Deriving relationships between number of observed short gamma ray bursts and viewing angle

by:

John T. Micallef

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Dr. Davide Lazzati

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Abstract

Gamma ray bursts are some of the brightest events in the entire observable universe. Since the late 1960's, thousands of gamma ray bursts have been observed and they have been researched extensively. However, there are still many mysteries which remain unsolved. One such mystery is whether or not the viewing angle, more specifically the *off-axis viewing angle*, of short gamma ray bursts affects the distribution of bursts. It has been suspected that at least some of the features of the gamma ray burst distribution are due to off-axis viewing (meaning the burst was observed at large angles relative to its propagation direction). In this study, Monte-Carlo simulations were used to determine the relationships between the number of observable short gamma ray bursts and the flux of those bursts based on randomly generated viewing angles. Here we show that the viewing angle is a primary contributor of the shape of the distribution of short gamma ray bursts. Other effects include the geometry of the universe, its expansion rate, and its star formation rate. Understanding this relationship will allow for better contextualization of the experimental data that has been gathered for the past 50+ years. Further studies should be done to exclude some of the other parameters that affect the distribution, such as distance, in order to further refine the relationship between viewing angle, flux, and number.

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1. Introduction

There are many mysterious cosmological phenomena that have been observed over the last few thousand years. Among the most fascinating, and mysterious, are gamma ray bursts (GRBs). GRBs were first observed in the late 1960s by United States spy satellites that were looking for the tell-tale gamma ray flashes that would surely accompany nuclear weapons tests by the Soviet Union. It was soon realized that these bursts of gamma rays were not only not coming from Earth, but were coming from other galaxies. In order for these signals to be from that far away the source of bursts must have been immensely strong. In fact, they must be the brightest events in the entire observable universe. Since the late 1960's, thousands of GRBs have been observed. Many insights have been made over the years; such as the bimodal duration distribution of GRBs that has led to the subclasses of "short" and "long" GRBs, as well as the definitive mechanism behind long GRBs being core-collapse supernovae. Most research into short GRBs is focused on determining the exact mechanism behind their progenation. Most of this research points to compact binary mergers such as a pair of neutron stars colliding or a neutron star colliding with a black hole. A lot is currently known about long GRBs. However, short GRBs remain fairly shrouded in mystery. One such mystery is how many short GRBs we should expect to see given the most probable mechanism behind their progenation and the accepted cosmological model (the Big Bang Theory with Cosmic Inflation), which is what will be explored in this study.

1.1 Objectives

My goal was to create a computer simulation based on theoretical models of short gamma ray bursts and then to extract a relationship between the number of observed bursts and their isotropic flux. Observing the relationships that arise from the accepted models and comparing them to known experimental data will provide insight into the validity of those models.

1.2 What is a gamma ray burst

A Gamma Ray Burst, or GRB, can be thought of as a pulse from a powerful space laser (note that GRBs are not coherent, and therefore not actually lasers, this is just an analogy). These events are non-repeating and the front-end of these "space lasers" are in the gamma ray spectrum, hence the name: "Gamma Ray Burst". GRBs are some of the most energetic events in the universe. A single GRB, which usually lasts only a few milliseconds to a few seconds, outputs more energy than our sun has in its entire lifetime thus far [1]. A GRB is a dipolar event which consists of a massive burst of gamma rays and a relativistic particle jet that exudes from the magnetic poles of some generation mechanism [2]. The particles that make up these jets are moving so quickly through the local interstellar medium that they give off synchrotron radiation in the x-ray band, this effect is commonly called the "afterglow" of the burst [3].

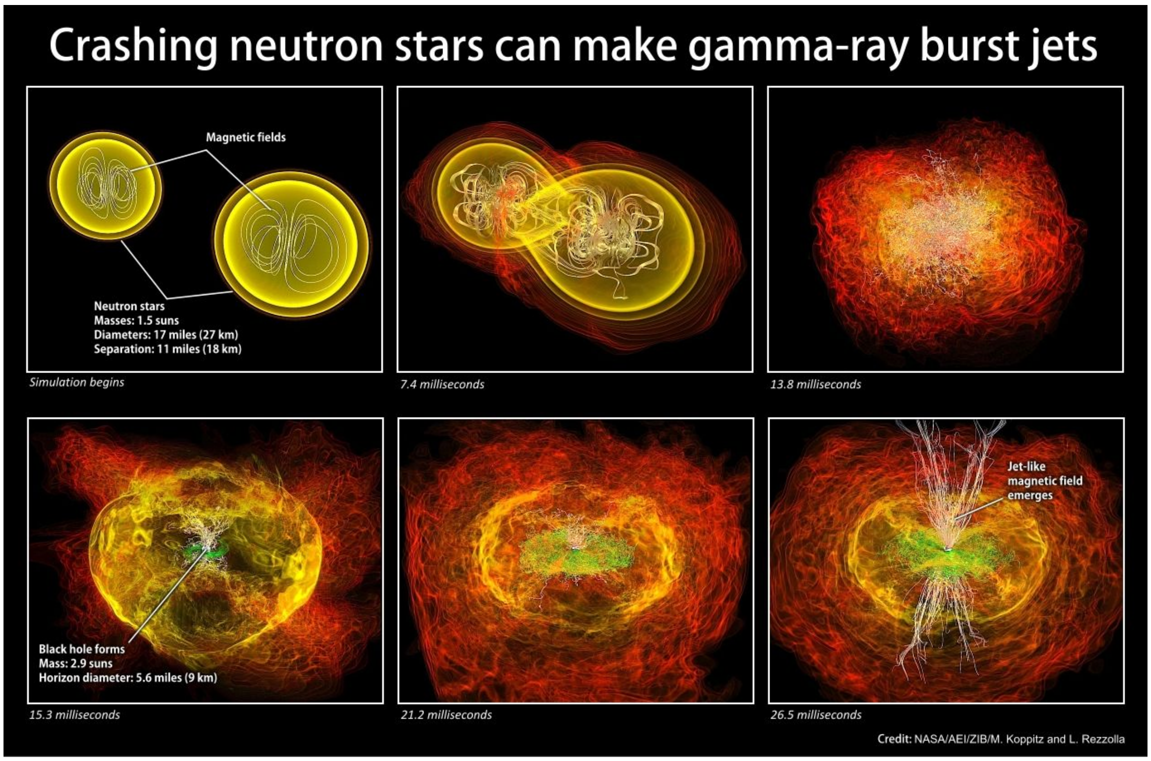

Figure 1.2.1: gamma ray burst diagram A six stage process is shown. First, the magnetic fields of each object (each is a neutron star here, but it could be a black hole and neutron star as well) start to interact as the objects approach each other. Second, the objects start to merge. Third, the objects become one. Fourth, the black hole forms at the centre of the new object. Fifth, the new object begins to show axial magnetic fields. Sixth, the new object's magnetic fields shoot outwards from their central axis and form the collimated energy bursts we call "gamma ray bursts". [5]

There appears to be two distinct populations of GRBs, and they can be differentiated solely by their duration. "Long" GRBs have a minimum duration of 2 seconds and an average duration of 30 seconds [1]. "Short" GRBs have a maximum duration of 2 seconds and an average duration of 300 milliseconds [1]. In *Figure 1.2.1* above one of the ways short GRBs are thought to form can be seen. Showing the viability of the models by comparing what they predict with what is observed is a basic tenet of science. This allows for integrity to be tested and preserved.

A lot more is known about long GRBs than short GRBs. It is generally accepted that Long GRBs are caused solely by core-collapse supernovae as there is ample evidence and studies that have shown this [2]. However, short GRBs are more obscure [2]. The mechanism behind them is less certain, though there are several that are widely accepted [2]. Of these, the most popular is a binary merger [2]. That is to say a merger between a black hole and a neutron star, or a merger between two neutron stars [2]. The lack of certainty in the mechanism behind short GRBs leads to further gaps in the associated knowledge base.

1.3 Measurable properties of short gamma ray bursts

The basic properties of a GRB include its duration, viewing angle, light curve, peak isotropic luminosity, and distance [1]. Duration of burst will not be variable in this simulation. Instead, the average duration of a short GRB, 300 milliseconds [1], will be assumed for all bursts. Viewing angle is the angular difference between the central axis of the burst and the observational axis, this angle can be seen in *Figure 1.3.1*. In my simulations viewing angle will be generated pseudo-randomly, following a sine distribution.

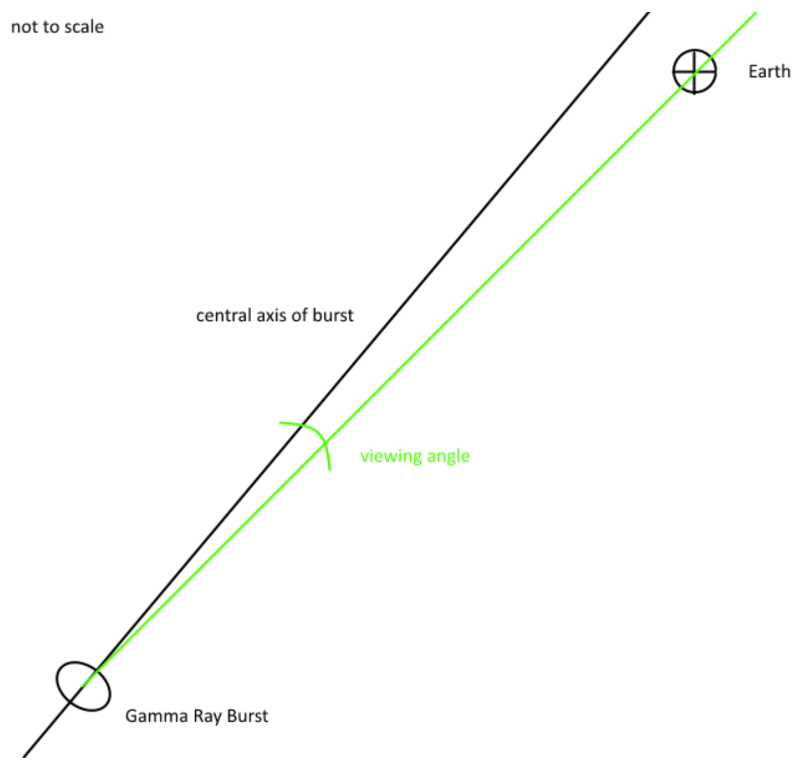

Figure 1.3.1: viewing angle diagram A diagram showing the meaning of viewing angle.

The light curve of a GRB describes the brightness of the burst as a function of time. Peak luminosity describes the maximum value of the light curve observed for the burst, and will be assumed to be approximately equal to the radiated isotropic energy in this simulation. In actuality, the energy is the time-integral of the luminosity curve over its duration. However, since the goal of this project is to determine the general shape of the number per flux versus flux curve, and not its exact parameterization, the brightness curves for all bursts were assumed to be single peaked gaussian distributions with identical durations. Making this simplification allows the distribution shape to better show the effects of the viewing angle by reducing the number of inputs . Radiated isotropic energy describes the total energy released for a gamma-ray burst if it were spherically symmetric (which it is not). This value is very easy to measure experimentally, even though it is not actually a good representation of the actual burst's energy. Radiated isotropic energy will be derived from the viewing angle given the relationship in observed short GRBs. Distance is simply the distance between the observer and the GRB's source. Distance will be calculated based on the redshift of the burst. Redshift is a measure of the doppler shift between the observed wavelength, or frequency, of light released by an object versus its (known) wavelength, or frequency, at the source. Doppler shift happens when a source (of light) is moving relative to its observer. The measure is called 'redshift' because the wavelength of the light becomes longer, that is to say 'more red' in the case of visible light, if the source is moving away from the observer. Because the universe is expanding, and has been since its beginning, we can use redshift as a cosmological ruler. Redshift will be derived from the relationship given by Porciani et al between star formation rate and redshift [3]. Star formation rate is generated based on the accepted cosmological models [4].

1.4 Monte Carlo simulations

The results of the roulette table at the Monte Carlo casino in Monaco were regularly published in a local newspaper [7]. Karl Pearson famously used this dataset to test statistical mechanics. Interestingly, Pearson found that the data was extremely biased and must have be corrupted, that is to say "rigged" [7]. However, it turns out the fault didn't lie with the casino, but with the journalists recording the numbers. They had not been observing the tables at all, they had been sitting at the casino's bar and making the numbers up [7]. It is from this story, and a few others, that the term “Monte Carlo simulation” comes. A Monte Carlo simulation is any simulation which relies on randomly generated inputs.

Monte Carlo simulations use randomized initial conditions and model what would happen, given those initial conditions, in the real world [6]. Having randomized initial conditions is especially helpful in simulations that are trying to model behavior of large systems, such as in cosmology and astrophysics.

Monte Carlo simulations have four general steps [6]. First, define the allowed values for the inputs [6]. Second, randomly generate these inputs utilizing some realistic probability distribution over the previously found domain [6]. Third, use the models to determine the results of your simulation given these random inputs [6]. Fourth, use the results to adjust the domain of inputs, probability distributions of the inputs, and model [6].

2. Methods

2.1 Research methodology

This simulation was coded using Python 3 in Geany. All the code was written, compiled, and run on my personal computer by me. However, the code was regularly reviewed by Dr. Lazzati at weekly meetings where he would give his input, advice, and corrections. The code and results from the simulation were backed up to a shared folder on my personal Google Drive, which Dr. Lazzati also has access to.

2.2 Coding methodology

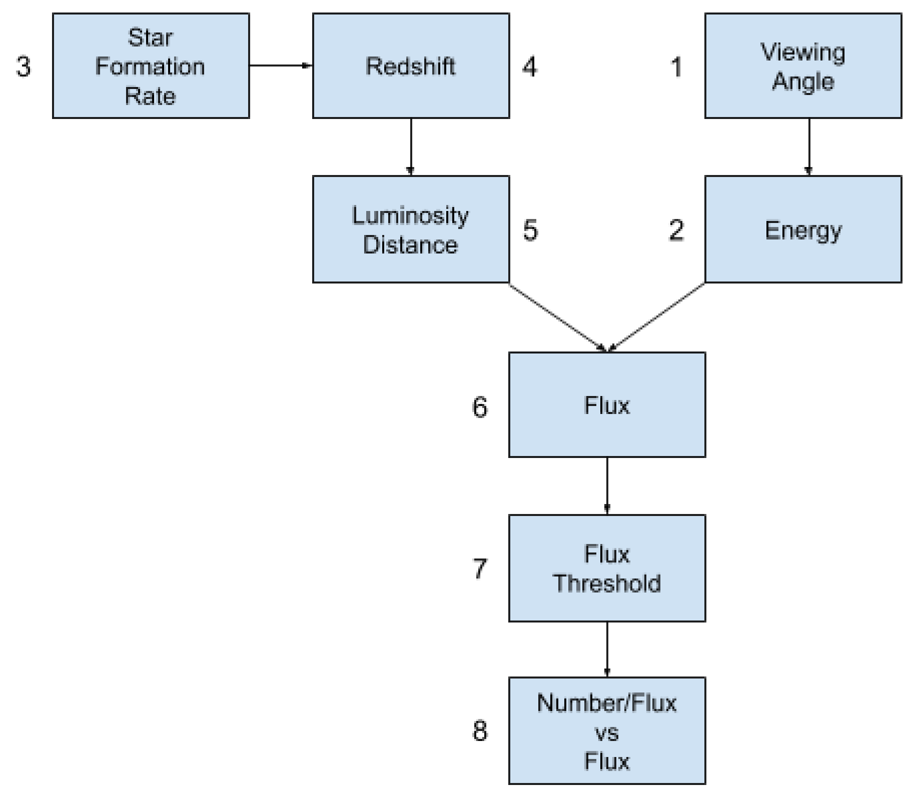

Figure 2.2.1: flowchart of code logical structure This flowchart shows the eight separate modules which make up the code for this simulation. Two processes are shown running in parallel and then combining for the finale three modules, though the actually code currently runs serially. Each module feeds into the next, shown by the inter-module arrows.

The code for this simulation relies on eight modules whose relationships are shown in figure 2.2.1 Each module utilizes random number generation either directly, or from its inputs. Running the simulation for one million total events took about two weeks. While the first two modules take a trivial amount of time to run, modules 3, 5, 6, 7, and 8 take a few hours each; module 4 taking approximately a week to run. The fourth module took so much longer because it was both mathematically complex, doing multiple complicated integrals, and it had a threshold cutoff that caused most of the values to be rejected.

2.3 Coding schema

Generate a random viewing angle using a sine distribution. A sine distribution was chosen because of the way a sphere, and therefore a solid angle, grows suggests that a significantly higher number of events will be observed at higher viewing angles.

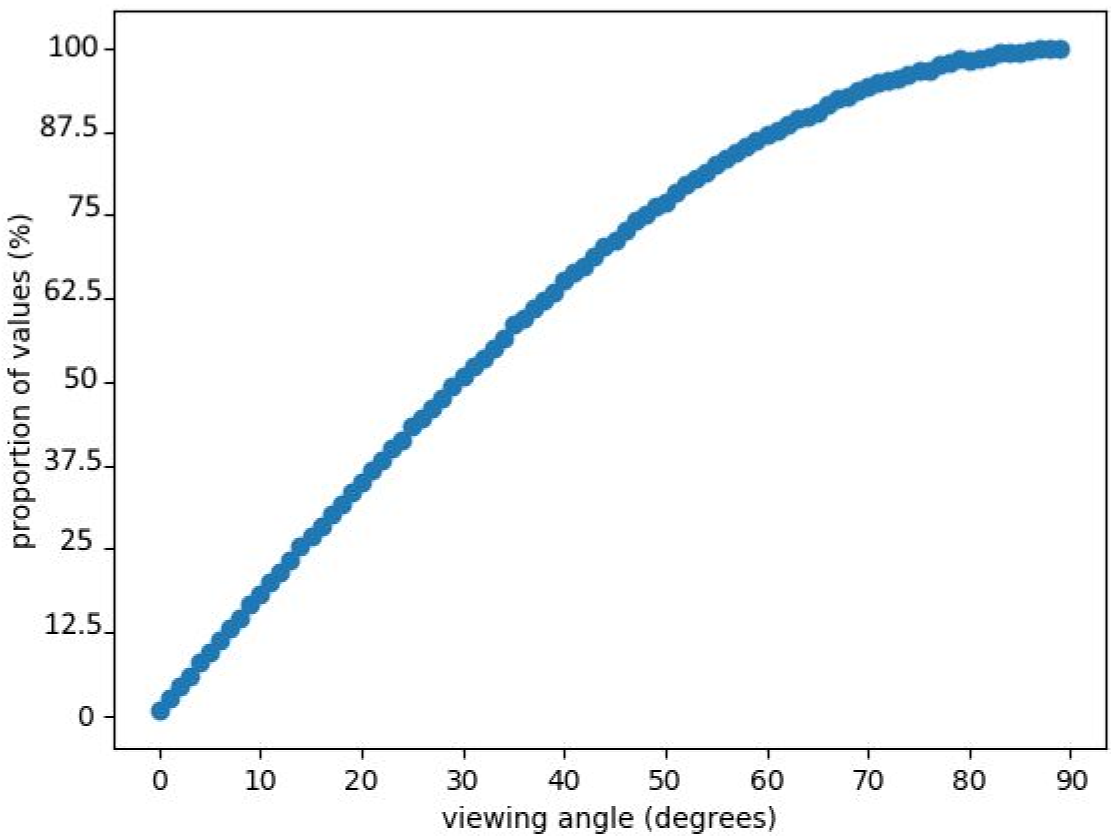

Figure 2.3.1: sine distribution of viewing angles This graph shows the distribution of angles used as a sequence of the angles used in this simulation bucketed into their nearest degree.

A table of energies, which had gave the relationships between viewing angle and radiated isotropic equivalent kinetic energy as determined by numerical simulations, was then loaded into memory and used to find the associated energy for each burst by interpolating their viewing angle with the table's data.

Star formation rate describes the rate at which stars form. It depends on universal constants as well as time. Since it is time dependent, and the speed of causality is constant, the star formation rate is proportional to the distance between the object and observer. However, on cosmological scales distance and time are not measured directly but are instead measured from redshift. This module generated a redshift versus star formation rate curve using the model derived by *Porciani et al* in *On the association of gamma-ray bursts with massive stars: implications for number counts and lensing statistics*.

$$R_{SF}\{z\} = 0.3 * h_{65} * \frac{e^{3.4*z}}{e^{3.8*z}+45} \quad \textbf{(1)}$$

This distribution can be seen in both equation, *equation 1*, and graphical, *Figure 2.2.3*, forms below. In this equation: z is the redshift, *Rsf* is the star formation rate, and h_{65} is the hubble constant, assumed to be 65 kilometer/Megaparsec/second [3].

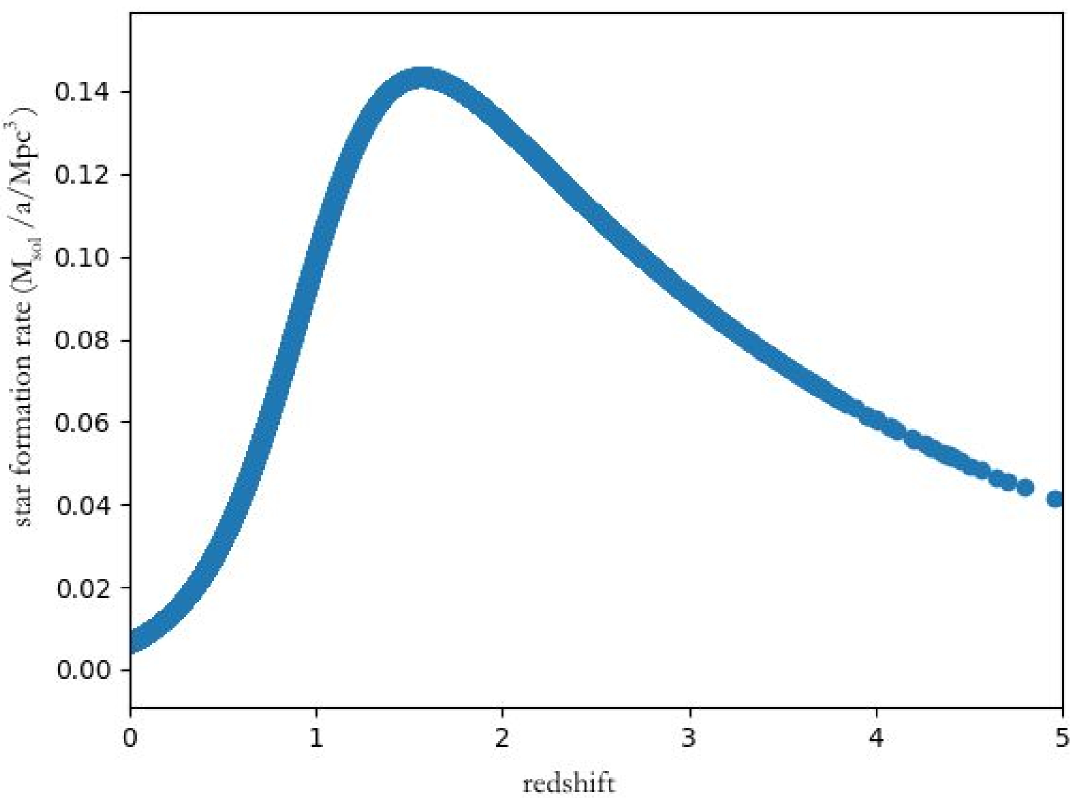

Figure 2.3.2: redshift versus star formation rate distribution This figure shows the relationship between the star formation rate and redshift. This distribution can be thought of as a histogram of redshifts. Showing, explicitly, that there are significantly more medium redshifts than high or low redshifts.

This module generated a set of random redshifts and star formation rate pairs. Check these values against the star formation rate versus redshift distribution to see if the pair is possible, that is to say, underneath the curve seen in *Figure 2.3.2*. The redshifts and star formation rate pairs which were possible were stored, the rest were ignored.

$$d_L\{z\} = \frac{c_0*(1+z)}{H_0} * \int_0^z \left\{ \frac{1}{\sqrt[2]{\Omega_r*(1+z')^4+\Omega_m*(1+z')^3+\Omega_\Lambda}} \right\} dz' \qquad \textbf{(2)}$$

Find luminosity distance from redshift using *equation 2*. In this equation, c_0 is the speed of causality, H_0 is the current day Hubble parameter, z is the redshift, Ω_r represents the equivalent mass density of relativistic particles, Ω_m represents the matter density of the universe, and Ω_Λ represents the dark energy density [4]. The formula is in its simplest form since the curvature term, Ω_k, is *0*. This corresponds to the current model, Lambda-CDM (bigbang + inflation + cold dark matter + dark energy acceleration), of the cosmos which has flat spacetime curvature in the current epoch.

Flux is generally given in small scale units while luminosity distance is generally given in cosmological distances, so distances are converted from megaparsecs to centimeters for better data visualization. Flux is then calculated from energy and luminosity distance. The equation for this transformation can be seen in *equation 3*. In this equation: Φ_{iso} is the isotropic flux, Y_{iso} is the isotropic luminosity (which is proportional to radiated isotropic equivalent energy), and d_L is the luminosity distance. Note that since we are ultimately concerned with the shape of the $\frac{dN}{d\Phi_{iso}}$ vs Φ_{iso} curve and not its actual values, we can just let the luminosity be equal to the radiated isotropic equivalent energy. This is allowable because the simulation assumes that all the bursts are single peaked gaussians with the same duration.

$$\Phi_{iso}\{d_L, E\} = \frac{Y_{iso}}{4*\pi*(d_L)^2} \qquad \textbf{(3)}$$

Check each burst to see if its flux would be observable. Save events that are observable, delete values which are not.

Finally, the differential number of bursts per flux can be calculated and plotted against flux.

3. Results

3.1 Simulation

As mentioned in the *Methods* section, the bursts generated in this simulation were simplified. They have a distance, a viewing angle, and a flux. All the other properties are just intermediaries. This simulation ran for about two weeks and produced roughly one million events. The observable events were then binned, scaled by the size of their bin, and graphed on a log-log plot. This graph, *Figure 3.2.1*, can be thought of like a histogram of the fluxes, though it is not a typical histogram.

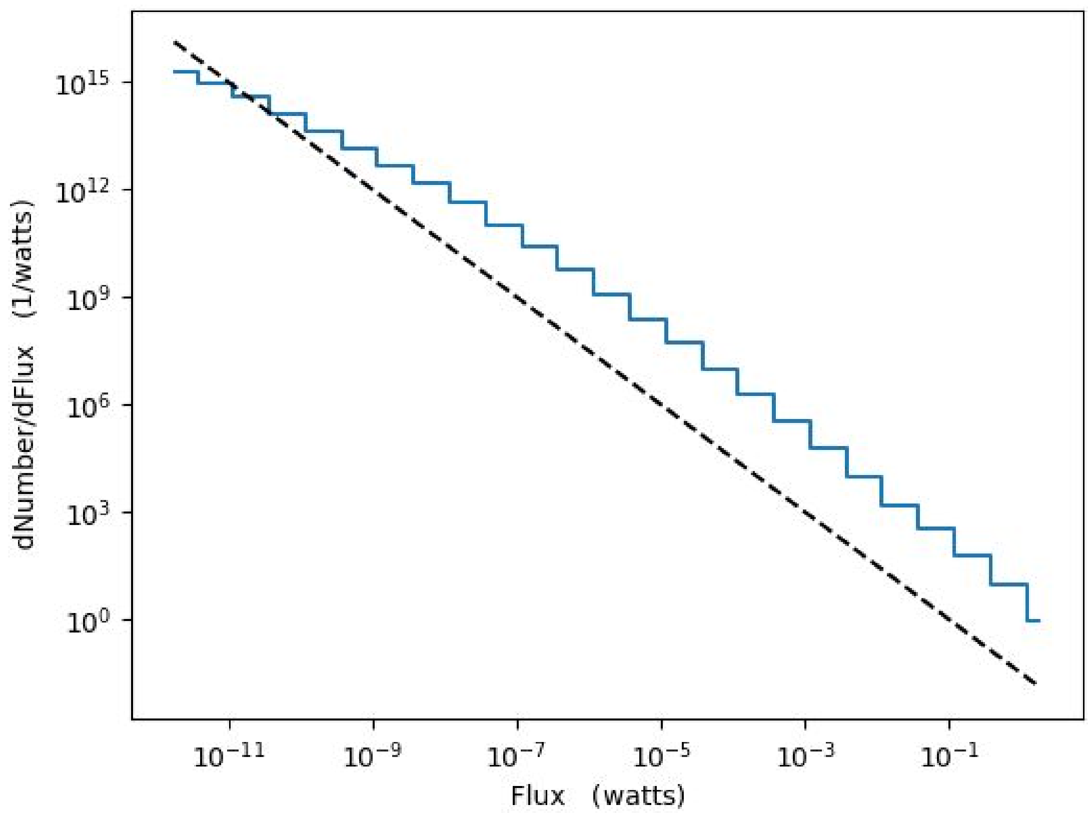

Figure 3.2.1: log/log plot of differential number per flux versus flux The blue line shows the binned and logged distribution of fluxes which are scaled by the size of the bins. The black line shows the centre line.

Next, the FERMI satellite data was plotted alongside my simulated data [9]. Since there are significantly less events recorded by the FERMI satellite, the domain of events was further reduced to only include those which would have been observable by the FERMI satellite. This was done by using a simple minimum flux threshold. About 15% of the simulated events were also observable by this metric.

3.2 Comparison to experimental data

Then the simulated data was correlated to the experimental data to determine how accurate it was. A graph of both the modified simulation data and the experimental data can be seen below in *Figure 3.2.2*.

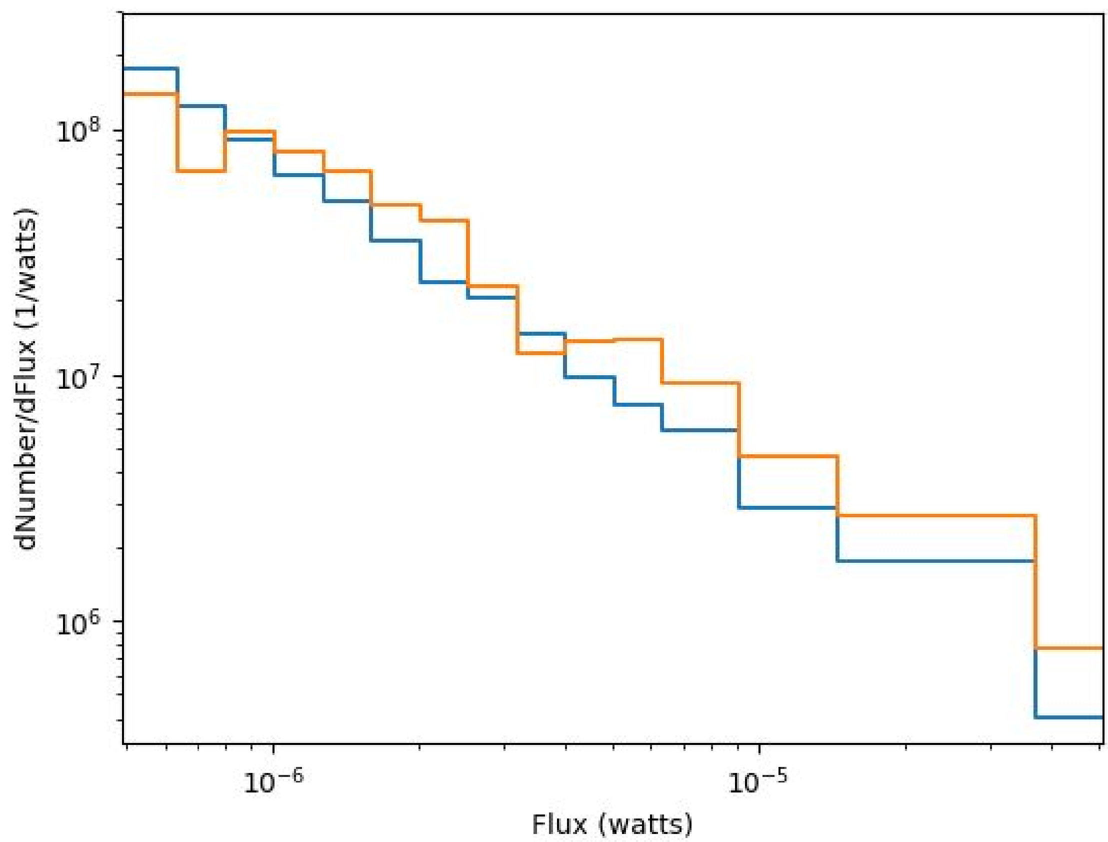

Figure 3.2.2: log/log plot of differential number per flux versus flux for both data sets The blue line represents results of my simulation while the orange line represents the experimental data from the FERMI satellite [9]. The counts for each bin is scaled by the bin size.

Unfortunately, the experimental data covers too small of a range to show all of the interesting features of the simulated curve. In *Figure 3.2.2* the shapes of both the simulated and experimental curves can be seen tracking together quite well. Because of this, we can can say that the models used to derive the equations that were used in this simulation are, in fact, consistent with experimental data. The chi square was minimized to find a shifting factor to align the experimental data with the simulated data in *Figure 3.2.2*. This was necessary because the population of observed bursts for the FERMI satellite is significantly smaller than that of the population of the simulated bursts. The minimum chi square was found to be *1.662* for a shifting factor of *-1.580*. Meaning that the $dN/dFlux$ results of the histogram were multiplied by a factor of $10^{-1.580}$.

3.3 Interpretation of results

As can clearly be seen in *Figure 3.2.1*, there is a very high degree of negative logarithmic correlation between the number of observed bursts and their flux. That is to say, there are significantly more bursts at lower fluxes than at higher fluxes. There are two main astrophysical effects that should contribute to this. The first is that the flux is much larger when the burst is closer, meaning at lower redshift. This is due to the fact that the burst sends out a finite number of photons in a dome (partial sphere), the further away you are from the source, the lower the density of these photons. Flux is, in its essence, a measure of photon density. The caveat to this simplification being that it is a density over a two-dimensional surface and a rate with respect to time. The second contributor to this effect is that there are a lot more bursts further away. This is due to the volume of a three-dimensional space increasing by the cube of its radius. So each *step* outward is massive increase in volume. Assuming the universe is fairly homogeneous, which is an assumption of modern cosmology, the number of GRB progenitors would also increase proportionally as you move outward. There is a lot of space out there for GRBs to occur in.

In *Figure 3.2.1*, the distribution also tends to decline towards the smaller flux end. There are two main cosmological effects that could cause this. According to the *Lambda CDM* model, the universe is expanding and has been since its inception at the *Big Bang*. So, because the universe used to be much smaller, there are much fewer bursts at high redshifts (remember that high redshift is proportional to high distance from the observer). The second effect is the fact that the speed of light in a vacuum is constant. Because of this, the further away we look the further back in time we are looking. This time-travelling effect means that at very high redshifts the universe will have been so much younger that far fewer GRBs will have been able to have occured.

The shape of the distribution curve is definitely affected by many factors and the effects discussed above definitely contribute to this shape. However, looking at the actual data from the simulations, and from the FERMI satellite, you can see that all of the redshifts, and therefore the distances, are extremely small (on cosmological scales). Because of this it can be assumed that the cosmological effects are minimal, if not negligible. If the cosmological effects are minimal, the viewing angle must be the source of the declination in the curve. This can be assumed because the model used for this simulation depends only on the viewing angle, the distance, and the cosmological equations. As discussed above, the effects of distance are purely inversely proportional. Or, more specifically, a negative power law distribution.

4. Conclusion

The shapes of both the simulated and experimental curves track together quite well. Because of this, we can say that the models used to derive the equations that were used in this simulation are, in fact, consistent with experimental data. The chi square for the experimental versus simulated data was found to be *1.662*. Such a low value for the chi square further backs up the visual intuition that the shapes of the simulated and experimental data are very similar, which means the models used are quite accurate.

From the data the simulation generated, it can be concluded that the viewing angle of the burst is highly relevant to the associated flux. Since the viewing angle was the primary input of this simulation, the correlation between the simulated and experimental data is evidence that the viewing angle is a major factor in the observed data distribution.

5. Future research

A good next step would be to simulate sets of bursts all at the same distance to completely eliminate the distance correlation and focus only on the viewing angle. It may also be interesting to take this simulation further and see what effect *inflation* or *varying speed of light* would have on the distribution of GRBs.

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