# Teaching Gravitational Potential Energy: Student Interaction with Surface Manipulatives

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## ABSTRACT

Past studies have shown that students struggle with main concepts of potential energy, such as reading graphs, understanding that potential energy can be negative, and, most pertinent to this research project, connecting potential energy to force. I designed an activity to help introductory physics students address these struggles. The activity required students to draw graphs of both gravitational potential energy (GPE) and gravitational force, which enabled them to see that GPE was completely negative, and to make mathematical and graphical connections between GPE and force. The activity took place during a weekly recitation, during which students worked in groups of 2-3 and used a plastic 3D surface manipulative and accompanying contour map to assist in answering the prompts. The surface and contour map were both representative of the GPE of an Earth-object system. I filmed 3 groups as they worked on the activity and examined the ways in which the students interacted with the surface and what purpose each interaction served.

I found that students most often interacted with the surface by pointing to or tracing components of the surface, showing that the students were unfamiliar with the surface and less comfortable with moving, turning, grabbing, and drawing on the surface. The students most often used the surface to examine the functional behavior of both GPE and force, discuss the rate of change of GPE, and to compare values, slopes, or signs (+ or -) between different locations. One group even discovered that they could use a pen to act as the tangent to the surface (specifically the tangent pointing only in the radial direction), a physical representation of force. In future surfaces activities, it might benefit students to do a small activity as an entire class. This would allow the students to watch how the instructor manipulates the surface in different ways and apply those strategies to subsequent activities. I found that all the students still struggled with one or more aspect of GPE, but both students and TA frequently used the surface to explain that aspect to the struggling student(s) and to make sense of their own difficulties.

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# 1 Introduction

## 1.1 Overview

Past studies have shown that students struggle with main concepts of potential energy, such as reading graphs, understanding that potential energy can be negative, and, most pertinent to this research project, connecting potential energy to force. I designed an activity to help introductory physics students address these struggles. The activity required students to draw graphs of both gravitational potential energy (GPE) and gravitational force, which enabled them to see that GPE was completely negative, and to make mathematical and graphical connections between GPE and force. The students were provided with a contour map and plastic surface model of GPE to use during the activity [Figure 1]. I wanted to know "How do students interact with the surface and each other to accomplish the activity?" The answers to this question will provide information about how students work with each other and their materials that can help future instructors in planning similar lessons.

***Figure 1:*** *This surface represents a quarter-section of the gravitational potential energy for an Earth-object system.*

### 1.1.1 Purpose

The purpose of this study is to examine the ways in which students interact with a plastic surface model during an instructional activity about gravitational potential energy and gravitational force as well as gather information on student reasoning about GPE. The surface is a 3D model of the GPE of an Earth-object system. The students are in an introductory calculus-based physics class.

### 1.1.2 Gravitational Potential Energy and Force

Gravitational potential energy (*U*) is the energy due to the gravitational attraction between objects; in this study we will be focusing on GPE due to two objects (the Earth and a fictitious space station), approximated as point particles.

$$U = -\frac{GMm}{r} \tag{1}$$

Gravitational force is the force between two objects due to their gravitational attraction to each other.

$$\vec{F} = -\frac{GMm}{r^2}\hat{r} \tag{2}$$

*G* is the gravitational constant, $6.67 * 10^{-11}\frac{m^3}{kg*s^2}$, *M* and *m* represent the masses of the two objects, and *r* is the distance between the objects. The negative sign indicates the direction of the force. Since gravitational force points into the center of an object, and the standard r-direction points out from the object, the force is negative. Force is related to potential energy by the following equation:

$$\vec{F} = -\vec{\nabla}U \tag{3}$$

In the case of GPE, which is spherically symmetric,

$$\vec{F} = -\frac{dU}{dr}\hat{r}. \tag{4}$$

Later in the study, we ask students to draw graphs of both GPE and force with respect to r. Two important things to note are that the graphs look very similar and both graphs lie entirely below the x-axis.

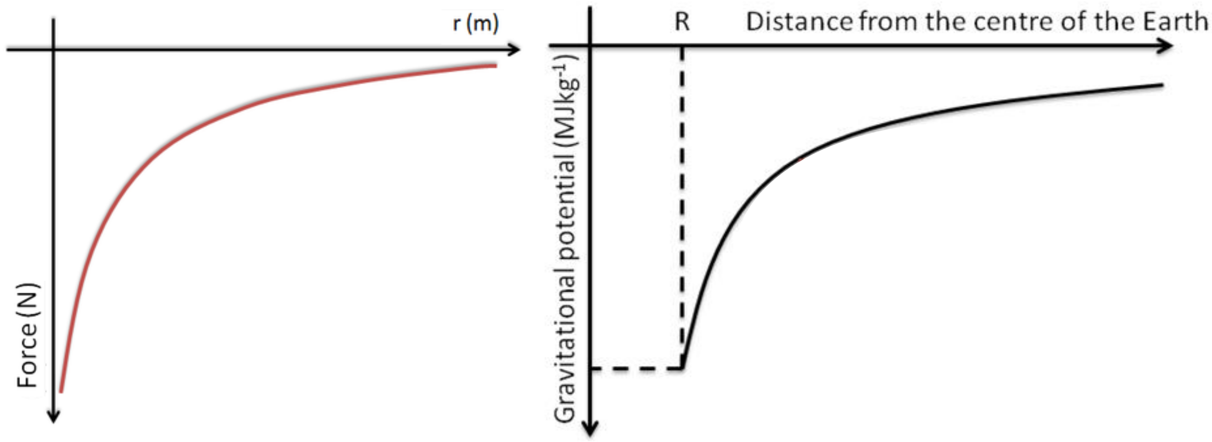



***Figure 2:*** *(a) A graph of gravitational force with respect to distance from the center of the Earth. The graph has been adjusted to reflect the coordinate system of the instructional activity [1]. (b) A graph of gravitational potential energy with respect to distance from the center of the Earth [1]. The graph only depicts the GPE outside of the radius of the Earth, as that is the focus of this research.*

# 2 Theory and Previous Research

## 2.1 Student Difficulties with Potential Energy

A resources framework for learning recognizes that all students bring previous knowledge and skills with them to the classroom that can be tapped into as resources. These resources can be applied either correctly or incorrectly to physics topics, and the same idea can be used in different ways [2]. For example, if a student said that "pushing a cart makes it go faster," a resources framework suggests that the student knows that more force means faster, so we need to go one step further to teach the student that more force means more acceleration, which in turn means the maximum speed of the cart is faster. For the purposes of this study, student difficulties are defined as resources applied incorrectly or to an incorrect situation. Students have common difficulties when they are working with potential energy, gravitational or otherwise: equating $\frac{dU}{dx}$ to acceleration, equating the sign of $\frac{dU}{dx}$ to the sign of the force (it is opposite), treating the minimum value of potential energy as zero (and subsequently believing that potential energy cannot be negative), and ignoring or misusing mathematical expressions for potential energy (*i.e.*, using $U = mgh$ instead of $U = \frac{-GMm}{r}$) [3,4].

Additionally, students often believe that potential energy is inherent to an object rather than a characteristic of a system. For example, when presented with a pendulum swinging near the surface of the Earth, students can apply knowledge about the change in potential energy (the potential energy decreases as the pendulum swings down and increases as the pendulum swings up) but they often describe the potential energy as being something the pendulum has ("the pendulum's potential energy increases" or "the potential energy of the pendulum decreases as the pendulum swings down"), rather than something the system has ("the potential energy of the Earth and pendulum system increases") [5].

When relating potential energy to force, students tend to use the strength of the force to reason about the value of the gravitational potential energy; the stronger the force, the greater the gravitational potential energy. This leads students to incorrectly state that GPE increases as two objects get closer to one another. For example, in one study, a student built on that explanation to say that "since $PE = -\frac{GMm}{R}$, as you increase R, the total potential energy of the system will decrease.

Notice that as R becomes closer and closer to infinity, the potential energy of the system becomes zero" [5]. However, GPE actually increases as R increases, even though GPE does go to zero as R approaches ∞ (Figure 3).

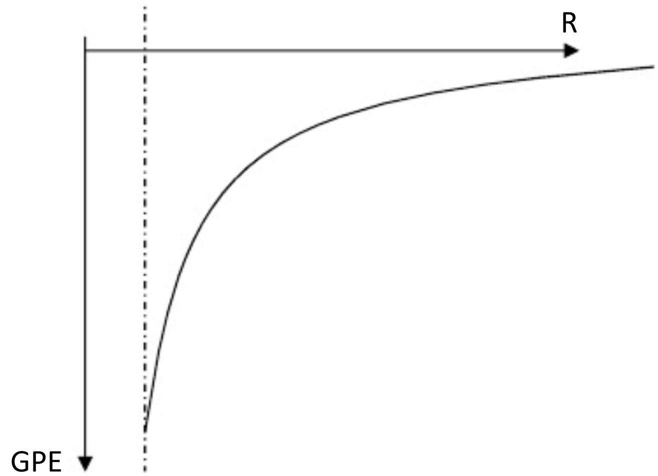



***Figure 3:*** *Gravitational potential energy as a function of distance. The dashed line represents the surface of the Earth [6]. The GPE inside of the Earth's surface is not depicted as it is not the focus of this research.*

## 2.2 How Students Incorporate Data That Doesn't Match Their Reasoning

When presented with data that does not fit the student's own reasoning, students will either ignore the data or change their reasoning to accommodate the new data. The strategy a student chooses is influenced by other students' interaction with the data (students are more likely to ignore the data if everyone else ignores it too), social status (students with more friends in their groups are more likely to be able to convince their friends to re-examine the data), and where the students are in their discovery (if students are past the part of the assignment the anomalous data fits into, they are more likely to ignore it) [7].

## 2.3 Manipulatives in Education

Representations, whether internal or external, are likenesses or simulations of ideas, concepts, or objects. External representations are those available in the environment (maps, graphs, models, pictures, equations, etc.) and internal representations are available only in the mind of the learner (memories, expectations, mental models, recollections) [8,9]. Each representation has two meanings: what it is meant to represent and what it actually is. For example, a bunch of lines on paper can represent a map of the world, even though it's actually a bunch of lines on a piece of paper. Novices have a more difficult time interpreting the representation, while experts often have a difficult time seeing what the representation actually is [9]. For example, before you learn to read a language, every word looks like a lot of lines and dots and slashes, but once you know what those lines, dots, and slashes mean, it's nearly impossible not to see them as letters, words, and sentences. Thus, for a representation to work effectively, students must be able to understand the form of the representation, how the representation relates to the information, and how to move between that representation and another (equations, graphs, the student's mental model, etc.) [9].

A manipulative, like the plastic surface used in this study, is a type of external representation. Similarly, the graphs students are asked to draw in the instructional activity (Appendix 1) are external representations of a different type (because they are made of different materials and have different dimensionality). Learning with multiple external representations, both provided and student-created, "support different cognitive processes…, constrain interpretation options, and promote deep level understanding" [8].

## 2.4 Surface Manipulatives

Plastic surfaces [Figure 4] like the one used for this project were created by the Raising Physics to the Surface project team (NSF DUE Grant No. 1612480).

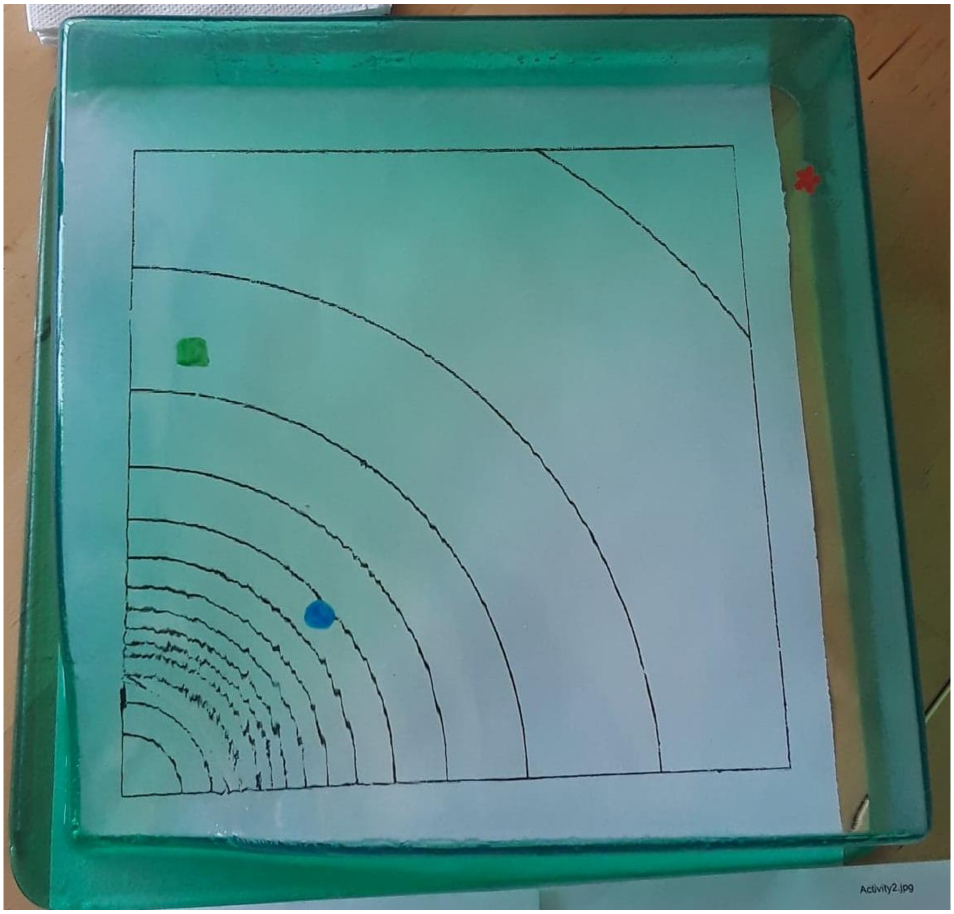



***Figure 4:*** *A surface used to represent the gravitational potential energy of an Earth-object system and the accompanying contour map.*

During the design portion of this experiment, the OSU surfaces group talked with Dr. Aaron Wangberg via Skype to discuss his process for making the surfaces. Wangberg first designed the surfaces while he was a graduate student teaching multivariable calculus. At the end of the course, he noticed that students didn't understand mathematical concepts like gradient and divergence, and they couldn't apply the information in multiple ways. So, Wangberg began making papier-mâché models from footballs, books, and other household objects to create wavy surfaces for students to physically interact with these mathematical concepts. The surfaces helped students understand the meaning behind gradient and divergence, but they weren't durable, smooth, or reusable once they were written on.

Wangberg stopped using the surfaces when he accepted his current position at Winona State University, but several years later met a non-traditional student who was a mold maker. Wangberg asked the student for help in refining the surfaces, and the student suggested using a CNC machine (a machine designed to precisely cut material in three dimensions [10]). So, Wangberg began to create wooden surfaces. These surfaces were more durable than papier-mâché but required constant upkeep. Since the wood was spray-painted, the surfaces were pseudo-dry-erasable, but they had to be repainted at the end of each term and cured for 30 days afterwards.

Eventually, Wangberg began research on these surfaces and he and his research team discovered a new way to create the surfaces – with thermal formation. Thermal formation is a process that shapes heated plastic to a mold; this process is less expensive and faster to make than previous models and the plastic surfaces are transparent, so students can place contour maps underneath, and dry-erasable, so students can write on top of the surface. Now, Wangberg prints a 3D mold, sands the mold until smooth, then thermal forms plastic over top.

## 2.5 Working in Groups

The activity in this study was performed in groups of 2-3, based both on the size of recitations (approximately 4-15 students) and the recommended group size for activities [11]. There are two types of groups: cooperative and non-cooperative. For the purpose of this study, a cooperative group is a group where the students are working together to answer prompts, as opposed to a non-cooperative group where either the students do not work together, one or more students is working on their own or otherwise not participating, or one or more students pushes forward without waiting for the other group members to understand.

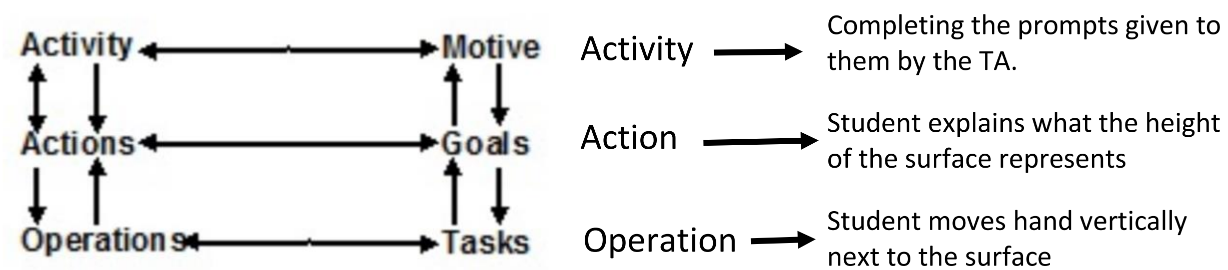
In cooperative groups, the group tends to perform better than the comparative work of the best student in the group. In several studies where students were placed in cooperative groups throughout a course, students in the top, middle, and bottom third all improved in problem solving ability by the end of the course [12,13]. This is because students are required to elaborate, explain, or defend their position to the other group members, resulting in increased comprehension. “When students talk through what they are learning, they not only learn more, but they are also more likely to develop a strategy for learning the material” [12]. In groups, students create more useful descriptions with less conceptual difficulties than they would on their own because they are forced to explain their reasoning and work through each step as they go [13].

## 2.6 Activity Design: Theory

Beneficial texts have the following characteristics: concise sentences with no complicated words or phrases when possible, clearly arranged text with visible structure, and text that is interesting to the reader [8]. These were the guidelines I tried to follow when designing the activity. When students solve these types of problems, their “solutions exhibit more expertlike [sic] characteristics” including drawing useful diagrams and exhibiting logical mathematical progressions [13].

## 2.7 Activity Theory

During the activity, students will interact with the surface, each other, and the worksheet. The Russian psychologist, Leontiev, determined that interactions students have during an activity can be split into three levels based on complexity: the activity, the actions, and the operations (Figure 5a). Starting with the most basic, operations are the things students do: pick up a pencil, turn to a classmate, or rotate the surface. Actions are the reasons for doing the operation, not to be confused with “interactions,” which I also use to describe “operations.” The activity is the overarching motivation for setting goals and performing operations. More concisely, actions are *what* is done, operations are *how* it is done, and the activity is *why* it is done [14]. An example is shown below in Figure 5b.




(a) (b)

***Figure 5:*** *(a) Leontiev’s activity hierarchy. The vertical arrows represent possible movement between categories and the horizontal arrows connect Activity Theory terms with corresponding synonyms. [14]. (b) An example of Leontiev’s hierarchy based on data from the research in this paper.*

# 3 METHODS

## 3.1 SURFACE DESIGN

Liz Gire and Paul Emigh were primarily responsible for the design of the plastic surface [Fig 6], while the manufacturing was done by Aaron Wangberg.


***Figure 6: Gravitational Potential Energy Surface**. This surface represents a quarter-section of the gravitational potential energy for an Earth-object system. There are three marked points for use in the activity: a blue circle, a green triangle, and a red star. The Earth is located in the corner where the surface is lowest (shown above) and the Earth's surface is etched into the plastic.*

The size and color of the surface were chosen purely for convenience. The surface is slightly smaller than a piece of paper, so that a corresponding contour map could be printed on an 8.5”x11” sheet of paper. The color was chosen only to distinguish it from other surface models for different physical systems. Several mathematical decisions were also made for convenience. The ratio of the Earth to the rest of surface (in other words, how much of the Earth vs. how much empty space was included in the surface) was chosen to give students adequate workspace both inside and outside of the Earth. The Earth was modeled as having a linear mass density that never reaches zero.

The surface was chosen to be a quarter-plane (rather than a full plane with the Earth in the middle) for several reasons: to keep the size approximately a sheet of paper while showing a significant amount of the potential energy trend and because gravitational potential energy is symmetrical, so having a full plane would be redundant. The marked points on the surface were chosen to be at approximate distances of 2 Earth radii ($r_e$) from the center of the Earth, $3r_e$, and $6r_e$ so that the difference in potential energy between the first and second points was equal to the difference in potential energy between the second and third points. The Earth's surface was etched into the surface so that students could easily identify where the potential energy began to be affected by the changing mass.

## 3.2 Activity Design: Methods

I began designing the instructional activity by identifying the main learning goals: students can switch between representations (surface, graph, contour map, equations, concepts), students can recognize that potential energy can be negative, students can recognize that gravitational potential energy increases to 0 at infinity, and students can recognize that gravitational force is the negative gradient of gravitational potential energy. The activity went through several revisions in which the questions were changed to be more general and the activity was given more context. The final copy of the activity is located in Appendix 1.

The prompts in the activity were ordered in such a way that students could draw on their previous knowledge in the beginning of the activity, then apply that knowledge to new situations and answer a previously unknown question at the end ("What is the relationship between potential energy and force?"). Each sub-prompt was designed to give students hints on how to answer the prompt and help students check their answers.

I also created an instructor's guide to provide structure for the TA and provide possible follow-up questions that TA could ask the students about their work (Appendix 1) and a solution to the activity (Appendix 2). The contour map in Appendix 3 was created by Aaron Wangberg.

## 3.3 Implementation

The activity was implemented in the PH212 recitation course. The PH21x series is introductory calculus-based physics and is composed of 3 terms (PH211, PH212, and PH213), typically beginning in the spring of the students' first year and continuing to the fall and winter of their second year. Each course in the series has a lecture and corresponding lab, studio, and recitation. The lectures meet twice a week, on Tuesdays and Thursdays, and the labs, studios, and recitations meet once a week. The students in the first few recitations had not had any lecture on GPE or gravitational force due to spherical bodies, but by the last recitation, every student had lecture on these topics. In PH211, students learn the near-Earth approximations for GPE and gravitational force, $U = mgh$ and $F = mg$.

The lab and studio give students hands-on experience with conceptual ideas from the lecture and expand on the course material. The recitation is a supplementary course to the lecture course and allows students to work in small groups on homework, activities, and other textbook problems, as well as get clarification on lecture, lab, or studio material from the teaching assistant (TA). Unlike the lecture, lab, and studio, the recitation is optional for the course, however, several STEM majors (like physics) require the recitation.

During the class, students were split into groups of 2-3. The TA introduced the material, as some students had not had a lecture on gravitational potential energy yet. Then, the handout (Appendix 1), surface (Fig 6), and contour map (Appendix 3) were passed out to the students and they began working on the prompts. Summaries of the videos are located in Section 4.2 and Appendix 4.

## 3.4 Data Collection and Storage

To collect data, I attended three recitations and filmed one group of students in each class. I also scanned the handouts from each student in the filmed group. I stored the videos and scans on an external hard drive in the PER lab at Oregon State University as well as on a cloud-based drive called Box. The students' names have been removed from the transcriptions and replaced with code names or letters. I then transcribed the videos and recorded the interactions the students had with the surface. I then sorted the students' interactions into categories based on type of interaction and reason for interaction.

# 4 RESULTS

## 4.1 PRESENTATION OF RESULTS

I transcribed the audio of each group's activity as well as the interactions the students had with the surface. I then separated the interactions into 6 categories, each with a varying number of subcategories (Section 4.3 and Figure 7), to describe the type of operation each student performed. The interactions were also separated into categories describing the underlying reason behind each operation (Section 4.4 and Figure 7). (Figure 7 is located on the next page to save space).

I counted the number of times each group and each student within each group interacted with the surface (Section 4.5). I also coded the interactions between students within the group based on whether the interaction was indicative of a cooperative group or a noncooperative group (Section 4.6).

In this section, I will first give a brief summary of each video and introduce the student pseudonyms. The groups are titled Patterns, Fruits, and Colors, and each student's name matches the group title; The Pattern Group contains the students Paisley, Plaid, and Stripes, The Fruit Group contains the students Apple, Peach, and Grape, and The Color Group contains the students Pink and Purple. Next, I will describe each category and subcategory of the operations and present the relevant results. I will also describe each category of the actions and present those results as well. Then, I will present the data on individual students' interactions. Finally, I will present the cooperative vs. noncooperative data.

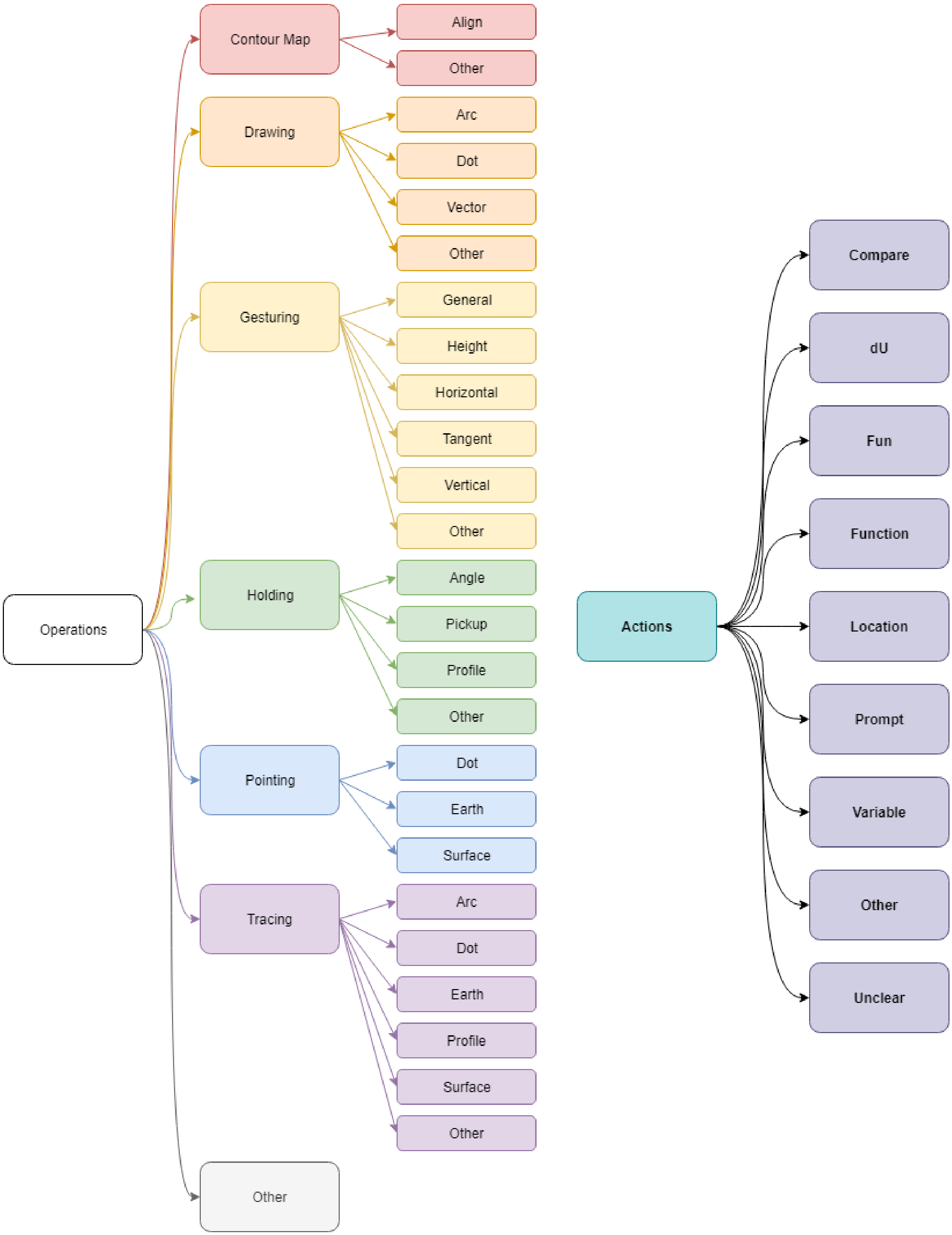



***Figure 7:*** *Diagrams of the categories of operations (left) and actions (right) used for this research.*

## 4.2 Video Summaries

### 4.2.1 Brief Summaries

See Appendix 4 for extended video summaries and students' written responses to the prompts.

The TA introduces the activity in similar ways for each group, so I will only summarize the introduction once. The TA tells the students that the surface "represents the gravitational potential energy of space around Earth" and that height indicates the GPE. The TA explains that the surface and contour map are dry-erasable and that the students are welcome to draw on them.

**Group 1/Patterns (Paisley, Plaid, Stripes):** This is the first recitation of the week. The students' pseudonyms are Paisley, Plaid, and Stripes. The group starts with Paisley and Plaid; they answer questions 1 and 2 (see Appendix 1) before Stripes arrives. Paisley and Plaid continue with the questions while Stripes catches up. The group struggles with drawing the axes for their graphs in question 3; Stripes suggests a polar coordinate system, but is persuaded otherwise by Paisley and Plaid, then the group labels their axes with GPE on the horizontal and distance on the vertical (opposite of how the surface is laid out). After struggling to make sense of the graph, they realize that the graphs should be drawn with GPE on the vertical and distance on the horizontal. All three draw their graphs above the horizontal axis, making GPE positive. The group answers questions 4 and 5 using conceptual knowledge from previous experiences. The TA pauses the class to discuss the graph of GPE. During the discussion, Paisley points out that the graph looks like the profile of the surface. After the discussion, the group realizes they should have drawn their graph below the horizontal axis rather than above; they change their graphs.

While answering question 6, the group struggles with determining the sign of the rate of change. Paisley thinks the rate of change will be negative since the GPE is decreasing, but Plaid points out that the slope is always positive, even though it's decreasing. For question 7, the group draws the graph of force as a horizontal flip of the GPE graph, citing that kinetic energy is the opposite of potential energy and force is related to kinetic energy, therefore force must be the opposite of GPE. The group references their graphs when answering question 8; they decide they agree that "force is the negative gradient of gravitational potential energy." The TA discusses the group's results and reminds the students that it's the *change* in GPE that determines the force, not the *value* of GPE. The students soon forget this and continue talking about GPE as the opposite of force. The whole class discusses the graph of force vs. distance and discovers that the force graph should also be below the horizontal axis. The TA discusses coordinate systems as a way to determine the sign of the force; since the force points in the opposite direction of the positive radial coordinate, the force is negative. End of class.

**Group 2/Fruits (Apple, Peach, Grape):** This is the second recitation of the week. Two out of the three lecture sections have happened, but some students are in the afternoon lecture (which has not happened yet) and some students won't get to the gravity material until later in the week. The students' pseudonyms are Apple, Peach, and Grape. The group begins by determining where the Earth is located on the surface. Each student agrees that GPE increases, but Apple believes that GPE will only increase to a specific point before "flattening out." Peach and Grape answer question 2 by saying that "as long as the radius stays the same," the GPE will stay constant as well. Apple is confused by the wording of the question; I used the terms "gravitational potential energy" and "potential energy" to refer to GPE, which causes Apple to consider them two separate quantities. Apple tries to equate the expression $-\frac{GMm}{r^2}$ (which is force and not GPE), which Grape remembered from lecture, and the expression $mgh$, which Apple remembered from previous physics classes. This leads to a discussion between all group members as they try to determine how those expressions can represent the same quantity. The TA gets involved and somewhat resolves the situation, though I can tell from the video that Apple is not convinced. The group answers questions 3 and 4 with little discussion. The discussions for questions 5 and 6 are led by Peach; Apple and Grape seem to take everything Peach says as correct, without asking for much clarification. No spoken agreement is reached about either question, though each person has a written answer (except for Apple, who does not answer question 5). While drawing the graph of force, this group reaches the same conclusion as The Pattern Group; force is the opposite of GPE. The TA brings the class together to discuss the graphs of GPE and force. The discussion is nearly identical to that of The Pattern Group's, though the Fruit Group does not seem to reach the same conclusions drawn by The Pattern Group afterwards. At the end of the discussion, the TA tells the class that anyone who is finished may leave. The Fruit Group leaves, even though they are not finished. End of class.

**Group 3/Colors (Pink, Purple):** This is the fifth out of six recitations of the week. This took place two days after the first groups' recitations, so every student has had at least one lecture that week, most have had two lectures. The students' pseudonyms are Pink and Purple. Pink and Purple initially determine that the GPE decreases as the distance from Earth increases, citing the equation $\frac{GMm}{r^2}$, and never verbally correct themselves to say that GPE increases, though their written answers both say that

GPE increases as distance increases. The two students quickly draw contour lines and align the surface with the provided map. They both draw a graph of GPE, citing the equation $-\frac{GMm}{r}$. Neither Pink not Purple write anything down on their worksheets for the 4th or 5th prompt, though they do mark on the surface the direction of the force (towards the Earth) and a point with a greater force than the blue point. The two begin discussing the sign of the rate of change of GPE, saying that the answer "depends on which way your positive is." They call over the TA and discuss direction and what the rate of change means. Purple says that the rate of change should be positive and Pink agrees after acting out the slope with a pencil. Pink and Purple both draw their graphs for the seventh prompt, first drawing them as an entirely positive force. Initially, Pink and Purple both disagree with the statement on the eighth prompt because the negative sign doesn't match their graphs for force. However, after explaining the predicament to the TA, they discover that the force should be negative since it is pointing in the opposite direction of $\hat{r}$. Pink ends the discussion by saying "it doesn't really matter, as long as you're, like, careful about which way you're pointing it [the force]." The class ends with a group discussion similar to the end-of-class discussion in The Patterns Group.

## 4.3 Operations

When organizing the students' interactions with the surface, I found 5 major categories of interactions: holding, drawing, pointing, gesturing, and tracing. I made a 6th category called contour map to track when and how the students interacted with the provided contour map. First, I will describe each category, give an example, and display individual data. The overall data will be included after each category has been introduced.

### 4.3.1 Holding

This category includes interactions where students lift the surface, turn the surface, or move themselves around the surface. The three subcategories are angle, profile, and pick up. Angle means moving the surface or one's self to see the surface from another angle. In Figure 9a, Purple is looking through the surface to the contour map below to make sure that the two pieces are lined up correctly. Profile means specifically looking at the surface from the side highlighted in Figure 5. Students typically use this method when drawing the graph of GPE. In Figure 9b, the TA is holding up the surface to a graph of GPE drawn on the board to compare the shape of the surface's profile to the shape of the graph.

***Figure 8:*** *The surface used in the activity. The side containing Earth on the right is highlighted, demonstrating what is called the "profile" in the rest of the paper.*

Pick up means to move the surface for any reason, usually to retrieve something from underneath or to relocate the surface. In Figure 9c, Pink is picking up the surface to move it on top of the contour map.

(a) (b) (c)

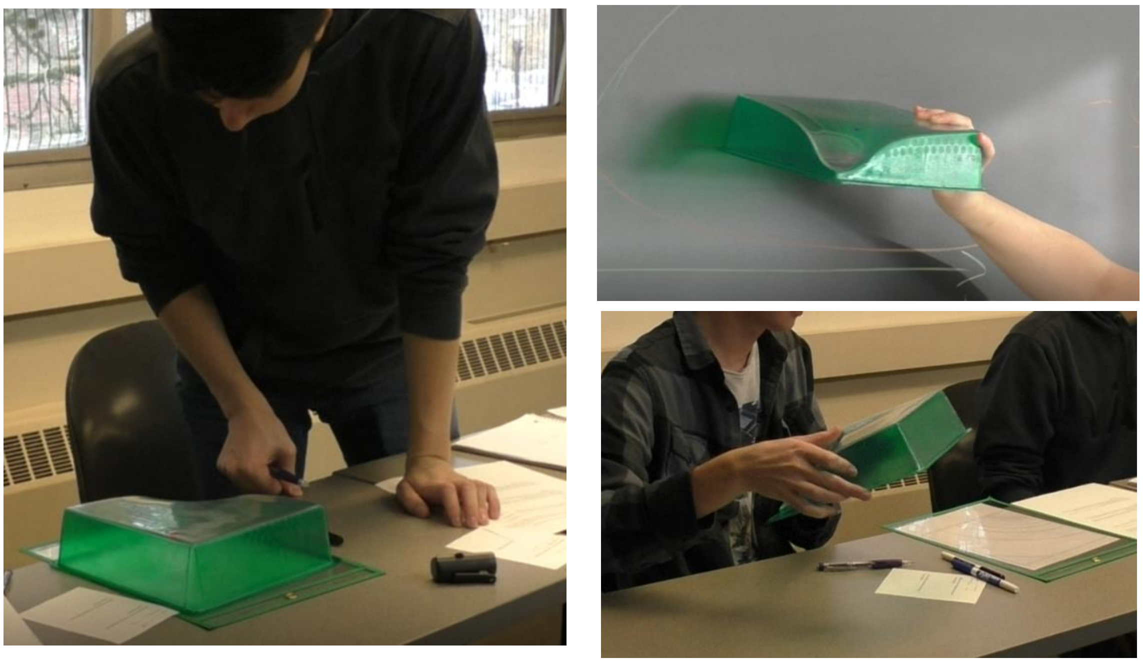


***Figure 9:*** *Examples of (a) angle, (b) profile, and (c) pickup.*

Students most often picked up the surface to relocate it or looked at the surface from a different angle [Figure 10]. There was only one instance of a student looking at the profile of the surface; when Peach was drawing the graph of GPE.

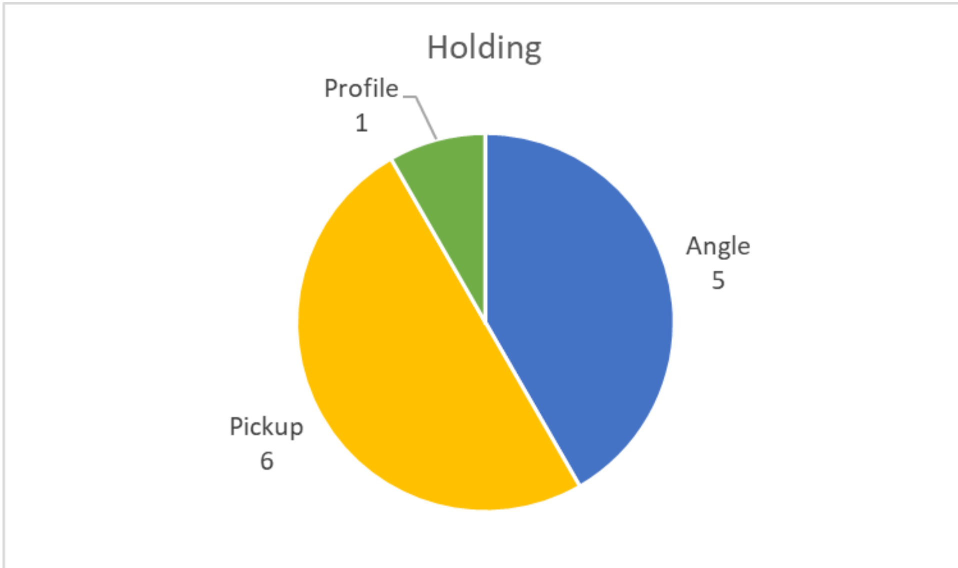



***Figure 10:*** *The number of each interaction within the holding category.*

### 4.3.2 Drawing

Students were given dry-erase markers to write and draw on the surface and contour map. Students drew arcs (lines of constant radius, as in Figure 11), dots, and vectors.

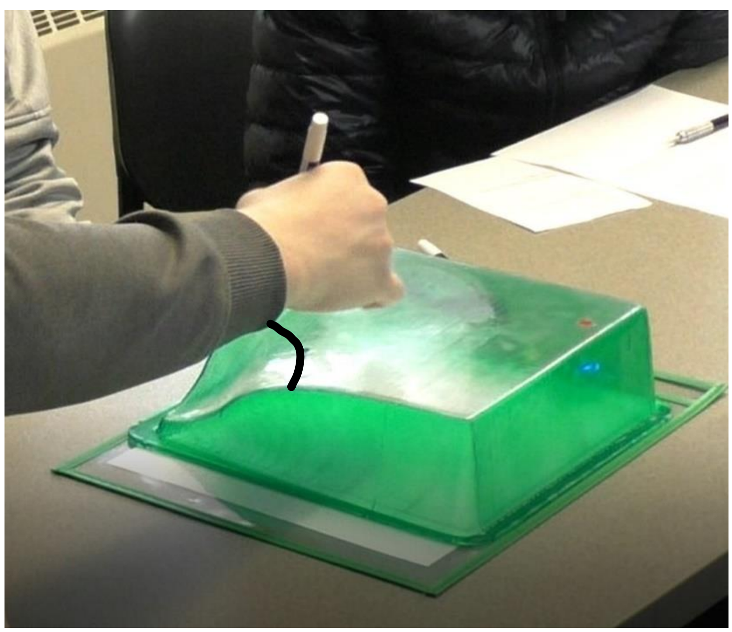


***Figure 11:*** *A student drawing on the surface in response to question 2 (see Appendix 1). I have emphasized a previously drawn arc.*

Students most often drew arcs on the surface, followed by vector and dots [Figure 12].

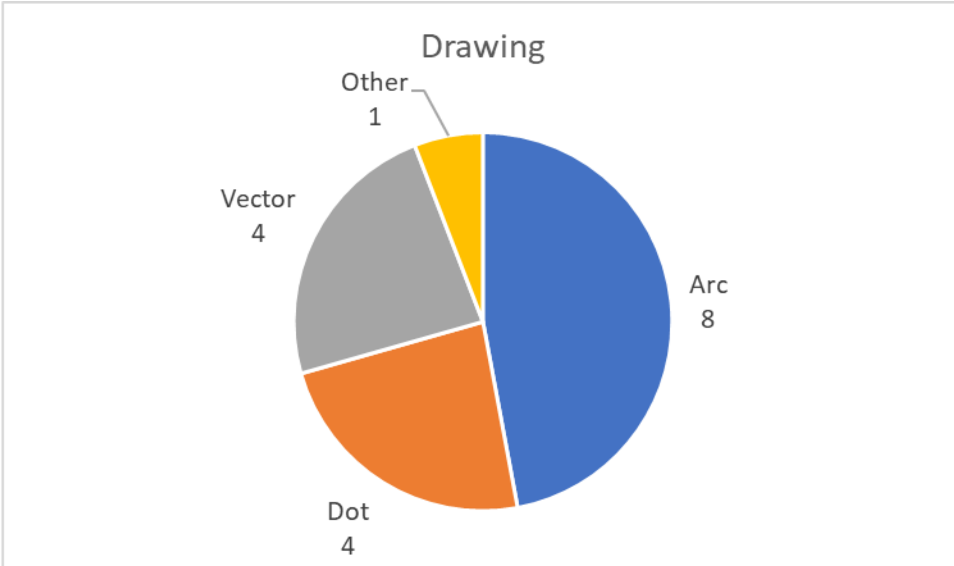



***Figure 12:*** *The number of each interaction within the drawing category. The section "Other" includes students doodling on the surface.*

### 4.3.3 Pointing

Students used fingers or other objects to point to dots (either self-drawn or included on the surface), the Earth, a self-drawn arc on the surface, and the surface in general. In Figure 13, Grape is pointing at the blue marked point on the surface while comparing the force there to the force at the green marked point.

***Figure 13:*** *A student pointing to a dot on the surface.*

Students most often pointed to dots (marked points or points the students drew) during the instructional activity [Figure 14].

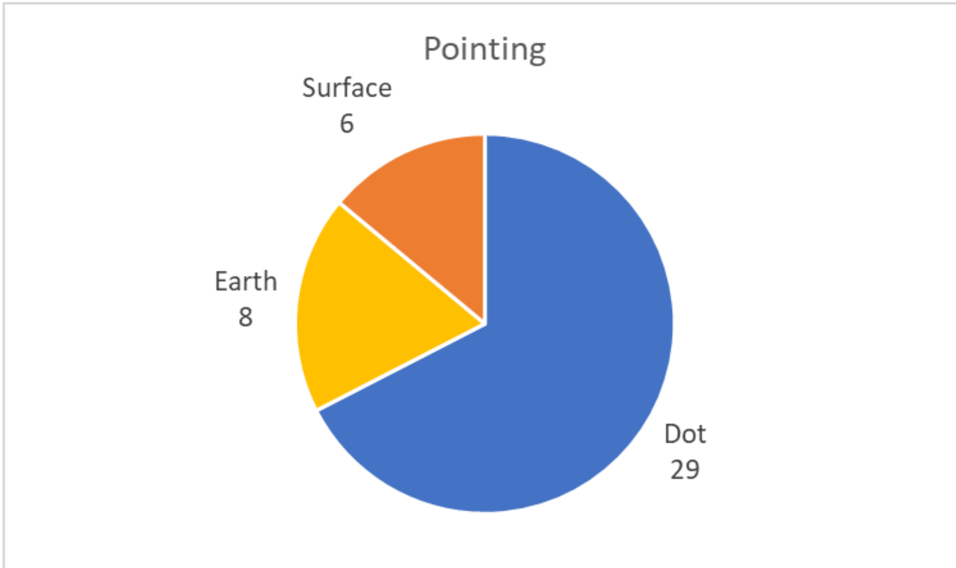
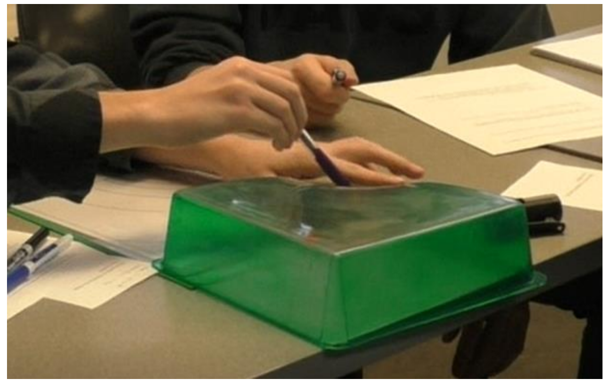



***Figure 14:*** *The number of interactions within the pointing category.*

### 4.3.4 Gesturing

Students used their hands or fingers to gesture at or near the surface while describing certain features of the surface. Horizontal and Vertical mean students moving their hands horizontally or vertically, respectively, over or near the surface. Height is when students use fingers in a pinching motion to represent the height of the surface. In Figure 15a, Pink places a thumb at the top of the surface and the rest of the hand at the bottom of the surface to indicate the height of the surface. General represents the students gesturing at the surface in general, without performing a specific operation or discussing a particular feature. Tangent is when students use a finger or other straight object to show the tangent to the surface at a specific location or along a specific path. In Figure 15b, Pink places a pencil on the surface at the blue marked point to represent the tangent line, and thus the force, at that location.

(a)



(b)

***Figure 15:*** *(a) A student spreads the thumb and pointer finger to show the height of the surface. (b) A student uses a marker to demonstrate the tangent to the surface at a specific point.*

Students gestured in each category somewhat equally [Figure 16]. The "tangent" category had the most interactions, however, the only group to perform those operations was The Color Group.

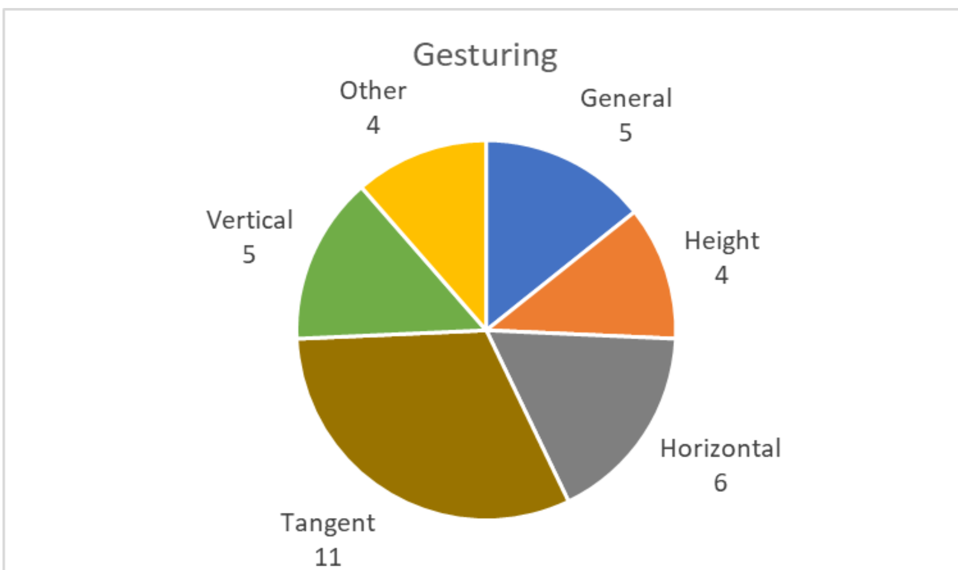



***Figure 16:*** *The number of interactions within the gesturing category. Gestures in the "Other" category include students spanning the width of the surface with their thumb and pointer finger and placing a finger and thumb at two points on the surface, then bringing the finger and thumb together.*

#### 4.3.5 Tracing

Tracing is when a student uses a finger or other object to trace a feature of the surface, whether touching the surface or not. The subcategories are Arc, Dot, Earth, Profile, and Surface. When a student traces an arc on the surface or the profile of the surface, the operation is labelled Arc and Profile, respectively. Tracing a Dot or Earth means to trace a path to or from a dot or the Earth, respectively.

Tracing the surface means that the student was tracing around the edges of the surface, excluding the profile edge (Figure 8), or on top of the surface in no particular direction. In Figure 17, Apple is tracing a line of constant radius intersecting the blue marked point to depict an equipotential line. This is in response to question 2 (see Appendix 1).

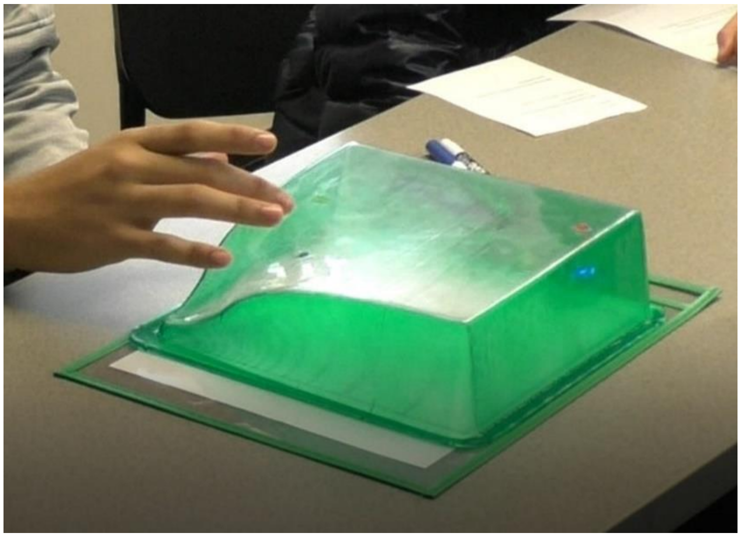


***Figure 17:*** *A student tracing an arc on the surface with the middle finger. This operation would be categorized as Arc.*

Students most often traced arcs, the etched surface of the Earth, and the surface in general [Figure 18].

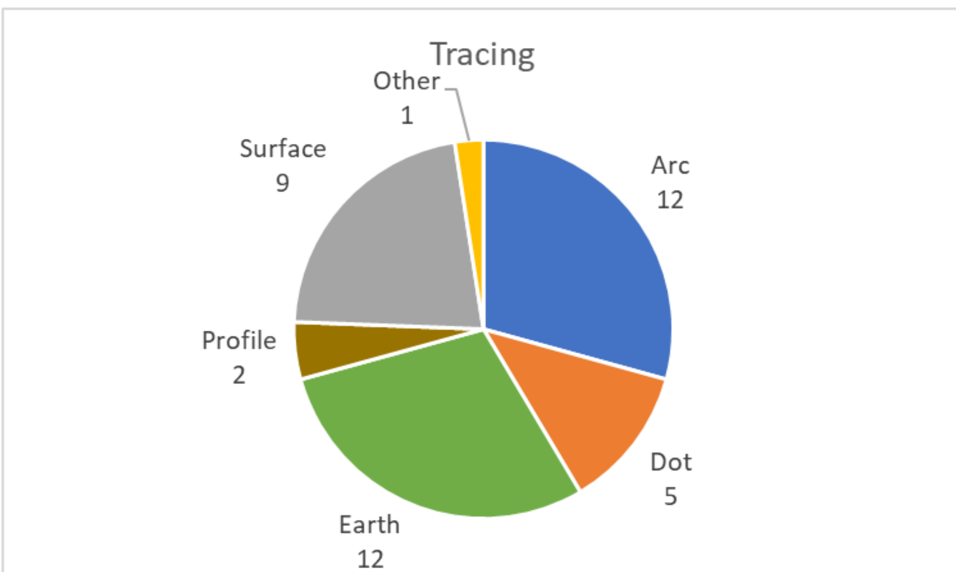



***Figure 18:*** *The number of interactions within the tracing category. The category "Other" includes tracing vertically on the side of the surface.*

### 4.3.6 Contour Map

Contour Map describes an interaction with the contour map in any way. Most often, the students only interacted with the contour map to align it with the surface as instructed in question 2 (Appendix 1).

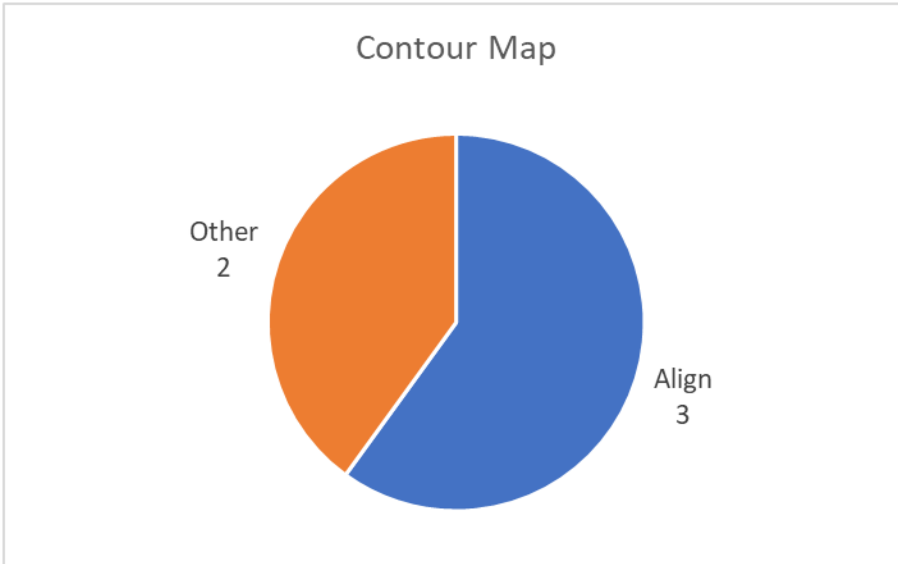



***Figure 19:*** *The percentage of interactions within the contour map category. "Other" includes picking up the contour map and looking at the contour map through the surface.*

Overall, students most often traced on, pointed at, or gestured around the surface (Figure 20).

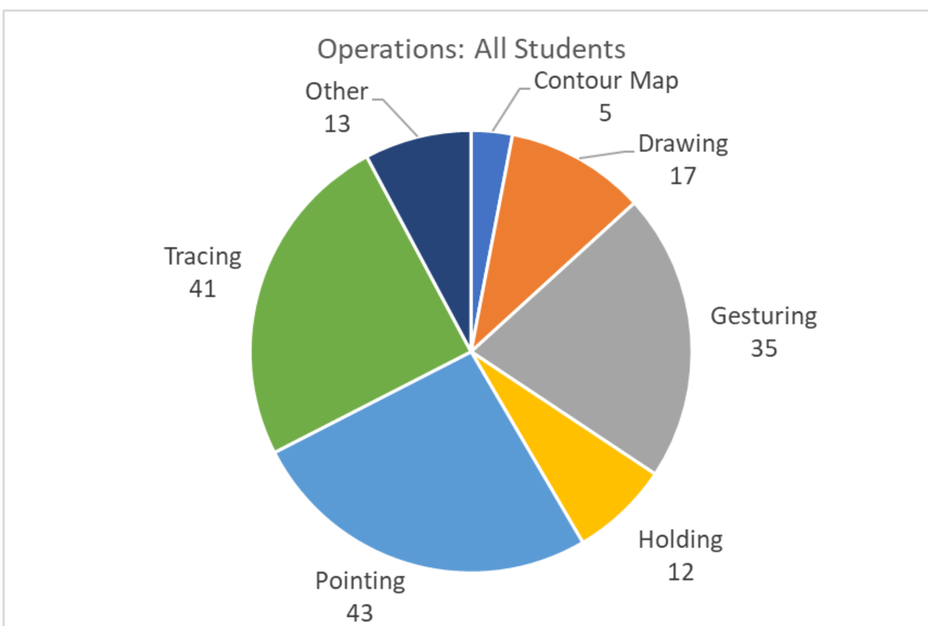



***Figure 20:*** *The number of operations that all groups had split by category.*

## 4.4 ACTIONS

I separated the operations into 8 categories, plus one category for actions that were unclear to me. The categories are: compare, dU, fun, function, location, prompt, variable, other, and unclear. I will present this data in a similar fashion to the operations data; first I will describe each category, provide examples, and then present the data.

### 4.4.1 Compare

This category includes operations where students were comparing characteristics of two or more places on the surface. Often, students would compare the magnitude of the force at two different points in order to answer questions 4, 5, and 6 (Appendix 1).

### 4.4.2 dU

The category "dU" represents the operations where students looked at the rate of change, derivative, or slope of the surface at specific places. It is called dU to mimic the derivative expression $\frac{\partial U}{\partial r}$. Despite derivative, rate of change, and slope commonly used interchangeably, I have included each of them separately because I want to be clear about what counts as a "dU" action. This action mostly occurred when students were examining the force at various places.

### 4.4.3 Fun

The "fun" category includes interactions students had with the surface for the sole purpose of fun. This includes when Pink placed a whiteboard marker on the surface and caught it as it rolled off and when Paisley drew a circle on the surface and called it a planet.

### 4.4.4 Function

The "function" category includes operations that students used to examine functional behavior, either of GPE or gravitational force. Typically, students would talk about how GPE increases as distance increases and would try to determine how quickly GPE or force increased. This action occurred most often when students were drawing their graphs (questions 3 and 7, Appendix 1).

#### 4.4.5 Location

The "location" category was used to classify interactions that students used to show the location of an object or specific place on the surface. Typically, this was students clarifying the location of the Earth on the surface or pointing out which marked point they were talking about in a conversation.

#### 4.4.6 Prompt

The category "prompt" describes interactions that students did because the activity told them to. For example, question 2 asks students to draw lines of equal GPE through the marked points, so when a student performed those operations, they were classified as "prompt."

#### 4.4.7 Variable

The category "variable" includes interactions where students were discussing the characteristics of a specific variable, for example, discussing the sign or direction of $\hat{r}$. It also includes discussing a function (like GPE or force) without talking about functional behavior. This occurs several times when students point out that the height of the surface represents GPE or that the vector drawn on the surface represents force.

#### 4.4.8 Other

The "other" category includes all actions that I could distinguish a purpose for, but that didn't fall into any of the other categories. This includes when Paisley shows Stripes the work Paisley and Plaid did earlier, when Paisley explains the materials to Stripes, and when students moved the surface.

#### 4.4.9 Unclear

The "unclear" category contains all the operations I couldn't distinguish a purpose for. The operation in this category were "Pink traces diagonally through the surface," "Pink appears to be drawing over the blue dot," "Plaid traces the edge [not the profile] of the surface with a pencil," "Paisley traces diagonally through the surface," "Paisley traces the side [not the profile] of the surface with a pencil," "Paisley has a hand resting on the surface," and "Apple rubs the surface near the Earth [I couldn't determine if Apple was erasing something]." Most of the time, these were idle operations that may not have had a purpose other than to occupy the hands of the student during the activity.

As you can see in Figure 21, the students most often compared, discussed the functional behavior, and examined the derivative (or rate of change or slope).

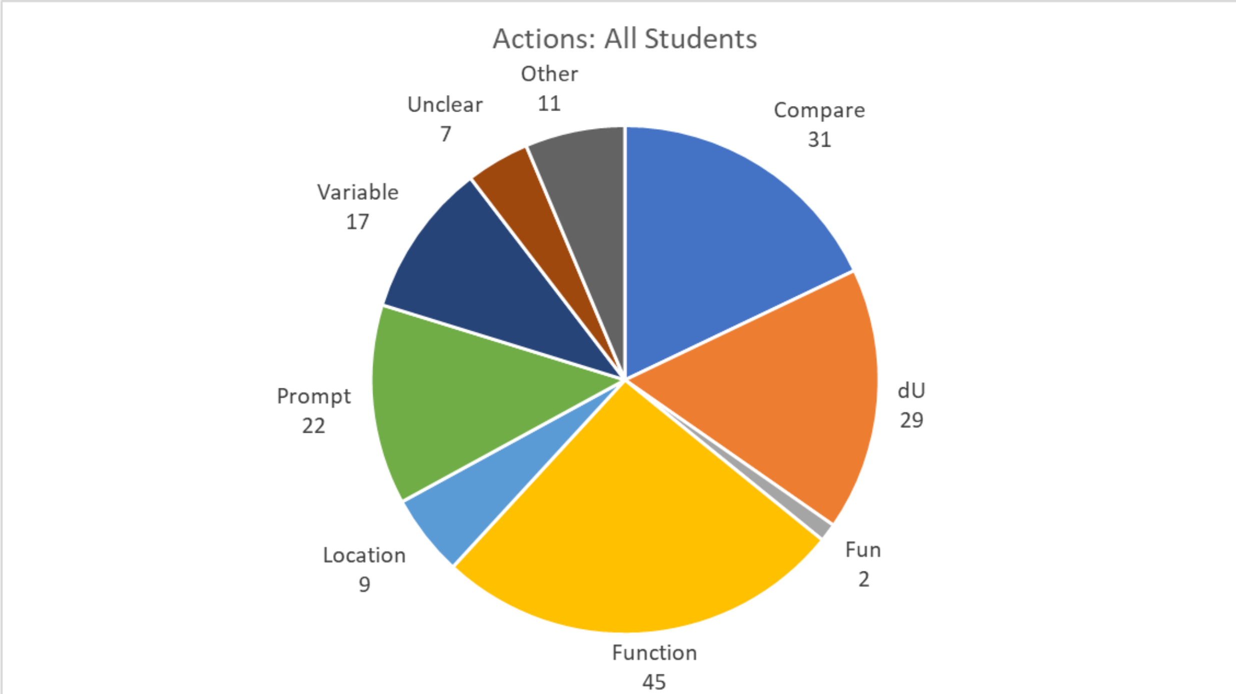



***Figure 21:*** *The actions of every group, split into categories. See above for descriptions of each category.*

## 4.5 Students

The Pattern Group interacted the most with the surface manipulative, 67 times, while the Fruit Group interacted only 54 times, and the Color Group interacted 45 times [Figure 22].

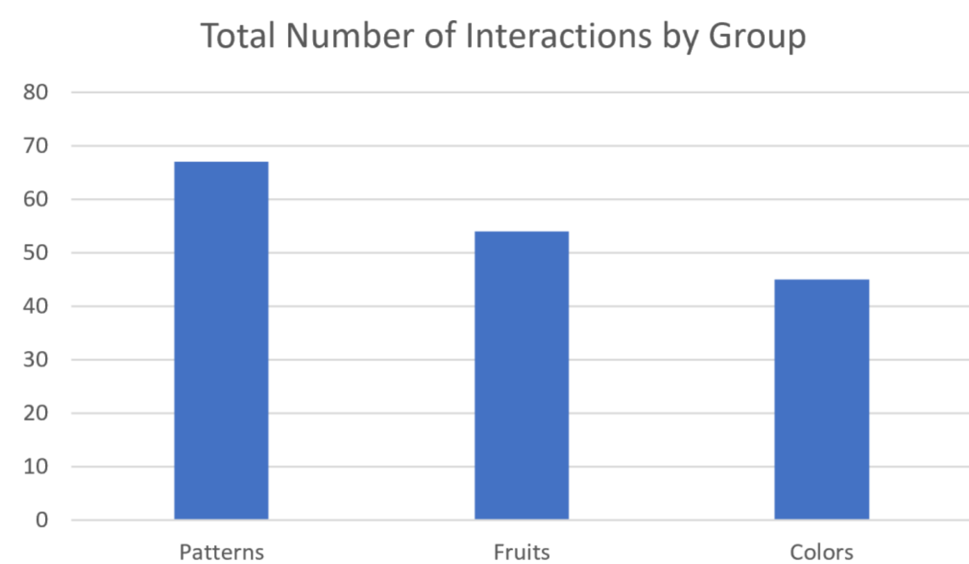



***Figure 22:*** *The total number of times each group interacted with the surface during the activity.*

However, the Color Group also had 2 people in it instead of the usual 3 people, therefore it was expected that they would interact less than the other groups. If we look at the number each individual person interacted with the surface [Figure 23], we can see that the Pattern and Fruit Groups have one person taking over a large portion of the interactions (Paisley with 44 and Peach with 26), one person interacting a very small amount (Stripes and Grape both with 8), and one person somewhere in the middle (Plaid with 15 and Apple with 20).

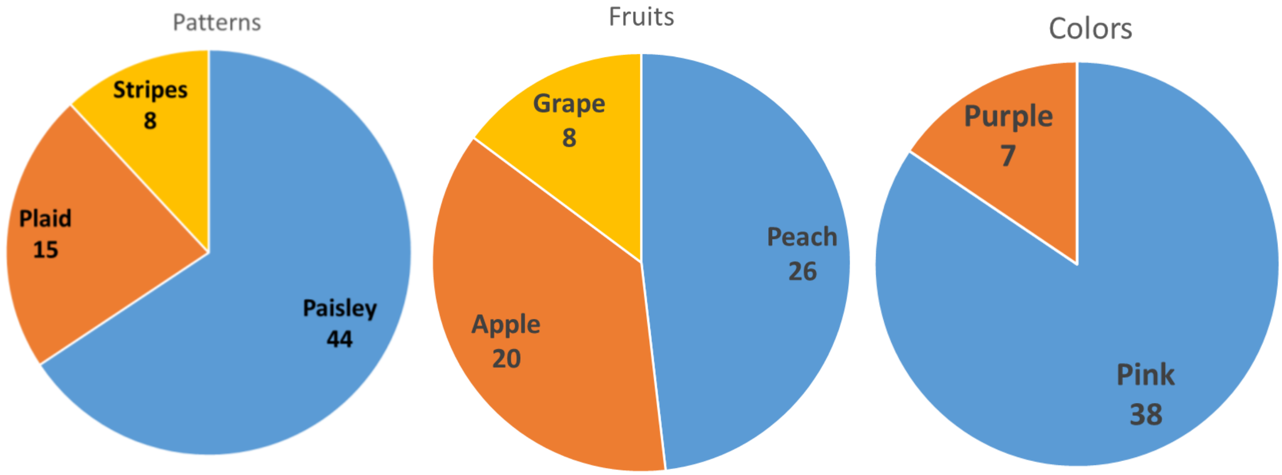



***Figure 23:*** *The distribution of interactions between each student within a group.*

The Color Group is missing that person in the middle. So, to make the Color Group's interactions comparable to the Pattern Group and 2, we can add an imaginary person that interacts between 15 and 20 times with the surface [Figure 24].

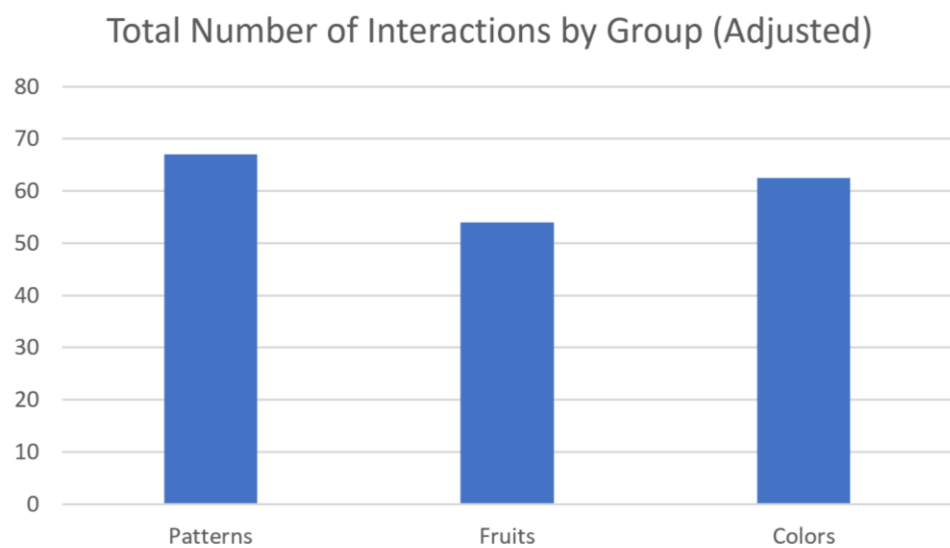



***Figure 24:*** *The number of times each group interacted with the surface. The Color Group's number has been adjusted by +17.5 to account for the smaller group.*

## 4.6 Cooperative vs. Noncooperative Grouping

While I was analyzing the transcripts and videos, I noticed that the groups of students showed characteristics of cooperative and noncooperative groups. A cooperative group is a group where the students work together to answer prompts, having discussion about the answer, making sure everybody is on board, and explaining topics to any student who doesn't understand. A non-cooperative group where either the students do not work together, one or more students is working on their own or otherwise not participating, or one or more students pushes forward without waiting for the other group members to understand. Sometimes, the groups in this study would act more cooperatively and sometimes more noncooperatively. I determined which behaviors and discussions demonstrated cooperative behavior and which demonstrated noncooperative behavior and recorded the amount of time each group spent in either category [Figure 25].

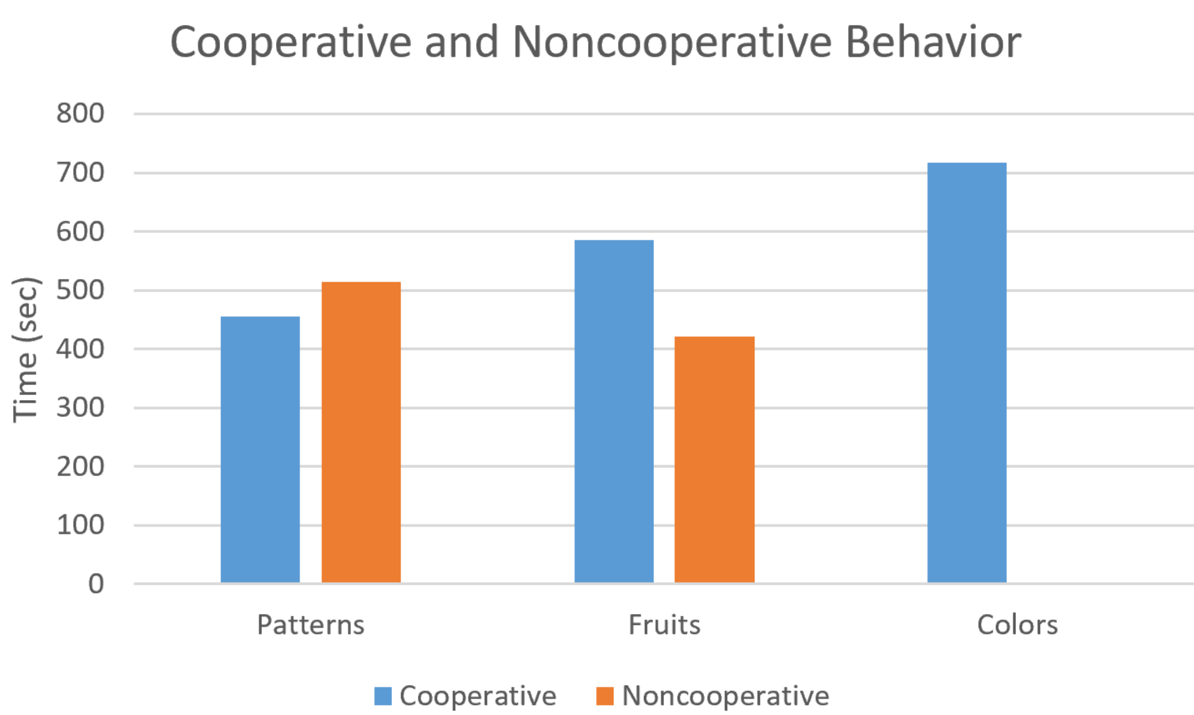



***Figure 25:*** *The amount of time each group spent enganging in cooperative and noncooperative behaviors during the instructional activity.*

Many of the actions and discussions were neither cooperative nor noncooperative, but rather neutral, so those were purposefully not coded as either cooperative or noncooperative. If I was unsure about whether a behavior qualified as cooperative or noncooperative, it was coded as neutral. Figure 26 shows the amount of time each group spent demonstrating cooperative, noncooperative, and neutral behaviors as a percentage of the total time taken on the activity.

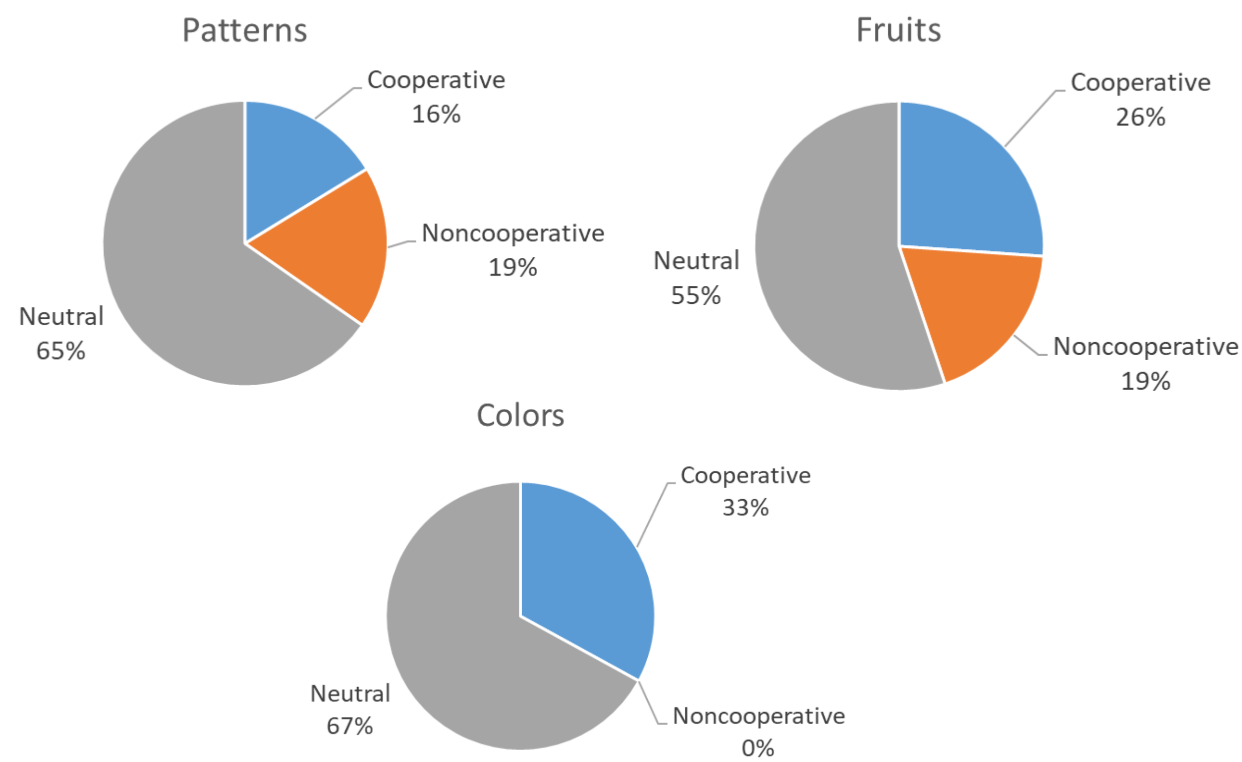



***Figure 26:*** *The amount of time each group spent demonstrating cooperative, noncooperative, and neutral behaviors as a percentage of the total time taken on the activity.*

When coding cooperative group behavior, I purposefully did not consider whether the students were getting the correct answer or heading in the right direction, only whether there was discussion, cooperation, or expression of difficulties between all group members. This is because correct and incorrect answers do not automatically imply cooperative or noncooperative behavior (or vice versa). Groups can be cooperative and get the wrong answer and they can noncooperative and answer correctly. To qualify as cooperative behavior, discussions about answers needed to be more than just a statement of one student got, but rather how the student got to the answer or what the answer means. For example, the following conversation about prompt 5 was coded as cooperative:

> STRIPES: So pretty much, black dot? (Stripes is confirming the answer the Patterns Group got for prompt 5, Appendix 1).
>
> PAISLEY: Ya, cause we said like this [pointing at the surface] is potential energy, but this [points at the vector drawn for prompt 4, Appendix 1] is like force.
>
> STRIPES: And it gets greater the lower it goes. And I guess you could think of that conceptually, like there's way more gravity on Earth than in outer space and assuming that's Earth [points to Earth]

> PLAID: And, okay, because this is potential energy, is, okay, kinetic energy and potential energy kinda do the opposite thing, right? The force kinda matches up better with the kinetic. So the lower the potential, higher the kinetic, better force, high force, potentially?
>
> PAISLEY: We kind of reconciled it with if you hold a ball up here, it has more gravitational potential energy that if it's like at the ground.

Paisley, Stripes, and Plaid all discussed their answers, even though they all agreed with each other. They explained why they thought the force would be larger closer to Earth and made sure the whole group was on the same page. In contrast, the following conversation about prompt 5 was coded as noncooperative:

> APPLE: K, so what did you answer?
>
> PEACH: Um, closer than the blue dot, the gravitational force would be bigger.
>
> APPLE: Mmmm, ya. Well, isn't the gravitational force the same always, because Earth has always the same mass? Or maybe when it talks about force, like the force, no.
>
> PEACH: I mean –
>
> APPLE: The pull would be like the same, but what changes is like the, ya I don't know.
>
> PEACH: Ya, I don't – I'm not sure. Um. Ya.

Not only does this conversation not include Grape, the third member of the group, but Apple was unable to express the difficulties with the prompt and Peach was unable to explain the answer. For all three of these reasons, this discussion was coded as noncooperative.

For the Pattern and Fruits Groups, since they both had three group members, it was more difficult to code cooperative behavior because there were sometimes productive conversations between two of the three students while the other was not included. I purposefully did not code any discussion involving the TA since a classroom authority figure would likely change the group dynamic. I also coded non-relevant conversations as noncooperative, since the group members were not working towards the goal of the activity.

# 5 Discussion

## 5.1 Interaction with Surface

As seen in Figure 21, most interactions with the surface were non-contact (tracing, pointing, gesturing), meaning that the students typically did not touch, move, or draw on the surface. I found that students typically only drew on the surface when they were told to by the activity. Out of the 17 times students drew on the surface, 15 of those were instructed by the activity and the other 2 were small drawings for fun (one student drew a tiny circular "planet" and another colored over a pre-marked dot). One reason that the students may have been disinclined to make contact with the surface is that this is the first time these students have interacted with surface models. Another reason may be that it's easier to point to or gesture at the surface rather than drawing, moving, or otherwise manipulating the surface.

Students also only interacted with the contour map (Appendix 3) when told specifically to align the contour map (4 times) or when explaining the materials to other students (when Stripes showed up late, Paisley held up the contour map to explain that it represented GPE). Even when specifically instructed to draw a force vector on the contour map in question 4 (Appendix 4), all the students drew the force vector on the surface instead. The lack of interaction with the contour map is probably due to students' inexperience with contour maps, compounded by the lack of information on the contour map. We purposefully made the contour map minimalistic by not including numbers or variables; we thought this would make the map easier to use, especially since the instructional activity doesn't involve any calculations. However, the lack of numbers and variables may have deterred students from using the map since it didn't provide them with any information they couldn't get from the surface.

Since students interacted with surface between 50 and 70 times during the 50 minutes instructional activity, it can be assumed that the surface aided the students in their completion of the activity. Out of the 173 interactions, only 22 were explicitly prompted by the activity. This implies that most of the interactions were done because the students decided that was the best way to explain their thinking.

## 5.2 Student Dynamics

As shown in Figure 21, each group of students has a dominant student, one person who performs most of the interactions with the surface. In the Color and Patterns Groups, the dominant student performed well over half of the interactions, while in the Fruit Group, the dominant student performed nearly half.

In each group, the dominant student was the one who had the surface placed in front of them by the TA, implying one of two things: either the student is consistently dominant in group work and the TA consciously or subconsciously placed the surface in front of the dominant student or the student became the dominant student of the activity because the surface was placed in front of them. I believe that student became the dominant interactor because the surface was placed in front of them. These students have not worked with these surfaces before, so they were uncomfortable with them and unlikely to move them to a location more easily accessible to all group members. The Fruit Group, however, did move the surface to the middle of the table around 4 minutes into the activity, resulting in the dominant student interacting less compared to the others. Each group also has a non-dominant student; a student who interacted with the surface between 12%-16% of the time. In The Pattern Group, the non-dominant student, Stripes, was the student who arrived late to class; in The Fruit Group, the non-dominant student was the one seated furthest away from the dominant student; and in The Color Group, the non-dominant student was the student who wasn't dominant (since there were only two students in this group).

The only group to introduce the operation "tangent" was The Color Group. This group was the most math-oriented of the three groups; whenever Purple was confused about a question, Purple would look for an equation in the lecture notes. Pink was more inclined to think about the functional behavior (inverse or inverse square) rather than exact equations for force and GPE. Pink was not the only student in the class that used a finger or other object to mimic a tangent line, but Pink was the only student I videotaped that used the tangent. Other students talked about the tangent line, used their arms to mimic the tangent to graphs, but none of them used the surface to show the tangent line.

## 5.3 Cooperative vs. Noncooperative Behavior

The distribution of interactions as well as several of my observations from the videos seemed to show that these students were working in noncooperative groups. In the Pattern Group, Stripes arrived late. Paisley and Plaid made no real effort to catch Stripes up with the group and only shared information and answers when asked by Stripes. Several times throughout the activity, Stripes was behind on answering the questions, while the other two moved forward. In the class that the Fruit Group was in, the students were formed into groups based on playing cards they were given at the beginning of class. Grape, Apple, and Peach did not seem to know each other very well and there were gaps of missing information in Apple's and Grape's physics knowledge as well as their interpretations of the activity prompts. Peach worked on the activity essentially alone, ignoring the fact that Apple and Grape were behind except to answer questions they had about the previous questions. Peach was the dominant student in the group as far as interacting with the surface, but Apple was dominant in the conversations. Apple voiced several misunderstandings and difficulties with the activity, but Grape usually replied that they didn't know, and Peach often didn't reply except to acknowledge that Apple had spoken. In the Color Group, the two students seemed to work together well, although Pink was the more visual talker (more movements, gestures, and interactions when speaking) and the one doing most of the explaining, both the Purple and to the TA. This caused Pink to be the dominant interactor.

However, in coding the cooperative and noncooperative behaviors, the Pattern and Fruit Groups each only spent 19% of the time engaging in noncooperative behaviors and the Colors Group spent no time at all. Despite this, no group spent more than a third of the time interacting in cooperative ways. A large part of the reason the Colors Group had no noncooperative behaviors was because the group only had two members; it's extremely difficult to leave one person out of the conversation when there are only two people. Without knowing the social backgrounds of the students in each group (whether they were friends outside of class or had interacted before this activity), it is difficult to determine the root causes of the noncooperative behaviors. Initially, I thought that the way the groups were formed impacted the amount of cooperative/noncooperative behavior. The first two members of the Patterns Group, Paisley and Plaid, were allowed to choose their groups for the assignment and since they were sitting next to each other and conversing before class, it's likely that they were at least acquaintances. However, when Stripes arrived late to class, Stripes was assigned to their group. Most of the noncooperative behaviors in the Patterns Group were caused by Stripes being left behind during the first section of the activity.

The members of the Fruits Group were assigned to work together using playing cards; each person arriving in class got a playing card and groups were determined by the number on the card. None of the students in the Fruits Group were sitting near to or talking to each other before class, though this doesn't mean they don't know each other. Most of the noncooperative behavior in the Fruits Group was caused by a breakdown in communication between all three members. Despite this, the Fruits Group did spend more time engaging in cooperative behaviors than the Patterns Group. Two possible reasons for this may be the set up of the classroom and the difficulties the group members had with the problems. For the Patterns Group, the students were sitting in individual desks that they had pushed near each other to create a semicircle. For the Fruits Group, the students were seated along one side of a table; the Fruits Group may have been more inclined to work together because they were sharing the same workspace. The Fruits Group also struggled more with the prompts in this activity. This caused the group members to have to ask each other and the TA for assistance since they did not know the answer themselves. In contrast, the Patterns Group frequently communicated to confirm their answers with the rest of the group rather than to come up with an answer. Conversations to confirm answers and explain reasonings with someone who already agrees with your answer are shorter than conversations where you must teach someone something new or persuade them to change their answer.

Examining these two groups caused me to believe that the noncooperative behavior was caused, or at least exacerbated, by assigning group members to work together rather than letting the students choose who to work with. However, the members of the Colors Group were also assigned to work together. They had not been sitting near to or talking to each other before class, though again, this doesn't mean they didn't know each other. It could be that this group was an outlier that happened to work well together, or it could be that the other two groups were outliers that did not work well together. Either way, without more data I cannot draw a conclusion about the relationship between cooperative and noncooperative behavior and the creation of the groups.

Despite my initial thoughts that the amount of cooperative behavior would correlate with the number of interactions the group had with the surface, that was not the case [Figure 27].

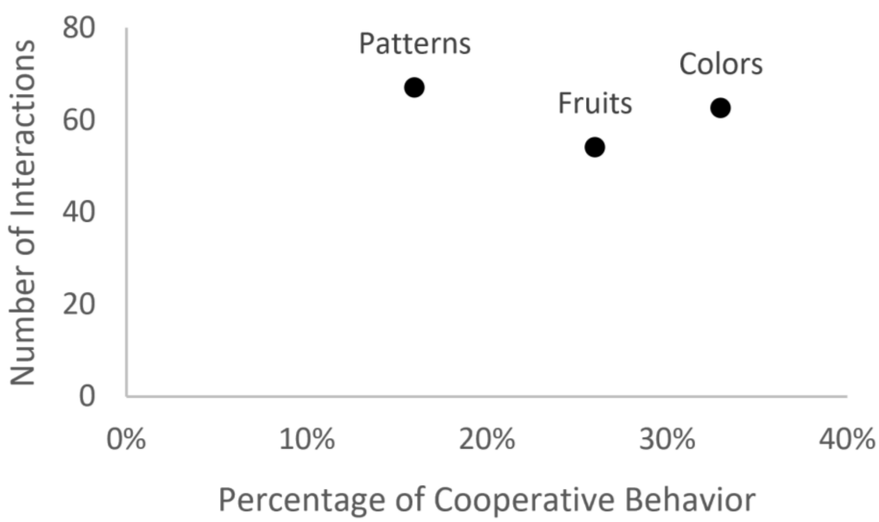



***Figure 27:*** *The number of interactions plotted against the percentage of cooperative behavior the group engaged in.*

# 6 Conclusion

Since students interacted with surface between 50 and 70 times during the 50 minutes instructional activity, it can be assumed that the surface aided the students in their completion of the activity. Out of the 173 interactions, only 22 were explicitly prompted by the activity. This implies that most of the interactions were done because the students decided that was the best way to explain their thinking.

The groups worked more cooperatively throughout the week, though none of the groups were entirely cooperative. Each group had one dominant student which interacted with the surface the most during the instructional activity and one student who rarely interacted with the surface. The students in this study had no experience with surface models before this activity.

Based on this study, it seems that students work more cooperatively in groups of two, through future research could be done to examine the ideal group make-up. Future research could look at how the ways students interact with the surface change depending on whether the students are novices or experts on the topic. It could also look and how the ways students interact with the surface change depending on the amount of experience students have with surfaces.

# REFERENCES


[1] "Gravity Fields and Potentials", Physics A-Level. http://www.physbot.co.uk/gravity-fields-and-potentials.html

[2] D. Hammer and A. Elby, "Tapping Epistemological Resources for Learning Physics", The Journal of the Learning Sciences 12(1), 53-90 (2003).

[3] B. M. Stephanik, "An Investigation of Student Understanding of Classical Ideas Related to Quantum Mechanics: Potential Energy Diagrams and Spatial Probability Density", Dissertation at the University of Washington (2015).

[4] B. M. Stephanik, P. S. Shaffer, N. S. Rebello, P. V. Engelhardt, and C. Singh, "Examining Student Ability to Interpret and Use Potential Energy Diagrams for Classical Systems", 2011 Physics Education Research Conference, 367-370 (2012).

[5] B. A. Lindsey, "Student Reasoning about Electrostatic and Gravitational Potential Energy: An Exploratory Study with Interdisciplinary Consequences", Physical Review Special Topics – Physics Education Research 10(1), 013101-1–013101-6 (2014).

[6] P. Mills, "Gravitational Fields", presentation (2010). https://www.slideshare.net/drpmills/gravitational-fields-3591431

[7] D. Hammer and T. R. Sikorski, "Implications of Complexity for Research on Learning Progressions: Research on Learning Progressions", Science Education 99(3), 424-431 (2015).

[8] D. F. Treagust, R. Duit, and H. E. Fischer, "Multiple Representations in Physics Education", Models and Modeling in Science Education 10, Springer International Publishing AG (2017).

[9] J. K. Gilbert, M. Reiner, and M. Nakhleh, "Visualization: Theory and Practice in Science Education", Models and Modeling in Science Education 3, Springer Science+Business Media (2008).

[10] ShopBot, "What is CNC, Anyway?", ShopBot Tools, Inc. (2017). Shopbottools.com

[11] P. Heller and M. Hollabaugh, "Teaching Problem Solving Through Cooperative Grouping. Part 2: Designing Problems and Structuring Groups", American Journal of Physics 60(7), 637-644 (1992).

[12] M. Gardner, J. G. Greeno, F. Reif, A. H. Schoenfeld, A. diSessa, and E. Stage, "Toward a Scientific Practice of Science Education", Lawrence Eribaum Associates, Inc. (1990).

[13] P. Heller, R. Keith, and S. Anderson, "Teaching Problem Solving Through Cooperative Grouping. Part 1: Group versus Individual Problem Solving", American Journal of Physics 60(7), 627-636 (1992).

[14] H. Hasan and A. Kazlauskas, "Activity Theory: who is doing what, why and how", Being Practical with Theory: A Window into Business Research, 9-14 (2014).

# APPENDIX 1: ACTIVITY AND INSTRUCTOR'S GUIDE

**Name:** ________________________

## Gravitational Potential Energy and Gravitational Force

*Your group has a plastic surface and a contour map that represent the gravitational potential energy of a space station-Earth system as a function of the position of the space station relative to Earth. The gravitational potential energy is zero infinitely far away from Earth. Solve the following problems together and discuss the results.*

*You are employed by a company called SpaceY. SpaceY wants to put a space station at the blue circle.*

(1) If the space station moves further away from Earth, how will the gravitational potential energy change? What if the space station moves closer to Earth?

(2) Identify other points on the surface where the gravitational potential energy is the same as the potential energy at the blue circle and draw a line to connect them. Do the same for the orange star and the green square.

  (a) Align your surface with the contour map. How are you making your alignment?

  (b) How could the space station move so that the gravitational potential energy remains constant?

(3) Sketch a graph of the gravitational potential energy ($U$) vs. distance from the center of the Earth ($r$). Remember to label your axes.

  (a) Does your graph match your answers to (1)? If not, reconcile any differences.

  (b) Why is it reasonable to represent the information from the surface in a graph with only 2 axes?

(4) What direction will the gravitational force at the blue circle be? Indicate the direction of the force at this point on the contour map.

(5) Locate a point where you would expect the gravitational force to be larger than at the blue circle.

(6) Is the rate of change of gravitational potential energy with respect to $\boldsymbol{r}$ positive, negative, or zero? Compare $\frac{dU}{dr}$ at the two points from (4) and (5). Which one has a larger magnitude?

(7) Sketch a graph of the gravitational force vs. distance from the center of the Earth. Remember to label your axes.

(8) The SpaceY employee handbook states:

> There is a relationship between gravitational potential energy and force; force is the negative gradient of gravitational potential energy. $F = -\nabla U(r)$

Do you agree? Support your answer with evidence from this activity. Recall that the gradient of a function, $\nabla U(r)$, is equal to $\frac{\partial U}{\partial r}\hat{r}$ as long the function depends only on $r$.

**Name:** ________________________

## Gravitational Potential Energy and Gravitational Force

*Each prompt will have the goal of the prompt and additional questions you may want to ask each group (or the whole class) as they work. The horizontal direction of the surface represents distance,* $\boldsymbol{r}$ *in this case. The surface of the Earth is marked by an indent in the surface near the lowest corner. The height of the surface corresponds to the value of gravitational potential energy. Read the introduction below to the class before beginning. In the last 5 minutes of class, pass out the Post-Activity Evaluation.*
*Important things to note (you may choose to put this on the board): The surface and the contour map represent the SAME system and the SAME variables (distance and GPE). GPE goes to 0 when r goes to infinity.*

*Your group has a plastic surface and a contour map that represent the gravitational potential energy of a space station-Earth system as a function of the position of the space station relative to Earth. The gravitational potential energy is zero infinitely far away from Earth. Solve the following problems together and discuss the results.*

*You are employed by a company called SpaceY. SpaceY wants to put a space station at the blue circle.*

(1) If the space station moves further away from Earth, how will the gravitational potential energy change? What if the space station moves closer to Earth?

**Goal:** General trend of GPE. How does GPE change with respect to distance?

**Additional Questions:** Prompt students to look at the continuous change of the potential energy (i.e. not just picking one point and comparing the two values) or have them choose several points to compare.

(2) Identify other points on the surface where the gravitational potential energy is the same as the potential energy at the blue circle and draw a line to connect them. Do the same for the orange star and the green square.
  (a) Align your surface with the contour map. How are you making your alignment?
  (b) How could the space station move so that the gravitational potential energy remains constant?

**Goal:** Introducing contour lines - places where the value of GPE is the same. What do contour lines represent? Explicit combination of representations.

**Guide:** How can you tell the value of potential energy? What on the surface represents potential energy? Find one spot where the potential energy is the same as at the blue circle and draw a dot. Do this again for several other points and connect them with a smooth line. What do these lines mean? Why are they drawn on the map? What information do the contour lines give you? How could you relate this to the lines you just drew on your surface? What main features could you use to align the surface to the map?

(3) Sketch a graph of the gravitational potential energy ($U$) vs. distance from the center of the Earth ($r$). Remember to label your axes.
  (a) Does your graph match your answers to (1)? If not, reconcile any differences.
  (b) Why is it reasonable to represent the information from the surface in a graph with only 2 axes?

**Goal:** Visualize the function U(r)

**Guide:** What are you graphing? What axes should you plot? What will the axes look like? Where is GPE zero? How does GPE change with respect to distance? How could you represent that on your graph? Remember that height represents the value of GPE.

**Additional Questions/Comments:** Note where students are putting GPE=0. Ask them to label it on their graph. If students have trouble with the graph inside of the Earth, ask them "do you need to think about inside the Earth if we are talking about a space station?" If they want to graph within the Earth, they can use the surface to determine what the graph should look like.

(4) What direction will the gravitational force at the blue circle be? Indicate the direction of the force at this point on the contour map.

**Goal:** Start the students with something intuitive. They should know gravitational force points toward Earth.

**Guide:** Locate the point on the contour map. What direction is the gravitational force in? What is the qualitative magnitude (i.e. is the force very very small? very, very big? somewhere in between?)? Remember that this is a sketch of the vector, so the magnitude does not need to be exact.

(5) Locate a point where you would expect the gravitational force to be larger than at the blue circle.

**Goal:** Compare force at two different points. Begin to connect force to change in GPE.

**Guide:** What direction is the force at the blue circle? Does gravitational force change depending on the position of the space station? Does gravitational force get smaller or larger as the space station moves closer to the Earth?

**Additional questions:** In what way is the change in GPE different at this point?

**Note:** Students may locate a point on one representation (graph, contour map, surface) - have them locate the point on all representations.

(6) Is the rate of change of gravitational potential energy with respect to $\boldsymbol{r}$ positive, negative, or zero? Compare $\frac{dU}{dr}$ at the two points from (4) and (5). Which one has a larger magnitude?

**Goal:** Introduction to relating potential energy to force. We already talked about the force, now we will examine the change in potential energy. Compare $\frac{dU}{dr}$ and begin to make a connection between magnitude of force and magnitude of $\frac{dU}{dr}$.

**Guide:** Rate of change is represented by the slope.

(7) Sketch a graph of the gravitational force vs. distance from the center of the Earth. Remember to label your axes.

**Goal:** Visualize the function F(r)

**Guide:** What are you graphing? What axes should you plot? How does force change with respect to distance? How could you represent that on your graph? Is force positive or negative? Note that if students make the force positive, they are plotting the magnitude of the force. The component of the force is negative since it points opposite to increasing $\boldsymbol{r}$.

(8) The SpaceY employee handbook states:

> There is a relationship between gravitational potential energy and force; force is the negative gradient of gravitational potential energy. $F = -\nabla U(r)$

Do you agree? Support your answer with evidence from this activity. Recall that the gradient of a function, $\nabla U(r)$, is equal to $\frac{\partial U}{\partial r}\hat{r}$ as long as the function depends only on $r$.

**Goal:** Develop a relationship between GPE and force based on knowledge gained in this activity.

**Guide:** Direct students to their answers from (4), (5), (6). Does the magnitude of the force seem to change w.r.t. a change in GPE? Which direction is force in?

# APPENDIX 2: ACTIVITY SOLUTIONS

**Name:** Solution Example

**Gravitational Potential Energy and Gravitational Force**

*Your group has a plastic surface and a contour map that represent the gravitational potential energy of a space station-Earth system as a function of the position of the space station relative to Earth. The gravitational potential energy is zero infinitely far away from Earth. Solve the following problems together and discuss the results.*

*You are employed by a company called SpaceY. SpaceY wants to put a space station at the blue circle.*

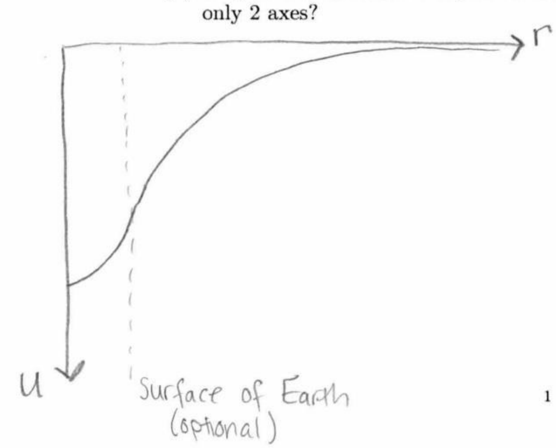
(1) If the space station moves further away from Earth, how will the gravitational potential energy change? What if the space station moves closer to Earth?

If the space station moves further away, the GPE will increase (get less negative).
If the space station moves closer, GPE will decrease (get more negative).

(2) Identify other points on the surface where the gravitational potential energy is the same as the potential energy at the blue circle and draw a line to connect them. Do the same for the orange star and the green square.
   (a) Align your surface with the contour map. How are you making your alignment?
   (b) How could the space station move so that the gravitational potential energy remains constant?

ⓐ I made my alignment using the lines I just drew to match with the lines on the contour map
ⓑ The space station could move in a circle w/ Earth in the middle (along one of the lines I drew).

(3) Sketch a graph of the gravitational potential energy ($U$) vs. distance from the center of the Earth ($r$). Remember to label your axes.
   (a) Does your graph match your answers to (1)? If not, reconcile any differences.
   (b) Why is it reasonable to represent the information from the surface in a graph with only 2 axes?




ⓐ yes, U increases as r does.

ⓑ Because U is only dependent on distance from Earth (as long as you are talking about the same object).

1

Solution Example

(4) What direction will the gravitational force at the blue circle be? Indicate the direction of the force at this point on the contour map.

Towards the Earth

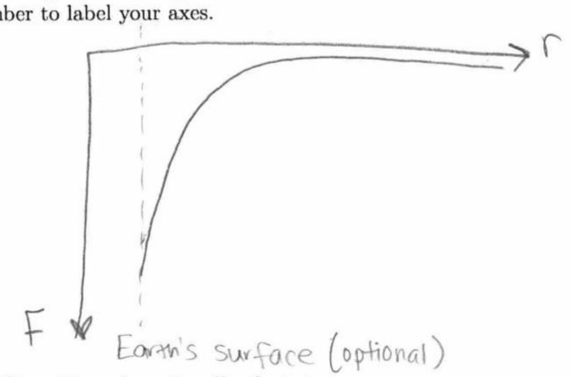
(5) Locate a point where you would expect the gravitational force to be larger than at the blue circle.

Anywhere closer to Earth than the blue circle.

(6) Is the rate of change of gravitational potential energy with respect to $\boldsymbol{r}$ positive, negative, or zero? Compare $\frac{dU}{dr}$ at the two points from (4) and (5). Which one has a larger magnitude?

$\frac{dU}{dr}$ is positive (U is increasing) $\frac{dU}{dr}$ is larger at (5)

(7) Sketch a graph of the gravitational force vs. distance from the center of the Earth. Remember to label your axes.




(8) The SpaceY employee handbook states:

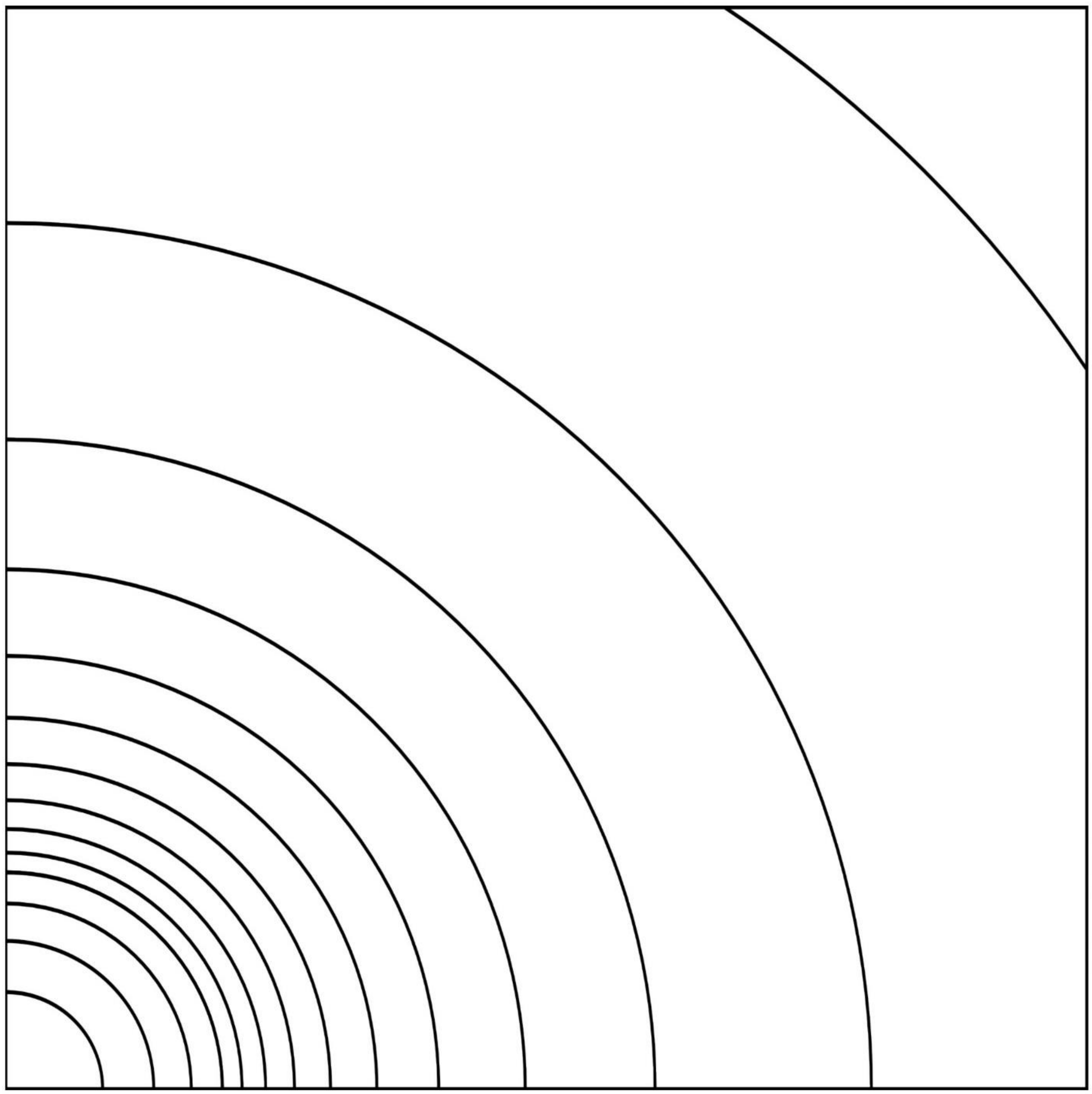
> There is a relationship between gravitational potential energy and force; force is the negative gradient of gravitational potential energy. $F = -\nabla U(r)$

Do you agree? Support your answer with evidence from this activity. Recall that the gradient of a function, $\nabla U(r)$, is equal to $\frac{\partial U}{\partial r}\hat{r}$ as long the function depends only on $r$.

I agree. The force seems to be related to $\frac{dU}{dr}$ — they both increase and decrease similarly. The force is the negative gradient since $\frac{dU}{dr}$ is positive and the force is negative.

$$F = -\frac{dU}{dr}\hat{r}$$

# APPENDIX 3: CONTOUR MAP



# APPENDIX 4: LONG VIDEO SUMMARIES

## LONG SUMMARY: PATTERNS

Students: (Paisley, Plaid, Stripes)

This is the first recitation of the week. None of the students have had lecture about gravitational force or potential, besides the near-Earth approximations $U = mgh$ and $F = mg$.

The TA introduces the activity and tells the students that the surface "represents the gravitational potential energy of space around Earth" and that the height indicates the GPE. The TA lets the students choose groups to work in. There are 4 students in the class at the beginning, so they split into two groups of two. The TA tells the students that the surfaces and contour maps are dry erasable, and they are free to draw on them if they would like. The TA hands out the surfaces, contour maps, and worksheets – one of each per group.

The group starts with just Paisley and Plaid. They examine the worksheet and surface, discussing what the TA has just said. They start going through the prompts on the worksheet, reading each one aloud as they get to it.

> *Your group has a plastic surface and a contour map that represent the gravitational potential energy of a space station-Earth system as a function of the position of the space station relative to Earth. The gravitational potential energy is zero infinitely far away from Earth. Solve the following problems together and discuss the results.*
>
> *You are employed by a company called SpaceY. SpaceY wants to put a space station at the blue circle.*
>
> (1) If the space station moves further away from Earth, how will the gravitational potential energy change? What if the space station moves closer to Earth?

Question 1: Paisley and Plaid place their fingers on the surface and trace outward to mimic the path of the space station. They determine that GPE would increase as the space station moves away from the Earth. Plaid notices that the worksheet says "GPE is zero infinitely far away from Earth" and tells this to Paisley. Paisley responds by saying that only happens *infinitely* far away, and since they can't see or get to infinity, it doesn't really matter. Plaid and Paisley decide that GPE goes up, then goes down at some point past what is shown on the surface to reach zero at infinity. Paisley uses previous knowledge of planets ("really far away planets aren't affected by gravity") to understand that GPE should go to zero at infinity. Plaid then brings up the idea that gravity increases as planets get closer to Earth, so shouldn't GPE be going up? Paisley relates the idea to raising and then dropping a ball. At the top, the ball has the greatest GPE, and at the bottom, the ball has none. Paisley finishes the thought with "to an extent," relating back to the discrepancy between their ideas about GPE and the fact that GPE goes to zero at infinity.

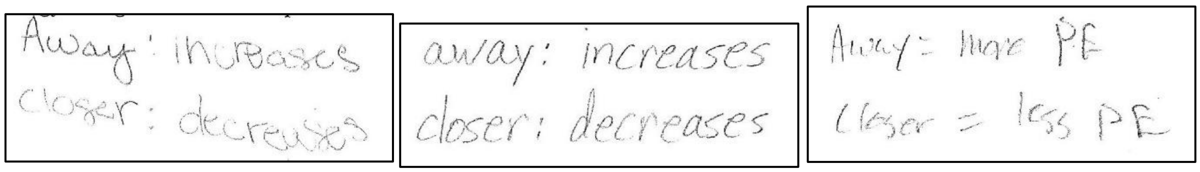



***Figure 28:*** *The answers the Pattern Group wrote down for question 1. From left to right, the answers belong to Paisley, Plaid, and Stripes.*

Stripes then arrives in the classroom, the TA gives a brief description of the activity, and then Stripes sits in a desk next to Paisley and Plaid. Paisley and Plaid move on to question 2 while I get information from Stripes.

(2) Identify other points on the surface where the gravitational potential energy is the same as the potential energy at the blue circle and draw a line to connect them. Do the same for the orange star and the green square.
   (a) Align your surface with the contour map. How are you making your alignment?
   (b) How could the space station move so that the gravitational potential energy remains constant?

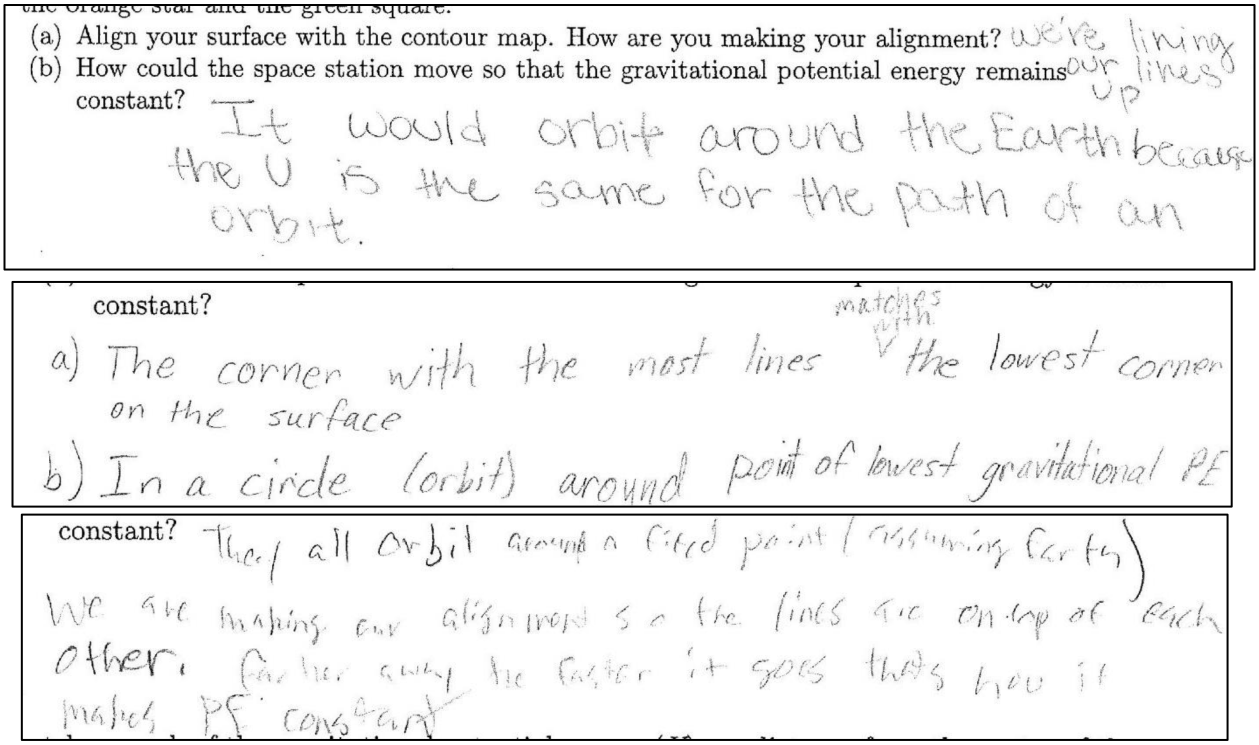
Question 2: Paisley and Plaid read the question, Paisley draws an arc of constant radius intersecting the blue dot, then does the same for other marked points. There is little to no discussion about this part since both Paisley and Plaid seemed to already be on the same page about the answer. Paisley examines the contour map and surface, then rotates the contour map 180 degrees to match the map to the surface.

> Paisley and Plaid take a quick break to fill in Stripes about the activity as a whole (the surface and map represent GPE) and their previous problems. Stripes reads questions 1 and 2, checking with Paisley and Plaid about the answers while Plaid and Paisley discuss the alignment of the map.

Paisley and Plaid discuss the contour map. Paisley begins to determine what the spacing of the contour lines means but doesn't seem to figure it out. Plaid says each line might be an equal change in GPE and Paisley agrees.

> Stripes is still behind, working on question 2. Paisley explains the markings previously drawn on the surface and their reasoning for their answer.

Plaid tells Paisley that the space station would move in orbit and Paisley agrees.


the orange star and the green square.

(a) Align your surface with the contour map. How are you making your alignment? we're lining our lines up

(b) How could the space station move so that the gravitational potential energy remains constant? It would orbit around the Earth because the U is the same for the path of an orbit.

constant?

a) The corner with the most lines matches with the lowest corner on the surface

b) In a circle (orbit) around point of lowest gravitational PE

constant? They all orbit around a fixed point (assuming Earth)

We are making our alignment so the lines are on top of each other. Farther away the faster it goes that's how it makes PE constant

***Figure 29:*** *The Pattern Group's written answers to the second prompt. From top to bottom, the answers belong to Paisley, Plaid, and Stripes.*

(3) Sketch a graph of the gravitational potential energy ($U$) vs. distance from the center of the Earth ($r$). Remember to label your axes.
   (a) Does your graph match your answers to (1)? If not, reconcile any differences.
   (b) Why is it reasonable to represent the information from the surface in a graph with only 2 axes?

Question 3: The group is finally all caught up with each other and they discuss axes for their graphs. Plaid asks if Paisley has put U on the y-axis. Stripes wonders if they should make an x-y graph or a spiral graph. The group decides on the x-y graph because the prompt says to label the axes. Paisley decides that U should go on the x-axis because of the order of the variables; the prompt says to make a graph of U vs. r and "they usually say x vs. y, so U is our x." The group agrees and draws their graphs with U on the x-axis and r on the y-axis. Paisley begins to draw the graph by looking at overall behavior first: "U increases as r increases." Stripes and Paisley say that the graph should look exponential or parabolic. The group struggles to draw the graph due to their axes. Paisley and Plaid return to the fact that GPE is zero infinitely far away from Earth – Plaid asks if Paisley is going to draw the graph decreasing to zero, and they both joke about going infinitely far away from Earth before deciding not to draw the end behavior.

After reading 3a, which involved connect their graph to their answers for question 1, Plaid realizes that the graph makes more sense with the axes flipped, so the group decides to change their axes to U on the vertical and r on the horizontal. The group connects their graph to their answers by saying that as r increases, U increases, and as r decreases, U decreases. Plaid answers 3b by saying that the third dimension is constant, while Paisley answers by saying that there aren't any other dimensions. Plaid explains that since there is symmetry in the x and y directions on the surface, you only need to represent space as one variable (r). The group finishes the discussion with "it's only different in two dimensions."

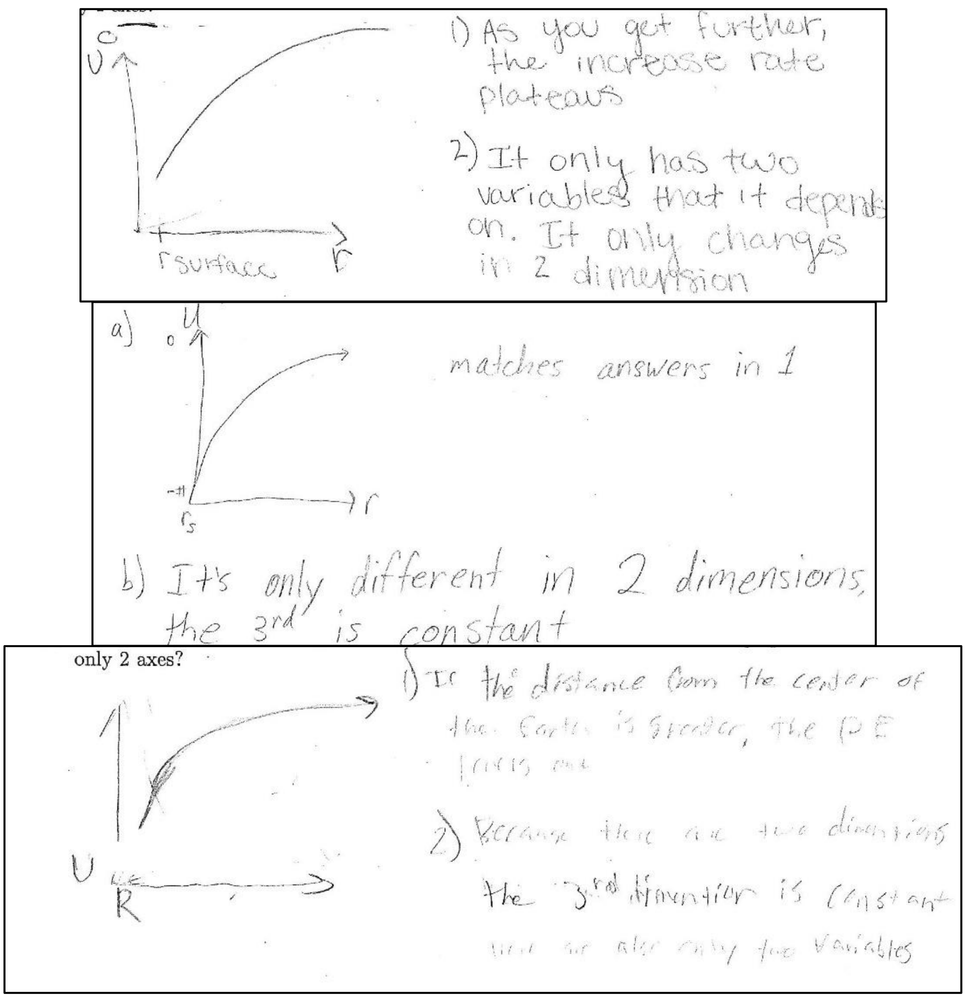



***Figure 30:*** *The Pattern Group's written answers to the third prompt. From top to bottom, the answers belong to Paisley, Plaid, and Stripes.*

(4) What direction will the gravitational force at the blue circle be? Indicate the direction of the force at this point on the contour map.

Question 4: Plaid and Paisley draw a vector from the blue dot towards Earth. Stripes agrees with the direction of the force. They make sense of the answer by remembering past knowledge of gravity – force points towards whatever is pulling the object.

It points inwards towards Earth.

he force at this point on the contour map.
The force will point in toward Earth.

Pulling inwards toward what ever is being Pulled toward it (Earth)

***Figure 31:*** *The Pattern Group's written answers to the fourth prompt. From top to bottom, the answers belong to Paisley, Plaid, and Stripes*

(5) Locate a point where you would expect the gravitational force to be larger than at the blue circle.

Question 5: Plaid answers the question by pointing to a place where GPE is larger. Paisley initially agrees, then remembers that the question asks about force, not GPE, so the answer would be closer to Earth. The group agrees that the force is larger at any point with a smaller *r* than the blue dot. Stripes says "it [the force] will be greater the lower it [the surface, and thus GPE] goes." Stripes makes sense of the answer conceptually: "I guess if you think about it conceptually, there's more gravity in Earth than in space and assuming that's [points to lowest part of surface] Earth…" Plaid introduces the concept of kinetic and potential energy to make sense of the prompt. Plaid relates force to kinetic energy and says that as potential energy goes down, kinetic energy goes up, so it makes sense that the force goes up as GPE goes down. Paisley brings the group back to the example of holding a ball above the ground.

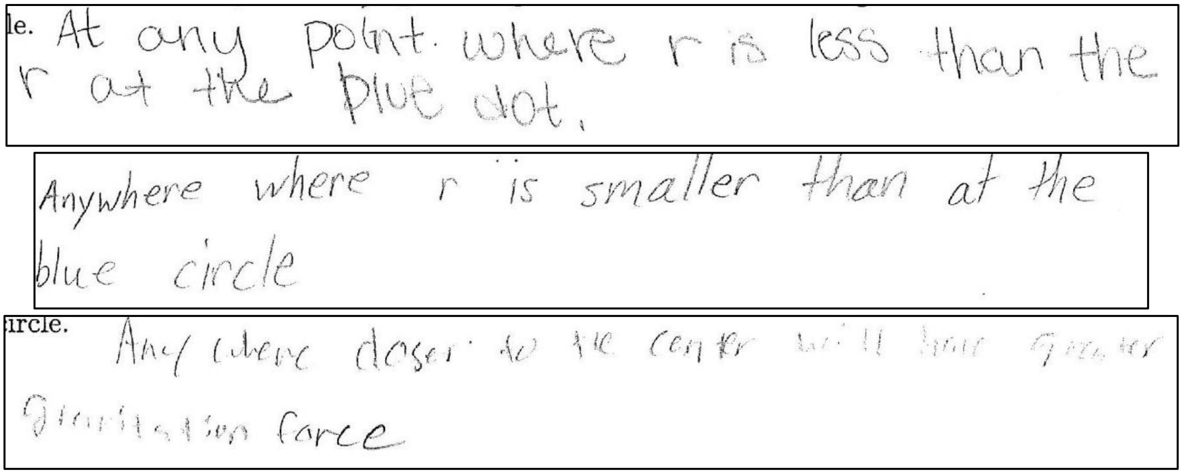



***Figure 32:*** *The Pattern Group's written answers to the fifth prompt. From top to bottom, the answers belong to Paisley, Plaid, and Stripes.*

Interlude: The TA talks to the group as a whole about drawing a graph of U(r). TA draws first quadrant of a graph with vertical axis labelled U and horizontal axis labelled r, the asks the students what characteristics the graph would have. The group discusses the what happens at the origin of the graph, first conceptually (which proves difficult because it can be confusing to talk about potential from an object while you are in the object), then using the surface profile as a reference. One other struggle the students have is reconciling the shape of the graph (constantly increasing) and the end behavior of GPE (goes to zero). Eventually, the TA shows the students that zero is above the graph. TA draws a line above the graph and labels it 0, explaining that if potential energy is increasing and goes to zero at infinity, then it must be increasing to zero. One student suggests a decreasing graph since the potential energy goes to zero. The same student introduces an expression:

$$-\frac{GMm}{r^2}.$$

(Note that this is the equation for gravitational force, not GPE). TA shows the students that since the equation is negative, they want a graph that is negative as well, so the correct graph is the one initially drawn, not the new one suggested by the student. Paisley suggests the TA use the profile of the surface to match the graph as well. TA finishes the origin discussion by telling the students that they don't ever need to be concerned about the potential field in the center of the object, so it doesn't matter what happens in the origin. The students solidify the concept that GPE is always negative ("So it [GPE] goes from a really negative value to a less negative value").

Paisley, Plaid, and Stripes discuss the equation written on the board during the class discussion. Paisley and Plaid aren't sure if that equation is for the force or the GPE, but they know that the equation for GPE is something similar. Stripes had never seen an equation like that.

(6) Is the rate of change of gravitational potential energy with respect to $\boldsymbol{r}$ positive, negative, or zero? Compare $\frac{dU}{dr}$ at the two points from (4) and (5). Which one has a larger magnitude?

Question 6: The group discusses the rate of change. Paisley thinks the rate of change will be negative because "it's [the potential energy is] changing less, like the slope [of the U(r) graph] is decreasing." Stripes initially disagrees, but changes immediately to agreeing, then back to disagreeing again because Stripes isn't sure whether the rate of change will stay negative throughout the whole area or if the rate changes when the slope flattens out. Plaid points out that the slope goes from being more positive to less positive, so the slope "is decreasing, but it's not negative." Paisley asks if the derivative, or the rate of change, is negative. (Note that this shows a lack of connection, at least at the moment, between slope, rate of change, and derivative). Plaid and Paisley eventually agree that the rate of change is positive. Stripes is silent throughout most of the discussion. The discussion about comparing the rate of change at two of the points is minimal since the group immediately agrees that the slope at the dot from (5) is larger than the slope at the dot from (4).

1) The rate of change is positive
2) The slope at 5 is much greater than 4, so 5 has the largest magnitude.

The rate of change is positive but decreasing
5 > 4 → 5 has the larger magnitude

Slope is negative and eventually plateaus into 0
but from point 4-5 slope will increase because one is way closer to the center (larger magnitude)

***Figure 33:*** *The Pattern Group's written answers to the sixth prompt. From top to bottom, the answers belong to Paisley, Plaid, and Stripes.*

(7) Sketch a graph of the gravitational force vs. distance from the center of the Earth. Remember to label your axes.

Question 7: At the very beginning of the discussion, Plaid revisits the idea that force is "the opposite" of GPE, going back to the idea that force is related to kinetic energy. Once again, the group starts drawing the graph by examining the overall behavior. The force is large near the Earth and smaller further away. Then the group considers the slope of the graph – does the graph start decreasing quickly, then plateau, or does it decrease slowly at first, then quickly? They decide to draw the graph as a mirror image of the GPE graph (flipped horizontally).

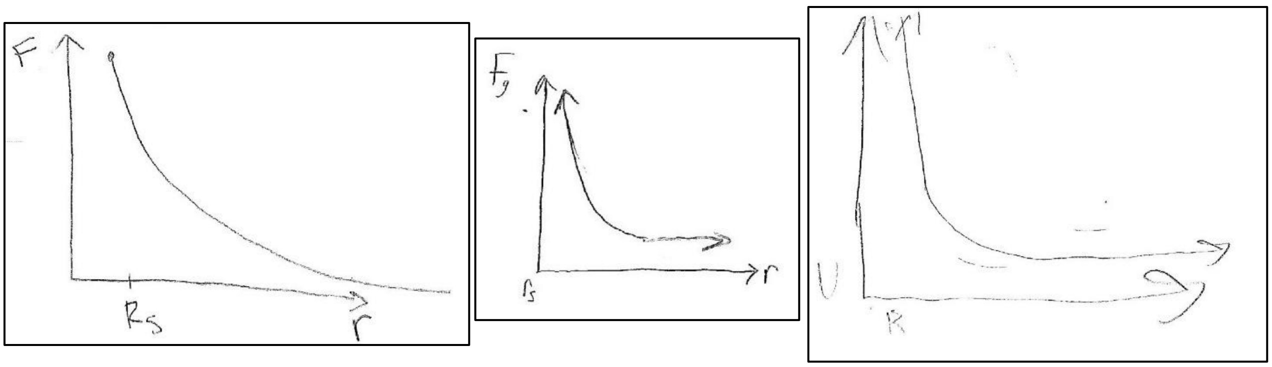


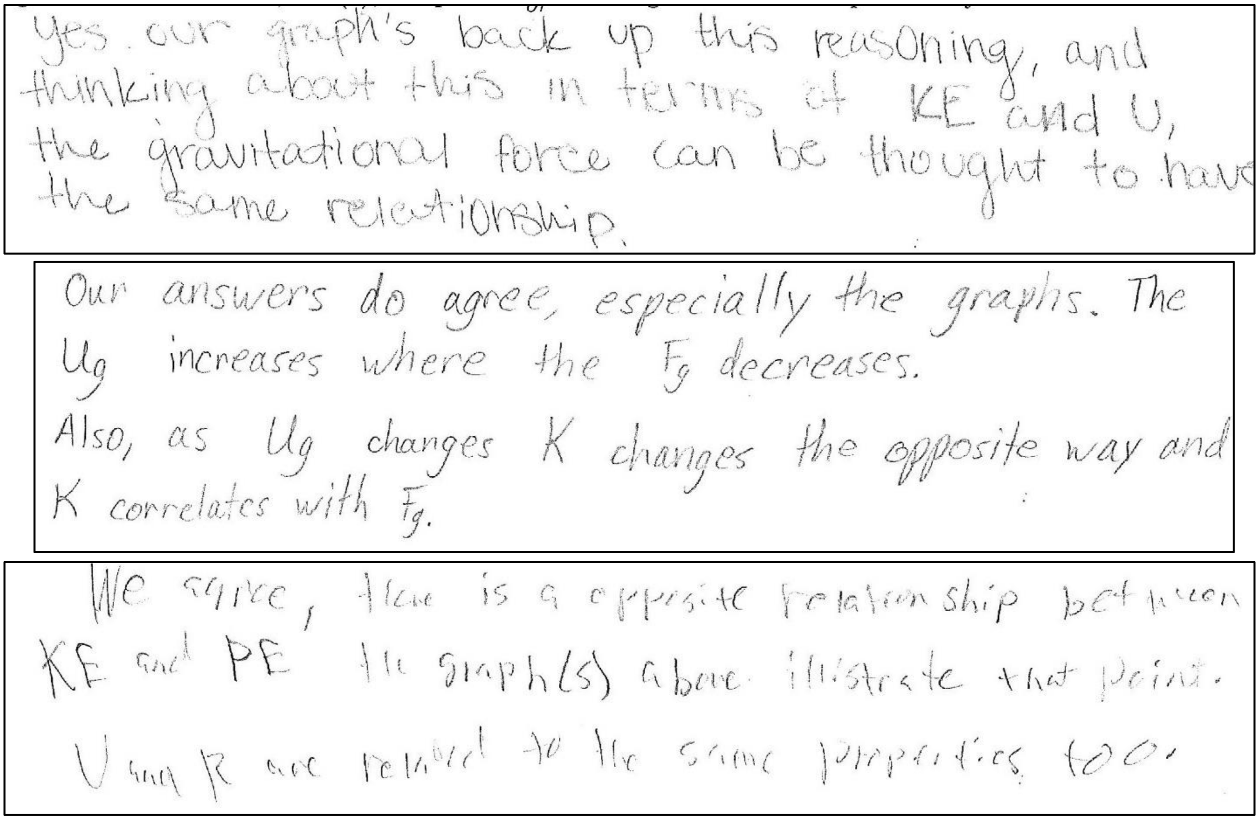

***Figure 34:*** *The Pattern Group's written answers to the seventh prompt. From left to right, the graphs belong to Paisley, Plaid, and Stripes.*

(8) The SpaceY employee handbook states:

> There is a relationship between gravitational potential energy and force; force is the negative gradient of gravitational potential energy. $F = -\nabla U(r)$

Do you agree? Support your answer with evidence from this activity. Recall that the gradient of a function, $\nabla U(r)$, is equal to $\frac{\partial U}{\partial r}\hat{r}$ as long the function depends only on $r$.

Question 8: The group reads the question and initially agrees with the statement, saying "it seems right." Plaid notices that their graphs agree (but provides no further explanation of how or why they agree). Paisley returns to the fact that PE and KE are opposite of each other and force is related to KE. The group begins silently writing without further discussion. A minute or two later, Stripes confirms the answer with Paisley and Plaid; "there's an opposite relationship between kinetic and potential energy and the graphs illustrate that."


Yes. our graph's back up this reasoning, and thinking about this in terms of KE and U, the gravitational force can be thought to have the same relationship.

Our answers do agree, especially the graphs. The $U_g$ increases where the $F_g$ decreases.
Also, as $U_g$ changes K changes the opposite way and K correlates with $F_g$.

We agree, there is a opposite relationship between KE and PE the graph(s) above illustrate that point. U and K are related to the same properties too.

***Figure 35:*** *The Pattern Group's written answers to the last prompt. From top to bottom, the answers belong to Paisley, Plaid, and Stripes.*

The TA comes over to the group to discuss their findings. The group uses end behaviors (very near to and very far away from Earth) to makes sense of their results. The instructor interjects to ask, "is it the potential going up that tells you about the force, or is it how steep the potential is that tells you how strong the force is?" The students respond with "the steepness," though they all add modifiers to their answers. Stripes says, "that's just me throwing a shot in the dark, Paisley says, "we talked a lot about the steepness," and Plaid says, "I was gonna say [the steepness] mostly because this is physics with calculus and steepness is the derivative." They all agree that context clues gave them the answer.

The students go on to compare their graphs of GPE and force, noticing that the steepness of one graph corresponds to the steepness of the other. (Notice the students stop thinking about the steepness of GPE determining the value of force, but rather shift their view from the value of GPE determining the value of force to the steepness of GPE determining the steepness of the force).

The TA discusses the direction of the force with Plaid, Paisley, and Stripes. They have all drawn their graphs with an entirely positive force, so the TA asks them to consider the direction of the force. They discuss the which way the force is pointing (inward) and compare that direction to the direction of $\hat{r}$ (outward).

The TA meets with the class as a whole to wrap up the activity. The TA discusses the direction of the force with the entire class before drawing the graph on the board. After a similar discussion to the one had between the TA, Paisley, Plaid, and Stripes, the class agrees that the force should be entirely negative. One student struggles with the relationship between the force and GPE graphs, stating that "they should be flips of each other." The TA explains that force is the negative derivative of GPE. The TA draws tangent lines at various points on the GPE graph, then compares the magnitudes to the magnitude of the force graph.

Class ends.

## Long Summary: Fruits

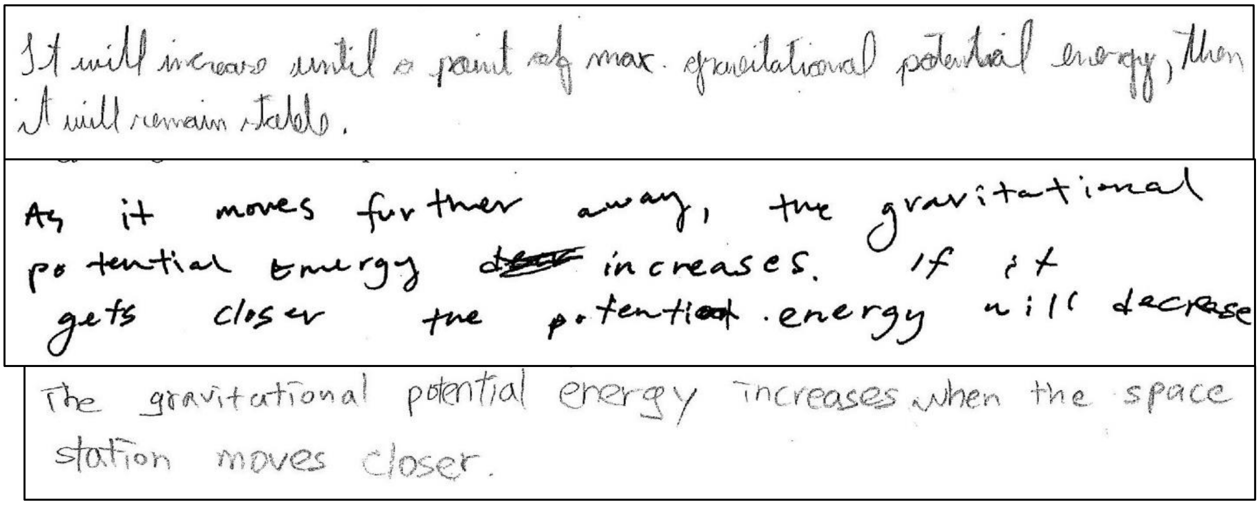
Students: Peach, Apple, and Grape.

This is the second recitation of the week. 2 lectures have happened at 9 and 10 am, but some students have the 4 pm lecture, and some of the gravity content won't be addressed until Thursday.

The TA introduces the activity in the same way as in The Pattern Group and then passes out the materials.

> *Your group has a plastic surface and a contour map that represent the gravitational potential energy of a space station-Earth system as a function of the position of the space station relative to Earth. The gravitational potential energy is zero infinitely far away from Earth. Solve the following problems together and discuss the results.*
>
> *You are employed by a company called SpaceY. SpaceY wants to put a space station at the blue circle.*
>
> (1) If the space station moves further away from Earth, how will the gravitational potential energy change? What if the space station moves closer to Earth?

Question 1: The students begin by orienting themselves with the materials. They write their names on the papers and discuss the properties of the surface. Peach asks if the height of the surface represents GPE, Apple agrees, then they discuss the location of the Earth. Peach believes that the Earth is in the lower corner of the surface, while Apple believes it is elsewhere since "as you get further away from Earth, gravitational potential energy decreases." It's likely that Apple thinks this because the introduction to the activity says that GPE goes to zero as distance goes to infinity. Grape asks if one of the marked points on the surface is the Earth. The group reaches a consensus that the Earth is at the lowest part of the surface after Apple says that "it's like bending space." They decide that the potential energy must increase as the space station moves further away from Earth. Apple believes that the GPE increases until a certain point (the blue dot on the surface), then flattens out.



It will increase until a point of max. gravitational potential energy, then it will remain stable.

As it moves further away, the gravitational potential energy ~~dec~~ increases. If it gets closer the potential energy will decrease

The gravitational potential energy increases when the space station moves closer.

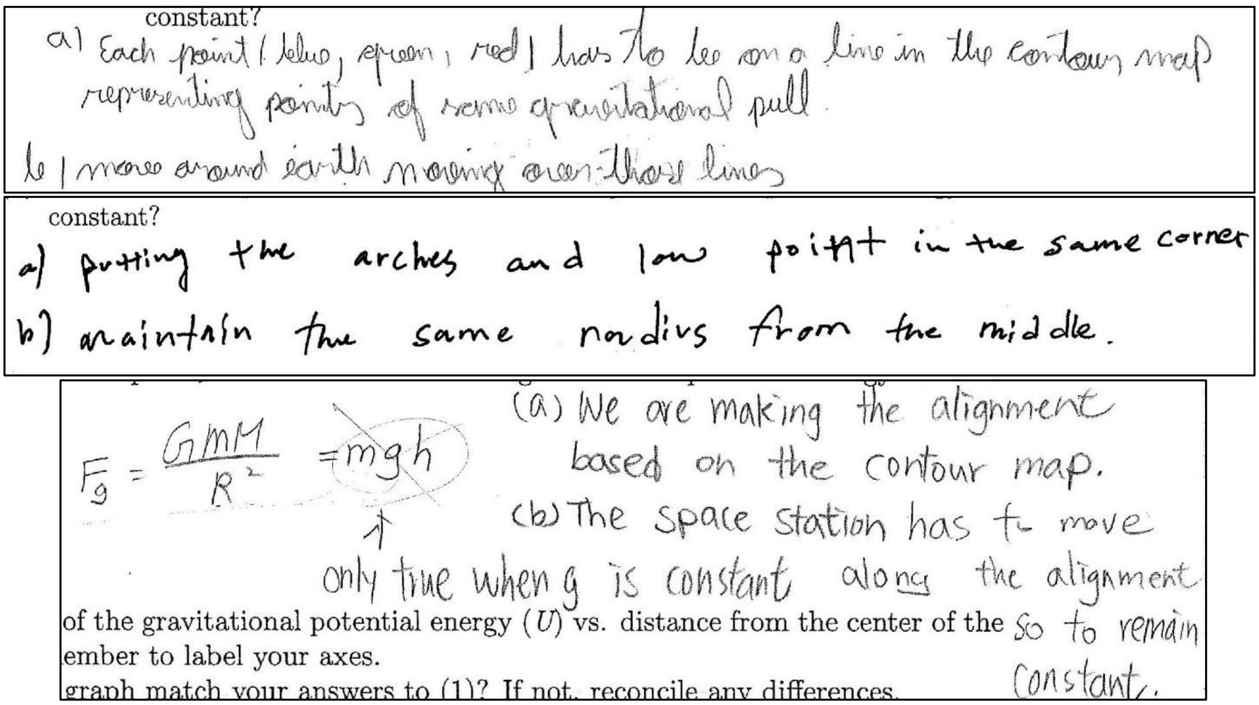
***Figure 36:*** *The Fruit Group's written answers to the first prompt. From top to bottom, the answers belong to Apple, Peach, and Grape.*

(2) Identify other points on the surface where the gravitational potential energy is the same as the potential energy at the blue circle and draw a line to connect them. Do the same for the orange star and the green square.

(a) Align your surface with the contour map. How are you making your alignment?

(b) How could the space station move so that the gravitational potential energy remains constant?

Question 2: Peach states that as long as the radius stays the same, the GPE will stay the same as well. Peach draws an arc of constant radius through the blue mark. The wording of the question confuses Apple; I used both "gravitational potential energy" and "potential energy" to refer to GPE in the question which causes Apple to consider them as two separate entities rather than one. Instead of comparing the GPE at the blue point to the GPE at another point, Apple tries to compare GPE at the blue point to potential energy at the blue point (even though GPE is a type of potential energy, and also the only potential energy we are considering in this problem). This causes difficulties in discussion about this question between group members and with the TA. About 6 minutes into the activity, they begin to discuss the equation for GPE. Grape asks Peach for the equation and Peach replies with $\frac{GMm}{r^2}$ (Notice that this is the equation for gravitational force, and it is missing a negative sign). Peach moves on to 2a and discusses the alignment of the contour map – the rest of the group seems behind. Peach states that the contour map was aligned with the surface by matching one of the circles with the etched divot on the surface representing the surface of the Earth. Apple returns to the comparison between GPE and potential energy. Apple and Grape try to compare the equation for GPE told to them by Peach, $\frac{GMm}{r^2}$, with the equation for GPE they are most used to, $mgh$. Apple struggles to make sense of the two equations, noting that *h* would be the radius, but one equation is multiplied by *h* while the other is divided by $h^2$. Grape comments that one has only one mass, while the other has two. Peach explains that *G* is the gravitational constant, while *g* is based on *G* and *M*, which is why the equations are different. Peach then wonders if $\frac{GMm}{r^2}$ is an equation for something else instead, like kinetic energy. Grape recalls using that equation in a homework assignment and concludes that it must be right, or at least relevant. The group calls the TA over to discuss their dilemma, but they do not decide on an answer.


constant?
a) Each point (blue, green, red) has to be on a line in the contour map representing points of same gravitational pull.
b) move around earth moving over those lines

constant?
a) putting the arches and low point in the same corner
b) maintain the same radius from the middle.

$F_g = \frac{GMM}{R^2} = mgh$
only true when g is constant
(a) We are making the alignment based on the contour map.
(b) The space station has to move along the alignment so to remain constant.
of the gravitational potential energy ($U$) vs. distance from the center of the
ember to label your axes.
graph match your answers to (1)? If not, reconcile any differences.

***Figure 37:*** *The Fruit Group's written answers to the second prompt. From top to bottom, the answers belong to Apple, Peach, and Grape.*

(3) Sketch a graph of the gravitational potential energy ($U$) vs. distance from the center of the Earth ($r$). Remember to label your axes.
   (a) Does your graph match your answers to (1)? If not, reconcile any differences.
   (b) Why is it reasonable to represent the information from the surface in a graph with only 2 axes?

Question 3: Apple, Peach, and Grape work quietly on this question for several minutes. Peach then explains an easy way to draw the graph; simply look at the profile of the surface and sketch the same shape. Apple asks Peach how to answer 3b, and Peach replies that it's because GPE is spherically symmetric.

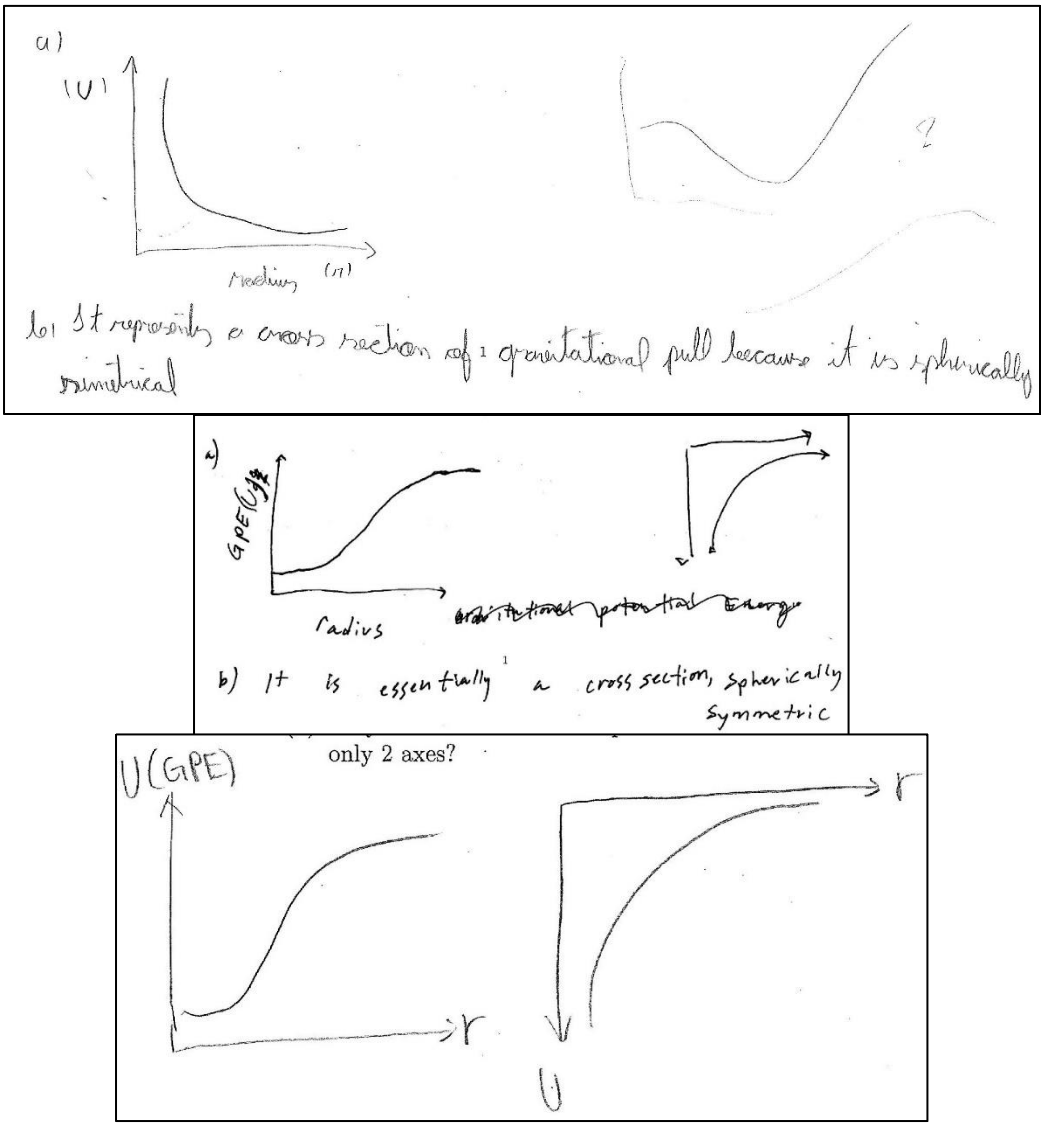



***Figure 38:*** *The Fruit Group's written answers to the third prompt. From top to bottom, the answers belong to Apple, Peach, and Grape. The graphs on the left of Peach and Grape's answers are the original graphs they drew, and the ones on the right are the graphs drawn after the whole class discussion. The graph on the right of Apple's answer is a graph from a pre-activity questionnaire that Apple had recalled.*

(4) What direction will the gravitational force at the blue circle be? Indicate the direction of the force at this point on the contour map.

Question 4: Grape points towards the Earth on the surface and asks if that's the direction the force would go. Both Apple and Peach agree.

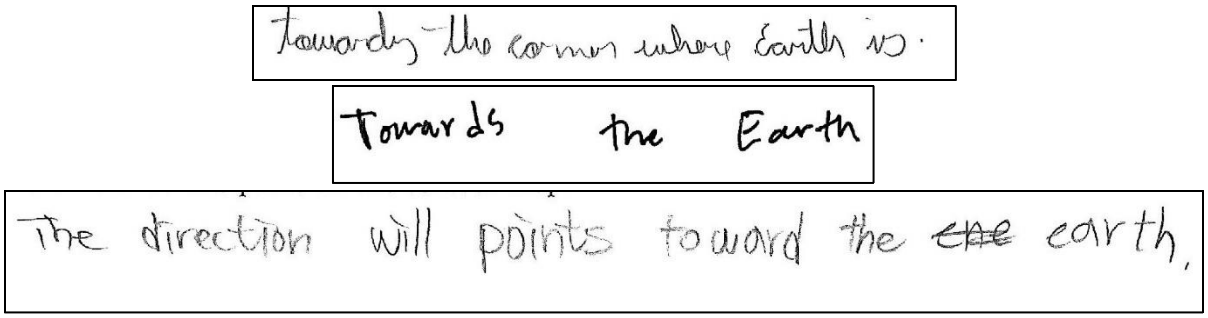



***Figure 39:*** *The Fruit Group's answers to the fourth prompt. From top to bottom, the answer's belong to Apple, Peach, and Grape.*

(5) Locate a point where you would expect the gravitational force to be larger than at the blue circle.

Question 5: Apple reads the question aloud and Grape responds by tracing over the arc previously drawn by Peach in question 2 and stating that any point outside that line (meaning further from Earth) would have a larger gravitational force. The other two agree. Grape elaborates by stating that at the green point (a marked spot further from the Earth than the blue point), the Earth and the green point would be pulled together faster than the blue point. Apple disagree but isn't sure how to explain since the GPE is larger at the green point. Peach brings up that the question asks about force, not gravitational potential. Grape comments that the force should depend on the mass of the planet and Peach replies "I don't know, I'm not sure." Peach thinks that the bigger force will occur inside of the circle (the arc drawn through the blue mark. Apple thinks that doesn't make sense based on the graph drawn in question 3. Peach repeats that the graph is of potential, not force. Grape continues to struggle with the missing mass in the formula – "so then the mass is… massless?" Peach responds that "[we aren't] worrying about the mass in this. We're just talking about points. Kinda massless, maybe?" Apple thinks that the gravitational force "is the same always because Earth has the same mass." The group is silent for several minutes

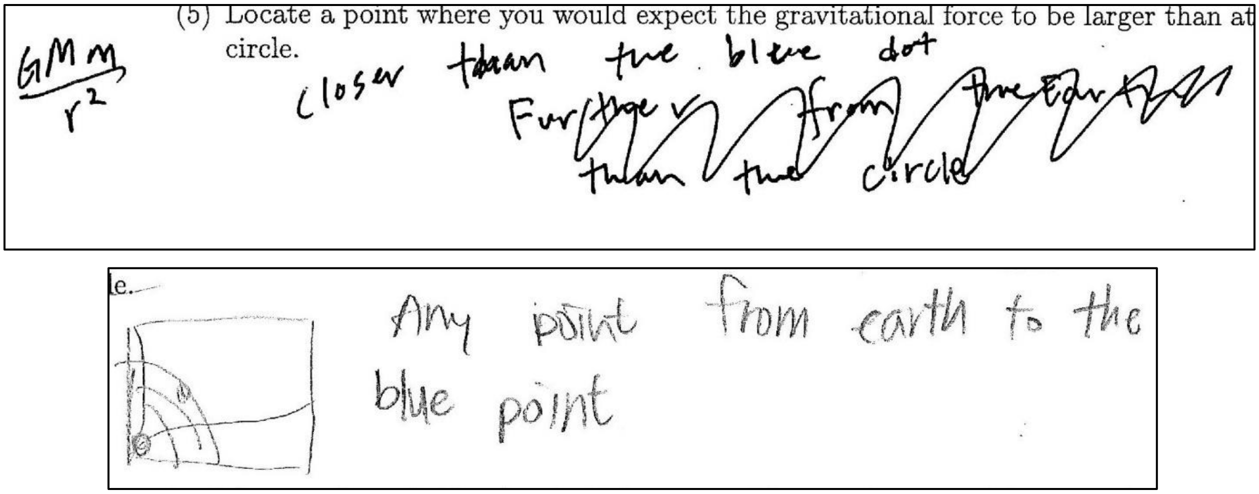



***Figure 40:*** *The Fruit Group's written answers to the fifth prompt. The top answer belongs to Peach and the bottom answer belongs to Grape. Apple did not write anything down for this prompt.*

(6) Is the rate of change of gravitational potential energy with respect to $\boldsymbol{r}$ positive, negative, or zero? Compare $\frac{dU}{dr}$ at the two points from (4) and (5). Which one has a larger magnitude?

Question 6: While sitting silently, the group picks up on a conversation the TA is having with another group about the relationship between force and the derivative of GPE. Peach summarizes the information "you know, like the slope, it's always positive, but it's decreasing. So, the rate of change is negative then, right? $\frac{dU}{dr}$ is negative, um, and then, like, point, the closer point would have a larger magnitude, right? So like the slope is changing a lot more between here [points to section closer to Earth on surface] than it is between here [points to section further from Earth on surface]. So the closer point would have a larger magnitude"

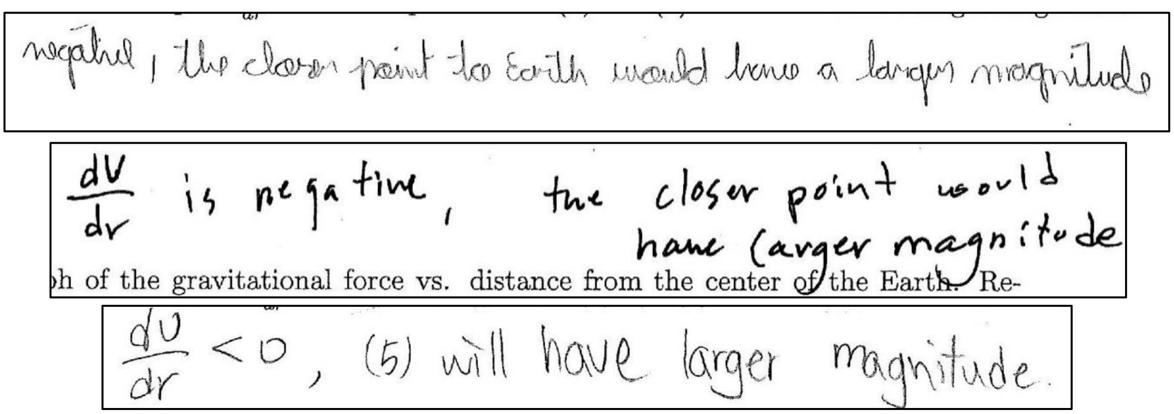



***Figure 41:*** *The Fruit Group's written answers to the sixth prompt. From top to bottom, the answers belong to Apple, Peach, and Grape.*

(7) Sketch a graph of the gravitational force vs. distance from the center of the Earth. Remember to label your axes.

Question 7: Apple and Peach begin discussing the graph of gravitational force. Apple draws a graph of force. Peach draws the same graph, confirming with Apple that it is correct. Grape agrees. Apple makes sense of the force graph by comparing it to the GPE graph, saying that "it makes sense that the potential energy would be opposite," indicating that Apple knows the force graph is right because force is the opposite of GPE.

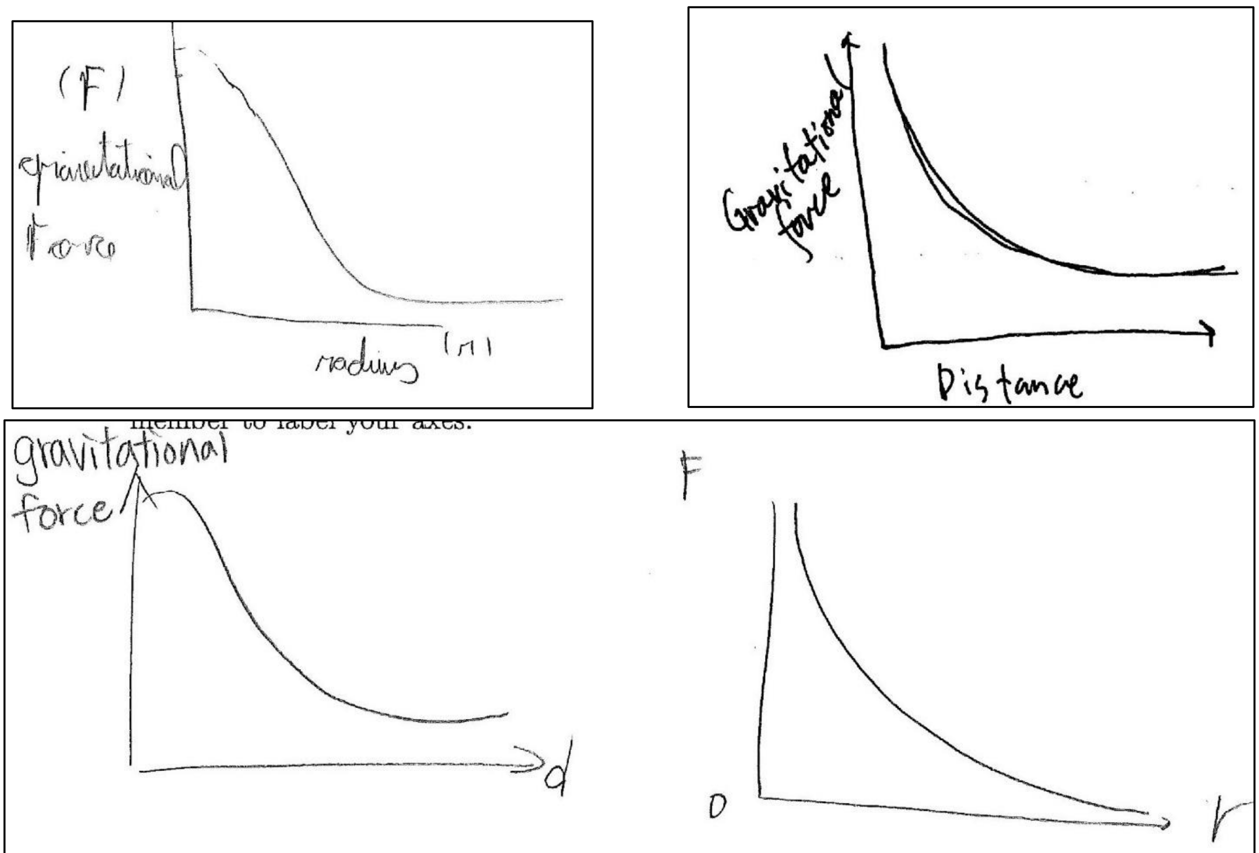
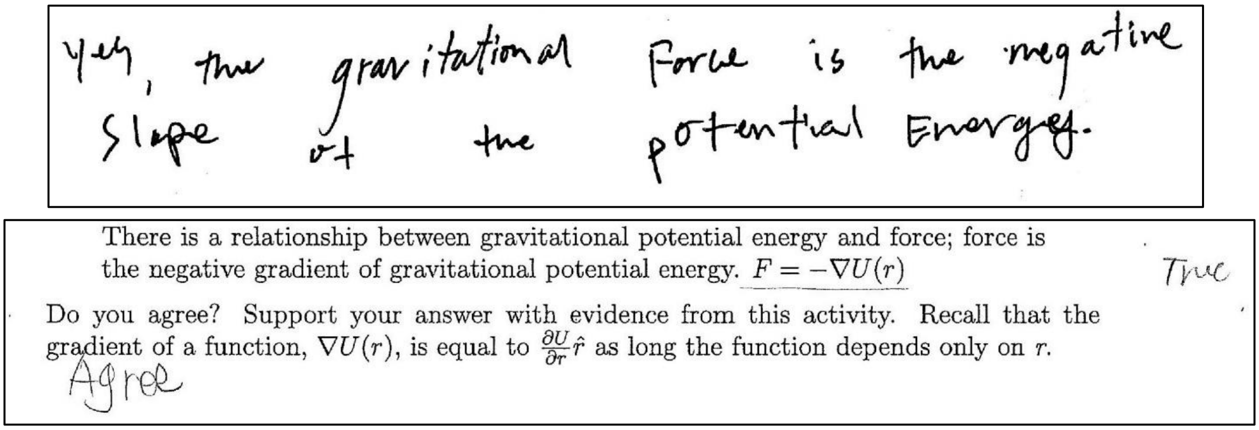



***Figure 42:*** *The Fruit Group's written answers to the seventh prompt. The top left answer belongs to Apple, the top right to Peach, and the bottom answer to Grape.*

Interlude: The TA discusses the graphs of GPE and force with the whole class. The discussion is nearly identical to that of The Pattern Group's whole class discussions, so I will not re-summarize the information. During this discussion, Apple, Grape, and Peach realize that they drew their force graph upside down. Apple doesn't understand how potential energy and force can be maximum at the same time. The group doesn't finish this discussion or clarify the issue as the TA tells the class they may leave if they have finished the assignment. The group leaves. Despite not discussing the last prompt, several of the group members still have written answers (see below).


Yeh, the gravitational Force is the negative slope of the potential Energy.

There is a relationship between gravitational potential energy and force; force is the negative gradient of gravitational potential energy. $F = -\nabla U(r)$ True

Do you agree? Support your answer with evidence from this activity. Recall that the gradient of a function, $\nabla U(r)$, is equal to $\frac{\partial U}{\partial r}\hat{r}$ as long the function depends only on $r$.
Agree

***Figure 43:*** *The Fruit Group's written answers to the last prompt. The top answer belongs to Peach and the bottom answer belongs to Grape. Apple did not write anything down for this prompt.*

End of class.

## Long Summary: Colors

Students: Pink and Purple.

This is the 5th of 6 recitations this week. Everyone has had Tuesday lecture and most of the class had Thursday lecture as well, so most students have at least been introduced to gravitation.

The TA introduces the activity in the same way as in The Pattern Group, then hands out the materials.

Pink and Purple read the introduction on the worksheet, then begin to answer the first question.

> *Your group has a plastic surface and a contour map that represent the gravitational potential energy of a space station-Earth system as a function of the position of the space station relative to Earth. The gravitational potential energy is zero infinitely far away from Earth. Solve the following problems together and discuss the results.*
>
> *You are employed by a company called SpaceY. SpaceY wants to put a space station at the blue circle.*
>
> (1) If the space station moves further away from Earth, how will the gravitational potential energy change? What if the space station moves closer to Earth?

Question 1: Their initial response is that GPE gets lower as the space station moves farther away. Pink traces a path from the blue dot outward and agrees that GPE decreases. Pink looks at the surface for a while and asks if it's the other way around. Purple says no, writes down the equation $\frac{GMm}{r^2}$ (which is the equation for force with a missing negative sign), and says that "if the distance increases, then the potential energy will get smaller as r increases, right?" Pink responds that the equation might be the one for force, so Purple spends about a minute looking through notes to confirm. Purple finally says that "it's still the same for, that's the potential energy."

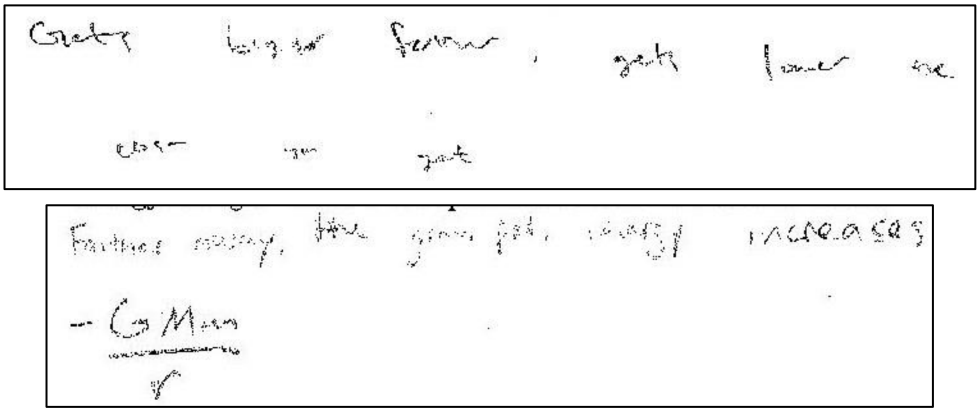


***Figure 44:*** *The Color Group's written answers to the first prompt. The top answer belongs to Pink and says, "Gets bigger farther, gets lower the closer you get." The bottom answer belongs to Purple and says, "Farther away, the grav. pot. energy increases.* $-\frac{GMm}{r}$*."*

(2) Identify other points on the surface where the gravitational potential energy is the same as the potential energy at the blue circle and draw a line to connect them. Do the same for the orange star and the green square.
  (a) Align your surface with the contour map. How are you making your alignment?
  (b) How could the space station move so that the gravitational potential energy remains constant?

Question 2: Pink draws a quarter circle on the surface and looks at Purple for confirmation. Purple agrees. Pink draws the other two arcs, then lines up the contour map with the surface. Purple stands up to look through the surface at the contour map and says that "it's pretty close, actually, to your circles." Neither Pink nor Purple wrote anything down for this prompt.

(3) Sketch a graph of the gravitational potential energy ($U$) vs. distance from the center of the Earth ($r$). Remember to label your axes.
  (a) Does your graph match your answers to (1)? If not, reconcile any differences.
  (b) Why is it reasonable to represent the information from the surface in a graph with only 2 axes?

Question 3: Purple draws a horizontal and vertical axis and asks Pink if "by graph, does it just mean this?" Pink agrees and draws the same axes. Purple labels the horizontal axis U and the vertical axis r, then gets out a calculator. Pink says, "it's an inverse square, ya?" Purple writes down the equation $-\frac{GMm}{r}$ next to the graph; Pink follows suit, stating that it's just a regular inverse and not an inverse square relationship. Pink draws an inverse graph. Purple switches the labels on the axes so that U is on the vertical axis and r is on the horizontal axis, then draws hashmarks on each axis. Purple asks Pink how far the r value should go and what units they should use on their graphs. Pink says that units probably don't matter. Purple draws a similar graph to Pink's graph.

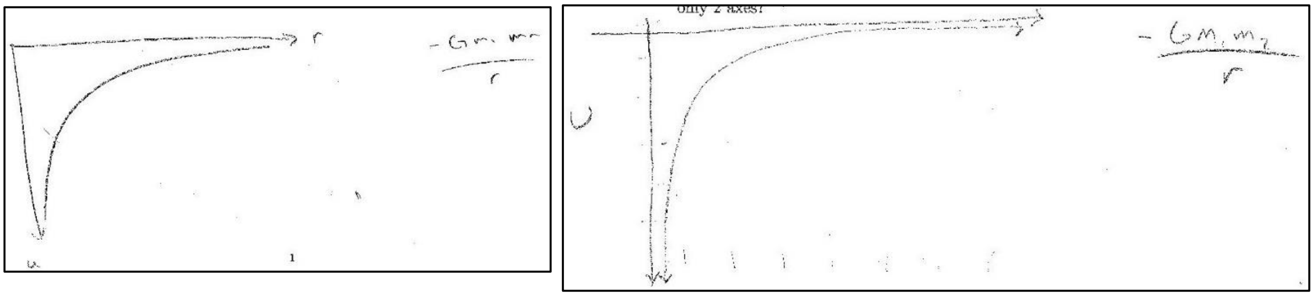


***Figure 45:*** *The Color Group's written answers to the third prompt. The answer on the left belongs to Pink and the answer on the right belongs to Purple. Both have written the equation* $-\frac{Gm_1m_2}{r}$.

> (4) What direction will the gravitational force at the blue circle be? Indicate the direction of the force at this point on the contour map.

Question 4: Purple confirms that the Earth is located in the lowest corner of the surface, then says that the force will point "directly forward," pointing towards the Earth. Pink draws an arrow on the surface in that direction. Neither Pink nor Purple wrote anything down for this prompt.

> (5) Locate a point where you would expect the gravitational force to be larger than at the blue circle.

Question 5: Pink marks a spot on the surface closer to the Earth than the blue dot. Purple agrees. Neither Pink nor Purple wrote anything down for this prompt.

> (6) Is the rate of change of gravitational potential energy with respect to $\boldsymbol{r}$ positive, negative, or zero? Compare $\frac{dU}{dr}$ at the two points from (4) and (5). Which one has a larger magnitude?

Question 6: Pink and Purple initially discuss the rate of change of force between the two locations. They discuss that the answer "depends on which way your positive is." They begin talking about the rate of change of potential energy before flagging down the TA for help. Purple begins the discussion with the TA by saying that, "we said positive cause as it gets closer, it's growing, it's a larger, uh, potential energy." Purple asks if that's what the question is asking, and the TA clarifies that the question asks them to compare the instantaneous rate of change at the two locations. Pink compares the slopes at the two points. The TA says that the first part of the question just asks if the slope is positive or negative. The group discusses whether towards the Earth is positive or negative and decides that the positive direction is away from Earth. Purple asks, "what does it mean by rate of change?" The TA demonstrates by picking up the surface and holding the profile (Figure 8) towards Pink and Purple, tracing the edge of the profile, and telling the group that the rate of change is how quickly the curve is changing at different points. Purple responds with, “Oh, then wouldn’t it be positive? Cause as r grows, the slope gets more positive?” The TA has Pink act out the slope at one of the points and asks the group if that slope is positive or negative.

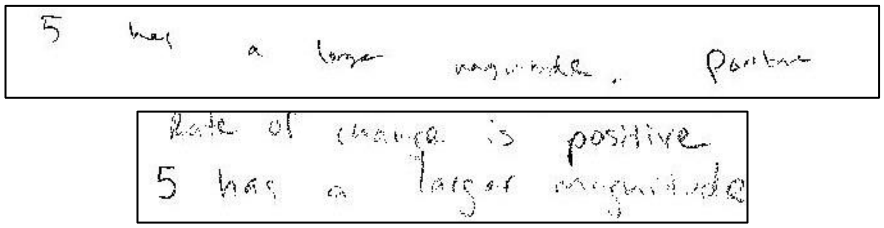



***Figure 46:*** *The Color Group’s written answers to the sixth prompt. The top answer belongs to Pink and says “5 has a larger magnitude. Positive.” The bottom answer belongs to Purple and says, “Rate of change is positive. 5 has a larger magnitude.”*

(7) Sketch a graph of the gravitational force vs. distance from the center of the Earth. Remember to label your axes.

Question 7: Purple draws axes, inputs something into the calculator, then draws the graph. Purple labels the horizontal axis r and the vertical axis $F_g$.

Interlude: The TA talks to the whole class about their graphs for potential energy. One of the students speaking during the class discussion did not consent to be recorded, so I will not be summarizing this part, but it was a very similar discussion to The Pattern Group’s first class discussion.

Question 7 (Continued): Pink and Purple realize they drew the graph of GPE upside down. Pink erases the graph and talk about the discovery. Pink says, “I was saying that it gets lower the further away you go, but that’s the other way. Which makes sense because like if you’re just on Earth, and you’re like 50 feet above the ground, you have more potential energy than if you’re like, 5 feet above the ground.” Purple agrees and Pink both redraws the graph.

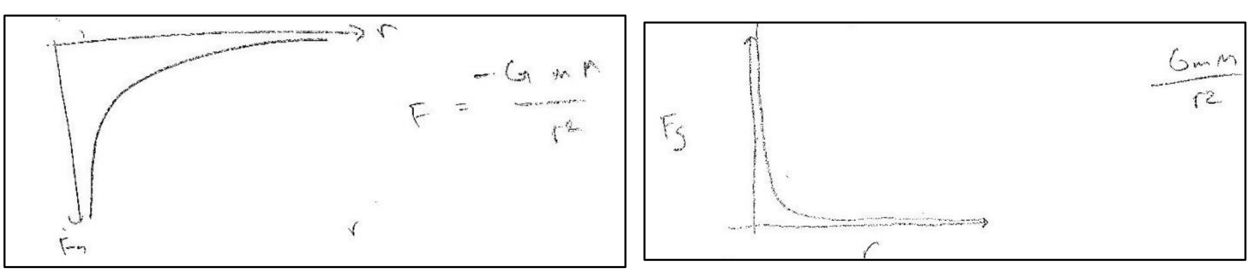
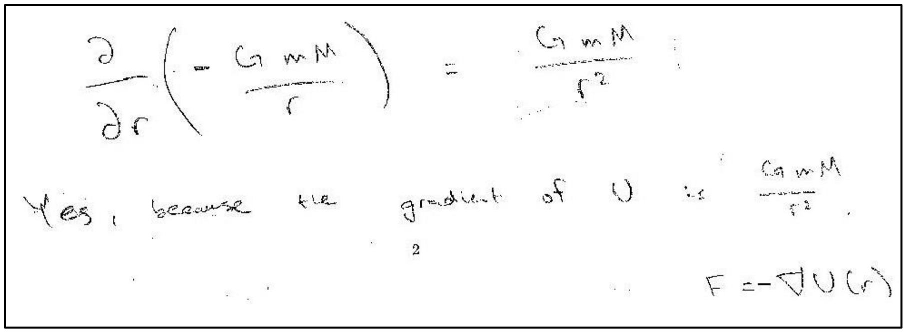


***Figure 47:*** *The Color Group’s written answers to the seventh question. The left answer belongs to Pink and the right belongs to Purple. Pink has written the equation* $F = -\frac{GmM}{r^2}$ *and Purple has written* $\frac{GmM}{r^2}$.

> (8) The SpaceY employee handbook states:
>
> There is a relationship between gravitational potential energy and force; force is the negative gradient of gravitational potential energy. $F = -\nabla U(r)$
>
> Do you agree? Support your answer with evidence from this activity. Recall that the gradient of a function, $\nabla U(r)$, is equal to $\frac{\partial U}{\partial r}\hat{r}$ as long the function depends only on $r$.

Question 8: Pink asks if "the gradient is like the vector that's tangent to the – the field, right?" Pink acts out the tangent with a marker while explaining the gradient to Purple. Purple asks if that's "like the derivative of this graph [GPE vs. r]." Purple and Pink then argue about the negative sign for a while. They each drew their force graph positive, so the sign doesn't agree with their graphs. They try to make sense of this before deciding that they don't agree with the statement. The TA comes over to ask about their answer and the two say that "we disagreed…cause when you take the derivative, it'll be, it'll turn to that [points to force graph], but then since it's negative, it wouldn't be the negative gradient of the gravitational potential energy." The TA explains that "negative and positive just indicate direction." The group talks about whether force is negative or positive in this scenario; they start by looking at the direction of the vector they drew and comparing it to the direction of $\hat{r}$. Then, the TA asks if the force vector always points in the same direction. Pink and Purple say that force will always point in the same direction and therefore force will always be negative. The TA tells them that their plot would be correct if they chose $\hat{r}$ to point inward instead of outward. Pink redraws the graph, but Purple looks back at the equation for gravitational force (Purple wrote it as $\frac{GMm}{r^2}$) and questions the sign again. Purple says "the gravitational force equation is that [points to paper]…. I mean, that's what I've been getting from like lecture and stuff, but I'm not…" Pink says that "it doesn't really matter, as long as you're, like, careful about which way you're pointing it." Purple seems to agree.



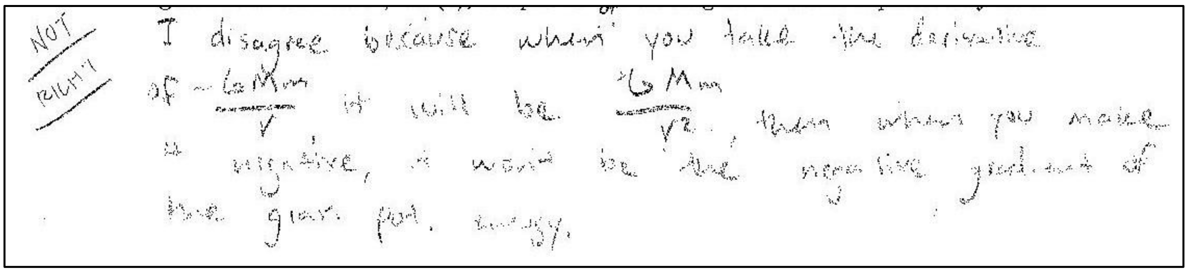


**Figure #:** The Color Group's written answers to the last prompt. The top answer belongs to Pink and says, "$\frac{\delta}{\delta r}\left(-\frac{GmM}{r}\right)=\frac{GmM}{r^2}$. Yes, because the gradient of U is $\frac{GmM}{r^2}$. $F=-\nabla U(r)$." The bottom answer belongs to Purple and says, "<u>NOT RIGHT</u>" on the side and, "I disagree because when you take the derivative of $-\frac{GMm}{r}$, it will be $\frac{GMm}{r^2}$, then when you make it negative, it won't be the negative gradient of the grav. pot. energy." Purple wrote the "<u>NOT RIGHT</u>" section after discussing the sign of force with the TA rather than rewriting the whole section.

Interlude: The TA brings the whole class in for a final discussion. Once again, I will not be summarizing this discussion as a non-consenting student was speaking. The discussion was similar to the final discussion in The Pattern Group.

End of class.

# LIST OF FIGURES