

18 higher or equal to 57% will result in a lowest price selection. Highways agencies should be aware of how weights and
19 scores impact the best-value selection so that they can align these elements with their selection objectives.

20

21 **Keywords:** Design-build; Best-Value Procurement; Weighted Criteria; Bidding results; Highways.

22

23 **INTRODUCTION**

24 In design-build project delivery, best-value procurement is a selection method that enables public agencies to choose
25 the proposer that provides the most advantageous offer for a particular project (AGC of America & NASFA 2008;
26 Douglas and Michael 1997). In highway projects, the most advantageous offer relates to adding value in regards to
27 schedule, technical merit, management options, and past performance (Molenaar and Tran 2015). This paper refers
28 to these aspects as non-cost factors. The importance of including non-cost factors in the selection of project teams
29 was suggested more than two decades ago. Holt et al. (1995) claimed that clients should select contractors based on
30 the value for money rather than accepting the lowest bidder; they recommended weighting criteria related to skill,
31 experience, and past performance. Egan (1998) reinforced this idea by arguing that procuring design and
32 construction teams entirely based on price was one of the most significant obstacles to meeting project goals. Since
33 then, several research have empirically demonstrated the benefits of this approach in projects' cost, schedule, and
34 quality performance (Scheepbouwer et al. 2017).

35 Highway agencies have used best-value procurement in the United States (U.S.) for the last two decades (Tran et al.
36 2017). However, a recent study analyzing 305 projects from 18 Departments of Transportation (DOTs) procured
37 using best-value between 2005 and 2018 has shown that 80% of the times, projects are awarded to the lowest bidder
38 (Gaikwad 2019). Thus, U.S. design-build highway best-value procurement is biased toward price, suggesting that
39 current practices may be missing an opportunity to balance cost and non-cost factors. Actual best-value selection
40 results are misaligned with the concept of best-value itself, as it fundamentally differs from the lowest bid paradigm
41 by seeking awards "on the basis of something other than the lowest cost alone" (Gransberg 2020; Ojiako et al.
42 2014).

43 Highway agencies award best-value contracts by using evaluation criteria, weights, scores, and award algorithms
44 (Scott et al. 2006) . The evaluation criteria establish what should be measured in the proposals; this includes cost and
45 non-cost factors. The weights represent the relevance of each criterion in the proposal's assessment, whereas the
46 scores constitute the evaluation results for each criterion. The award algorithm refers to the formula used to combine
47 evaluation criteria, weights, and scores to obtain an overall score.

48 Previous research has proposed different award algorithms (or multicriteria decision methods) to select the most
49 suitable team for developing the contract (Alarcón and Mourgues 2002; Chen et al. 2008; Chua et al. 2001; Dobi et
50 al. 2010; Nguyen 1986; Paek et al. 1992; San Cristóbal 2011; Scöttle et al. 2015; Seydel and Olson 1991). However,
51 in practice, highway agencies use simplified approaches such as adjusted score (i.e., multiplying the non-cost score
52 by the estimated project price and dividing it by the price proposal), adjusted bid (i.e., dividing bid price by the non-
53 cost score) and weighted criteria (i.e., applying weighted sum) award algorithms. From these, the weighted criteria
54 algorithm is the most intuitive and transparent approach because of how the evaluation criteria are weighted and
55 scored (Molenaar and Tran 2015).

56 In weighted-criteria best-value procurement, weights measure the relative importance of cost and non-cost factors.
57 Usually, these weights are based upon the relevance that each agency gives to the related criteria. General guidelines
58 on best-value procurement leave open to the highway agency the determination of weights and scores in their best-
59 value procurements (AASHTO 2018; U.S. Federal Government 2002). As a result, agencies differ in the weight
60 ranges they apply, as explained later in this paper.

61 Previous research have analyzed how to determine evaluation criteria based on project characteristics (Abdelrahman
62 et al. 2008); how weights and scores can influence the proposers' behavior (Ballesteros-pérez et al. 2016); and how
63 subjectivity in scoring and weights might be removed using normalization and graphical models (Asmar et al. 2010).

64 Overall, these studies have helped practitioners and academics to better understand and improve best-value
65 procurement and have contributed to the increased use and success of this procurement method in the last years.

66 However, none of the previous studies have analyzed whether best-value selection is balancing cost and non-cost
67 factors and how score and weighting practices might influence this balance.

68 This research aims to fill this gap by addressing the question: What ranges of weights and scores can better balance
69 cost and non-cost evaluation criteria in weighted criteria best-value procurement? To this end, this study: (1)

70 characterizes current scoring practices and explores their influence in the balance between cost and non-cost factors;
71 and (2) identifies ranges of weights and scores that enable non-cost factors to be more influential in the selection.

72 This research constitutes a unique contribution to both scholarly literature and current practice because it presents a
73 comprehensive analysis of 347 transportation projects procured with best-value procurement in the United States
74 between 2002 and 2020. The results of this research will help improve existing highway agency practice by
75 facilitating the selection of weights to use in the procurement. In summary, this research contributes to minimizing
76 bias toward price in best-value procurement by recommending ranges of weights that help balance cost and non-cost
77 factors in best-value selection.

78 The following sections include a literature review on best-value procurement, the research methodology, results, and
79 discussion. The final section offers conclusions, contributions, recommendations for practical implications, and
80 needs for future research.

81

82 **REVIEW OF RELEVANT WORK ON BEST-VALUE PROCUREMENT**

83 Best-value procurement aims to balance cost and non-cost factors in design-build projects. This balance provides
84 for an evaluation of design and other non-cost factors that add value to project proposals. Thus, non-cost factors
85 should play an essential role in the process of selecting the design-builder (DBIA 2019). However, the analysis of 15
86 years of best-value bidding results shows that 80% of best-value projects were awarded to the lowest bidder (FMI
87 2018; Gaikwad 2019). This means that best-value procurement is biased towards price and is almost operating
88 somewhat as a low bid procurement.

89 Best-value procurement can be thought of as a multicriteria decision-making process that aims to answer the
90 question of “given a set of alternatives and a set of decision criteria, what is the best alternative?” (Triantaphyllou
91 2000). In best-value procurement, the alternatives are the different design-builder proposals, and the decision criteria
92 are the cost and non-cost factors that highway agencies establish to evaluate those proposals. Best-value selection
93 requires balancing multiple factors, making it necessary to construct a model that considers the decision-maker’s
94 preferences and assessments of each evaluation criterion (Belton and Stewart 2002). The next sections summarize
95 relevant work related to each of the components needed to obtain the overall score of proposals in best-value
96 procurement: the award algorithm, weights, and scores.

97 **Award algorithm**

98 The weighted criteria is one of the award algorithms used in best-value procurement of highway projects because it
99 is intuitive and transparent (Molenaar and Tran 2015) and has the advantage of “distinctly communicating the
100 agency’s perceived requirements for a successful proposal through the weights themselves” (AASHTO (American
101 Association of State Highway and Transportation Officials) 2018). The weighted criteria algorithm (Equation 1)
102 considers that having “ m ” alternatives (i.e., proposers) and “ n ” evaluation criteria, the best alternative is the one that
103 satisfies:

104
$$FS = \text{Max } i \sum_{j=1}^n W_j * S_{ij}, \text{ for } i = 1, 2, 3, \dots, m. , \text{ with } \sum_{j=1}^n w_j = 1. \text{ (Equation 1)}$$

105 Where: FS is the Final Score of the best alternative, n is the number of decision criteria, S_{ij} , is the score of criterion j
106 in the assessment of proposal i , and w_j is the weight of importance of the j criterion.

107 The weighted criteria algorithm works under the implicit assumption that there exists a decision-maker’s cardinal
108 utility function, which is additive over the criteria. This means that equal FS can be obtained with very different
109 proposers’ performance regarding the different criteria. In other words, what is lost on one criterion is compensated
110 by what is gained on the other (Pomerol and Barba-Romero 2000). This might lead to selections based on
111 unbalanced criteria. To illustrate this, we can consider a best-value procurement with two evaluation criteria (i.e.,
112 cost and non-cost) and two proposers (A and B). In this particular example, let’s consider that the final score of both
113 proposers A and B is equal to 1. Based on this, both proposals have the same right to win. However, the proposer
114 A’s final score breakdown is 0.3 for cost and 0.7 for non-cost criteria, while for proposer B, it is 0.7 for cost and 0.3
115 for non-cost criteria. These results do not lead to a best-value selection.

116 Previous research on the weighted criteria algorithm has raised this issue and has proposed alternative mathematical
117 techniques to obtain a more balanced decision (Granat et al. 2006; Pomerol and Barba-Romero 2000; Wierzbicki et
118 al. 2000). However, these studies are theoretical in nature, and they do not address the limitations of the weighted
119 criteria algorithm in practical approaches such as best-value procurement.

120 **Weights**

121 Weights should represent the relative importance of the related criterion, according to the decision-maker
122 preferences.

123 Agencies adjust the weights of each evaluation criterion to reflect the needs and objectives of a particular project
124 (Scott et al. 2006). This results in heterogeneous ranges of weights used by different public agencies. For example, a
125 study developed in the United Kingdom found that public and private construction representatives assigned more
126 than 60% of importance to price, with authors suggesting that assigning a maximum weight of 70% to price might
127 help defend decision-makers from public criticism and accountability (Wong et al. 2000). In Sweden, a study
128 analyzing 386 public bidding documents found that the weight of cost was usually set to 70% (Waara and Brochner
129 2006). In Australia, the Tasmanian government establishes guidelines on weighted criteria and recommends to use a
130 weight for cost between 40% and 70% (Department of Treasury and Finance 2019).

131 In the U.S., general recommendations for best-value suggests a weight of cost over 50% if the cost is more
132 important than non-cost factors (AGC of America & NASFA 2008). A report elaborated by South Carolina DOT,
133 which summarizes design-build practices in different states, documents that South Carolina DOT typically sets a
134 weight of cost between 50% and 70%. In contrast, Virginia DOT considers a weight for the cost of 70%, and
135 Georgia DOT has commonly used between 50% and 80% (SCDOT 2018). Despite this, highway agencies' design-
136 build manuals rarely recommend specific ranges to use. Some of them suggest testing the weights against different
137 scenarios so that decision-makers can feel comfortable in case the lowest bidder is not selected (Colorado DOT
138 2016; LaDOT 2017).

139 Overall, these recommendations report the current state of best-value practice in regard to weight determination, and
140 they show a lack of research-based criteria to set these weights. Therefore, there is a need to determine what ranges
141 of weight for cost are more adequate to reach a best-value selection that evenly balances cost and non-cost factors.

142 **Scores**

143 Scores measure the level of accomplishment of each proposal towards each evaluation criterion. Public agencies
144 have used a variety of scoring systems, from the commonly called “go/no go” to direct point assignment (Scott et al.
145 2006). Non-cost scores are established by a technical evaluation team. The potential bias in this evaluation might
146 cause a significant concern (Asmar et al. 2010), leading to public mistrust and protest by bidders (Shane et al. 2006).
147 Thus, previous research has focused on studying this potential bias—generally through case studies—and proposing
148 methods and practices to minimize it (Asmar et al. 2010; Molenaar and Tran 2015; Tran et al. 2017). Other studies
149 have analyzed how score rules might influence the competitiveness of bidders (Ballesteros-pérez et al. 2016), how

150 the different types of economic scoring formulas can be categorized (Ballesteros-pérez et al. 2015), and what are the
151 mathematical and statistical relationships between scoring parameters (González-cruz 2012).

152 Best-value transparency has generally been analyzed using case studies (Asmar et al. 2010; Molenaar and Tran
153 2015; Tran et al. 2017), while the specific analysis of scoring has been conducted in more theoretical research
154 (Ballesteros-pérez et al. 2015, 2016; González-cruz 2012). Now that best-value has become an established
155 procurement method in the U.S.; this study uses the opportunity to collect historical data on these procurements and
156 analyze how best-value procurement weights and scores influence the design-build highway selection in practice.

157 **RESEARCH METHODOLOGY**

158 The research followed a four-step process (Fig.1). First, the authors collected and normalized historical data from
159 347 best-value procurements. This data was used to characterize current scoring practices using preliminary
160 statistical analysis and distribution fitting. Based on current scoring practices, the authors analyzed the balance
161 between cost and non-cost factors and derived recommendations on the ranges of weights and scores that enable a
162 better balance between cost and non-cost factors. The ultimate goal of these recommendations is to inform
163 practitioners on how non-cost factors can become more influential in the selection.

164 < FIGURE 1 >

165 **Data Collection and Normalization**

166 The authors gathered cost and non-cost scores from bidding results of 347 design-build best-value highway projects
167 procured between 2002 and 2020 by 22 DOTs. Bidding results from these projects were collected from DOTs'
168 websites and from direct requests to DOTs' representatives. Each bid comprised two to seven cost and non-cost
169 scores from submitted proposals. The number of scores available in each project depended on the number of firms
170 that placed proposals for that specific project. The data set from the 347 best-value procurements included 822 non-
171 cost scores and 1,158 cost scores.

172 New graph

173 All the scores were normalized to a common zero-to-one scale to facilitate the comparison among projects. Cost
174 scores were normalized on the basis of the lowest bidder (Equation 2), which is a common practice in weighted
175 criteria best-value procurement. With this normalization, the lowest bid was assigned a normalized cost score of 1,

176 whereas the other proposers obtained a normalized score between 0 and 1 depending on how their bid compared to
177 the lowest bid.

$$178 \quad S_{C_i} = \frac{\text{Lowest bid}}{\text{Proposer } i \text{ bid}} \quad (\text{Equation 2})$$

179 Where:

180 S_{C_i} is the normalized cost score in a scale 0-1, for the proposer i .

181 *Lowest bid* is the minimum price bid among all the proposers in the procurement.

182 *Proposer_i bid* is the bid price of proposer i .

183 Non-cost scores were also normalized to a zero-to-one scale. The initial non-cost scores had different scales (e.g., 1-
184 100, 1-1,000, 1-1,200) depending on the scale used in the procurement. The authors normalized each bid's non-cost
185 scores to a common 0-1 range using Equation 3, which conserves the proportionally between scales.

$$186 \quad S_{nc_i} = \frac{\text{Initial nc score}_i}{\text{Max initial nc score}} \quad (\text{Equation 3})$$

187 Where:

188 S_{nc_i} is the normalized non-cost score in a scale 0-1 for proposer i .

189 *Initial nc score_i* is the non-cost score of proposer i based on the initial procurement scale.

190 *Max initial ns score* is the maximum value of non-cost scores based on the initial procurement scale.

191 **Characterization of Current Scoring Practices**

192 The characterization of current scoring practices consisted of a preliminary analysis of normalized scores (including
193 basic descriptive analysis, outlier detection, and the analysis of scores variability) and the fitting of probability
194 functions that best represent the score dataset.

195 Preliminary Analysis

196 A descriptive analysis was conducted to determine the main statistics for cost and non-cost scores and their
197 distribution in histogram diagrams. Following this, a variability analysis was performed to ensure that the scoring
198 data were homogeneous in terms of the project scope and geographic distribution. The projects considered in the
199 research had varying scopes, including bridges, highways, and interchanges. Therefore, a variability analysis was

200 performed to determine whether the project scope impacted scoring. A similar analysis was performed to determine
201 the potential impact of the geographic distribution of data. This analysis was necessary because some DOTs had a
202 significantly larger experience in best-value than other DOTs and, therefore, contributed to a larger set of data. The
203 variability analysis sought to identify potential differences in scoring practices among states. Both analysis
204 (variability based on project scope and state) were based on the Mann-Whitney U test (Conover 1980).

205 Finally, an analysis of outliers was conducted to quantify, characterize, and determine how to treat this type of data.
206 A score data was identified as an outlier if it was outside the range ($Q1 - 1.5IQR$, $Q3 + 1.5IQR$), where $Q1$ is the first
207 quartile, $Q3$, the third quartile and IQR the interquartile range. Once the outliers were identified, the authors
208 analyzed the potential reasons why each of these data points were outliers and derived conclusions on whether they
209 should be removed or not from the analysis on a case-by-case basis.

210 Goodness-of-fit Analysis

211 The authors developed a statistical analysis to determine the probability distributions that best fit the cost and non-
212 cost scores. These probability distributions were used in the simulation process to characterize current practices in
213 scoring and simulate their impact on the final evaluation. To find the probabilistic distributions, goodness-of-fit
214 techniques were used to measure the fitness of the sample with a set of hypothesis distributions (D'Agostino and
215 Stephens 1986). Akaike Information Criteria (AIC) and the Bayesian Information Criteria (BIC) were used to
216 identify the distribution providing the best fit for the data. AIC (Akaike 1974) is a technique based on in-sample fit
217 to estimate the likelihood of a model to predict future values. BIC (Stone 1979) is another criterion for model
218 selection that measures the trade-off between the model complexity and fit. These metrics do not have physical
219 meaning, except in relative terms, with the lower parameter being indicative of a better fit (Yoe 2019). The
220 probability distributions selected were those with the lowest parameters for both AIC and BIC methods.

221 **Analysis of the Balance between Cost and Non-Cost Factors**

222 The cost and non-cost probability distributions resulting from the previous analysis were used to analyze the
223 weighted criteria algorithm (Equation 1) under different weighting scenarios. In each scenario of weight, the balance
224 between cost and non-cost factors was assessed.

225 Metric to measure balance

226 To understand how cost and non-cost factors impact the final score in weighted criteria algorithm, the research
227 considered a two-component algorithm comprising cost and non-cost criteria (Equation 4). The balance between
228 cost and non-cost factors was measured in terms of the ratio RC , which represents the proportion of cost over non-
229 cost factors in the final score (Equation 5).

230
$$FS = Wc * Sc + Wnc * Snc, \text{ with } Wc + Wnc = 1 \quad (\text{Equation 4})$$

231
$$RC = \frac{Wc * Sc}{Wnc * Snc} \quad (\text{Equation 5})$$

232 FS is the final score; Wc and Wnc represent the weights of cost and non-cost factors and, Sc and Snc account for the
233 score of cost and non-cost factors, respectively. The cost component ($Wc * Sc$) relates to the weight of cost multiplied
234 by the cost score. Similarly, the non-cost component ($Wnc * Snc$) is obtained by multiplying the weight and the score
235 assigned to the non-cost factors. RC constitutes the ratio between the cost and the non-cost component.

236 The ratio between cost and non-cost components is relevant because high or low RC values are indicators of
237 unbalanced selections (Fig.2). High RC values represent cases where the cost component is notably larger than the
238 non-cost component, leading thus, to a cost-driven selection. On the contrary, low RC values imply that the non-cost
239 component has more significant importance than cost, which may lead to a non-cost-driven selection. Best-value
240 procurement aims to select the best contractor on the basis of a balanced evaluation of cost and non-cost factors.
241 Therefore, extreme RC values should be avoided to ensure a best-value selection.

242 < FIGURE 2 >

243 Monte Carlo Simulation

244 Each procurement might have different values for weights and scores, resulting in a balance that can be
245 deterministically calculated. However, the analysis of particular cases does not enable researchers to find a fairly
246 accurate estimate of that balance of cost and non-cost in the best-value practice as a whole. For this reason, this
247 research used Monte Carlo simulation. By using this technique, it is possible to estimate a deterministic quantity by
248 using a large and random sample (Brandimarte 2014). In this study, two probability distributions built upon
249 empirical score data were considered to simulate final project scores and find the balance between cost and non-cost
250 factors in different weighting scenarios.

251 According to Johnson (2013), simulation is a “way of forming an educated guess about the most likely outcomes or
252 the range of possibilities.” Through Monte Carlo simulation, researchers can obtain enough large set of results that
253 enables them to make statistical inferences (Kroese et al. 2014). In this research, the authors used Monte Carlo
254 simulation to replicate a large number of weighted criteria best-value procurements using different sets of weights
255 and scores. In each iteration, the relative contribution of the cost component, and the non-cost component (i.e., RC)
256 was analyzed.

257 The authors determined 0.3 and 0.7 as the extreme values for the weight of cost. This range contains 70% of the
258 dataset weighted criteria cases. Further, this range consider a variation of $\pm 20\%$ in regards 50%. Fifty percent (50%)
259 is the limit suggested by AGC of America % NASFA (2008) when cost is more important than non-cost factors.

260 Within this range, the authors performed two analyses. The first one, considering equal weights ($W_c = W_t = 0.5$),
261 aimed to better understand the impact of current scoring practices in the overall evaluation. The second analysis
262 considered 41 scenarios with weights varying in centesimal increments (e.g., in the scenario 1, $W_c=0.3$; scenario 2,
263 $W_c=0.31$; scenario 3, $W_c=0.32$; etc). For each of these scenarios, the weight of non-cost factors (W_{nc}) was
264 determined by considering that $W_c+W_t=1$ (Equation 4). In each scenario, the simulation run iterations in which the
265 cost and non-cost scores were obtained from the probability distributions. The Monte Carlo simulations were
266 performed using @Risk software, considering a seed to guarantee replicability. To reach validity in the results, each
267 weight scenario comprised 10,000 iterations to ensure the convergence of the output mean and standard deviation
268 with a 95% confidence level.

269 **Recommendations of Ranges of Weights and Scores**

270 Finally, the simulation results were analyzed to derive recommendations of the ranges of weights and scores that
271 should be used to make non-cost factors more influential in best-value selection.

272

273 **RESULTS**

274 **Characterization of Current Scoring Practice**

275 Preliminary Analysis

276 The original data distribution is characterized by descriptive statistics (Table 1) and histograms (Fig. 3 and Fig. 4)..
277 When analyzing these results it is important to remember that all the scores were normalized to a common zero-to-
278 one scale (using Equations 2 and 3) to facilitate the comparison among projects. Results from the preliminary
279 descriptive analysis show that cost scores are slightly more skewed toward “1” than non-cost values (median 0.879
280 vs. 0.865 in Table 1). The data spread is higher in cost scores (standard deviation of 0.103 vs. 0.079).

281 < TABLE 1 >

282 To understand the shape and the spread of the sample, the data are displayed using histograms in the form of relative
283 frequency graphs. The cost score data pattern is skewed to the left, with 90% of the cost scores ranging from 0.652
284 and 0.987. Fig.3 shows a gradual variation of the data frequency from the peak (0.88-0.93) to the minimum and
285 maximum values (0.53 and 0.99).

286 < FIGURE 3 >

287 Non-cost scores are slightly skewed to the left, with 90% of the non-cost scores ranging from 0.610 and 0.958. In
288 contrast with the cost scores, Fig.4 shows a sharp variation of the data frequency from the peak (between 0.77 and
289 0.93) to both high and low ends. The values lower than 0.77 and higher than 93 are much unlikely.

290 < FIGURE 4 >

291 The variability analysis shows that neither project type nor project location have a significant impact on scoring. The
292 results of the Mann Whitney U test (p-value > 0.2) show that there is no statistically significant difference among
293 project scopes for both cost and non-cost scores. Concerning the project’s geographic distribution, Florida’s projects
294 constitute 35% of the whole sample. Given this, it was tested whether there was a statistically significant difference
295 in the scores of Florida’s projects and the overall sample. The results of the Mann Whitney U test (p-value 0.154)

296 show that there is not a statistically significant difference. Therefore, the data set is considered consistent and was
297 not divided based on project type nor location.

298 Finally, the authors identify the outliers in the score sample. This analysis determines that cost scores have 16
299 outliers, accounting for roughly 2% of the cost scores' sample. These data points represent bids with prices 70%
300 higher or more than the lowest bid (i.e., cost scores lower than 0.579). In this research, the outliers in the cost score
301 sample correspond to six (6) projects, with three (3) of them having two (2) outliers. Procurements with two (2) or
302 more outliers indicate that more than 60% of the proposers bid outside the expected range. This suggests a very high
303 variance in bid prices and may result from specific project-case circumstances that do not represent standard
304 practices. This might happen, for example, when one company bid with a very low and unrealistic price aiming to
305 win the contract. As a result, other companies score very low (because they are costly) as compared with the lowest
306 bidder. The authors considered that having two (2) or more outliers do not reflect the general scoring trend. Thus,
307 the scores of three (3) projects containing six (6) outliers were removed.

308 The non-cost scores contain 52 outliers, accounting for 6% of the sample. All the outliers have values lower than
309 0.66. Outliers in non-cost scores might correspond to specific project-case circumstances. It may be the result of
310 vague Request for Proposals (RFP) that lead most of the proposers to not adequately prepare their proposals. In this
311 study, non-cost scores outliers correspond to 17 projects from six (6) DOTs. Twelve (12) of these projects have
312 more than two (2) outliers, representing a total of 34 outliers. One DOT contributed the most to the outlier set, with
313 five (5) projects having 17 outliers. In this particular case, projects were delivered between 2014 and 2016, just
314 when this DOT began using best-value procurement. The second DOT with significant contribution provided three
315 (3) projects with a total of 7 outliers. These projects were delivered between 2010 and 2012, also in the early years
316 of using best-value procurement. The remaining four DOTs provided only one project each. These projects were
317 delivered at different times (2007, 2012, 2015, 2018), suggesting that each specific case's circumstances might
318 explain the outliers.

319 Overall, these cases do not correspond to the general scoring trend, which is what this research aims to simulate.
320 Therefore, the scores associated with the 12 projects that contained the 34 outliers were removed for further analysis.

321 Goodness-of-fit analysis

322 The Beta distribution is the probability distribution that better fit cost scores. They had thus the lowest values for
323 both AIC and BIC parameters (Table 2).

324 < TABLE 2 >

325 The Beta distribution is commonly used to describe variability over a limited range, being naturally defined over 0
326 and 1 (Yoe 2019). The Beta distribution is widely known as the foundation of the Program Evaluation and Review
327 Technique (PERT) method. The PERT method is usually adopted to model task duration in construction
328 management by using three values, the most optimistic (shorter), the most pessimistic (longest), and the most likely
329 (mode) (Damjanovic and Reinschmidt 2020). Cost scores could indeed be characterized in this way by considering
330 a maximum value of 1, a minimum value of approximately 0.65, and a most likely range between 0.87-0.93.

331 Non-cost scores were fitted to a Gumbel distribution. The Gumbel distribution is a limiting extreme value
332 distribution that serves to model the maximum and minimum values of any set of data (Gumbel 1955). The Gumbel
333 distribution is used to model extreme events as well as construction design elements (Mun 2002). The non-cost
334 scores distribute at the high end of the evaluation scale, with 70% of the data between 0.77 and 0.93. It seems thus
335 reasonable that non-cost scores are well suited for the Gumbel distribution because they do not normally vary
336 around one value.

337 **Analysis of the Balance between cost and non-cost factors**

338 The balance between cost and non-cost factors was measured in terms of the RC ratio, which represents the
339 proportion of cost over non-cost factors in the final score (Equation 5). RC values were obtained using Monte Carlo
340 simulation and the Beta and Gumbel distributions to characterize current scoring practices and explore different
341 weighting scenarios. For the simulation, both distributions were truncated in the maximum and minimum value of
342 “1” and “0”, respectively.

343 RC with Equal Weights

344 Intuitively, when setting equal weights to both factors, the decision-maker would expect a balanced contribution of
345 cost and non-cost factors in the overall score. However, the simulation showed that, based on current scoring
346 practices, overall scores did not follow this intuition. Although the weights were equal, the cost component had a

347 more significant contribution to the overall score than the non-cost component. This is shown in the probability
348 density graph depicted in Fig.5, where RC is higher than “1” in 68.6% of the cases.

349 < FIGURE 5>

350 If both components were to contribute evenly to the overall score, the relative frequency graph would be symmetric
351 and centered in 1 (implying that the cost and non-cost component have equal relative importance). However, the
352 results show a relative frequency skewed to the right, meaning that when assigning equal weights, the cost
353 component has a larger contribution to the overall score compared to the non-cost component in 68.6% of the cases.
354 These results are explained because the cost scores are statistically higher than the non-cost scores (Fig.3 and Fig.4).
355 Ultimately, this led to a counterintuitive result in which equal weights do not result in the equal importance of cost
356 and non-cost factors in the overall score. Therefore, there is a need to better understand how current scoring
357 practices are impacting the relative importance of cost and non-cost factors in best-value procurement. To do so, the
358 next section analyzes the impact of different weighting scenarios on the relative importance of cost and non-cost
359 components.

360 RC with Different Weighting Scenarios

361 Fig.6 synthesizes the results from the simulation of 41 weighting scenarios. The X-axis represents the weight of cost
362 considered in each scenario (W_c), whereas the Y-axis represents the proportion of cost over non-cost factors in the
363 overall score (RC). For each value of weight of cost, a Monte Carlo simulation with 10,000 iterations is run, and the
364 values of RC for the median and 5th and 95th percentile are recorded. These values, plotted in Fig.6 with solid black
365 lines, account for 90% of the cases in each scenario of weight. The grey dashed line represents the ratio defined by
366 the weights of cost (W_c) and non-cost factors (W_{nc}). In other words, it is the contribution of the weights to the RC
367 ratio (W_c/W_{nc}). If RC is higher than the value defined by the grey dashed line, this means that the cost score is
368 higher than the non-cost score. The median (percentile 50%) is above this line; therefore, in 50% of the
369 procurements, cost scores are higher than non-cost scores.

370 < FIGURE 6 >

371 The area defined above $RC = 1$ shows a cost-driven selection, where the cost component is higher than the non-cost
372 component. On the contrary, the area below $RC = 1$ represents the non-cost-driven selection, where the non-cost
373 component is higher than the cost component. Given this consideration and the results obtained in the simulation,

374 Fig.6 shows that weights of cost lower than 0.43 result in a non-cost-driven selection. On the contrary, weights of
375 cost higher than 0.57 lead to a cost-driven selection. In the range of weights defined between 0.43 and 0.57, the
376 result of the selection can be either cost or non-cost driven.

377 Another interesting result relies on the increasing distance between percentiles 5th and 95th as the weight of cost
378 increases. This relates to the relative effect of weights and scores on the RC ratio. Low weights of cost (W_c), result
379 in low ratios between the weights (e.g., for $W_c = 0.3$, W_c/W_{nc} is equal to $0.3/0.7 = 0.43$; while for $W_c = 0.7$, W_c/W_{nc}
380 is equal to 2.33). This trend is represented by the grey dashed line in Fig.6. As the weight of cost increases, the ratio
381 of the scores (S_c/S_{nc}) is therefore multiplied by a higher number, resulting thus in more spread RC values and larger
382 distance between the 5th and 95th percentile lines.

383 **DISCUSSION**

384 This research aimed to find the ranges of weights that lead to a better balance between cost and non-cost factors in the
385 weighted criteria best-value procurement. By analyzing historical cost and non-cost scores under different weighting
386 scenarios, the study provided insight into what ranges might lead to a better balance in best-value selection.

387 Furthermore, this research raises awareness about how the trade-off between cost and non-cost criteria can vary
388 depending on the award algorithm, the weights and the scores used. Specifically, the weighted criteria algorithm
389 (because of the nature of the formula) might lead to selections based on unbalance criteria. Previous research proposed
390 mathematical techniques to obtain more balance selections when using the weighted criteria algorithm (Granat et al.
391 2006; Pomerol and Barba-Romero 2000; Wierzbicki et al. 2000). This research contributes to scholarly research on
392 weighted criteria algorithm by proposing a range of weights that limit the relative variation of cost and non-cost
393 criteria. Thus, this research addresses the weighted criteria limitation under the practical approach of best-value
394 procurement selection.

395

396 **Weights**

397 The analysis showed that, under current scoring practices, weights lower than 0.43 and higher than 0.57 do not
398 enable highway agencies to make a best-value selection, as the selection is skewed toward either non-cost or cost
399 factors. A range of weight of costs between 0.43 and 0.57 allows decision-makers to have chances of having both

400 cost and non-cost-driven selections. In this range, the scores can determine whether the non-cost component is
401 higher or lower than the cost component. Further, this range of weights ensures that in 90% of the cases the RC is
402 between 0.6 and 1.8. This means that the proportion of the cost component regarding the non-cost component is
403 limited within a range that minimizes selections based on unbalanced criteria

404 It is relevant to note that the proposed range of weight of cost between 0.43 and 0.57 is due to the existing scoring
405 tendencies (represented by Fig. 3 and Fig.4). This range could be wider for a specific highway agency if this agency
406 followed a wider pattern in the scoring of the proposals, as suggested in the following section.

407 Overall, the range of weights that this research proposes helps to minimize unbalance selections in the weighted sum
408 algorithm when applied in best-value procurement. Further, previous studies and the current state-of-the-practice
409 showed a lack of specific criteria to determine the weights to use in weighted criteria best-value procurement (AGC
410 of America & NASFA 2008; Colorado DOT 2016; LaDOT 2017; SCDOT 2018). This research contributes to this
411 current state of practice by proposing the use of weights of cost between 0.43 and 0.57 in order to reach an adequate
412 balance between cost and non-cost factors in the selection. These research-based recommendations might help
413 highway agencies to overcome the fear suggested by Wong et al. (2000) about using weight for cost lower than
414 70%.

415 ScoresAs well as weights, the scores might play an essential role in characterizing the best-value selection. Previous
416 research analyzed best-value scoring using case studies (Asmar et al. 2010; Molenaar and Tran 2015; Tran et al. 2017),
417 or theoretical approaches (Ballesteros-pérez et al. 2015, 2016; González-cruz 2012). This research contributes to
418 scholarly research on best-value scoring by collecting and analyzing a dataset of 882 non-cost scores and 1,158 cost
419 scores from 347 best-value highway projects.

420 The cost and non-cost data distributions (represented by Fig. 3 and Fig.4) served to obtain the best-value range of
421 weight of cost between 0.43 and 0.57. This range could vary for a specific highway agency if this agency had another
422 scoring pattern. In this regard, this research proposes the following recommendations for non-cost and cost scores.

423 Non-cost scores.

424 Widening the range of non-cost scores can help to make the non-cost component more influential. Under a stochastic
425 approach (such as the one developed in this research), having a more spread non-cost score distribution (similar to the
426 cost scores' one) would lead to a balance between cost and non-cost factors comparable to the one established by the

427 weights. In other words, taking the example of equal weights shown in Fig.5, similar spread in cost and non-cost
428 distributions would lead to a more symmetric and centered in “1” RC distribution.

429 Under a deterministic approach, considering a wide range of non-cost scores in each procurement would enable
430 highway agencies to make a more meaningful differentiation between the technical proposals. In other words, if all
431 the proposers score equally in the technical evaluation (meaning a narrow range of non-cost scores so that there is no
432 “technical” distinction among proposers), the differentiator would not be the non-cost component, but solely the cost.
433 Thus, this research recommends widening the non-cost scores range in best-value technical evaluation in order to
434 make non-cost factors more influential.

435 Cost-scores

436 Expectations on cost scores dispersion can help highway agencies to decide the weight of cost to use in each
437 procurement. The results of this research are based upon a historic cost dispersion. However, agencies might expect a
438 very “tight” or a very “wide” range of price proposals depending upon each project’s scope, risks, or innovation.

439 Expecting ranges of cost scores that are narrow, and close to “1” implies that all the bids are close to the lowest bidder.
440 Having all cost scores on the upper side of the evaluation scale makes it more likely that non-cost scores are lower
441 than cost scores. In this case, highway agencies might wish to weight up non-cost factors by using a lower weight of
442 cost. On the contrary, expecting a wide range of cost scores suggest a more likely trade-off between cost and non-cost
443 factors in each proposer’s evaluation. In this case, highway agencies should consider the recommendations given for
444 both weights and non-cost scores in previous sections.

445

446 **CONCLUSIONS**

447 The goal of balancing cost and non-cost criteria in best-value selection is not being realized. Highway agencies use
448 non-cost criteria to evaluate and select design-builders. However, in more than 80% of the cases in our dataset, the
449 best-value selection award the contract to the lowest bidder. This evidence shows a bias toward price of best-value
450 selection; in other words, a lack of balance between cost and non-cost factors. This research aimed to solve this
451 problem by addressing the following research question: What ranges of weights and scores can better balance cost
452 and non-cost evaluation criteria in the weighted criteria best-value procurement?

453 The findings showed that the weight of cost and the ranges of scores used in the evaluation play an essential role in
454 having cost or non-cost-driven selections. Indeed, weights of cost higher than 57% always lead to a cost-driven
455 selection; that is, a low bid selection. A weight of cost ranging between 43% and 57% strengthens the best-value
456 selection by enabling highway agencies to reverse the driver of the selection depending on the difference between
457 cost and non-cost scores. In this range of weight, the selection might be cost or non-cost-driven. Further, by using
458 this range of weights, highway agencies will prevent selections based on unbalanced criteria. This is because this
459 range of weights limits the cost component to be bounded into 1.8-0.6 times the non-cost component.

460 Weights between 43% and 57% not only enable highway agencies to balance cost and non-cost factors in the
461 selection. Also, they send the message that both cost and non-cost factors are important. This might minimize the
462 proposers' tendency to cut bid prices in order to be the lowest bidder. Instead, by using this range of weights, the
463 idea of providing "the best-value for dollar spent" is encouraged.

464 It is relevant to note that the proposed range of weight of cost between 43% and 57% is based on the existing
465 scoring tendency. The historical scoring pattern used in this research showed a skewness toward "1" for both cost
466 and non-cost scores and a more widespread distribution of cost scores as compared with non-cost scores. This
467 tendency leads the cost component in the evaluation to be more influential than the non-cost component.

468 However, the proposed range of weight of cost could vary if the scoring trends were different. Highway agencies
469 could consider a wider range of weights of cost by using a wider pattern in the scoring of the proposals. Specifically,
470 non-cost factors could be more influential if the proposals' evaluation led to non-cost scores within a range wide
471 enough to enable the differentiation among the proposers. Further, another aspect to consider would be the cost
472 score dispersion. Non-cost factors could be more influential if highway agencies considered the effect of potential
473 economic bid dispersion when selecting the weight of cost. Low dispersion of cost scores suggests more probability
474 of having cost scores higher than non-cost scores. Thus, in these cases, highway agencies might adopt a lower
475 weight of cost in order to balance cost and non-cost factors.

476 In summary, this research constitutes a unique contribution to both scholarly literature and current practice by
477 recommending the weight of cost range that should be used to properly balance cost and non-cost factors in the
478 weighted criteria best-value procurement. This recommendation is derived from the analysis of 347 highway

479 projects using best-value procurement over the last two decades, reflecting thus existing trends in current practice.
480 The use of this range of weights will contribute to minimizing bias toward cost in best-value procurement.
481 This research establishes the first step to minimize the bias toward cost of best-value practice, which will make more
482 influential non-cost factors in the selection and, in turn, will increase the likelihood of achieving project goals. This
483 research, however, did not address all the elements that might influence best-value results and the balance between
484 cost and non-cost factors. These topics are thus suggested for future research. This includes the analysis of different
485 award algorithms and scoring systems and how they might impact the balance between cost and non-cost factors.
486 Further, the evaluation criteria selected to represent the non-cost factors might also influence the scoring trend and,
487 therefore, the balance between cost and non-cost factors. Another practical and future research will consider case
488 studies to evaluate the range of weights proposed in this research.

489

490 **DATA AVAILABILITY STATEMENT**

491 Some or all data, models, or code generated or used during the study are available from the corresponding author by
492 request.

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TABLES

Table 1 Descriptive statistics

| | Mean | Mode | Median | Variance | Std. Deviation |
|-----------------------|-------------|-------------|---------------|-----------------|-----------------------|
| Cost Scores [Sc] | 0.857 | 0.896 | 0.879 | 0.011 | 0.103 |
| Non-cost Scores [Snc] | 0.853 | 0.820 | 0.865 | 0.013 | 0.079 |

618
619

Table 2 Cost and Non-Cost Scores. Goodness-of-fit-parameters

| | Cost Scores | | Non-Cost Scores | |
|---------------------------------|--------------------|---------------|------------------------|------------|
| Distributions/parameters | AIC | BIC | AIC | BIC |
| Beta | -1,612 | -1,593 | n/a | n/a |

| | | | | |
|---------------|--------|--------|---------------|---------------|
| Gumbel | -1,546 | -1,537 | -1,930 | -1,921 |
| Logistic | -1,407 | -1,397 | -1,858 | -1,849 |
| Normal | -1,370 | -1,361 | -1,727 | -1,718 |
| Laplace | -1,347 | -1,338 | -1,860 | -1,851 |
| Triangular | -940 | -926 | -1,222 | -1,209 |
