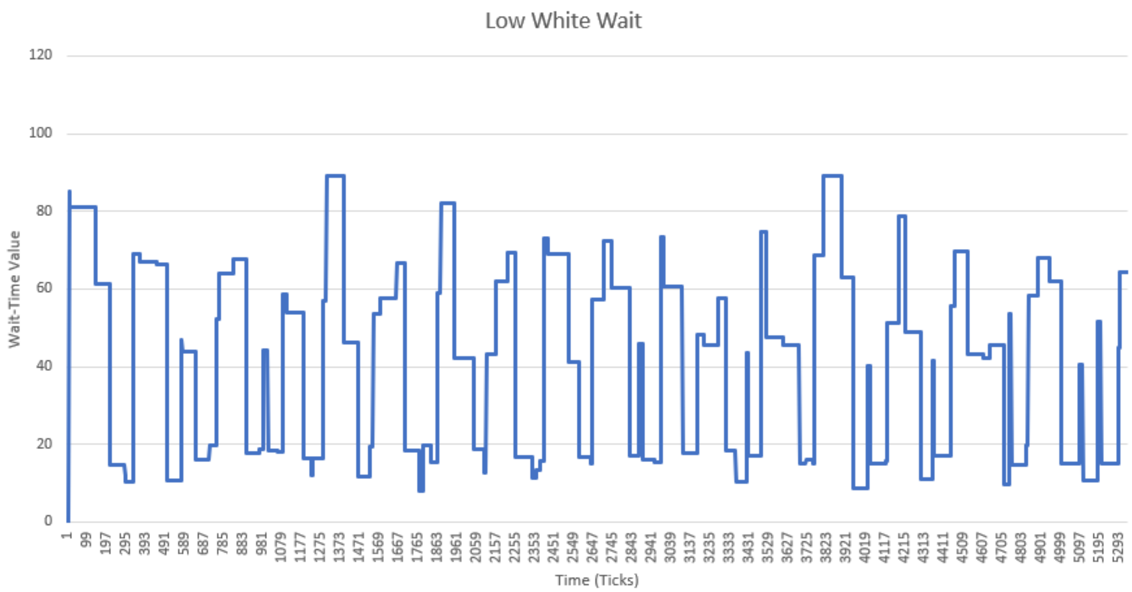
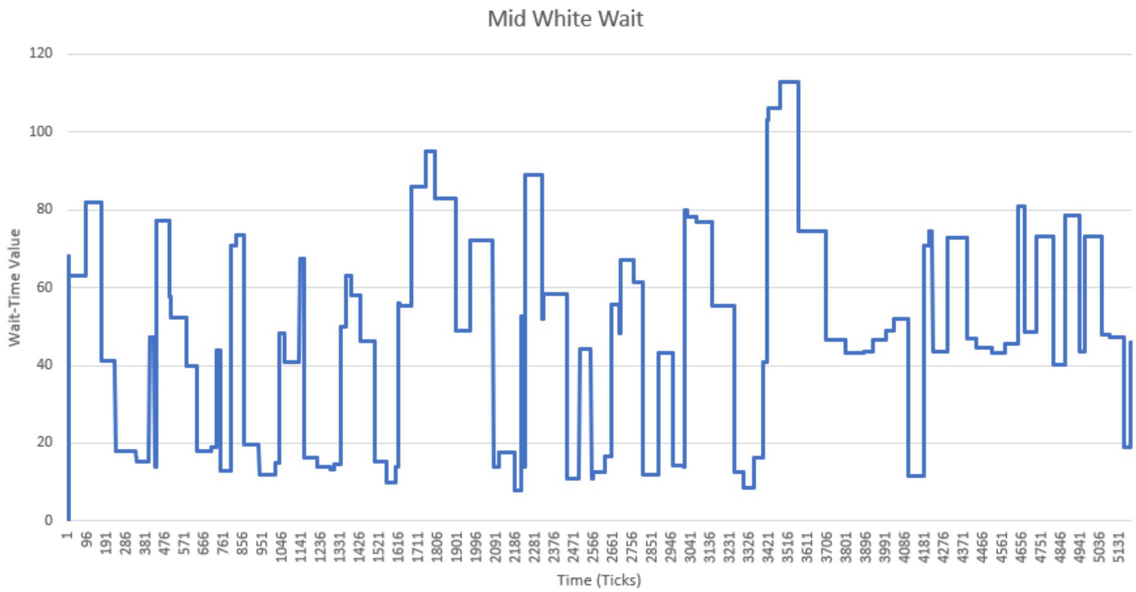
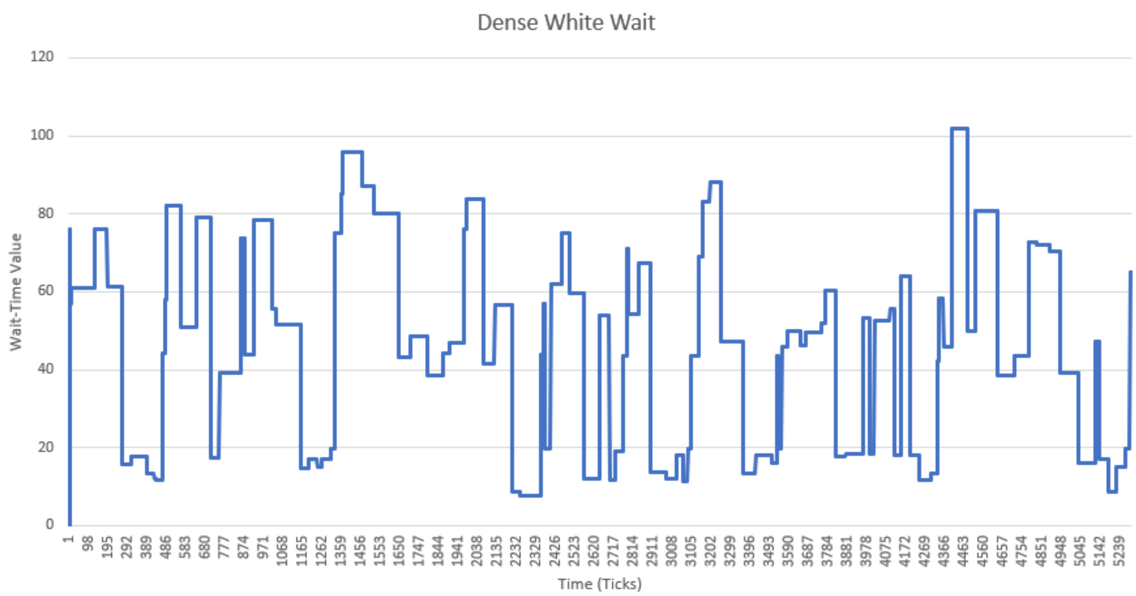

VeryDense Yellow Wait
Wait-Time Value
Time (Ticks)
0
20
40
60
80
100
120
1
98
195
292
389
486
583
680
777
874
971
1068
1165
1262
1359
1456
1553
1650
1747
1844
1941
2038
2135
2232
2329
2426
2523
2620
2717
2814
2911
3008
3105
3202
3299
3396
3493
3590
3687
3784
3881
3978
4075
4172
4269
4366
4463
4560
4657
4754
4851
4948
5045
5142
5239

Figure 32. White Car Wait-Time Graphs






Mid White Wait
120
100
80
60
40
20
0
Wait-Time Value
1 96 191 286 381 476 571 666 761 856 951 1046 1141 1236 1331 1426 1521 1616 1711 1806 1901 1996 2091 2186 2281 2376 2471 2566 2661 2756 2851 2946 3041 3136 3231 3326 3421 3516 3611 3706 3801 3896 3991 4086 4181 4276 4371 4466 4561 4656 4751 4846 4941 5036 5131
Time (Ticks)


Dense White Wait
120
100
80
60
40
20
0
Wait-Time Value
1 98 195 292 389 486 583 680 777 874 971 1068 1165 1262 1359 1456 1553 1650 1747 1844 1941 2038 2135 2232 2329 2426 2523 2620 2717 2814 2911 3008 3105 3202 3299 3396 3493 3590 3687 3784 3881 3978 4075 4172 4269 4366 4463 4560 4657 4754 4851 4948 5045 5142 5239
Time (Ticks)

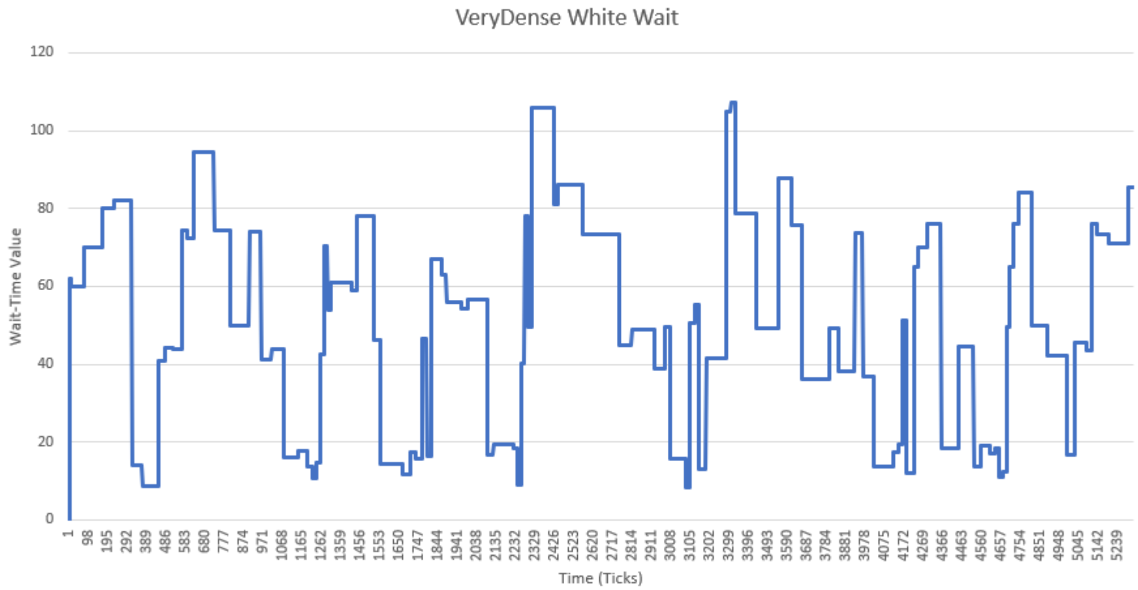
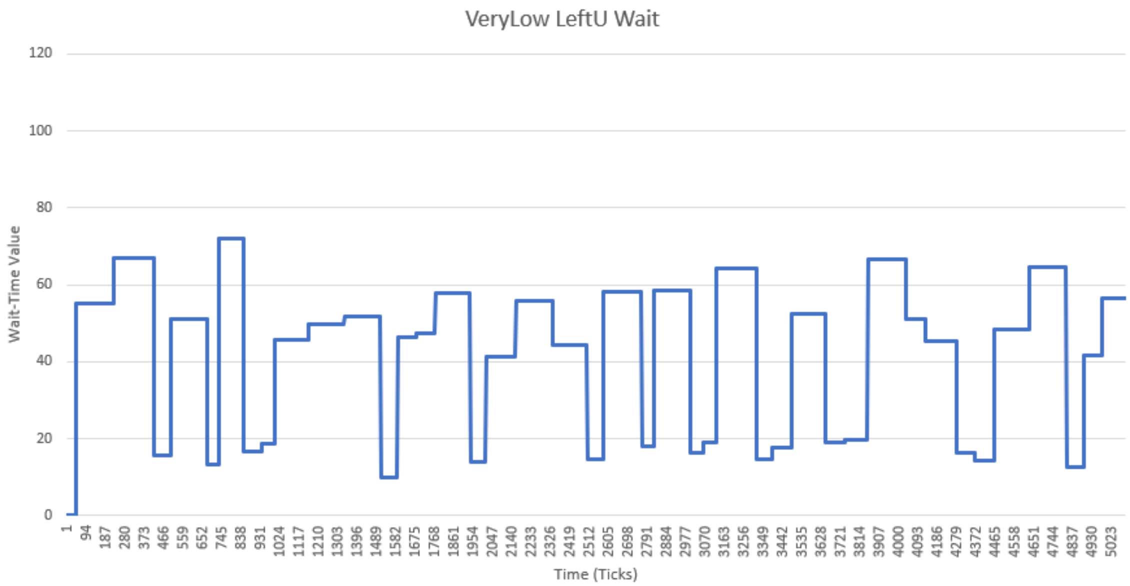
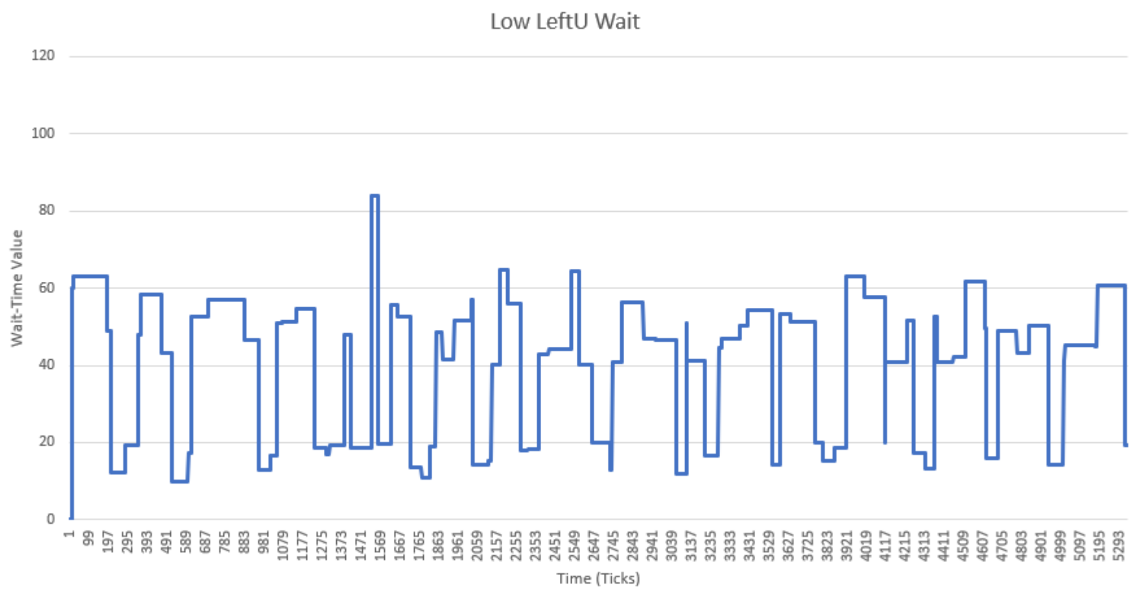
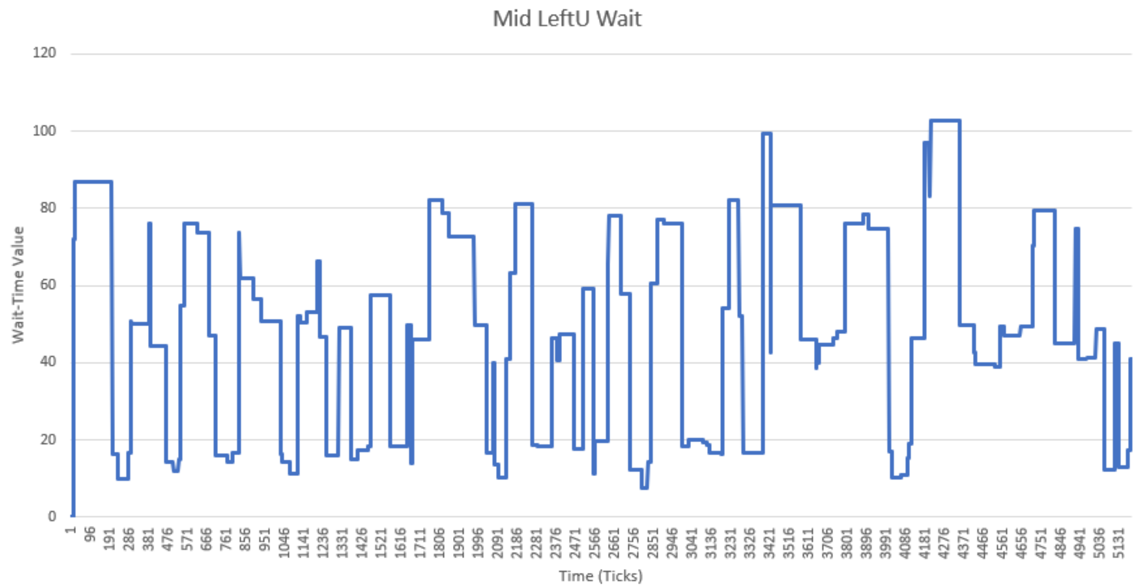
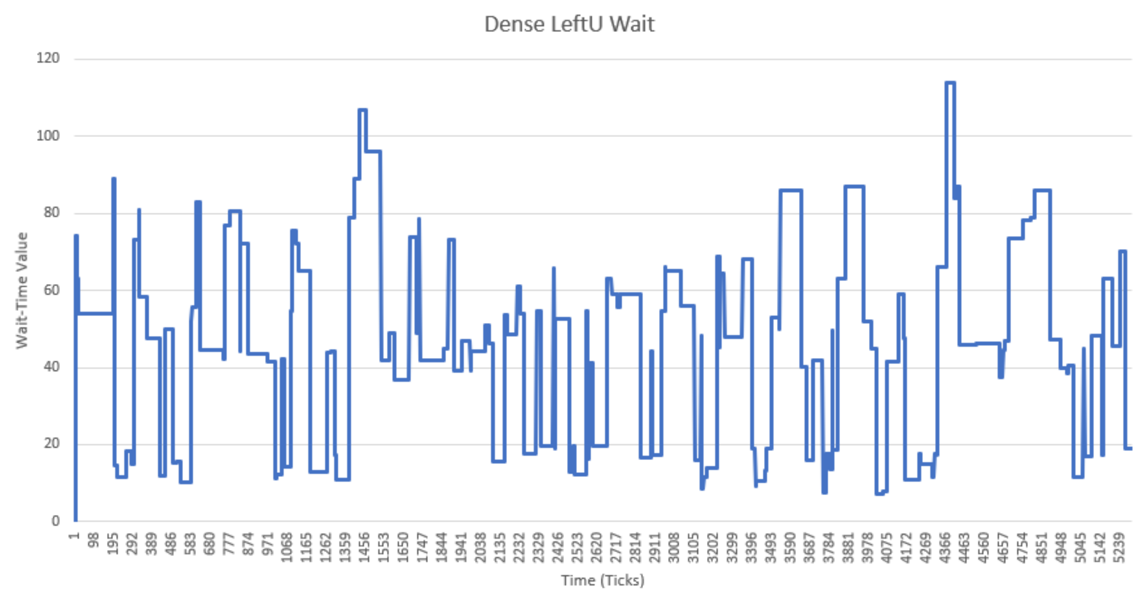

VeryDense White Wait
Wait-Time Value
Time (Ticks)
0
20
40
60
80
100
120

Figure 33. Left (Upper) Pedestrian Wait-Time Graphs






Mid LeftU Wait
Wait-Time Value
Time (Ticks)
0
20
40
60
80
100
120


Dense LeftU Wait
Wait-Time Value
Time (Ticks)
0
20
40
60
80
100
120

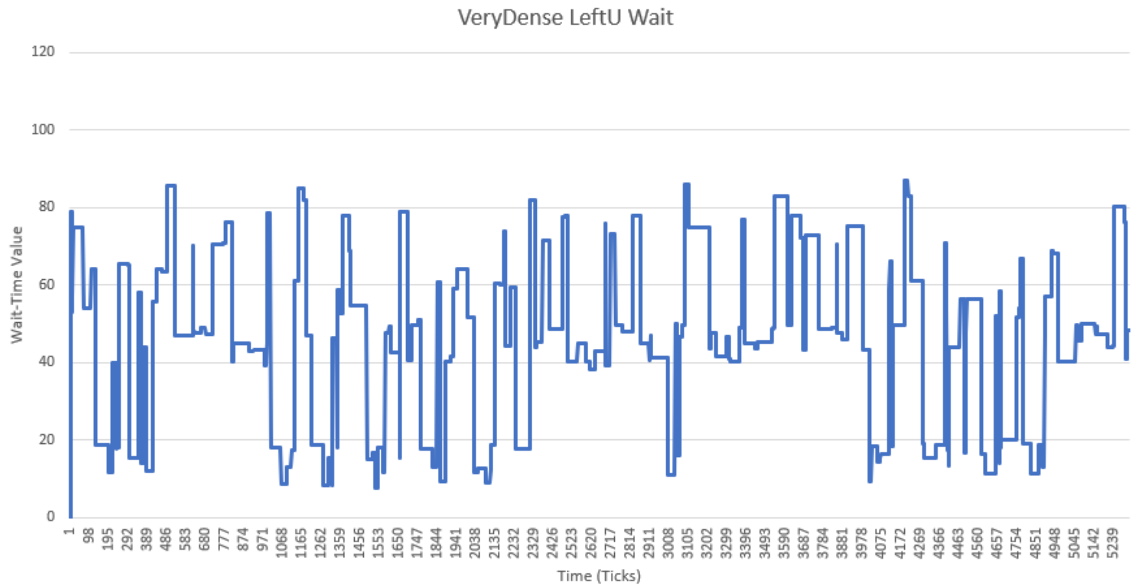
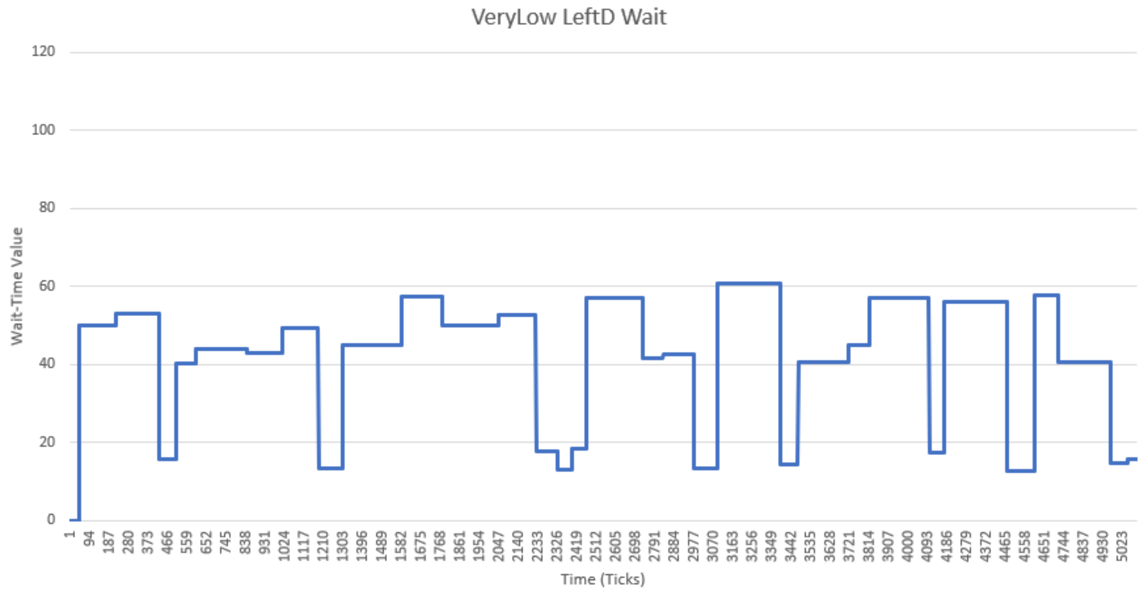
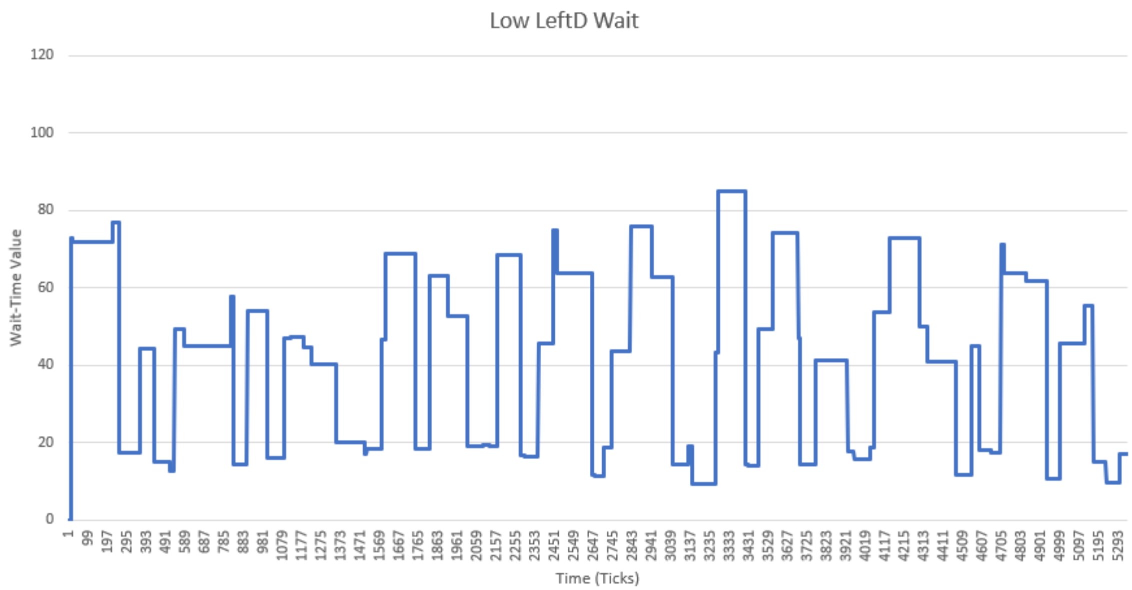
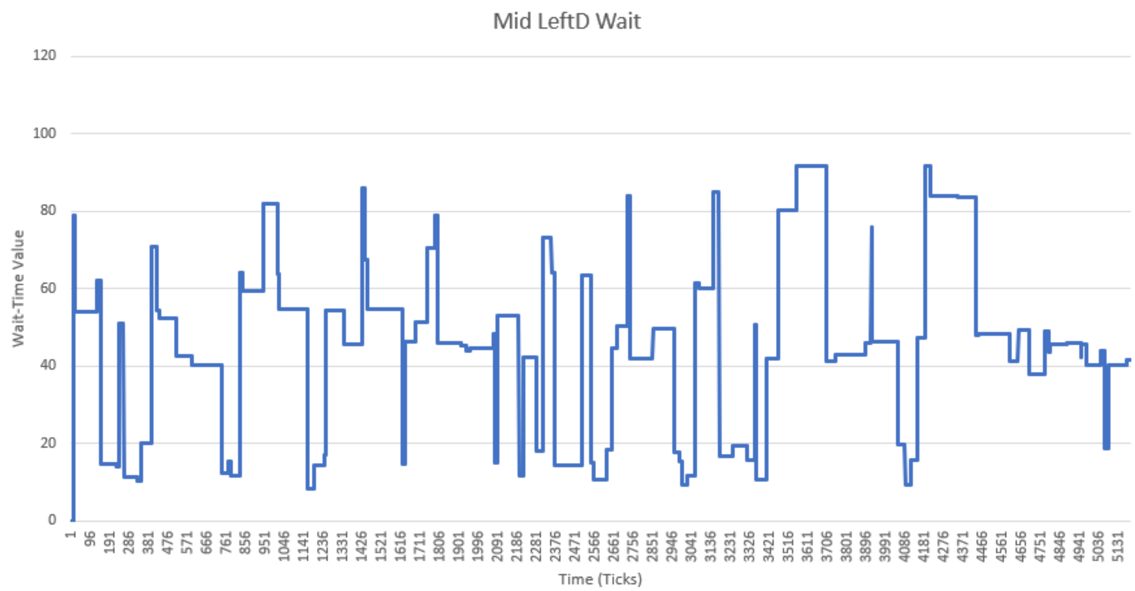
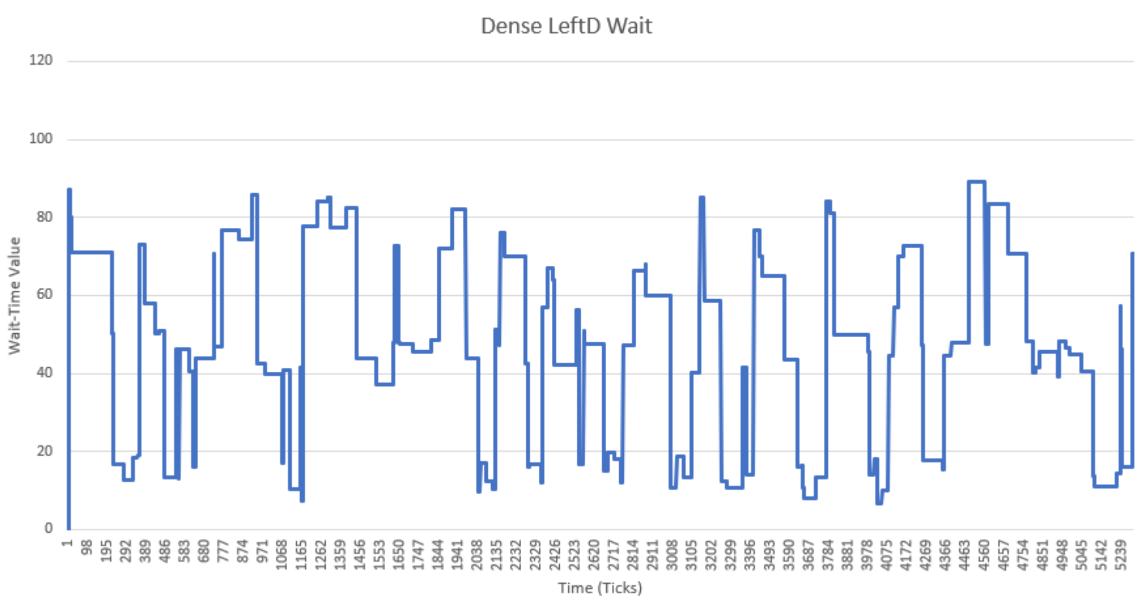

VeryDense LeftU Wait
Wait-Time Value
Time (Ticks)

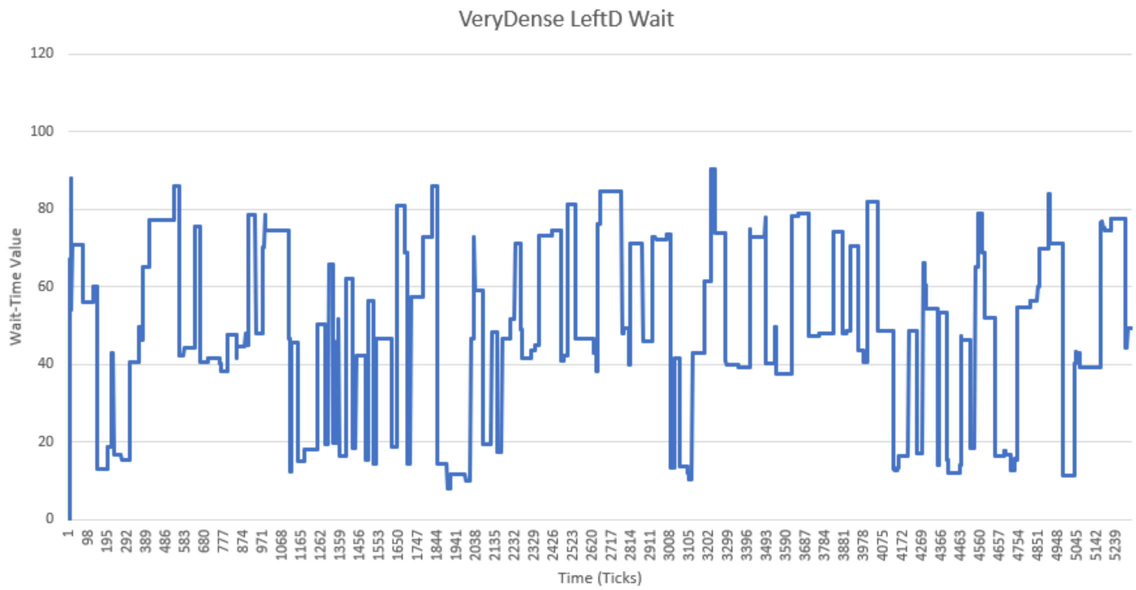
Figure 34. Left (Down) Pedestrian Wait-Time Graphs





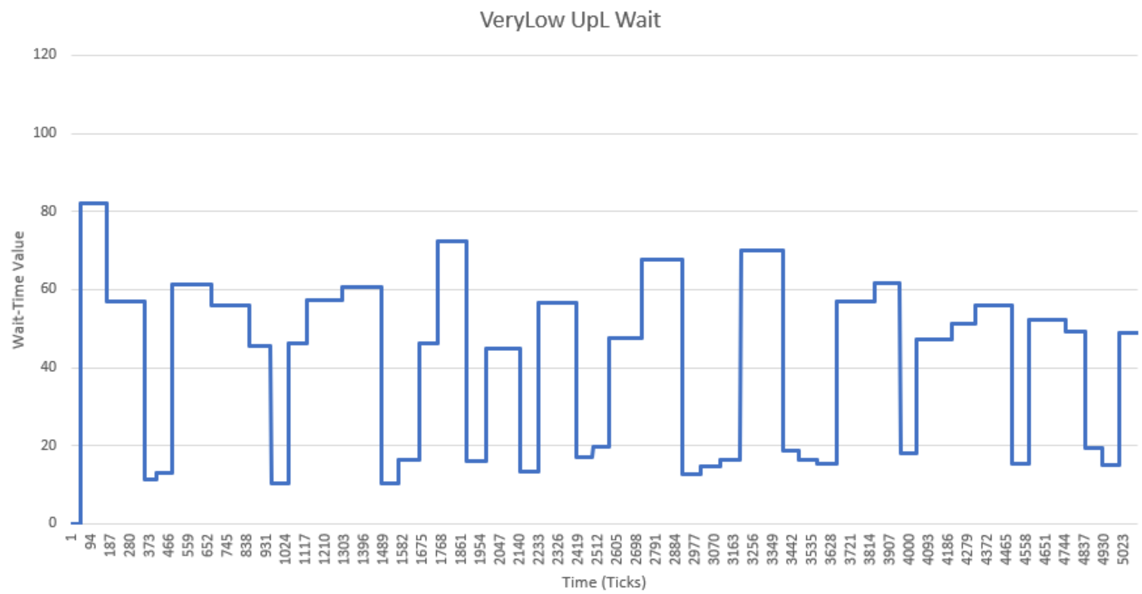

Mid LeftD Wait
Wait-Time Value
Time (Ticks)
0
20
40
60
80
100
120

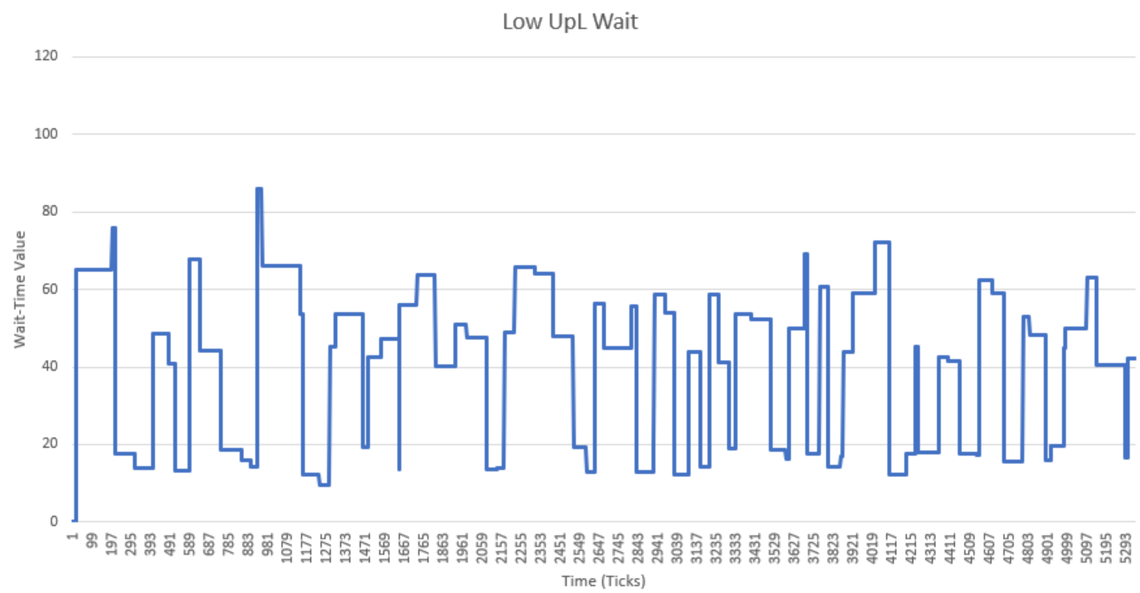

Dense LeftD Wait
Wait-Time Value
Time (Ticks)
0
20
40
60
80
100
120

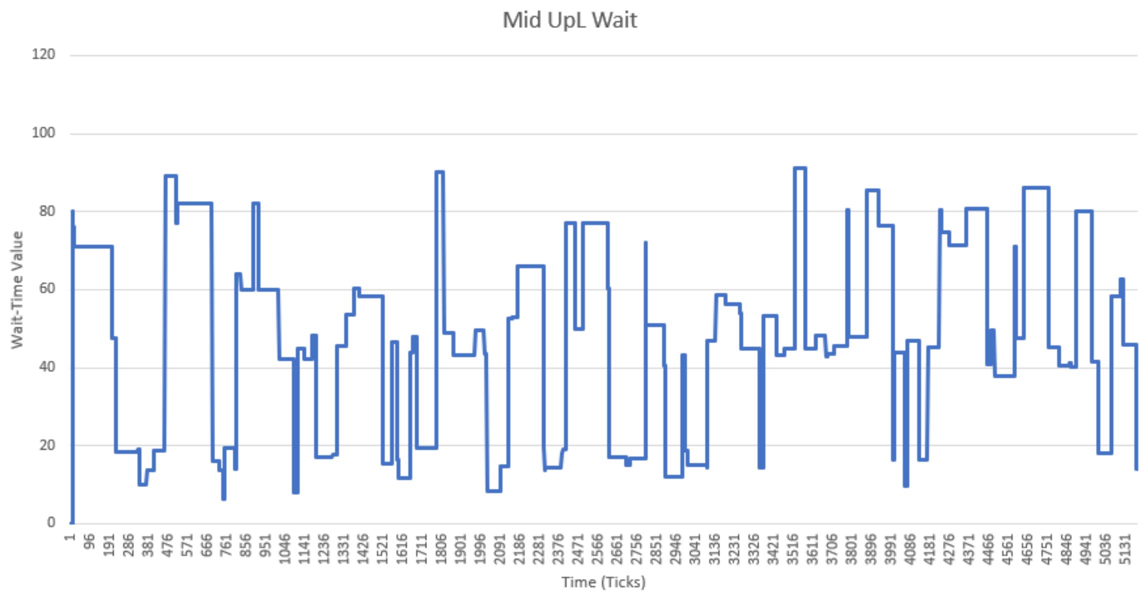


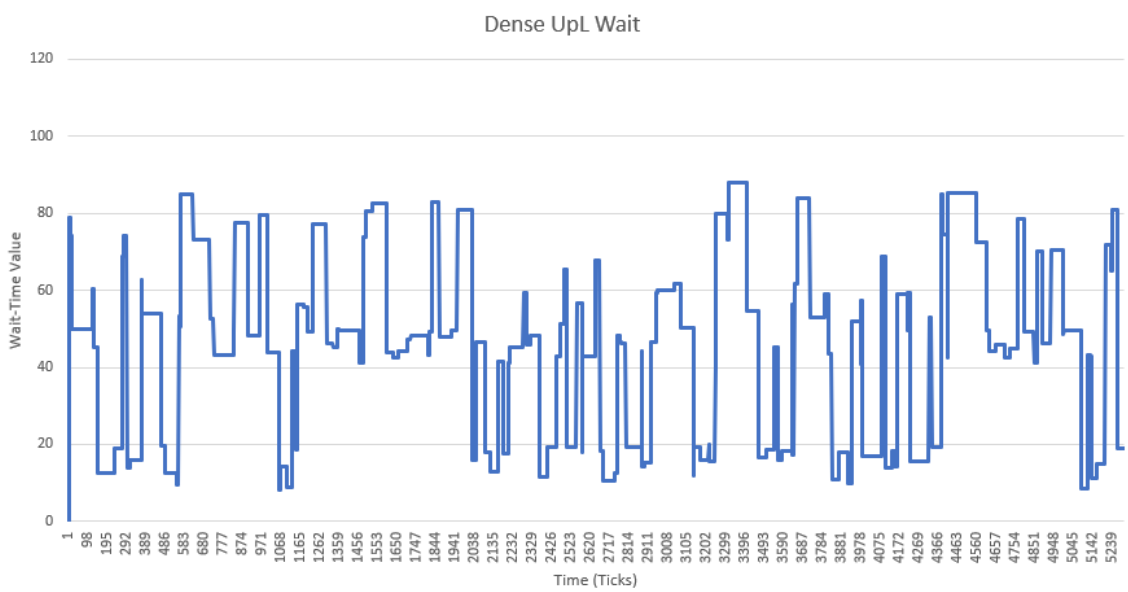
VeryDense LeftD Wait
Wait-Time Value
Time (Ticks)

Figure 35. Up (Left) Pedestrian Wait-Time Graphs






Mid UpL Wait
Wait-Time Value
Time (Ticks)


Dense UpL Wait
Wait-Time Value
Time (Ticks)

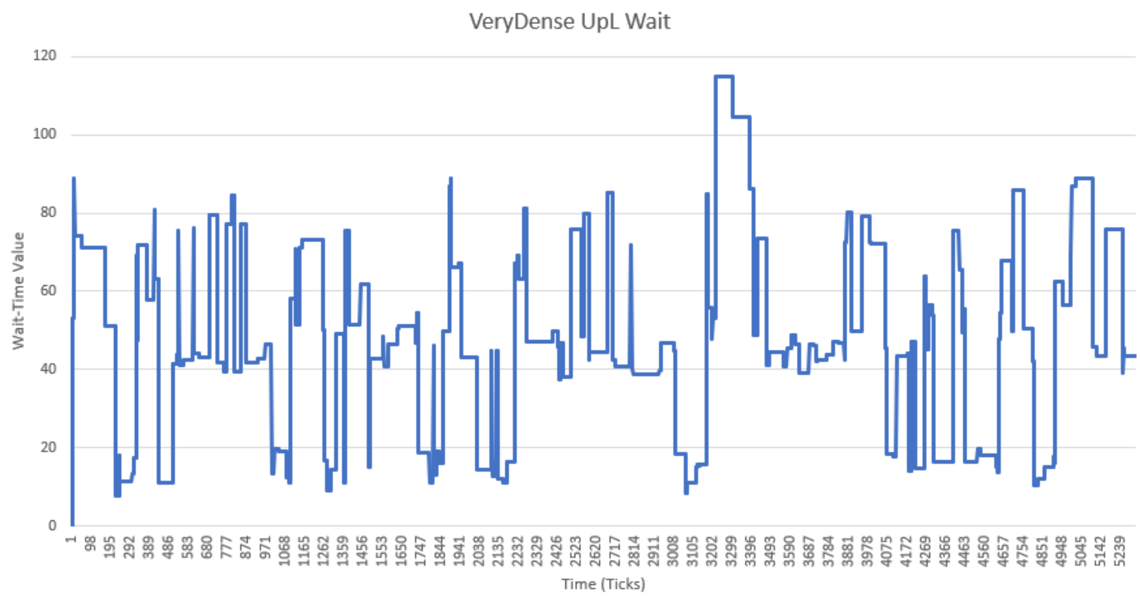
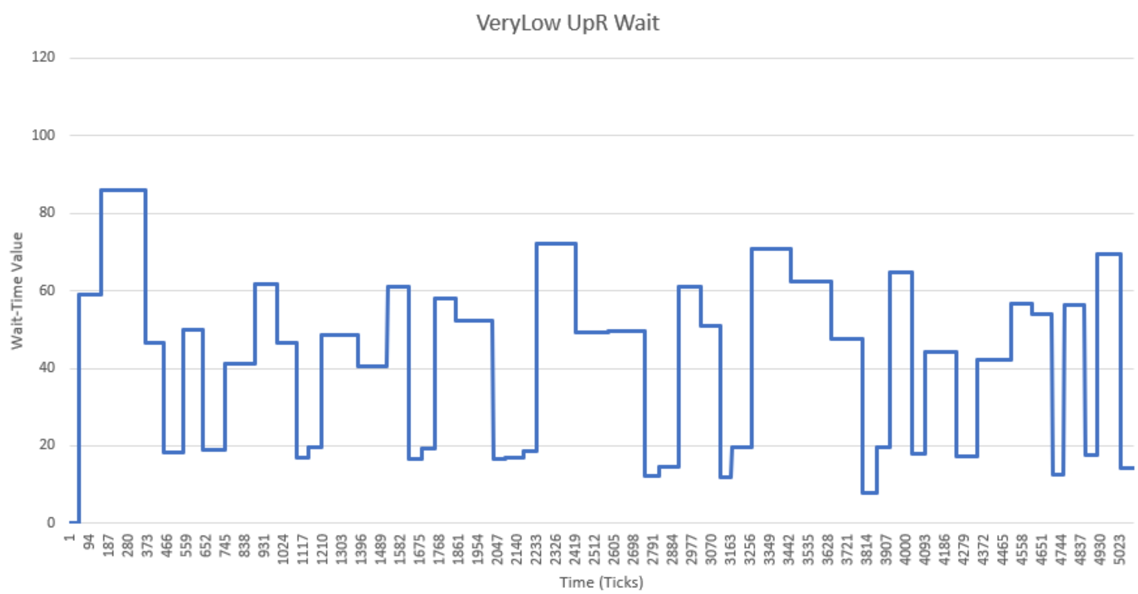
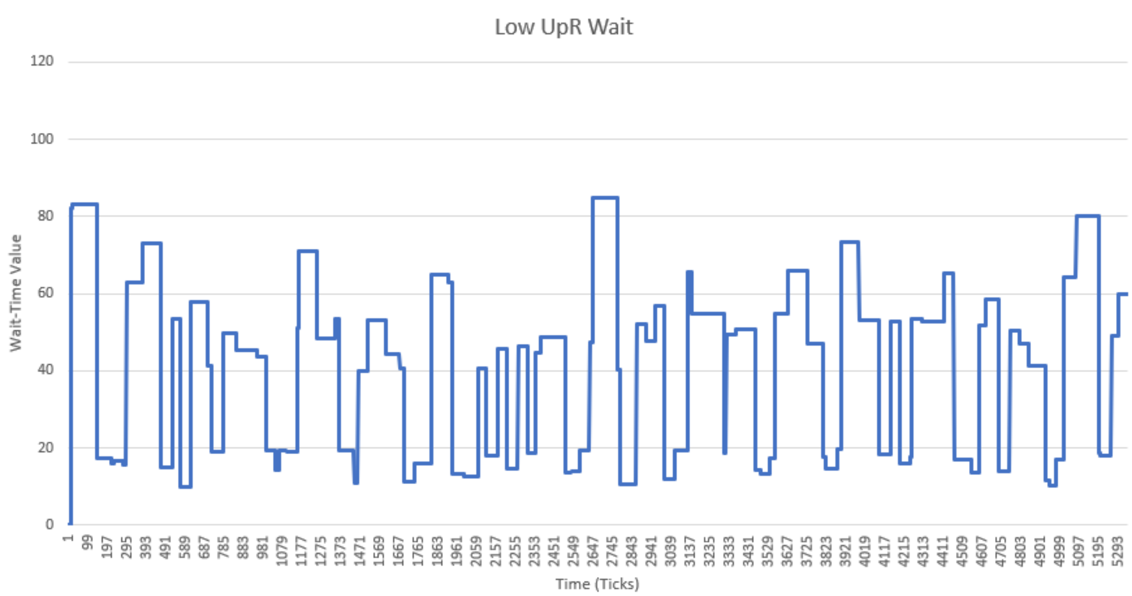
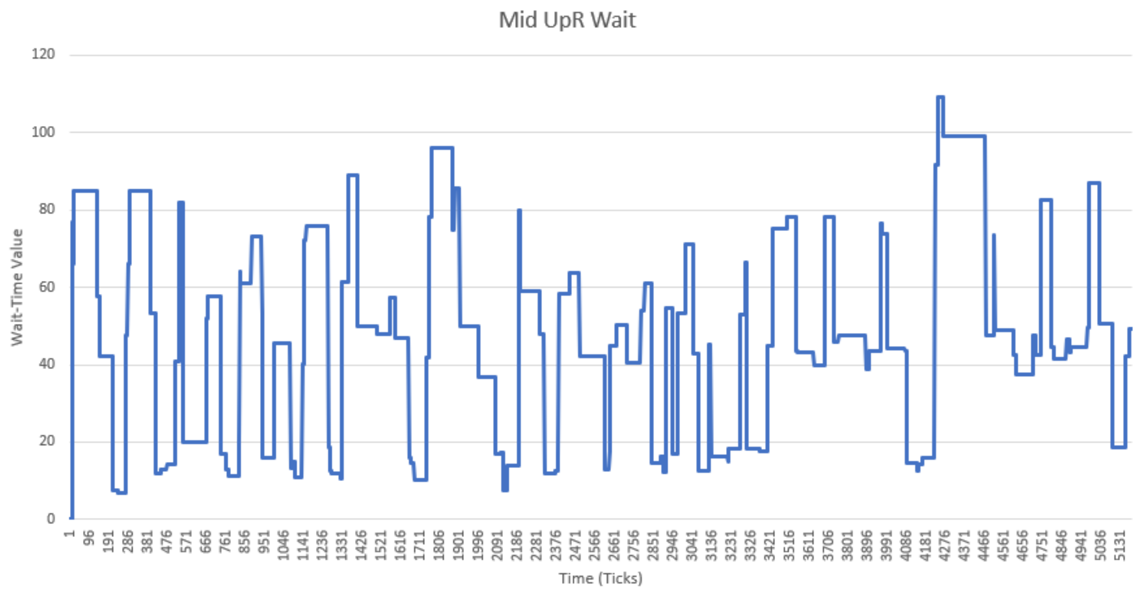
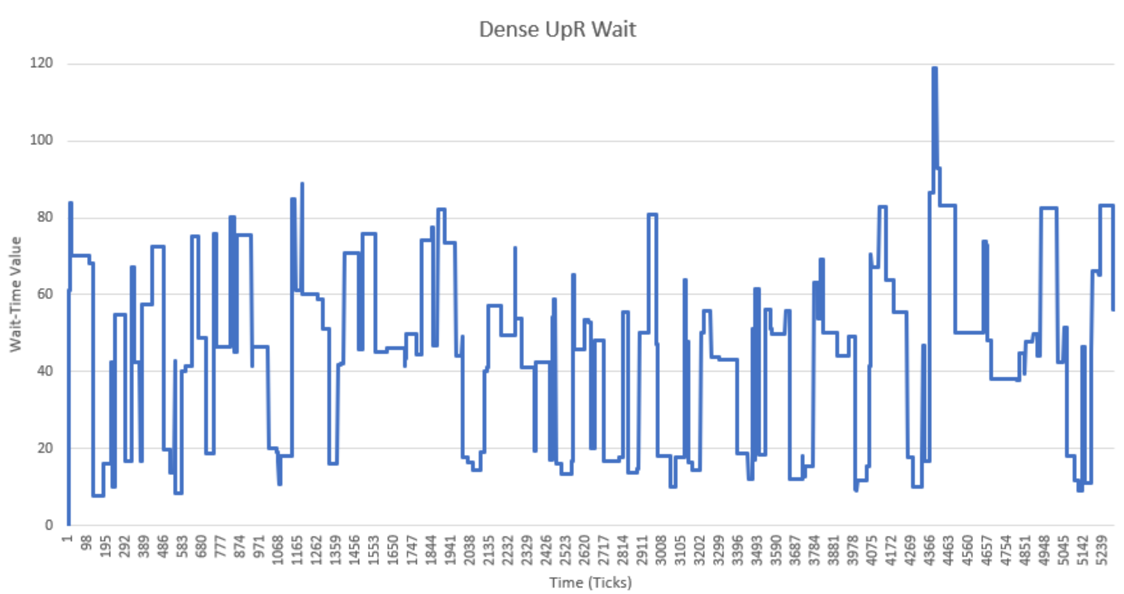

VeryDense UpL Wait
Wait-Time Value
Time (Ticks)
0
20
40
60
80
100
120

Figure 36. Up (Right) Pedestrian Wait-Time Graphs






Mid UpR Wait
Wait-Time Value
Time (Ticks)
0
20
40
60
80
100
120


Dense UpR Wait
Wait-Time Value
Time (Ticks)
0
20
40
60
80
100
120

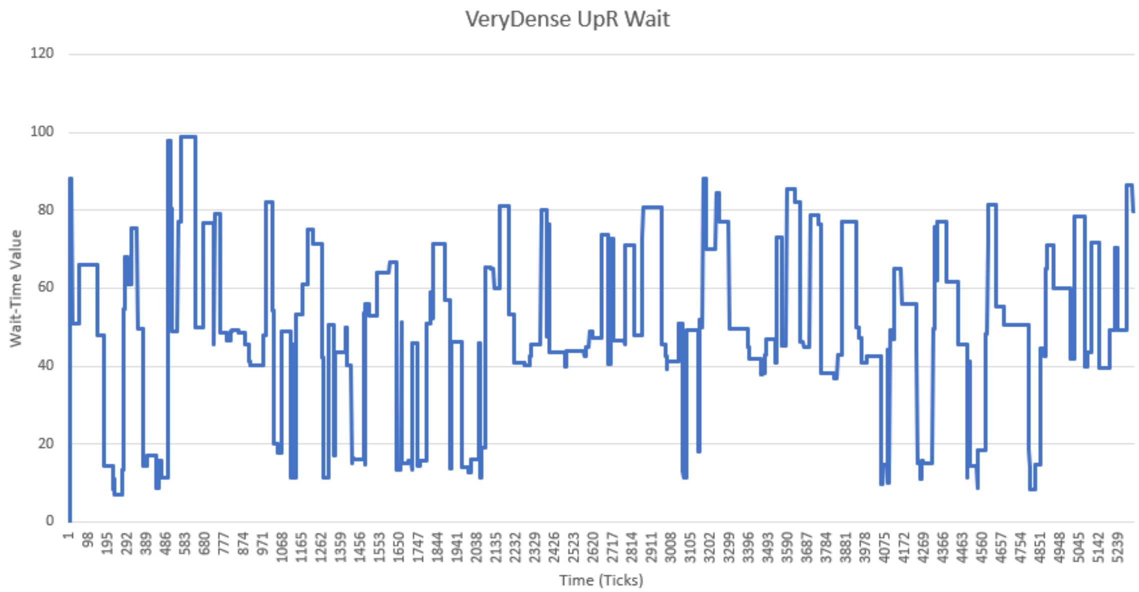
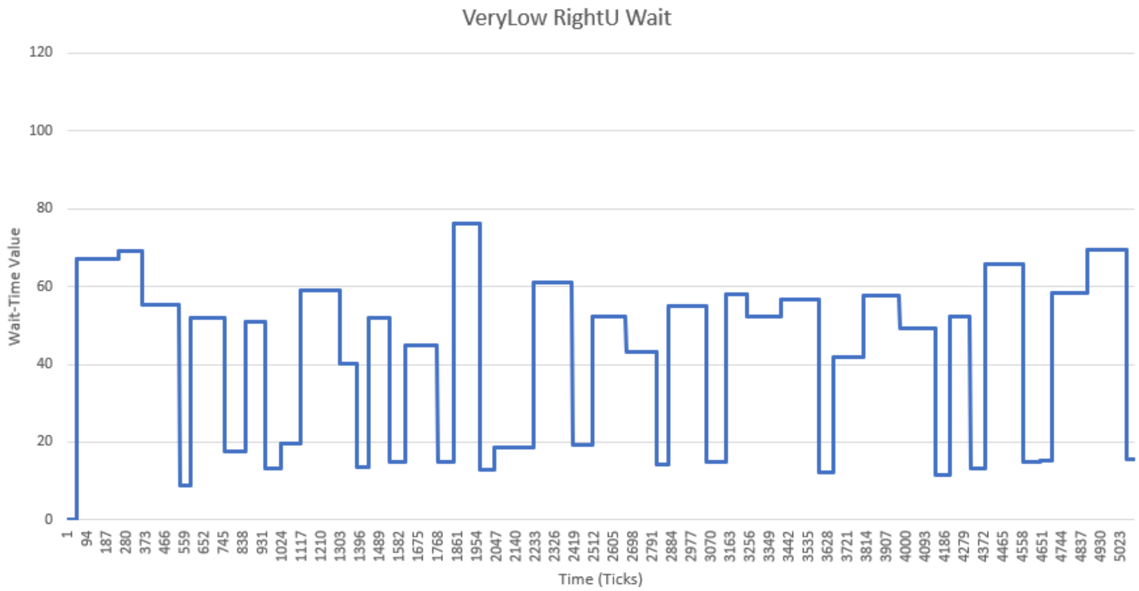
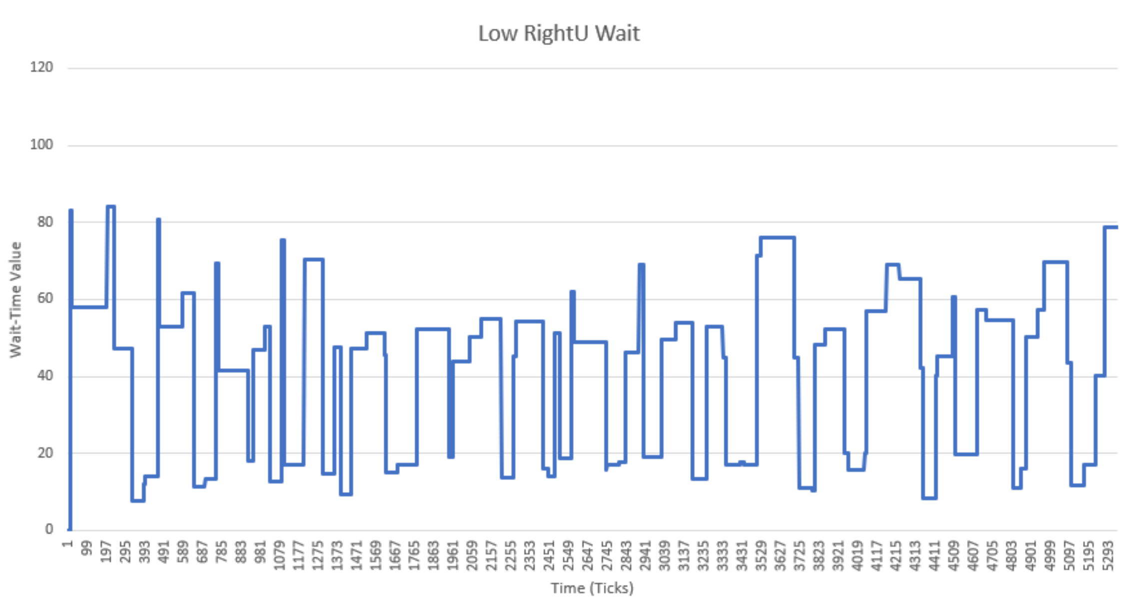
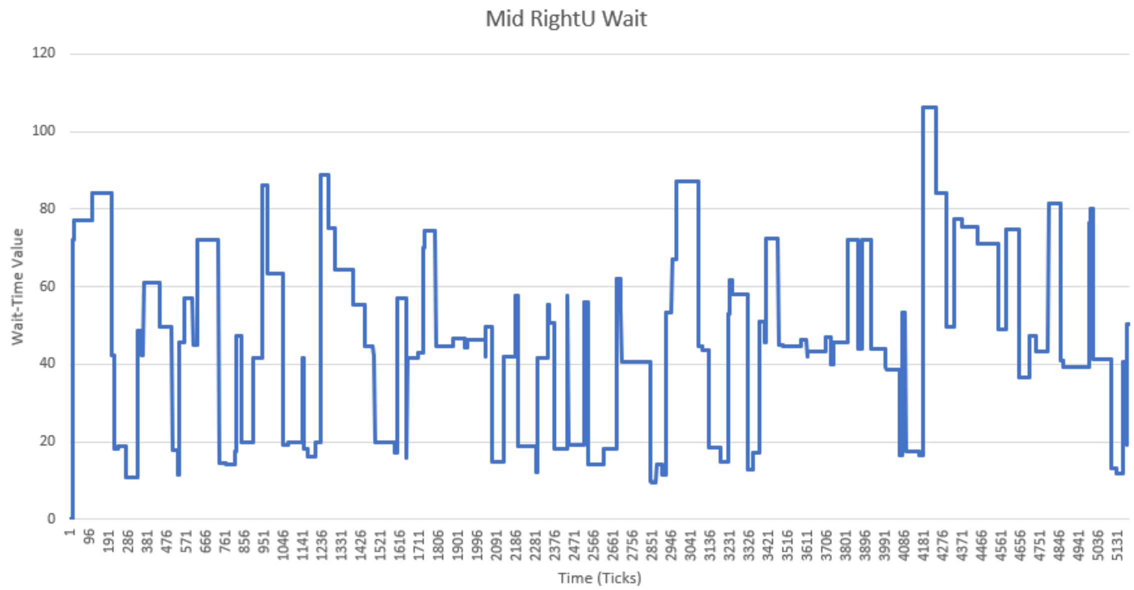
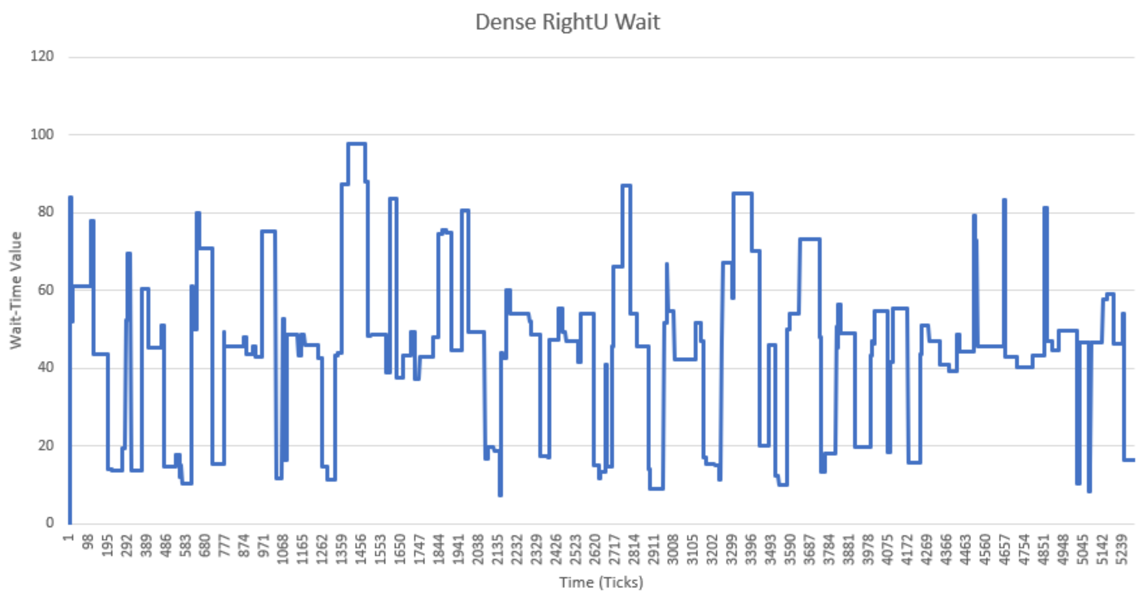

VeryDense UpR Wait
Wait-Time Value
Time (Ticks)
0
20
40
60
80
100
120

Figure 37. Right (Up) Pedestrian Wait-Time Graphs






Mid RightU Wait
120
100
80
60
40
20
0
Wait-Time Value
1 96 191 286 381 476 571 666 761 856 951 1046 1141 1236 1331 1426 1521 1616 1711 1806 1901 1996 2091 2186 2281 2376 2471 2566 2661 2756 2851 2946 3041 3136 3231 3326 3421 3516 3611 3706 3801 3896 3991 4086 4181 4276 4371 4466 4561 4656 4751 4846 4941 5036 5131
Time (Ticks)


Dense RightU Wait
120
100
80
60
40
20
0
Wait-Time Value
1 98 195 292 389 486 583 680 777 874 971 1068 1165 1262 1359 1456 1553 1650 1747 1844 1941 2038 2135 2232 2329 2426 2523 2620 2717 2814 2911 3008 3105 3202 3299 3396 3493 3590 3687 3784 3881 3978 4075 4172 4269 4366 4463 4560 4657 4754 4851 4948 5045 5142 5239
Time (Ticks)

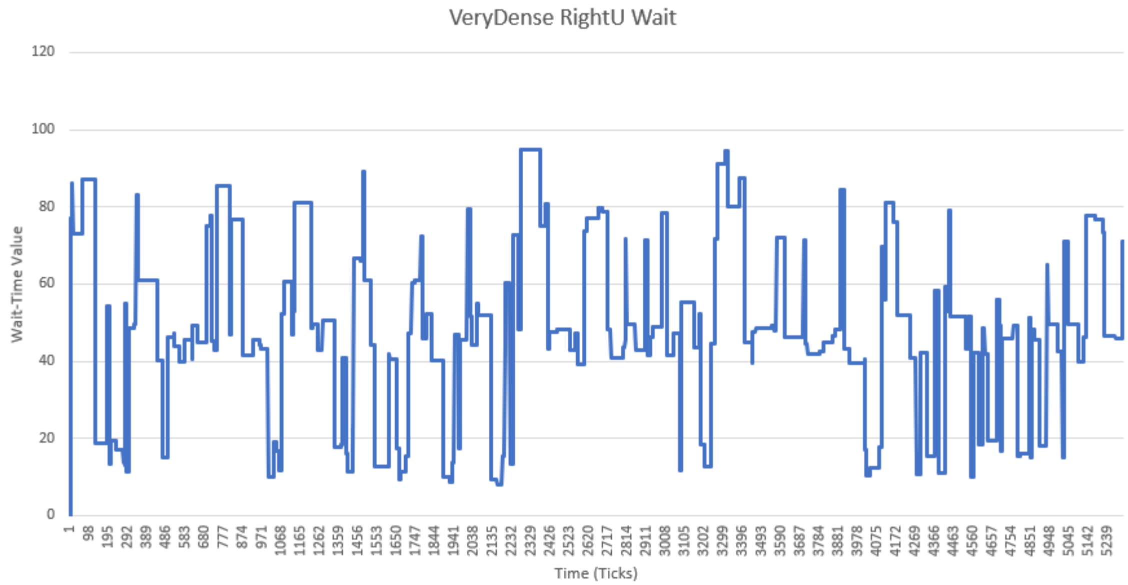
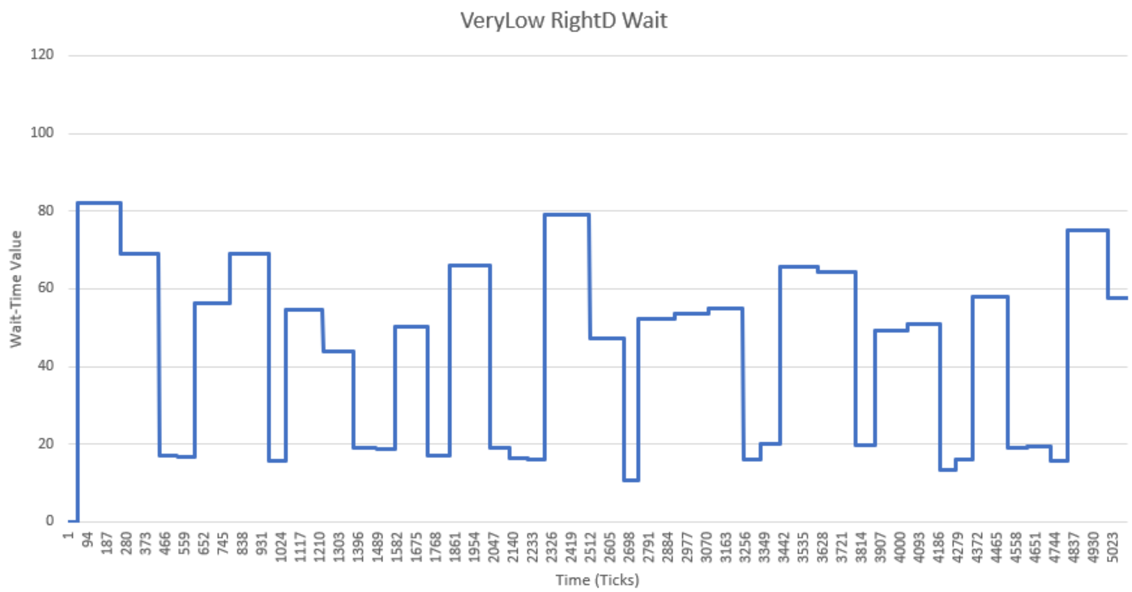
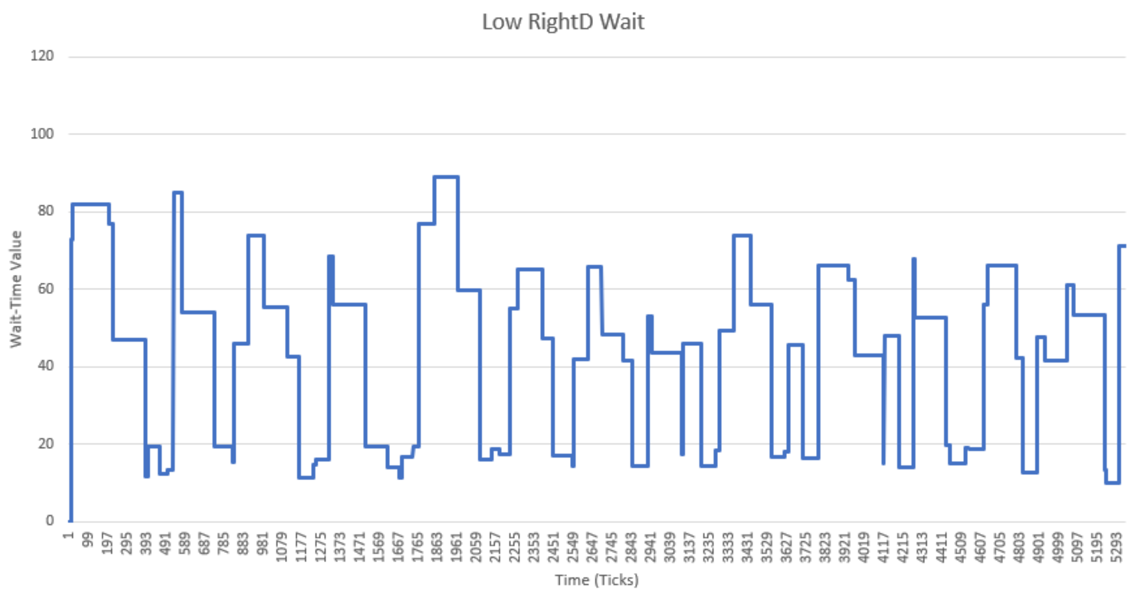
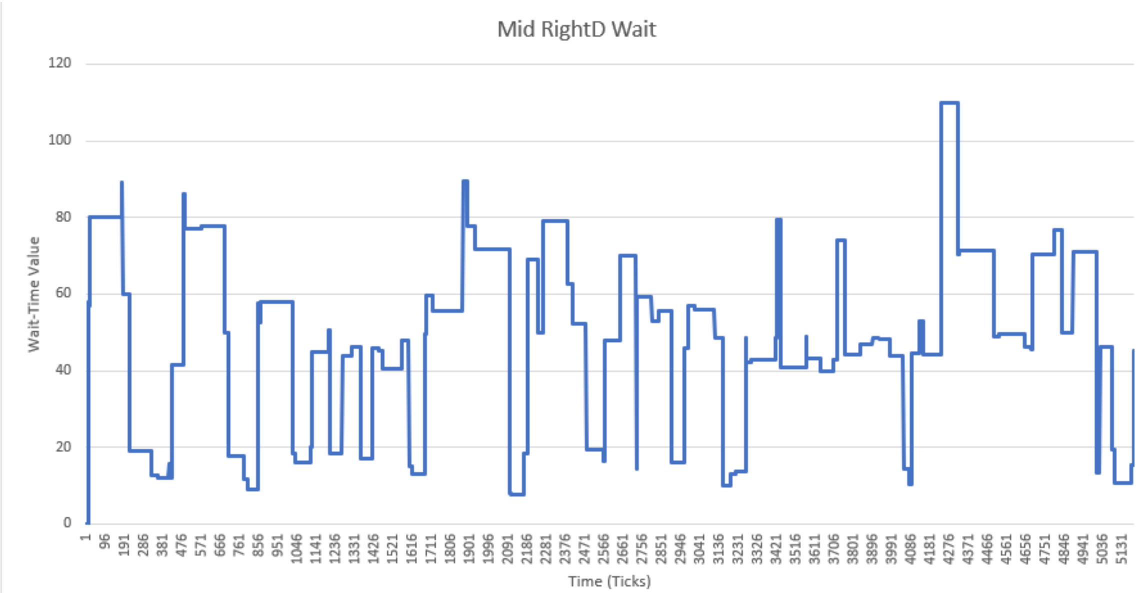
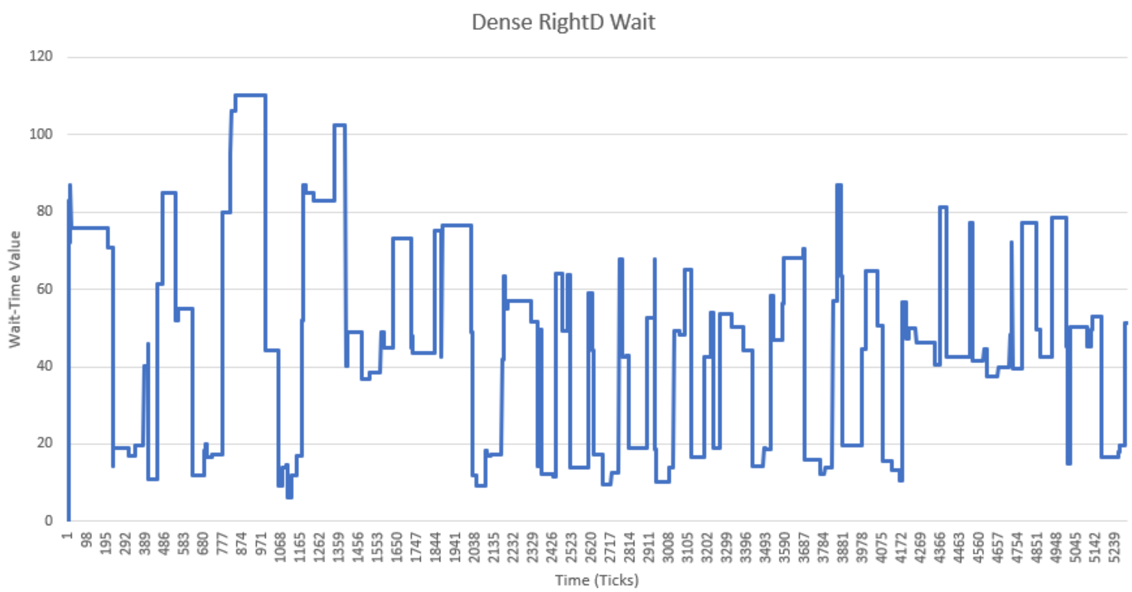

VeryDense RightU Wait
Wait-Time Value
Time (Ticks)
0
20
40
60
80
100
120
1
98
195
292
389
486
583
680
777
874
971
1068
1165
1262
1359
1456
1553
1650
1747
1844
1941
2038
2135
2232
2329
2426
2523
2620
2717
2814
2911
3008
3105
3202
3299
3396
3493
3590
3687
3784
3881
3978
4075
4172
4269
4366
4463
4560
4657
4754
4851
4948
5045
5142
5239

Figure 38. Right (Down) Pedestrian Wait-Time Graphs






Mid RightD Wait
Wait-Time Value
Time (Ticks)


Dense RightD Wait
Wait-Time Value
Time (Ticks)

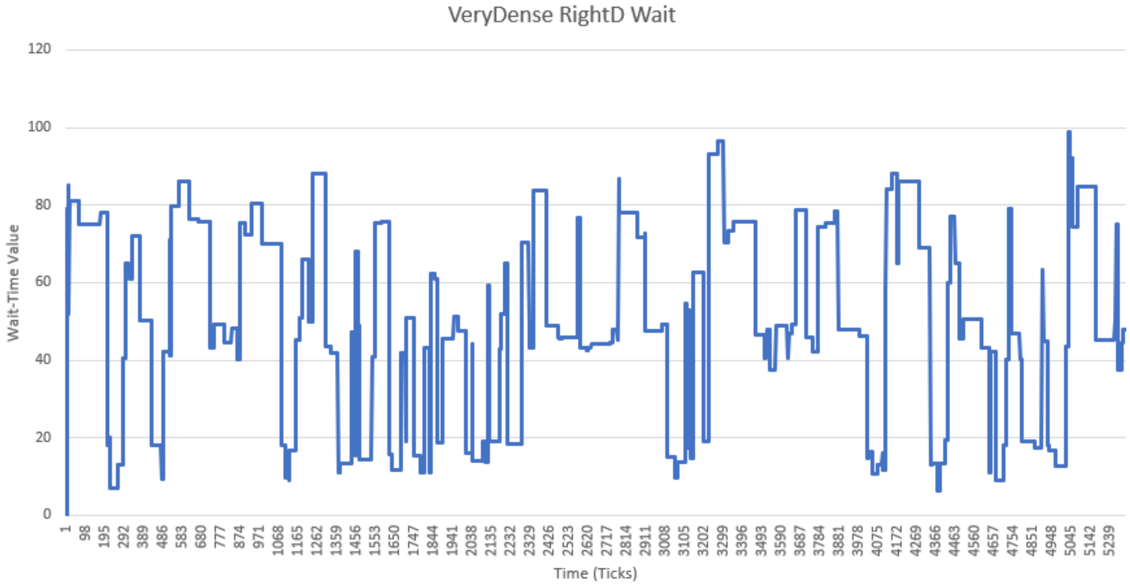
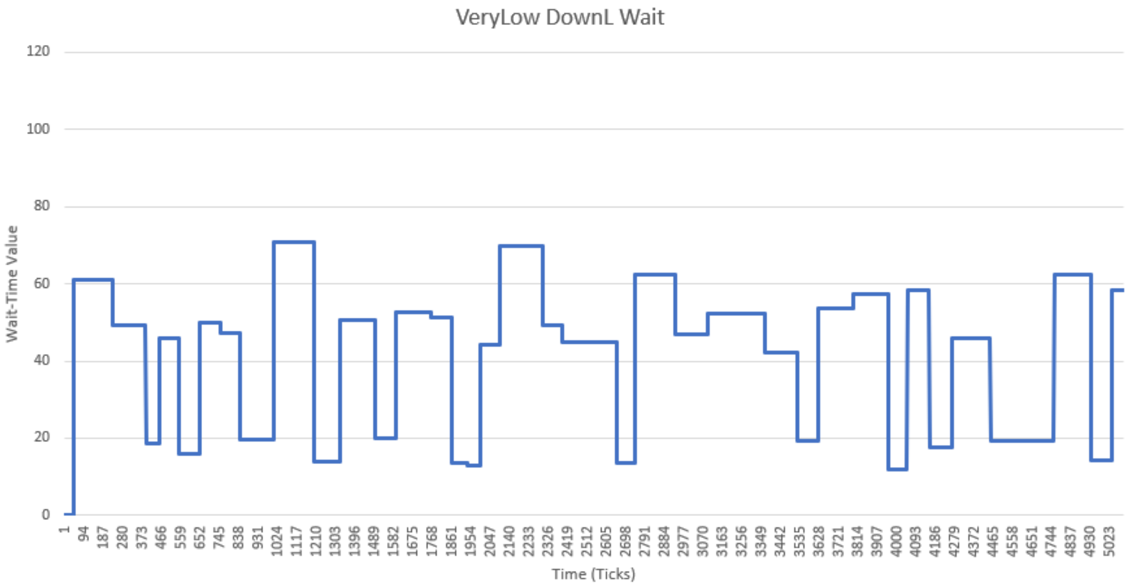
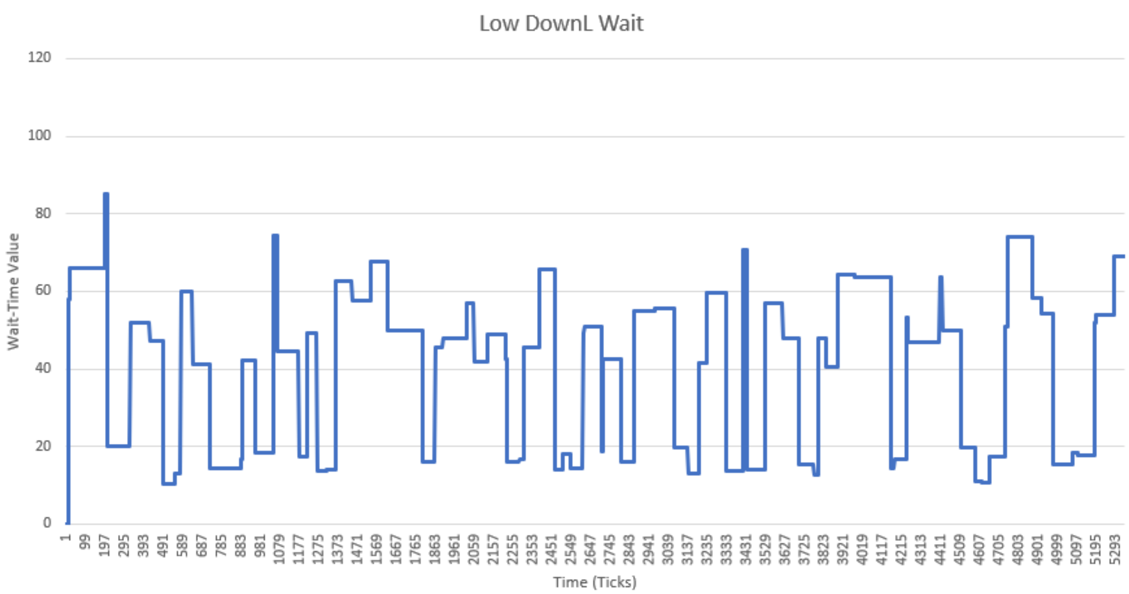
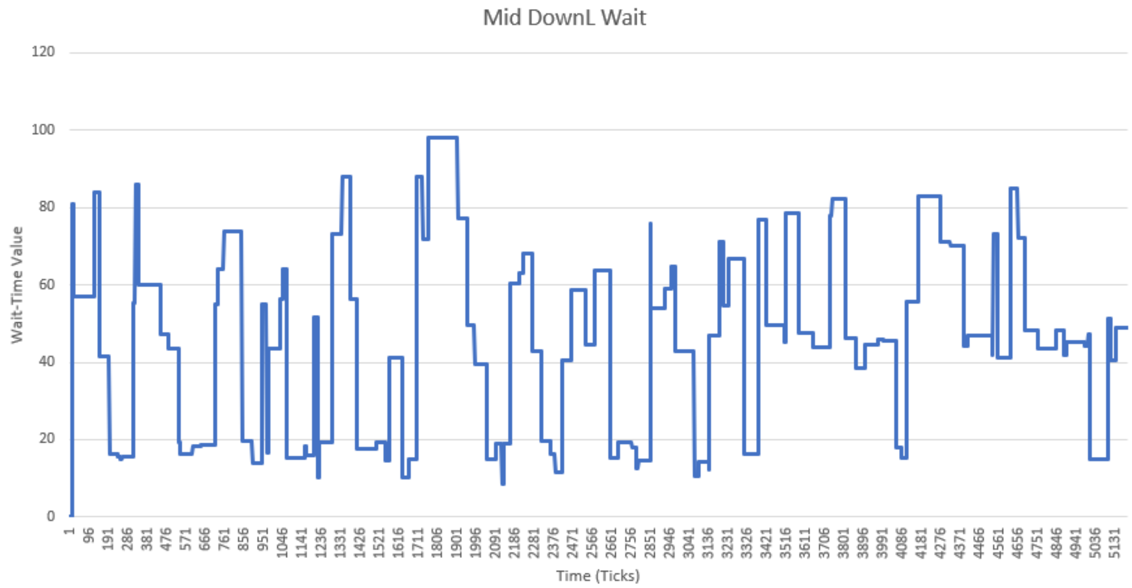
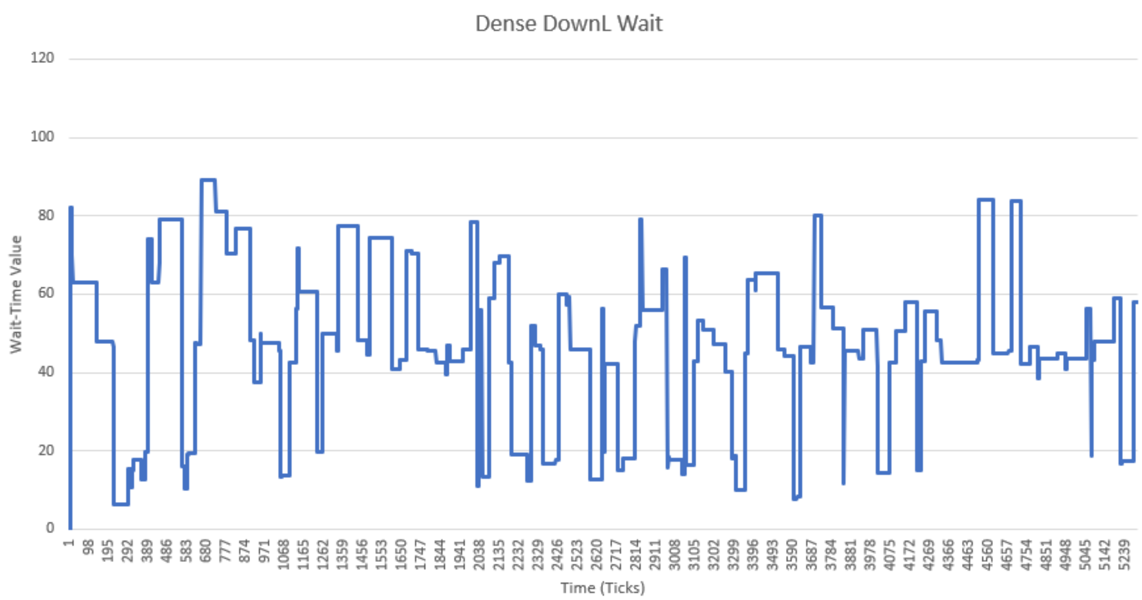

VeryDense RightD Wait
Wait-Time Value
Time (Ticks)

Figure 39. Down (Left) Pedestrian Wait-Time Graphs






Mid DownL Wait
Wait-Time Value
Time (Ticks)


Dense DownL Wait
Wait-Time Value
Time (Ticks)

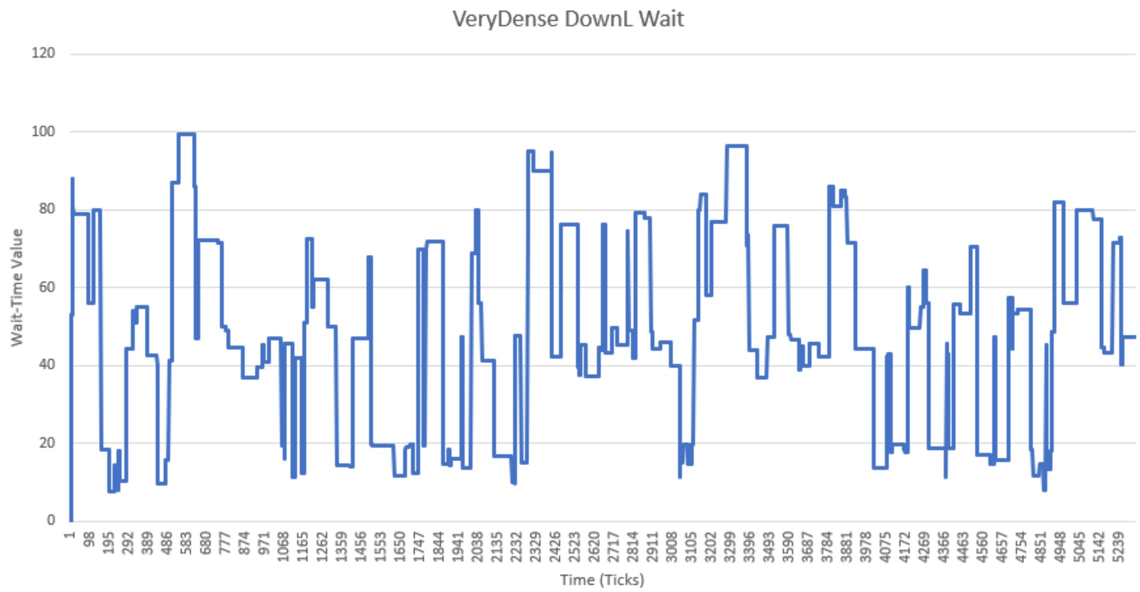
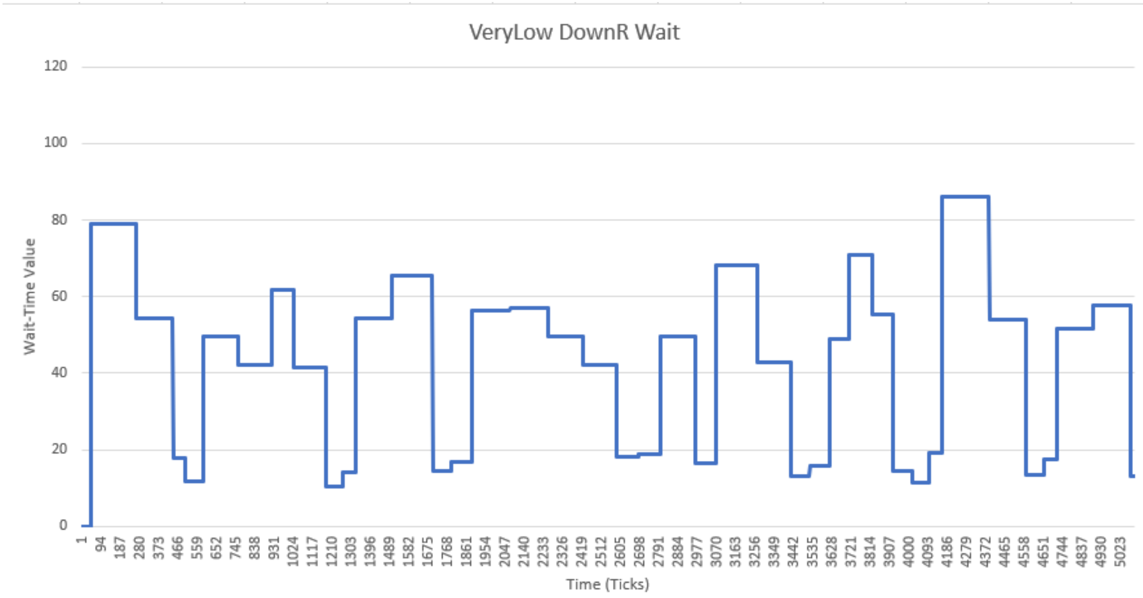
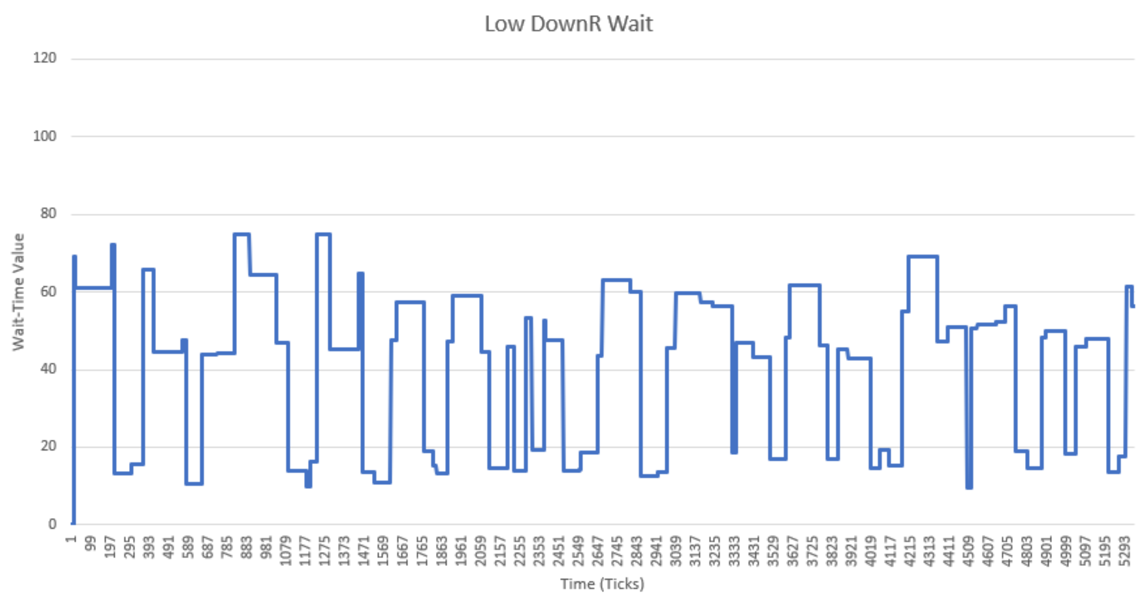
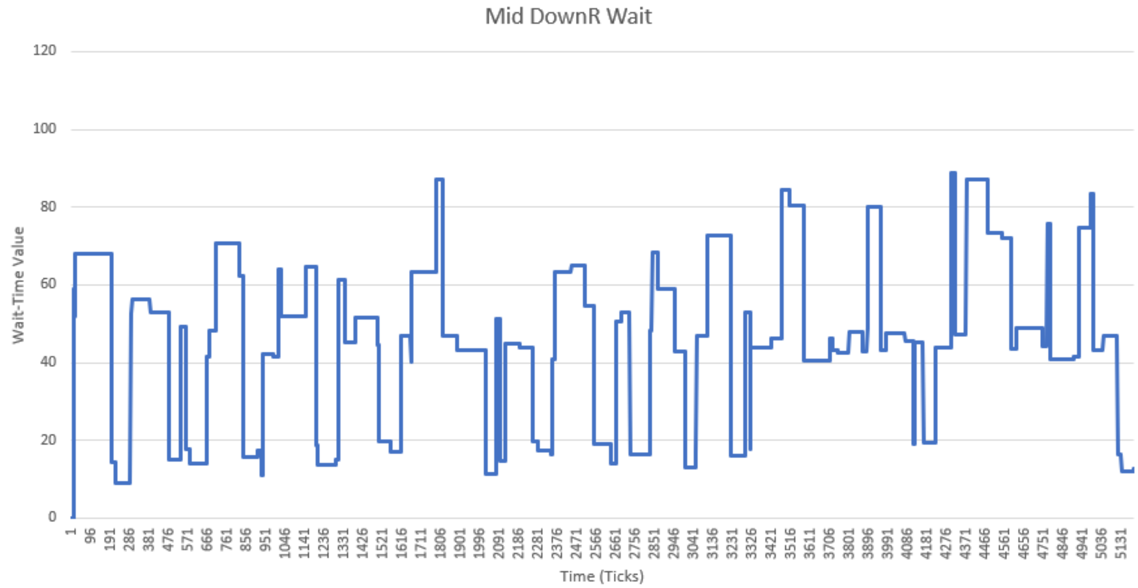
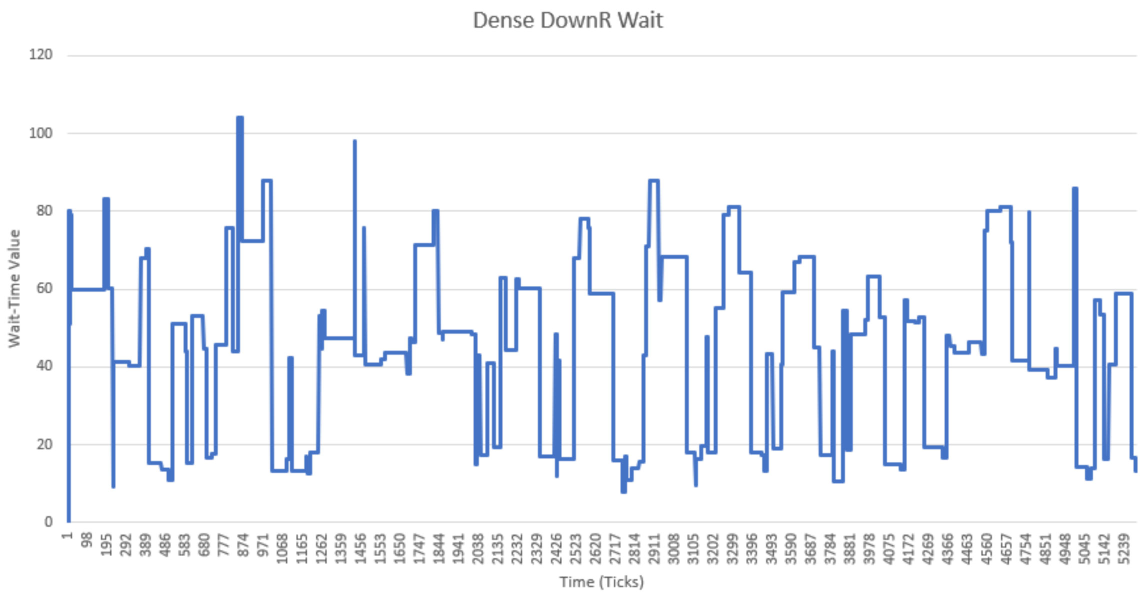

VeryDense DownL Wait
Wait-Time Value
Time (Ticks)
0
20
40
60
80
100
120

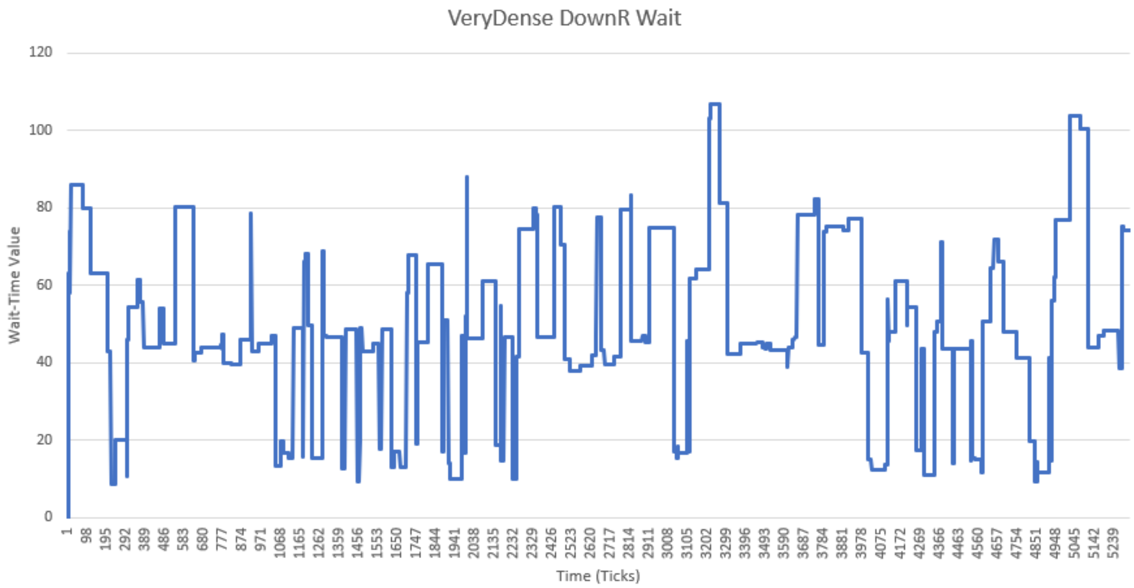
Figure 40. Down (Right) Pedestrian Wait-Time Graphs






Mid DownR Wait
120
100
80
60
40
20
0
Wait-Time Value
1 96 191 286 381 476 571 666 761 856 951 1046 1141 1236 1331 1426 1521 1616 1711 1806 1901 1996 2091 2186 2281 2376 2471 2566 2661 2756 2851 2946 3041 3136 3231 3326 3421 3516 3611 3706 3801 3896 3991 4086 4181 4276 4371 4466 4561 4656 4751 4846 4941 5036 5131
Time (Ticks)

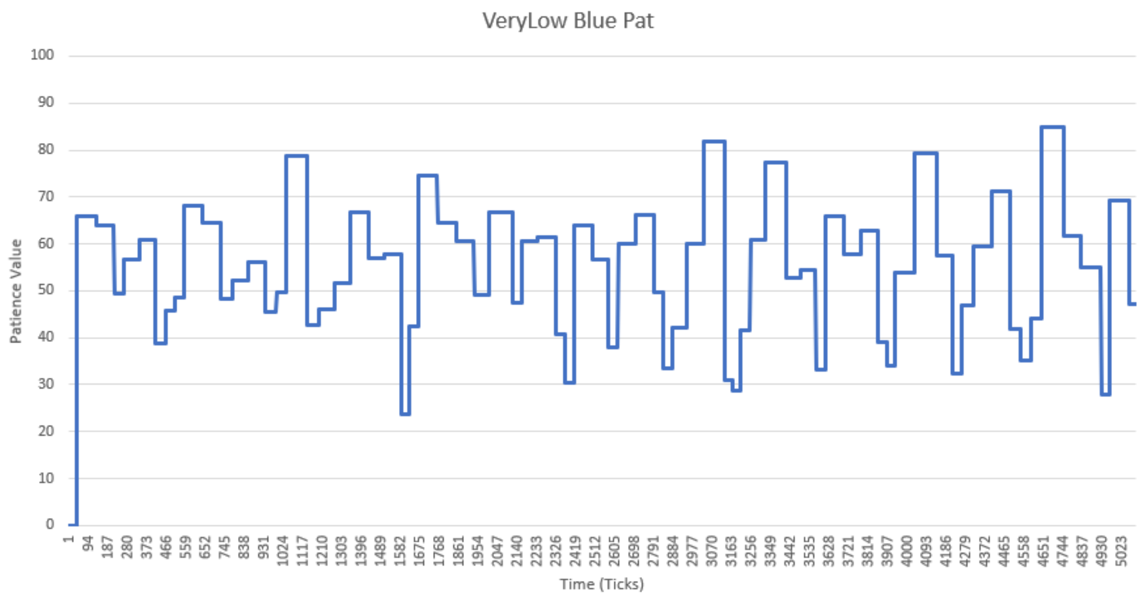

Dense DownR Wait
120
100
80
60
40
20
0
Wait-Time Value
1 98 195 292 389 486 583 680 777 874 971 1068 1165 1262 1359 1456 1553 1650 1747 1844 1941 2038 2135 2232 2329 2426 2523 2620 2717 2814 2911 3008 3105 3202 3299 3396 3493 3590 3687 3784 3881 3978 4075 4172 4269 4366 4463 4560 4657 4754 4851 4948 5045 5142 5239
Time (Ticks)

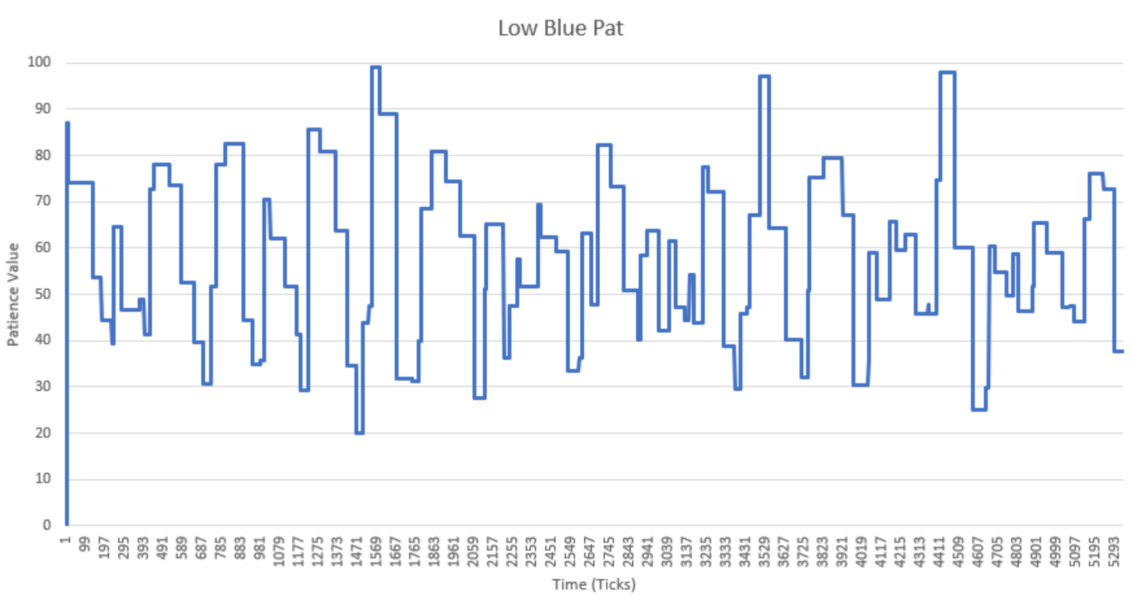

VeryDense DownR Wait
Wait-Time Value
Time (Ticks)

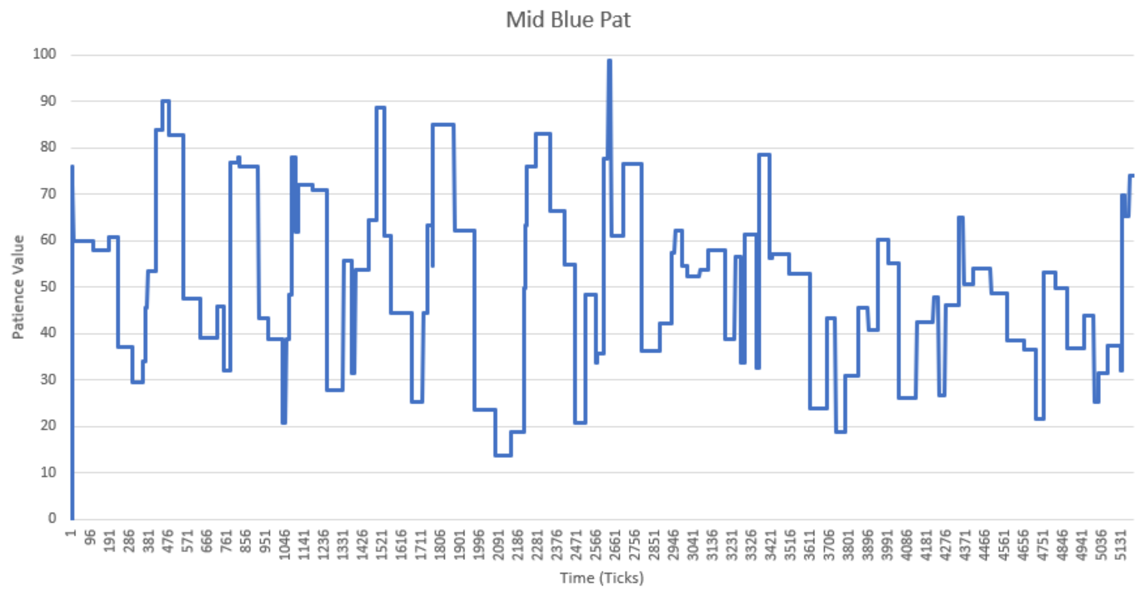
Appendix D: Patience Graphs

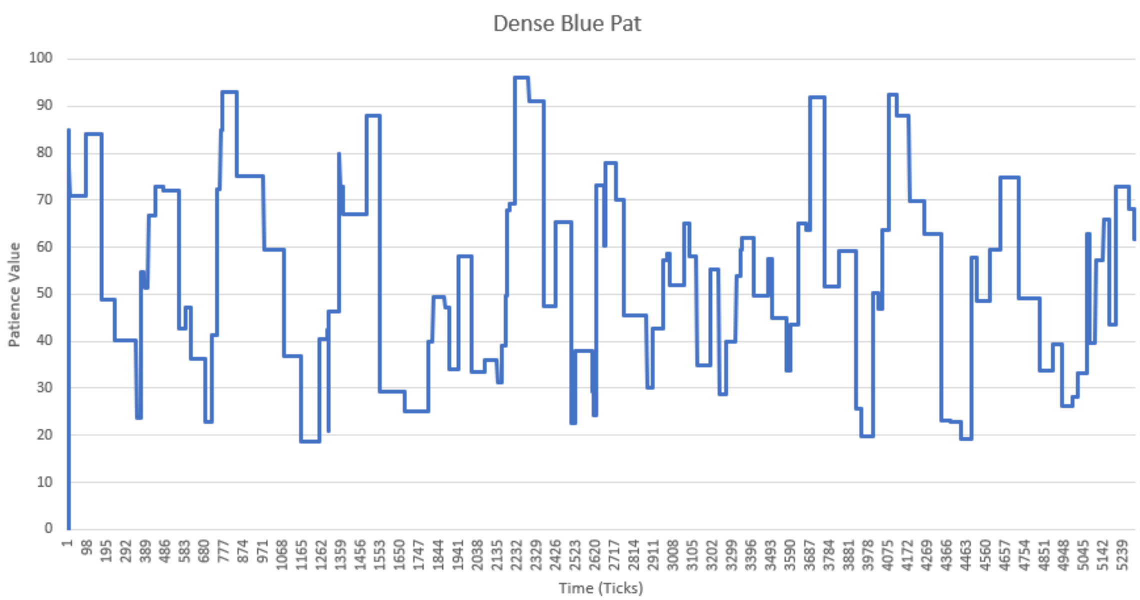
Figure 41 presents all the patience graphs of the blue car agents, Figure 42 presents all the patience graphs of the red car agents, and the pattern repeats until all other patience graphs are presented. Each graph is labeled with what test the data originates from.

Figure 41. Blue Car Patience Graphs






Mid Blue Pat
Patience Value
Time (Ticks)


Dense Blue Pat
Patience Value
Time (Ticks)

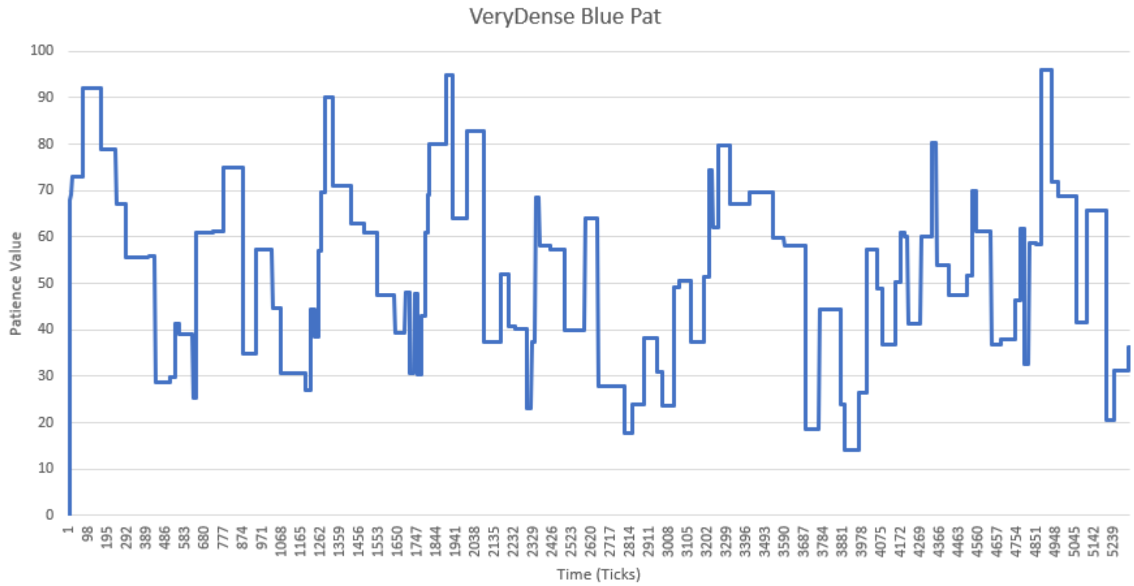
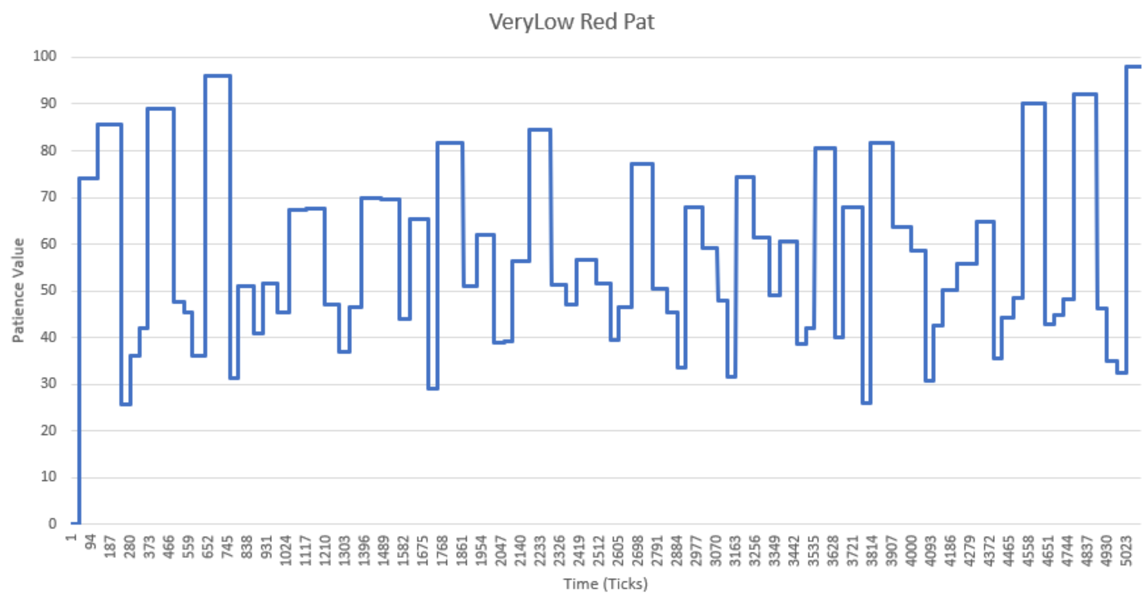
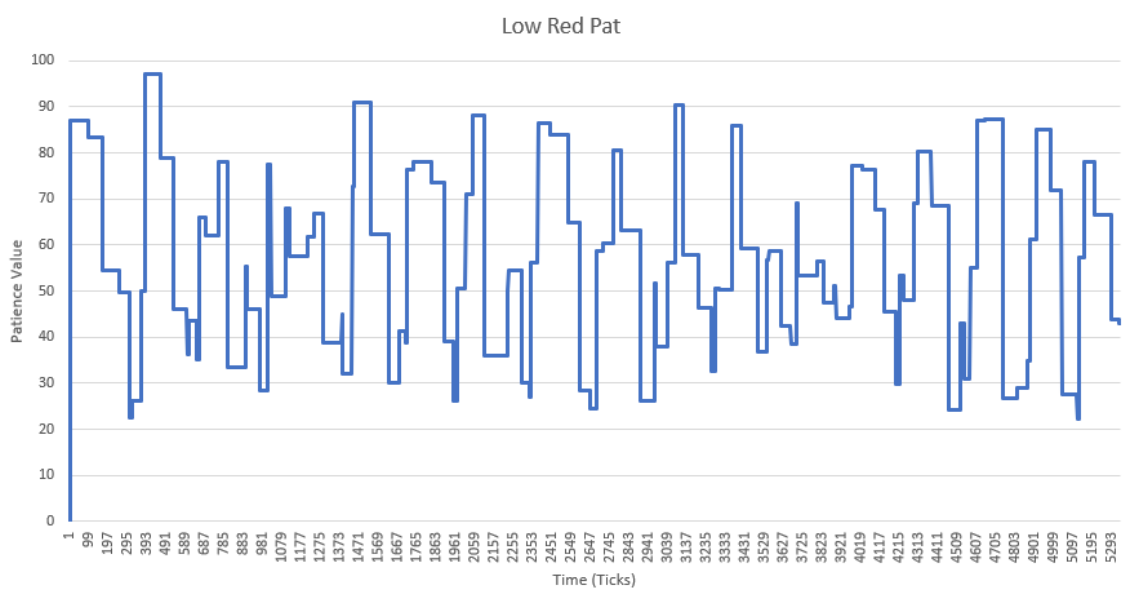
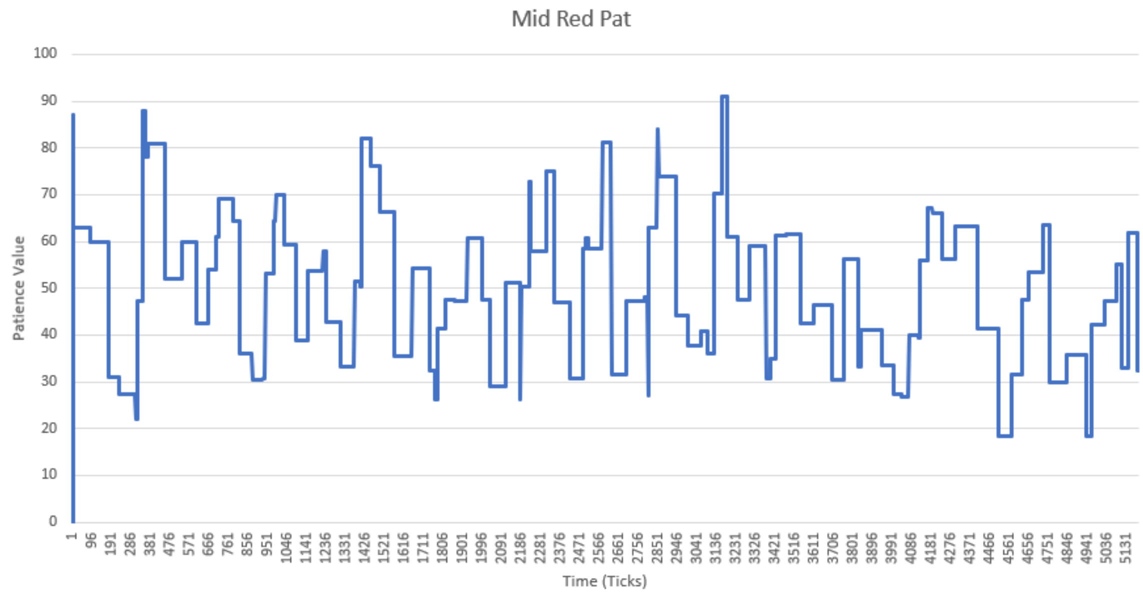
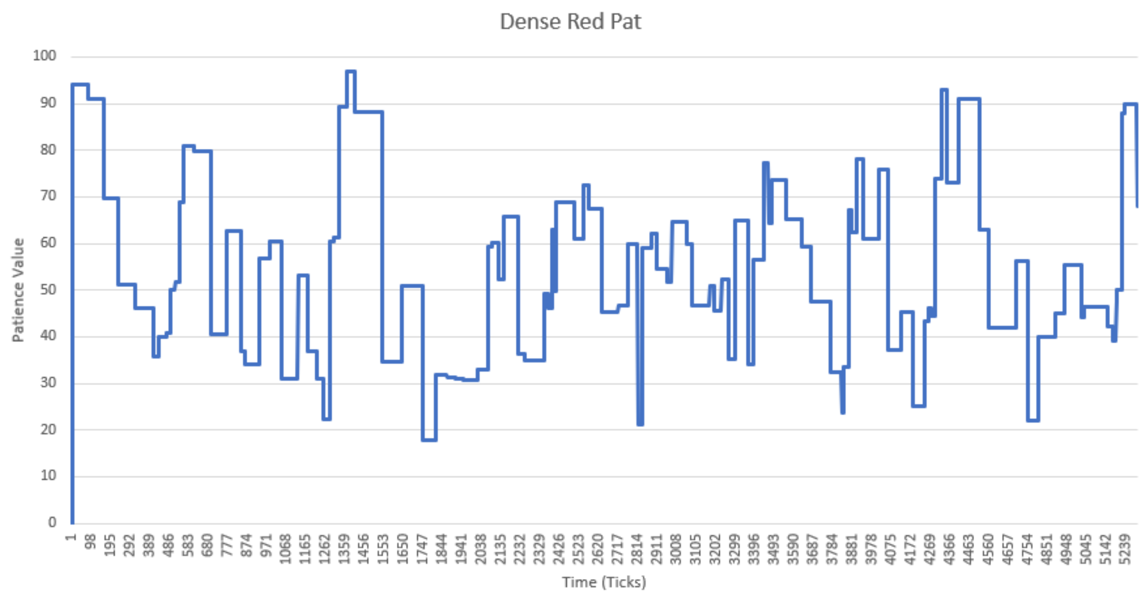

VeryDense Blue Pat
Patience Value
Time (Ticks)
0
10
20
30
40
50
60
70
80
90
100
1
98
195
292
389
486
583
680
777
874
971
1068
1165
1262
1359
1456
1553
1650
1747
1844
1941
2038
2135
2232
2329
2426
2523
2620
2717
2814
2911
3008
3105
3202
3299
3396
3493
3590
3687
3784
3881
3978
4075
4172
4269
4366
4463
4560
4657
4754
4851
4948
5045
5142
5239

Figure 42. Red Car Patience Graphs






Mid Red Pat
Patience Value
Time (Ticks)


Dense Red Pat
Patience Value
Time (Ticks)

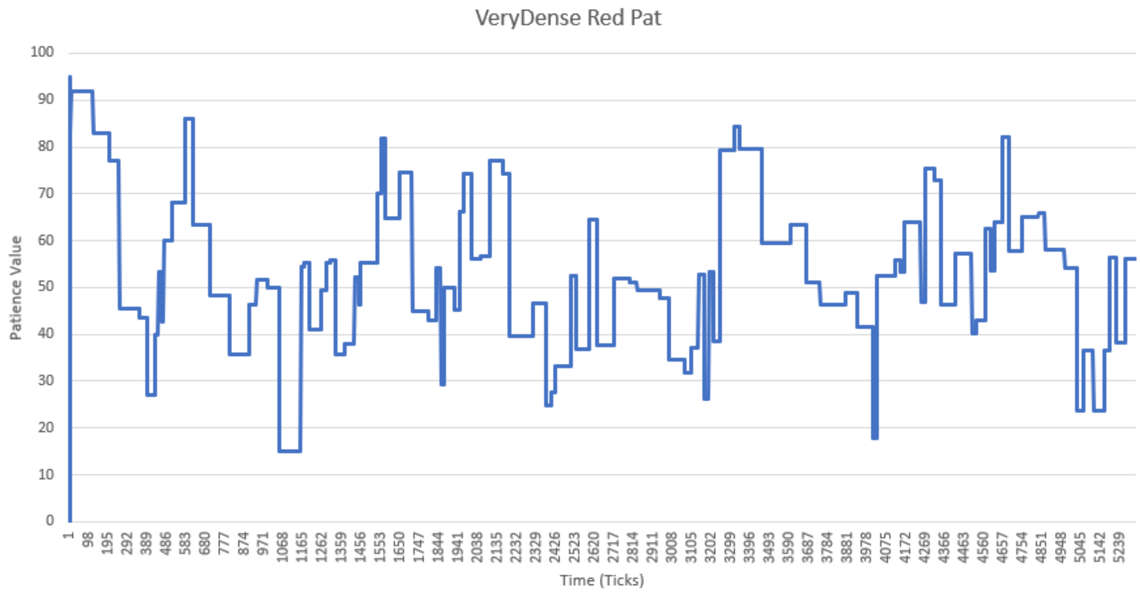
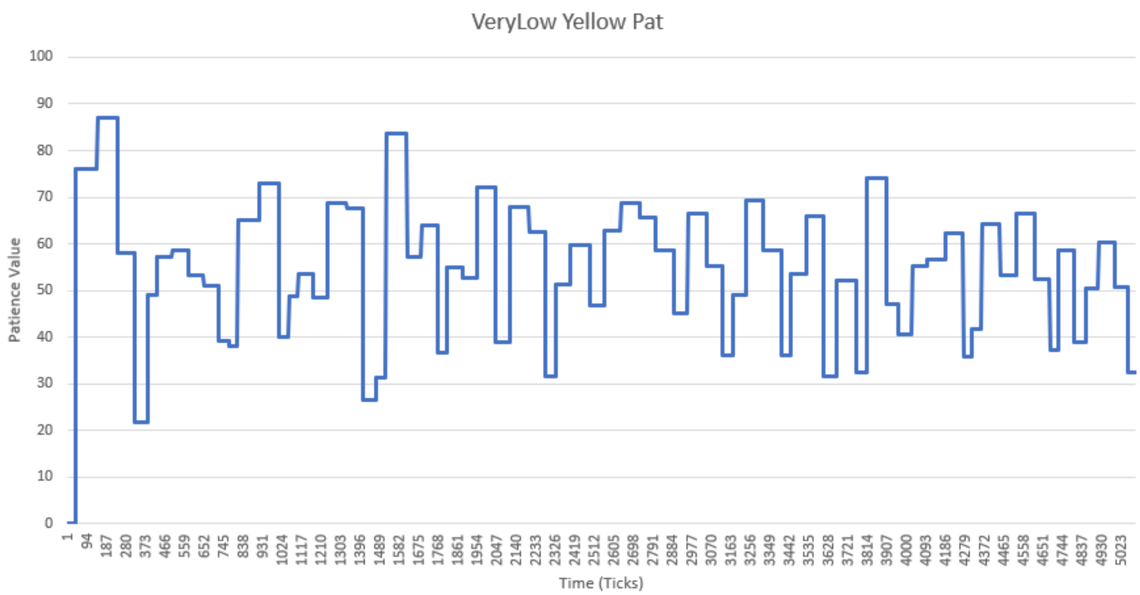
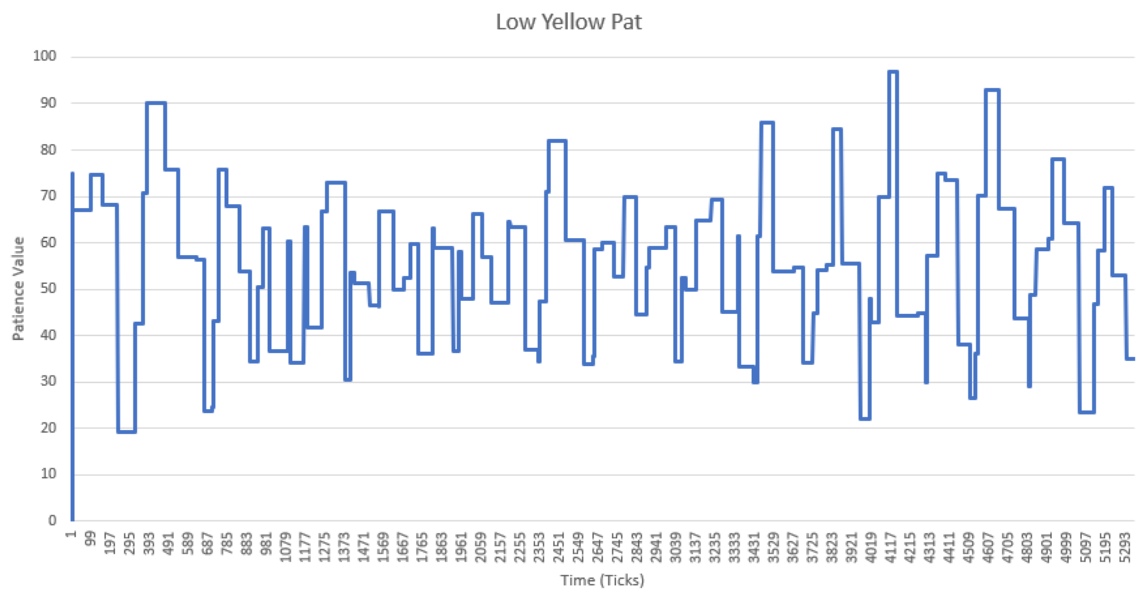
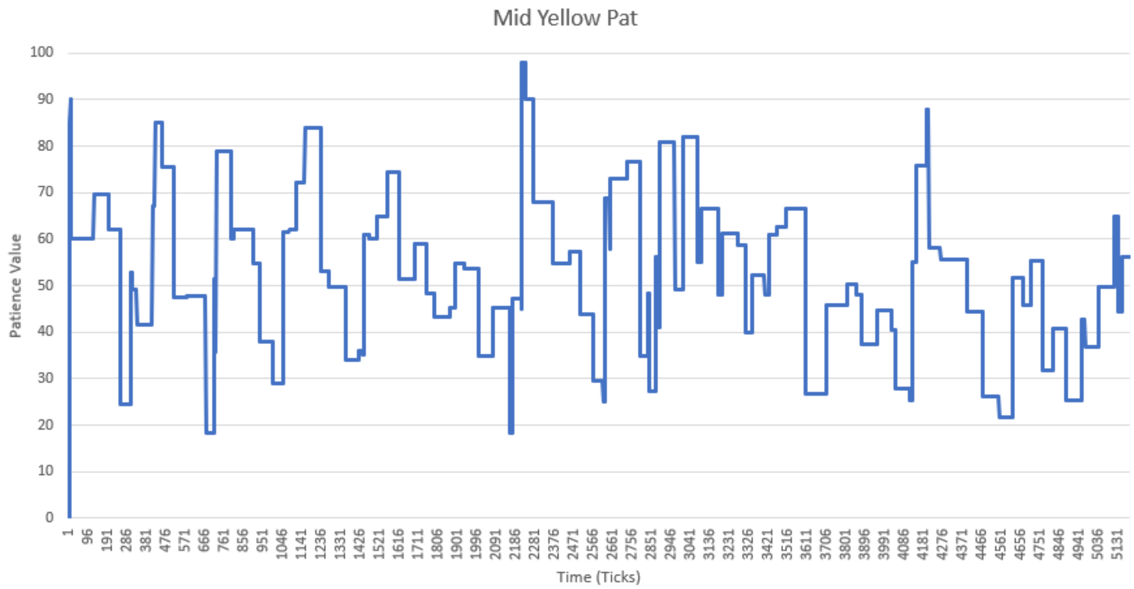
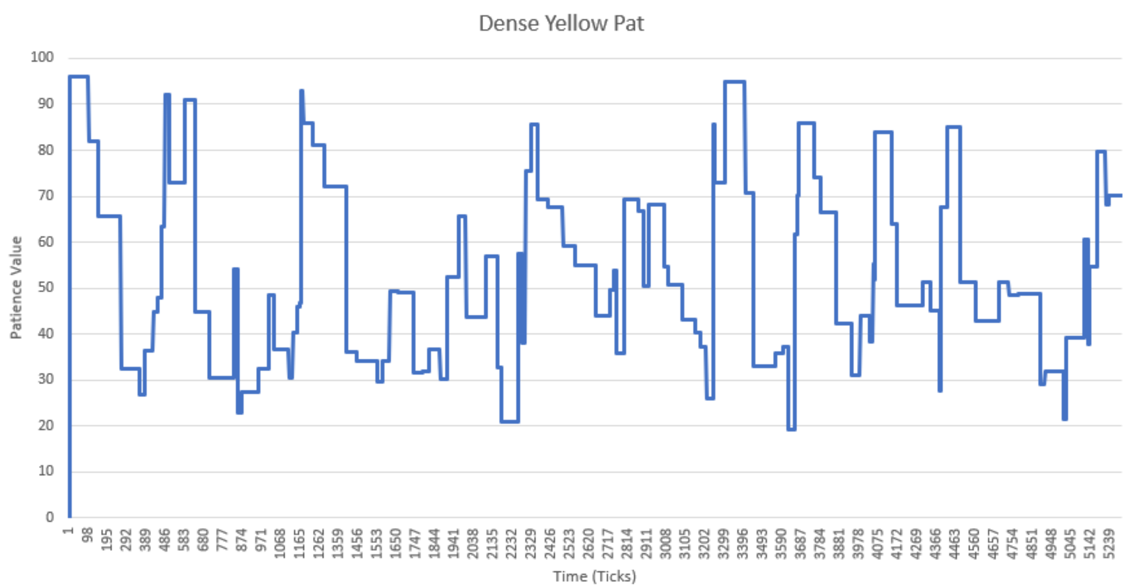

VeryDense Red Pat
Patience Value
Time (Ticks)
100
90
80
70
60
50
40
30
20
10
0
1
98
195
292
389
486
583
680
777
874
971
1068
1165
1262
1359
1456
1553
1650
1747
1844
1941
2038
2135
2232
2329
2426
2523
2620
2717
2814
2911
3008
3105
3202
3299
3396
3493
3590
3687
3784
3881
3978
4075
4172
4269
4366
4463
4560
4657
4754
4851
4948
5045
5142
5239

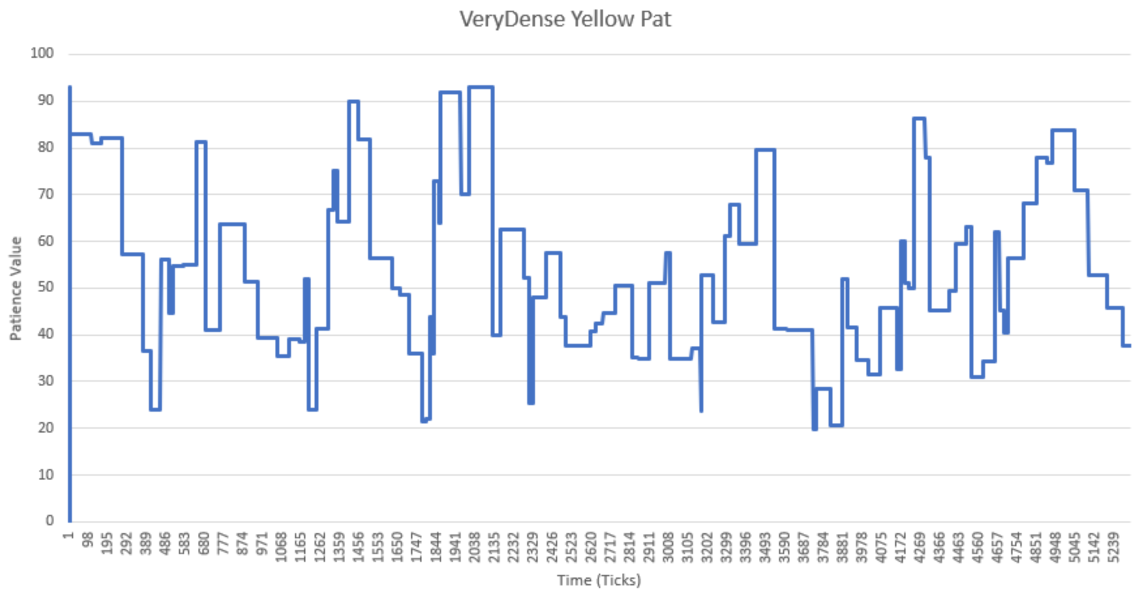
Figure 43. Yellow Car Patience Graphs





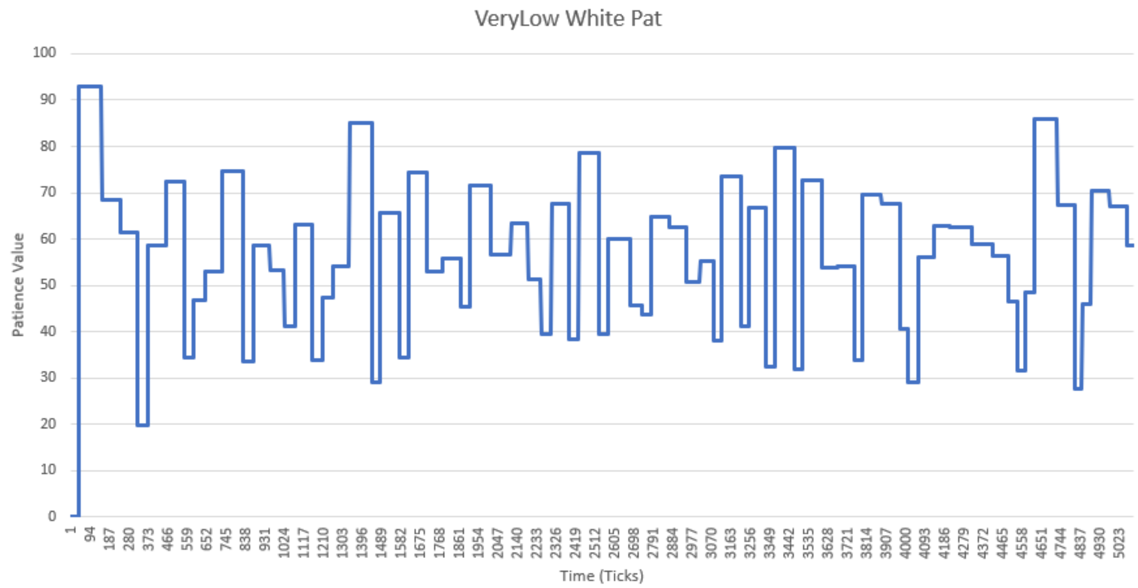

Mid Yellow Pat
100
90
80
70
60
50
40
30
20
10
0
Patience Value
Time (Ticks)

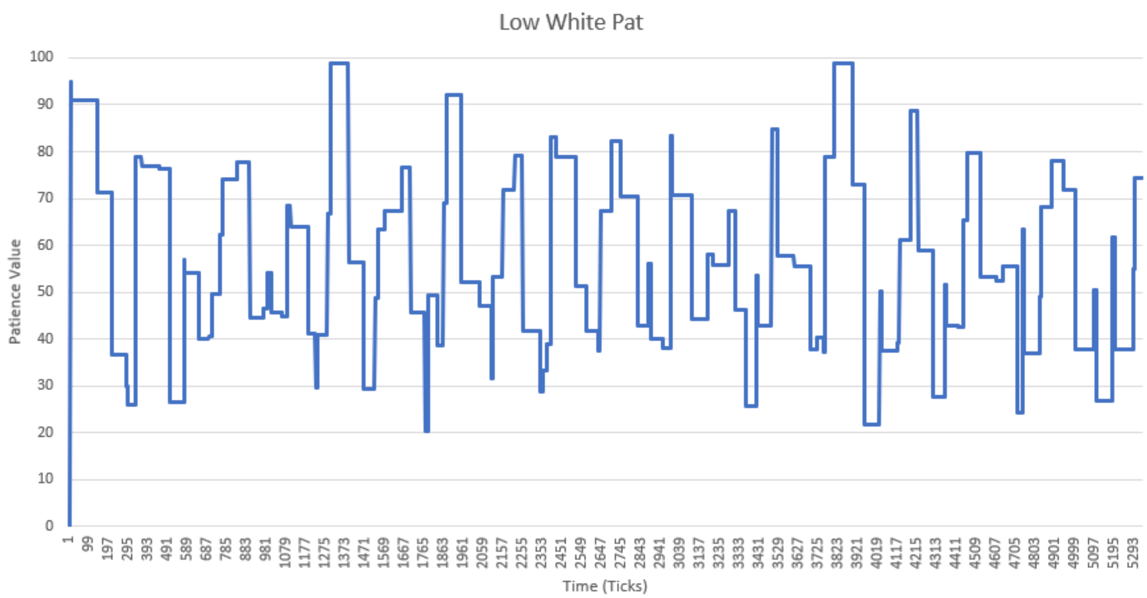

Dense Yellow Pat
100
90
80
70
60
50
40
30
20
10
0
Patience Value
Time (Ticks)

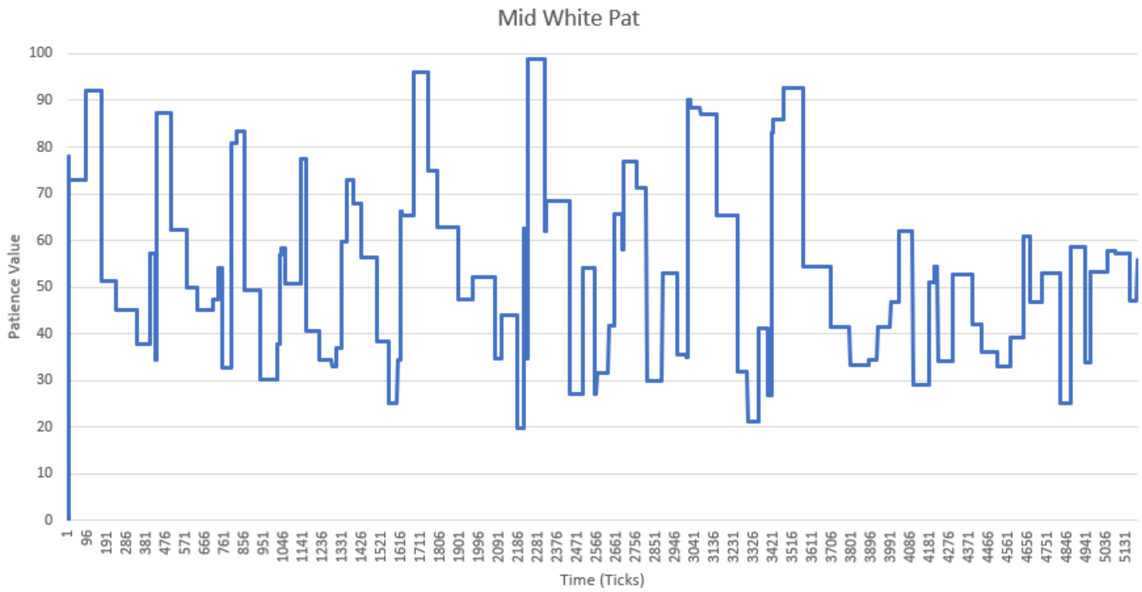


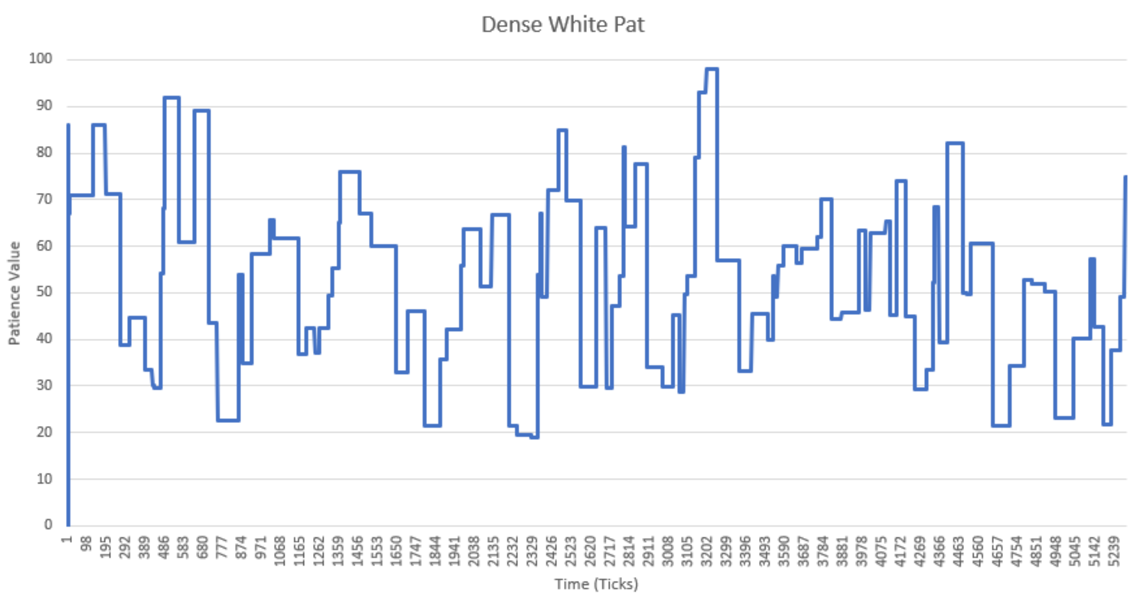
VeryDense Yellow Pat
Patience Value
Time (Ticks)
0
10
20
30
40
50
60
70
80
90
100
1
98
195
292
389
486
583
680
777
874
971
1068
1165
1262
1359
1456
1553
1650
1747
1844
1941
2038
2135
2232
2329
2426
2523
2620
2717
2814
2911
3008
3105
3202
3299
3396
3493
3590
3687
3784
3881
3978
4075
4172
4269
4366
4463
4560
4657
4754
4851
4948
5045
5142
5239

Figure 44. White Car Patience Graphs






Mid White Pat
Patience Value
Time (Ticks)


Dense White Pat
Patience Value
Time (Ticks)

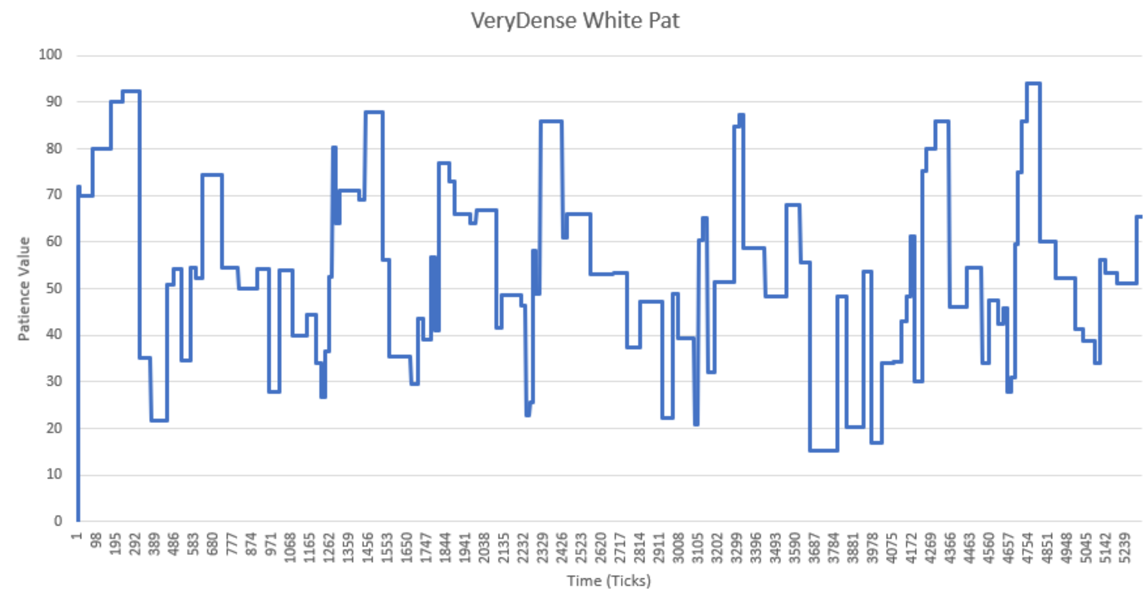
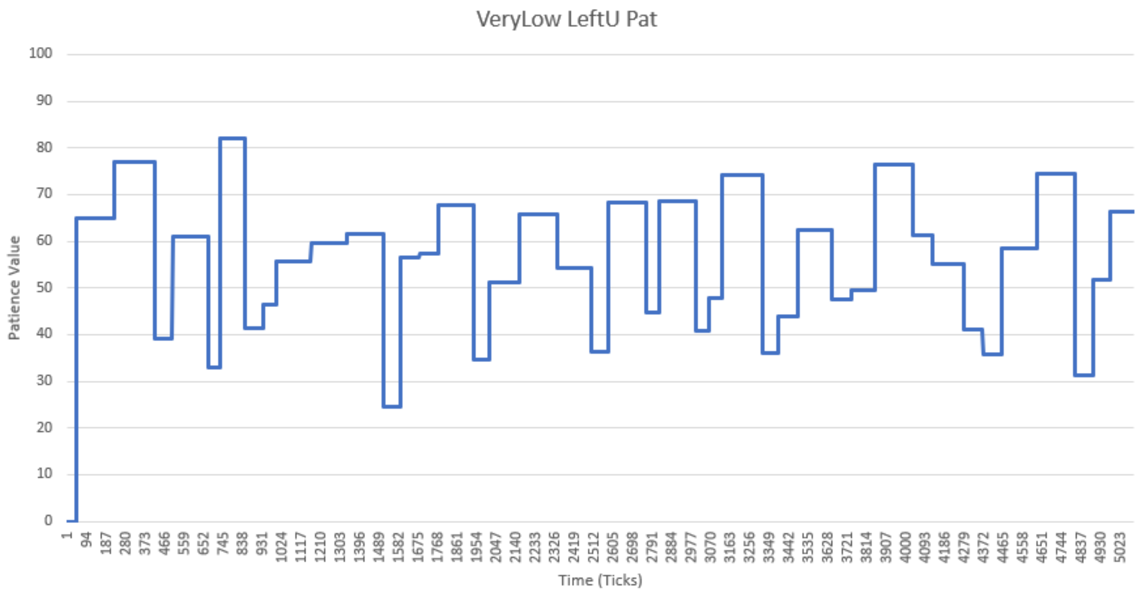
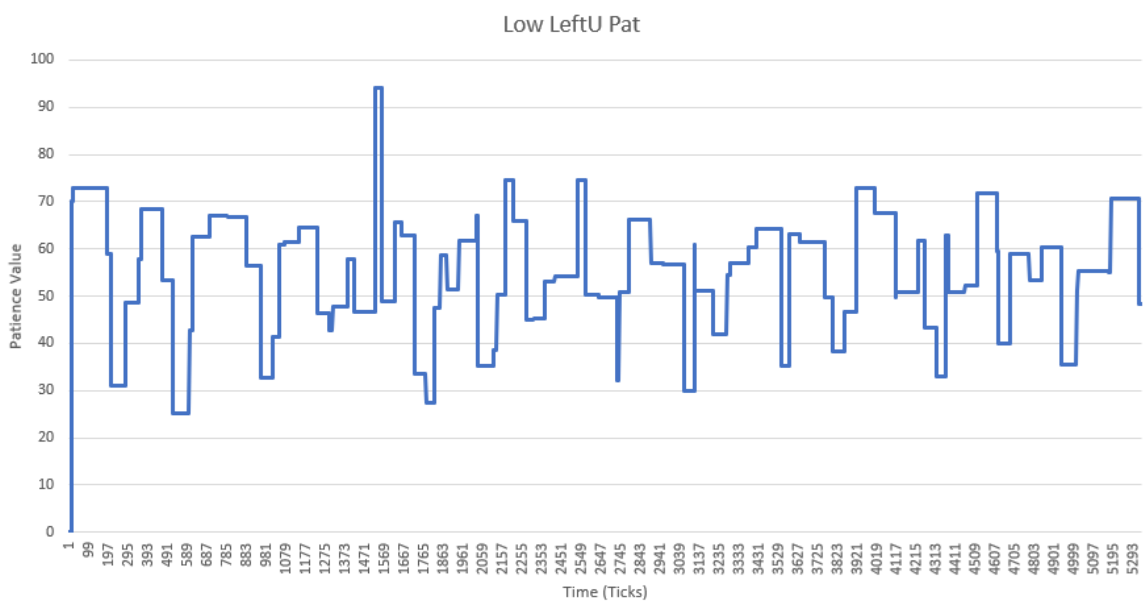
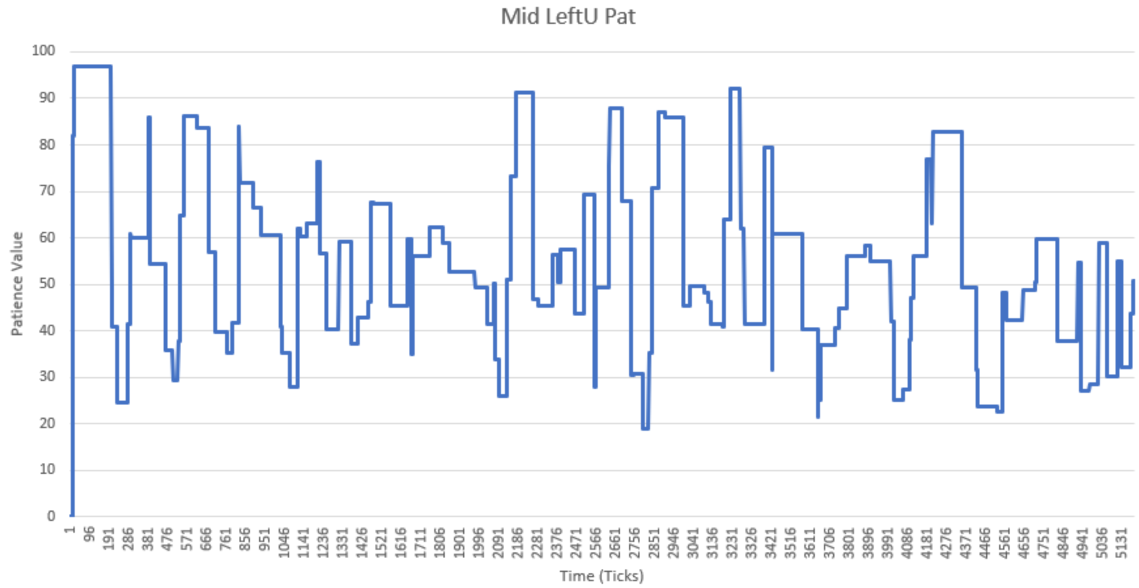
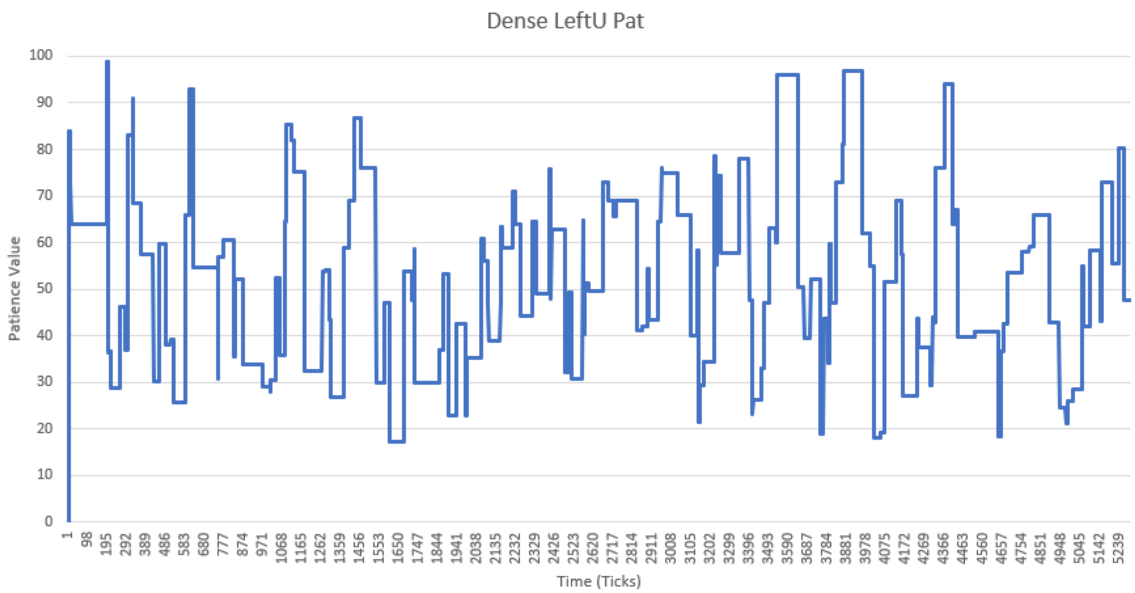

VeryDense White Pat
Patience Value
Time (Ticks)
100
90
80
70
60
50
40
30
20
10
0
1
98
195
292
389
486
583
680
777
874
971
1068
1165
1262
1359
1456
1553
1650
1747
1844
1941
2038
2135
2232
2329
2426
2523
2620
2717
2814
2911
3008
3105
3202
3299
3396
3493
3590
3687
3784
3881
3978
4075
4172
4269
4366
4463
4560
4657
4754
4851
4948
5045
5142
5239

Figure 45. Left (Upper) Pedestrian Patience Graphs






Mid LeftU Pat
Patience Value
Time (Ticks)


Dense LeftU Pat
Patience Value
Time (Ticks)

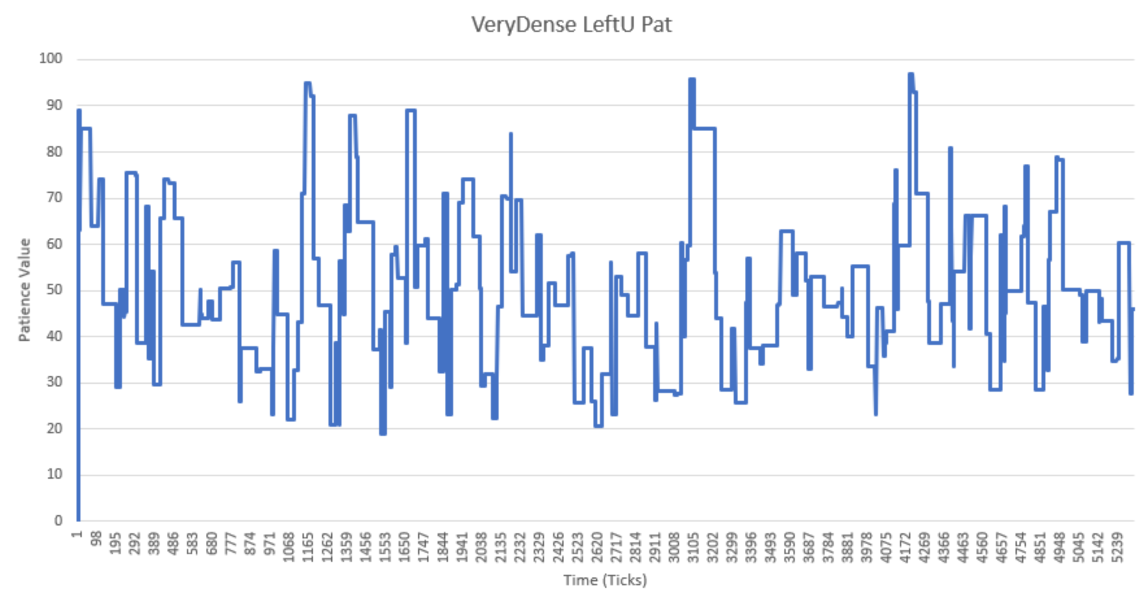
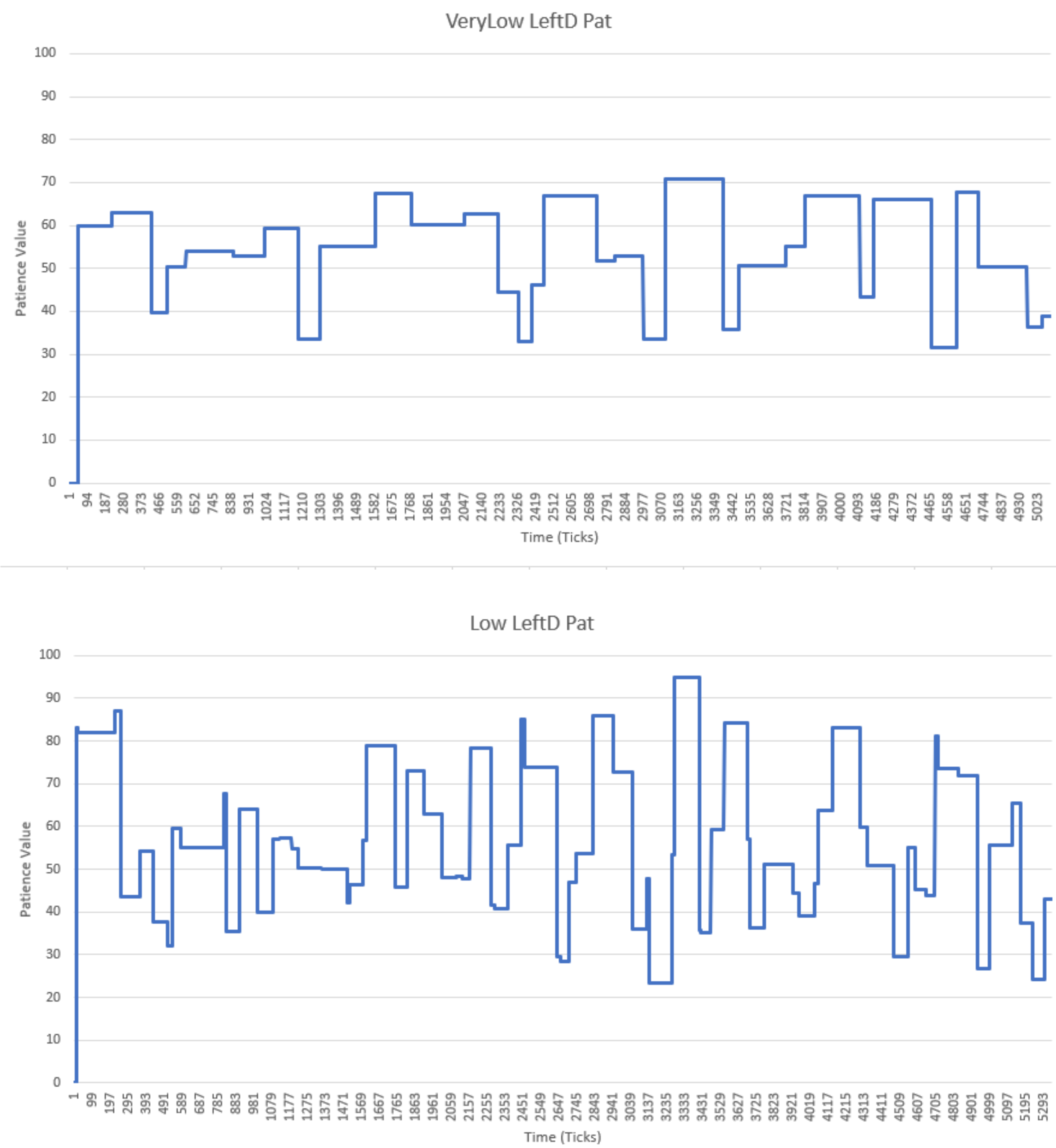
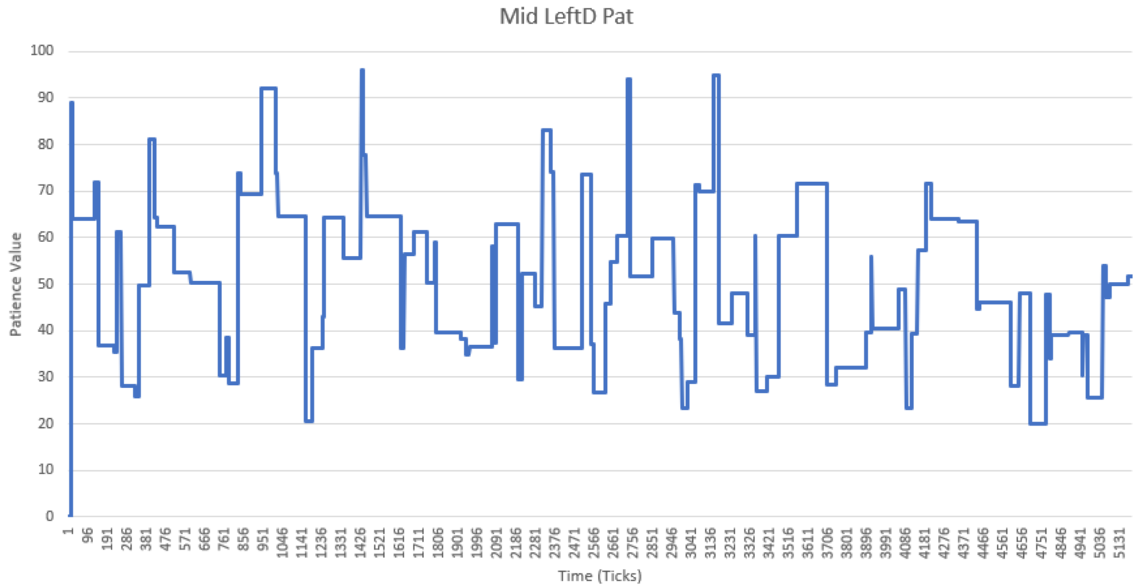
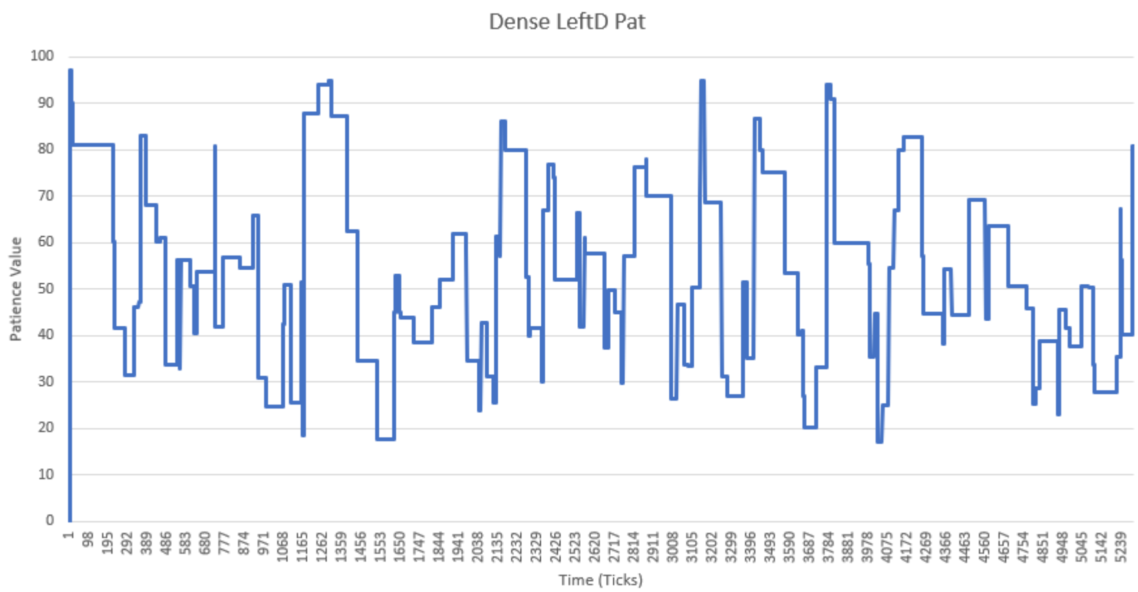

VeryDense LeftU Pat
Patience Value
0
10
20
30
40
50
60
70
80
90
100
1
98
195
292
389
486
583
680
777
874
971
1068
1165
1262
1359
1456
1553
1650
1747
1844
1941
2038
2135
2232
2329
2426
2523
2620
2717
2814
2911
3008
3105
3202
3299
3396
3493
3590
3687
3784
3881
3978
4075
4172
4269
4366
4463
4560
4657
4754
4851
4948
5045
5142
5239
Time (Ticks)

Figure 46. Left (Down) Pedestrian Patience Graphs




Mid LeftD Pat
Patience Value
Time (Ticks)


Dense LeftD Pat
Patience Value
Time (Ticks)

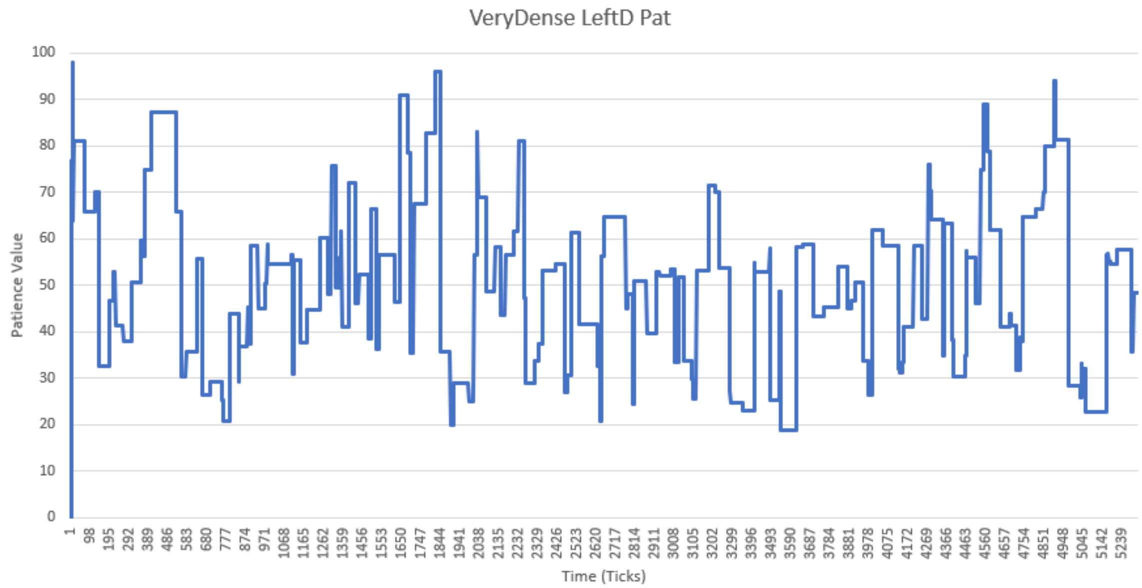
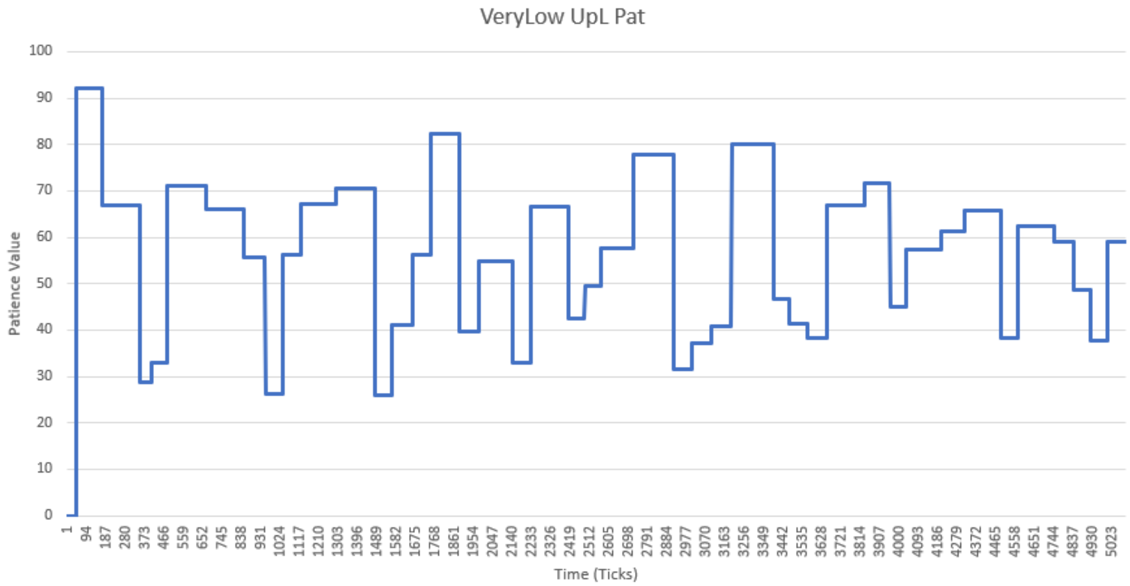
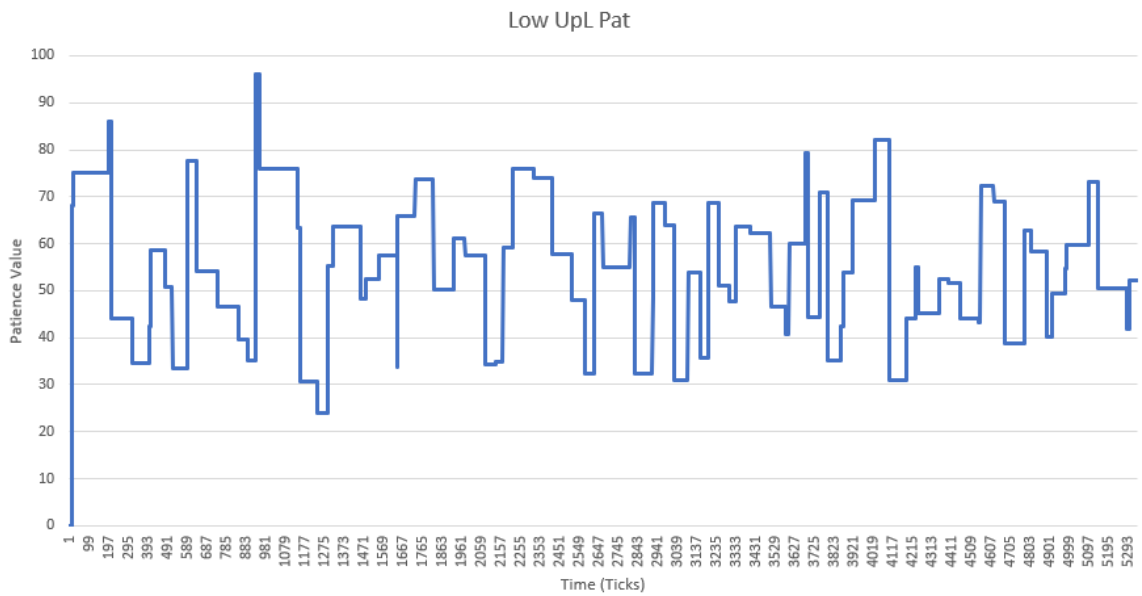
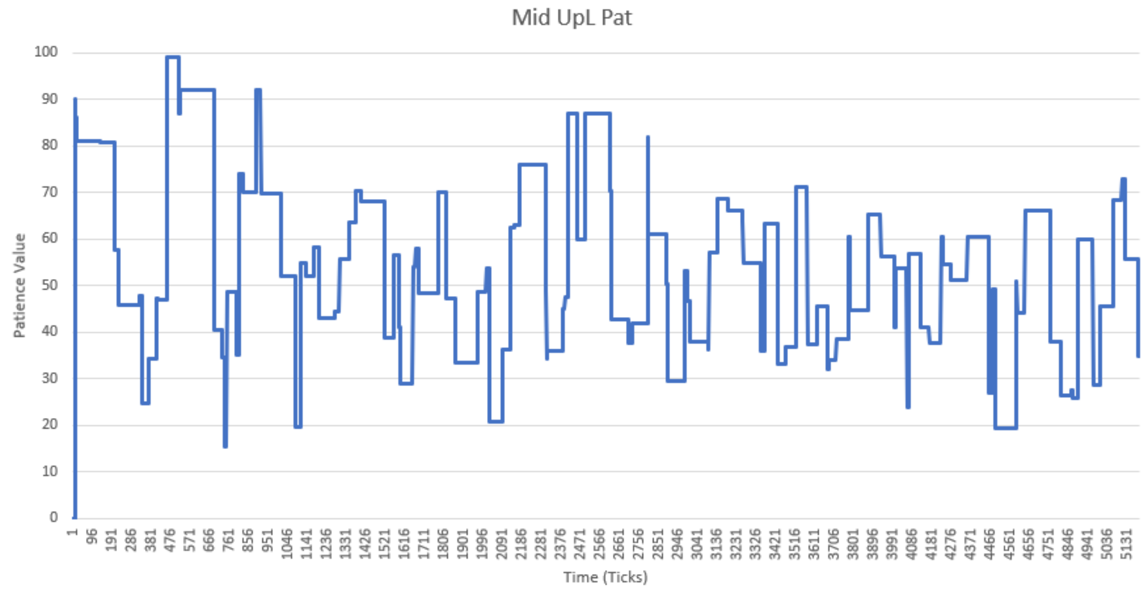
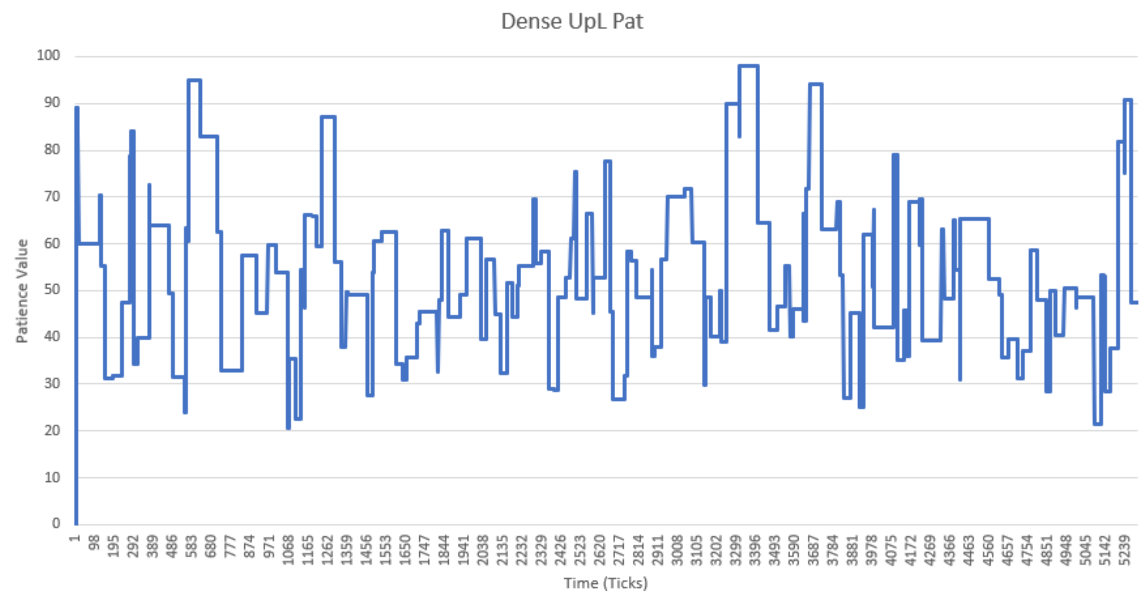

VeryDense LeftD Pat
Patience Value
Time (Ticks)
100
90
80
70
60
50
40
30
20
10
0
1
98
195
292
389
486
583
680
777
874
971
1068
1165
1262
1359
1456
1553
1650
1747
1844
1941
2038
2135
2232
2329
2426
2523
2620
2717
2814
2911
3008
3105
3202
3299
3396
3493
3590
3687
3784
3881
3978
4075
4172
4269
4366
4463
4560
4657
4754
4851
4948
5045
5142
5239

Figure 47. Up (Left) Pedestrian Patience Graphs






Mid UpL Pat
100
90
80
70
60
50
40
30
20
10
0
Patience Value
1 96 191 286 381 476 571 666 761 856 951 1046 1141 1236 1331 1426 1521 1616 1711 1806 1901 1996 2091 2186 2281 2376 2471 2566 2661 2756 2851 2946 3041 3136 3231 3326 3421 3516 3611 3706 3801 3896 3991 4086 4181 4276 4371 4466 4561 4656 4751 4846 4941 5036 5131
Time (Ticks)


Dense UpL Pat
100
90
80
70
60
50
40
30
20
10
0
Patience Value
1 98 195 292 389 486 583 680 777 874 971 1068 1165 1262 1359 1456 1553 1650 1747 1844 1941 2038 2135 2232 2329 2426 2523 2620 2717 2814 2911 3008 3105 3202 3299 3396 3493 3590 3687 3784 3881 3978 4075 4172 4269 4366 4463 4560 4657 4754 4851 4948 5045 5142 5239
Time (Ticks)

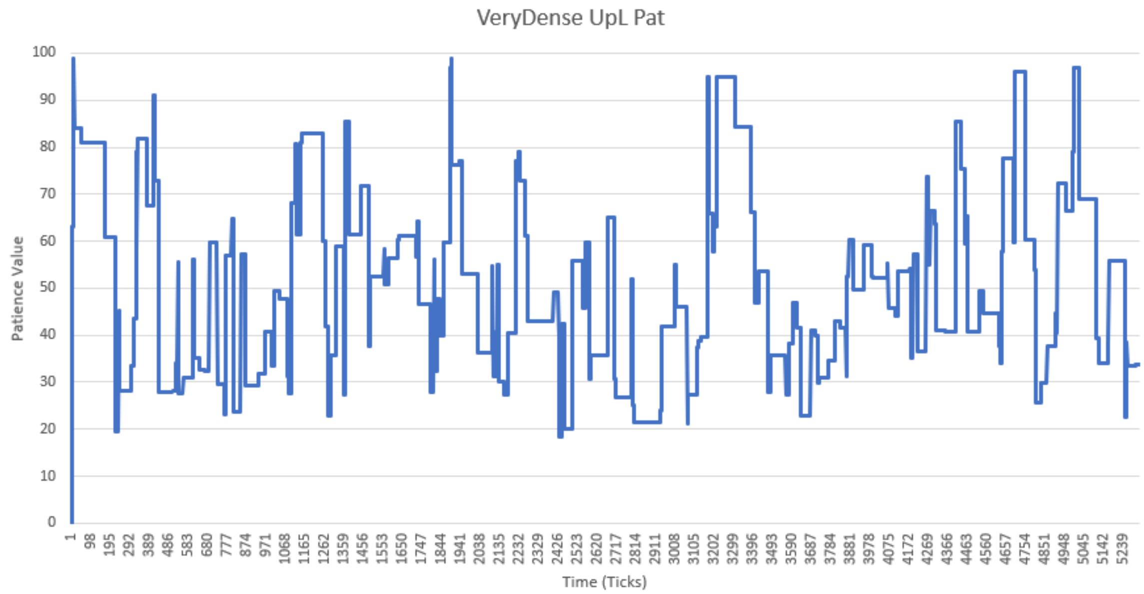
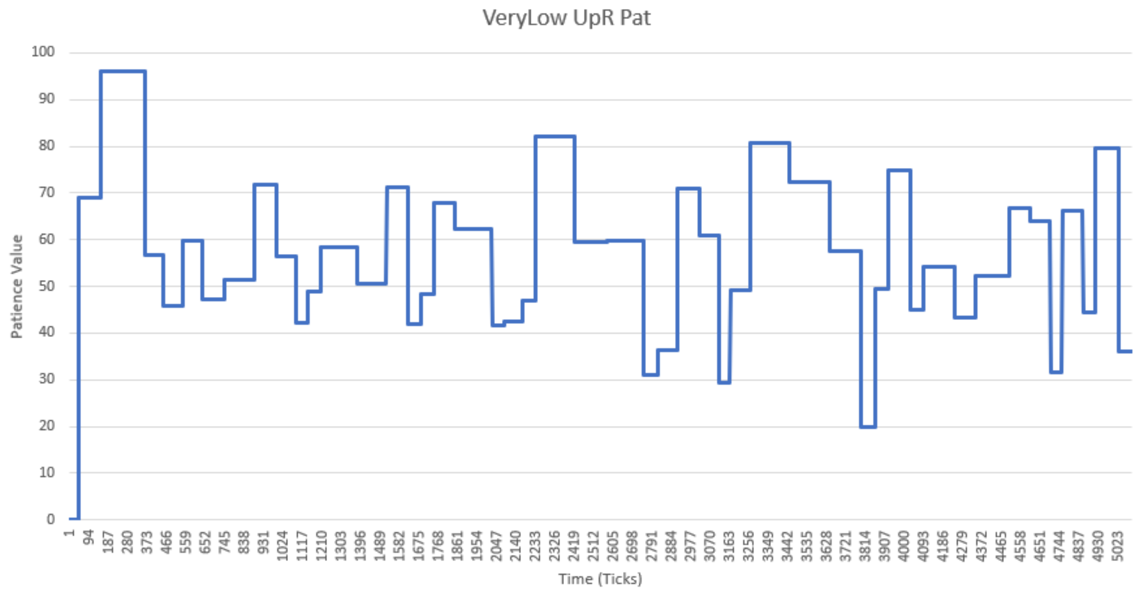
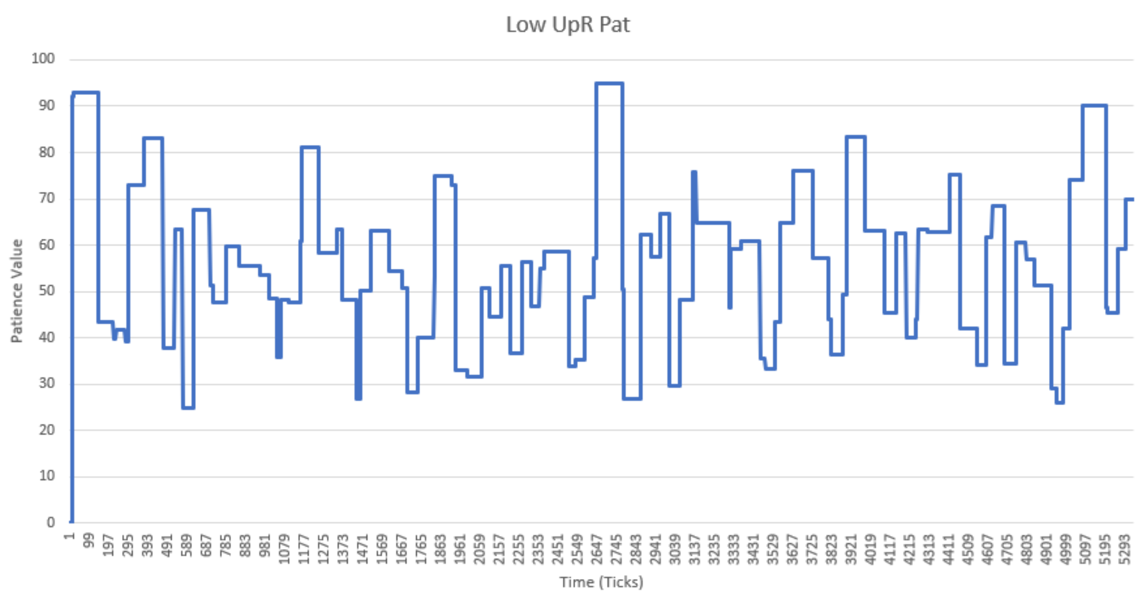
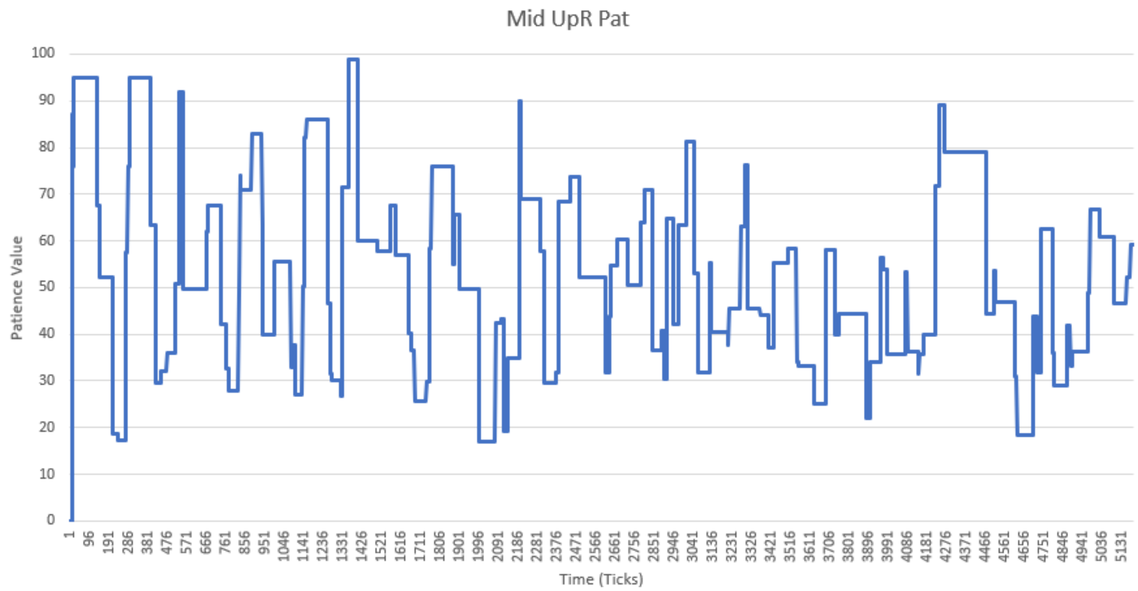
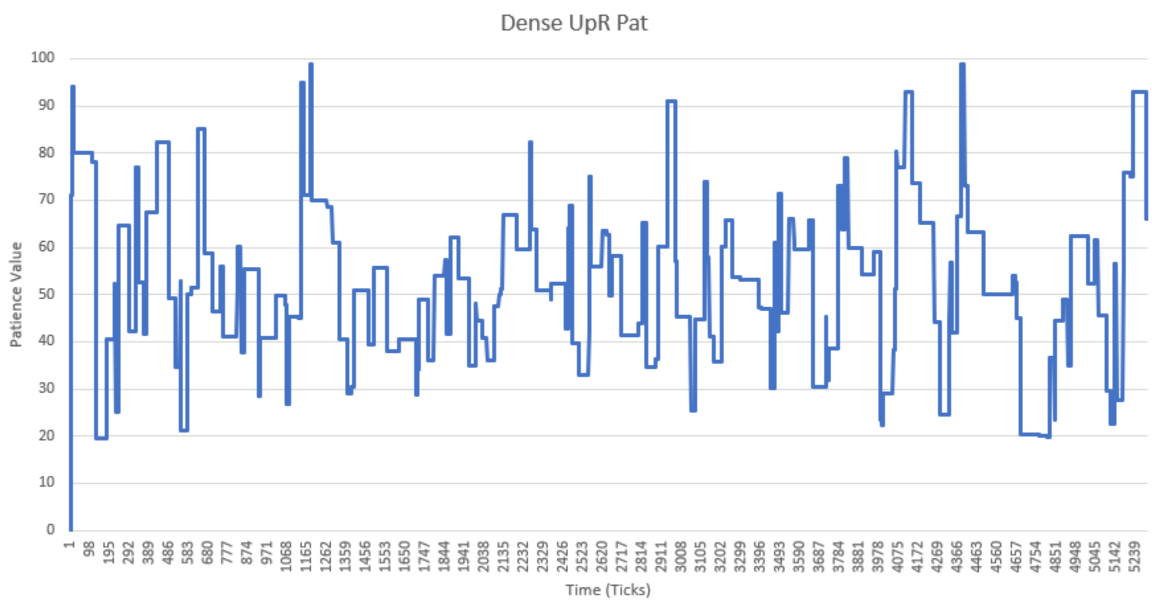

VeryDense UpL Pat
Patience Value
Time (Ticks)
100
90
80
70
60
50
40
30
20
10
0

Figure 48. Up (Right) Pedestrian Patience Graphs






Mid UpR Pat
Patience Value
Time (Ticks)


Dense UpR Pat
Patience Value
Time (Ticks)

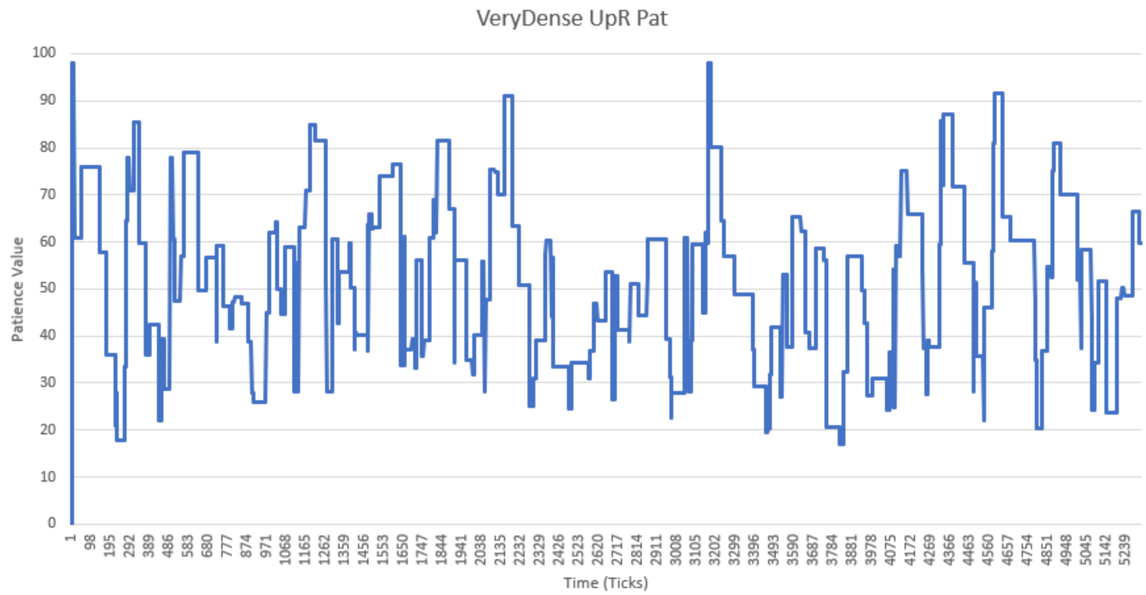
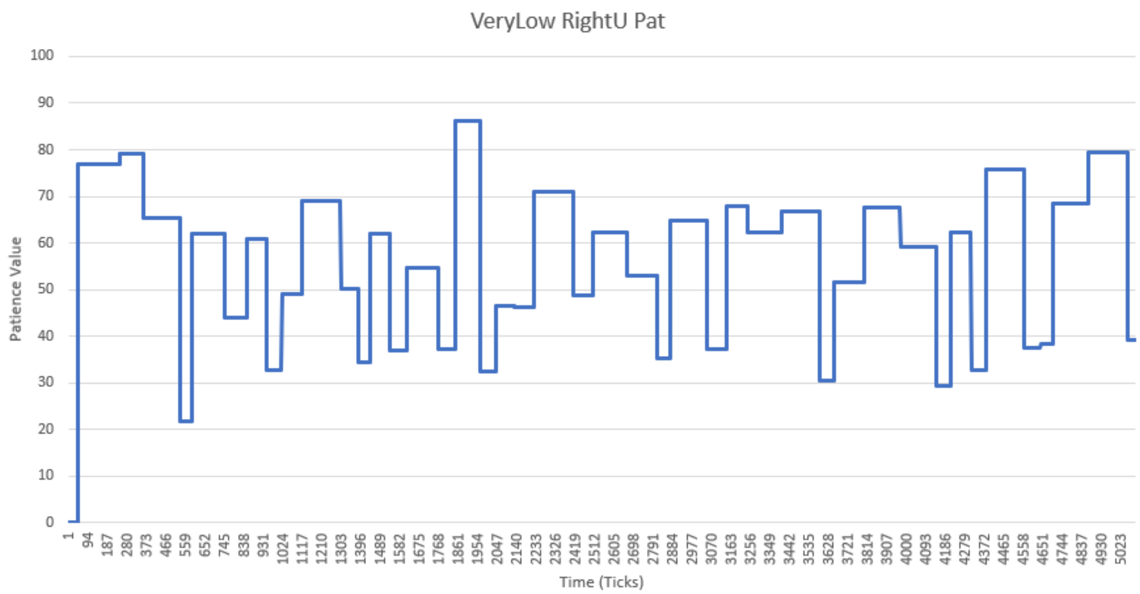
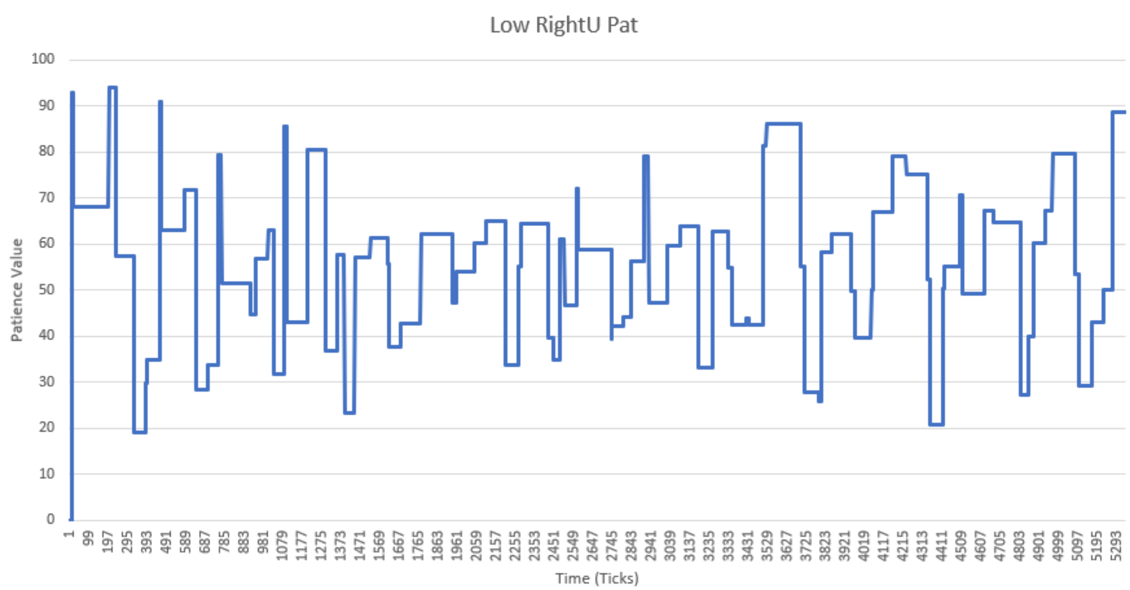
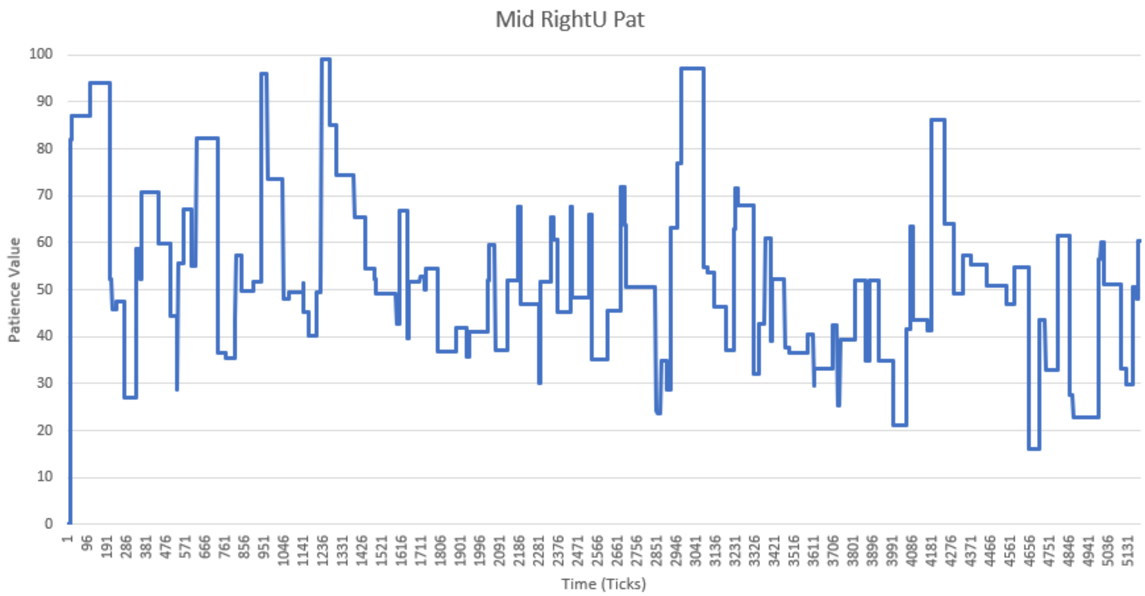
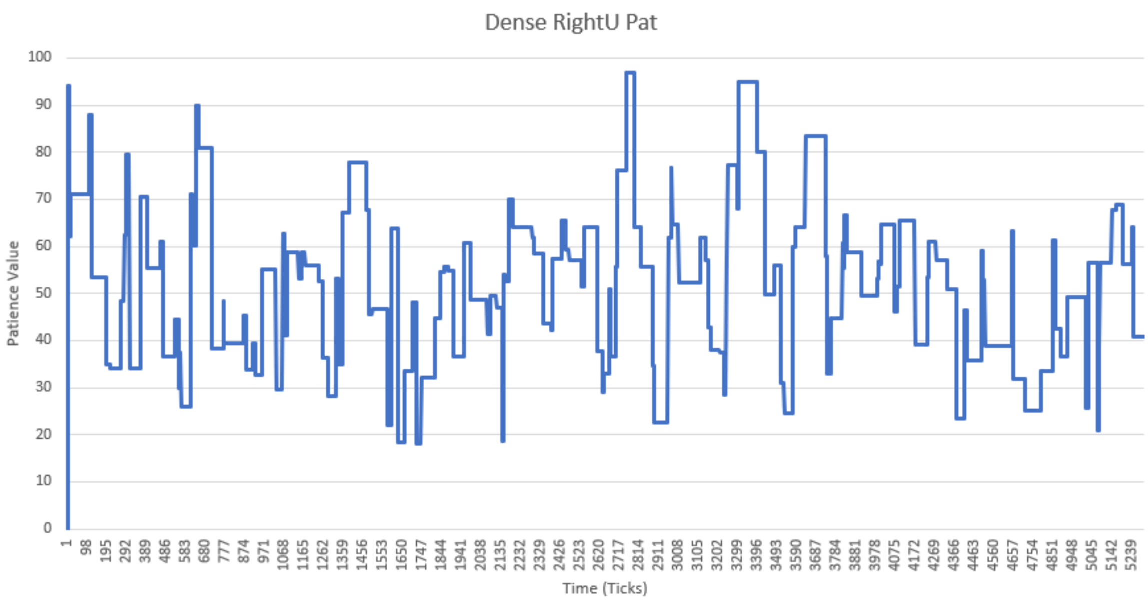

VeryDense UpR Pat
Patience Value
Time (Ticks)

Figure 49. Right (Up) Pedestrian Patience Graphs






Mid RightU Pat
Patience Value
Time (Ticks)


Dense RightU Pat
Patience Value
Time (Ticks)

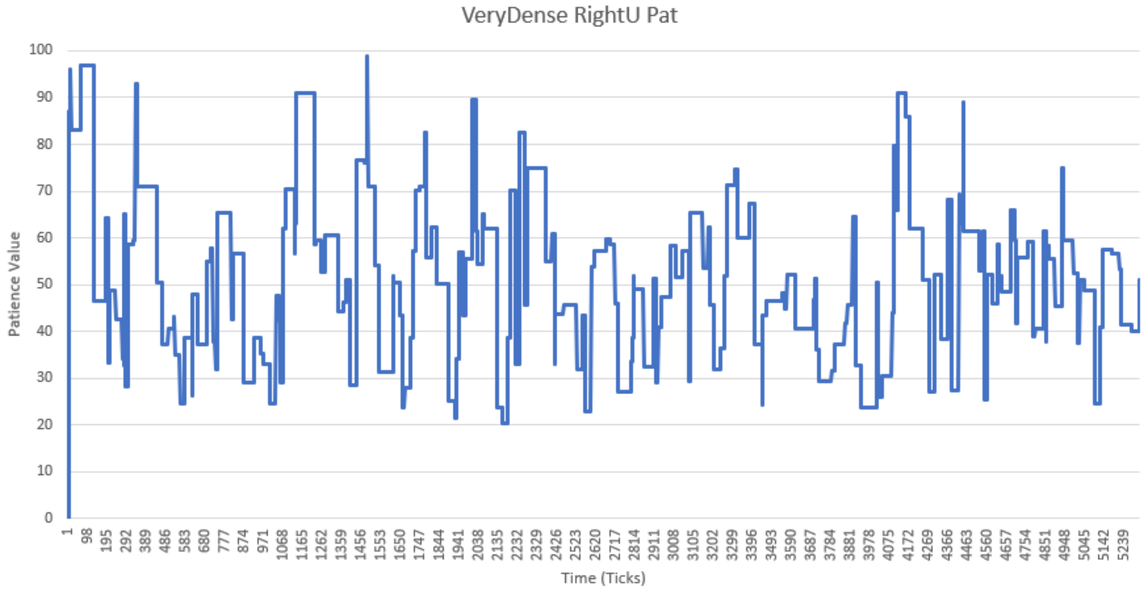
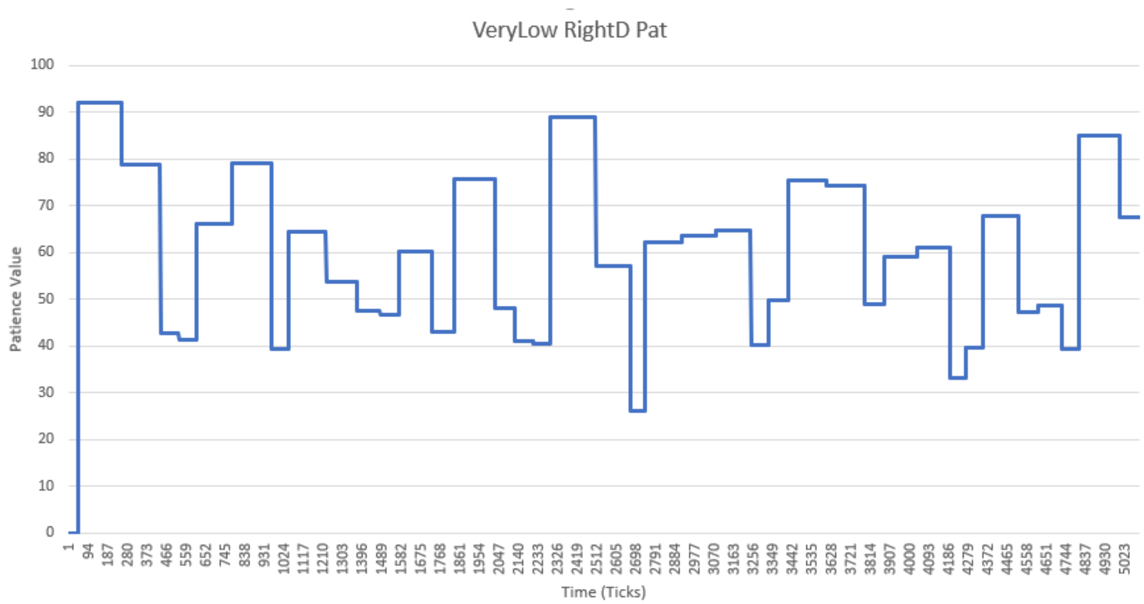
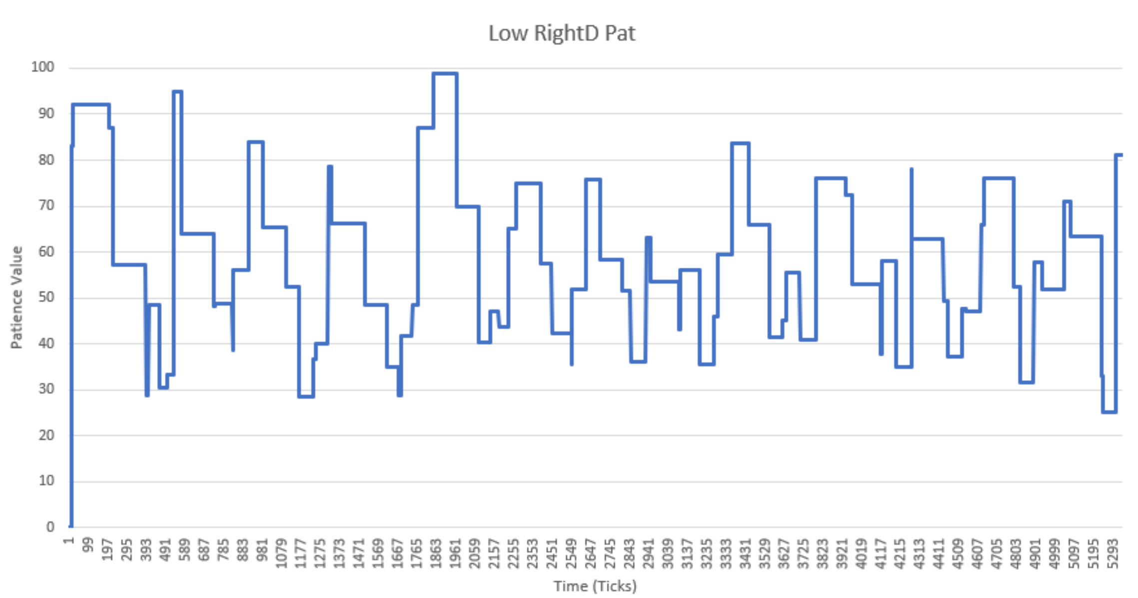
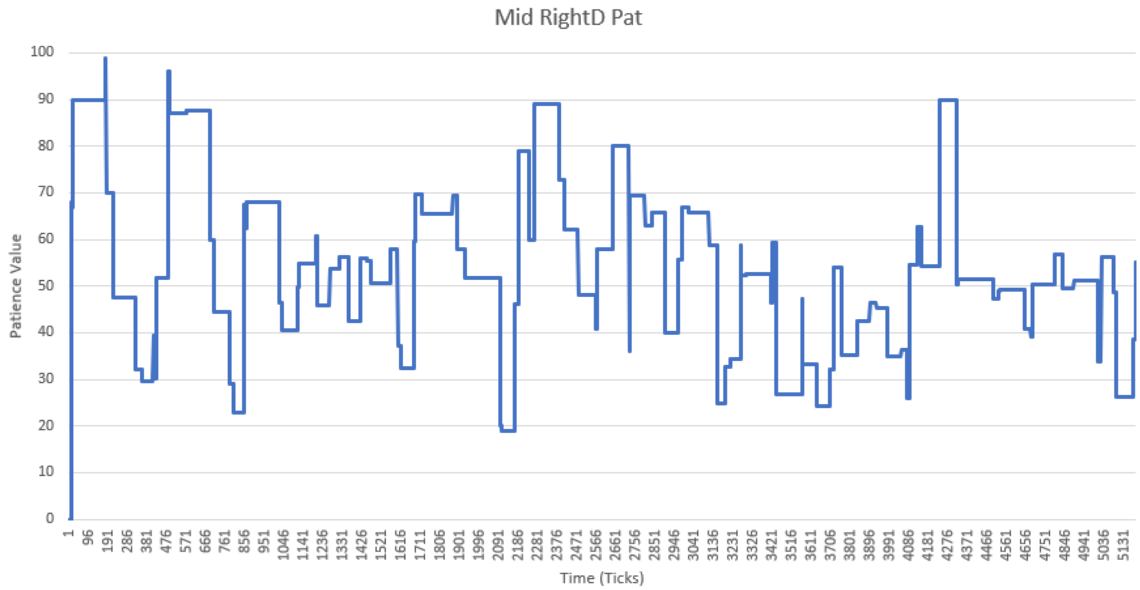
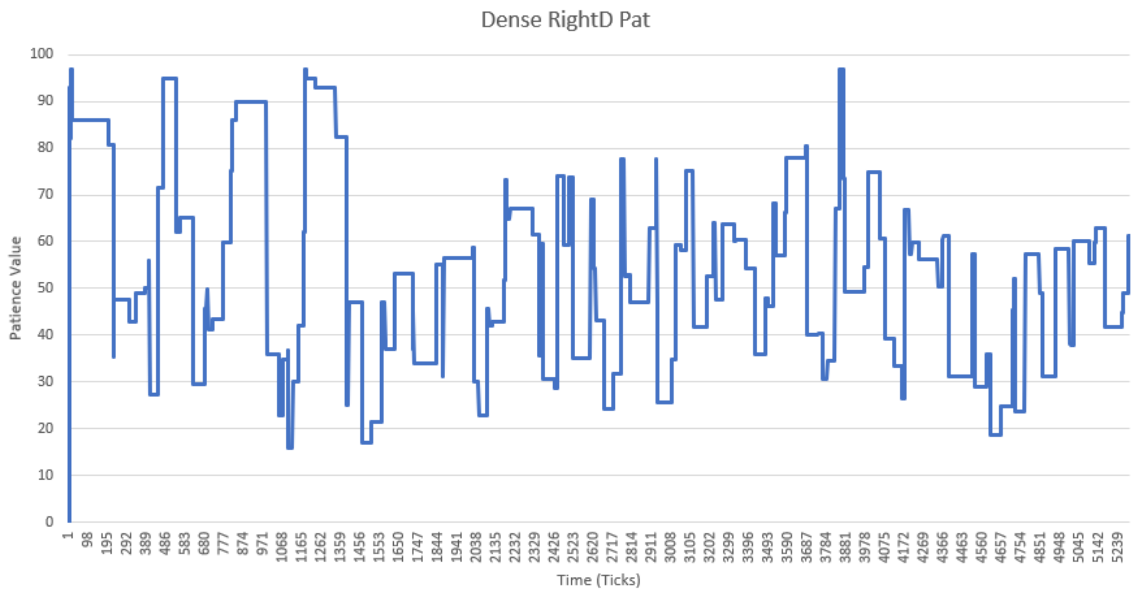

VeryDense RightU Pat
Patience Value
Time (Ticks)

Figure 50. Right (Down) Pedestrian Patience Graphs






Mid RightD Pat
Patience Value
Time (Ticks)


Dense RightD Pat
Patience Value
Time (Ticks)

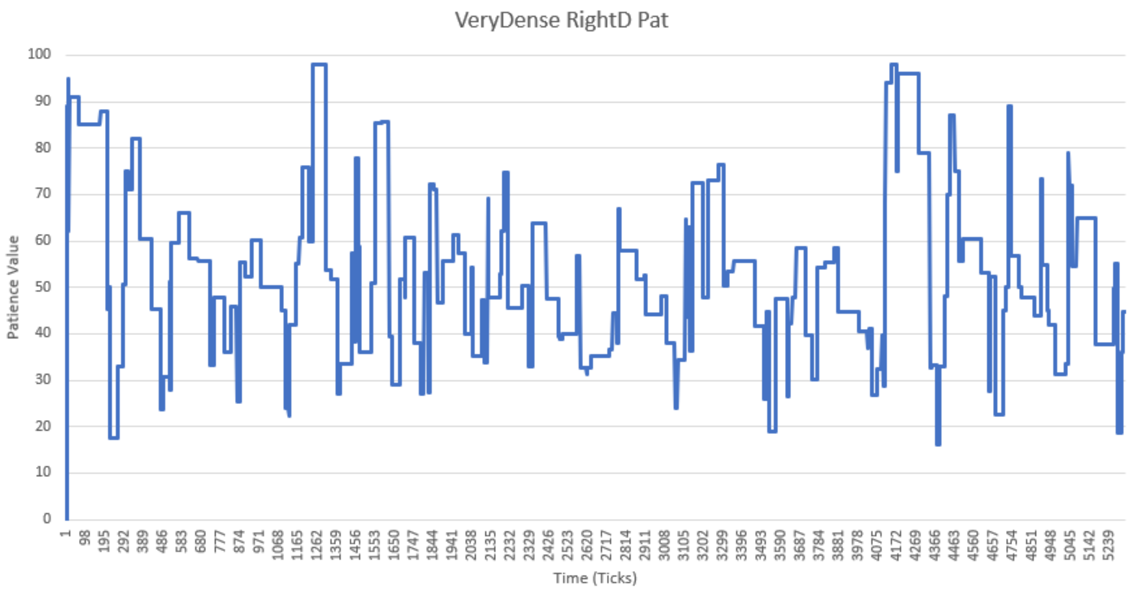
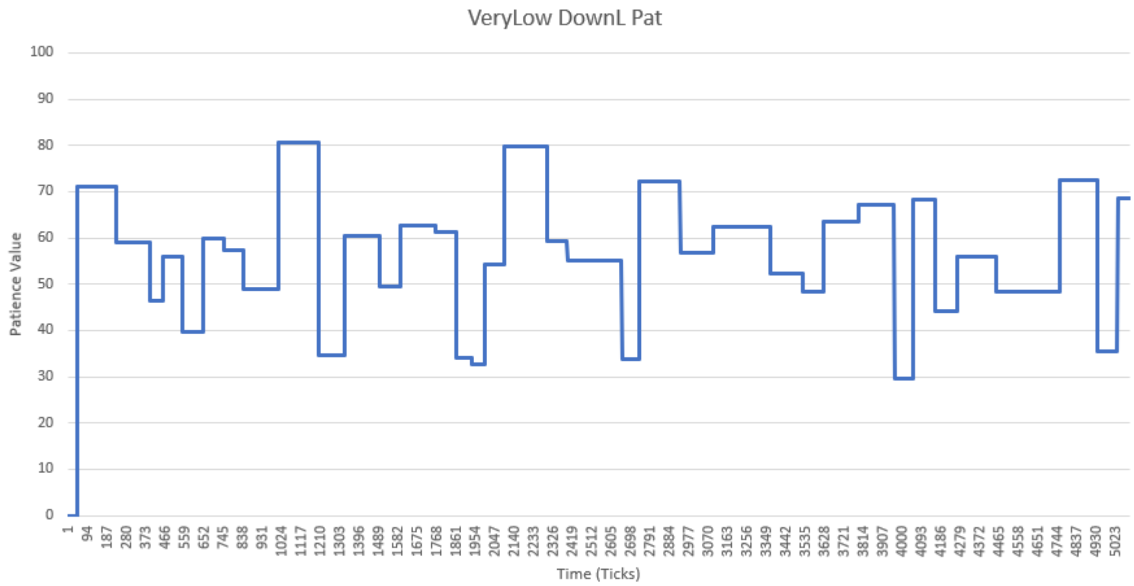
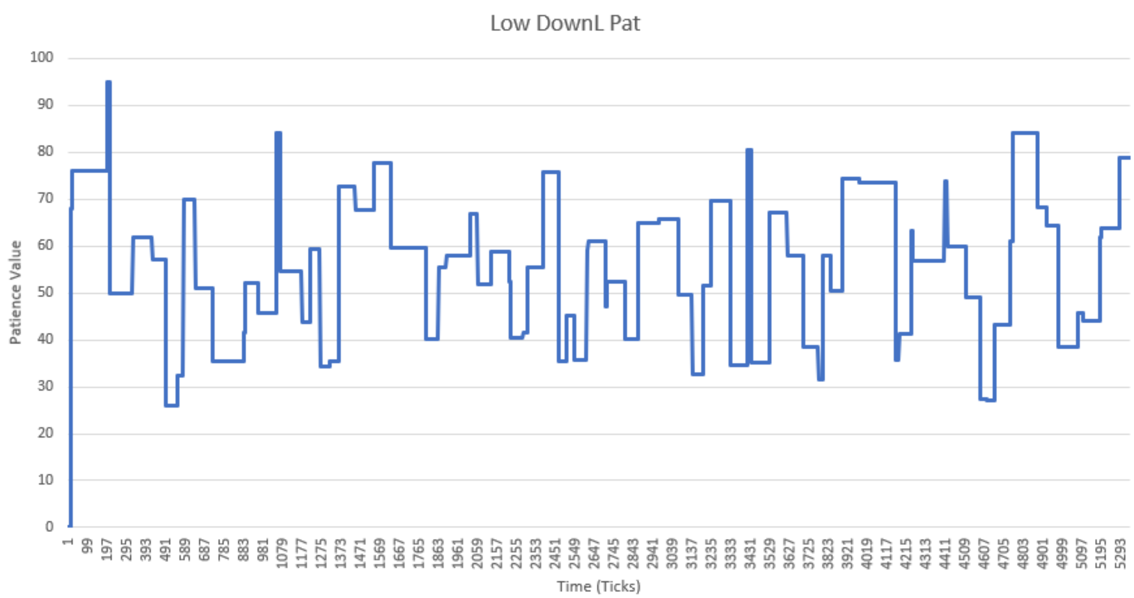
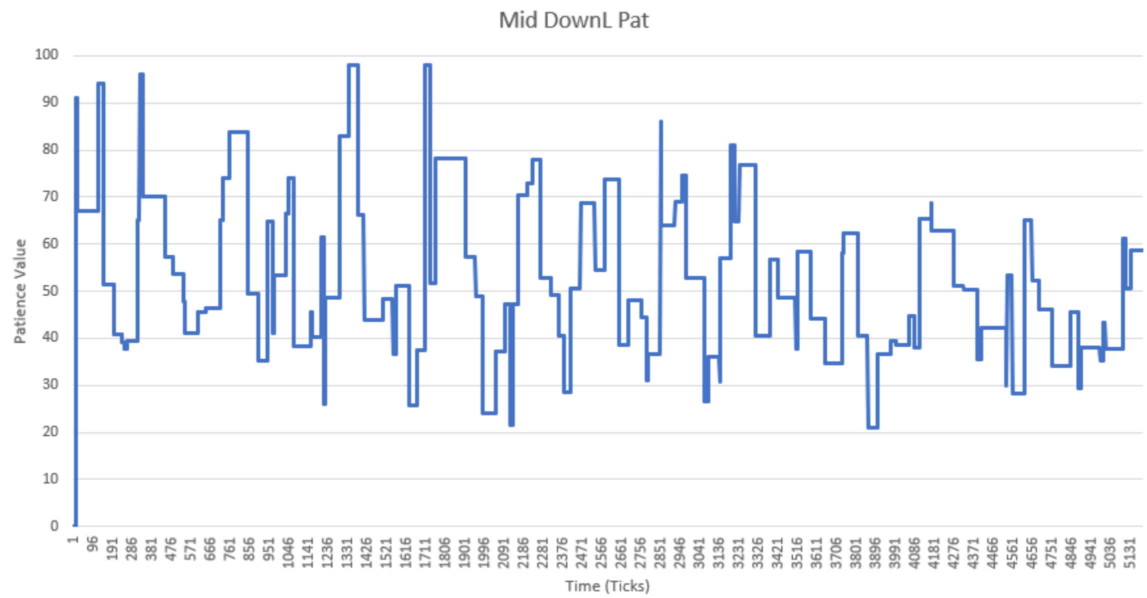
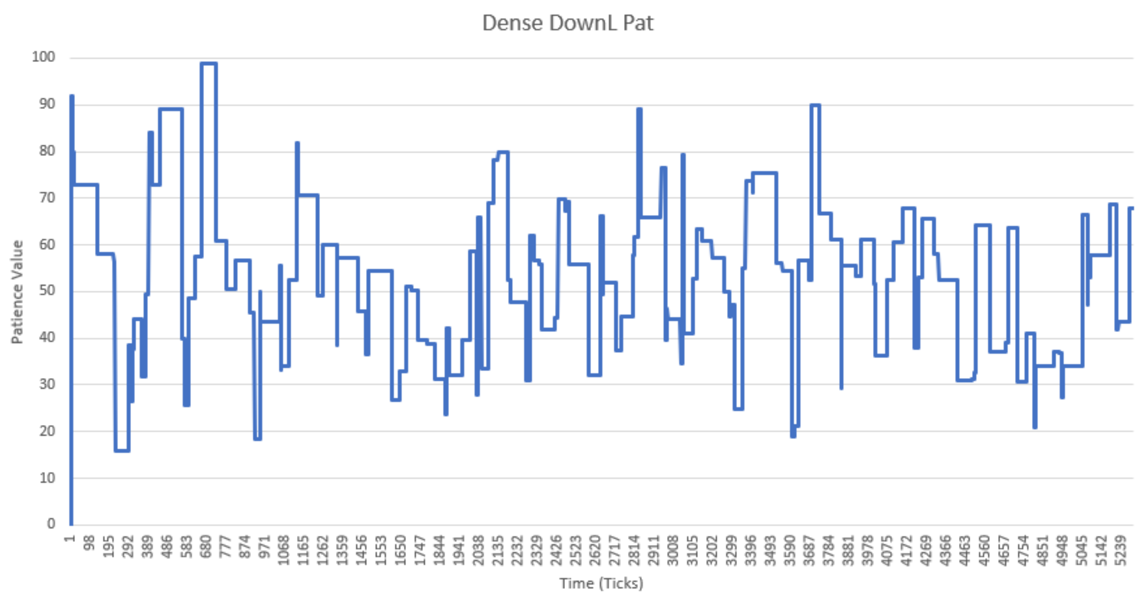

VeryDense RightD Pat
Patience Value
Time (Ticks)

Figure 51. Down (Left) Pedestrian Patience Graphs






Mid DownL Pat
Patience Value
Time (Ticks)


Dense DownL Pat
Patience Value
Time (Ticks)

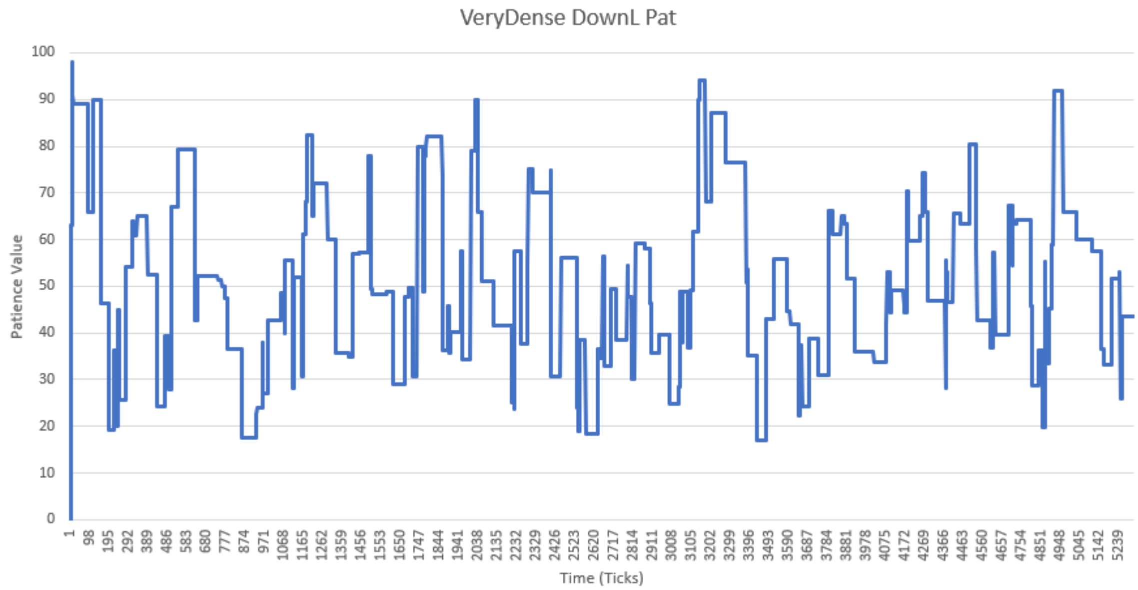
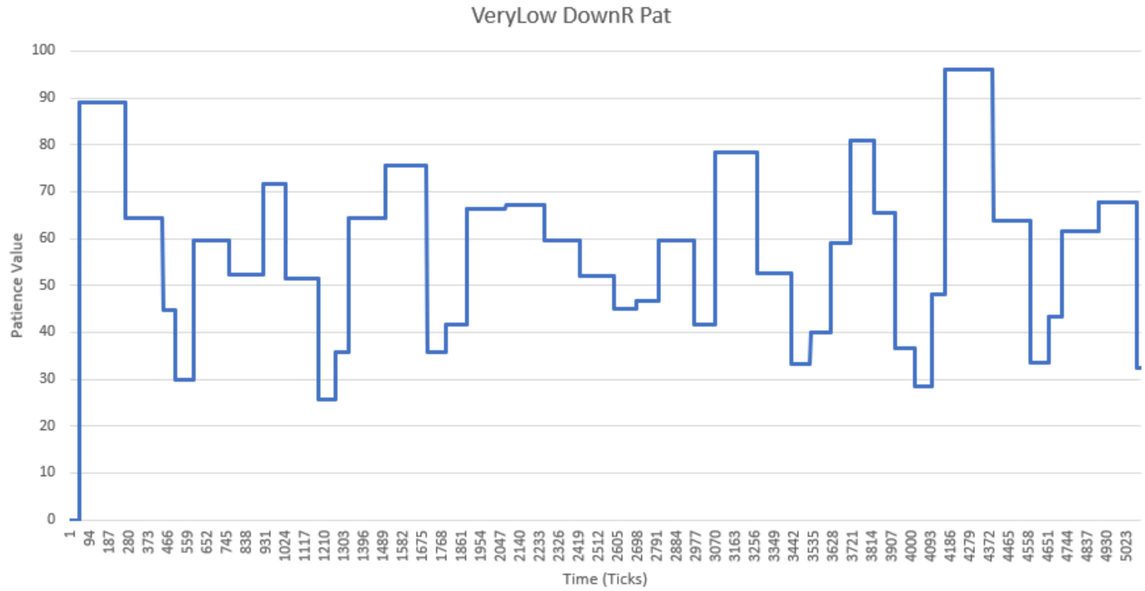
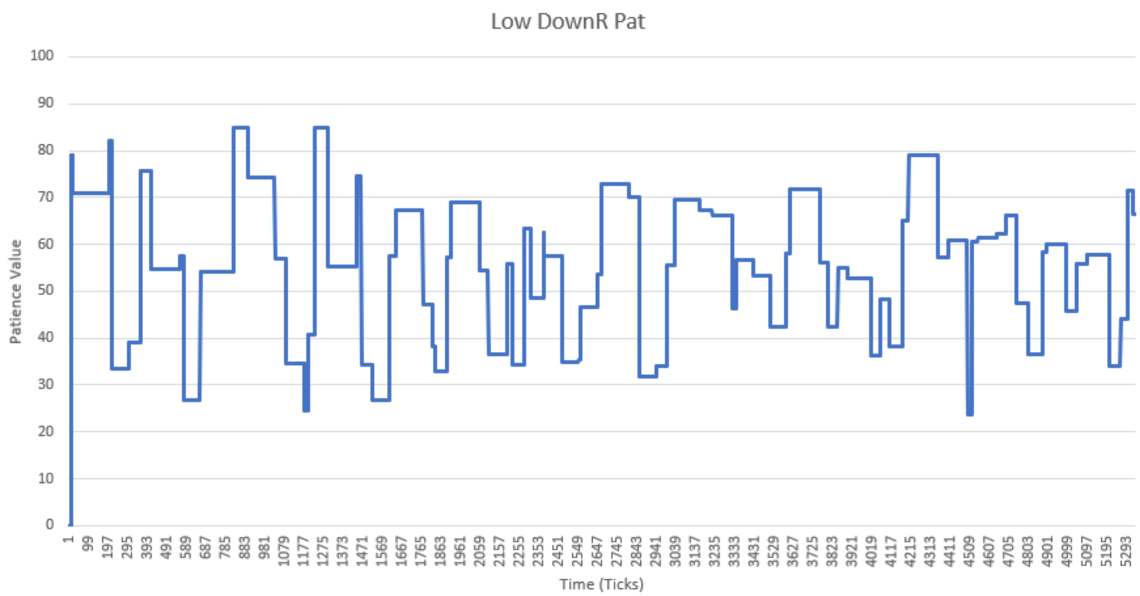
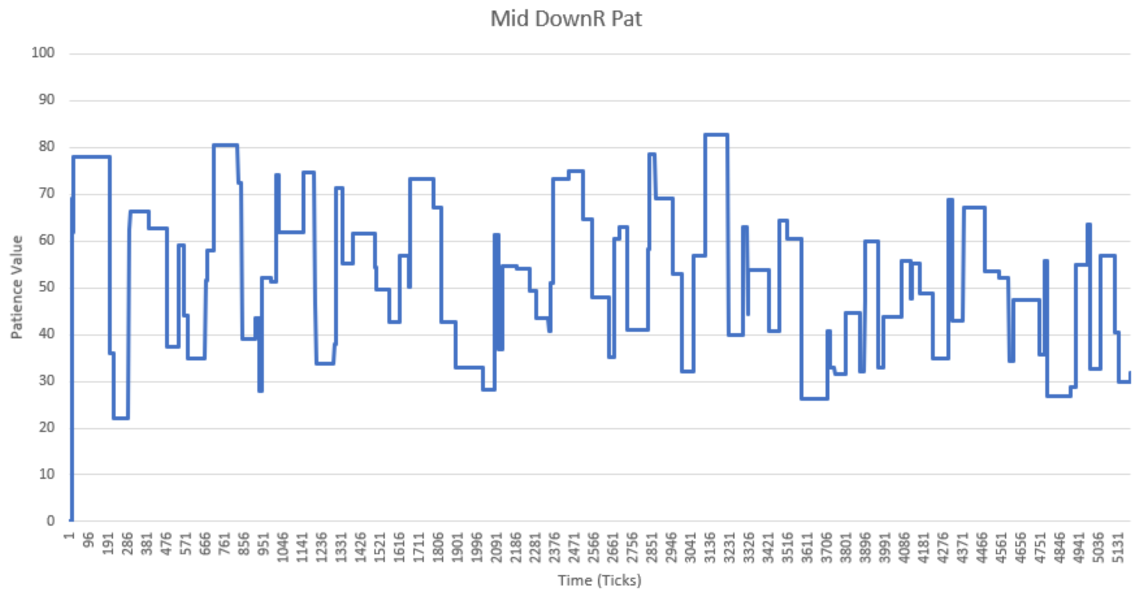
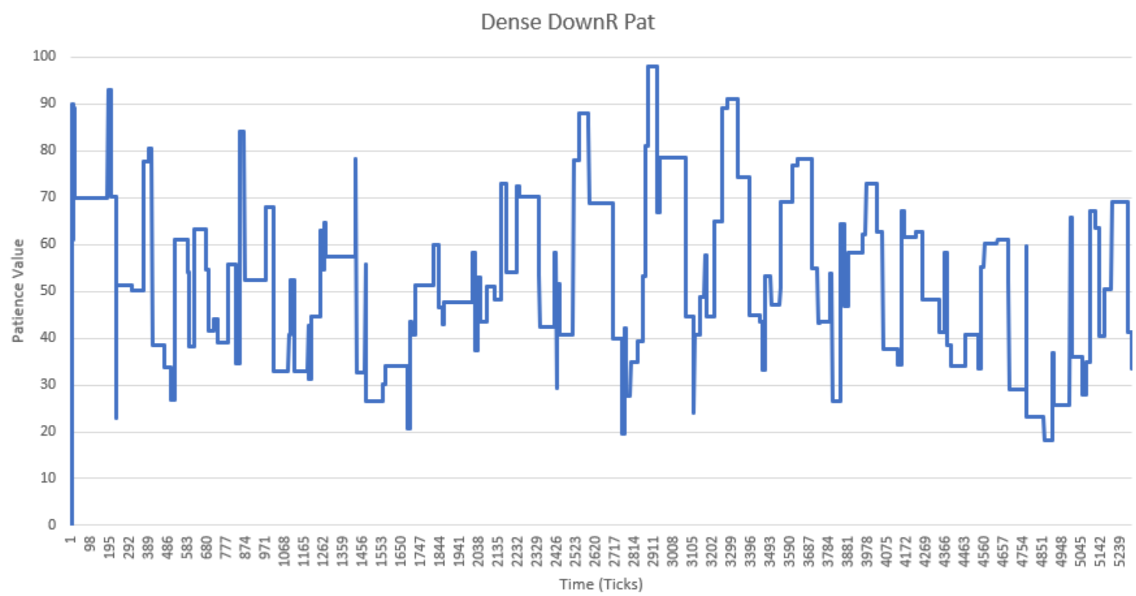

VeryDense DownL Pat
Patience Value
Time (Ticks)
0
10
20
30
40
50
60
70
80
90
100
1
98
195
292
389
486
583
680
777
874
971
1068
1165
1262
1359
1456
1553
1650
1747
1844
1941
2038
2135
2232
2329
2426
2523
2620
2717
2814
2911
3008
3105
3202
3299
3396
3493
3590
3687
3784
3881
3978
4075
4172
4269
4366
4463
4560
4657
4754
4851
4948
5045
5142
5239

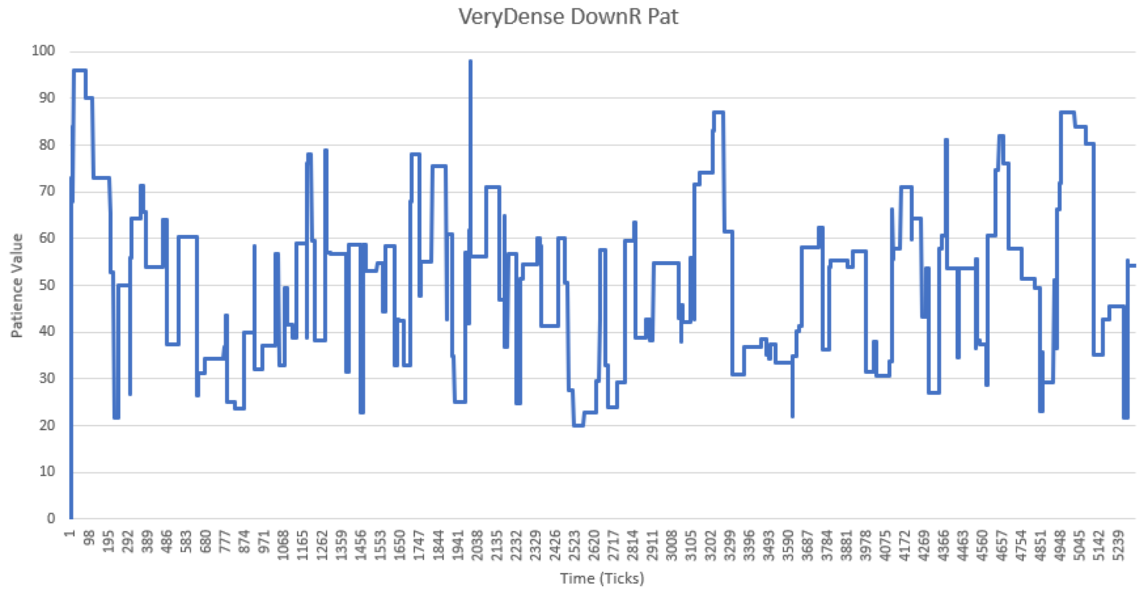
Figure 52. Down (Right) Pedestrian Patience Graphs






Mid DownR Pat
100
90
80
70
60
50
40
30
20
10
0
Patience Value
1 96 191 286 381 476 571 666 761 856 951 1046 1141 1236 1331 1426 1521 1616 1711 1806 1901 1996 2091 2186 2281 2376 2471 2566 2661 2756 2851 2946 3041 3136 3231 3326 3421 3516 3611 3706 3801 3896 3991 4086 4181 4276 4371 4466 4561 4656 4751 4846 4941 5036 5131
Time (Ticks)


Dense DownR Pat
100
90
80
70
60
50
40
30
20
10
0
Patience Value
1 98 195 292 389 486 583 680 777 874 971 1068 1165 1262 1359 1456 1553 1650 1747 1844 1941 2038 2135 2232 2329 2426 2523 2620 2717 2814 2911 3008 3105 3202 3299 3396 3493 3590 3687 3784 3881 3978 4075 4172 4269 4366 4463 4560 4657 4754 4851 4948 5045 5142 5239
Time (Ticks)


VeryDense DownR Pat
Patience Value
Time (Ticks)

Appendix E: Enlarged Version of Table 1

Entity Type	Data Type - Distribution Fit	VeryLow Test			Low Test			Mid Test			Dense Test			VeryDense Test			
		ChSq	KS	SE	ChSq	KS	SE	ChSq	KS	SE	ChSq	KS	SE	ChSq	KS	SE	ChSq = Chi Square Test p-value
																	KS = Kolmogorov-Smirnov Test p-value
Blue Car	Inter-Arrival - Exponential	<0.005	N.A.	0.0211	<0.005	<0.01	0.0572	<0.005	0.0307	0.0299	0.0311	0.06	0.0139	0.0422	>0.15	0.0171	SE = Square Error
	Patience - Normal	<0.005	<0.01	0.0137	<0.005	<0.01	0.0118	<0.005	<0.01	0.0118	<0.005	<0.01	0.0107	<0.005	<0.01	0.0109	
	Wait Time - Triangular	<0.005	<0.01	0.0377	<0.005	<0.01	0.038	<0.005	<0.01	0.0296	<0.005	<0.01	0.0279	<0.005	<0.01	0.0236	
Red Car	Inter-Arrival - Exponential	<0.005	N.A.	0.0144	<0.005	<0.01	0.0314	<0.005	<0.01	0.0499	<0.005	<0.01	0.0446	<0.005	<0.01	0.0453	
	Patience - Normal	<0.005	<0.01	0.0108	<0.005	<0.01	0.0097	<0.005	<0.01	0.0203	<0.005	<0.01	0.012	<0.005	<0.01	0.0093	
	Wait Time - Triangular	<0.005	<0.01	0.0448	<0.005	<0.01	0.0321	<0.005	<0.01	0.036	<0.005	<0.01	0.0345	<0.005	<0.01	0.04	
Yellow Car	Inter-Arrival - Exponential	<0.005	N.A.	0.0186	<0.005	<0.01	0.0414	<0.005	0.0288	0.0339	<0.005	<0.01	0.0403	0.0344	0.0907	0.0185	
	Patience - Normal	<0.005	<0.01	0.016	<0.005	<0.01	0.0112	<0.005	<0.01	0.0083	<0.005	<0.01	0.0154	<0.005	<0.01	0.0163	
	Wait Time - Triangular	<0.005	<0.01	0.0255	<0.005	<0.01	0.0375	<0.005	<0.01	0.0229	<0.005	<0.01	0.0339	<0.005	<0.01	0.0452	
White Car	Inter-Arrival - Exponential	<0.005	N.A.	0.014	<0.005	<0.01	0.0359	<0.005	<0.01	0.0557	<0.005	<0.01	0.035	<0.005	0.0122	0.0168	
	Patience - Normal	<0.005	<0.01	0.0078	<0.005	<0.01	0.0149	<0.005	<0.01	0.0143	<0.005	<0.01	0.0132	<0.005	<0.01	0.0145	
	Wait Time - Triangular	<0.005	<0.01	0.0213	<0.005	<0.01	0.0381	<0.005	<0.01	0.0326	<0.005	<0.01	0.0245	<0.005	<0.01	0.0288	
LeftU Ped	Inter-Arrival - Exponential	<0.005	0.094	0.0829	<0.005	<0.01	0.0378	0.0701	>0.15	0.0102	0.331	>0.15	0.0065	0.0282	N.A.	0.0076	
	Patience - Normal	<0.005	<0.01	0.018	<0.005	<0.01	0.0123	<0.005	<0.01	0.0125	<0.005	<0.01	0.009	<0.005	<0.01	0.0111	
	Wait Time - Triangular	<0.005	<0.01	0.0297	<0.005	<0.01	0.0334	<0.005	<0.01	0.0389	<0.005	<0.01	0.0294	<0.005	<0.01	0.0292	
LeftD Ped	Inter-Arrival - Exponential	<0.005	>0.15	0.0594	0.027	0.145	0.0127	0.0465	>0.15	0.0102	0.399	>0.15	0.0047	<0.005	>0.15	0.0045	
	Patience - Normal	<0.005	<0.01	0.0567	<0.005	<0.01	0.0192	<0.005	<0.01	0.033	<0.005	<0.01	0.0093	<0.005	<0.01	0.0091	
	Wait Time - Triangular	<0.005	<0.01	0.0615	<0.005	<0.01	0.0334	<0.005	<0.01	0.0343	<0.005	<0.01	0.0303	<0.005	<0.01	0.0277	
UpL Ped	Inter-Arrival - Exponential	<0.005	0.0191	0.1152	<0.005	<0.01	0.0501	0.126	0.0498	0.007	0.0375	>0.15	0.0103	0.326	>0.15	0.0033	
	Patience - Normal	<0.005	<0.01	0.0258	<0.005	<0.01	0.0154	<0.005	<0.01	0.0111	<0.005	<0.01	0.0082	<0.005	<0.01	0.0125	
	Wait Time - Triangular	<0.005	<0.01	0.0642	<0.005	<0.01	0.0436	<0.005	<0.01	0.0321	<0.005	<0.01	0.0378	<0.005	<0.01	0.0286	
UpR Ped	Inter-Arrival - Exponential	0.0053	0.1	0.0393	<0.005	<0.01	0.0427	0.0677	>0.15	0.0069	0.136	>0.15	0.0069	0.59	>0.15	0.0023	
	Patience - Normal	<0.005	<0.01	0.019	<0.005	<0.01	0.0127	<0.005	<0.01	0.0109	<0.005	<0.01	0.0082	<0.005	<0.01	0.0079	
	Wait Time - Triangular	<0.005	<0.01	0.0339	<0.005	<0.01	0.0331	<0.005	<0.01	0.0258	<0.005	<0.01	0.0328	<0.005	<0.01	0.0224	
RightU Ped	Inter-Arrival - Exponential	0.0096	>0.15	0.0474	0.0148	<0.01	0.0167	0.159	>0.15	0.0093	0.21	>0.15	0.0035	<0.005	N.A.	0.0084	
	Patience - Normal	<0.005	<0.01	0.0314	<0.005	<0.01	0.0167	<0.005	<0.01	0.0136	<0.005	<0.01	0.0129	<0.005	<0.01	0.0075	
	Wait Time - Triangular	<0.005	<0.01	0.0354	<0.005	<0.01	0.0351	<0.005	<0.01	0.03	<0.005	<0.01	0.0355	<0.005	<0.01	0.02866	
RightD Ped	Inter-Arrival - Exponential	<0.005	0.0427	0.0598	0.0164	0.0477	0.0253	0.416	>0.15	0.0046	0.413	>0.15	0.0032	0.0852	0.066	0.012	
	Patience - Normal	<0.005	<0.01	0.0306	<0.005	<0.01	0.0214	<0.005	<0.01	0.0206	<0.005	<0.01	0.0128	<0.005	<0.01	0.0103	
	Wait Time - Triangular	<0.005	<0.01	0.0532	<0.005	<0.01	0.0389	<0.005	<0.01	0.0347	<0.005	<0.01	0.0277	<0.005	<0.01	0.0363	
DownL Ped	Inter-Arrival - Exponential	<0.005	0.0574	0.0953	<0.005	<0.01	0.0391	<0.005	<0.01	0.034	0.244	>0.15	0.0041	0.509	N.A.	0.0056	
	Patience - Normal	<0.005	<0.01	0.0309	<0.005	<0.01	0.0151	<0.005	<0.01	0.0144	<0.005	<0.01	0.0093	<0.005	<0.01	0.0074	
	Wait Time - Triangular	<0.005	<0.01	0.0402	<0.005	<0.01	0.0394	<0.005	<0.01	0.0359	<0.005	<0.01	0.0337	<0.005	<0.01	0.0289	
DownR Ped	Inter-Arrival - Exponential	<0.005	0.0478	0.0907	<0.005	0.0104	0.0331	0.0084	0.088	0.0166	0.331	>0.15	0.0055	0.183	>0.15	0.0057	
	Patience - Normal	<0.005	<0.01	0.0278	<0.005	<0.01	0.0216	<0.005	<0.01	0.0116	<0.005	<0.01	0.0134	<0.005	<0.01	0.0177	
	Wait Time - Triangular	<0.005	<0.01	0.0599	<0.005	<0.01	0.0482	<0.005	<0.01	0.0324	<0.005	<0.01	0.0371	<0.005	<0.01	0.042	

Table 1. A table detailing the two p-values and the square error value for the fit of each data type. Enlarged from Section 4.3.5.