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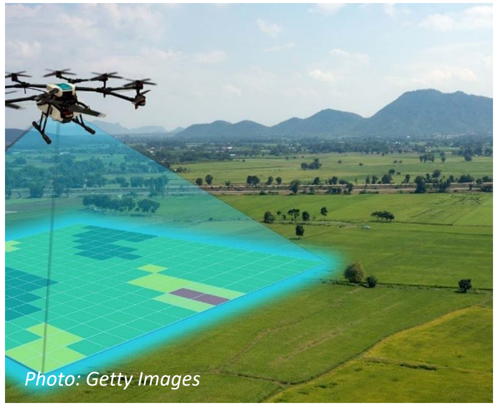
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UAS FOR SURVEYING AND MAPPING

Module C: Positioning Techniques

On-Ramp to Innovation
every day counts

Topic 6:
Geodesy & Coordinate Systems

A review of terrestrial techniques and geodetic principles applicable to sUAS

Overview of Content:

- → *Introduction to Geodesy*
- → *Horiz. & Vert. Datums*
- → *GPS's Ellipsoid*
- → *Coordinate Systems*

U.S. Department of Transportation
Federal Highway Administration

Using Geospatial data

- ***Four things we should be thinking of when using geospatial data:***
  - » Geodetic datum definitions & reference coordinates

    How are the data connected to the Earth?
  - » Grid coordinate systems and computations

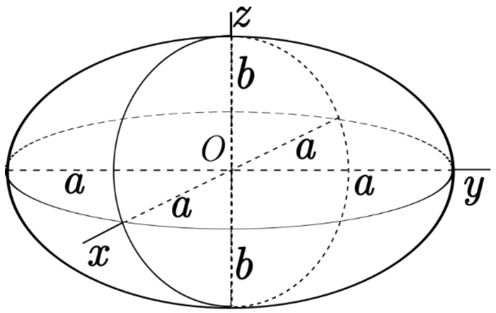
    How are the data displayed? How are the data used?
  - » Vertical datums and height systems

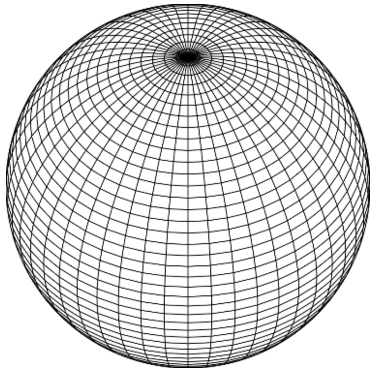
    How high is it? How deep is it? Where will water go?
  - » Accuracy estimation and reporting

    Is it in the right place? By how much? How do you know?

Geodesy

- GNSS has forced surveyors to learn geodesy
- Geodesy:
  - Also known as geodetics or geodetic engineering
  - A branch of applied mathematics and earth sciences
  - Scientific discipline that deals with the measurement and representation of Earth, includings its gravitational field, in a 3D time-varying space [Wikipedia]
  - Studies geodynamical phenomena such as crustal motion, tides, and polar motion
  - Literally the science of measuring and monitoring the size and shape of Earth, and the location of points on its surface [NOAA]





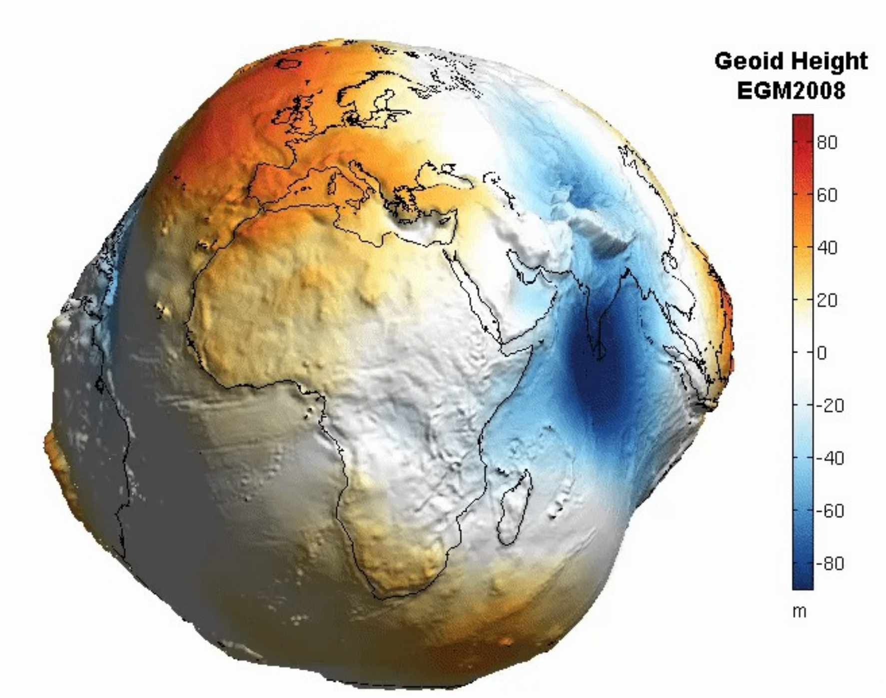
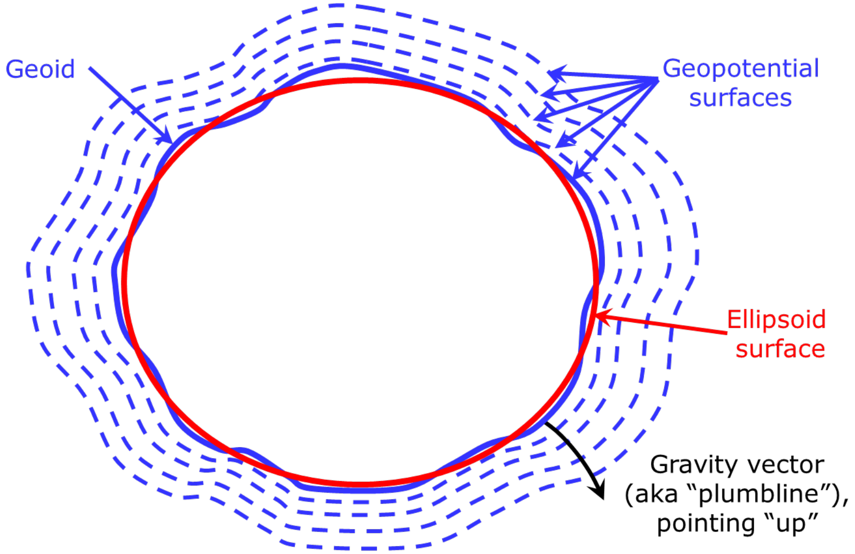


Image Credit: National Geospatial-Intelligence Agency

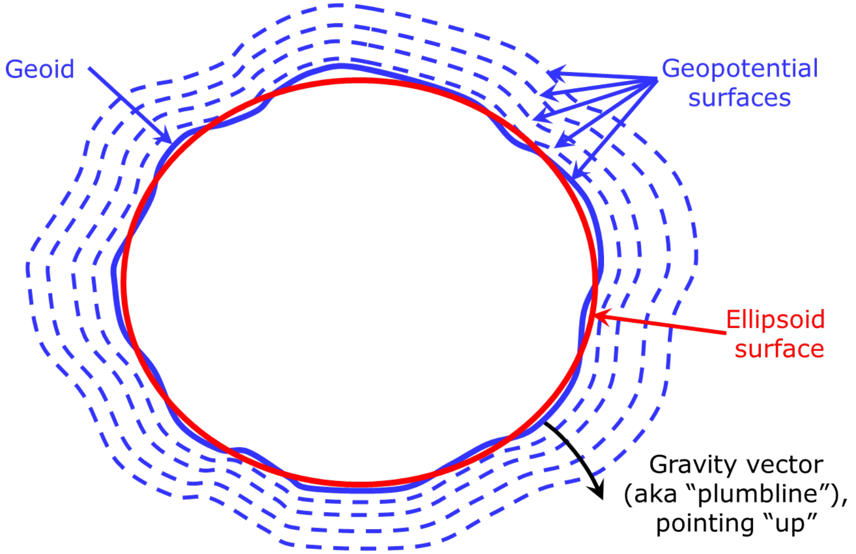
GNSS Derrived Heights

- GNSS yields heights relative to the ellipsoid (h)
- We are much more interested in physical heights
  - » Heights related to gravity (I.e. orthometric heights)
  - » The best fit of earth's gravity field is called the "geoid"
- Geoid is an imaginary surface
  - » Fits the height of the oceans if all forces but gravity are removed
  - » level surface relating to mean sea level surface
    - – But wait.. Mean sea leveling is rising? How does that affect our heights?
  - » Geoid separates from modern ellipsoids by up to 100 m in some places



Types of Heights

- Orthometric height, H
  - » The distance between the geoid and a point on the Earth's surface measured along the plumb line.
- Geoid Separation (height), N
  - » The distance along a perpendicular from the ellipsoid of reference to the geoid
- Ellipsoid height, h
  - » The distance along a perpendicular from the ellipsoid to a point on the Earth's surface.

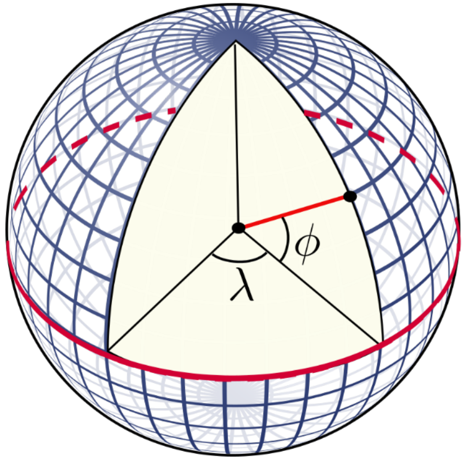


H = Orthometric Height (NAVD 88)
h = ellipsoidal height (NAD83(2011))
(note: book calls this geodetic height)
N = geoid height (GEOID 12B or Geoid18)

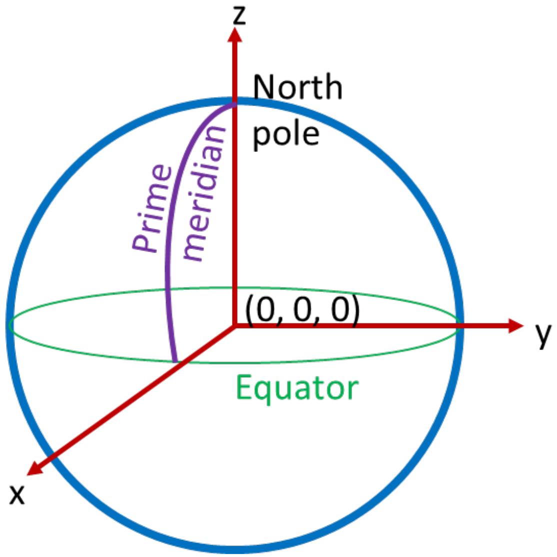
$$\boxed{H \approx h - N}$$

Geodetic and Geocentric Coordinates

- Geodetic latitude, longitude and height
  - » Ø = geodetic latitude
  - » λ = geodetic longitude
  - » h = ellipsoid height
  - » Disadvantages: not as intuitive as grid coordinates, and math is more complicated (e.g., inversing between points involves a much lengthier series of calculations)

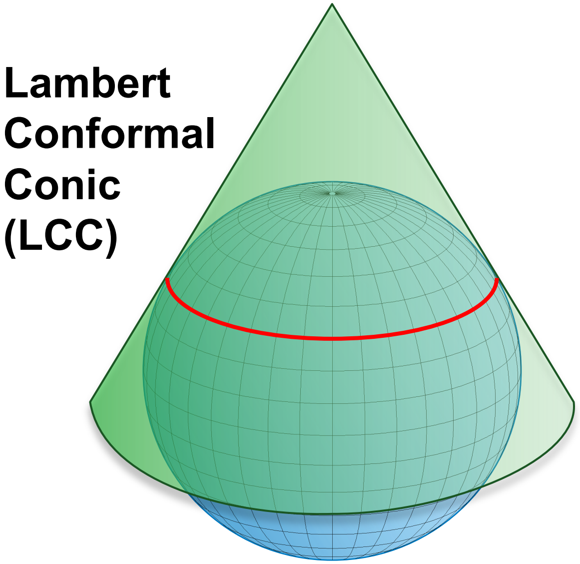


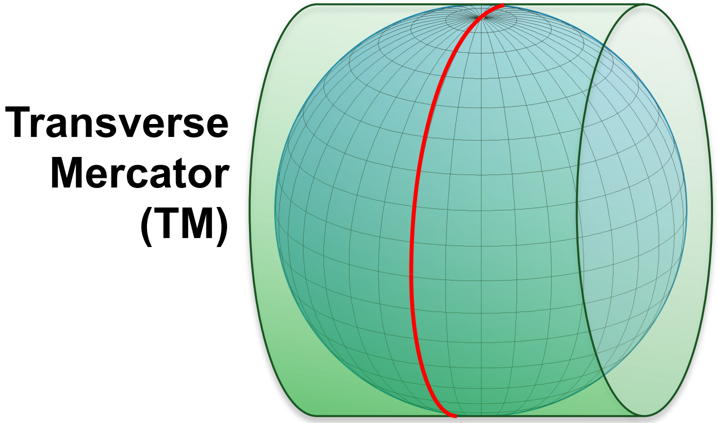
- ECEF XYZ
  - » Origin is at center of mass of Earth
  - » z axis is aligned with Earth's rotation axis and passes CTP
  - » x axis passes through intersection of equatorial plane and Greenwich meridian
  - » y axis completes a right-handed coordinate system
  - » Disadvantages: coordinate values are very large, and axes are not aligned with local east and north directions, and, therefore, are not intuitive. Z is not "height."



Map Projects

- “Developable surface”
  - » Envision flat surface that can be wrapped around Earth somehow and then unrolled and laid flat
- Imperfect
  - » There will be distortions in one or more of the following:
    - Angles
    - Azimuth
    - Distance
    - Area
  - » Conformal projections (e.g., Lambert conformal conic and transverse Mercator) minimize distortion in shapes and angles
    - Will have distortion in distance



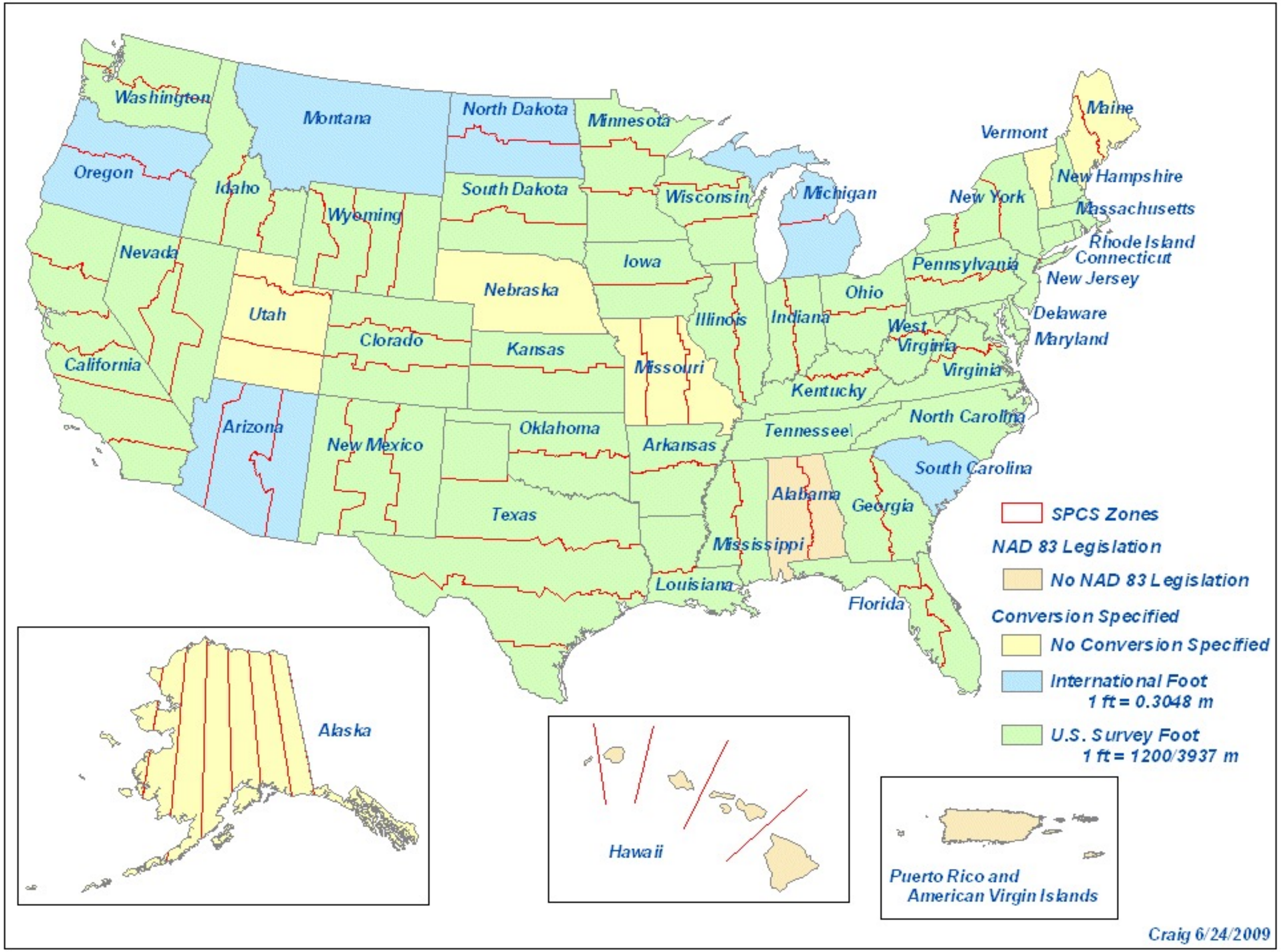


State Plane Coordinate Systems

Actually 2 different systems: SPCS27 and SPCS83

Differences go beyond reference datum (NAD27 vs. NAD83)

- SPCS27
  - » All coordinates in U.S. Survey Feet
- SPCS83
  - » Some states reduced number of zones in new system
  - » All values in meters: conversions to feet defined by individual state legislation (can be either International Feet or U.S. Survey Feet)
- Soon to be 3!
  - » The soon to be released ***SPCS2022*** to be discussed by Michael Dennis on Nov. 30th!!!
    - Will only use International feet
    - Referenced to NATRF 2022


Washington
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North Dakota
Minnesota
Maine
Vermont
New Hampshire
Oregon
Idaho
South Dakota
Wisconsin
Michigan
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Connecticut
Nevada
Iowa
Pennsylvania
New Jersey
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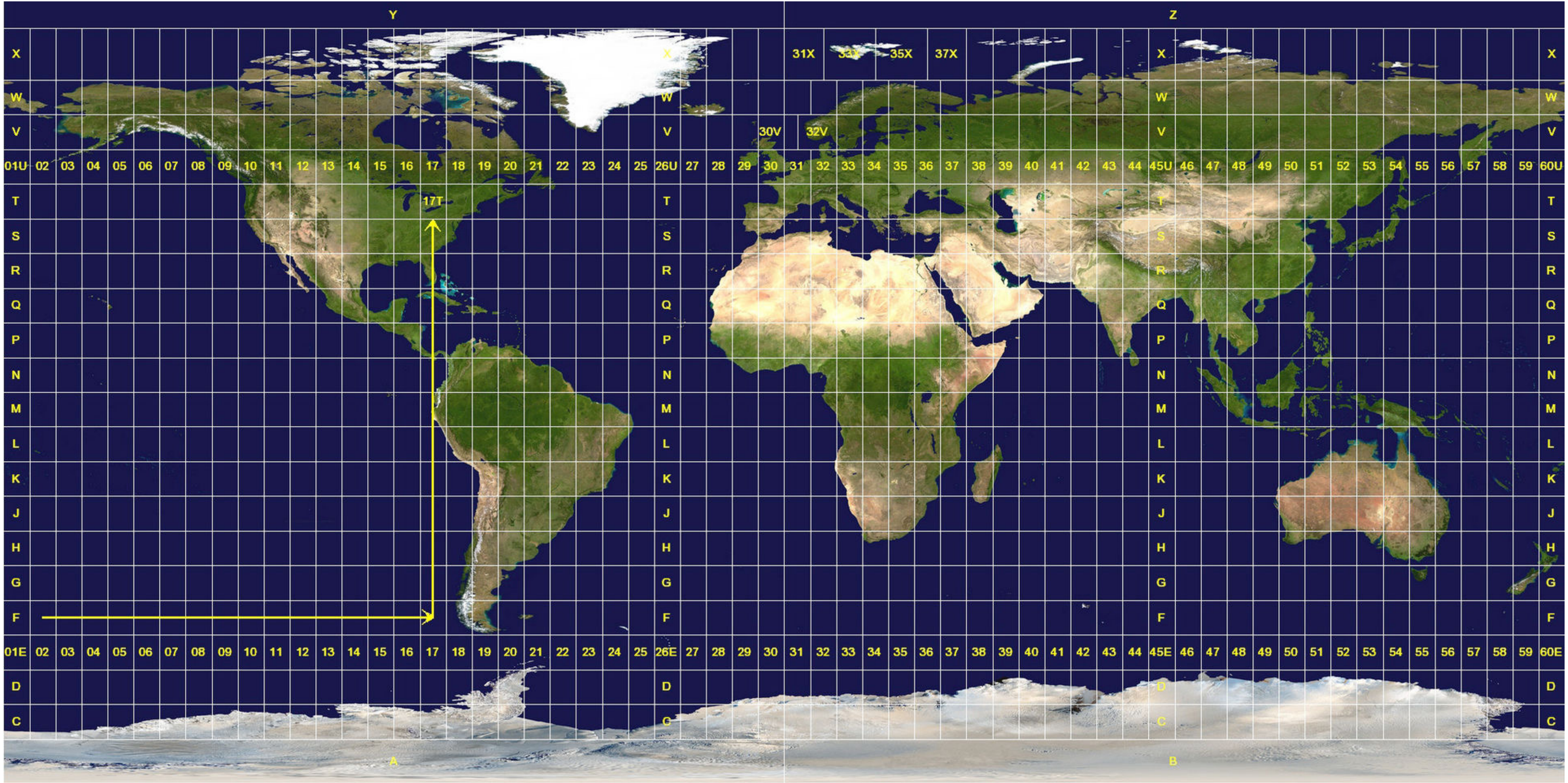
California
Missouri
Virginia
Kentucky
North Carolina
Arizona
Oklahoma
Tennessee
New Mexico
Arkansas
South Carolina
Alabama
Georgia
Texas
Mississippi
Louisiana
Florida
SPCS Zones
NAD 83 Legislation
No NAD 83 Legislation
Conversion Specified
No Conversion Specified
International Foot
1 ft = 0.3048 m
U.S. Survey Foot
1 ft = 1200/3937 m
Alaska
Hawaii
Puerto Rico and American Virgin Islands
Craig 6/24/2009

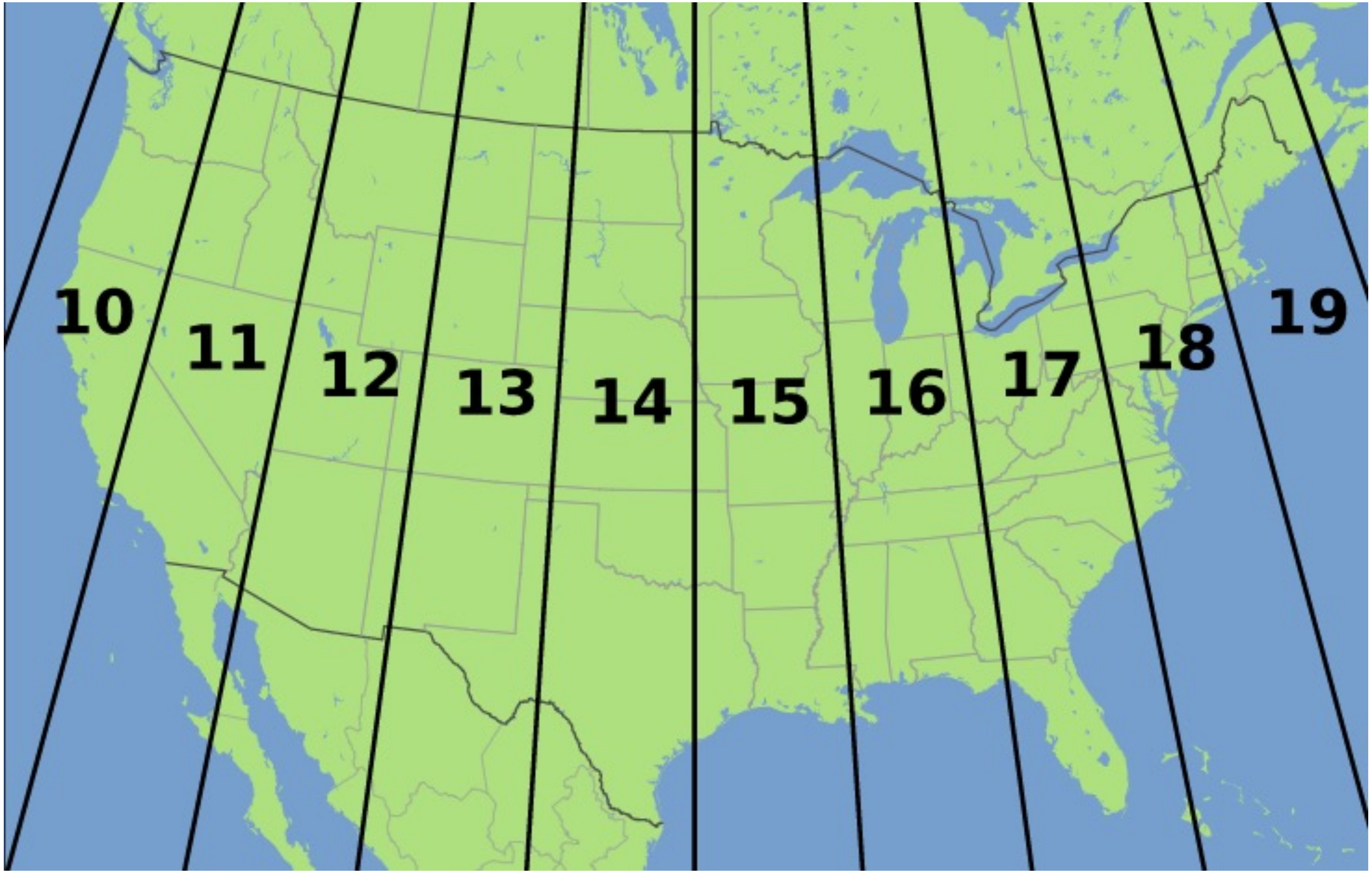
Universal Transverse Mercator (UTM)

- Another important map projection system
- Covers whole globe
- Divides the Earth into 60 zones, each 6° wide (in longitude)
  - » Uses secant transverse Mercator projection in each zone
- Larger zones than SPCS, so distortion is greater: 1 part in 1,000

Universal Transverse Mercator (UTM)



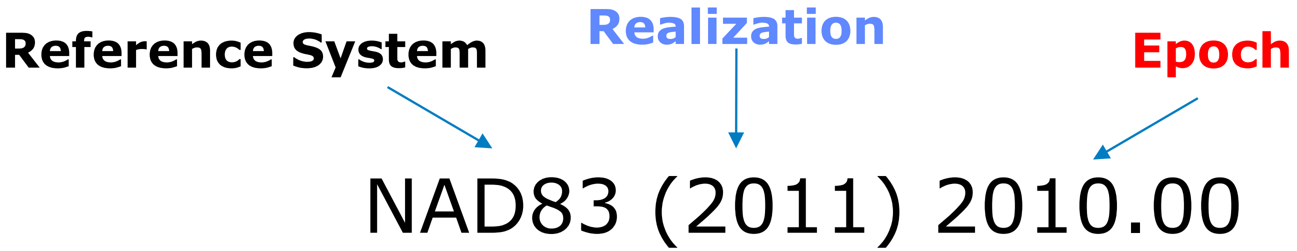
Universal Transverse Mercator (UTM)



Datums

- Although there are many datums the official Datums primarily used in the US are:

- Horizontal:



Reference System

Realization

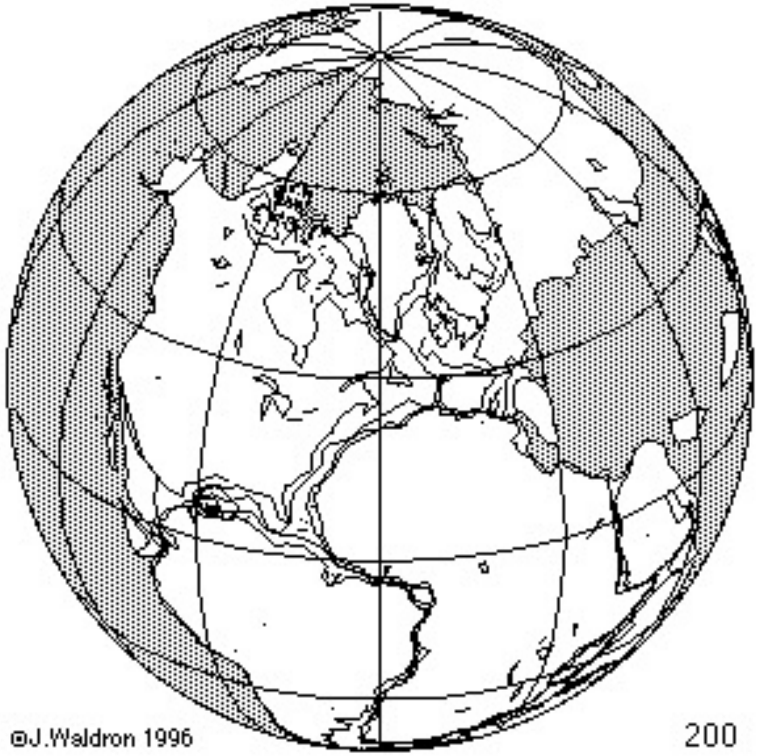
Epoch

NAD83 (2011) 2010.00

- Vertical:
  - » North American Vertical Datum of 1988 (NAVD88)

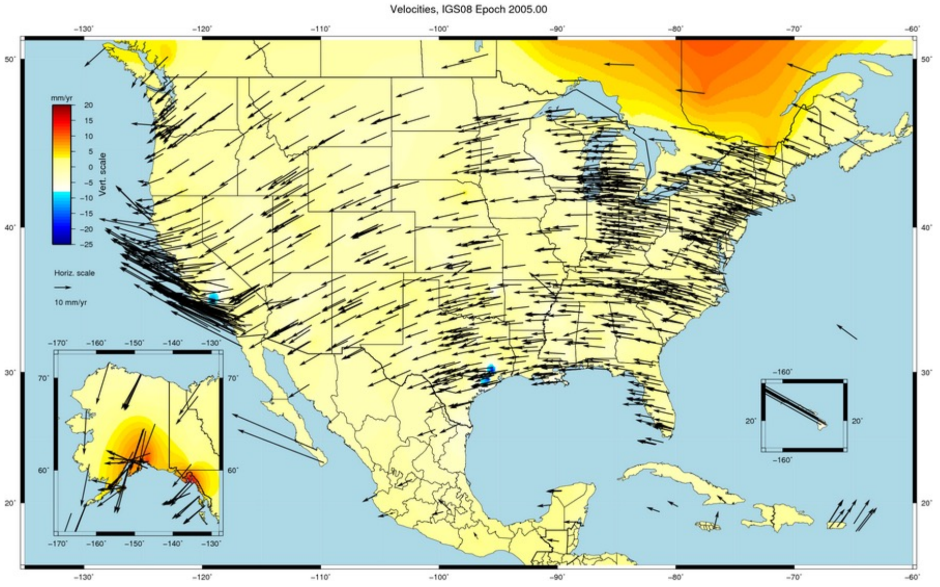
Epochs & Realizations

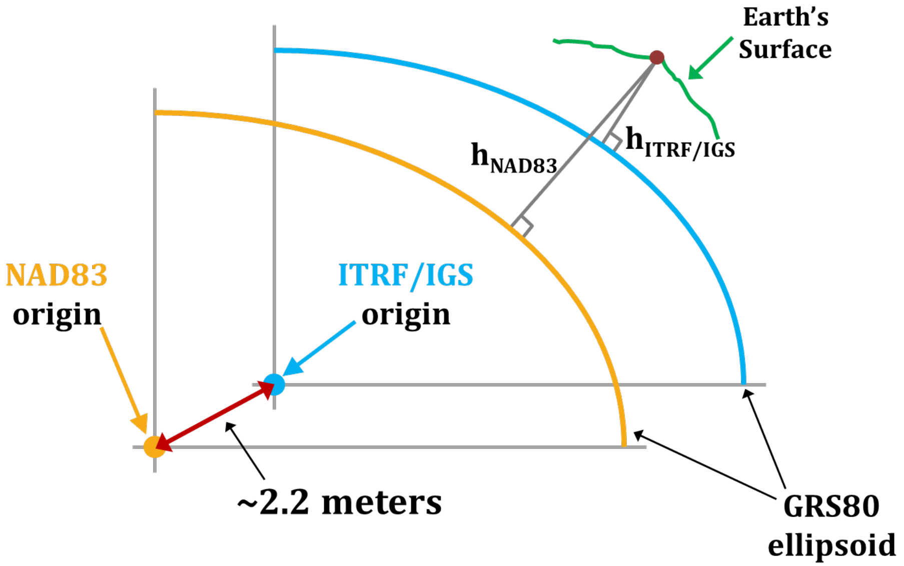
- The definition of a coordinate system does not change
  - Shape of the ellipsoid defining a coordinate system does not change!
  - Points on the surface of the earth just get new coordinates or new “realizations”
- Realizations make the coordinate system practical
  - Coordinates are “established” on sets of stations
  - Use of GPS, VLBI, SLR measurements
  - Use of numerous, globally scattered active GPS stations that monitor the satellites
  - Realizations tend to improve over time



Current U.S. Horizontal Reference System

- NGS decided to fix NAD83 to the North American Plate, which moves 10/20 mm per year in relation to WGS84 (uses IGS08 as reference ellipsoid)
- Most recent realization of NAD83 is NAD83(2011) Epoch 2010.00
  - » NAD83(2011) Epoch 2010.00 differs with WGS84(G1762) by 2 meters within the U.S.
  - » The geocenter of NAD83 differs by 2 meters from the true geocenter of Earth





Vertical Datums

A Set of fundamental elevations to which other elevations are referred

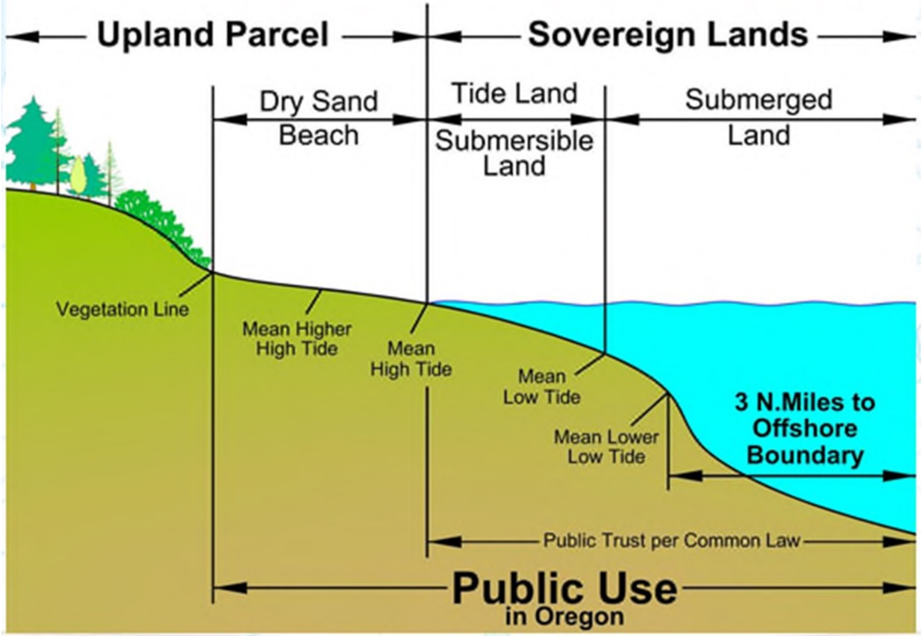
Vertical Datum Types

Tidal

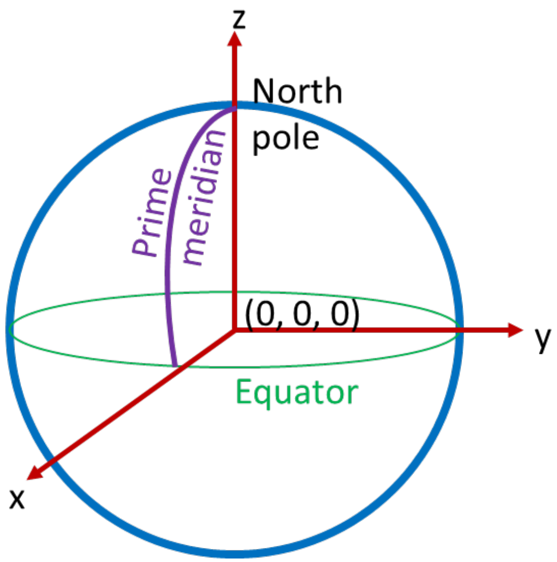
- Defined by observation of tidal variations over some period of time
  - (MSL, MLLW, MLW, MHW, MHHW etc.)

Geodetic

- Either directly or loosely based on Mean Sea Level at one or more points at some epoch
  - (NGVD 29, NAVD 88, etc.)



GPS's Ellipsoid: WGS84



- WGS84(G1762) was aligned to the IGb08 realization of ITRF2008
  - » Enables consistency in standards and interoperability with other GNSS
  - » Used active GPS stations from the U.S. Airforce and the IGS
  - » Adopted measured velocities at the IGS stations
  - » IGS active stations were held fixed while the U.S. Airforce and NGA stations were adjusted.
  - » Can conclude that coordinates in WGS84(G1762) ≈ IGS2008
- The International Earth Rotation Service also maintains its own reference frame, known as ITRF
  - » Most recent reference system is named ITRF2014.
    - – Uses GRS-80 ellipsoid
  - » International GNSS Service maintains an ITRF realization

GPS's Ellipsoid

Name	Epoch	Remarks	Shift	Accuracy
WGS84	1984	First realization established by DoD in 1987 using Doppler observations. Also known as WGS84 (1987), WGS84 (original), WGS84 (TRANSIT). For surveying purposes, original WGS84 is identical to NAD83 (1986).	N/A	1-2 m
WGS84 (G730)	1994.0	Realization introduced by DoD on 1994-06-29 based on GPS observations.	0.70 m	10 cm
WGS84 (G873)	1997.0	Realization introduced by DoD on 1997-01-29 based on GPS observations.	0.20 m	5 cm
WGS84 (G1150)	2001.0	Realization introduced by DoD on 2002-01-20 based on GPS observations.	0.06 m	1 cm
WGS84 (G1674)	2005.0	Realization introduced by DoD on 2012-02-08 based on GPS observations.	0.01 m	< 1 cm
WGS84 (G1762)	**2005.0**	**Realization introduced by DoD on 2013-10-16 based on GPS observations. Aligned to ITRF2008 reference system**		**< 1 cm**

Errors from Geodetic Datums and Refence Systems

Positioning error examples	**Magnitude***
Using NAD 27 when NAD 83 required	Varies from ~250 to 330 ft (horizontal)
Using "WGS 84" when NAD 83 required (e.g., using WAAS or CORS ITRF/IGS coordinates)	~4-5 ft (horizontal) ~1-2 ft (vertical)
Using NAD 83 (1986) when NAD 83 HARN/HPGN (1991) required	Up to 4.5 ft (horizontal)
Using NAD 83 (2007/CORS96) when NAD 83(2011) epoch 2010 required	Up to 0.5 ft (horiz) Up to 0.6 ft (vert)

*Typical Values for Oregon

Topic 7:
Terrestrial Surveying Techniques *(Review)*

A review of terrestrial techniques and geodetic principles applicable to sUAS

Overview of Content:

- → *Terrestrial Surv. Techniques*
- → *Definition of Accuracy*
- → Establishing Project Control

Survey/Mapping Methods

There are many ways to acquire data to satisfy a surveying/mapping operation but most often a project will require 2 or more of the options below:

- Levels
- Total Stations
- GNSS

Still need a fundamental understanding of these methods

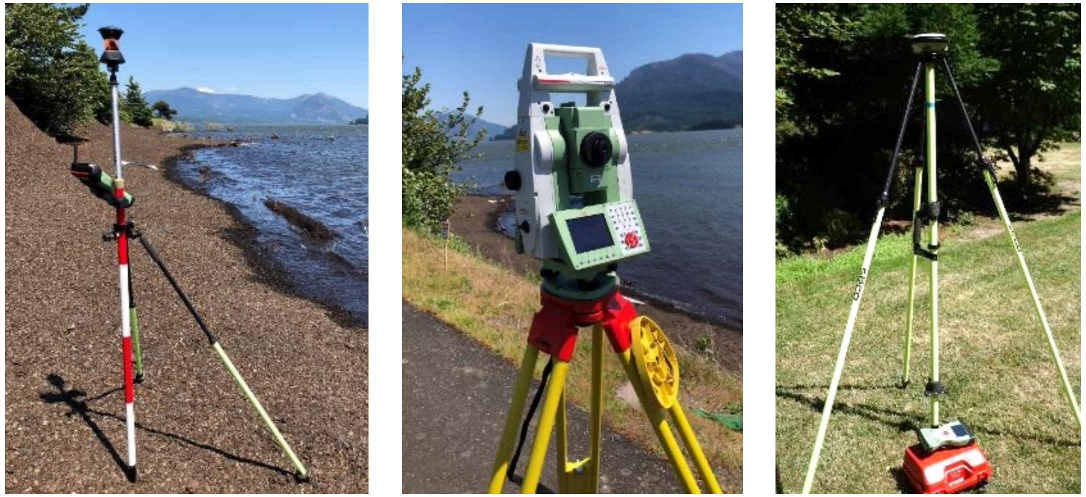
- Lidar (active Sensor)
  - » Terrestrial, Mobile, Aerial
- Photogrammetry (Passive sensor)
  - » Terrestrial, mobile, aerial
- Sonar (active sensor)
  - » Bathymetric Mapping

Photo: Pheonixlidar.com

Photo: dji.com

Ground Control Methods

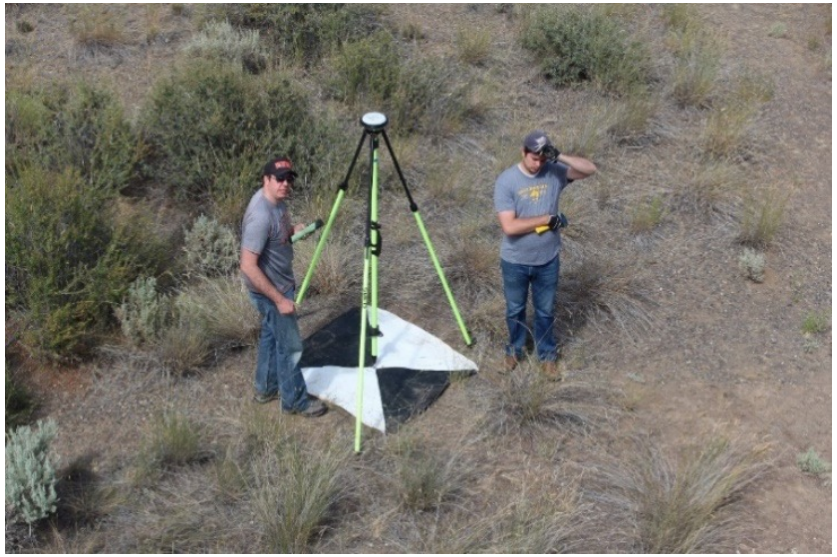
- Most Control Surveys Completed using 1 or more of the following technologies
  - » Levels
  - » Total Stations
  - » GNSS
- Also used to establish check points used to assess accuracy of final products
  - » ASPRS Recommendations for CP quantity location
    - – All check points (CP's) on flat or uniformly sloped open terrain
    - – Evenly distributed throughout project
    - – 20 or more for confident statical assessments



Control Surveys

Control Surveys can be broken into two categories

- Conventional Surveys
  - » Performed using traditional precise survey techniques and instruments (e.g. Total stations, and levels)
  - » Trilateration, Triangulation, Traverse, Level Loops, etc.
- GNSS Surveys
  - » Utilize GNSS receivers to determine geodetic positions and is becoming the go to approach for performing control surveys
  - » Static-GNSS, Real-Time Networks, PPP, etc.
- So which of these approaches should we use?
  - » It depends completely on our desired accuracy requirements



Definitions

Error: difference between observed value of a quantity and its true value:

$$\varepsilon = m - \mu$$

Residual: is the difference between the most probable value and the measured value:

$$v = m - \bar{M}$$

Accuracy: measure of the absolute nearness of measured quantities to their true value

Precision: the degree of consistency between measurements

- If multiple observations are made and the discrepancies are all small, this indicates high precision

Discrepancy: the difference between any two observed values of the same quantity

Dispersion: Range of Measurements

ε = error
v = residual
m = measured value
$\bar{M}$ = most probable value
μ = true value

Accuracy

Which survey methods should we use to establish control?

» It depends completely on our desired accuracy requirements!

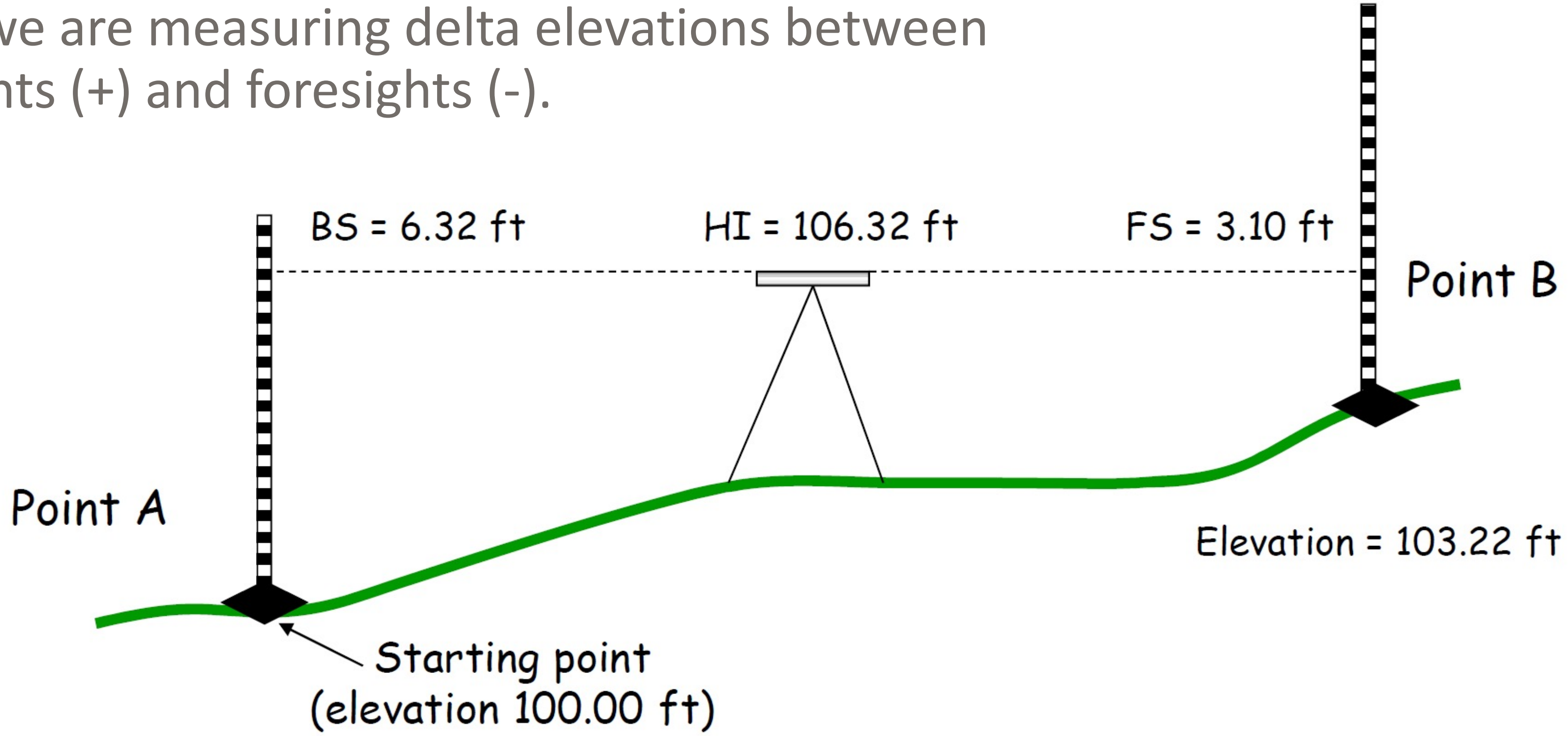
Accuracy can be broken down into two categories:

- Relative Accuracy *(local accuracy)*
  - » Represents the accuracy of a position with respect to nearby adjacent points
- Absolute Accuracy *(network accuracy)*
  - » Used to define the uncertainty of a position relative to a datum ore reference system

Leveling

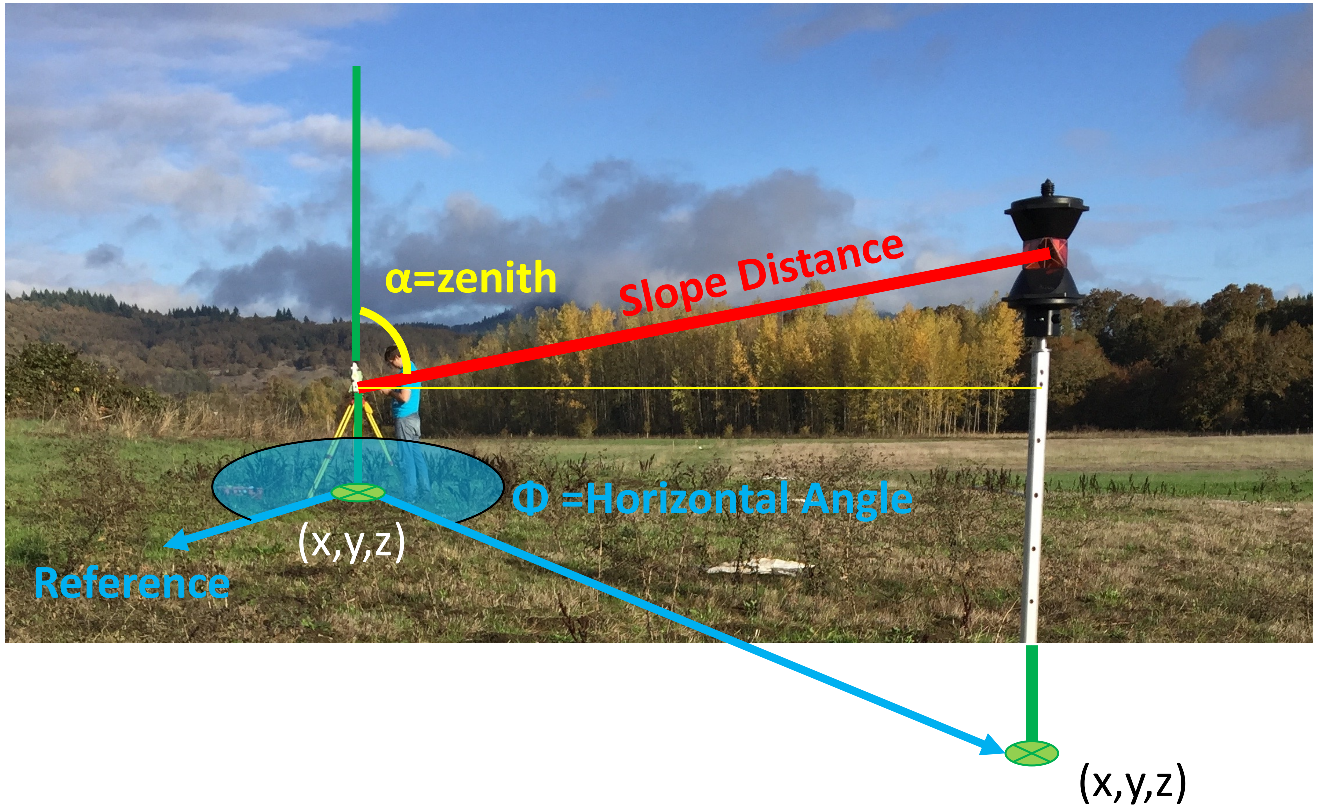
Used to determine elevations of unknown points relative to a known points

- Really, we are measuring delta elevations between backsights (+) and foresights (-).



Total Stations

- Combine 3 Basic Components:
  - Electronic Distance Measuring (EDM)
  - Electronic Angle Measuring
  - Microprocessor
- Observe:
  - Horizontal angle
  - Zenith angle
  - Slope Distance
- Automatically compute
  - Average of multiple angle and distance observations
  - Corrections (e.g., prism constants, atmospheric corrections)
  - C&R corrections
  - Reduction of slope distances to horizontal and vertical components
  - Coordinates from angle and distance observations



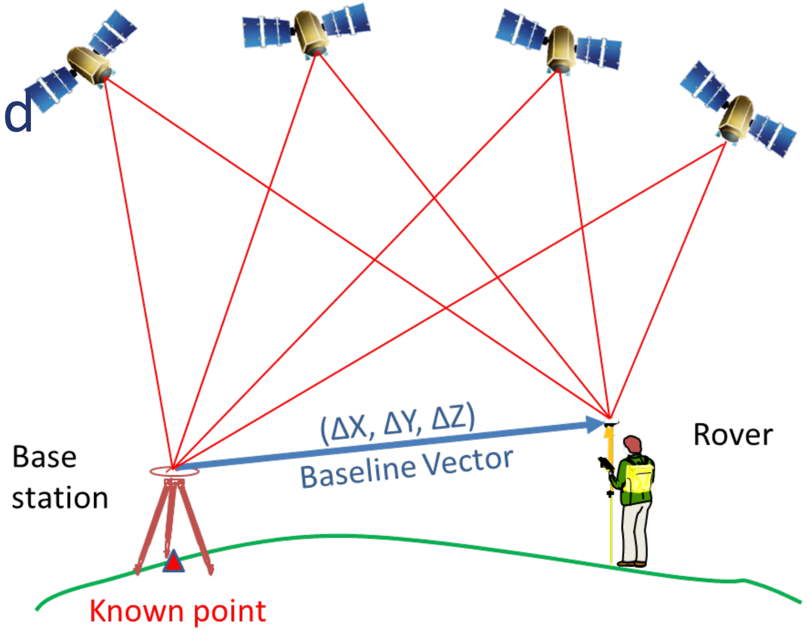
GNSS

Generally, want accuracies of ~0.5 – 4.0 cm, depending on type of survey

  - » Key elements is to use carrier-phase based positioning, rather than relying on code-based pseudoranges

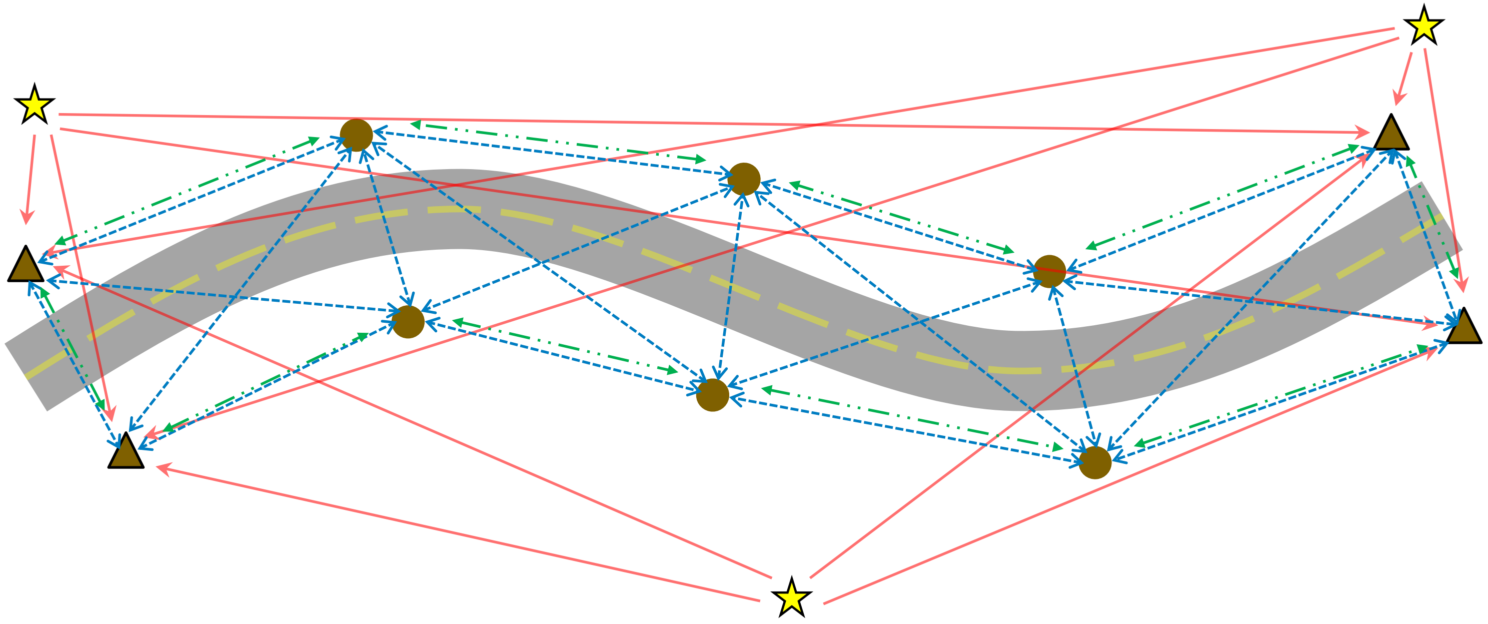
- Use relative positioning - multiple receivers: at least 1 base + 1 rover
  - » Post processed or RTK
  - » Can use dedicated base station(s) that you set up or continuously operating reference stations (CORS)

- Use appropriate care in survey planning, observations and processing
  - » Minimize PDOP
  - » Make sure you are initialized in RTK
  - » Use care in centering
  - » Use appropriate post processing, as applicable
  - » Network adjustment, when applicable



Which Method Should we use?

- In a perfect world where we have all the time and resources to complete the survey, using all three methods would be preferred:
  - Differential leveling → Precise delta elevations between each GCP
  - Total station → Precisely horizontal positions of each
  - GNSS survey → Align control to a desired coordinate system
- Method selection dependent on desired accuracy



Legend

● Ground control Point

▲ GNSS Passive Station

☆ CORS

←·–·→ Differential Leveling Obs.

←----→ Total Station Network Obs.

⟶ GNSS Baseline

GNSS vs. Total Station

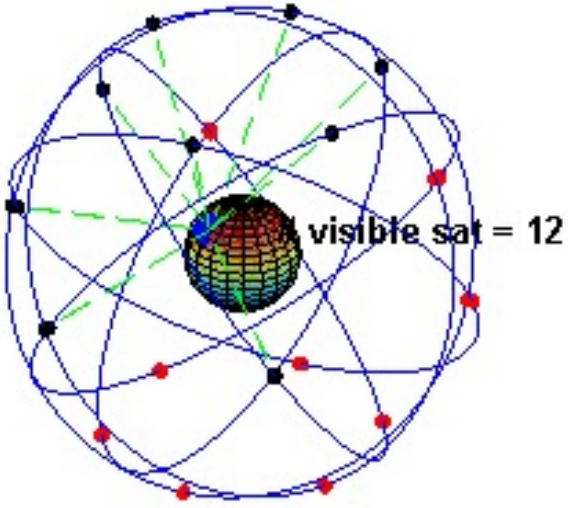
Both methods determine 3D positions, but…

Observations & Measurements

- *TS: Directly observes vector components*
  - *(slope distance, zenith and horizontal angles)*
- *GNSS: Observes satellite signal,* computes vector components
  - *No direct measurements made on the ground!!*

Computations

- *TS: Can compute coordinates with plane trigonometry*
- *GNSS: Requires geodetic methods for computations*



Reference frame

- *TS: Referenced to plumbline passing through instrument*
- *GNSS: Referenced to global coordinate system*

Do you ALWAYS need GCPs?

- It depends
- Skipping ground control points may yield perfectly fine results, but your reconstruction might not have the correct scale, orientation, or absolute position information.
  - » Scale issue can be alleviated by incorporating scale constraints
    - – Measure between two or more photo-identifiable features within the AOI.
    - – Identify scale constraint in SfM Software (e.g. Agisoft, Pix4D, etc.)
    - – Applicable for volumetric analysis or when absolute accuracy is not required

Importance of Accuracy

- As a surveying and mapping professional (or enthusiasts) we should know accuracy is key to the success of just about any project.
- Using a UAS for aerial photography can yield impressive imagery
- But to perform UAS-photogrammetry an understanding of accuracy and precision is required

OK, but why is uncertainty so important, anyway?

- “Data without uncertainty are usually useless.” [1]

  “The knowledge of …. uncertainty in a test result is as important as the result itself…Results should never be reported without also reporting their … uncertainty.” [2]

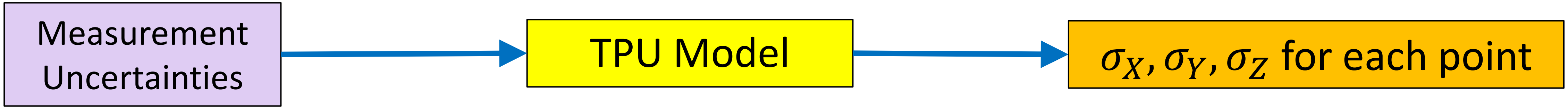
- Without uncertainty
  - No way to evaluate whether it is appropriate for use in a particular project
  - Managers/decision makers can’t make informed decisions based on your output without knowing its uncertainty
  - Cannot be defended, if challenged (e.g., in court)
  - Don’t know how much to trust the data (or weight the data, if being combined with other data sources)

1. Rumble, John Jr. (NIST), 1999. Scientific Data in the Internet Era. World Conference on Science, 26 Jun – 1 July, Budapest, Hungary.
2. Dieck, R.H., 2007. *Measurement Uncertainty: Methods and Applications*. ISA, Research Triangle Park, NC.

Accuracy Assessments

- There are two primary methods used to assess accuracies of geospatial data:

1. Empirical Assessment
   - » ASPRS Positional Accuracy Standards for Digital Geospatial Data
   - » Requires a Comparison against check points from an **independent source of higher accuracy**
   - » Will be discussed further in Topic 14
2. Total Propagated Uncertainties (formal error propagation)
   - » Gives us per-point uncertainties of lidar (or SfM, or sonar, etc.) point coordinates
   - » Measure or model uncertainties in inputs: observations (e.g., ranges, scan angles, position and orientation, etc.), calibration data, etc.
   - » Propagate those uncertainties through the geolocation process to uncertainties in computed, georeferenced points in the point cloud

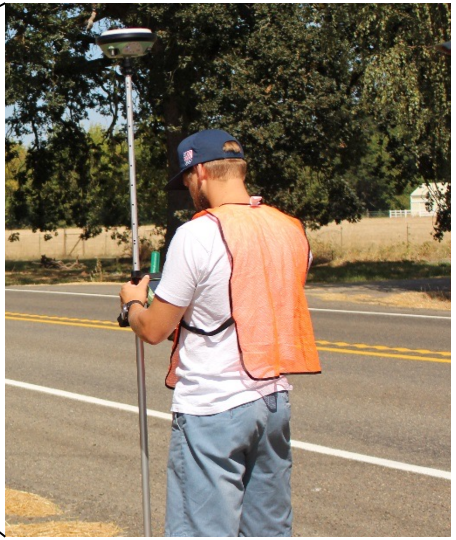


Measurement Uncertainties

TPU Model

$\sigma_X, \sigma_Y, \sigma_Z$ for each point

Why do we need TPU when we can just do an empirical accuracy assessment?



RTK GNSS

* *Not actual location*

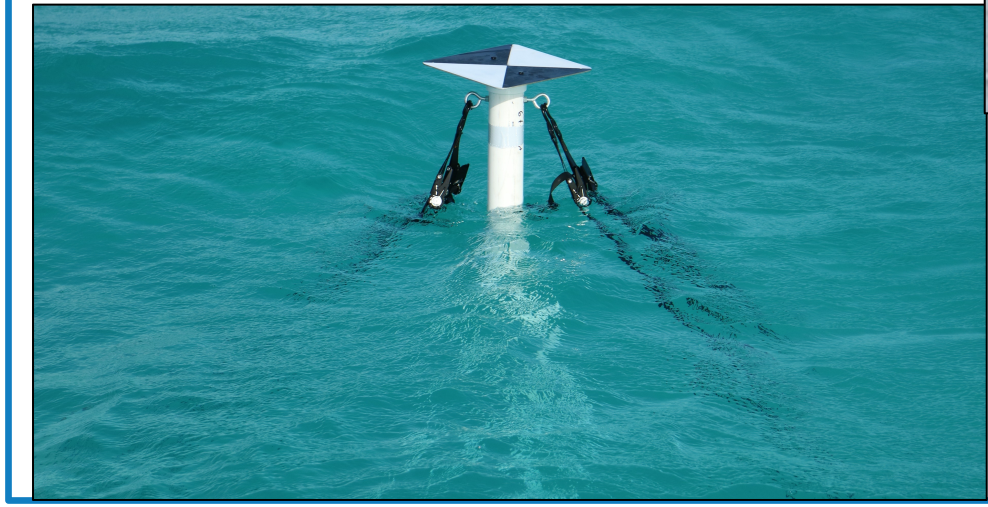
$$RMSE_z = \sqrt{\frac{\sum \left(z_{datai} - z_{checki}\right)^2}{n}}$$

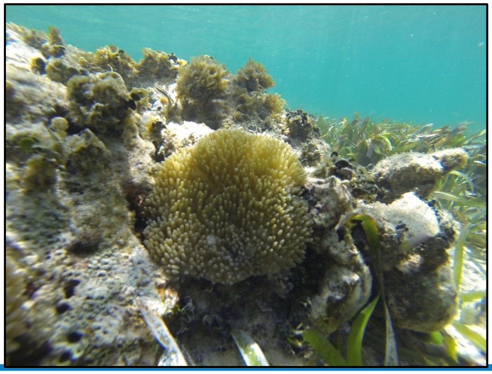
$$Accuracy_z = 1.96\left(RMSE_z\right)$$

≥20 well-distributed checkpoints

Seems easy enough. Why not just do this everywhere…

What if your site is inaccessible (e.g., due to being underwater, remote, dangerous to access, etc., etc.)?

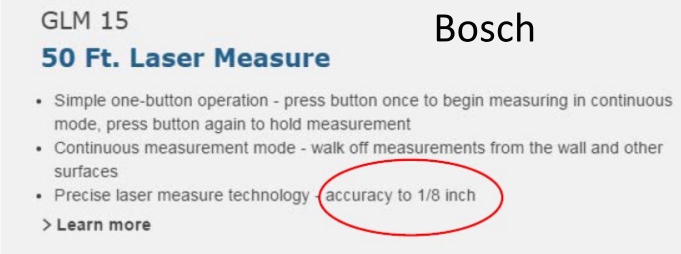




“Boots on the ground” field surveys to establish independent, higher-accuracy checkpoints are not always possible, due to logistics, costs, travel budget, being underwater, etc.

Uncertainty in a measured value vs. in a computed value

- Obtaining uncertainties for directly-measured values is *relatively* easy
  - » How?
    - – Manufacturer specs:



GLM 15

Bosch

50 Ft. Laser Measure

- Simple one-button operation - press button once to begin measuring in continuous mode, press button again to hold measurement
- Continuous measurement mode - walk off measurements from the wall and other surfaces
- Precise laser measure technology - accuracy to 1/8 inch

> Learn more

Unless otherwise stated (and it usually isn't), you can generally assume these manufacturer-stated accuracies are standard errors (i.e., 1σ). So, in this case, σ = $\pm\frac{1}{8}$ inch)

    - – Empirically: repeat measurement 30 times, compute spread (sample standard deviation)

- OK, but how about the uncertainty in something that we don't directly measure, but, rather, compute from directly measured things?
  - » Isn't this much more common?
    - – Example: does a lidar system actually measure terrain elevation?
  - » This is where uncertainty *propagation* comes in
  - » Propagate the measurement uncertainties into uncertainty in the computed quantity

Quick note on terminology

- **In Geomatics, we often talk about "error propagation." Is this different from "uncertainty propagation"?**
- Same thing, just different terms
  - » In keeping with emerging conventions (ISO/IEC), I tend to refer to "uncertainty" rather than "error" or "accuracy"
  - » Also consistent with hydrographic surveying community (e.g., IHO S-44)

"uncertainty" "error" "accuracy"

???

Philosophical Musings on Error Theory:

- Are random errors (or “random uncertainties”) really random?
  - If we can move some amount of error from the random to systematic category simply by doing increasingly better calibrations, how could the error have been random to begin with?
  - Are random errors really just all the left over systematic errors (due to our imperfect knowledge of the instrument, measurement conditions, etc.), after we feel we’ve done a sufficiently good calibration?
    - Based on answer to this, how logical is it to assume “random” errors will be normally distributed? (Side note: from my experience, they often are, although certainly not always.)
  - For that matter, are things we think of as inherently random, such as the outcome of rolling dice or flipping a coin, really random? Or, are they just mechanics, but with imperfect knowledge of the starting conditions and forces that make the outcome appear random?

References

Diaconis, P., Holmes, S. and Montgomery, R., 2007. Dynamical bias in the coin toss. SIAM review, 49(2), pp.211-235.

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On-Ramp to Innovation
every day counts

Topic 8:
Introduction to GNSS Positioning

A review of positioning via GNSS

Overview of Content:

- → *History of GPS*
- → *Importance of Time*
- → Code vs. Carrier Ranging
- → Relative GNSS Positioning
- → Error Sources

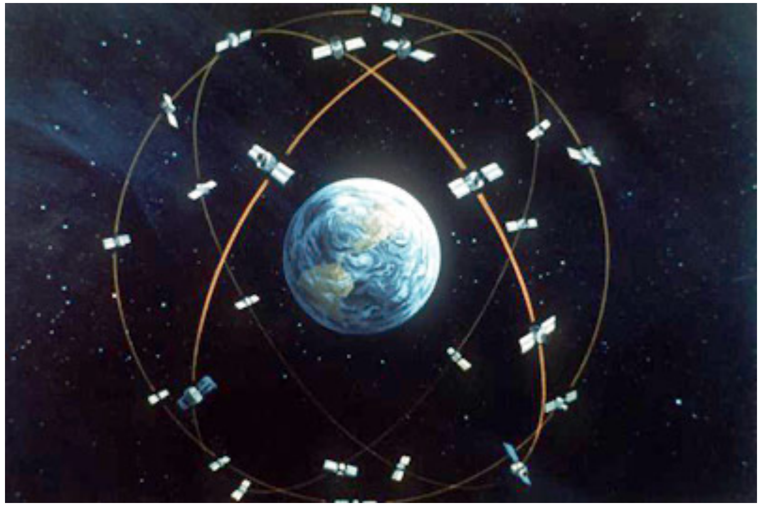
U.S. Department of Transportation
Federal Highway Administration

Brief History of GPS: 1940-Now

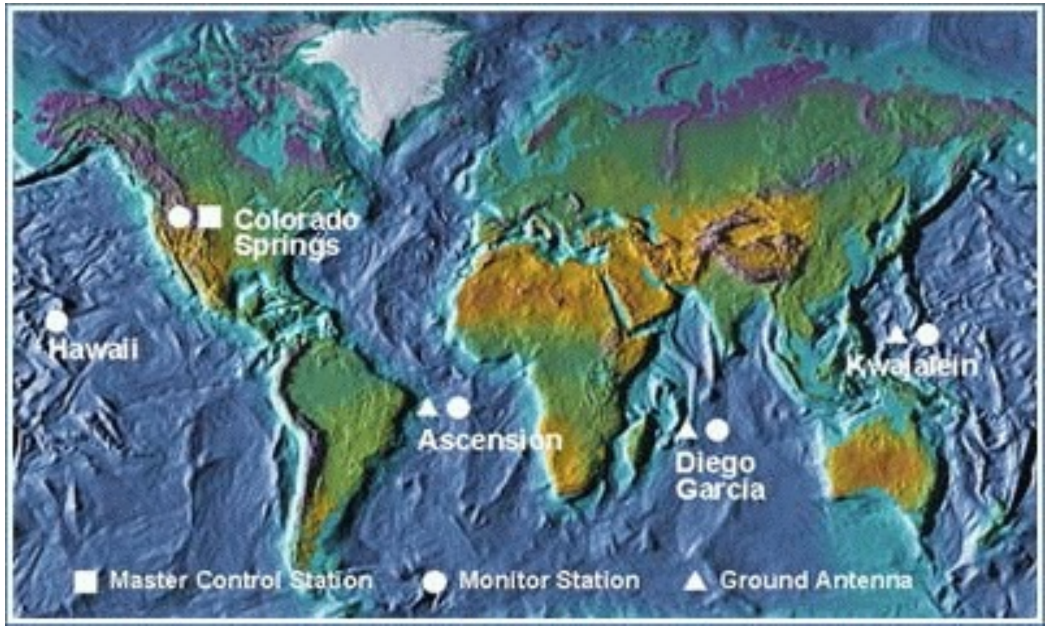
- 1940s: ground-based radio navigation systems (e.g., LORAN)
- 1957: Sputnik
- 1960: US Navy TRANSIT system
  - First satellite based navigation system
  - Based on Doppler effect
- 1973: DOD sponsored NAVSTAR program
- 1978: first satellite launched
- Mid 1990s: 24 satellites (fully operational in 1993)
  - Cost: ~$12B
- 2000: S/A turned off
- 2008-Now: GPS modernization
  - Block IIM and Block III satellites
  - L2C and L5 frequencies
  - M-code

Three Segments of GPS

Space segment:
satellite constellation



Control segment:
monitoring stations and
master control station



User segment:
- SPS: available to all users
- PPS: available to military

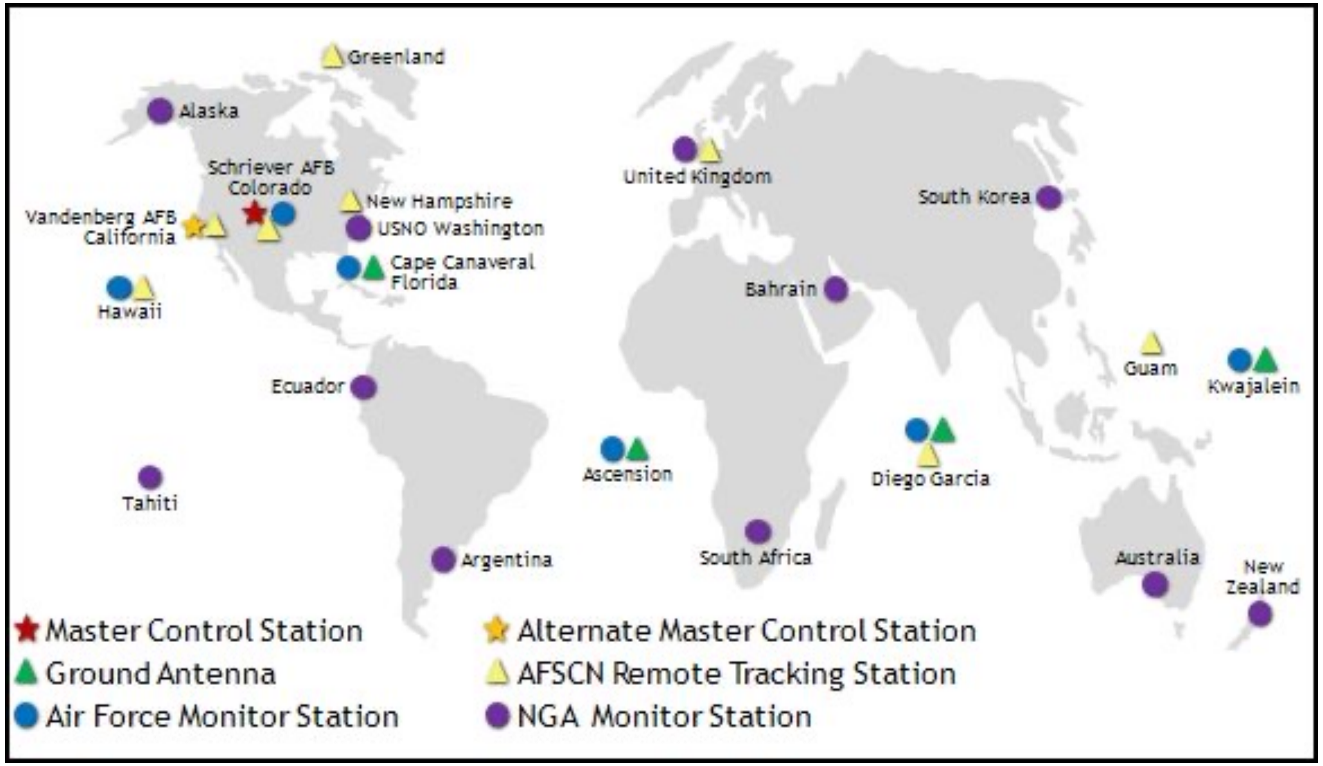
Three Segments of GPS: The Control Segment

Monitored by Department of Defense - All locations perform monitor functions

- » Receive all satellite signals
- » Collect Meteorological data (used for ionospheric modeling)
- » Transmit data to MCS

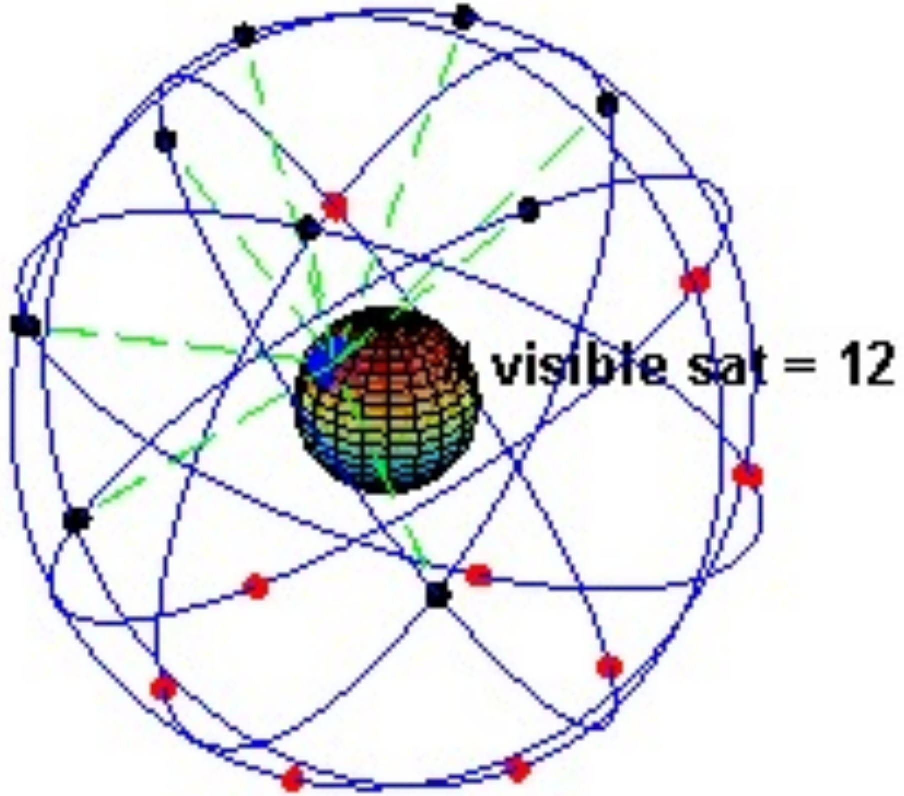
Master Control Station Upload to Satellites

- » Orbital prediction parameters
- » SV Clock corrections
- » Ionospheric models
- » SV commands



Three Segments of GPS: The Space Segment

- 24 + 5 satellites in final constellation
  - » 6 planes rotated 55° with the equator
  - » each plane has 4/5 satellites
  - » Size: 5 meters wide, 900 kg mass
  - » Lifespan: 7.5 years
  - » Medium-earth orbit (MEO)
    - – 20,183 KM, 12,545 miles
    - – approximately 1 revolution
      - ✓ *in 12 hours (i.e., 11 hr 58 min.)*
  - » For accuracy
  - » survivability
  - » coverage



Three Segments of GPS: The Space Segment

LEO:

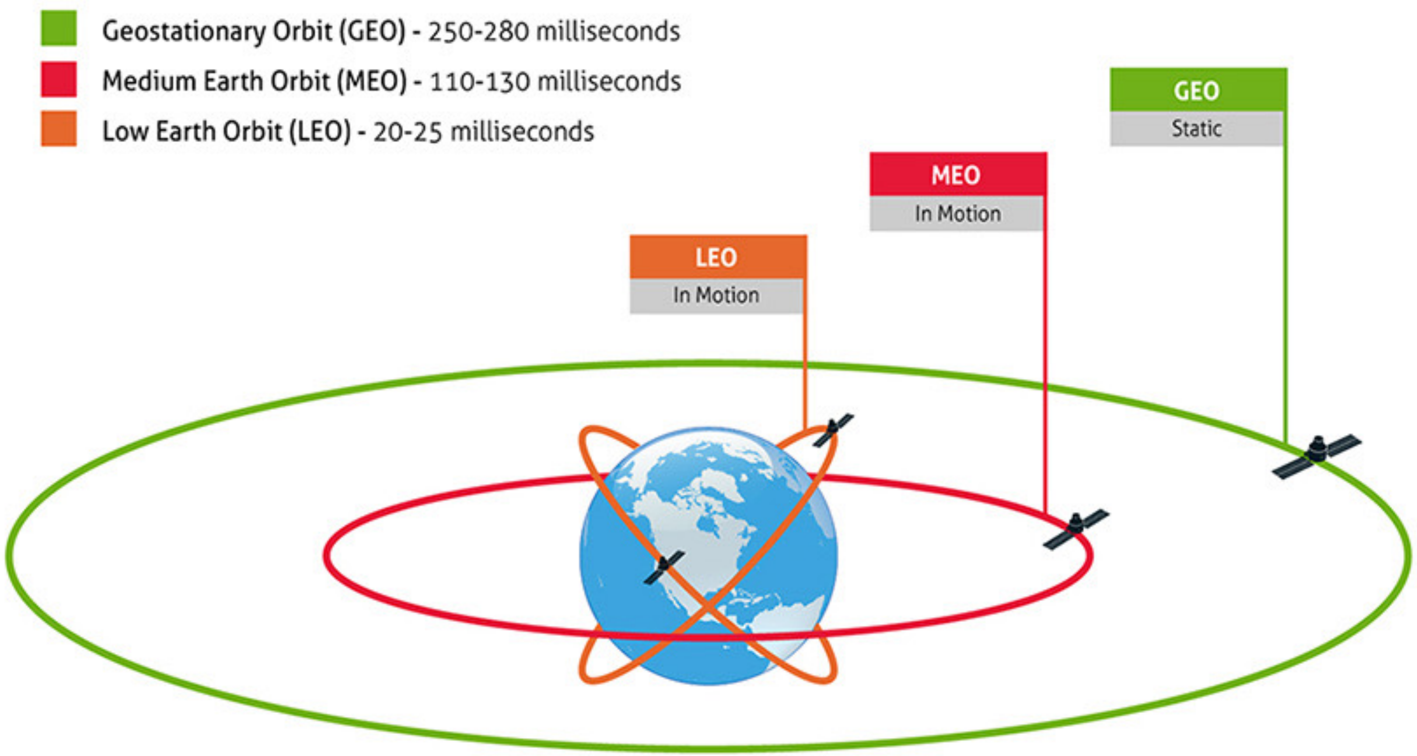
- Less than 2000 km
- Telecommunications
- Earth Imaging Satellites
- International Space Station

MEO:

- 2000 – 35786 km
- GPS (~20,350 km)
- Other positioning constellations

GEO:

- Greater than 35786 km
- Geostationary orbit
  - Rotates at approximately the same angular rate as the earth
- WAAS
- Regional Satellite systems (QZSS)

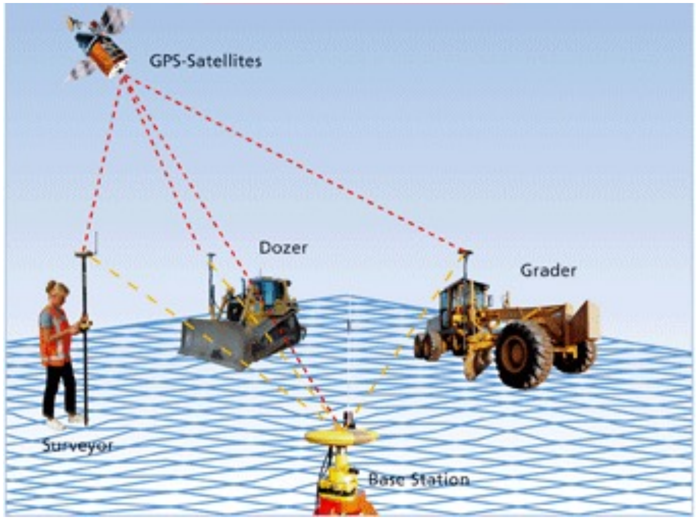


Note: Not drawn to scale

Three Segments of GPS: The User Segment

User Segment:

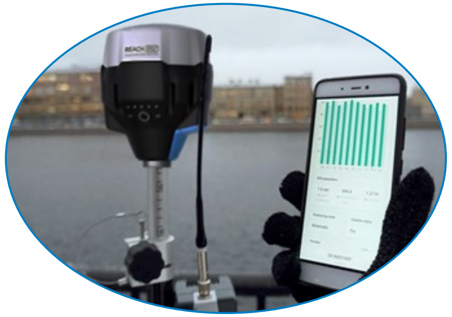
- Surveyor, engineer, GIS, military, public
- Anyone with GPS equipment!
  - » Hardware and Software can be application specific



GPS Receiver “Grades”

- Recreational Grade
  - <$100-$1000 1-10 meters
- Mapping/GIS
  - $1,000-$6,000 submeter - 3 meter
- Survey Grade
  - $10,000 +
  - Multifrequency

    Most likely multi-constellation 0.5 – 2 cm



Caveat: these general “rules of thumb” are changing: equipment is rapidly improving and costs are rapidly decreasing!

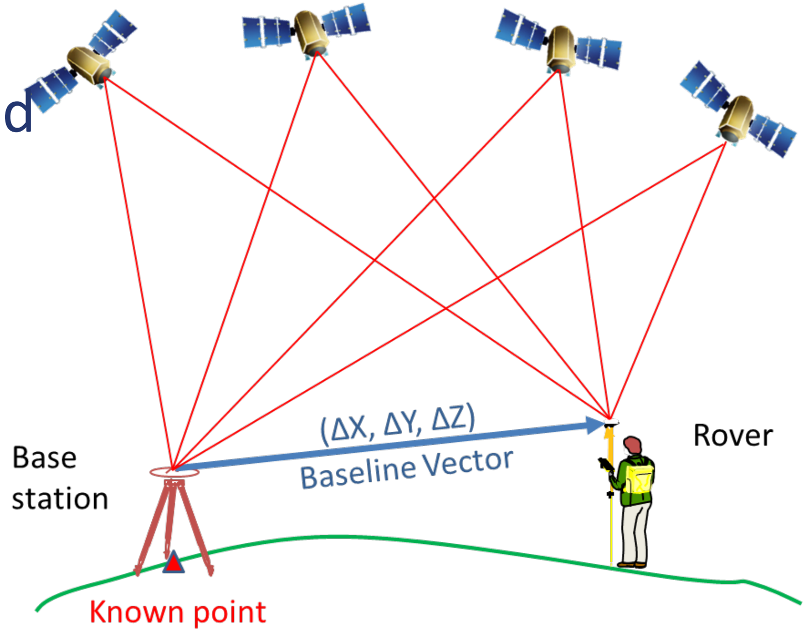
GNSS

Generally, want accuracies of ~0.5 – 4.0 cm, depending on type of survey

  - » Key elements is to use carrier-phase based positioning, rather than relying on code-based pseudoranges

- Use relative positioning - multiple receivers: at least 1 base + 1 rover
  - » Post processed or RTK
  - » Can use dedicated base station(s) that you set up or continuously operating reference stations (CORS)

- Use appropriate care in survey planning, observations and processing
  - » Minimize PDOP
  - » Make sure you are initialized in RTK
  - » Use care in centering
  - » Use appropriate post processing, as applicable
  - » Network adjustment, when applicable



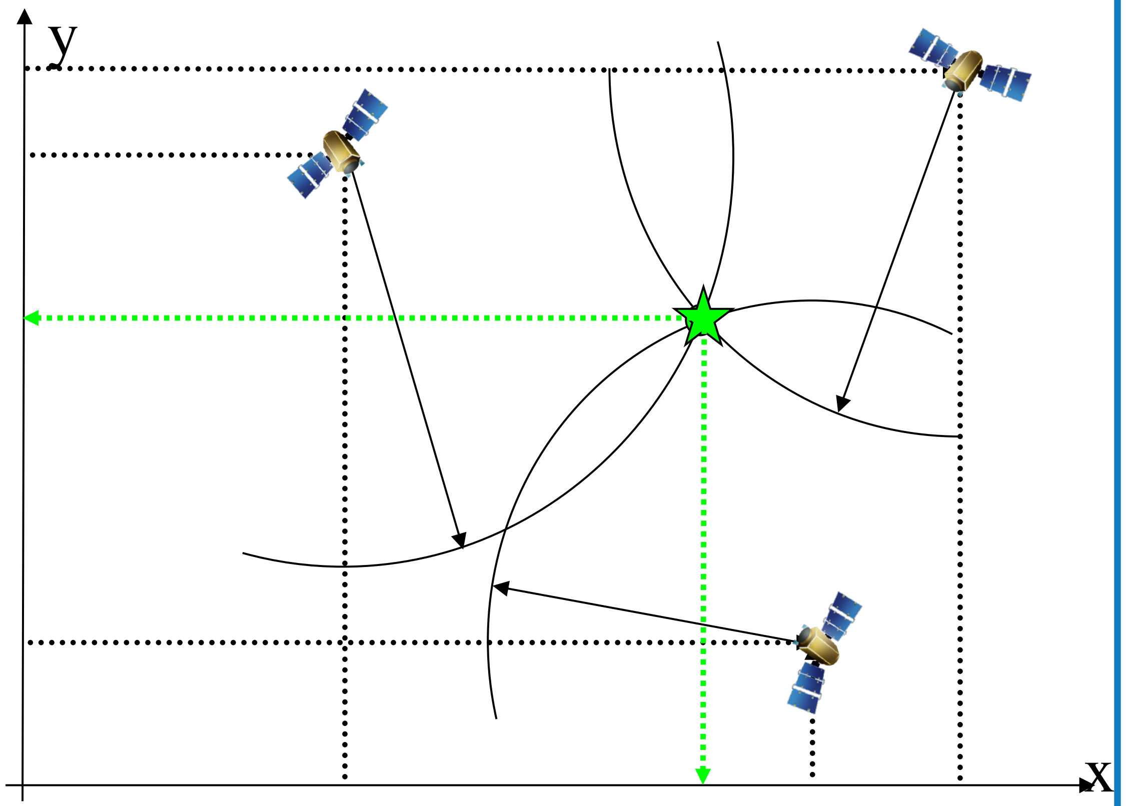
GNSS Positioning

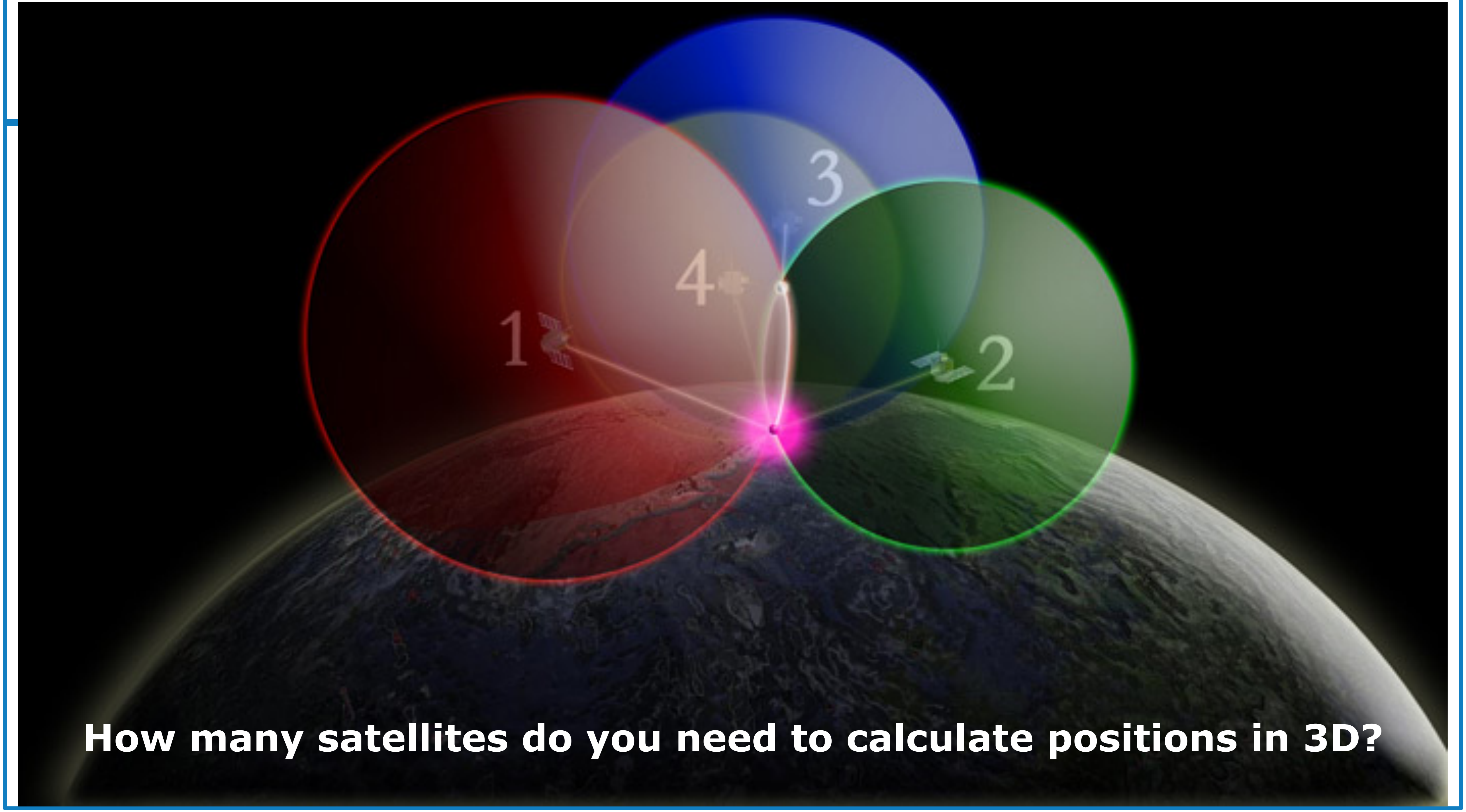
- One of the most common methods for establishing control for your UAS surveys is to use GNSS positioning.
  - » Note, it is also required for direct georeferencing!
- Similar to positioning via trilateration using an EDM
- Receivers passively receive SV signal and measure time for signal to reach it. Then distances from the satellites are computed by:

Distance = Rate x Time

- Rate = speed of light
- Time needs to be very accurate
  - » Satellites use atomic clocks accurate to 1 nanosecond (0.000000001 second)
  - » Receivers use crystal clocks – much more affordable but also have more time drift
  - » An error of 0.01 sec. leads to an error of 2,993 km (1,860 miles)!

GNSS positioning in 2D:
Trilateration of pseudoranges



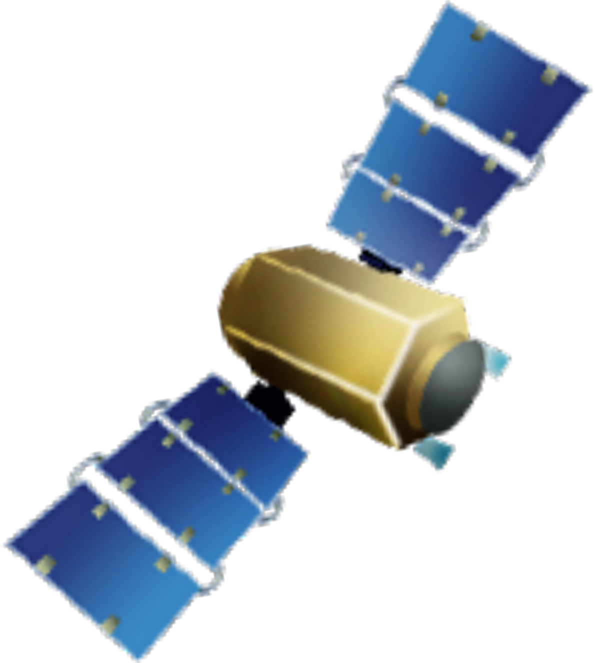


How many satellites do you need to calculate positions in 3D?

GNSS Positioning

How many satellites do you need to see to calculate position in 3D?

- In theory, 3 (if no clock error)
- But, receiver clock error introduces another unknown
  - » Need 4 pseudoranges to solve for:
    - – 3 components of position
    - – Receiver clock error, Ci
- More than 4 provides redundancy (a good thing!)



GNSS Need for Time!

Definition of a second:

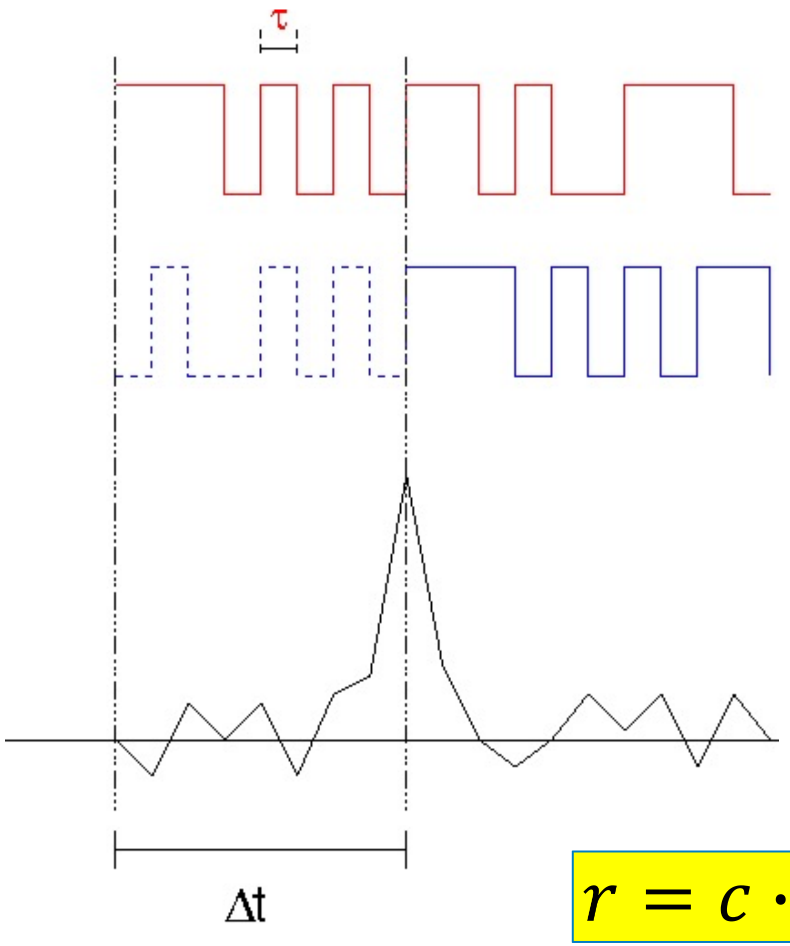
1 second is equal to 9,192,631,770 oscillations of caesium-133!

Need accurate clocks for this process to work!

Necessary to measure travel time…

- » Making sure both receiver and satellite are synchronized
- » Whole system depends on very accurate clocks
- » Satellites have atomic clocks… accurate but expensive
  - – measure time by using the resonance frequency of a particular atom as a metronome
    - ✓ *Rubidium is great; but cesium is even better!*
  - – Each satellite has 4 atomic clocks (Redundancy!)
  - – ~$100k per clock
- » Ground receivers just need consistent clocks
  - – Typically use a quartz clock – measure
  - – Constantly being calibrated by the satellites
  - – **The secret is in the extra satellite measurement that adjusts the receiver clock**

C/A Code Ranging



C/A PRN code from satellite

Code replica generated by receiver

Correlation

$$r = c \cdot \Delta t$$

Δt = travel time
c = speed of light

One of the Clever Ideas of GPS:

- » Use same code at receiver and satellite
- » Synchronize satellites and receivers so they're generating same code at same time
- » Then we look at the incoming code from the satellite and see how long ago our receiver generated the same code

Autocorrelation function:

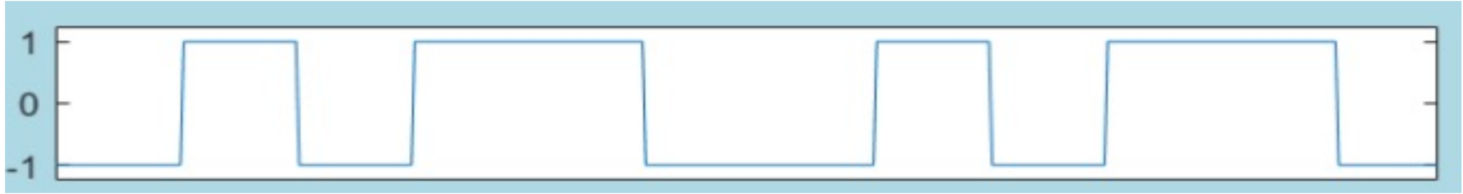
$$R_f(\tau) = \frac{1}{T}\int_0^T f(t)f(t-\tau)d\tau$$

- » This is called a pseudorange (as opposed to a “true” range), because of clock biases and other sources of error

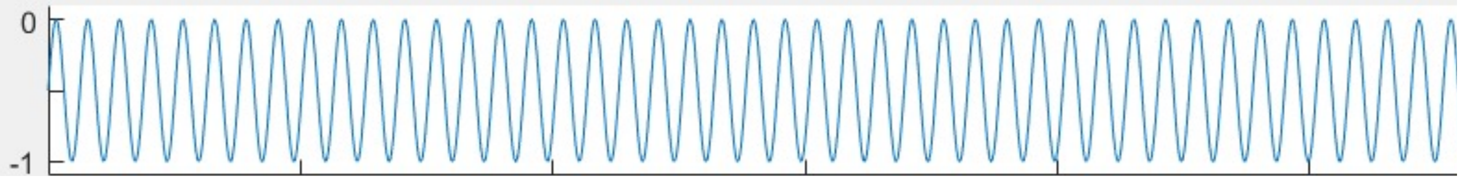
Code vs. Carrier Frequencies

- Again, use basestations (like DGNSS), but use *carrier phase measurements* instead of code measurements

C/A code: 1.023 MHz => 300 m chip-length



L1 carrier wave: 1575.42 MHz => 19 cm wavelength



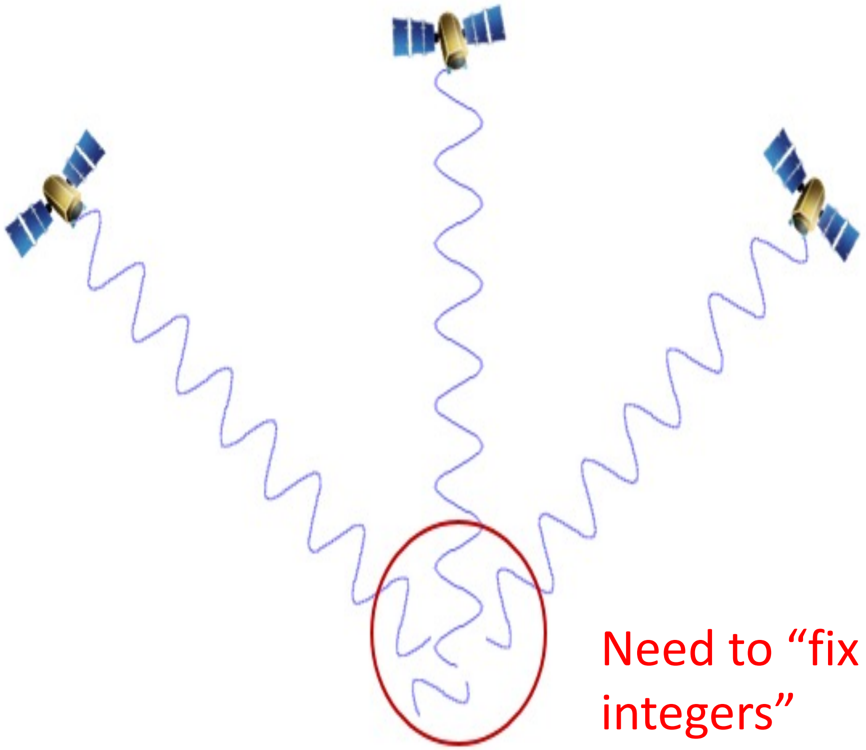
N.T.S.

- Typical accuracy of RTK and PPK: ±1-4 cm
  - » Difference is that PPK is *post-processed* and can be a bit more accurate

GNSS Positioning: Integer Ambiguity

Ranging based on carrier phase measurements

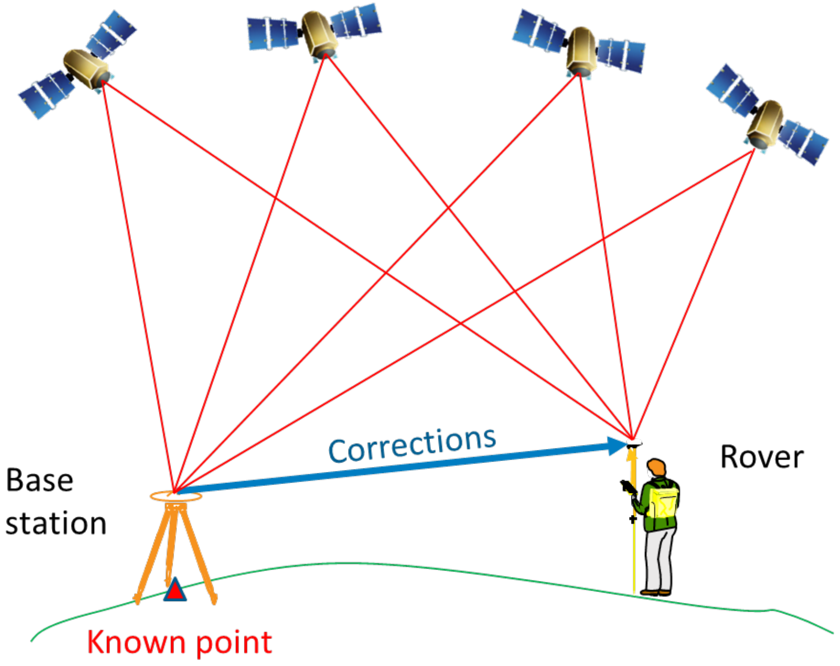
- Why?
  - » Wavelength of the carriers is much smaller than that of the code modulations, thereby enabling more accurate positioning
- But…problem
  - » “Integer ambiguity”: the full number of wavelengths of the carrier between the satellite and the receiver is initially unknown



Need to “fix integers”

Differential GNSS (DGNSS)

- Exploiting spatial and temporal correlation between errors at a reference receiver (base station) and nearby rover
- Enables systematic errors common to both the base and rover to be reduced
- Types of errors that can be reduced
  - » Ionospheric delays
  - » Tropospheric delays
  - » Ephemeris errors
  - » Clock errors
- Error sources that cannot be corrected
  - » Multipath
- Typical accuracy of C/A-code DGNSS: ±0.5 – 2 m



GPS Code and Phase Ranging Equation

GPS Receivers Collect 2 “observables”

- Pseudorange determined from the codes
- Carrier Phase measurements determined from the carrier wave

Pseudorange equation:

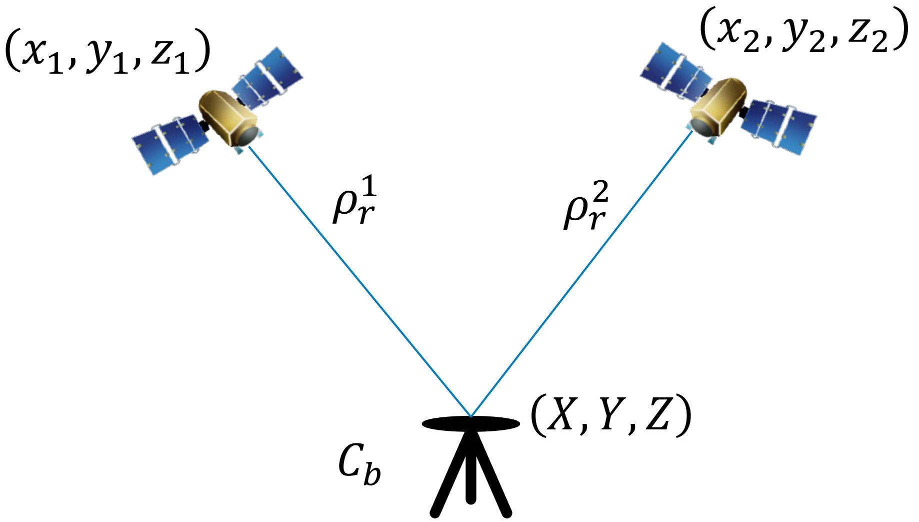
$$Z_\rho^i = \rho_r^i = \sqrt{(x_i - X)^2 + (y_i - Y)^2 + (z_i - Z)^2} + C_b$$

ρ_r^i = psdueorange to i^{th} satellite

C_b = receiver clock bias

(X, Y, Z) = East, North, Up components of user′s antenna location

(x_i, y_i, z_i) = known satellite coordinates

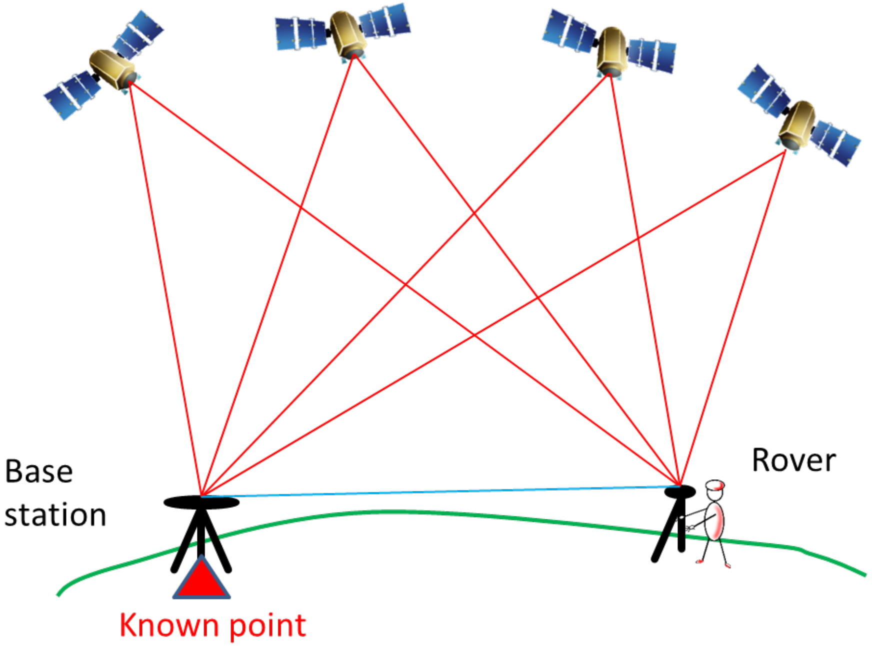


Note…
We need a minimum of 4 satellites to solve for our antenna location even though I am only showing 2 in this figure.

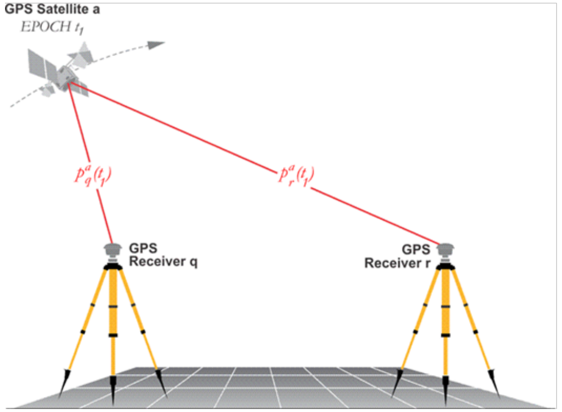
Point Positioning vs Relative Positioning

Relative Positioning:

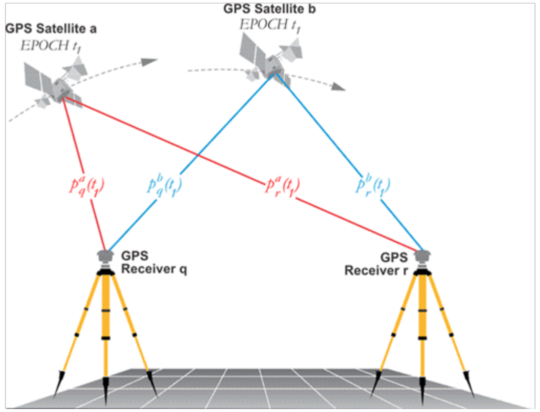
- Requires a minimum of two receivers
- For code-only receivers, this method is also commonly known as differential GPS (*DGPS*)
- Results from two receivers are differenced to cancel out common errors. A "baseline" between stations is thereby formed by this differencing
- Multiple methods available
  - (i.e. single, double, or triple differencing)



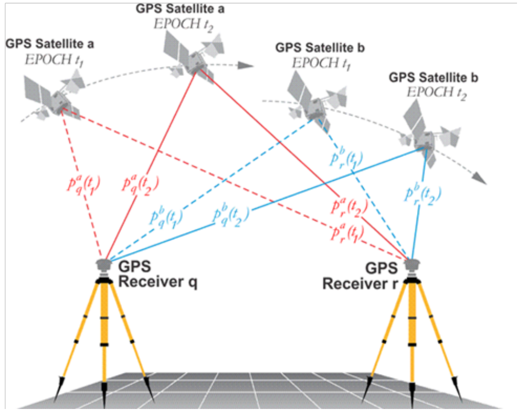
Single Differencing



Double Differencing



Triple Differencing



GPS Carrier Ranging Equation

Carrier Phase Observation:

$$\Phi = \rho + d_\rho + c(dt - dT) + \lambda N - d_{ion} + d_{trop} + \varepsilon_{m\Phi} + \varepsilon_\Phi$$

where :

Φ = carrier phase measurement

ρ = true range

d_ρ = satellite orbital error

c = speed of light

dt = satellite clock bias from GPS time

dT = receiver clock bias from GPS time

λ = carrier wavelength

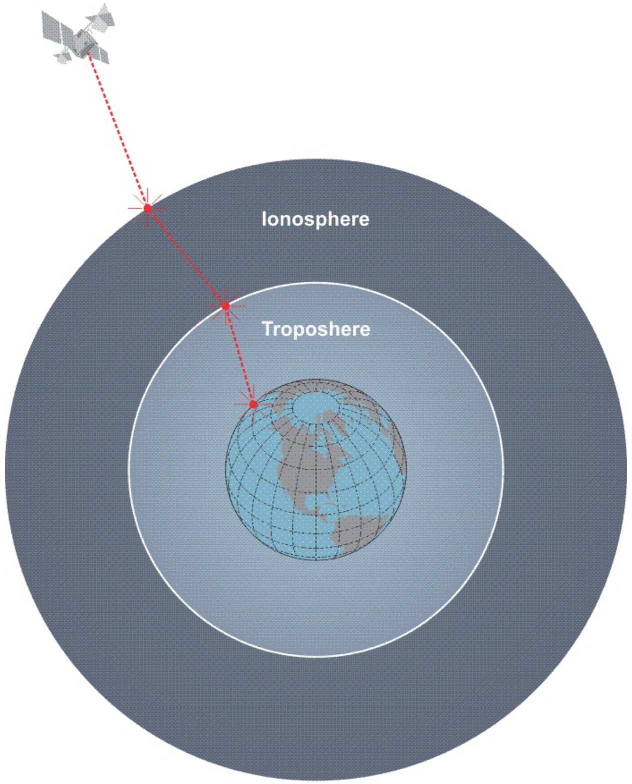
N = integer ambiguity (in cycles)

d_{ion} = ionospheric delay

d_{trop} = tropospheric delay

$\varepsilon_{m\Phi}$ = multipath

ε_Φ = receiver noise



Relative positioning: cancelling out satellite and receiver cock errors

- Assume we’re using pseudoranges (for now):
- The way relative positioning works is different than the way DGPS works
  - » Instead of transmitting errors, the reference receiver transmits its observations to the rover
    - – (pseudo-ranges and/or carrier phase measurements)
  - » The rover receiver can form linear combinations of the pseudo-range equations or the carrier phase equations to get some of the errors to cancel out

$$\mathrm{r} = \rho + \Delta \mathrm{c}^{\mathrm{s}} + \Delta \mathrm{c}^{\mathrm{r}} + \Delta \mathrm{r}$$

$$\rho = [(\Delta X)^2+(\Delta Y)^2+(\Delta Z)^2]^{1/2}$$

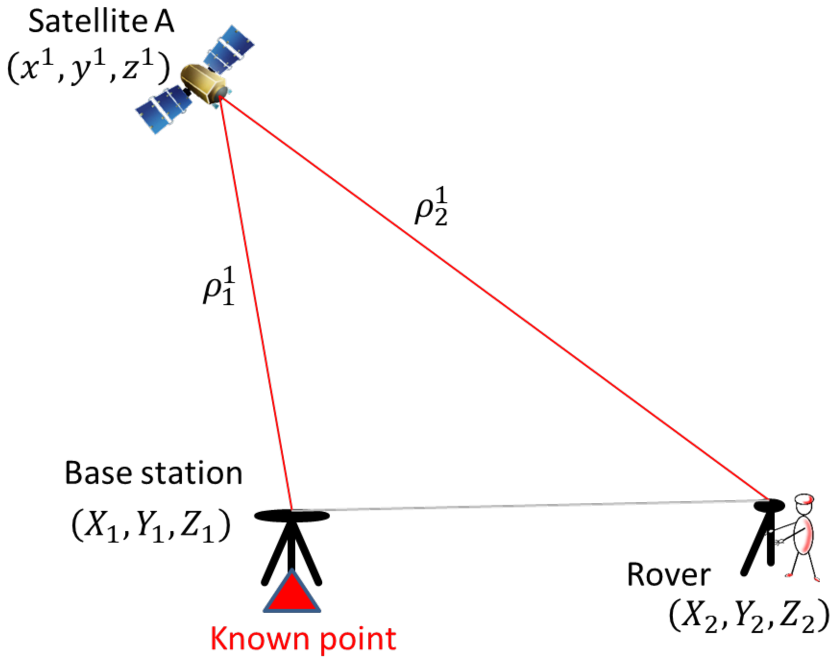
Satellite clock error

Receiver clock error

Catch-all error term that accounts for other error sources

Add subscripts and superscripts to denote i^{th} receiver and j^{th} satellite:

$$\mathrm{r}_{\mathrm{i}}^{\mathrm{j}} = \rho_i^j + \Delta \mathrm{c}^{\mathrm{s}_{\mathrm{j}}} + \Delta \mathrm{c}^{\mathrm{r}_{\mathrm{i}}} + \Delta \mathrm{r}_{\mathrm{i}}^{\mathrm{j}}$$



Relative positioning: example of cancelling out satellite and receiver cock errors

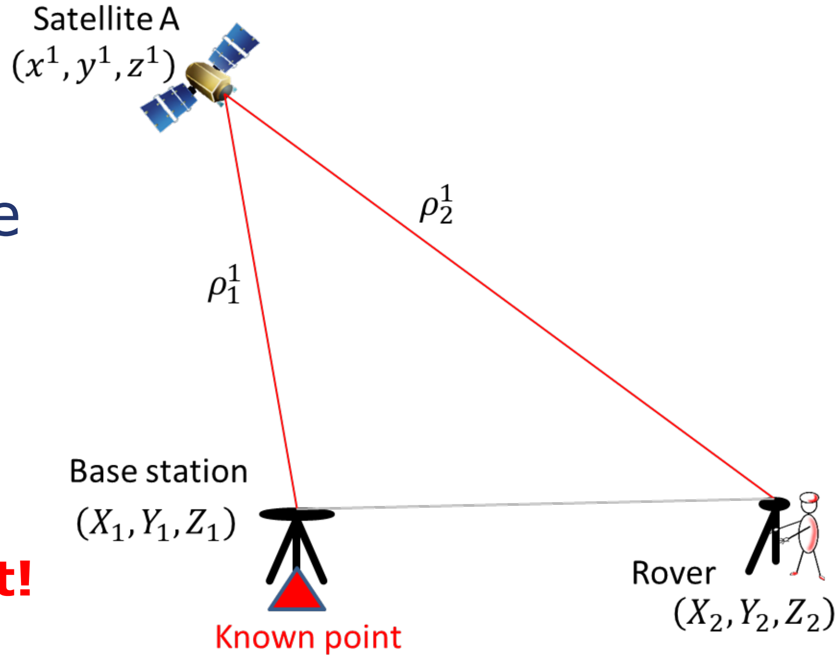
The two pseudoranges between receiver 1 and satellite 1 and receiver 2 and satellite 1 are:

$$r_1^1 = \rho_1^1 + \Delta c^{s_1} + \Delta c^{r_1} + \Delta r_1^1$$

$$r_2^1 = \rho_2^1 + \Delta c^{s_1} + \Delta c^{r_2} + \Delta r_2^1$$

Subtracting the 1st equation from the 2nd gives the single difference

$$R_{12}^1 = r_2^1 - r_1^1 = \rho_2^1 - \rho_1^1 + \Delta c^{r_2} - \Delta c^{r_1} + \Delta r_2^1 - \Delta r_1^1$$



Important: the satellite clock error term has cancelled out!

Relative positioning: example of cancelling out satellite and receiver cock errors

- Next, if we consider the same two receivers (1 and 2) tracking a second satellite, we can write a second single-difference equation:

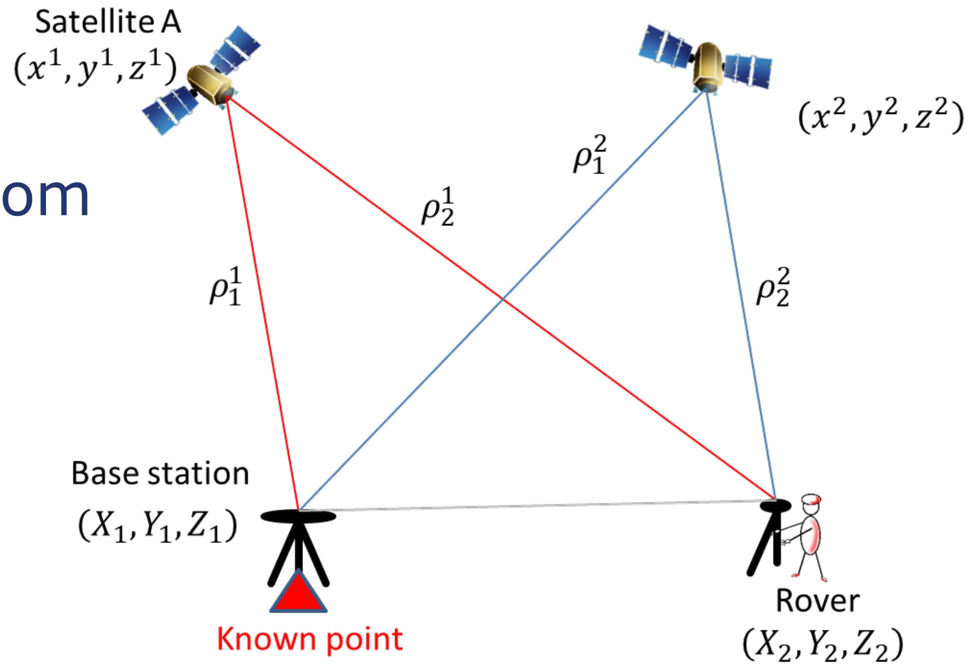
$$R^{1}_{12} = r^{1}_{2} - r^{1}_{1} = \rho^{1}_{2} - \rho^{1}_{1} + \Delta c^{r_2} - \Delta c^{r_1} + \Delta r^{1}_{2} - \Delta r^{1}_{1}$$

$$R^{2}_{12} = r^{2}_{2} - r^{2}_{1} = \rho^{2}_{2} - \rho^{2}_{1} + \Delta c^{r_2} - \Delta c^{r_1} + \Delta r^{2}_{2} - \Delta r^{2}_{1}$$

- Subtracting one single difference equation from the other gives the double difference:

$$R' = R^{2}_{12} - R^{1}_{12} = \rho' + \Delta r'$$

Important: the receiver clock error terms have cancelled out!



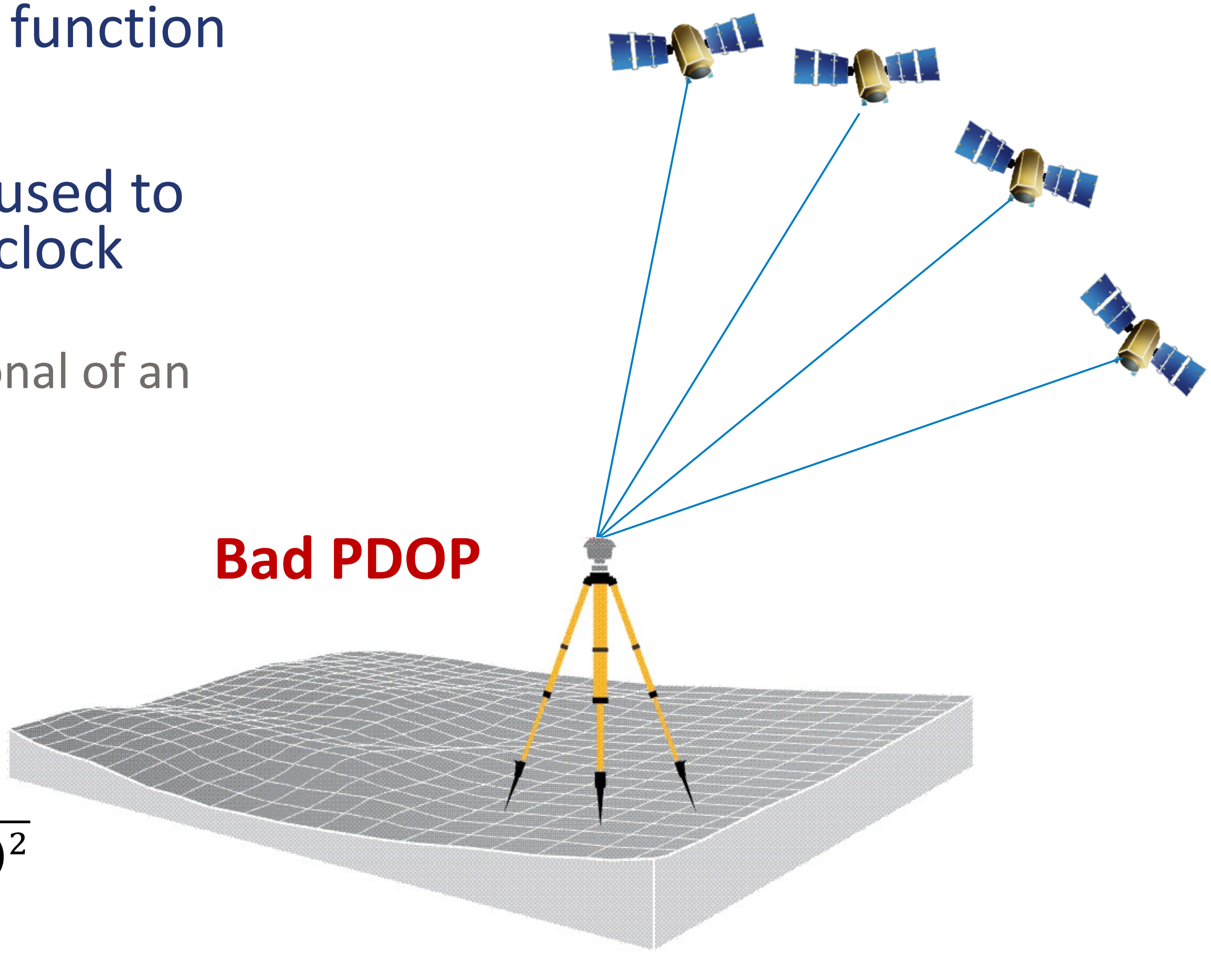
GNSS Positioning Techniques

GNSS type	How it works	"Typical" achievable accuracy
Code-based pseudoranging	Only need one, inexpensive receiver; ranges based on PRN codes	±10 m (±30 ft)
Code-based DGNSS	Errors (differential corrections) are transmitted from the reference station to the rover; these are corrections to C/A code pseudoranges	±0.5-3 m (±2-10 ft)
RTK	Similar to DGPS, except using carrier phase measurements and dual (or multi)-frequency receivers.	Achievable accuracy: ±1-4 cm (±0.03-0.1 ft)
PPK	Same as RTK, but post processed	Generally same level as RTK, but can be a bit better, due to post processing (e.g., forward-backward processing and use of precise ephemerides)
PPP	Requires only a single receiver; need precise orbits + code & carrier phase obs, multi-freq, multi-constellation receiver + PPP processing algorithm	"Near survey grade"

Dilution of Precision (DOP)

- An Indicator of GNSS position quality as a function of satellite geometry
- Product from a least squares adjustment used to determine the receivers coordinates and clock offset
    - They are just a combination of the cross diagonal of an unscaled error-covariance matrix

$$\begin{bmatrix} (\text{east DOP})^2 & & \text{Cross terms} & \\ & (\text{north DOP})^2 & & \\ & & (\text{vertical DOP})^2 & \\ \text{Cross terms} & & & (\text{time DOP})^2 \end{bmatrix}$$



$$PDOP = \sqrt{(East_{DOP})^2 + (North_{DOP})^2 + (Vertical_{DOP})^2}$$

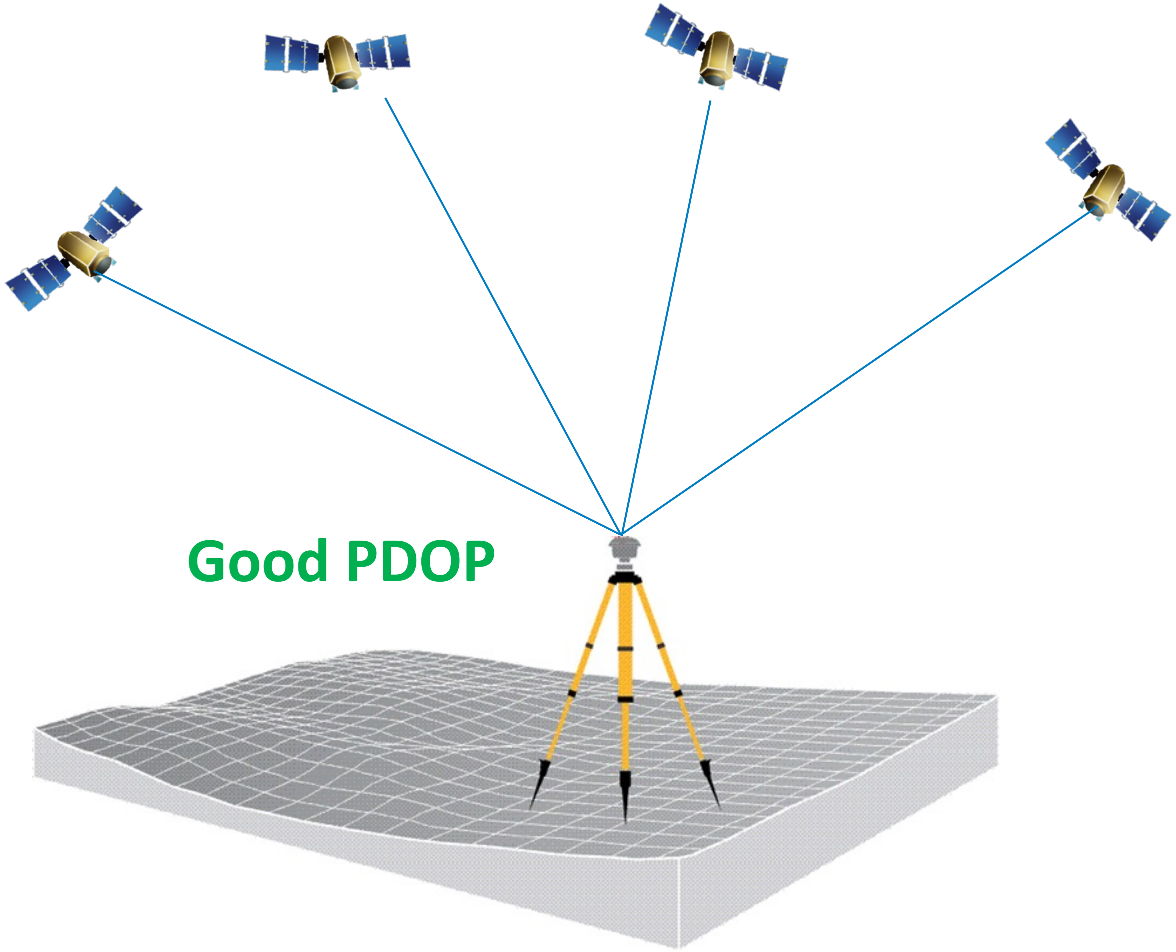
$$HDOP = \sqrt{(East_{DOP})^2 + (North_{DOP})^2}$$

$$VDOP = \sqrt{(Vertical_{DOP})^2}$$

Positional Dilution of Precision

- (PDOP)

PDOP Value	Interpretation
1	Ideal
1-2	Excellent
2-5	Good
5-6	Fair
> 6	Suspect



UERE

- Estimating your ranging uncertainties

User Equivalent Range Error

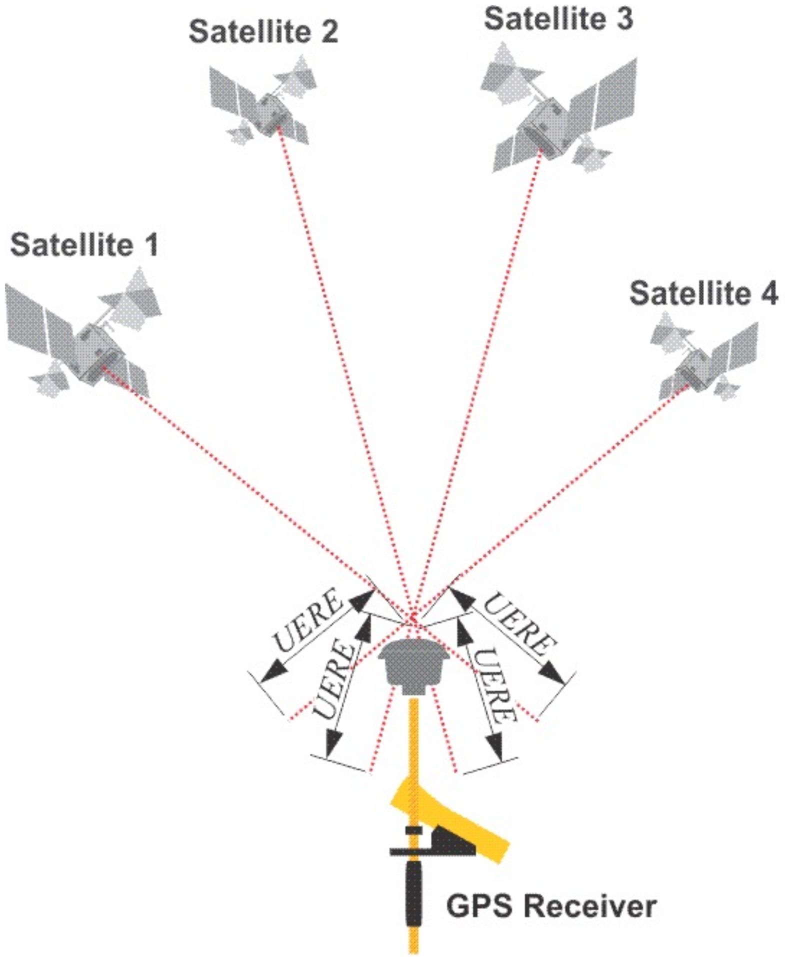
$$\sigma_{CR} = \text{DOP factor} \times \text{UERE}$$

Code ranging uncertainty

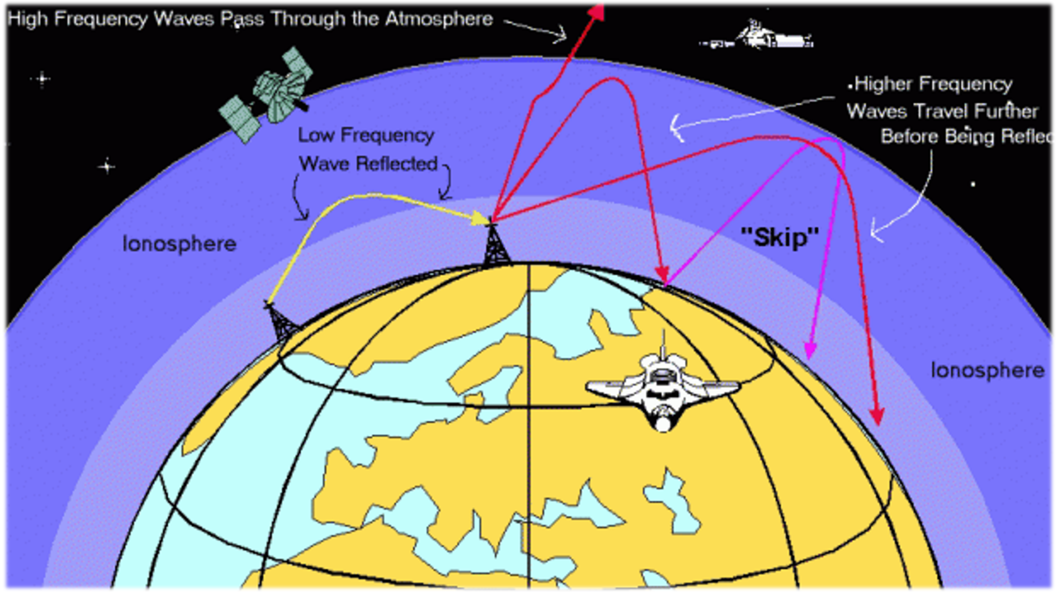
Just talked about these

User Equivalent Range Error

$$\text{UERE} = \pm\sqrt{\sigma_{orbit}^2 + \sigma_{iono}^2 + \sigma_{tropo}^2 + \sigma_{receiver}^2 + \sigma_{other}^2}$$



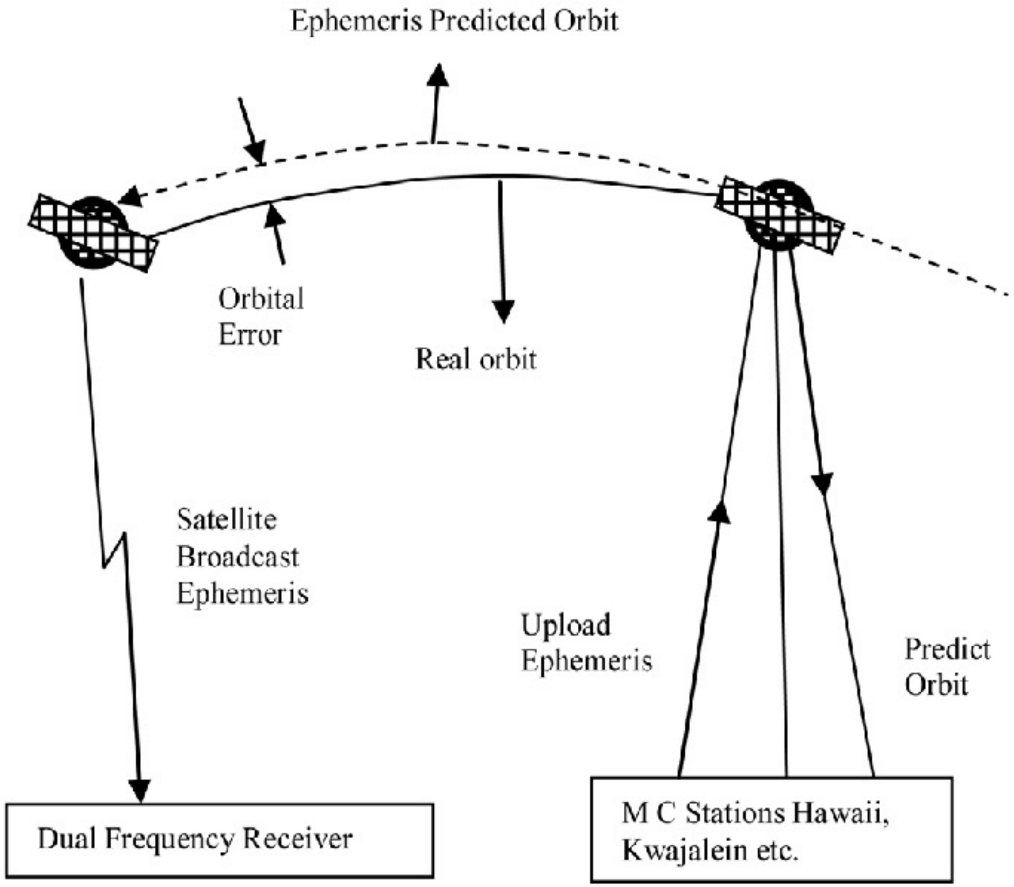
Error Sources



Error Source	“Typical” magnitudes (m)	How to reduce
Satellite clock	±1.5-2.0	Differential GPS
Orbits	±2.5	Precise ephemeris
Ionosphere	±5-7	Dual frequency (L1/L2)
Troposphere	±0.2-0.5	Model
Receiver noise	±0.6	Survey-grade receivers
Multipath	Difficult to estimate	Choke-ring antennas

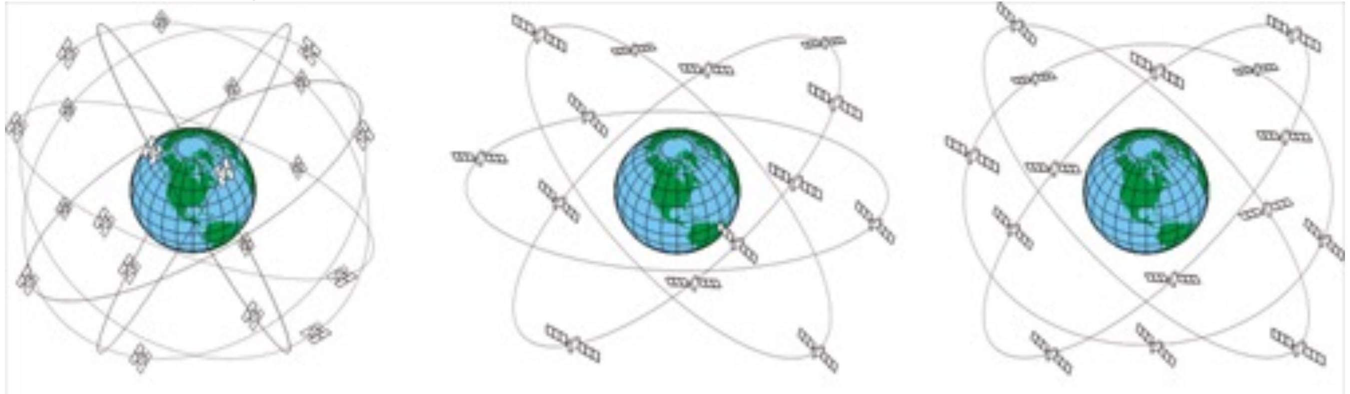
Satellite Ephemerides

Type	Orbit Error	Availability
Broadcast	~100 cm	Real-Time
Ultra-Rapid (Predicted Half)	~5 cm	Real-Time
Ultra-Rapid (Observed Half)	~3 cm	3-9 Hours
Rapid	~2.5 cm	17-41 Hours
Precise (Final)	~2.5 cm	12-18 Days



Next Generation: GNSS

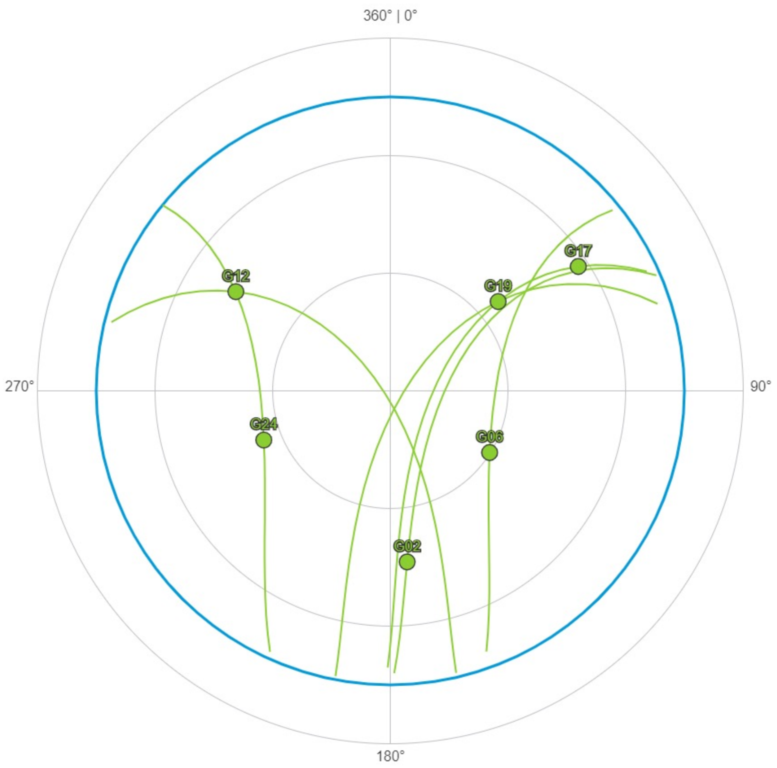
- Next generation = GNSS
  - » Other countries are developing systems similar to GPS
    - – GLONASS- Russian system (now has full constellation)
    - – GALILEO – European system (Now has full contellation)
    - – BEIDOU – Chinese system



GPS	**Galileo**	**GLONASS**
- 6 Orbital planes	- 3 Orbital planes	- 3 Orbital planes
- 24 Satellites + Spare	- 27 Satellites + 3 Spares	- 21 Satellites + 3 Spares
- 55^{0} Inclination Angle	- 56^{0} Inclination Angle	- 64.8^{0} Inclination Angle
- Altitude 20,200km	- Altitude 23,616km	- Altitude 19,100km

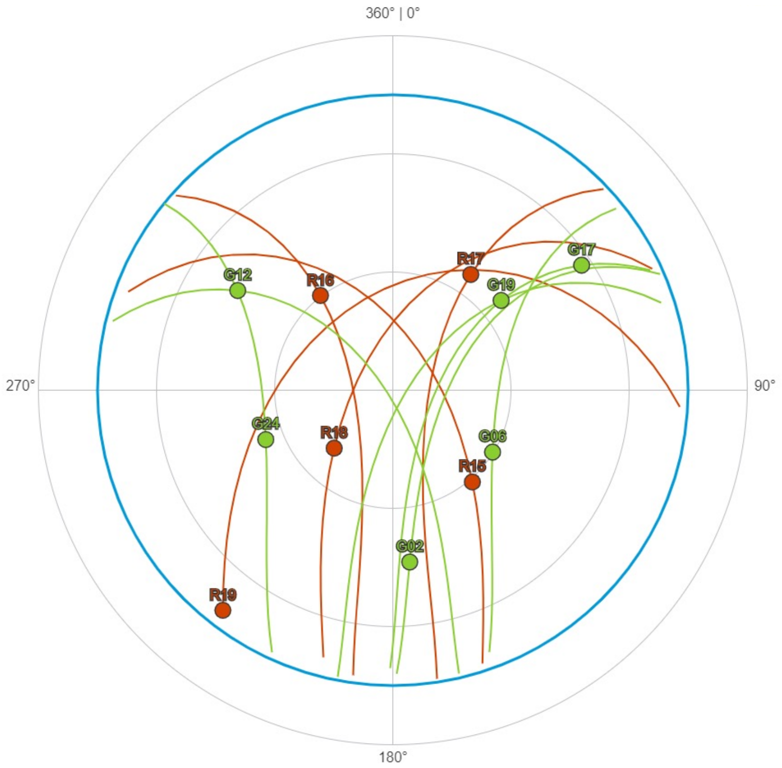
How can we improve our precision?

Estimated DOP's using
Trimble GNSS Planning Tool



@ 13:50 PDT

Constellations:	GPS Only
# Satellites:	6
PDOP:	4.41



@13:50 PDT

Constellations:	GPS + GLONASS
# Satellites:	6 + 5 = **11**
PDOP:	2.11

On-Ramp to Innovation
every day counts

Topic 9:

Kinematic Positioning and Navigation

Introducing Fundamental Concepts of Kinematic Positioning for sUAS

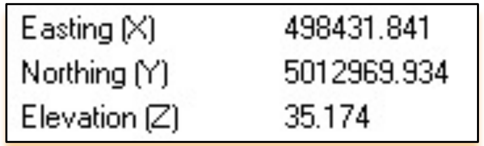
Overview of Content:

- → *Direct Georeferencing*
- → Inertial Navigation
- → *GNSS-INS Integration*

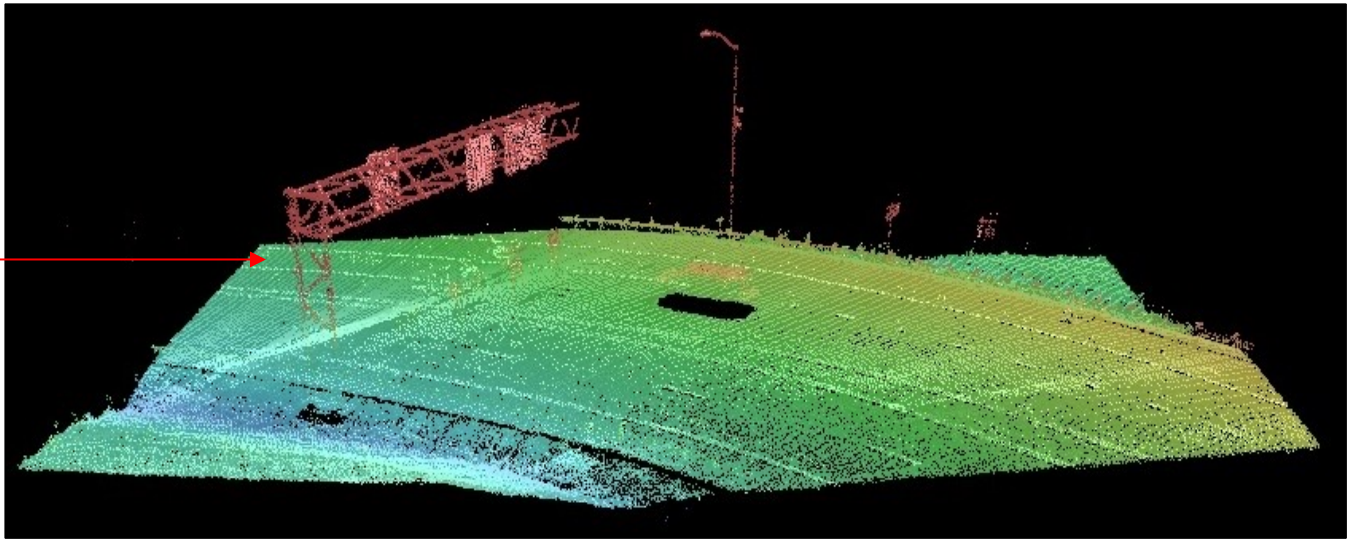
U.S. Department of Transportation
Federal Highway Administration

Point Clouds

UTM (NAD83(2011)), Zone 10N, NAD83(2011) ellipsoid heights (meters)



Easting (X)	498431.841
Northing (Y)	5012969.934
Elevation (Z)	35.174



- This is a point cloud
  - » Maybe created from UAS imagery with SfM/MVS software, maybe from UAS lidar. It doesn’t matter (for this discussion), so don’t worry about it!

Question: How did the coordinates get in there?

A. Magic

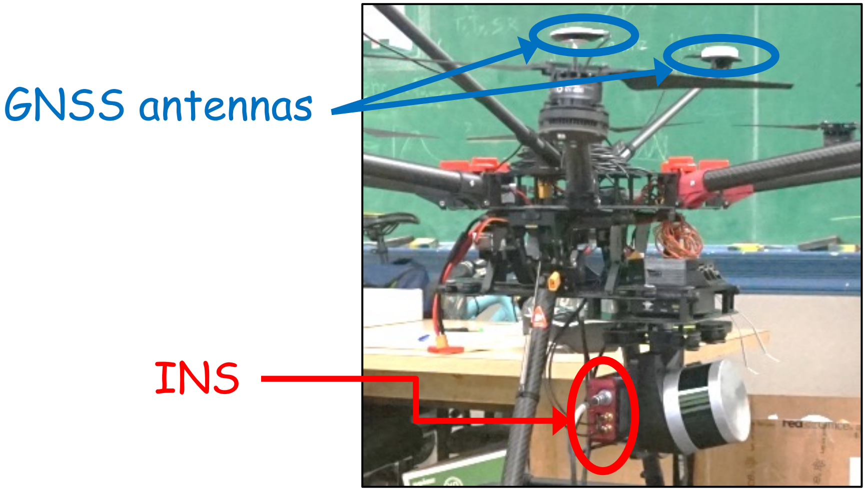
B. Ground control

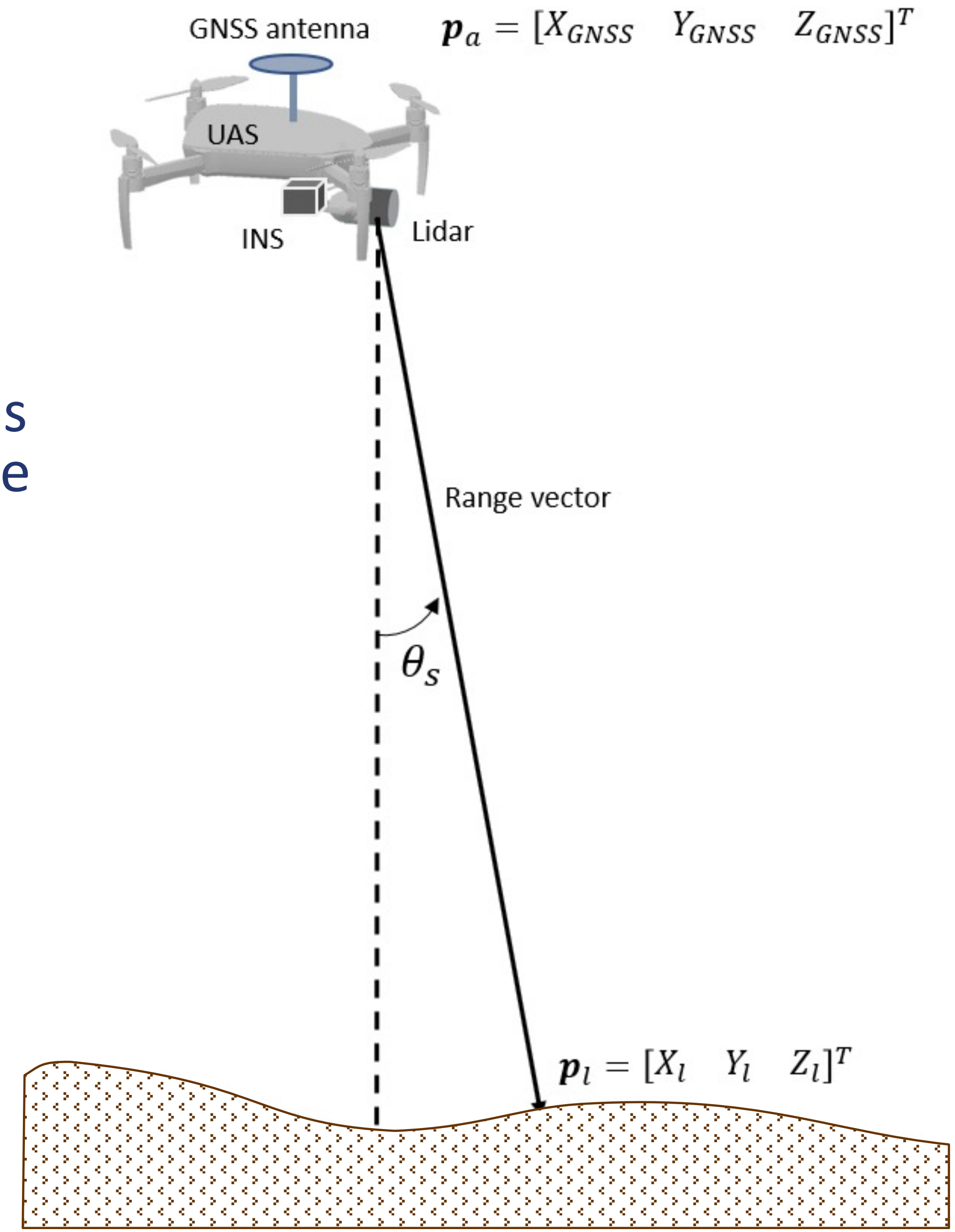
C. Direct georeferencing

D. What’s direct georeferencing?

Direct Georeferencing

- Direct determination of the position and orientation of aircraft/sensor
- Enables direct calculation of spatial coordinates output data products (point clouds, orthoimage
- Minimizes need for ground control
- Primary enabling technology: **GNSS-aided INS**





Direct Georeferencing

- Why is Direct Georeferencing important for UAS?
- Establishing GCPs is time consuming, expensive, and, at times, even dangerous or impossible
- Gives you position and orientation
- Necessary for lidar
  - » Scanning lidar doesn't have redundant (overlapping) geometry, like in stereo photogrammetry
    - – Each point needs to be georeferenced individually

GNSS Aided Inertial Navigation System

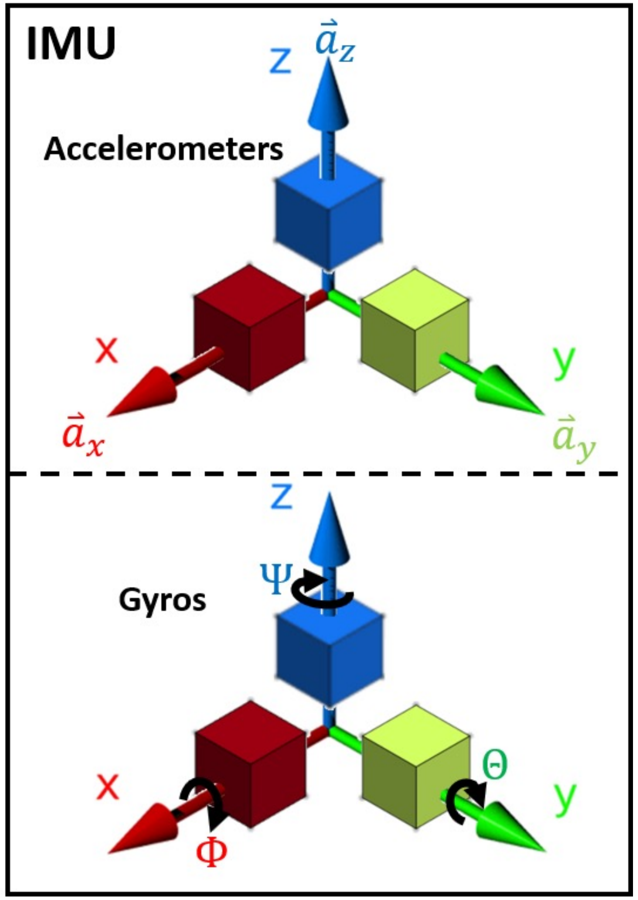
Why Combine GNSS and Inertial Navigation Systems (INS)?

- Complimentary technologies

- GNSS + INS:
  - High data rates
  - High accuracy (short term and long term)
  - Get position *and* orientation
  - Can overcome GNSS signal loss/cycle slips, due to obstructions

	Advantages	Disadvantages
INS	• High measurement rates (e.g., 200 Hz) • Short-term errors are small • Provides position and orientation	• Errors grow with time • INS in standalone mode is performing dead reckoning, so it's subject to drift
GNSS	• Errors do not accumulate with time	• Relatively low measurement rates (e.g., 1 Hz) • Signals can be blocked in urban canyons, forested areas, etc.

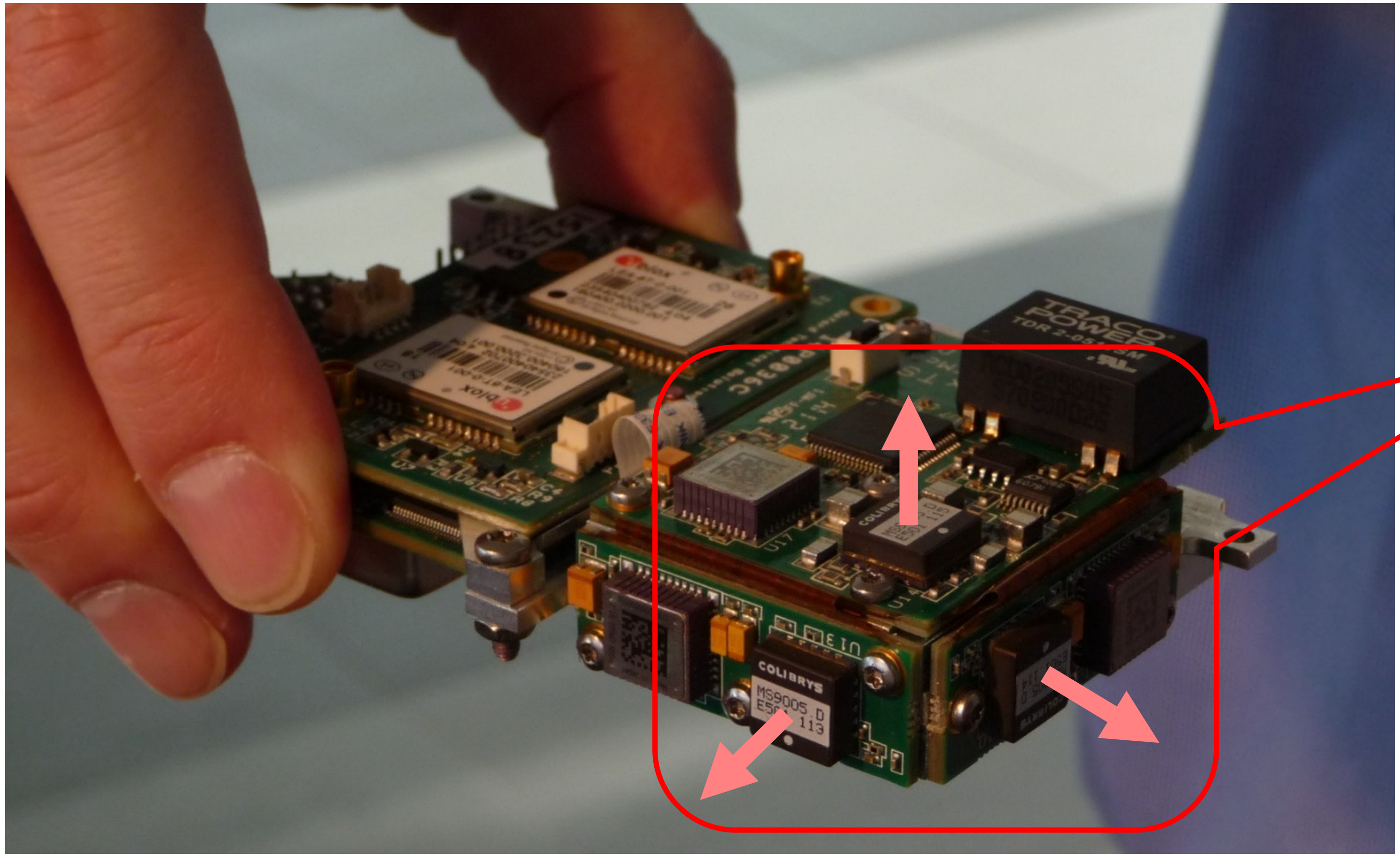
Inertial Navigation System (INS)



- Quick terminology note: what’s the difference between an INS and an IMU (inertial measurement unit)?
  - » A: an INS is a complete system (S = system); an IMU is the primary component of an INS
- IMU
  - » Orthogonal triads of gyros and accelerometers
    - – Sense 3-axis acceleration (well, really specific force) and 3-axis angular rate
    - – Try to fix mount to sensor (camera or lidar)
    - – Measure at high rates (e.g., 200 Hz)

Inertial Sensor Assembly

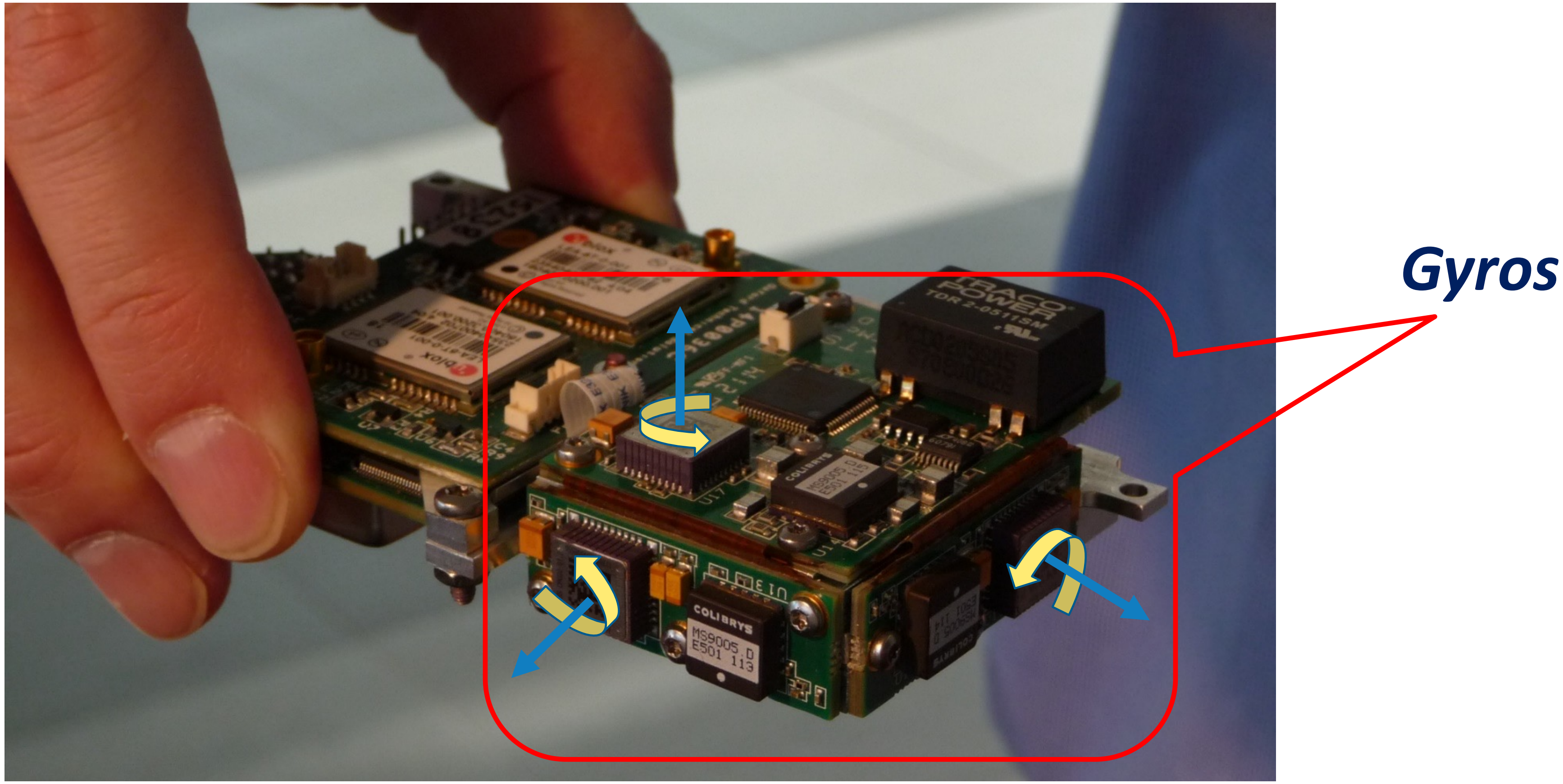
Example of an inertial sensor assembly



Accelerometers

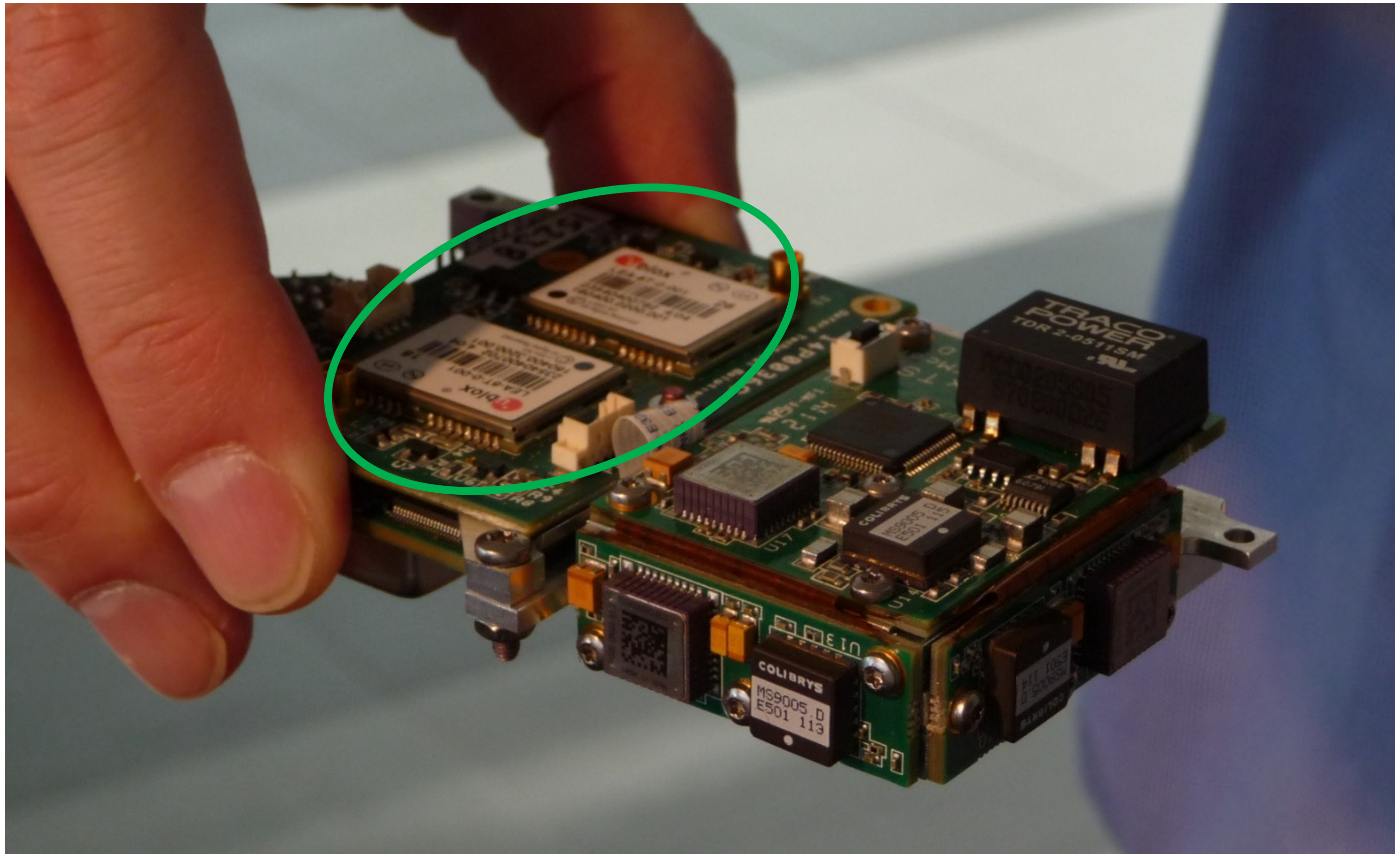
Inertial Sensor Assembly

Example of an inertial sensor assembly



Inertial Sensor Assembly

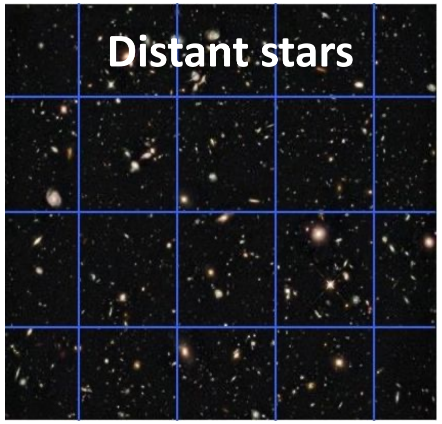
Example of an inertial sensor assembly



GNSS Receivers

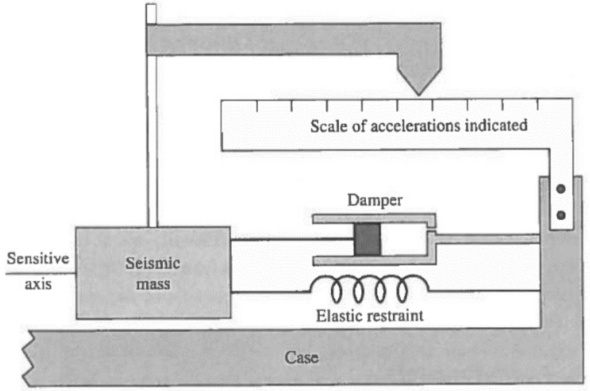
Inertial Sensors (Gyros & Accelerometers)

Make measurements with respect to inertial space



Simple Mechanical Gyro

Simple Mechanical Accelerometer



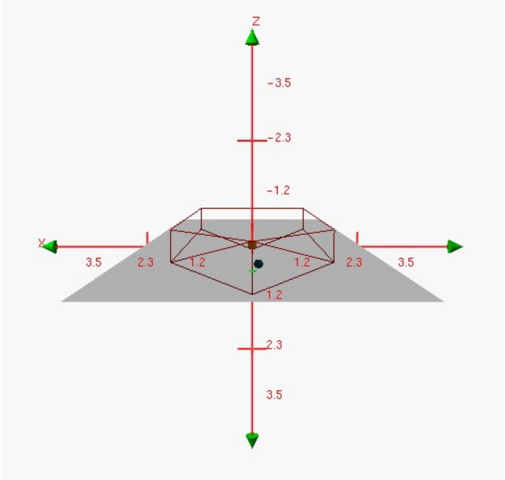
Units

- Gyros (rate gyros)
  - deg/s
  - rad/s
- Accelerometers
  - m/s^2 (the SI unit for acceleration)
  - g (1 g = 9.81 m/s^2; Earth's gravity at sea level)
  - μg (10^{-6} g)

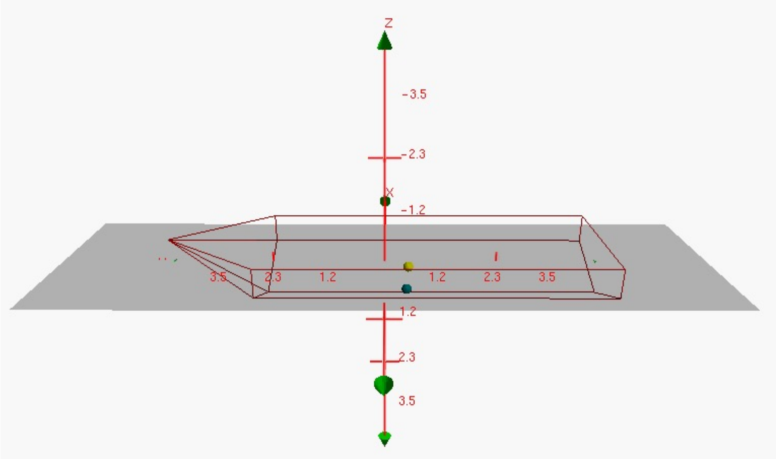
Image credits: Feynman; http://en.wikipedia.org/wiki/Euler_angles#mediaviewer/File:Gyroscope_operation.gif

Body Orientation “Attitude”

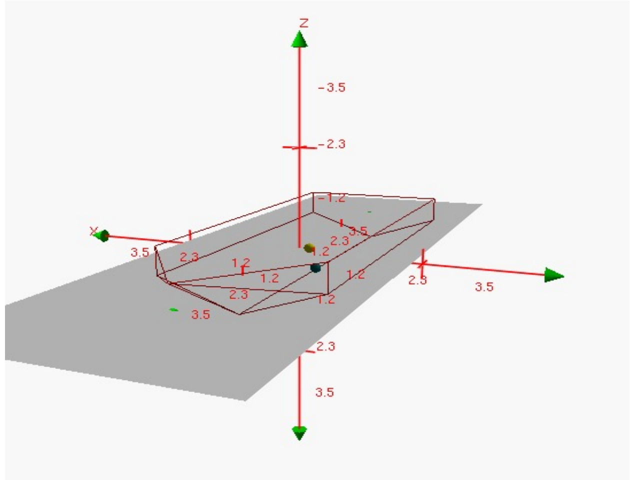
VEHICLE DYNAMICS



PITCH

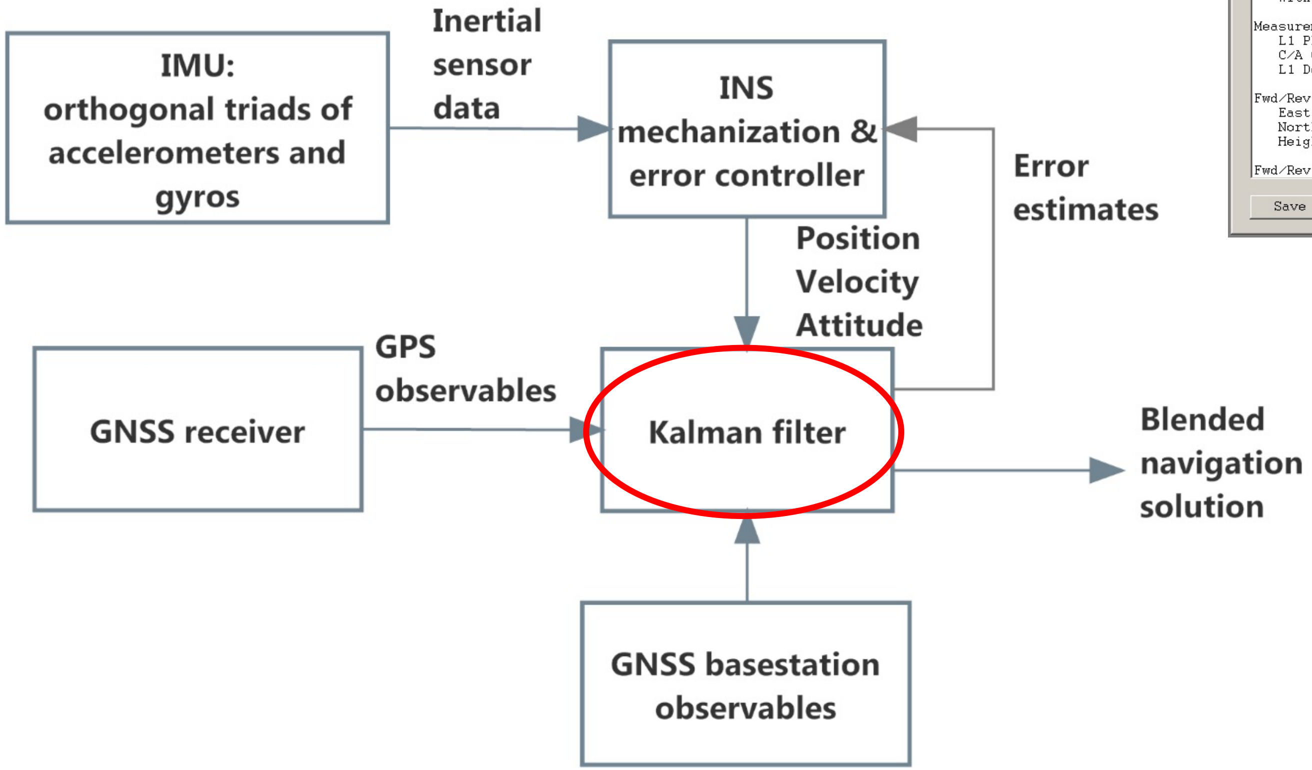


ROLL



YAW

GNSS-INS Integration via Kalman filter



IMU: orthogonal triads of accelerometers and gyros

Inertial sensor data

INS mechanization & error controller

Error estimates

Position

Velocity

Attitude

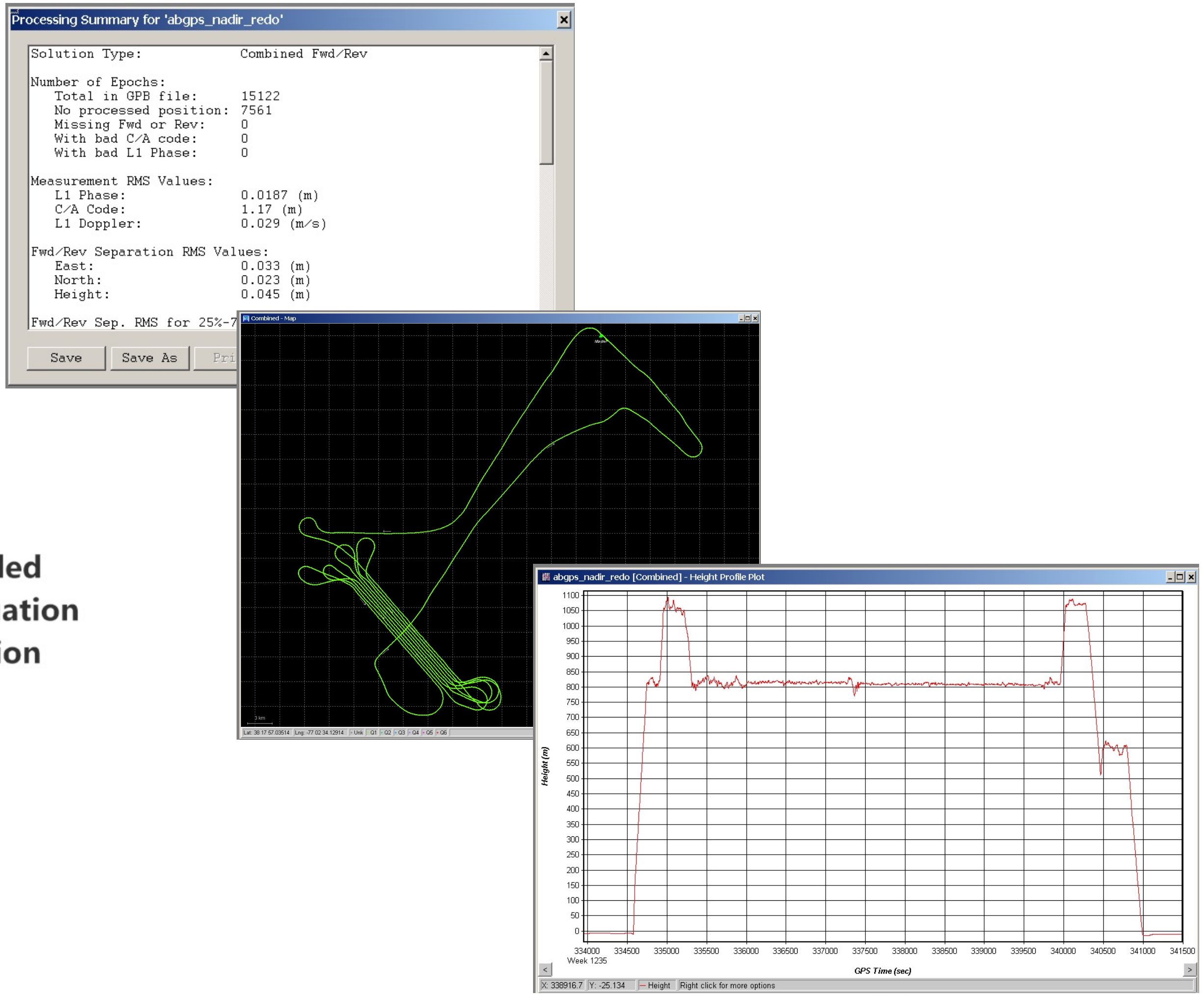
GNSS receiver

GPS observables

Kalman filter

Blended navigation solution

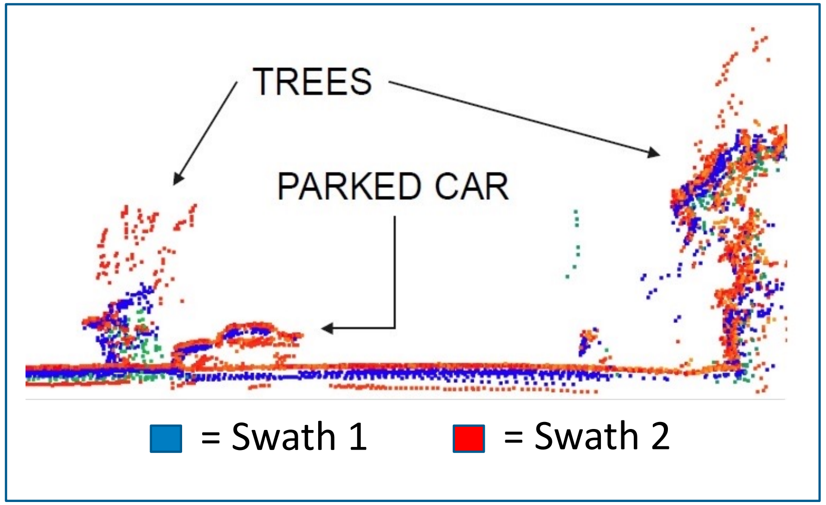
GNSS basestation observables



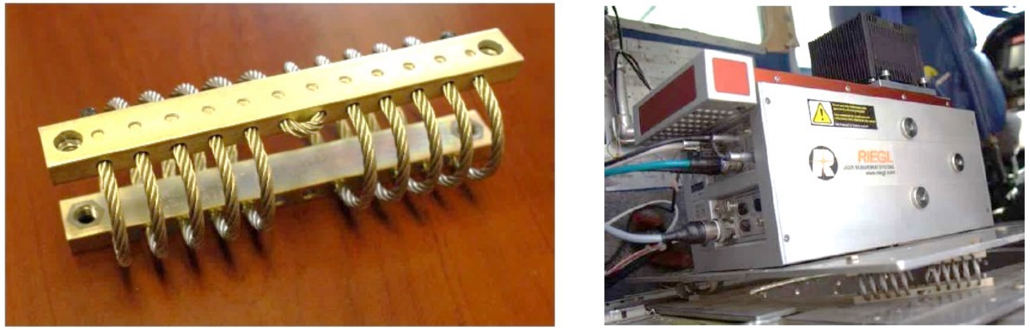
Processing Summary for 'abgps_nadir_redo'

```
Solution Type:              Combined Fwd/Rev

Number of Epochs:
   Total in GPB file:       15122
   No processed position:   7561
   Missing Fwd or Rev:      0
   With bad C/A code:       0
   With bad L1 Phase:       0

Measurement RMS Values:
   L1 Phase:                0.0187 (m)
   C/A Code:                1.17 (m)
   L1 Doppler:              0.029 (m/s)

Fwd/Rev Separation RMS Values:
   East:                    0.033 (m)
   North:                   0.023 (m)
   Height:                  0.045 (m)

Fwd/Rev Sep. RMS for 25%-7
```

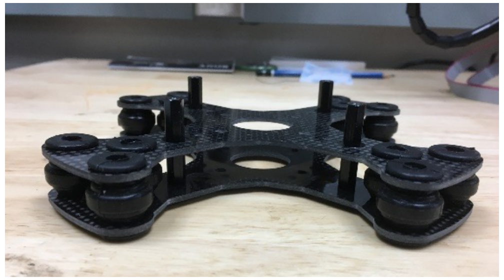
Practical issues/challenges



- Challenging trajectories
- Sensor misalignment
  - » Need for boresight calibration
- Need high-quality antenna
  - » High-accuracy data from “survey- grade” receiver can be negated by low-quality antenna
- High-frequency vibrations and need for dampening
- Datums (horizontal and vertical), realizations, epochs
  - » Ex: NAD83 (2011) 2010.00 ellipsoid heights → NAVD88 orthometric heights, or, say, MLLW tidal datum heights



Conventional aircraft vibration dampeners/isolators



UAS vibration dampener example