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Video in the Middle: Purposeful Design of Video-Based Mathematics Professional Development

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In this article the authors described their exploration of a particular design element they labeled "video in the middle." As part of the video in the middle design, the viewing of carefully selected video clips from teachers' classrooms is sandwiched between pre- and postviewing activities that are expected to support teachers' engagement in and learning from the video. These three elements (prevideo, video, and postvideo), taken together, comprise a videocase. Videocases can then be further sequenced to create a specified professional development (PD) curriculum. Purposeful selection of each video clip allows for coherence between the prevideo, video viewing, and postvideo activities, which in turn, supports the link between a given videocase and identified teacher learning goals. Incorporating a video in the middle design within a video-based mathematics PD environment can promote a detailed and focused examination of complex mathematical content, the relationship between pedagogical decisions and practices, and an unpacking of students' mathematical thinking. It is essential to underscore the major role that facilitators play in video-based PD, and that the effective application of the video in the middle design is, in large part, dependent on skillful facilitation. The video in the middle design can be useful across different content areas and teacher education settings.

The power of using video in professional development (PD) to elicit critical reflection and to support teachers' learning of new content and skills has been widely documented (e.g., Borko, Koellner, Jacobs, & Seago, 2011; Brophy, 2004; Harford & MacRuairc, 2008; Rich & Hannafin, 2009; Rosaen, Lundeberg, Cooper, Fritzen, & Terpstra, 2008; Santagata, Zannoni, & Stigler, 2007; Sherin, 2007). Video allows for the complexities of classroom practice to be stopped in time, unpacked, and thoughtfully analyzed, helping to bridge the perpetual theory-to-practice divide and support instructional improvement. Whereas in the classroom teachers must constantly make individual in-the-moment decisions, viewing video during PD allows teachers the opportunity to deconstruct collectively and discuss familiar experiences and to actively generate new understandings about content, pedagogy, and student thinking (Cullen, 1991; Korthagen, Kessels, Koster, Lagerwerf, & Wubbels, 2001).

Video-based PD typically incorporates selected clips for teachers to discuss and analyze collaboratively. These PD programs can be classified as those that use clips from the teachers' own or peers' classrooms (i.e., those participating in the program) or from other teachers' classrooms (i.e., those not participating in the program). (See Seidel, Sturmer, Blomberg, Kobarg, & Schwindt, 2011; Zhang, Lundeberg, Koehler & Eberhardt, 2011.)

When teachers view video clips from their own classrooms, video analysis usually takes place within small groups of teachers who are already colleagues, at regular intervals and over an extended period of time (Borko, Jacobs, Koellner & Swackhammer, 2015; van Es & Sherin, 2008). In this type of professional learning group, teachers are generally guided by a facilitator or teacher educator who focuses the viewing on an instructional strategy, new content, or student thinking.

Alternatively, commercial or specified professional development materials typically incorporate video clips from other teachers' classrooms, either as exemplars or representations of authentic practice. Facilitators who guide the viewing of these clips are expected to support a focus on the specific objectives highlighted in the materials using approaches recommended by the developers (Jacobs, Seago & Koellner, 2017).

Interestingly, although many PD models utilize video either as the centerpiece of the program or as a supplementary component, there is dearth of research on the design features and, in particular, the articulation of how video is intended to contribute to teacher learning. This paper articulates and illustrates a purposeful design decision to place video viewing between pre- and postviewing experiences to promote a defined teacher learning trajectory.

The video in the middle design uses video clips strategically selected as examples (rather than exemplars) from a variety of classrooms to address particular mathematical topics in an authentic manner that supports specific, incremental goals for teacher learning. This design moves beyond having a facilitator set the instructional context prior to video viewing and leading a conversation afterwards. An intentionally designed prevideo experience can motivate teachers to attend and react to particular ideas and activities that unfold within the selected clip. Similarly, an intentionally designed postviewing experience can support movement toward a particular learning goal (or goals) through guided discussion and reflection.

Next is a review of some of the literature on how video can be effectively incorporated into a PD program — including ways clips should be purposefully selected and knowledgeably used by facilitators. Because this article's focus is on video-based mathematics PD, we drew largely on research from mathematics education. We then address some remaining questions, such as precisely what activities should come before and after viewing video clips and how they can support teacher learning and the aligned goals of the PD materials. We also provide ideas and examples from the Learning and Teaching Geometry PD materials.

Conceptual Framework: Teaching Noticing in Professional Development

Mathematics teachers come to PD workshops with varying levels of knowledge, much like the K-12 students who come to their mathematics classrooms. One unique aspect of teachers' knowledge is their "professional vision," which refers to their ability to notice and analyze features of classroom interactions, make connections to broader principles of teaching and learning, and reason about classroom events (van Es & Sherin, 2002; Sherin, 2007).

Over the years, diverse conceptions of noticing have emerged in the literature, but in general, most considerations of mathematics teacher noticing involve two main processes: (a) Attending to particular events in an instructional setting (i.e., teachers choose where to focus their attention and for how long) and (b) Making sense of events in an instructional setting (i.e., teachers draw on their existing knowledge to interpret what they notice in classrooms). Sherin, Jacobs, and Philipp (2011) argued that these two aspects of noticing are not discrete, but rather interrelated. Teachers attend to events based on their sense-making and the way they interpret classroom interactions influences where they choose to focus their attention.

The conceptual frame of noticing is relevant to the design of video-based PD aimed at providing coherent, incremental learning opportunities. It is well established that teacher education programs incorporating video foster the development of teachers' noticing skills (Koellner & Jacobs, 2014; Roller 2016; Santagata & Yeh 2013; van Es & Sherin, 2002). As they attend to and make sense of PD focused on cases of instruction, teachers are also likely to consider the implications for their own improvement of practice (Koh, 2015). In other words, what teachers notice appears directly relevant to the way they elect to carry their learning into their classrooms (Sherin & van Es, 2009).

Participants in PD do not all make sense of their experiences in the same way; rather, individuals bring differing knowledge and beliefs about teaching and learning, students, content, and curriculum to bear on what they notice (Erickson, 2011; van Es, 2011). This individual diversity impacts what they notice, how they engage in the professional development, and what they take and use in their own practice. It also has implications for the purposeful design of video-based PD and teacher education (Hatch, Shuttleworth, Jaffe, & Marri, 2016).

Purposeful Selection of Video Clips

Video-based PD is premised on the notion of using a concrete, authentic artifact of practice to examine teaching and learning. For the developers of video-based PD materials, the purposeful selection of video clips is central to ensuring that teachers will have a meaningful learning experience. A number of researchers have provided insight and guidance regarding how to select and use video clips for a variety of PD purposes — including community building, meeting specific learning goals, and promoting noticing and reflection.

For example, at the beginning of a long-term PD effort, clips may be selected with an eye toward building a strong professional community and supporting teachers to learn reflective and analytic skills (Borko, Jacobs, Eiteljorg, & Pittman, 2008, van Es, 2012a; Zhang et al., 2011). Later, clips may be purposefully chosen to guide teachers toward one or more identified learning objectives, such as to understand more deeply student thinking, to become more cognizant of specific instructional moves, or to learn targeted subject matter (Borko et al., 2015; Kersting, Givvin, Sotelo, & Stigler, 2010; Roth et al., 2011).

Several articles provide detailed information regarding the selection of video clips for mathematics PD settings. For example, Sherin, Linsenmeier, and van Es (2009) identified three dimensions of video clips that promote productive discussions of student mathematical thinking: (a) the degree to which the clip offers a window into student thinking, (b) the depth of student thinking shown in the clip, and (d) the clarity of student thinking in the clip.

Seago (2004) offered suggestions with respect to selecting clips based on content (it should be “mathematically important”), reality (“believable” to teachers), and length (not more than 6 minutes). This type of information is critical for PD material developers and facilitators who must make choices about what video clips to include in their program or show teachers during workshops. Moreover, this guidance is likely to prove helpful across a variety of PD efforts based on video — including those that use video from teachers’ own classrooms or from other teachers’ classrooms.

Embedding Video Within the Broader PD Program

Video clips, by themselves, are unlikely to foster teacher learning without being intentionally integrated into a PD program or course (Blomberg, Sherin, Renkl, Glogger, & Seidel, 2014). Along with the purposeful selection of video clips, a central component of designing effective PD materials is determining how to embed the video within the broader curriculum. It is essential to situate the video in a framework that supports detailed analysis and interpretation, thereby providing access and opportunities for teacher learning across the totality of the PD experience. Both the video and the activities around the video should be designed to target predetermined learning goals for the PD curriculum as a whole and each individual workshop or class session (Blomberg Renkl, Sherin, Borko, & Seidel, 2013).

Many, but not all, video-based mathematics PD programs are structured such that teachers engage in specific activities before and after watching the focal video (e.g. Borko et al., 2015; LeFevre, 2004; Santagata 2009). For example, prior to watching a clip, PD facilitators may ask teachers to solve and discuss the math problem shown in the video in order to develop content knowledge, motivate teachers to notice particular elements of the content contained within the clip, and attend to specified activities, such as a unique solution method or teacher questions that prompt extended student reasoning. After viewing the video, there may be a facilitator-led discussion and, perhaps, follow-up activities in which the teachers relate what they have seen on the video to their own classroom practice. The discussion and follow-up activities extend teachers’ thinking and analysis by probing more deeply into topics or issues presented within the video.

We label this type of intentional sequencing of video viewing that occurs between designated activities with specified learning goals a “video in the middle” design. In video-based mathematics PD that incorporates this design feature, video is located in the middle of the learning experience, sandwiched between activities such as mathematical problem-solving and pedagogical reflection.

Our use of this sequence as part of a larger set of curricular materials will be described in more detail in the discussion of the Learning and Teaching Geometry project, which is the focus of this paper. An illustrative vignette will be included from a workshop in which teachers were engaged with the Learning and Teaching Geometry PD materials to depict how a specific sequence looks in action. This design feature is not new to PD, but our goal is to highlight and label it and discuss how the design is likely to support teachers' learning.

The Importance of Knowledgeable Facilitation

Even in a carefully designed PD program that includes purposefully selected video and intentionally sequenced pre- and postvideo activities, knowledgeable facilitation is essential to foster a productive learning environment. Although viewing video is, by nature, a self-directed exercise, in which teachers attend to topics of their own interest and construct personally relevant knowledge (Ebsworth, Feknous, Loyet & Zimmerman, 2004), the importance of a facilitator who asks guided questions, uses observation protocols, and provides targeted viewing tasks is consistently emphasized in the research literature (Baecher, Rorimer, & Smith, 2012; Borko, Koellner & Jacobs, 2011; Groschner, Seidel, Pehmer, & Kiemer, 2014). Skillful facilitation helps to focus teachers' attention in order to identify, interpret, and reason about what they see in the video (Gaudin & Chaliès, 2015; van Es & Sherin, 2008). Without such guidance, research indicates that teachers may focus on more superficial features of the video clip and neglect to make meaningful insights into practice (Calandra, Gurvitch & Lund, 2008; Harford, MacRuairc & McCartan, 2010; Laycock & Bunnag, 1991).

Attending carefully to student thinking as seen in video clips, in particular, has been identified as challenging for mathematics teachers (van Es, 2012a). Van Es (2012b) argued, "It is important to note that it is not natural for teachers to attend to the particulars of student ideas.... Thus, teachers need to learn how to problematize student thinking and develop discourse practices for engaging in this work" (p. 104).

Pressing teachers to incorporate details from the video into their discussion has been identified as a facilitation move during video-based PD that encourages teachers to reason about student thinking, promotes an inquiry stance, and deepens mathematical thinking (van Es, Tunney, Goldsmith, & Seago, 2014; Zhang et al., 2011). At the same time, video enables a shared space within which teachers and PD facilitators can work with "interpretive flexibility" (Star, 2010), enabling multiple viewpoints and ideologies to surface from multiple parties (Miller & Zhou, 2007). The main point here is that a video in the middle design does not stand on its own to support teacher learning; it depends heavily on the application of appropriate and effective facilitation moves.

Learning and Teaching Geometry PD Materials

The Learning and Teaching Geometry (LTG) materials (Seago et al, 2017) use video as a centerpiece in PD workshops. The LTG materials are designed to improve the teaching and learning of mathematical similarity based on a robust understanding of geometric transformations through engagement in a series of five modules (18 three-hour sessions). The sessions follow a specified learning trajectory and offer access to specific and increasingly complex mathematical concepts that are presented within the dynamics of classroom practice (Seago, Driscoll & Jacobs, 2010).

Each session contains at least one videocase, constructed using a video in the middle design. This use of this design was based on our conjecture that viewing and discussing the video footage required a highly intentional surrounding framework in order to meet the goals specified within the LTG materials' learning trajectory. These videocases form the backbone of the materials and serve as a holistic basis for supporting learning from representations of practice (Seago et al., 2017).

All of the video clips in the LTG materials are unedited segments selected from real classroom footage of 15 teachers' unstaged mathematics lessons, representing a range of grade levels, geographic locations, and student populations across the United States. The clips offer a window into a variety of issues related to content, student thinking, and pedagogical moves. By focusing on classroom video from across multiple and varied contexts, the materials provide insight into what an emerging understanding of similarity looks like as well as specific instructional strategies that can foster this understanding.

A field test of the LTG materials was conducted in 8 sites throughout the United States in order to generate both formative and summative data. The field test took place over the 2010-11 and 2011-12 school years and involved 126 participants (87 treatment teachers and 39 comparison teachers), including in-service and preservice teachers, teacher leaders, and mathematics coaches.

Three pre/post instruments were used to examine impacts of the LTG Foundation Module on teachers' mathematical knowledge for teaching related to geometric similarity: a content assessment and two sets of embedded assessments. On each instrument, the teachers who take part on LTG workshops demonstrated significant knowledge gains, whereas the comparison teachers did not. In addition, students of the treatment teachers demonstrated larger gains on a content knowledge assessment compared to students of the comparison teachers (Borko, Jacobs, Seago & Mangram, 2014; Seago et al., 2013). These field test results offer evidence of the promise of the LTG materials for achieving the intended learning outcomes for both teachers and their students.

Video in the Middle Design Within the LTG Materials

The LTG materials purposefully utilized a video in the middle design. The larger materials development process actually began well before the collection of video (Seago et al., 2010). The first stage of this larger process included determining the content to be covered and generating a mathematics learning trajectory to ensure the materials would support a vetted sequence of learning goals for teachers. This initial stage was followed by developing a set of mathematical tasks that aligned with the targeted topic areas. The teachers who were selected for videotaping either used one of these project-generated tasks or a task from their curriculum that was determined by the developers to be an appropriate fit for the materials.

After filming the lessons, the LTG materials development team selected video clips that highlighted critical math content within the context of authentic instructional practices and students' ideas about the content. Once clips were selected, activities were generated and sequenced to frame the viewing to support the trajectory of teacher learning goals.

Each video clip, combined with the framing of pre- and postvideo activities, constitutes a videocase. Last, the videocases were organized into 3-hour PD sessions. After this process was completed for each module (or set of sequenced sessions), the module went through multiple rounds of piloting and revision.

Although it is situated in the middle of the user experience, the video clip is, in fact, the primary ingredient from a design perspective, serving as a focal point of the videocase. Once the video clip has been selected, activities can be designed around it to ensure that teachers will engage deeply with the targeted mathematics content, instructional components, or student thinking depicted in the clip. The activities surrounding the video also serve as transitions to and from other activities (or videocases) within a given PD experience to ensure an appropriate flow and integration across the learning trajectory.

In the LTG materials, the activity that most commonly comes before watching a video clip is working on and then discussing a mathematical task that is relevant to the clip. Solving the same task as the students in the video allows teachers to develop an adequate understanding of the mathematical demands faced by the students and helps them to better engage with and then interpret the student thinking and the pedagogical moves captured by the video clip. In some videocases, the pre-activity may prompt teachers to make predictions about how students will solve the problem or consider the types of mistakes they think students might make. The assumption behind this type of activity is that teachers need a period of time to become sufficiently immersed in and familiar with the mathematics content they are about to see, so that they can readily follow the pertinent (and perhaps subtle) issues that arise in the video episodes.

Video clips were selected for inclusion in the LTG materials based on the expectation that they would provoke extended and productive discussion within a PD setting. Some clips contain challenging mathematics content, a conceptual hurdle, student misunderstanding, or interesting pedagogical moves.

As they move into postvideo activities, facilitators of the LTG materials are encouraged to promote a culture of inquiry and reflection, and support teachers to offer alternative and dissenting viewpoints (as in Little, 1993). For example, the materials suggest that facilitators use a language of tentativeness, soften teachers' definitive statements, and model a stance of wondering, questioning, and inquiry (LeFevre, 2004; Seago, 2004). Facilitators can probe teachers for evidence of their claims, seeking multiple perspectives, and refraining from judgment, as appropriate. In addition, the materials urge facilitators to model language use and instructional behaviors that are respectful of the teachers and students shown in the video clips.

Postvideo viewing activities in the LTG materials include careful analysis of the ideas presented in the video clip, considering how those ideas apply in different mathematical contexts, discussing the pedagogical strategies that were contained within the video clip, and reflecting on how teachers can apply their emerging insights to make improvements to their own lessons. Certainly not all of these topics are discussed during each videocase, but they are generally part of each session. The materials contain guiding discussion questions for facilitators, but they are also free to improvise based on their understanding of the teachers' needs and interests (Jacobs, Seago, & Koellner, 2017).

The Randy Videocase: A Vignette

In order to illustrate what the video in the middle design looks in practice, we created a vignette based on one group of teachers who participated in the LTG in-service PD. These teachers were part of a study seeking to demonstrate the efficacy of the LTG materials. Twenty-four teachers attended the workshop from which the vignette is drawn, which was conducted in August 2016. All of the teachers were practicing secondary mathematics teachers in a large, urban school district on the East Coast of the United States. The workshop was videotaped, and the vignette is based on our repeated viewing of this videotape along with our field notes. Although some license has been taken with the dialogue (e.g., some utterances are written verbatim while others are summarized or edited for clarity), all of the teachers' activities and conversations are described as literally and accurately as possible.

The facilitator of the workshop, Hannah, was an experienced mathematics teacher, PD facilitator, curriculum developer, and university professor. Hannah took part in an extensive training led by the LTG materials developers and demonstrated fidelity in her use of the materials (Jacobs, Seago & Koellner, in press). In addition, Hannah's eighth-grade mathematics class was videotaped as part of the LTG project, and video clips from her class were incorporated into the materials. One of these video clips is the focus of the vignette.

The vignette is set in the beginning portion of Session 3 of the LTG materials, which for these teachers was the second morning of a 5-day (10 session) PD workshop (see Figure 1). During the prior two sessions, the teachers explored geometric transformations based on rigid motions (rotation, reflection, and translation).

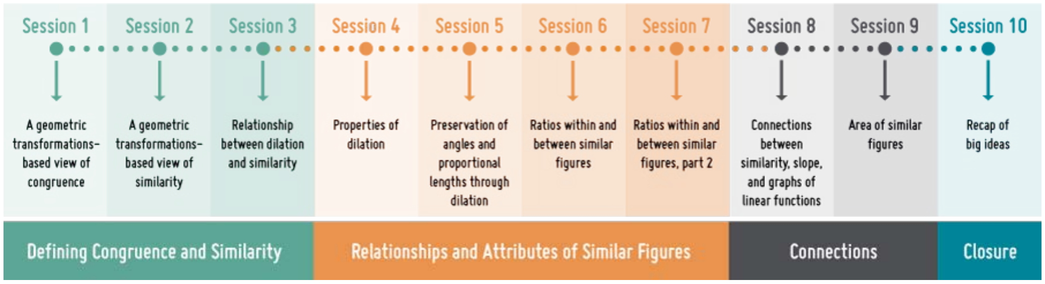

Figure 1. *Mathematical trajectory of 10 sessions in the LTG materials.*

In Session 3, they were beginning to explore another transformation, dilation, in depth. Dilation is a similarity transformation that is generally less familiar to both teachers and students in the United States, as it has not previously been a part of most K-12 mathematics curriculum or standards (Teuscher, Tran & Reys, 2015). However, dilation is now a central part of the geometry standards in the Common Core State Standards for mathematics (Seago et al., 2013).

A key goal of Session 3 is to provide teachers with opportunities to gain an understanding of dilation and the fluency to make instructional decisions that support students' learning of dilation. The teachers continue to explore various aspects and applications of dilation in the remaining 7 sessions of the module.

Overview

The following vignette, based on what we call the Randy Videocase, is an example of how the video in the middle design can play out with in-service mathematics teachers using the LTG materials. The Randy Videocase and video are publicly available part of a set of free PD materials focused on illustrating the Standards for Mathematical Practice.

At the core of this videocase is a clip showing a student, Randy, solving the Rectangle Problem mathematics task (see Figure 2). This task was part of Hannah's eighth-grade curricular materials, which were designed to promote a transformations-based approach to similarity [a]. The Rectangle Problem asks students to examine a set of rectangles and decide which ones are similar. Prior to being videotaped, the students in Hannah's eighth-grade class had a number of experiences working with geometric transformations, including dilation.

The video of Randy was purposefully chosen to highlight his use of dilation as a tool for solving the given similarity problem. The activities that were designed to occur before and after viewing the clip encourage teachers to explore various mathematical approaches to determining the similarity of rectangles and to investigate how and why dilation works.

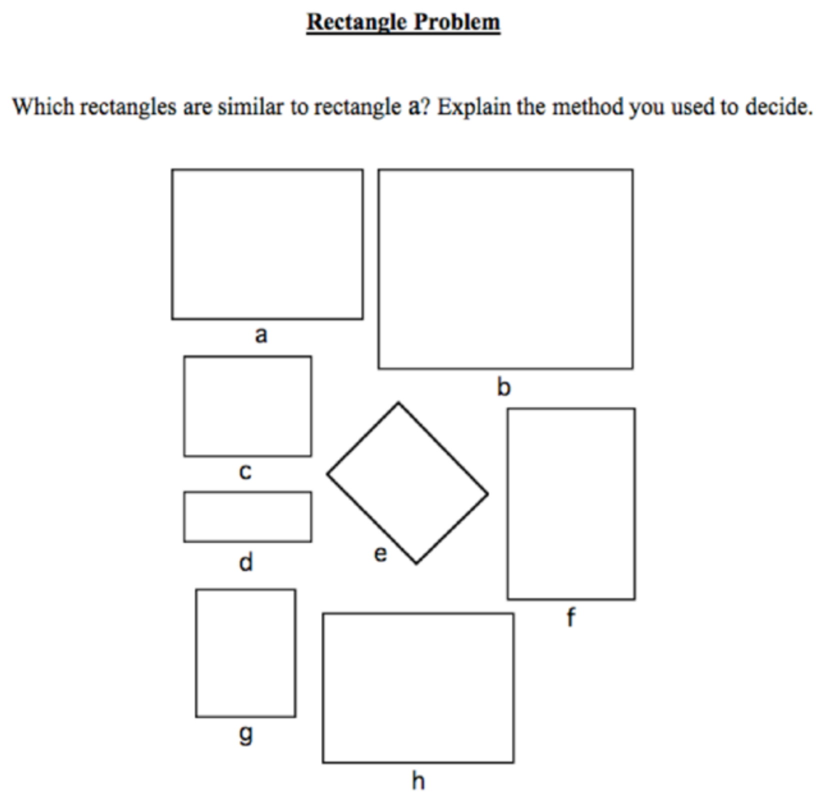

Figure 2. *The Rectangle Problem*.

As they engage in the Randy videocase, teachers are likely to learn how dilation can be used to solve similarity problems in general, how tracing paper can be used as a tool for creating a dilation and determining similarity, and how attending to and unpacking student thinking can support pedagogical content knowledge.

In terms of the mathematical practice standards in the Common Core State Standards, these professional learning experiences are designed to help teachers to better support their own students to engage in reasoning that involves dilation, to use precise mathematical language in their explanations, and to make use of geometric structure given problems without numeric values.

In the Randy Videocase, the prevideo activity prompts teachers to work on the Rectangle Problem and then share a variety of solution methods (see Figure 3). The learning goal for this activity is to explore different mathematical approaches to solving a similarity task. The design assumption is that engaging with and discussing the task prior to viewing Randy's video will help teachers (a) develop a deeper and more connected understanding of the content and (b) analyze and interpret Randy's thinking in accurate and substantive ways. Furthermore, based on piloting, our materials development team learned that many teachers do not use Randy's dilation strategy when initially presented with the Rectangle Problems, and they are often surprised to see a middle school student applying dilation to solve the task with fluency.

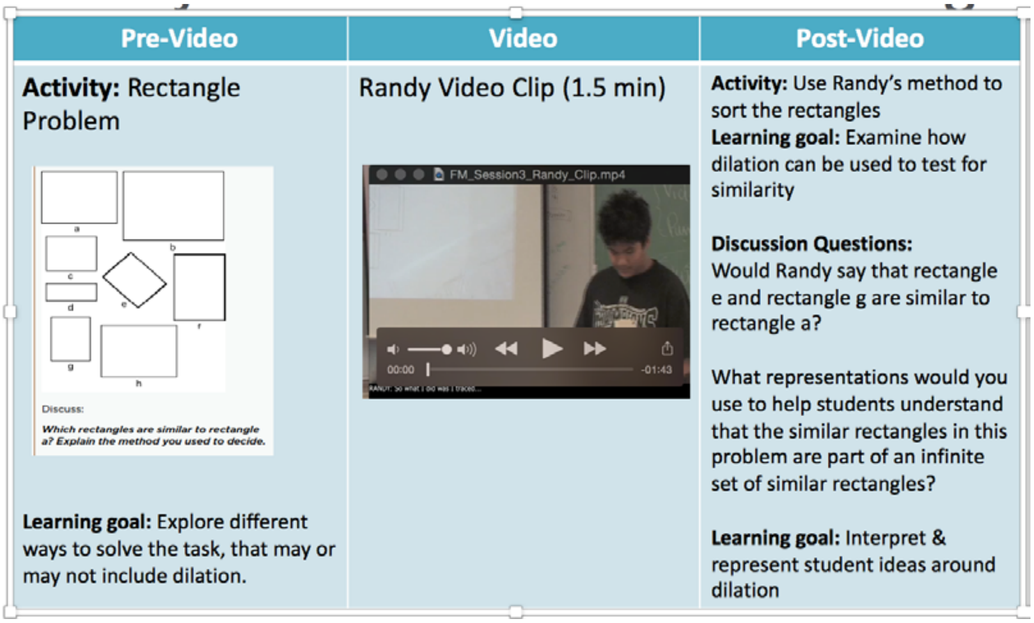

Figure 3. *The Randy Videocase: An example of the video in the middle design.*

As a postvideo activity during the Randy Videocase, teachers are asked to use Randy's dilation strategy and then engage in a discussion of this method, including his use of patty paper (a form of tracing paper). This activity was designed with two learning goals in mind: (a) to interpret and represent a student's idea involving dilation and (b) to examine how dilation can be applied as a mathematical test for determining similarity across figures. These two learning goals are at the intersection of content and pedagogy; as such, they represent an opportunity to learn the type of knowledge identified as highly critical to effective mathematics instruction.

This knowledge has been labelled "mathematical knowledge for teaching" and can be understood as the knowledge teachers need to effectively carry out the work of teaching math to students (Ball & Bass, 2000; Ball, Hill & Bass, 2005; Ball, Thames & Phelps, 2008). Research has shown that gains in mathematical knowledge for teaching improves the quality of teachers' classroom work and positively predicts gains in their students' mathematical achievement (Hill, 2010; Hill, Rowan, & Ball, 2005).

As part of the Randy Videocase, working through the sequence of the prevideo activity, video viewing, and the postvideo activity provides teachers with the opportunity to use and interpret various methods for solving a similarity task, as well as to explore and unpack a specific students' dilation method. As a whole, the experience promotes conversation and reflection on how to teach challenging content in a manner that supports student learning, thereby fostering mathematical knowledge for teaching. The videocase is followed by more videoclips (and videocases) from the same group of students in Hannah's class, as they continue to explore the use of dilation in other geometric contexts.

Prevideo Activity and Conversation

In the video Hannah, the facilitator, welcomes the teachers to the second day of their 5-day PD workshop using the LTG materials. She explains that they are going to begin with a prevideo activity before viewing a video clip selected from her eighth-grade math class.

Hannah instructs the group:

> We are going to try another math task. The LTG materials call this task, Hannah's Rectangle Problem. That is because I used this task with my eighth-grade class in Hawaii, and they filmed me. You will be able to see a clip from my class after we have worked and discussed this problem as a group. Why don't you take 10 minutes or so to work with a partner or in small groups to solve The Rectangle Problem.

Hannah walks around the room while the teachers collaboratively engage in the task. A materials table is set up in a corner of the room with a variety of tools that teachers can access throughout the workshop, including straight edges and rulers, compasses, and patty paper. Some teachers choose to use pencils and straight edges (or rulers) while others select patty paper. Once most teachers appear to be finished working on the problem, Hannah brings the group together:

Hannah: There were a lot of great conversations happening. So let's share some of your ideas as a group. Which rectangles are similar to Rectangle A?

George: I crossed off Rectangles D, F, and G because I don't think they are similar.

Hannah: Why do you think they're not similar? What method did you use?

George: I took the ratio of length and width, but I just used a visual check.

Hannah: So you were looking at the within ratio. Did you measure or did you just look? What were you looking for?

George: I didn't measure it. I was comparing the two sides visually.

Hannah: So you determined that they weren't similar by inspection. That is a strategy I heard some other tables use. Let's see, how many of you threw out Rectangle D based on visual inspection? [Several hands go into the air.] Yes, that is an appropriate method, and in this case you can tell just by looking that certain rectangles are not similar to Rectangle A. That is a good method, but sometimes but it can also fool you, for example, when things are turned. Did someone else use a different method?

Susana: When the rectangles were oriented the same way, I tried to connect corresponding vertices.

Hannah: [Encourages Susana to show her paper to the group.] Can you come show us?

Susana: [Takes her paper and goes up to the document camera.] I'll show you one.

Susana begins to explain how she compared Rectangles A and G by drawing lines connecting the vertices from one rectangle to the other, and then determining where those lines intersect. However, Susana's paper contains many lines, leading John to comment, "I don't get it. Can you start again using a blank worksheet? Susana starts over with a new paper, drawing lines and explaining her strategy as she works (Figure 4):

> I drew lines through the corresponding vertices of rectangles that had the same orientation, for each of the four vertices. If I do it perfectly, you can see they all intersect at this common point. So this process creates a center of dilation.

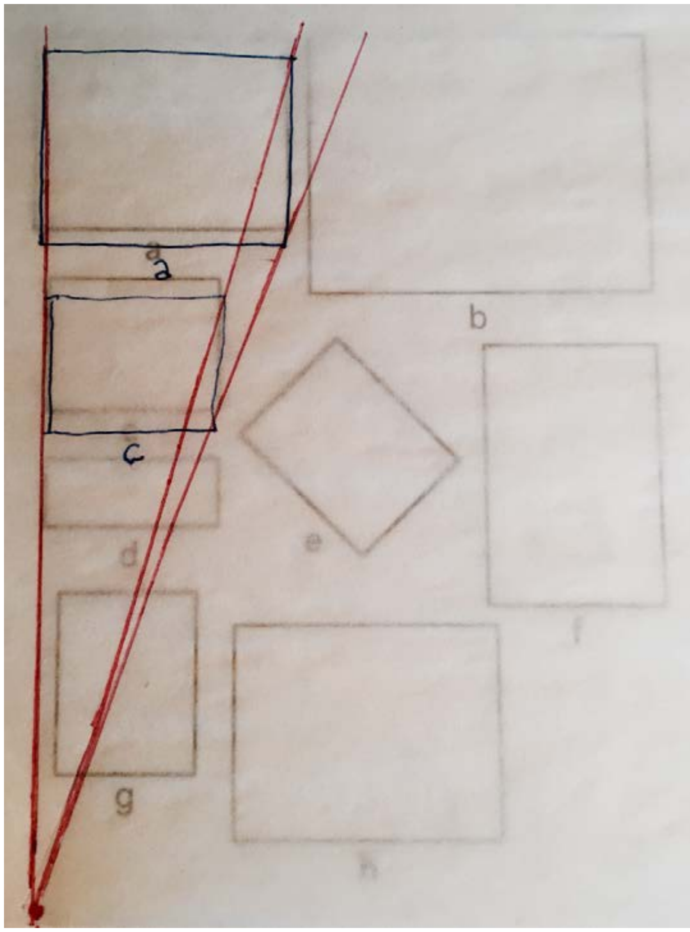

Figure 4. *Susana's method.*

A teacher questions Susana, "I understand your method, but how can we use it for the rectangles that are not oriented the same way?"

Susana responds:

> I was thinking about that and I could probably use tracing paper to orient Rectangle F the same way as Rectangle A. Then I could check if the lines would intersect at the same point. But I'm also wondering if there's another way to do it.

Several other teachers begin considering how they can compare rectangles when the rectangles are not oriented the same way — including doing a visual inspection, measuring, or rotating them and drawing lines, as Susana suggested.

Tiffany goes back to Susana's method and points out, "This method only works if the lines have the same scale factor between the two shapes. Because you could put rectangles anywhere on the lines, but if distances between them aren't constant they wouldn't necessarily be similar."

There is a bit of discussion around this idea and Hannah comments, "We'll see more of that. Keep that in mind." Violet jumps in at this point and goes up the document camera. Using a blank worksheet she tells the group,

> When you dilate, you extend the line for all the vertices. So, for Rectangle A, I used the top left corner as the center of dilation. I used tracing paper and drew dilation lines from the top left corner through the other three vertices (Figure 5). Then I took those lines and placed them next to the other rectangles, one by one, and checked to see if the vector was the same for the each of the rectangles. I just moved my tracing paper over each rectangle, turning it if I needed to, and I could see which ones worked. I knew when the dilation lines matched up the rectangles would be similar.

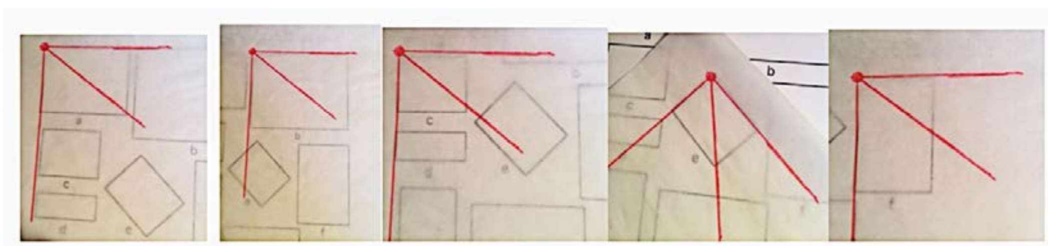

Figure 5. *Violet's method.*

During Violet's demonstration, a number of teachers make comments such as "Wow," and "Oh, I see." However, some teachers appear surprised and a bit puzzled by her strategy.

Hannah notes casually, "Ok. Well, we'll see that." Scott admits, "Wait, I don't understand. I am so lost." Instead of talking further about Violet's solution method, Hannah says, "I think rather than talking this through now, let's watch the video."

Scott smiles and remarks, "Oh great...of sixth graders solving the problem!" The class laughs and Hannah tells him, "Well they are eighth graders." There's more laughter, but Hannah adds in a serious tone, "I won't forget your question, Scott. We will definitely get more ideas and questions on the table and we'll address them."

As part of the prevideo activity during the Randy Videocase, teachers used the opportunity to explore the mathematics within the Rectangle Problem and shared a variety of methods to determine similarity among rectangles, several of which involved the application of dilation. This activity served to familiarize the teachers with the solution method they will be seeing in the video, while at the same time considering other potential methods and raising questions about the math content. Testing for similarity by drawing dilation lines is an unfamiliar strategy for several teachers in the group, and not everyone was able to follow the conversation and representations related to this topic. However, Hannah intentionally left some important questions on the table for the teachers to pursue in greater depth after watching the Randy video, as well as later in the PD workshop.

Video Viewing

Directly following the prevideo activity, the teachers watch a 1 m 45 s, video clip of Randy explaining how he solved the Rectangle Problem. As depicted in Figure 6, Randy first uses patty paper to trace Rectangle A. Then he takes the traced rectangle and moves it on top of Rectangle B, making sure the top left vertices are aligned. Randy tells the class that he used this top left vertex as the center of dilation, and drew lines from the center of dilation through all of the corresponding vertices. Because all the sides line up, and the rectangles share a common diagonal, he concludes that Rectangles A and B are similar.

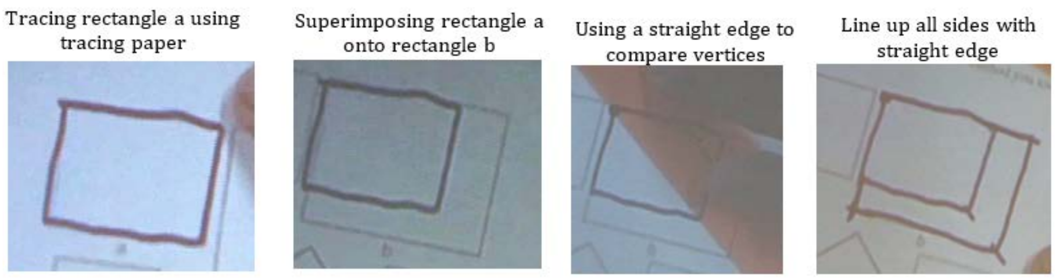

Figure 6. *Randy's method.*

Randy starts to use this method with another similar rectangle when his teacher, Hannah, asks him to show how the method looks when the rectangles are not similar. He moves his traced version of Rectangle A onto Rectangle D and again lines up the top left vertices. Drawing lines through the corresponding vertices, Randy demonstrates that the diagonals for Rectangles A and D are not in the same location; therefore, they are not similar.

The Randy video clip was selected to help teachers more deeply understand how dilation lines can be used to test for similarity of rectangles. Interestingly, the solution method that Violet shared during the prevideo activity is similar to the method used by Randy. Both drew Rectangle A on tracing paper and then overlaid the traced version of Rectangle A onto other given rectangles. They both used the top left corner of Rectangle A as the center of dilation and then constructed dilation lines. Furthermore, both Randy and Violet articulated that any rectangles that fell within those dilation lines would be mathematically similar.

By engaging in the preactivity and the video viewing, teachers were exposed twice to the accurate use of dilation in the context of The Rectangle Problem. Although teachers' solution methods certainly will vary across PD participants, in this case, the double exposure was likely to be particularly helpful for teachers like Scott, who did not understand how to use dilation lines based on Violet's initial explanation. Viewing the video of Randy provided him with a second opportunity to observe how dilation concepts can be applied to solve the problem. However, both Violet and Randy provided incomplete verbal descriptions of their work, which meant that more careful unpacking would be required to ensure the details of the approach were documented and well understood. The postvideo activity provides yet another opportunity for teachers to examine the dilation method and then to extend the mathematics behind this method.

Postvideo Analysis and Conversation

At the conclusion of the Randy video clip Hannah poses the questions, "What did Randy do? What was his method, in your own words?" Some teachers can readily describe what Randy did and attach mathematical language to his drawings. One teacher notes that she tried using Randy's method and found it much easier than her original method of measuring and comparison side lengths. However, Scott reiterates that he is still having trouble understanding the specifics around how and why dilation can be used to determine similarity between rectangles: "I don't understand why he was able to choose that point of dilation, and why the lines show similarity just because they go over the other shapes. I just don't get it."

Hannah records these questions on the white board, clarifying that Scott is asking about both the center of dilation and the lines of dilation. He continues, "I understand dilation when the lines are double or triple from the point of dilation, but that's not necessarily true here." Another teacher attempts to explain, "You can extend the lines of dilation indefinitely. So it forms a ray, and you can create an infinite number of similar rectangles." Hannah clarifies, "Right. You can extend the lines to create vectors."

Hannah shifts the conversation by asking the teachers how they might help their own students understand this method, and specifically what representations or tools they would use in their classrooms. Jason responds that he would have students look at pictures on the computer, and see that you can use dilation to enlarge them: "Usually you go from the top left to the bottom right corner. And if you extend that diagonal line, you are enlarging your picture. The computer creates that vector, and you can extend it."

Hannah follows up, "Right, if you don't use the diagonal it won't be proportional. You need to click on the diagonal to ensure all of the lines increase proportionally. That's a nice connection."

Hannah continues to encourage the teachers to identify the mathematics behind Randy's dilation approach by asking, "What mathematics does Randy's method draw on? There are mathematics principles that underlie his strategy, that explain why it works. What are some of these?"

Teachers suggest a number of possibilities, such as ratio and proportion, slope, scale factor, and angle-angle similarity. The teacher who mentions angle-angle similarity describes a proof that involves decomposing rectangles into triangles and using parallel lines and a transversal to demonstrate that corresponding angles are congruent.

Hannah concludes the discussion by noting that although she used the Rectangle Problem with her middle school students, high school teachers can use tasks like that one to lead into more sophisticated mathematical content with their students. She then moves to the next phase of the workshop, which continues to investigate properties of dilation.

In the discussion following the viewing of Randy's video clip, the teachers explored the mathematics behind Randy's method and attempted to articulate more precisely why dilation works to determine similarity. Randy's method provides a window into the idea that an infinite set of similar figures results from dilation. Specifically, similar rectangles can be formed along the horizontal and vertical lines of dilation, which are themselves an infinite collection of points. Each similar rectangle can be constructed from four points: the common center of dilation and one unique point on each of three lines of dilation. Furthermore, the set of similar rectangles share a common diagonal.

This postvideo activity provided a third opportunity for teachers to engage their prior knowledge and construct new knowledge about dilation lines and similar rectangles. Although Hannah documented Scott's questions and several teachers tried to address them, it seems clear that he (and perhaps other teachers) still lacked a solid understanding of dilation at the conclusion of the Randy Videocase. However, the LTG materials continue to pursue the relationship between similarity and dilation, and upcoming videocases target learning goals that should help to address Scott's concerns. By experiencing multiple and purposefully sequenced videocases, all of which are designed with video in the middle of particular learning experiences, the overaching goal of the LTG PD is for teachers with different levels of knowledge and skills to gain mathematical knowledge for teaching through skillfully facilitated and socially-mediated interactions with the materials.

Conclusion

The LTG PD materials contain a number of intentional design features, such as the use of video within highly collaborative environment, a detailed teacher learning trajectory, and extensive resources to promote effective facilitation (Jacobs, Seago & Koellner, in press; Seago et al., 2010). In this paper, we described the video in the middle design feature, which includes viewing of carefully selected video clips from teachers' classrooms sandwiched between pre- and postviewing activities that are expected to support teachers' noticing, engagement in, and learning from the video. The three elements of the learning in the middle design taken together comprise a videocase. Videocases can then be further sequenced to create a specified PD curriculum, as they are in the LTG materials.

Video clips are the key ingredient in video-based PD broadly, and in the video in the middle design, more specifically. Purposeful selection of each video clip allows for coherence between the prevideo, video viewing, and postvideo activities, which in turn, supports the link between a given videocase and identified teacher learning goals.

Incorporating a video in the middle design within a video-based mathematics PD environment can promote a detailed and focused examination of complex mathematical content, the relationship between pedagogical decisions and practices, and an unpacking of students' mathematical thinking. This type of experience, in which teachers observe and study the complexity of the mathematics that emerges from classroom interactions in a purposeful and sequenced manner, targets and has the potential to significantly improve teachers' mathematical knowledge for teaching (Ball et al., 2005; Ball, Lubienski, & Mewborn, 2001), a broad goal of mathematics PD and teacher education efforts, in general.

Facilitators play a major role in video-based PD, and the effective application of the video in the middle design is, in large part, dependent on skillful facilitation. Facilitators of PD curriculum such as the LTG materials are expected to become familiar not only with the content and resources, but also with the learning goals and theoretical underpinnings in order to enact the curriculum as intended by the developers (Koellner & Jacobs, 2015). The video in the middle experiences built into the LTG materials are intended to be used by facilitators to provide opportunities for collective inquiry, to support teachers' reflections on their own instructional practice, and to address specified content and pedagogical goals.

Last, while we have largely focused our discussion of the video in the middle design in relation to a particular type of video-based PD (that is, mathematics in-service PD), it can be a useful design feature applicable across a variety of content areas as well as in PD geared toward preservice teachers (Star & Strickland, 2008). In fact, in the development phase of the Learning and Teaching Geometry materials, three of the eight pilot sites were preservice (university) settings. The mathematics teacher educators who used these materials within their existing courses found the materials promoted the intended learning goals in a motivating and interactive environment. There are certainly more nuanced aspects of the video in the middle design that could be further elaborated and theorized, perhaps most effectively by a thorough consideration of this approach across PD types and purposes.

End Notes

[a] The Rectangle Problem is part of the curriculum materials created by the Curriculum Research & Development Group, University of Hawaii at Mãnoa, http://www.hawaii.edu/crdg/curriculum/

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