

Linguistic Research Institute

Report No. 22

CONCEPTUAL-NOTATIONAL DEVICES:

The PCF and Related Types

Peter G. Ossorio

1979

© Copyright 1979

ALL RIGHTS RESERVED

Linguistic Research Institute

Boulder, Colorado

When we acquire a concept there is an extended family of possible ways of acting on it. We may, for example, use it in recognizing instances or non-instances and treating them accordingly. We may also use it in hypothesizing, imagining, predicting, wondering, instructing, speculating, asking, wishing, demanding, denying, pondering, or asserting. And more. In general, the acquisition of a concept opens up behavioral possibilities. It adds to our behavior potential.

For example, the acquisition of the concept of a Person gives us the behavioral possibilities of a person. (Compare: the acquisition of the concept of chess gives us the behavioral possibilities of a chess player.)

In Descriptive Psychology the formulation of the Person Concept and of more detailed subject matter is accomplished by means of a number of notational devices. One reason for this strategy is that since concepts have no possible truth value, statements are never involved in an essential way in the conceptual formulations themselves, but at most appear in some accompanying commentaries or illustrations. As a result, the possibilities of straightforward discursive presentation are very limited, and so some alternative is required.

Because notational devices of verbal and other sorts are public and communicable, they play an essential part in the public and communicable character of concepts.

Because of this essential connection, and also for certain heuristic purposes, the notational devices used in Descriptive Psychology are generally designated as conceptual-notational devices. We also distinguish between particular conceptual-notational devices and the more general types of which they are exemplars. The latter are designated as conceptual-notational device types.

Our present concern is with four related device types. These are (1) the definition, (2) the paradigm case formulation, (3) the parametric analysis, and (4) the calculational system. Various exemplars of each of these are found in the literature of Descriptive Psychology. One reason for considering these as a group is that each has some relevance to the problem of introducing a subject matter without any essential reference to any other subject matter or dependence on any other subject matter. A second reason is that these four device types are systematically related to one another. The domain within which these relationships have a place will require further elucidation at a later time. I expect to have more to say about it in "The Behavior of Persons."

I. Definition

To define a term, "X," is, traditionally, to specify the necessary and sufficient condition(s), Y, for the correct use of the term. Certain additional qualifications are involved.

- (a) The satisfaction of condition(s) Y must be what makes the use of the term correct. (Having an angular sum of 180° is a necessary and sufficient condition for a plane polygon to be a triangle, but it is having three sides that makes it a triangle and is the defining characteristic. One might put it that its being a triangle is its having three sides, whereas its being a triangle is only logically equivalent to its having an angular sum of 180° .)
- (b) The term "X" is to be used to refer to the states of affairs which consist of the satisfaction of the necessary and sufficient conditions. (Thus, even if one could [and we can't] specify the necessary and sufficient condition[s] for the correct use of the term "Aha!," that would not be a definition, because "Aha!" is not used to refer to those conditions. Here, "correct use" paraphrases as "appropriate" rather than "true.")
- (c) The term "X" does not appear essentially in the specification of the condition(s), Y. If it does, the definition is "circular," and one would normally say that it wasn't really (didn't do the proper job of) a definition, but rather, only had the form of a definition. (The reference to "essentially" will legitimize recursive definitions.)

Reflection on this last qualification provides an interesting reminder. For any term, "X," for which a necessary and sufficient condition for applicability is relevant, there is a simple, direct, and rigorous way of specifying that condition, namely "X." For example, the necessary and sufficient condition for the correct application of the term "blue" to a thing is that the thing be blue; for the term "angry," that the individual be angry, and so forth.

Hence a traditional definition is not the way of specifying the necessary and sufficient conditions for the correct use of "X." Rather, it is a second way of doing so. Not too surprisingly, a second way is usually not available. (Just as one may argue that no two words are really synonymous, so one may argue that no real world phenomena are really definable except certain matters involving explicit conventions, i.e., those which are, in effect, created by a definition. It is instructive to try to formulate a really rigorous definition of something like a chair, a mountain, or a lemon, just as it is often instructive to try to give a set of directions for even a simple behavior such as tying one's shoes such that if only those directions are complied with, the behavior must be successful. Such exercises can do much to clarify the distinctive contributions of knowledge and competence.) Thus:

(a) Definitions are given for the sake of a listener who is not already clear enough about the way "X" is used or is to be used. They are not in principle necessary for identifying the necessary and sufficient conditions for the correct use of "X," nor are they necessary for picking out cases of X.

(b) The clarification attempted by recourse to a definition may not be successful. In that case some further definitions of the terms used in the first definition may help. But this procedure cannot be carried out ad infinitum. Definitions can be given in this recursive fashion but the success of a definition depends ultimately on the successful use of terms which do not need further clarification for the task at hand. Either the original term "X" or any preferred definition is ultimately an appeal to the competence of the listener, not a device for creating something out of nothing. Thus, any general 'requirement' that all terms be given (or even be capable of) definition is incoherent.

(c) Unless one already understands "X" there is no way to tell whether the specification of necessary and sufficient conditions given by "X," i.e., X, is the same as the specification given by a definition of "X," i.e., Y. Perhaps more often than not, there is a difference. In the case where we detect a merely appreciable difference we will generally say either that the definition is faulty or that a new word has been introduced in a misleading and possibly deceptive way, since it is spelled the same as "X." In the case where the difference is fundamental, we will generally say that the definition is a reductive one and that a new word has been introduced in an apparently deceptive way since it is spelled the same as "X" and clearly means something other than "X," yet no disclaimer is offered.

For example, to define "pleasure" as "the diffuse experience of the operation of reward mechanisms" would be of this latter sort.

So also would be the definition of "verbal behavior" as "behavior the reinforcement of which is mediated by another person" and, likewise the definition of "love" as "the reciprocal satisfaction of dependency needs." These definitions are genuinely reductive, and not merely fatuous, because they take us from the original concept to a different logical category or to a different conceptual system (and in the terms of "What Actually Happens," to a different "world," or domain of facts).

Ordinarily, a reductive definition is offered when its author believes that the specification given by the definition covers the same set or range of cases as "X." However, a definition requires more than coincidence in this respect for in that case "theory of" rather than "definition of" would be the appropriate characterization. Thus, the test of the 'definition' is to offer a hypothetical separation and see which way the decision goes. Of the author of our "pleasure" definition we would ask, "Suppose you had a case which by all existing standards was a case of pleasure ("If ever there was a case of pleasure, this is one.") but also, you had established to your satisfaction that it was not a case of diffuse experience of the operation of reward mechanisms. Would you then say, 'Then it can't be a case of pleasure, because it's not a case of experiencing the operation of reward mechanisms' or would you say 'Well, I guess pleasure isn't, after all, the experience of the operation of reward mechanisms'?" In the latter case we would conclude that the 'definition' was a bogus one because, as it turned out, it was not, even for its author, a specification of necessary and sufficient conditions for pleasure.

In the former case we would conclude that he had deceptively introduced a new word, and an associated concept, into the language and was pretending that the concept was that of pleasure. Conversely, the author of that genuinely reductive definition would also, it would seem, be committed to the customary follow-through of reductive definitions, namely the claim that "Properly speaking, there's no such thing as pleasure. What we (naively) call 'pleasure' is nothing but the diffuse experience of the operation of reward mechanisms. That's what there really is in the world."

The further importance of this reductive position is that if the definition is followed by investigation the latter will not be the investigation of pleasure (but instead, the investigation of the diffuse experience of et cetera), but it will be so presented to the scientific and general public.

Empirical data is heavily in favor of the prediction that a 'scientist' who offers such a definition of "pleasure" has access to some conventionally accepted technical means for studying the diffuse experience of the operation of reward mechanisms and none for studying pleasure. The definition will, therefore, offer the same advantage as looking for the lost coin under the lamp post because that's where the light is. Of course, I should not want to deny that we can study only what we can study, and only in the ways we have available. (Indeed, this is merely an instance of a general principle codified elsewhere as Maxim 5: "if a situation calls for a person to do something he can't do, he will do something he can do.") What is regrettable is that to date we should have been so unable to study our

professed subject matter and that our accounts of what it is we do as 'behavioral scientists' and what it is that we have found out should have been so obtuse or misleading.

If we hold to the requirement that our subject matter really be the behavior of persons, then we will be properly skeptical of the possibility of introducing that subject matter formally (explicitly and systematically) simply by means of either general definitions (e.g., of persons or behavior) or a collection of more particular ones (e.g., of "pleasure," "love," "verbal behavior," etc.), this in spite of the fact that our strongest proclivity in the matter, inculcated by academica is to begin with definition.

For, on the one hand, it would be almost impossible for an informative definition not to be a reductive one, which would lose us our subject matter at the first step (as it has happened in academica). And, on the other hand, we ought to have at least a nagging sense, if not a positive conviction, that if our hold on the subject matter were so tenuous that we had to anchor it with a second specification we should have no business claiming to study it. Conversely, one can imagine, for example, a 'behavioral scientist' announcing "I'm going to study the behavior of persons scientifically. Of course, I don't know what that is; I wouldn't know a case of behavior as such if I found one, and I can't tell a person from an organism, but I've got the experimental method and the hypothetico-deductive method and the theory of measurement on my side, and by cracky, there ain't anything that won't yield to those methods! Just give me twenty million dollars, six thousand subject hours, one

hundred years, and immunity from prosecution, and you'll see, I promise you!"

But of course, this isn't how to go about it, and the example is somewhat overdone. The "know nothing" approach has been viable in the "natural sciences" because the subject matters there are essentially completely invented ones, and so there were no antecedent reality constraints on what we might say, e.g., about electrons, ion exchanges, red blood corpuscles, et cetera. In contrast, when we study the behavior of persons, the course of their development, and the differences among them, there already is such a subject matter. It is what we need a better grasp of, and no amount of successful bait and switch tactics by our 'behavioral scientists' will meet that requirement. If we freely invent subject matters by means of definitions or theories of "behavior," "verbal behavior," "pleasure," et cetera we should not expect to have then a better grasp of what it was that we needed to understand better.

II. Paradigm Case Formulation (PCF)

A paradigm case formulation is possible in many cases where, in some relevant sense, a definition is not possible. For example a definition is not possible when there is no second set of necessary and sufficient conditions the satisfaction of which is what makes the use of the term correct. (Wittgenstein's example of games is a familiar heuristic in this regard.) A paradigm case formulation is accomplished in two major steps:

I. Introduce a Paradigm Case (of X)

II. Introduce one or more transformations of the Paradigm Case

As in the case of definition, the Paradigm Case must be either directly intelligible or finitely explainable in order for communication to be successful. The Paradigm Case will directly identify some portion of the cases which are to be picked out. Each transformation will pick out additional cases. Each transformation may be considered to be a constructional instruction or an indirect description:

"Change the Paradigm Case in this way (the transformation) and the result will still (also) be a case of X." Thus, if the PCF is successful, the Paradigm Case and the transformations will, collectively, pick out all and only those cases that one wants to pick out, even if there is no second thing that those cases have in common. Therefore, a PCF will accomplish the identification of a subject matter (a range of possible cases) no less effectively than would a definition.

A standard example of a paradigm case formulation is the following PCF for the concept of a family.

I. Paradigm Case: A husband and his wife living with their natural children, who are a seventeen-year-old son, a ten-year-old daughter, and a five-year-old daughter.

II. Transformations:

- T1. Eliminate one parent.
- T2. Change the number of children to N , $N > 0$.
- T3. Change the sex distribution of the children to any distribution other than zero boys and zero girls
- T4. Change the ages of the children to any values compatible with the ages of the parents.

- T5. Add any number of additional parents.
- T6. Add adopted and other legally defined sons and daughters.
- T7. Eliminate the requirement of living together.
- T8. Add zero children if husband and wife are living together.
- T9. Eliminate the requirement that the parents have the legal status of "married."

Note that constructing a PCF has a good deal in common with constructing a definition. For example, constructing a definition often involves some careful decisions and judgment in regard to what cases to include and what not to include. Correspondingly, what someone approves as a "good" definition someone else will disapprove as misleading, defective, or wrong. One constructs a definition in

the course of participating in some social practice(s) which give it a point or a purpose; depending on what one is up to, one or another specific construction may be correct, successful, adequate, et cetera. And to give a definition of a term already in use, e.g., "behavior," "emotion," or "family" is to run the risk of violating the existing use and hence to run the risk of degrading the language and misleading or manipulating one's audience. All of these possibilities are the case for paradigm case formulations, though the danger is not as great.

There are also important differences between a definition and a paradigm case formulation. First, of course, we have noted that a PCF will do the job of identifying a subject matter in circumstances where a definition would fail because there is no second set of necessary and sufficient conditions for the use of the term in question.

Second, a definition and a PCF are differently "keyed" to the behavioral context in which alone they are intelligible. A definition is a definition of a term. The concept which is to be associated with the term is identified by the specification given in the definition. The term can thereafter be used to mark that concept or instances of that concept. In contrast, a paradigm case formulation is a formulation of the concept or the range of its instances; it is not a formulation of the term. Thus, in connection with "family," a definition would be a definition of the term "family," whereas the PCF given above is a PCF of the phenomenon of "the family" or of the concept of a family.

Consequently, and thirdly, whereas it is almost impossible for a definition not to be reductive, if only in a weak sense and not in

the strong sense discussed above, it is essentially impossible for a paradigm case formulation to be reductive. A definition of "X," unless it is circular and hence defective, must be given in terms of something else, "Y" and "Z." Almost inevitably, one loses the X in favor of the Y and Z, because that is what X "really is," and, after all, if one gives a definition that implies that "X" was not a good enough way of saying what X really is. (I am speaking of definition primarily in scientific discourse, not in mathematics, logic, et cetera, where issues of reductiveness are not as salient.) If one is not prepared to sacrifice the X in this way, one normally does not define "X" but merely uses it to refer to X. In contrast, since a PCF begins with a genuine case of X and each transformation leaves us with something that is still (also) a case of X, we are in little danger of replacing X's with Y's and Z's.

Fourth, what is involved in a paradigm case formulation is not merely which cases are picked out, but also how that is done. In most PCF's the transformations reflect differences among cases, and these differences are at face value relevant and potentially important for the task at hand or in other contexts. By virtue of its structure, the PCF will often help to make clear why it makes sense to consider as one type of phenomenon a set of cases which are importantly different from one another and have no second and separately specifiable set of criterial attributes in common by reference to which we could codify the homogeneity of the set.

Thus, a paradigm case formulation will, paradigmatically, have some illuminating and explanatory power that makes it no less akin to

a theory or a conceptual analysis than to the usual sort of stipulative definition. Indeed, in the family example, a consideration of the ramifications of being legally married or legally adopted would take us very naturally into the whole area of kinship and kinship theory and social systems, so that these latter could be considered simply as extensions and elaborations of the PCF for the family.

Correspondingly, the PCF lends itself readily to certain research designs. Since each transformation both picks out a group of cases and reflects a possible important difference among cases, it would be natural and sensible on the whole to study each such group separately in order to decide whether the empirical regularities which were characteristic of one such group were equally characteristic of each of the others and of "the family" in general. [This is the PCF stratified sampling design.]

One of the reasons why this research design has a methodological interest is the issue of how usable one's results are by someone else in some other circumstances. And one reason why this is as important an issue as it is is that people will disagree. The issue of reductive definitions is an extreme case here, and there are less extreme cases as well. Looking back at the paradigm case formulation for the family, it seems pretty clear that few consumers would disagree with the Paradigm Case and the first four transformations but that T5 through T9 would each be likely to generate appreciable disagreement or dissatisfaction. For a consumer of research who, say, objected to T5, having the results stratified along these lines would permit him for his own purposes to discard the empirical

results for that group and make use of the remaining as findings about "the family" while still being able to understand and communicate effectively with someone who didn't reject T5. Neither a definition nor simple random sampling or 'representative' sampling would permit this happy result.

There is more to be said in connection with the fact that a paradigm case formulation involves issues of how cases are picked and not merely which cases are picked. Perhaps it is clear from the example of the family that in general, in a PCF any of the cases of X could conceivably be used as the Paradigm Case and, depending on that choice, the set of transformations would be different. In that sense, the choice of a Paradigm Case is conceptually arbitrary. It need not be methodologically arbitrary, however. In general it will make a difference, and sometimes a crucial difference which one chooses as the Paradigm Case, and there are some reasonable rules of thumb which provide a basis for the advantageous choice of a Paradigm Case.

The first rule of thumb is to choose the most complex case as the Paradigm Case. The reason for this choice is that then usually the transformations are simplified, becoming merely a series of deletions or something close to that. When that is not the case, there is no great advantage in this choice. In contrast, starting with the simplest case is generally a poor choice for a Paradigm Case because then usually the more complex cases cannot be generated as simple transformations but instead will require substantive additions which present the same decision problems as in the original choice of

Paradigm Case. This is particularly so when the simplest case is also a derivative, or parasitical, case (see below). There is an obvious moral to be drawn here in regard to the classic theoretical-experimental strategy of studying artificially simplified cases in the 'laboratory' in order to "get at the fundamentals of the phenomenon."

The second rule of thumb is to start with an indubitable case. This was the operative rule in the family PCF. "If ever there was a case of a family, that's one." The virtue of such a choice, when it has a virtue, is that it makes disagreement more easily managed. I mentioned that a consumer who disagreed with T5 in the family PCF could for his own purposes throw out the results for T5 and use the remainder if the research were stratified in accordance with the Paradigm Case and the transformations. It would generally be more inconvenient and it would be difficult or impossible to make use of partial results if the consumer objected even to the Paradigm Case, for then there would be a good chance that none of the groups defined by the transformations would be cleanly acceptable either.

The third rule of thumb is to start with a case which is in some relevant sense primary or archetypal. (The rule is applicable only if there is such a case, of course.) The relevant consideration is that one wants to give formal recognition to the fact that the other cases are cases because of their relation to the primary case. One thinks, for example, of legal precedents. This case is a case of violation of privacy because of the way it is related to the case of *Dukes vs. Wisconsin*, but not vice versa. Or again, one thinks of

the case of perceiving objects and hallucinating or imagining them or the case of genuine coins and counterfeit coins. A defective, partial, illusory, counterfeit, or imaginary something is that because of the way it resembles the real thing and not vice versa. In connection with the PCF for the concept of a family, the Paradigm Case is primary relative to T7 (eliminate the requirement of living together). Given the paradigm case we can (especially with some additional restrictions) accept T7 as an exception or variation because of the way it resembles the Paradigm Case. In contrast, if all cases of families were (conceptually) cases of people who were not living together, that would not be our concept of a family. (Note that a definition of a family which was broad enough to allow for T7 at all would automatically allow the possibility that all cases of families were cases of people living apart.) Yet T7 is what makes intelligible our notion of a family with a daughter in college or a son in the army and our notion of an extended family (kinship system).

Considered as a formal device, i.e., a device type, rather than as a substantive construction, the paradigm case formulation has a reflexive use as well as a recursive logic. That is, not only is it the case that some element of a PCF can be given by means of a PCF, but also the very notion of a paradigm case formulation as given above can be handled in PCF fashion. Let us construct an example. I introduced the paradigm case formulation by specifying a two-step procedure, namely, (a) First, introduce a Paradigm Case of X, then (b) Introduce some number of transformations of the Paradigm

Case such that when the Paradigm Case of X is transformed in that way the result will also (still) be a case of X. Let us designate this as PCF_1 . Now consider the following paradigm case formulation, which is designated as PCF_2 .

PCF_2 I. Paradigm Case: PCF_1 , i.e., A. Introduce a Paradigm Case of X
 B. Introduce transformations of
 Paradigm Case

II. Transformations:

- T1. Change the number of Paradigm Cases to N, $N > one$
- T2. Eliminate the requirement that the Paradigm Case is a case of X. (It will be sufficient if the transformations generate cases of X.)
- T3. Replace "transformation" with any functional equivalent thereof.
- T4. Allow transformations not only of the Paradigm Case, but also of the results of a previous transformation.

Here, we accomplish a bit of bootstrapping. The paradigm case formulation, as previously presented, can now be assigned the status of a Paradigm Case in a new PCF and the latter gives us a more complex and adequate representation of what a paradigm case formulation is (what the range of instances of a PCF is). And lest the transformations above appear to be merely bizarre or precious, we may note the following.

(a) There is currently in the design stage an interesting piece of empirical research on masculine-feminine relationships in which

the conceptualization of the phenomenon involves T1 (an archetypal variety of masculine-feminine relationships) and T2 (actual relationships generated by non-actual archetypal ones) and the rationale for the use of numerical scales involves T1 and T3 as an alternative to the traditional theory of measurement. (This is an example of both a recursive paradigm case design and the non-mathematical use of mathematical procedures.)

(b) T4 permits us to assimilate "family resemblance" structures, which are just a little looser than the original PCF in that one generates the family by moving from case to case rather than always returning to the same case. This device type is available when there is no guarantee that all the members of the set can be generated from a single Paradigm Case.

(c) T2, T3, and T4 extend the scope of the PCF to cover generative "Rewrite" systems such as those found in much current work in syntax and semantics. An arbitrary rewrite rule, "P may be rewritten as Q" will in general not correspond to any independently specifiable transformation. However, by anchoring in this way on P and Q we may thereby define a new transformation ad hoc, i.e., the "P-Q transformation." Thus, the "rewrite rule" is the functional equivalent (T3) of a transformation. (It is also the functional equivalent of an Operation in a calculational system together with the eligibility constraint that the Operation can only be performed on the Element P—see below.) In such systems T4 is what would permit derivations (rewriting the results of a rewrite) and T2 is what would permit

sentences, or surface structures, to be derived from non-sentences, or "deep structures."

Surprisingly, perhaps, the concept of a paradigm case formulation has some appreciable stylistic significance in the presentation of the Person Concept. Very often it is important to stick to the theme or task at hand without getting bogged down in details, side-tracks, qualifications, et cetera. And yet, saying something in a relatively simple and direct way is likely to be ineffective if there clearly are qualifications, amplifications, exceptions, et cetera which would need to be taken into account somehow if one wanted to be both as complete and as careful as possible. Frequently one can (and frequently, I do) try to minimize that sort of problem by using "paradigmatic" or "paradigmatically" as a standard marker to indicate that what is simply presented is to be understood as the (or a) Paradigm Case (frequently, of the primary, or archetypal, sort) in a paradigm case formulation and that the loose ends are presumed or known to be manageable via transformations and their existence is not crucial for the task at hand (or that they will be or have been dealt with explicitly elsewhere). For example, I commented above that "a paradigm case formulation will, paradigmatically, have some illuminating or explanatory power that makes it . . . akin to a theory or a conceptual analysis." Of course, one can't guarantee that a given PCF will be illuminating; triviality and awkwardness are just as possible here as with definition. But on the other hand, merely saying that a PCF may be illuminating, but then again, it may not would be like merely saying that the members of a family may live

together, but then again, they may not. Were the both possibilities on a completely even footing a PCF would be importantly different from what I have presented and a 'family' would be essentially different from a family.

I might mention, too, that the present concept of a Paradigm Case Formulation has no interesting connections to the body of literature in philosophy, dating mainly around 1955-65, in which the terms "paradigm case" and "paradigm case argument" occur. There, the term "paradigm case" is used to refer to "the very kind of case from which one could (or from which we normally do) acquire the concept of X" (where "X" might be "red," "angry," etc.). In the present terms, such a case would seem to have some aspects of the "indisputable" and "archetypal" cases indicated in the rules of thumb for choosing a Paradigm Case. A paradigm case argument, roughly, is an attempt to establish the indisputability of calling an exemplar of the paradigm case a case of "X" by reference to either (a) its indisputability in the course of acquiring the concept or (b) its presumed archetypal character by virtue of which it could serve as an indisputable case in the acquisition of the concept. Fortunately, the concept of Paradigm Case Formulation presented here does not depend either on that use of "paradigm case" or on the success, failure, or meaningfulness of that argument. In general in those rare cases where a reference to "the very kind of case from which one could acquire the concept" is intended, that will be indicated explicitly.

Returning to the PCF: Having constructed a Paradigm Case Formulation, one could wave one's hand over it and transform it into the verbal semblance of a definition. For example, one could say, "By 'a family' I mean one or more parents living with one or more of their children." Such a 'definition' would need explanation, qualification, commentary, elaboration, et cetera. ("I didn't mean that no children was a borderline case." "Of course, a family with an only child away at college is still a family—by 'living together' I mean" "Of course, by 'parents' I mean either natural or adoptive parents, and by 'children' I mean either natural or adopted children, and . . . and . . . and") These emendations tend collectively toward a scattered and cluttered unwieldiness so that even with such elaborations the augmented 'definition' would lack the crisp, recursive logical structure which, as noted above, both illuminates the phenomenon and facilitates the stratificational research strategy and is essential for the archetypal and reflexive-recursive use. In short, for a definition to have the advantages of the corresponding paradigm case formulation it would have to amount to a PCF in form as well as content, and so its being a definition at all would be immaterial. Indeed, because a definition has the (paradigmatic) verbal form of specifying a single set of necessary and sufficient conditions, when it amounts to a PCF it will have to be considered a "loose" or "defective" definition (not a paradigm case definition or an archetypal one), since no amount of definitional syntax will create a second defining criterion where there is none. So much for scholarly niggling about whether we aren't "really" or

"after all" talking about a definition. Qualifiers such as "really" and "after all" are characteristically ways of expressing transformations of implicit or informal Paradigm Cases.

In sum, the concept of a paradigm case formulation is a distinctive resource for both thought and action. It has a perspicuous and deceptively simple logical structure. Partly because of this, it is, paradigmatically, a remarkably effective device not merely for introducing or locating subject matter but also for generating more elaborate constructions which are suitable for the conceptual and empirical mapping of domains of fact. It has a special value in those logically complex behavioral domains wherein the structure and regularities are archetypically psychological rather than merely mathematical, physiological, or anything else.

III. Parametric Analysis

Like a definition and a paradigm case formulation, a parametric analysis is a device for notationally identifying a conceptual domain, or range of cases. Like a paradigm case formulation and unlike a definition, it also provides a way of dealing differentially with different cases within that range or domain. Like a paradigm case formulation, and unlike a definition, a parametric analysis deals with a domain, not a term or locution.

To give a parametric analysis of a given domain of cases is to specify the ways in which one of those cases could be the same as another of those cases as such or different from it. (Such a specification will also permit us to specify the ways in which one

kind of case could be the same as another kind of case within the domain or different from it.)

The ways in which two such cases as such could be different from each other are exactly the same as the ways in which they could be the same as each other except for those ways in which they are necessarily all alike, i.e., whatever makes them cases within the domain at all. Each such way is a parameter of each case and of the conceptualized "general case." Because a parameter represents a set of possibilities, a parameter has formally associated with it a set of "values" corresponding to that set of possibilities. One specifies (more or less completely) which of these possibilities is the case in a given instance by specifying (more or less completely) which value the parameter has in that instance. Thus, paradigmatically, one picks out cases more or less uniquely within the domain by specifying, more or less uniquely, values for each parameter.

This general feature of the parametric analysis is directly reflected in the formula notation used here to represent a parametric analysis. An example of such a formula is the formula for the "general case" of behavior which is presented elsewhere:

$$\langle B \rangle = \langle I, W, K, KH, P, A, PC, S \rangle$$

In such a formula the left-hand term represents the domain, the right-hand term represents the parametric analysis, and the terms within the brackets represent the parameters of the phenomenon. Thus, as with the transformations in the paradigm case formulation, such a formula can be read as a reference to a general procedure: "To

specify something about one of these (B's) you have to specify something about some of these (the parameters)." One can also take it in a simply factual way: "When something of this sort (B) is the case, something of each of these sorts (the parameters) is the case." Or, in more 'dynamic' form: "When something of this sort (B) occurs, something of each of these sorts is the case." Or, in the idiom of sets, "The set of B's is the set of octuples I, W, K,"

A more familiar example of a parametric analysis is the "color pyramid," commonly found in introductory textbooks in psychology. The domain is the domain, C, of visible colors. In the color pyramid, colors are arranged in a three-dimensional order as shown in Figure 1.

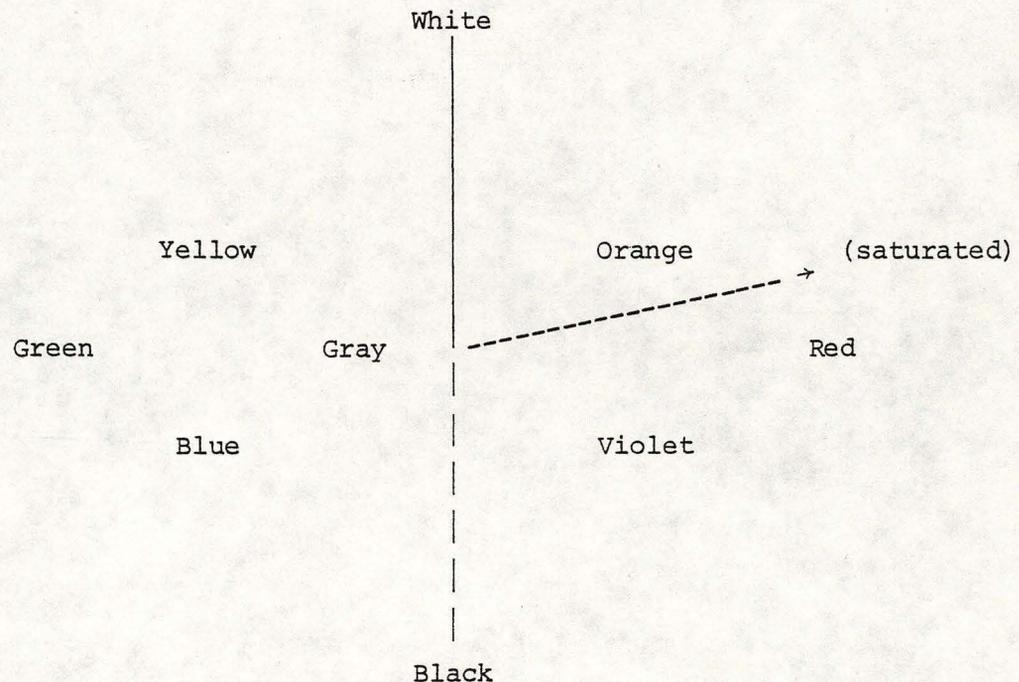


Figure 1. A Parametric Analysis of Color

There are three distinct orderings (parameters) in Figure 1. The first is the ordering from light colors to dark ones (White to Black). The second is the ordering from gray colors to brilliant, or intense, colors, as illustrated by the dotted arrow. (The actual ordering here is the perpendicular distance from the white-black axis.) The third is the familiar "rainbow color" ordering (circularly: red, orange, yellow, green, blue, violet). Ignoring some recently evolved niceties, these three orderings represent the ways in which one color can be the same as another color or different from another color. These three parameters are commonly designated as Brightness, Saturation (intensity, brilliance), and Hue. Thus, in formula form:

$$\langle C \rangle = \langle B, H, S \rangle$$

Because the concept of a parametric analysis is a purely formal (content-free) one there is no general restriction on, or prescription for, the kind of values that a parameter in such an analysis can have. For example, some parameters have numbers as their values; some have letters; some have facts; others have concepts; and so on. The only restriction is that all the values of a given parameter are of the same kind. (If it appeared that we had more than one kind of value for a given parameter, we would simply conclude that we were in fact dealing with more than one parameter or that we had conceived of the parameter too narrowly, etc.)

The nature of the domain is given in principle by what all the cases have in common. (Note that this no longer should suggest

that we ought to be able to define the domain.) Frequently, however, the distinctiveness of the domain is reflected in the distinctiveness of the parameters of the domain. Often enough, but less commonly, the distinctiveness of the domain is reflected in the distinctiveness of the values of the parameters of the domain. It is because the values of the parameters are in general the least distinctive aspect of the domain and because so much scientific effort is explicitly an effort to 'quantify' that we encounter so many cases where the parametric values are numbers.

The domain of colors is typical of a wide range of domains in that more than one parametric analysis is clearly possible. (For no domain is it demonstrably impossible that more than one parametric analysis could be given.) One could do it as in Figure 1, but one could do it by introducing any three-dimensional coordinate system, e.g., three orthogonal reference axes with an arbitrary base vector. Even if we consider only analyses in terms of brightness, hue, and saturation, there are still at least two ways of designating values for these parameters. The first is to provide categorical values, using qualifying adjectives (e.g., white, light gray, medium gray, dark gray, black), and this is typically done in ordinary discourse. (Grammatically, we normally use nouns for hues and adjectives for brightness and saturation if we talk about colors ["a light, toned-down green"], but we use adjectives for all three in talking about colored objects ["a light, gray-green chair"]; the latter reflects the fact that color is a parameter of material objects.) The second way is to specify the values of brightness, hue, and saturation with

numbers. This is frequently done in technical contexts, e.g., in manufacturing or in perceptual experiments. (Anyone who is bursting to ask, "But aren't you really just contrasting ordinal and ratio scales of measurement here?" is allowed to get it off his chest by writing it on the blackboard one hundred times.)

The two parametric analyses (categorical vs. numerical values) have different advantages and limitations. The advantage of the second, in principle, is that it opens up possibilities for numerical calculation (e.g., in predicting effects of color mixtures); it may or may not allow finer discrimination. The advantage of the first is that it is much more effective and efficient in picking out ranges of color which are in fact of interest and which would have to be represented in the three-dimensional space as irregular volumes. As Wittgenstein has reminded us, "Stand roughly there" is not a less precise instruction than "Stand exactly thirty inches north of that mark." Likewise, "a light, reddish brown" is not a less precise designation of a range of cases than "2.5-3.0; 3.3-3.7; 1.7-2.5." Indeed, picking out exactly that range of cases with a mathematical formula for an irregular volume in three dimensions is impossible, though we may jury-rig more or less adequate approximations. Note here the same kind of issue as with definition. Just as "pleasure" is not just a layman's naive way of referring to the diffuse experience of the operation of reward mechanisms, so "a light, reddish brown" is not a layman's naive way of talking about an irregular three-dimensional solid; rather, one might better say that the latter is the technician's naive way of talking about a light reddish brown.

The same considerations hold for "puce" or "the color of irises" and are even more to the point there.

Like the paradigm case formulation, the parametric analysis is essentially a non-reductive procedure. The specification of how one color may resemble another color or be different from it is, per se, not a reduction of color to something else. (But a parametric analysis may in turn be subjected to a reductive formulation, as when we say, "Hue is nothing but the wavelength of the electromagnetic excitation bouncing into your eye when you see the color.")

Like both a definition and a paradigm case formulation, the parametric analysis has a recursive grammar. The values of one of the parameters in a given parametric analysis may in turn be generated by a second parametric analysis. Indeed, our color example was just such a case. We noted that that color is one of the parameters of visible material objects, and it is the values of that parameter which are given by the parametric analysis of Figure 1.

A parametric analysis is more than just a substitute for a definition. It is also a preparation for dealing with the subject matter in a systematic way. In particular, patterns of various kinds or theoretical or empirical (real or hypothetical) regularities are potentially expressible as contingencies or other formal regularities or functional relationships connecting parametric values. For example, some important phenomena involving color can be represented as simply stated regularities in which the parametric values of a color mixture are expressed as a function of the parametric values and proportions of the component colors.

Such possibilities reflect the conceptual constraint that the only way a phenomenon or a historical particular can change is a change in the values of its parameters. Thus, it is unlikely that a sow's ear would change into a silk purse, but since they are both material objects, they have the same parameters, and so it could happen. In contrast, a mechanism could not become a motive, nor could an internalized parent or a history of reinforcement become a conscience or the ability to do arithmetic, and a flowerpot could not become the number 17.

Since a domain is at least as distinctive as its parameters, the latter are lower-limit determiners of what type ("logical category," "ontological category," "metaphysical category," etc.) of phenomenon (e.g., what type of object) the domain consists of. This is why, as indicated in "What Actually Happens," the specification of a set of "ultimate objects" also determines a self-contained "world" (a domain of possible facts), for the only possible happenings in that world will be changes in the values of the parameters of those objects and their aggregates as such, and the only possible states of affairs in that world will be the states of affairs identified by specifying the values of those parameters. In that world none of those changes or states of affairs will be the same thing as the existence of an object or process (etc.) of a different type. Only in the world of Persons is anything remotely resembling such an identity possible. Ironically, the major efforts of scientific and philosophical reductionists amount to an attempt to make the known world of Persons nothing but a world of non-persons, this under

curiously inverted theory of "naturalism." (See Sections V and VI of "What Actually Happens" for some elaboration.)

Like a paradigm case formulation, a parametric analysis is convertible into a "loose" definition. For example, "A color is anything that has some brightness, some hue, and some saturation."

But also, a parametric analysis and a paradigm case formulation will be formally inter-convertible. Thus, one could construct a PCF for color as follows:

- I. Paradigm Case: light grayish-green
- II. Transformations:
 - A. Change its brightness
 - B. Change its hue
 - C. Change its saturation

And the paradigm case formulation of the family could be converted to a parametric analysis along the following lines:

$$F = \langle P, NS, ND, LP, LC, A, AS, AD \rangle$$

where

F = family

P = Parents: M, F, M + F, M + F + O

NS = Sons: $N \geq 0$

ND = Daughters: $N \geq 0$

LP = Legal status of parents: M, C, U

LP = Legal status of children: N, A, O

A = Living arrangements: T, O

AS = Age of Sons: etc.

AD = Age of Daughters: etc.

In this analysis the values of the P parameter are (1) Mother, (2) Father, (3) Mother and Father, (4) Mother, Father, Other. The values of A are (1) Living together, (2) Other. And so on. Indeed, the 'definition' derived from the paradigm case formulation of the family ("some number of parents living with some number of their children") also corresponds to a crude parametric analysis in which the parameters P, NS, ND, and A are indicated but the nature of their possible values is left implicit (hence, in part, the need for commentary, elaboration, explanation, further definition, etc.).

Practically and aesthetically, paradigm case formulations and parametric analyses are not generally interchangeable, for all that there is a formal convertibility. A paradigm case formulation is, paradigmatically, suited to a situation where a certain general structure (including interrelationships among elements) is of interest. In contrast, a parametric analysis is more suited to a situation in which the range of variations among cases is of primary interest or in which a variety of patterns are to be mapped into the same domain. (This is not altogether dissimilar to the considerations affecting the choice of cluster analysis vs. factor analysis in multi-dimensional data-summarizing. Indeed, those who are familiar with these techniques may recognize them as hot-house varieties of paradigm case formulation.)

IV. Computational Systems

There are some number of ways of delineating what a calculational system is. The following "Element-Operation-Product" model is the one used in explaining the system of reality concepts in "What Actually Happens."

One constructs a calculational system by specifying explicitly a finite set of Elements and a finite set of Operations. An Operation is something to perform on an Element or set of Elements, and an Element is something upon which to perform operations. If there are constraints on which Operations may be performed on which Elements, these constraints are also specified. When an Operation is performed on an element, the result is a Product. Whatever is a Product is also an Element, hence something upon which an Operation can be performed. Paradigmatically, the construction of a calculational system will include a distinctive notation for an Element as such (the canonical form) and for a Product as such (i.e. for a product as an Element-Operation combination), since calculation consists of systematically eliminating the latter by substituting the former. (The same kind of duality is found in the "Name" and "Description" structure of the descriptive formats presented in "What Actually Happens" for use in directly representing real world phenomena or for constructing more elaborate structures which can also be used for representing real world phenomena.)

A heuristic example of a calculational system (though mathematicians or logicians are likely to be unhappy with it) is given in Figure 2.

Element	Operation	Product
0 (original Element)	Add one (only Operation)	
0	+1	0 + 1 (1)
1	+1	1 + 1 (2)
2	+1	2 + 1 (3)
3		etc.
etc.		

Figure 2. A Computational System and Its Operation

In Figure 2, the Element "zero" and the Operation "Add one" are explicitly specified. The first Product is (zero)(Add one), which is the same as $0 + 1$, which is the same as the Element 1. When we perform the Operation "Add one" to this Element, we have (One)(Add one), which is $1 + 1$, which is 2. And so on.

Another heuristic example is the game of chess. For our initial Element we specify the initial board position of the pieces. For Operations we specify the rules governing the movement of each piece, specifying that that Operation can be performed only on that piece. (Note the resemblance here to the "rewrite rule.") Specifying that the Operations are performed on pieces is a convenience in form. In point of fact, the rules for moving pieces are rules for generating one board position from another by changing it in a particular way (moving a piece).

Computational systems have certain characteristic or distinctive features. Not every computational system generates an infinite set

of Products. (The "Add one" system above does this; the chess rules do not generate an infinite set of board positions, though they do generate infinite sequences of changes in board positions.) However, only a calculational system can generate an infinite set. Thus, our understanding and representation of limitless sets of numbers, sentences, descriptions, forms of behavior description, real world configurations, et cetera reflect the mastery of calculational systems. Note that our understanding and mastery, such as it is, does not come from generating an infinite set of products and inspecting the set empirically, but rather from our being competent to generate any one of the Products, not every one.

I commented above that the paradigm case formulation, paradigmatically, carries some element of explanation or illumination because it clarifies the relationships among a set of otherwise disparate cases having nothing else in common than being cases of X. To this we can now add that the understanding provided by a calculational system is an extension of the understanding provided by a paradigm case formulation, and this is so because a calculational system is an extension of a paradigm case formulation.

The initial Elements in the calculational system correspond to the Paradigm Case(s) in the paradigm case formulation. The Operation(s) in the calculational system correspond to the transformation, in the paradigm case formulation. The difference is that in the calculational system we can operate in principle on the results (Product) of an Operation (transformation). To get from a PCF to a calculational system, it is not enough to allow transformation on the results of

transformations ad hoc—it must be in principle. For example in the PCF for the family, we allowed the deletion of one parent. We could, if we were so minded, take the result of that transformation, i.e., a one-parent family, and allow once more the deletion of one parent. However, we could not allow such a deletion indefinitely or, indeed, even one more time. Since there are finite number of parents there are a finite number of deletions of parents that could be allowed (and note that because of that the successive deletions could be replaced by a single transformation, i.e., "delete any number up to $N-1$ of the N parents").

To repeat, our grasp of an infinite set of products comes not from enumeration and not from empirical inspection or inductive encounter, but rather from having the concept of the totality of things having a certain relationship (corresponding to transformation or Operation) to each other and to some paradigmatic cases of which we have some direct acquaintance. In "What Actually Happens" we noted that this kind of understanding is what makes both feasible and intelligible a behavioral science which deals with all behavior, actual and possible, and that from this consideration there follow some constraints on what would qualify as behavioral theorizing.

Returning to our comparisons, a calculational system not only can be derived from a PCF, but also, it can, with some possible awkwardness, be transformed into a parametric analysis. For example, consider a two-parameter analysis with parameters E (Element) and O (Operation) such that specifying parametric values (of the domain of Products) is the functional equivalent of specifying which

Operations can be performed on which Elements. The problem, in the parametric analysis, of specifying which sets of parametric values (which Products) were the values for a given Element would correspond to the problem, in the calculational system of specifying which calculations (sequences of Operations on Elements) would generate the same Product.

Finally, a calculational system, like both a parametric analysis and a paradigm case formulation, can be replaced by a loose (very loose) definition, e.g., "A chess position is any position generated by the 'chess rule' system."

Like both the paradigm case formulation and the parametric analysis, the calculational system as such is essentially non-reductive. This is because the Products are related to the original Elements via the transformations (Operations) and hence will, in general, belong to the same logical category. (Conversely, it is because (a) things of the kind defined by the original Element are in principle transformable, and (b) the result of transformation is again a thing of that kind that we can say that the transformations are repeatable indefinitely.)