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Low noise electro-optic comb generation by fully stabilizing to a mode-locked fiber comb

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Abstract: A fully stabilized EO comb is demonstrated by phase locking the two degrees of freedom of an EO comb to a low noise mode-locked fiber comb. Division/magnification of residual phase noise of locked beats is observed by measuring an out-of-loop beat. By phase locking the 200 th harmonics of the EO comb and a driving cw frequency to a fiber comb, a record low phase noise EO comb across +/− 200 harmonics (from 1544.8 nm to 1577.3 nm) is demonstrated.

OCIS codes: (140.3425) Laser stabilization; (060.3510) Lasers, fiber; (060.2330) Fiber optics communications.

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1. Introduction

Optical frequency combs have been applied to many fields such as optical atomic clocks [1,2], lidar [3], biology [4], microwave generation [5], and astronomical calibration [6]. Although initially optical frequency combs were generated from mode-locked cavities (ML comb), electro-optic combs (EO comb) have recently attracted attention because they are cavity-less, and have tunable comb mode spacing. Especially, comb spacings of more than 10 GHz are very important for applications such as wavelength division multiplexing (WDM) coherent communication [7,8], microwave photonics [9,10], and astronomical calibration [11]. Indeed recently, EO combs were applied to astronomical calibration [11], and Tbit/s coherent communication systems [8].

EO combs are generated by phase/intensity-modulation of a cw laser, producing side modes with mode-spacing of f_m, centered at f_{cw}. Here, f_m and f_{cw} are the modulation, and the cw laser's optical frequency, respectively. Then, each comb mode is represented as $f_{n(EO)} = f_{cw} \pm nf_m$, where $f_{n(EO)}$ is the frequency of the integer ± n-th EO comb mode. The number of side modes is determined by the applied RF power into electro-optic phase modulators. To broaden the optical bandwidth, nonlinear spectrum broadening via highly nonlinear fiber (HNLF) [12] can be employed. Very recently, octave-spanning EO combs enabling self-referencing [13] and very flat 100 nm spanning EO combs for Tbit/s coherent communication were demonstrated [8].

When optical frequency combs are used as an ultra-precise gear to reference optical and microwave frequencies to each other, the phase noise of the comb modes should be considered. Comb modes from ML combs can be represented as $f_{n(ML)} = f_{ceo} + nf_{rep}$, where f_{ceo} is the offset frequency and n is integer. Here, n is about 10^6, whereas for EO combs n starts from zero. When using mode-locking and large bandwidth modulators for phase locking of f_{ceo} and nf_{rep}, the phase noise of $f_{n(ML)}$ can reach below −90 dBc/Hz despite the extremely large n [14,15]. On the other hands, although n for EO combs is much smaller than n for ML combs, the phase noise of $f_{n(EO)}$ is typically larger than that of $f_{n(ML)}$ because of the larger phase noise of f_m, which scales to n^2 for $f_{n(EO)}$ [16]. For example, standard commercial RF synthesizers with a 10 GHz carrier have a phase noise of about −120 dBc/Hz at 10 kHz – 100 kHz offset from the carrier, which scales to −60 dBc/Hz for the 1000 th harmonics of the EO comb spacing. Moreover, the phase noise of $f_{n(ML)}$ is much lower than that of $f_{n(EO)}$ at low frequency offset, once ML combs are stabilized to a cw laser which is referenced to an ultra stable optical cavity [5]. The phase noise magnifinication of the RF oscillator driving EO combs can be used to generate low noise RF generation [17,18].While the modulation format for WDM coherent communication by EO combs is limited to around 64 QAM over 50 km for each carrier [8], 2048 QAM over 150 km has been demonstrated when using narrow linewidth cw lasers for both the transmitter and the receiver ends [19], which indicates low phase noise EO combs would be necessary for WDM coherent communication with complex modulation formats.

In this work, we demonstrate phase locking of an EO comb to a fiber comb to overcome the phase noise limitations of the RF source of the EO comb. f_{cw} and nf_m are phase locked to selected fiber comb teeth, while measuring an out-of-loop beat between the EO comb and the fiber comb. In our recent conference paper [20], we demonstrated the proof-of-concept, but, here, a broader spectrum EO comb is generated and feedback bandwidth is improved. Because of these improvement, we clearly show division/magnification of the residual phase noise of nf_m, depending on the wavelength of the out-of-loop beat. In the best case, we demonstrate below 100 mrad integrated (up to 1 MHz) relative phase noise between the EO comb and the fiber comb across +/− 200 EO comb harmonics. In addition, we propose and demonstrate coherent addition of optical beats to obtain a lower noise floor, which is useful for low noise microwave generation or phase noise measurement when using an EO comb.

2. EO comb generation

The schematic setup of our EO comb is shown in Fig. 1(a). A single frequency cw laser with 1560.9 nm center wavelength and 10 mW power is modulated by three cascaded phase modulators and one intensity modulator, similar to ref [16,21,22]. A 10 GHz signal from a dielectric resonator (DRO) was used to drive the modulators. The total applied modulation to phase modulators is about $6V_\pi$, where V_π is the voltage to cause a π phase shift in the phase modulators. With appropriately applied RF power and bias voltage to the intensity modulator, the EO comb from the intensity modulator has a linear chirp, which can be compensated with a standard single mode fiber. The spectrum after the intensity modulator is shown in Fig. 1(b). In this experiment, instead of a standard single mode fiber, we used a fiber Bragg grating (FBG) with −8.8 ps^2 dispersion to improve the bandwidth of the phase locked loop. The output from the intensity modulator, after pre-amplification by an Er doped fiber amplifier (EDFA 1), passes an optical circulator, a FBG and another Er doped fiber amplifier (EDFA 2). After amplification to 250 mW by EDFA 2, the optical spectrum is broadened by a normal dispersion HNLF as shown in Fig. 1(c). Normal dispersion HNLF is used to avoid the amplification of ASE or input noise due to modulation instability in the HNLF, which makes generated broadened spectrum more coherent [8,13,22]. In the following experiments, up to the +/− 200 th harmonics (from 1544.8 nm to 1577.3 nm) of the EO comb are used.

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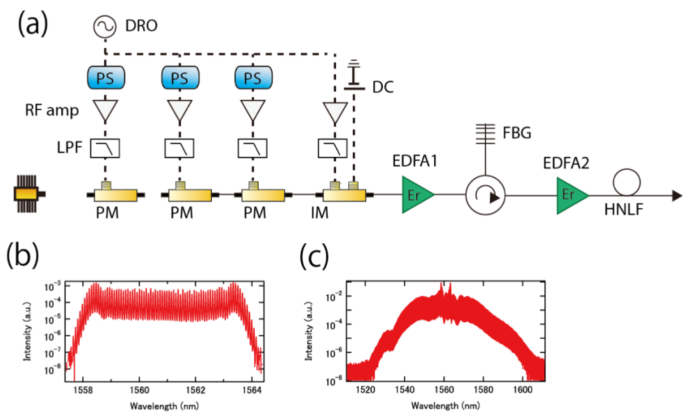

Fig. 1. (a) Schematic of EO comb. PM, phase modulators; IM, intensity modulator; LPF, low pass filter; PS, phase shifter; EDFA, Er doped fiber amplifier; FBG; fiber Bragg grating; HNLF, highly nonlinear fiber. (b) Optical spectrum after IM. (c) Optical spectrum after HNLF.

3. Phase locking of EO comb to fiber comb

Figure 2 shows the schematic of the locking setup. Three beats between the EO comb and the fiber comb are observed simultaneously. The output from the cw laser is split into two before generating the EO comb, and interfered with the optically bandpass-filtered fiber comb. The obtained beat ($f_{cw(beat)}$) is used to phase lock the cw laser to one of the fiber comb modes by feeding back to the cw laser. The output from the HNLF is split into two, optically bandpass filtered with less than 10 GHz bandwidth to avoid a beat between neighboring EO comb modes and the fiber comb, and interfered with the optically bandpass-filtered fiber comb. One of the generated beats ($f_{beat(in)}$) is used to phase lock the DRO to one of the fiber comb modes by feeding back to the DRO, i.e. stabilizing the comb spacing of the EO comb to that of the fiber comb. Note that stabilization of the DRO to f_{rep} with an RF frequency divider for the DRO or an RF multiplier for f_{rep} by using an analog mixer will not work for low phase noise levels because of poor sensitivity. Another beat is used to observe the out-of-loop RF spectrum ($f_{beat(out)}$) when both $f_{cw(beat)}$ and $f_{beat(in)}$ are phase locked. In addition, f_{rep} of the fiber comb is phase locked to an RF reference with locking bandwidth of below 100 Hz to avoid the wavelength-dependent slow drift of beats caused by a slow drift of f_{rep}.

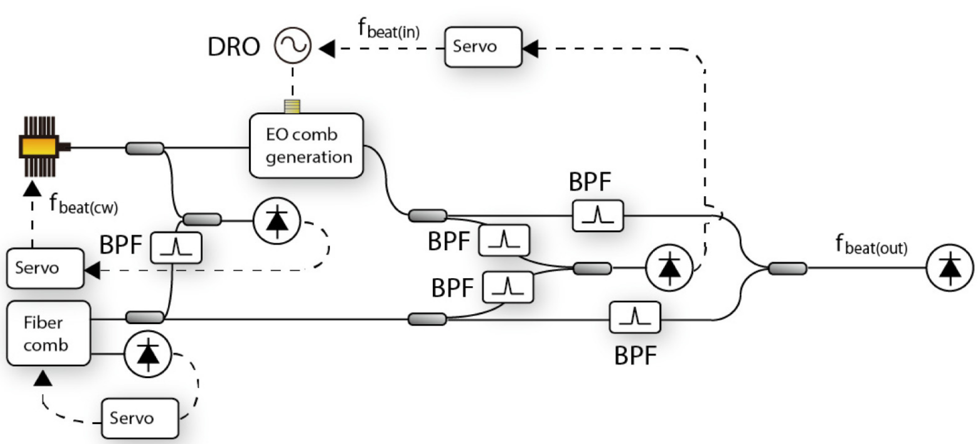

Fig. 2. Schematic of locking setup. BPF; bandpass filter; DRO, dielectric resonator.

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Figure 3 shows the schematic of selected possible locking configurations. For case 0, only f_{cw} is phase locked to the fiber comb, and out-of-loop beats at the 10 th, 50 th, and 200 th harmonics are measured to verify the phase noise of f_m is magnified. For case 1, not only f_{cw}, but also the 10 th harmonics are phase locked to the fiber comb, and out-of-loop beats at the 50 th and 200 th harmonics are measured. In this case, magnification of residual phase noise of $f_{beat(in)}$ is expected at $f_{beat(out)}$. For case 2, f_{cw} and the 200 th harmonics are phase locked to the fiber comb, and beats at the 10 th and 50 th harmonics are measured. In this case, division of the residual phase noise of $f_{beat(in)}$ is expected at $f_{beat(out)}$. For case 3, f_{cw} and the 200 th harmonics are phase locked to the fiber comb, and a beat at the –200 th harmonics is measured to confirm the –200 th harmonics has the same phase noise as the + 200 th harmonic.

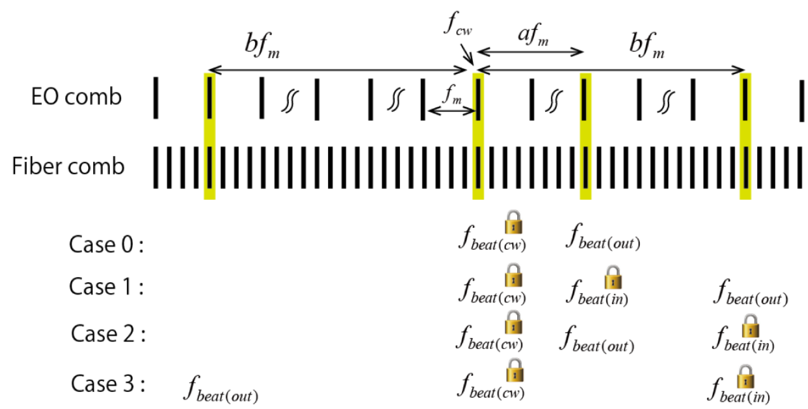

Fig. 3. Schematic of locking cases.

Figures 4(a) and 4(b) show the results for case 0, where the cw laser is phase locked to the fiber comb, and out-of-loop beats at the 10 th, 50 th, and 200 th harmonics are measured. As expected, the phase noise is clearly magnified with harmonic number. Phase noise should be magnified by 20 dB for the 10 th harmonics, 34 dB for the 50 th harmonics, and 46 dB for the 200 th harmonics. This is very different from ML combs. ML combs only cause 0.09 dB magnification compared with 1560.9 nm (n = 1922000) and 1577.3 nm (n = 1902000), when a 100 MHz ML comb is considered (i.e. 0.09 dB = 10 × log ($1922000^2/1902000^2$). Here, other excess noises such as ASE are ignored. Figures 4(c) and 4(d) shows results of case 1, where 10 th harmonics is phase locked to the fiber comb, and out-of-loop 50 th and 200 th harmonics are measured, while the cw laser is phase locked to the fiber comb. Note that purposely large feedback gain was applied to the DRO to accentuate the servo bumps, which helps with the observation of phase noise division/magnification. The contrast between the coherent carrier and the peak at the servo bump frequency is about the 31 dB at the 10 th harmonic and 18 dB at the 50 th harmonics. The residual phase noise was magnified, because the harmonic number for the out-of-loop beat is larger than for the phase-locked harmonic. The observed phase noise maginification factor of 13 dB fits well to the expected noise magnification factor (14 dB = 10 × log(50/10)). We could not observe a coherent carrier in the beat at the 200 th harmonics, because the phase noise magnification is too large. Figure 4(e) shows the results for case 2, where the 200 th harmonic is phase locked to the fiber comb, and out-of-loop beats at the 10 th and 50 th harmonics are measured, while the cw laser is phase locked to the fiber comb. The contrast between the coherent peak and the peak at the servo bump frequency is 19 dB at the 200 th harmonic, 30 dB at the 50 th harmonic, and 44 dB at 10 th harmonic. In contrast to case 1, the residual phase noise is divided, since the harmonic number of the out-of-loop beats is smaller than the harmonic number of the phase locked beat. The residual phase noise division factor of 11 dB (= 30 dB – 19 dB) and 25 dB (

= 44 dB – 19 dB) again agrees well with the expected noise division factor of 12 dB ($= 10 \times \log(200/50)^2$) and 26 dB ($= 10 \times \log(200/10)^2$). Figure 4(f) shows results for case 3, where the + 200 th harmonic is phase locked to the fiber comb and an out-of-loop beat at the −200 th harmonic is measured, while the cw laser is phase locked to the fiber comb. The contrast between the coherent peak and the peak at the servo bump frequency for the + 200 th and −200 th harmonics is almost the same. The phase noise increase of the −200 th harmonics at low frequencies comes from too much gain at low frequencies for in-loop phase locking. Actually, the in-loop phase locked beat has a lower power spectrum density (PSD) than the noise floor as shown in Fig. 5. From these results, when the EO comb is fully stabilized to the ML comb to reduce phase noise, large harmonic numbers should be used for phase locking, as also observed by Ishizawa et al [17,18]. Once the n th harmonics is tightly phase locked to the fiber comb, the phase noise of the EO comb within the +/− n th harmonics is lower than the residual noise of the phase locked n th harmonic.

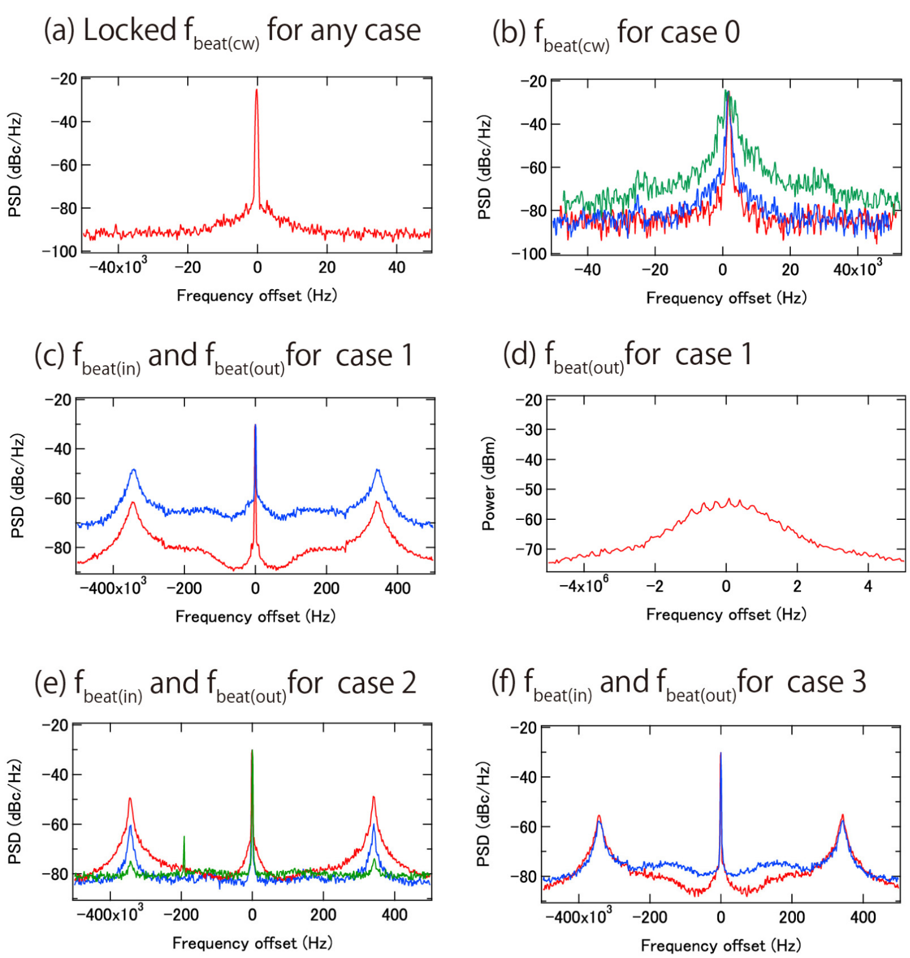

Fig. 4. (a) RF spectrum of phase locked $f_{beat(cw)}$ with 300 Hz resolution bandwidth and 10 kHz span. (b) RF spectra of $f_{beat(out)}$ for case 0. Red, blue, and green traces are $f_{beat(out)}$ at 10 th, 50 th, and 200 th harmonics with 300 Hz resolution bandwidth and 10 kHz span, respectively. (c) RF spectra of $f_{beat(in)}$ (red) and $f_{beat(out)}$ at the 50 th harmonics (blue) with 1 kHz resolution bandwidth and 1 MHz span for case 1. (d) RF spectrum of $f_{beat(out)}$ at the 200 th harmonics with 1 kHz resolution bandwidth and 10 MHz span for case 1. (e) RF spectra of $f_{beat(in)}$ (red) and $f_{beat(out)}$ at the 10 th (green) and 50 th (blue) harmonics with 1 kHz resolution bandwidth and 1 MHz span for case 2. (f) RF spectra of $f_{beat(in)}$ (red) and $f_{beat(out)}$ (blue) with 1 kHz resolution bandwidth and 1 MHz span for case 3.

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Figure 5 shows the RF spectra and the PSD of the phase locked $f_{beat(cw)}$ and $f_{beat(in)}$ at the 200 th harmonic. Here, the feedback gain was decreased to suppress the servo bump. Both beats are tightly phase locked to the fiber comb, and the integrated phase noise up to 1 MHz was below 100 mrad. The feedback bandwidth of the cw laser was limited by the mechanical resonances of the PZT inside the laser, and that of the DRO for the 200 th harmonics beat is limited by the length of the phase locked loop, mainly governed by the 100 m of HNLF. In principle, shorter, more highly nonlinear wavguides could be implemented to increase the feedback bandwidth.

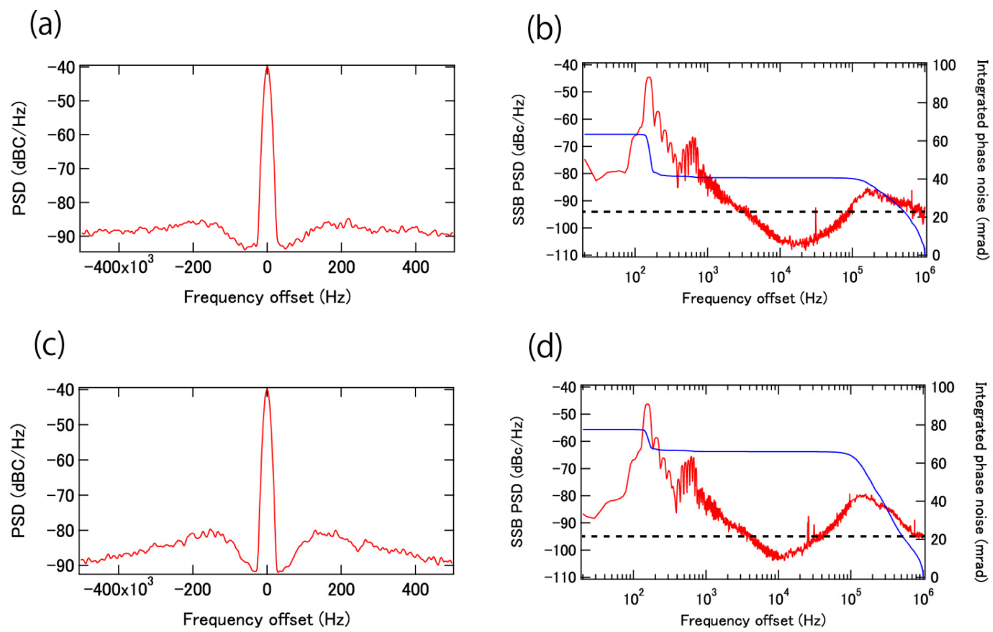

Fig. 5. (a) RF spectrum of phase locked $f_{beat(cw)}$. (b) Single sideband PSD (red) and integrated in-loop phase noise (blue) of phase locked $f_{beat(cw)}$ with 10 kHz resolution bandwidth. Black dotted line is the noise floor. (c) RF spectrum of phase locked $f_{beat(in)}$ at the 200 th harmonics with 10 kHz resolution bandwidth. (d) Single sideband PSD (red) and integrated in-loop phase noise (blue) of phase locked $f_{beat(in)}$ at the 200 th harmonics. Black dotted line is the noise floor.

4. Coherent addition of beats

The fact that the phase noise of the RF oscillator is magnified as shown in Fig. 4(b) can be used for low noise RF measurements or low noise RF generation [17,18]. The measurable noise floor or the obtainable RF phase noise when locking the EO comb to the ML comb is ultimately limited by the shot noise floor of the beat, i.e. the power per comb mode of the ML comb. For example, a few nW comb mode power produces as shot noise floor of about −100 dBc/Hz. Therefore, if a beat at the 1000 th harmonics were used, the measurable noise floor or the obtainable RF noise can reach as low as −160 dBc/Hz. In our case, the noise floor of the beat at the 200 th harmonics is about −94 dBc/Hz, resulting in −140 dBc/Hz (= −94 dBc/Hz – 46 dBc/Hz) of measurable or obtainable RF phase noise.

To improve the noise floor, we propose and demonstrate coherent addition of optical beats between the EO and ML combs. The idea is schematically shown in Fig. 6. If the comb spacing of the EO comb is close to an integer multiple of f_{rep}, several optical beats between the EO and ML combs can be observed at similar RF frequencies.

$$i_{sum} \propto \sum_{i=n+1}^{n+N} A_i \cos(2\pi\Delta_i t + \phi_i(t)) \quad (1)$$

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Here, i_{sum}, A_i, n, Δ_i, and $\varphi_i(t)$ represent the photo current, the product of electric amplitude of i-th EO comb ($A_{EO,i}$) and electric amplitude of neighboring ML comb mode ($A_{comb,i}$), EO comb mode number, beat frequency between the n th EO comb mode and neighboring ML comb mode, phase noise of i-th beat, respectively. N is the number of the EO comb mode, which will be coherently added. Here, DC and other frequency terms are omitted. Assuming A_i and $\varphi_i(t)$ are independent of i (A_i = A), and once those beats are phase locked to the same frequency (Δ), i_{sum} becomes

$$i_{sum} \propto NA\cos(2\pi\Delta t) \tag{2}$$

Here, for simplicity, we also assume that the residual phase noise and average phase in the locked beat note is zero. The shot noise for the coherent addition (N_{sum}) can be represented as,

$$N_{sum} \propto \sqrt{P_{combs} + \sum_{i=n+1}^{n+N} P_{EO,i}} = \sqrt{P_{combs} + NP_{EO,n}} \tag{3}$$

Here, P_{combs} is optical power of ML comb modes within a bandwidth of optical bandpass filter, and $P_{EO,i}$ is optical power of i-th EO comb. On the other hand, the shot noise for a single beat note, i.e. without coherent addition would be,

$$N_1 \propto \sqrt{P_{combs} + P_{EO,n}} \tag{4}$$

From these expressions, a power SNR improvement factor can be derived, and found to be proportional to N, i.e. the number of coherently added EO comb modes. This is valid as long as P_{combs} is much smaller than $P_{EO,n}$.

The number of available EO comb modes is limited by the saturation of the photo detector. In addition, as discussed in section 3, each EO comb mode has different phase noise, and therefore the achievable SNR enhancement factor can exceed this linear dependence of N.

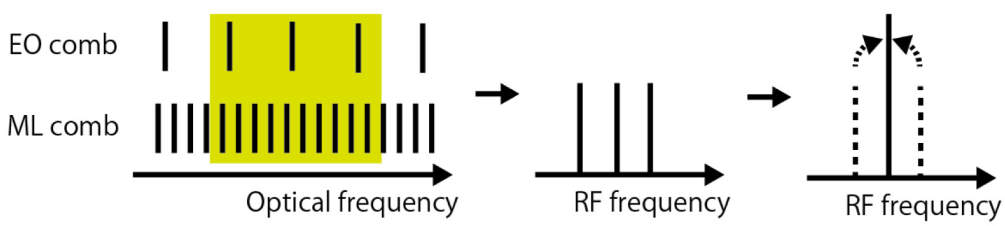

Fig. 6. The schematic of coherent addition of multiple beats. Yellow area shows the filtered spectrum.

For a proof-of-concept, we adjusted f_{rep} of the fiber comb to about 83.3 MHz (= 10 GHz / 120). Figure 7(a) shows several optical beats between the EO comb and the fiber comb. The noise floor of those individual beats is about −90 dBc/Hz. Once those beats are coherently added by adjusting the DRO frequency to be an exact integer multiple of f_{rep} by feeding back to the DRO, the noise floor became −103 dBc/Hz as shown by the red curve in Fig. 7(b). Here, seven optical beats were coherently added, which lead to an improvement of the noise floor by 13 dB. As discussed above, coherent addition of seven beats should improve the SNR by about 8.5 dB (= 10 × log(7)), as long as P_{combs} is much smaller than $P_{EO,n}$. However, this simple assumption is not strictly valid as it igonores the phase-noise multiplication discussed in section 3. We noticed that the carrier power of coherently added beats slowly fluctuated in ordinary laboratory conditions. We believe these slow fluctuations arise because the feedback loop only stabilizes the sum of the phase constant of all optical beats, i.e. $\sum_i(\phi_i(t)+\phi_{0,i})$, instead of keeping each term $(\phi_i(t)+\phi_{0,i})$ constant, which corresponds to standard phase locking of one optical beat. Here, $\phi_{0,i}$ is the phase of i-th beat. In unfortunate

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cases, some beats may interfere destructively, depended on $\phi_{0,i}$. The observed noise floor fluctuation are due to phase variations, and depend on EO comb mode frequency, i.e. chromatic dispersion. This problem can be solved by stabilizing the optical phase by changing the length of the optical fiber or by using an acousto-optic modulator inside the optical path. An error signal can then be obtained by taking the phase difference between $\phi_i(t)+\phi_{0,i}$ and $\phi_k(t)+\phi_{0,k}$ [23]. To further improve the SNR, combination of the coherent addition and the gated optical pulse noise reduction (GATOR) [24] can be employed. Then, more than 20 dB SNR improvement can be expected. Also, fiber comb oscillators producing a power per comb mode > 1 μW can be readily constructed, allowing further scope for improvement of the present system.

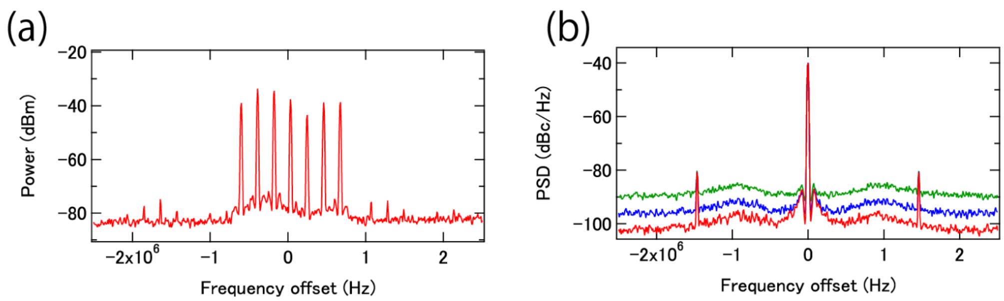

Fig. 7. (a) RF spectrum of multiple of optical beats between EO and ML comb at 10 kHz resolution bandwidth. The same variation of EO comb mode power can be observed in both optical and RF domain. (b) RF spectrum of coherently added beats. Green curve for one optical beat, blue curve for three optical beats, and red curve for seven optical beats.

5. Discussion

The results for phase noise showned in this paper are pertinent to relative phase noise between the fiber comb and the EO comb. In this section, we discuss the achievable absolute phase noise for EO combs by phase locking to a ML comb. As we mentioned in the introduction, the phase noise of EO arises come from two terms, phase noise of the cw laser and RF oscillator to drive the EOMs. For simplicity, we assume phase locking of a beat between a cw laser and the n th ML comb mode and a beat between the N-th harmonic of the EO comb and (n + Δ) th ML comb mode perfectly works, i.e. the residual phase noise from phase locking is zero. From this assumption, the phase noise of the cw laser and the RF oscillator can be represend as

$$\varphi_{cw}(f)=\varphi_{ceo}(f)+n^2\varphi_{rep}(f) \tag{5}$$

$$\varphi_m(f)=\frac{\Delta^2}{N^2}\cdot\varphi_{rep}(f)=\left(\frac{f_m}{f_{rep}}\right)^2\varphi_{rep}(f) \tag{6}$$

Here, $\varphi_{ceo}(f)$ and $\varphi_{rep}(f)$ are the phase noise of the offset frequency and fundamental repetition frequeny of the ML comb. Equation (6) means that achievable phase noise of the RF oscillator is determined by the phase noise of the repetition frequency of the ML comb after scaling to the RF frequency. In addition to this limitation, we have to take the shot noise limit into consideration as discussed in the previous section. Another practical limitation is the feeback bandwidth of the RF oscillator. Because of this, to obtain ultra low noise EO combs or generate ultra low noise RF [17,18], we need not only a low noise reference for the EO comb, i.e. a low noise ML comb, but also a high SNR for beats between the ML comb and the EO comb, and have to use a free-running low noise (or large feedback bandwidth) RF oscillator. Figure 8 shows an example of the achievable 10 GHz phase noise by locking the

EO comb to a ML comb. Here, we refer to a low noise fiber combs [14,15] with phase locking to a state-of-the-art cw laser [26], and assume that the feedback bandwidth of the RF oscillator is 1 MHz, i.e. feedback gain is zero at 1 MHz, and the feedback gain scales with the inverse of frequency squared. For the shot noise limit, we assume the beat between the 1000 th harmonics of EO comb mode and one of the ML comb has 50 dB SNR at 100 kHz RBW. The achievable phase noise is limited by the ML combs (or a cw laser as a reference for phase locking of the ML combs) at low frequency (< 100 Hz), by the shot noise at middle frequency (100 Hz to 10 kHz), and by the feedback bandwidth at high frequency (10 kHz to 1 MHz). As shown in section 4, the shot noise limit can be improved by using appropriate coherent addition or ML combs with higher comb mode power.

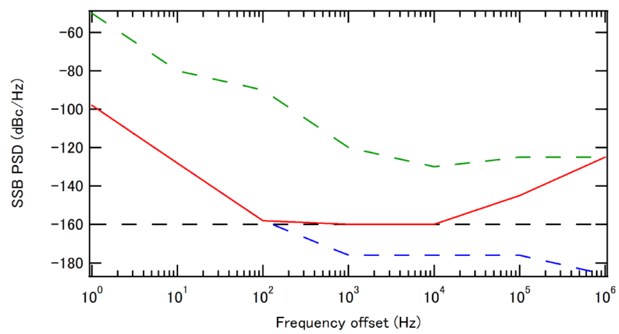

Fig. 8. Acheivable phase noise estimation of 10 GHz carrier by phase locking an EO comb to an ML comb. Green, black, and blue dotted curve shows phase noise of a free-running 10 GHz oscillator [25], the shot noise limit, and a low noise fiber comb with scaling to 10 GHz, respectively. Red curve shows the achievable phase noise of 10 GHz carrier.

6. Conclusion

In conclusion, by fully stabilizing an EO comb to a fiber comb, division/magnification of residual phase noise was clearly observed, and the obtained division/magnification factor agreed well with expected values. The stabilized EO comb had below 100 mrad integrated phase noise up to 1 MHz across +/– 200 harmonics (from 1544.8 nm to 1577.3 nm). In addition, coherent addition of optical beats was demonstrated, which can be exploited for low noise microwave generation or low noise measurements with an EO-based comb. The demonstrated low phase noise EO comb can also be used in future WDM coherent communication with very complex modulation formats such as 1024 QAM.

Acknowledgments

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