

**Interference Time Analysis for a Cognitive Radio on an
Unmanned Aircraft**

by

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The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.

Mysore Balasubramanya, Naveen (M.S., Electrical Engineering)

Interference Time Analysis for a Cognitive Radio on an Unmanned Aircraft

Thesis directed by Prof. Timothy X. Brown

This thesis considers a model consisting of a cognitive radio (CR) on an unmanned aircraft (UA) and a network of licensed primary users on the ground. The cognitive radio uses the same frequency spectrum as the primary users for its operation and hence acts as an interferer. This work analyzes the duration of interference in such a model. It defines two important metrics – the interference radius and the detection radius. The interference radius determines the boundary of the area within which a primary user might be subjected to harmful interference due to the operation of the CR. The detection radius determines the boundary of the area within which the presence of a primary user might be detected by the CR. The interference and detection radii might vary due to the dynamic nature of the radio environment. This thesis derives the dependence of these metrics on the radio propagation parameters like antenna gain, antenna height, path-loss exponents, etc. It uses these metrics and characterizes the model using an $M/G/\infty$ queue to determine the statistics of the interference time for the entire excursion of the unmanned aircraft. The key statistics determined are the distribution of the duration of interference periods, the mean and the total interference time. Firstly, this work analyzes a 1D system model where the primary users are distributed randomly along a straight line. The results are then extended to a 2D system where the primary users are distributed randomly over an area. The analysis is carried out for both sparsely-dense and highly-dense primary user ground network. This work gives a new dimension to analyze the effects of interference in terms of duration of interference. It also shows how these interference effects can be minimized on enhancing the detection capability of the cognitive radio. The results from this work can be used to determine the optimum setting for the cognitive radio system such that it restrains the duration of interference below tolerable limits.

Dedication

To my parents.

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Chapter 1

Introduction

The field of communication has developed rapidly over the past decades. With mobile phones, smart phones and portable devices like on-the-go data cards entering the market, and with the advent of 3G and 4G technologies, wireless communication services have become an integral part of our daily life. These wireless communication services require the radio frequency spectrum whose allocation and regulation is monitored by various government organizations. For example, the Federal Communications Commission (FCC) allocates and regulates the commercial usage of the radio frequency spectrum in the United States. With the extensive growth in wireless communication, there is an increasing demand for spectrum. But the useful spectrum between 3 kHz and 300 GHz range has already been allocated by the FCC for different communication services [6]. So, the new wireless services which are ready to enter the market might not obtain enough spectrum in order to operate successfully. Hence, these new services should overcome the shortage of spectrum by adopting a novel technology. One such technology is a cognitive radio [4]. A cognitive radio is a device that is intelligent enough to detect and use the unused parts of the already allocated spectrum. In many allocations, the spectrum is not used everywhere at all times [4]. For instance, TV channel 15 is only assigned in some markets. The so-called primary user who is assigned the channel, may not use their spectrum at all times and in all locations. For example, a channel 15 primary user may not operate late at night. So the cognitive radio can use the channel 15 spectrum whenever the primary user is not using it. Once it detects a primary user using the spectrum, the cognitive radio has to switch off or operate in another unused part of the

spectrum in order to avoid interference. But in many cases, the cognitive radio might not be able to detect a primary user in the spectrum until it comes close enough to the primary user. For example, consider a cognitive radio on an unmanned aircraft and a primary user at the ground level. The unmanned aircraft can be so far that it cannot detect the primary user, but near enough to cause interference to the primary user. Thus, modeling and analysis of interference caused by such intelligent devices becomes vital. This thesis attempts to answer some questions related to interference caused by cognitive radios to primary users.

1.1 The Unmanned Aircraft System

An Unmanned Aircraft System (UAS) consists of an Unmanned Aircraft (UA), the UA Control Station and the Control Link in-between. Using the Control Link, the UA control station pilots the unmanned aircraft through telecommands and the unmanned aircraft provides the responses through telemetry. The UAS also provides numerous interfaces for communication purposes. For example, it provides voice and data communication interfaces for the Air Traffic Control (ATC), interfaces for navigation control, interfaces for weather data, radar, “sense and avoid” information, etc. Figure 1.1 provides an overview of the UAS and the interfaces supported by it [10][7].

The UA control can be achieved in two ways [10] (Refer to Figure 1.2)

- (1) *Direct Control* - This uses dedicated links from the control station to the unmanned aircraft. Such links rely on line-of-sight (LOS) radio link or a satellite link.
- (2) *Network Control* - In this case, the UA Control Station has access to a shared network maintained by a Communications Service Provider (CSP). The UA control is achieved through the CSP since it provides the necessary infrastructure to radio stations and satellite links. It should be noted that the links within a CSP shared network may be wired or wireless.

The Radio Technical Commission for Aeronautics Special Committee (RTCA SC-203) is currently working on communication standards for the UAS. It includes the development of operational

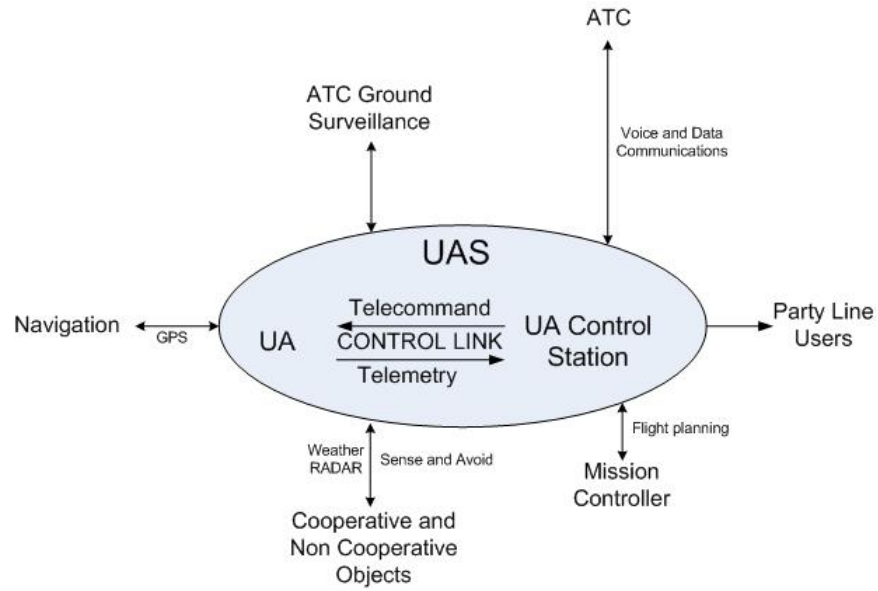


Figure 1.1: The Unmanned Aircraft System (UAS) (Adopted from [10])

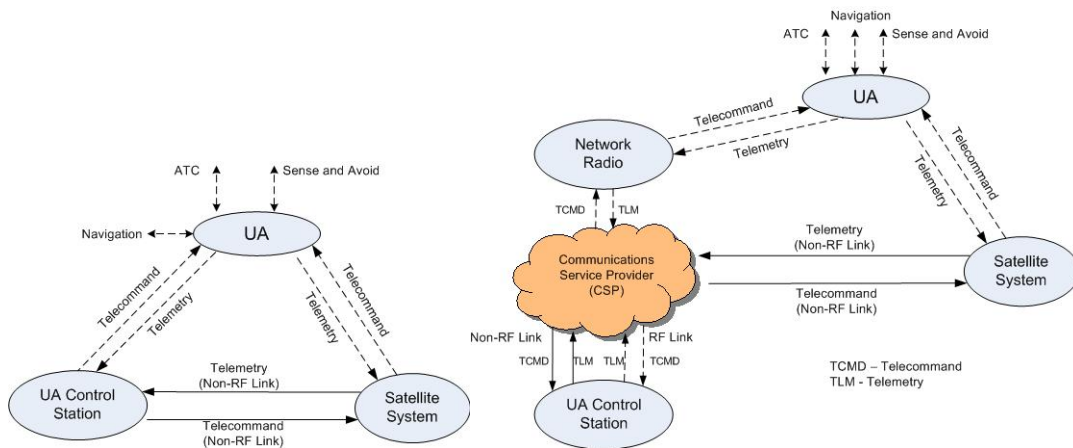


Figure 1.2: Direct Control and Network Control of the UA (Adopted from [10])

scenarios, control and communication architectures of the UAS products [15]. Hence, it becomes vital to understand and model the performance of different radio types including the cognitive radio in an UAS.

1.2 Cognitive radio

A “Cognitive Radio” is a radio that can change its transmitter parameters based on interaction with the environment in which it operates. [5]

From the above definition, one can derive two salient features for a cognitive radio - Cognitive capability and Reconfigurability. [1, 9, 11, 14]. These two features are discussed in detail below.

Cognitive capability: A cognitive radio should possess the ability to obtain information regarding the characteristics of the channel or the radio environment. Usually a cognitive radio does not have dedicated band for communication. It makes use of the unlicensed bands or uses the licensed bands opportunistically for its operation. This is called Dynamic Spectrum Access (DSA). Hence, finding a frequency band that causes minimal or no interference to primary users (users who are licensed to use that band) is of foremost importance. In most of the cases, the usage of a simple energy detector to determine the power in the band and deciding based on a threshold whether the channel is free or busy does not work. This is because the radio environment is dynamic. Radio propagation factors like shadow fading and path-loss strongly influence the power measured on a channel and one cannot be sure if the measured power reflects the actual channel occupancy. So, improved methods of sensing free channels and improved techniques for choosing the best available channel and the optimum operational parameters become necessary. So, the cognitive capability aspect can be classified into three major steps [1]

- (1) Sensing the spectrum: This involves searching for available bands, monitoring the frequencies (spectrum) in such bands and detecting spectrum holes. It should be noted that the sensing can be a direct act of the cognitive radio or indirectly through stored databases, feedback from other radios and sensors [3].

- (2) Analyzing the spectrum: This involves measurement and estimation of characteristics of the radio environment in the detected spectrum holes.
- (3) Deciding the best spectrum: This step covers the process of choosing the best spectrum for transmission based on parameters like maximum data rate that can be achieved, optimized power control, transmission modes and so on.

Reconfigurability: A cognitive radio is usually designed to adapt to the changing nature of the radio environment. Hence, it has to be reconfigured or dynamically programmed over the air for optimal operation without changing the underlying hardware design. Key parameters to be configured are the following

- (1) Operating frequency - The cognitive radio has to be able to operate over a range of frequencies. It should be able to shift or hop to another free channel as soon as it detects a licensed user communicating in the operating frequency.
- (2) Transmission Power - The operation of the cognitive radio adds to the interference power for primary users. Hence, the transmission power should be adjusted such that the interference is kept below the permitted level.
- (3) Radio Technology - The cognitive radio should be enabled to work over different radio technologies (modulation schemes, radio access and data protocols) and should be able to dynamically choose the best radio technology according to the radio environment.

1.3 Aim

From the characteristics of cognitive radios discussed above, we can note that it is very important to model the interference caused by a cognitive radios to existing primary users. In this thesis, we consider a cognitive radio (CR) placed on an unmanned aircraft (UA) which is interacting with a UA control station on the ground. The ground network consists of numerous primary transmitters and primary receivers communicating with them using an allocated frequency

spectrum. The CR on the UA detects spectrum holes in this allocated band for primary users and utilizes the best available frequency to communicate with the control station. Along with spectrum sensing and utilization, the CR also has the capability to detect a primary receiver which is broadcasting beacons within a radius R_{det} . Upon successfully detecting a primary receiver, the CR will switch off or hop to a different frequency. When such a strategy of switching off is adopted by the CR, it will result in durations where the CR cannot communicate with the control station. These are called detection outages. Hence, the entire excursion of the UA is distinguished by three kinds of periods – interference periods, detection outages and interference-free communication (Refer to Figure 1.3).

This thesis aims at modeling such a system, analyzing these three periods and determining the distribution of their duration. The thesis answers the following questions.

- (1) The UA propagation characteristics are different from terrestrial propagation. What is an interference radius and a detection radius between an unmanned aircraft and a primary user? How do these propagation parameters influence interference and detection radii?
- (2) The UA cares about the durations of time where it interferes with the primary receivers. What is the duration of interference per primary user? What is the mean and distribution for the duration of interference periods for a CR on the UA? What is mean and the total interfering time for the entire excursion of the UA?
- (3) The regulator cares about the overall impact of interference on the primary user. How helpful is primary user detection in minimizing the effect of interference from a primary user's perspective? What is the impact on the outages experienced by the UA due to primary user detection?
- (4) What is the effect on total interfering time due to varying interference and detection radii amongst the primary receivers?

This work derives the interference and detection radii are derived for a simple system consisting for

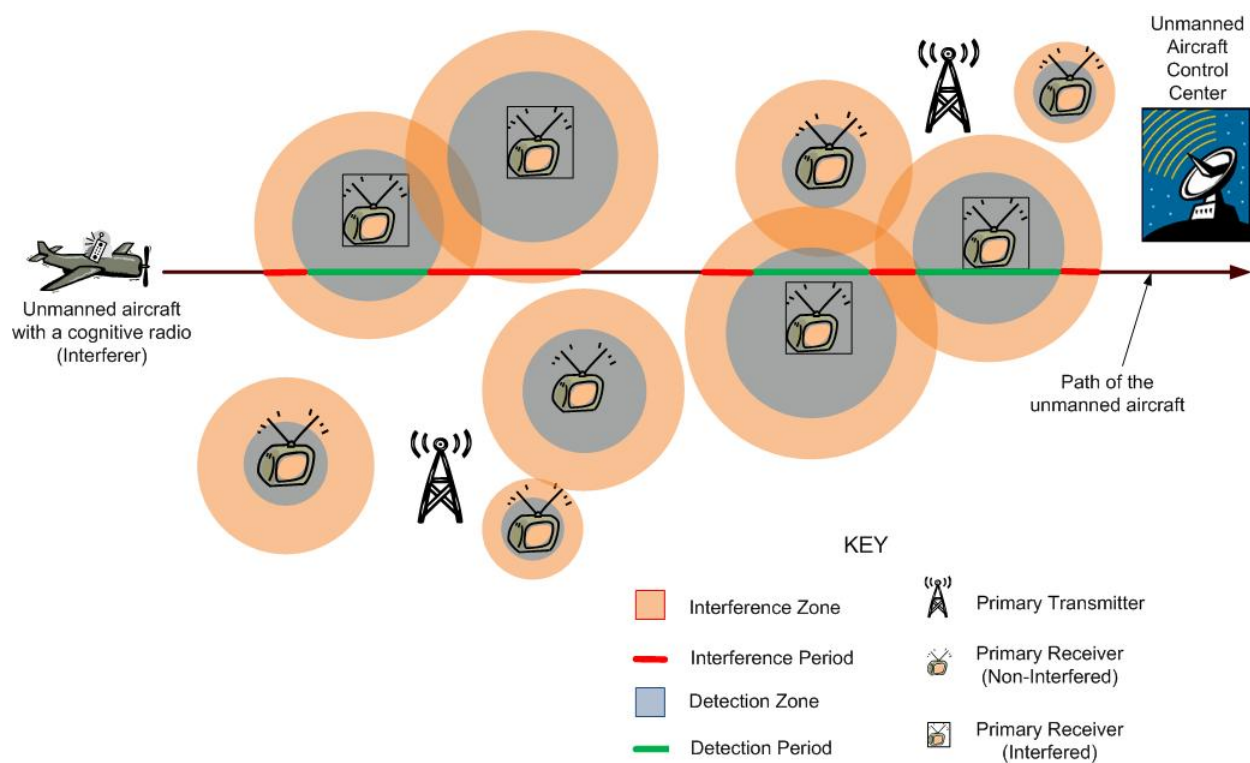


Figure 1.3: Interference and detection periods for a cognitive radio on an unmanned aircraft

one primary transmitter-receiver pair and one cognitive radio interferer. The results are used along with queueing theory aspects to analyze the statistics of interference time for one-dimensional and two-dimensional systems.

1.4 Thesis Outline

- Chapter 2 describes the interference and detection metrics for a cognitive radio. We define the interference and detection radii for a primary receiver and determine their relation with the radio propagation environment.
- Chapter 3 gives the interference time analysis for the one-dimensional systems. It uses queueing theory aspects discussed in Appendix C to provide an insight to the statistics of the interference and detection periods.
- Chapter 4 describes how the the results derived for the one-dimensional systems can be extended to the two-dimensional systems.
- Chapter 5 describes the applications of the results.
- Chapter 6 gives the conclusions and future work.

Chapter 2

Interference and detection metrics for a cognitive radio

This chapter describes how a cognitive radio can interfere with existing radio systems and how the cognitive radio can use its inherent detection capability to reduce interference. The chapter introduces a simple model consisting of a single primary transmitter-receiver pair and a single cognitive radio interferer. For instance, the cognitive radio might be on an unmanned aircraft at a certain height above the ground and can interfere with a primary receiver at the ground level if it comes within the receiver's interference radius. This chapter gives the mathematical analysis for interference radius and detection radius. It explains their relation to various system and radio propagation parameters like antenna gain, antenna height, path-loss, shadowing, etc.

2.1 A simple cognitive radio interferer model

As discussed in the previous chapter, a cognitive radio must be able to keep interference within limits. Hence modeling the interference becomes vital in such systems. A simple model to begin with consists of a primary transmitter, T_x , placed at co-ordinates (x_1, y_1) , which is communicating with a primary receiver, R_x , placed at the origin (Refer to Figure 2.1). The primary receiver is a licensed user and interference to him is considered harmful. Let us consider a single cognitive radio interferer, I , placed at (x_2, y_2) . Let R_T , R_I and R_{TI} be the distances between T_x and R_x , I and R_x , and T_x and I respectively.

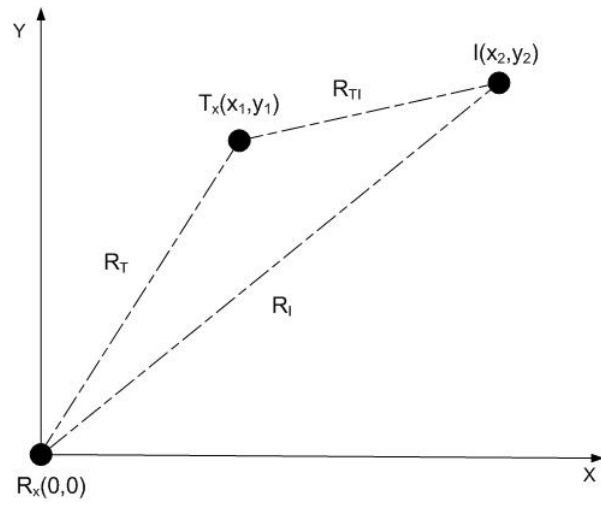


Figure 2.1: Simple Cognitive Radio Interferer Model

2.1.1 Interference radius, R_{int}

For the model shown in Figure 2.1, let R_0 denote the boundary radius i.e. the maximum distance possible between T_x and R_x for successful communication. In other words, the signal to noise ratio (SNR) required for successful communication between T_x and R_x is at its minimum threshold level Z at a distance R_0 [2]. Let P_S and P_N denote the received signal power and noise power at R_x . The received signal power, P_S is given by [2]

$$P_S = \frac{K_S \cdot S_S \cdot p_T}{R_T^a} \quad (2.1)$$

where,

K_S is the constant inclusive of antenna gain and antenna height between T and R_x ,

S_S is the shadow fading component between T and R_x ,

p_T is the transmission power of T_x , and

a is the path-loss exponent between T and R_x .

The received noise power P_N has two components — Noise, denoted by N and the interference power due the cognitive radio, denoted by P_I . Therefore, we have $P_N = P_I + N$, where

$$P_I = \frac{K_I \cdot S_I \cdot p_I}{R_I^b} \quad (2.2)$$

and where,

K_I is the constant inclusive of antenna gain and antenna height between I and R_x ,

S_I is the shadow fading component between I and R_x ,

p_I is the transmission power of I , and

b is the path-loss exponent between I and R_x .

At a distance R_0 from T_x , using the definition of the SNR threshold, Z , we have

$$\begin{aligned} Z &= \frac{P_S}{N} \\ &= \frac{K_S \cdot S_S \cdot p_T}{R_0^a \cdot N} \end{aligned}$$

Therefore, we can find the noise power N as below

$$N = \frac{K_S \cdot S_S \cdot p_T}{R_0^a \cdot Z} \quad (2.3)$$

For proper communication between T_x and R_x even in the presence of the cognitive radio interferer I , we should have

$$\begin{aligned} \frac{P_S}{P_N} &\geq Z \\ \left(\frac{\left(\frac{K_S \cdot S_S \cdot p_T}{R_T^a} \right)}{\left(\frac{K_I \cdot S_I \cdot p_I}{R_I^a} \right) + \left(\frac{K_S \cdot S_S \cdot p_T}{R_0^a \cdot Z} \right)} \right) &\geq Z \\ R_I &\geq \left(\frac{Z \cdot K_I \cdot S_I \cdot p_I \cdot R_0^a \cdot R_T^a}{K_S \cdot S_S \cdot p_T \cdot (R_0^a - R_T^a)} \right)^{\frac{1}{b}} \end{aligned}$$

Let $R_{int} = \left(\frac{Z \cdot K_I \cdot S_I \cdot p_I \cdot R_0^a \cdot R_T^a}{K_S \cdot S_S \cdot p_T \cdot (R_0^a - R_T^a)} \right)^{\frac{1}{b}}$. Thus, when the cognitive radio interferer I is at a distance $R_I \leq R_{int}$ from R_x , it causes harmful interference to the primary receiver. Hence R_{int} is called the interference radius of the primary receiver.

The radio propagation model between T_x and R_x can be chosen as a free-space path loss model or ground reflection model [13]. The received power, P_s , in the case of the free-space path-loss model is given by

$$P_s = \left(\frac{\lambda}{4\pi R_t} \right)^2 \cdot K \cdot S_s \cdot p_T \quad (2.4)$$

where,

λ is the wavelength used, i.e. if f is the frequency of operation and c is the velocity of light, $\lambda = \frac{c}{f}$, and

K denotes the constant corresponding to the operating parameters of the system.

Comparing equation (2.4) with equation (2.1), we can find that the path-loss exponent $a = 2$ and $K_S = \left(\frac{\lambda}{4\pi} \right)^2 \cdot K$.

In the case of the ground reflection model, the received power, P_s , is given by

$$P_s = \left(\frac{K \cdot G_T \cdot G_R \cdot H_T^2 \cdot H_R^2}{R_t^4} \right) \cdot S_S \cdot p_T$$

where,

K denotes the constant corresponding to the operating parameters of the system,

G_T denotes the gain of the T_x antenna

H_T denotes the height of the T_x antenna

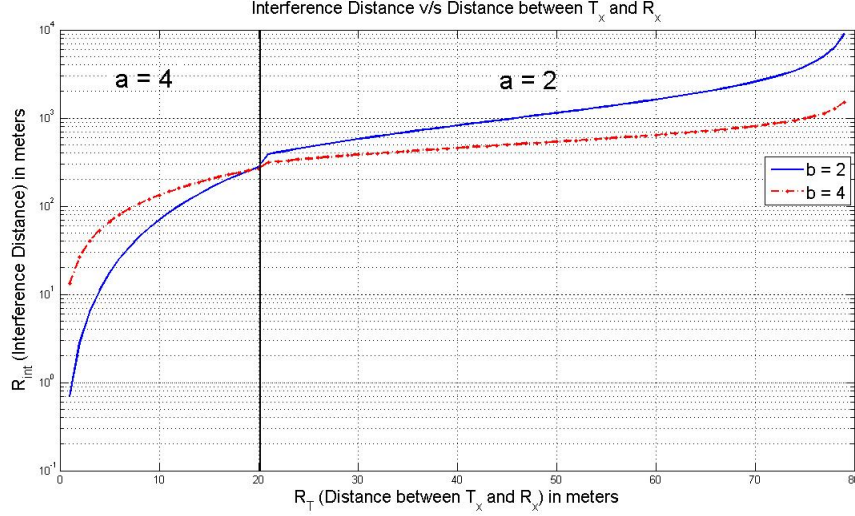


Figure 2.2: R_{int} (Interference Distance) vs. R_T (Distance between T_x and R_x)

G_R denotes the gain of the R_x antenna

H_R denotes the height of the R_x antenna

In this case, we have $a = 4$ and $K_S = K \cdot G_T \cdot G_R \cdot H_T^2 \cdot H_R^2$. Similarly, the radio propagation model between I and R_x can also be considered as a free-space path-loss model with $b = 2$ and $K_I = \left(\frac{\lambda}{4\pi}\right)^2 \cdot K$ or a ground reflection model with $b = 4$ and $K_I = K \cdot G_I \cdot G_R \cdot H_I^2 \cdot H_R^2$, where G_I and H_I denote the gain and height of the antenna on the interferer I .

Figure 2.2 shows the plot of the interference radius R_{int} as a function of R_T , the distance between T_x and R_x . It also depicts the dependency of the interference radius R_{int} on the path-loss exponents a and b . The simulation was conducted with the following values: $f = 482\text{MHz}$, $p_T = 15\text{dBm}$; $G_T = 2\text{dBm}$, $H_T = 1\text{m}$, $G_R = 0\text{dB}$, $H_R = 1\text{m}$, $S_S = 0\text{dB}$ (no shadowing), $p_{int} = 20\text{dBm}$, $G_I = 0\text{dB}$, $H_I = 10\text{m}$, $S_I = 0\text{dB}$ (no shadowing) and $K = 1$. The boundary radius, R_0 , is fixed to be 80m and the threshold $Z = 20\text{dB}$. It can be seen that R_{int} increases with increase in R_T . When R_x is close to T_x , the SNR at R_x is high. Hence, the interferer I should be very close to R_x so that it can contribute a large enough interference power P_I to reduce the SNR below the threshold Z and cause harmful interference to R_x . Therefore, the interference radius R_{int} is small for low values of R_T . With increase in separation of the primary receiver from the primary transmitter,

the received power decreases based on the path-loss exponent a . The ground reflection model with a pathloss exponent, $a = 4$ is suitable when $R_T \leq d_f$, where $d_f = \frac{4\pi H_t H_r}{\lambda}$. For $R_T > d_f$, the free-space pathloss model is applicable with the pathloss exponent, $a = 2$ [13]. With the transition to the free-space pathloss model, the constant K_S is now dependent only on the wavelength and is independent of antenna heights. This explains the increase in the slope of the curve for R_{int} at $R_T = d_f$. When R_x is far from T_x , the SNR at R_x is low. The interferer I can be quite far from R_x contributing a small interference power P_I . But this small amount of interference power P_I might be sufficient enough to further degrade the SNR below the threshold Z and cause harmful interference to R_x . Therefore, the interference radius R_{int} is large for higher values of R_T .

2.1.2 Detection radius, R_{det}

For the model shown in Figure 2.1, let us consider the scenario where the primary receiver, R_x , is broadcasting beacons. The cognitive radio interferer can detect this beacons and switch off his radio or hop to a different frequency to reduce interference to R_x . Let R_{bec} denote the maximum distance possible between R_x and I for successful detection of the beacons. In other words, the signal to noise ratio (SNR) required for successful detection between R_x and I is at its threshold level Z_{bec} at a distance R_{bec} [2]. Let P_{Sdet} and P_{Ndet} denote the received signal power and noise power at I respectively. Therefore, the received signal power, P_{Sdet} is given by

$$P_{Sdet} = \frac{K_I \cdot S_I \cdot p_{bec}}{R_I^b}$$

where p_{bec} is the beacon transmission power of R_x .

Since the beacons are almost always in a different band than the $T_x \rightarrow R_x$ channel, the received noise power P_{Ndet} is completely determined by the Noise, denoted by N_{det} . i.e. $P_{Ndet} = N_{det}$.

At a distance R_{bec} from R_x , using the definition of the SNR threshold, Z_{det} , we have¹

$$\begin{aligned} Z_{det} &= \frac{P_{Sdet}}{N_{det}} \\ &= \frac{K_I \cdot p_{bec}}{R_{bec}^b \cdot N_{det}} \end{aligned}$$

Therefore, we can find the noise power N_{det} as below

$$N_{det} = \frac{K_I \cdot p_{bec}}{R_{bec}^b \cdot Z_{det}}$$

For the cognitive radio interferer I , to properly detection of R_x , we should have

$$\begin{aligned} \frac{P_{Sdet}}{P_{Ndet}} &\geq Z_{det} \\ \left(\frac{\left(\frac{K_I \cdot S_I \cdot p_{bec}}{R_I^b} \right)}{\left(\frac{K_I \cdot p_{bec}}{R_{bec}^b \cdot Z_{det}} \right)} \right) &\geq Z_{det} \\ R_I &\leq R_{bec}(S_I)^{\frac{1}{b}} \end{aligned}$$

Let $R_{det} = R_{bec}(S_I)^{\frac{1}{b}}$. Thus, when the cognitive radio interferer I is at a distance $R_I \leq R_{det}$ from R_x , it successfully detects the primary receiver. Hence R_{det} is called the detection radius of the primary receiver. The values of the constants and pathloss exponents depend on the propagation model applied.

We can note that this interference and detection radius pair form concentric circles with the primary receiver as the center. We have two cases.

- (1) $R_{det} < R_{int}$ - In this case, there is some interference possible in spite of detection. This is because the interferer detects the primary receiver after entering its interference zone. But the amount of interference is less than having no detection.
- (2) $R_{det} \geq R_{int}$ - In this case, there is no interference since the interferer detects the primary receiver before entering its interference zone.

¹ Z_{det} is defined for $R_I = R_{bec}$ and $S_I = 1$

2.2 Discussion

This chapter explained the interference and detection radius of a primary receiver and their relation to various system and radio propagation parameters like antenna gain, antenna height, path-loss, shadowing, etc. In most cases, the radio link between the primary receiver and the primary transmitter is modeled using a ground reflection model with the pathloss exponent, $a = 4$ since the distance between them is quite small. For the unmanned aircraft which is high above above the ground, the radio link is more likely to be modeled as a free-space pathloss model with the pathloss exponents $b = 2$ and $c = 2$. The aircraft might encounter multiple primary receivers on its path. In such cases, what is the mean and total interference duration? How does the interference radii of multiple primary receivers determine the interference durations? How will the detection capability aid in reducing interference? These questions are answered in subsequent chapters.

Chapter 3

Interference Time Analysis for a one-dimensional system

In this chapter, we consider a cognitive radio placed on an unmanned aircraft which flies over a region consisting of primary receivers. This chapter covers the interference time analysis for a 1D system where the primary receivers are distributed on a straight line. Queueing theory aspects discussed in Appendix C are used to derive the statistics of interference time.

3.1 Interference Time Analysis for a 1D system with fixed interference and detection radii

3.1.1 Cognitive radio without any detection capability i.e $R_{det} = 0$

The setting of a 1D system model in this case is as shown in Figure 3.1. The path of the unmanned aircraft is a straight line with a total distance L . Assuming that the velocity of the aircraft is constant, let the total time required to cover this distance be T . The cognitive radio on the unmanned aircraft has no detection capability. There are N primary receivers distributed along the path of the plane according to a Poisson process with density λ' . This density λ' denotes the number of primary receivers per unit distance. Let λ denote the number of primary receivers per unit time. Then, $\lambda = \lambda'v$, where v is the velocity of the unmanned aircraft. Therefore, the expected number of primary receivers, $E(N)$ is given by, $E(N) = \lambda' L = \lambda T$. Each primary receiver has an interference radius of R_{int} , i.e. any device operating within a radius R_{int} causes interference to that primary receiver. An interference start time t_{is} is defined as the time at which the unmanned aircraft starts interfering with one or more primary receivers. An interference end time t_{ie} is defined

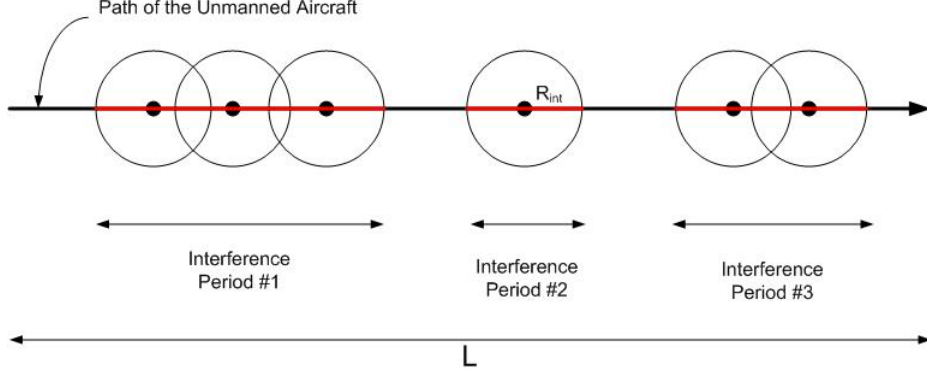


Figure 3.1: 1D scenario for a CR without detection

as the time at which the unmanned aircraft stops interfering with this set of primary receivers. The difference between the end time and the start time is defined to be the interference period t_i . i.e. $t_i = t_{ie} - t_{is}$.

This model is analogous to a queueing theory problem discussed in Appendix C. The primary receivers can be regarded as customers entering a queue from a Poisson arrival process (Markov) with density λ . The service time for each customer can be mapped to the interference period per primary receiver, which is $\frac{2R_{int}}{v}$, where v is the velocity of the unmanned aircraft. These interference periods can be overlapping, i.e. the cognitive radio on the unmanned aircraft does not wait for an interference period from one primary receiver to end before it begins interfering with the next one. This is analogous to having infinite number of servers in the queueing system. Hence this model can be analyzed using a $M/G/\infty$ queue, where G denotes the cdf of the interference period with the mean, $E(S)$, given by, $E(S) = \frac{2R_{int}}{v}$. The cdf of the interference period, G , is given by

$$G_I(i) = \begin{cases} 0; & i < \frac{2R_{int}}{v} \\ 1; & i \geq \frac{2R_{int}}{v} \end{cases}$$

In [8], the busy period of an $M/G/\infty$ queue is defined to be a time interval during which at-least one server is occupied. The paper provides the mean and distribution of the busy period along with the necessary and sufficient conditions for the results to hold. It is found that the busy

period approaches an exponential distribution for increasing λ with mean given by

$$E(B) = \frac{e^{\lambda E(S)} - 1}{\lambda}$$

where B and S denote the random variable for busy period and service time respectively. It should be noted that the expression for the mean busy period holds for any λ , while the exponential behavior of the distribution of busy period is an asymptotic result that holds for large values of λ [8]. The necessary and sufficient conditions for these results to hold are

- (1) The mean service time is finite. i.e. $E(S) < \infty$.
- (2) The service time is strictly positive. i.e. $P(S > 0) = 1$.
- (3) The distribution of service time should satisfy the criterion $(\log x) \int_x^\infty (1 - G(y)) \cdot dy \rightarrow 0$, as $x \rightarrow \infty$.

The interference period that is analogous to service time of an $M/G/\infty$ queue satisfies these conditions. Hence, the interference period for the entire excursion of the unmanned aircraft can then be mapped to the total busy period of this $M/G/\infty$ queue. Therefore, for this 1D model, the interference period follows an exponential distribution with mean

$$E(I) = \frac{e^{\lambda E(S)} - 1}{\lambda}$$

Using $E(S) = \frac{2R_{int}}{v}$,

$$E(I) = \frac{e^{\frac{2\lambda R_{int}}{v}} - 1}{\lambda}$$

It is further derived in [8] that the number of idle periods approaches a Poisson process of intensity $\lambda e^{-\lambda E(S)}$. Therefore, the number of idle periods in the interval $[0, T]$, N_{idle} for $E(S) < \infty$ is given by

$$N_{idle} = \lambda e^{-\lambda E(S)} T$$

Considering that the total time is large and ignoring the effects at the edges, a busy period is followed by an idle period, i.e. the expected number of busy and idle periods are equal [8].

$$N_{busy} = N_{idle} = \lambda e^{-\lambda E(S)} T$$

For this 1D model, the expected number of busy periods is nothing but the expected number of interference periods N_{int} . Therefore,

$$N_{int} = \lambda e^{-\lambda E(S)} T$$

Using $E(S) = \frac{2R_{int}}{v}$ and $E(N) = \lambda T$, we have

$$N_{int} = e^{-\frac{2\lambda R_{int}}{v}} E(N)$$

Let the total interfering time for the entire excursion of the unmanned aircraft be denoted by T_{int} . Then, the mean total interfering time $E(T_{int})$ can be calculated as below

$$\begin{aligned} E(T_{int}) &= E(N_{int}) \cdot E(I) \\ &= e^{-\frac{2\lambda R_{int}}{v}} \cdot E(N) \cdot E(I) \end{aligned}$$

where $N \sim \text{poiss}(\lambda T)$ and $I \sim \exp\left(\frac{e^{\frac{2\lambda R_{int}}{v}} - 1}{\lambda}\right)$ and

$$\begin{aligned} E(T_{int}) &= e^{-\frac{2\lambda R_{int}}{v}} \cdot E(N) \cdot E(I) \\ &= e^{-\frac{2\lambda R_{int}}{v}} \cdot \lambda T \cdot \frac{(e^{\frac{2\lambda R_{int}}{v}} - 1)}{\lambda} \\ &= (1 - e^{-\frac{2\lambda R_{int}}{v}}) T \end{aligned}$$

3.1.2 Cognitive radio with detection capability i.e $0 < R_{det} \leq R_{int}$

The setting of a 1D system model in this case is the same as the model in the previous section. But the cognitive radio on the unmanned aircraft can now detect any primary receiver within a radius R_{det} . It should be noted that if $R_{det} \geq R_{int}$, then there will be no interference periods. We consider the case where $R_{det} < R_{int}$. This is shown in Figure 3.2.

The expected number of interference periods, N_{int} , and the mean interference period, $E(I)$, due to the interfering radius R_{int} was calculated in Section 3.1.1.

$$\begin{aligned} N_{int} &= e^{-\frac{2\lambda R_{int}}{v}} E(N) \\ E(I) &= \frac{e^{\frac{2\lambda R_{int}}{v}} - 1}{\lambda} \end{aligned}$$

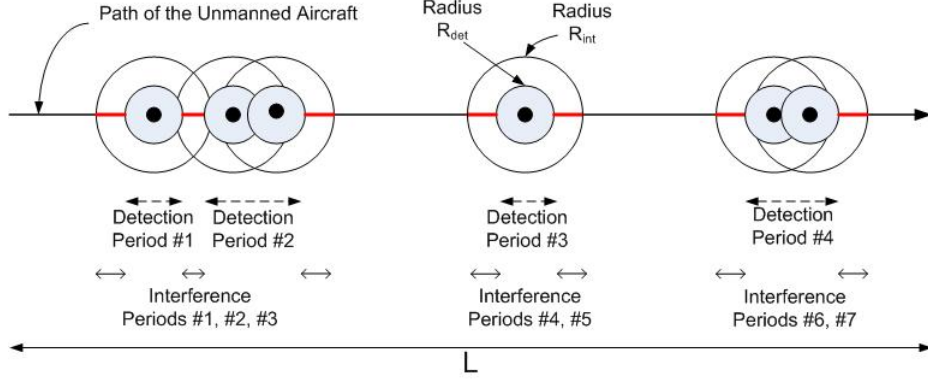


Figure 3.2: 1D scenario for a CR with detection capability

Similarly, the expected number of detection periods, N_{det} , and the mean detection period, $E(D)$, due to the detection radius R_{det} can be calculated as

$$N_{det} = e^{-\frac{2\lambda R_{det}}{v}} E(N)$$

$$E(D) = \frac{e^{\frac{2\lambda R_{det}}{v}} - 1}{\lambda}$$

The mean of the total interfering time for the entire excursion of the unmanned aircraft, $E(T_{ID})$, is the difference between the mean total interfering time and the mean total detection time.

$$E(T_{ID}) = E(N_{int})E(I) - E(N_{det})E(D)$$

$$= (1 - e^{-\frac{2\lambda R_{int}}{v}})T - (1 - e^{-\frac{2\lambda R_{det}}{v}})T$$

$$E(T_{ID}) = (e^{-\frac{2\lambda R_{det}}{v}} - e^{-\frac{2\lambda R_{int}}{v}})T \quad (3.1)$$

We can rewrite the above equation as $E(T_{ID}) = (e^{-\lambda E(S_{det})} - e^{-\lambda E(S)})T$, where $E(S_{det})$ and $E(S)$ denote the mean detection time and the mean interference time per primary user. In this case, $E(S_{det}) = \frac{2R_{det}}{v}$ and $E(S) = \frac{2R_{int}}{v}$. The value of λ for which this expected total interfering time is maximum can be derived from equation (3.1). Let us denote this by λ_f .

$$\lambda_f = \frac{1}{(E(S) - E(S_{det}))} \log \left(\frac{E(S)}{E(S_{det})} \right) \quad (3.2)$$

Let f denote the maximum fraction of the total time being interfered by the unmanned aircraft. It is calculated as the ratio of the mean total time, $E(T_{ID})$, evaluated at $\lambda = \lambda_f$ to the total time of the excursion, T .

$$\begin{aligned} f &= \frac{(e^{-\lambda_f E(S_{det})} - e^{-\lambda_f E(S)})T}{T} \\ &= e^{-\lambda_f E(S_{det})} \left(1 - e^{\lambda_f (E(S_{det}) - E(S))}\right) \end{aligned}$$

Using r to denote the ratio of the mean detection time to the mean interference time per primary user. i.e. $r = \frac{E(S_{det})}{E(S)}$ and substituting for λ_f using equation 3.2, we get

$$f = r^{\frac{r}{1-r}} (1 - r) \quad (3.3)$$

It can be seen that the fraction f approaches unity as the mean detection time approaches zero, i.e. as $E(S_{det}) \rightarrow 0$ or in other words as $r \rightarrow 0$, the fraction $f \rightarrow 1$. From Figure 3.3, we can note that the fraction f decreases with increase in the mean detection time per primary user. As $E(S_{det}) \rightarrow E(S)$ or in other words as $r \rightarrow 1$, the fraction $f \rightarrow 0$.

Now we discuss the idle (no-interference) and busy (interference) periods that occur in this model. These are used to calculate the mean interference time for the entire excursion of the aircraft. In this case, an idle period might occur due to the following two scenarios

- (1) The cognitive radio on the unmanned aircraft might have regions where it does not interfere with any primary receiver. These regions will result in idle periods. i.e the original set of idle periods generated due to the interference radius R_{int} .
- (2) The cognitive radio either switches off or hops to a different frequency on detecting the primary receiver within a radius R_{det} and does not cause any interference to it. So, the “busy” detection periods can also be regarded as idle periods. This is nothing but $E(N_{det})$.

We know that the expected number of idle periods due to R_{int} is equal to the expected number of busy periods due to R_{int} , given by $E(N_{int})$ [8]. Therefore, the expected total number of idle periods is the sum of these two kinds of idle periods. This is again same as the expected total

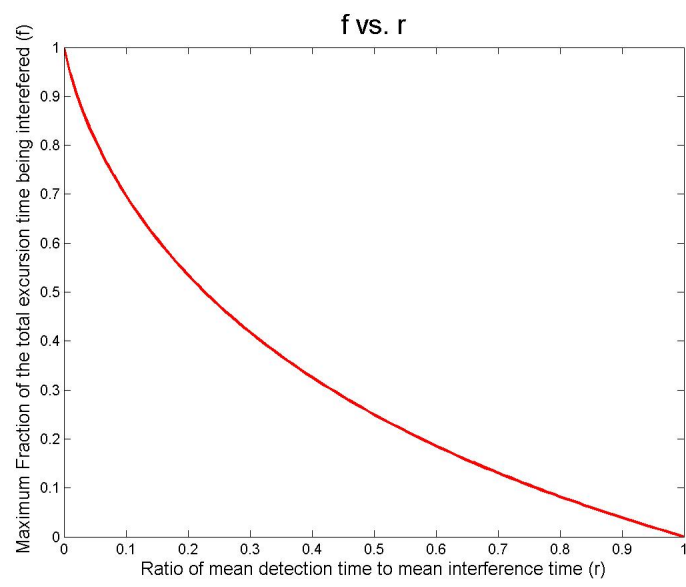


Figure 3.3: Maximum fraction of the total time vs. Ratio of the mean detection time to the mean interference time

busy periods. The mean interference period, $E(I_{det})$, is mean total interference time divided by the expected total busy periods. It can be calculated as below

$$\begin{aligned}
E(I_{det}) &= \frac{E(N_{int})E(I) - E(N_{det})E(D)}{E(N_{int}) + E(N_{det})} \\
&= \frac{(1 - e^{-\frac{2\lambda R_{int}}{v}})T - (1 - e^{-\frac{2\lambda R_{det}}{v}})T}{\lambda T(e^{-\frac{2\lambda R_{int}}{v}} + e^{-\frac{2\lambda R_{det}}{v}})} \\
&= \frac{(1 - e^{-\frac{2\lambda}{v}(R_{int} - R_{det})})}{\lambda(1 + e^{-\frac{2\lambda}{v}(R_{int} - R_{det})})} \\
&= \frac{1}{\lambda} \tanh\left(\frac{\lambda}{v}(R_{int} - R_{det})\right)
\end{aligned}$$

3.1.3 Cognitive radio with detection capability with $R_{det} > R_{int}$

In this case, there are no interference outages. Only detection outages occur. It is significant because the switch-off or alternate frequency operation mode of the cognitive radio on the unmanned aircraft can be modeled using the statistics of the detection outages. The results are similar to those derived in Section 3.1.1 with R_{det} substituted for R_{int} . For example, the distribution of the detection periods gives the distribution of the duration for which the cognitive radio is either switched off or operating in a different frequency band.

3.2 Interference Time Analysis for a 1D system with variable interference and detection radii

3.2.1 Cognitive radio without any detection capability i.e $R_{det} = 0$

The setting of a 1D system model in this case is as shown in Figure 3.4. It is similar to the one discussed in Section 3.1.1, but the the interference radius R_{int} is not from a deterministic distribution. It can be drawn from any general distribution G satisfying the necessary conditions stated in Section 3.1.1. Then, the mean interference period per primary user, $E(S)$, is given by $E(S) = \frac{2E(R_{int})}{v}$, where v is the velocity of the unmanned aircraft and $E(R_{int})$ is the expected interference radius.

Using $E(S) = \frac{2E(R_{int})}{v}$, the mean interference period, $E(I)$ and expected total interference

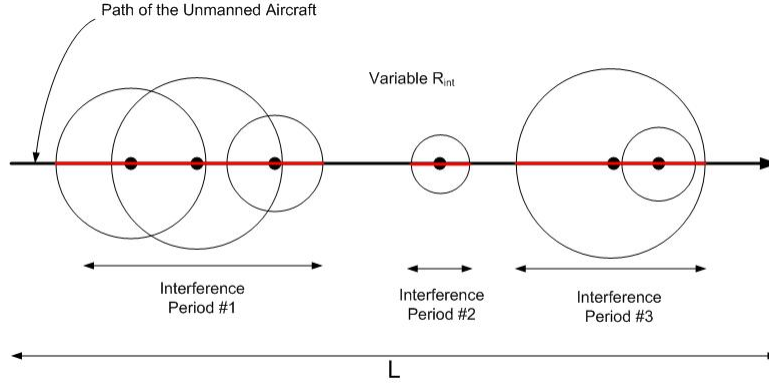


Figure 3.4: 1D scenario for primary receivers with variable interference radius R_{int} and a CR without detection

time, $E(T_{int})$, are calculated as below

$$E(I) = \frac{e^{\frac{2\lambda E(R_{int})}{v}} - 1}{\lambda}$$

$$E(T_{int}) = (1 - e^{-\frac{2\lambda E(R_{int})}{v}})T$$

3.2.2 Cognitive radio with variable detection capability i.e $0 < R_{det} \leq R_{int}$

The 1D system for this case is similar to that discussed in previous section. It is shown in Figure 3.5. But the cognitive radio can detect a primary receiver with a detection radius R_{det} . The detection radius, R_{det} , is not a constant. It is drawn from a distribution H which satisfies the conditions discussed in Section 3.1.1. Then, the mean detection period per primary user, $E(S_{det})$, is given by $E(S_{det}) = \frac{2E(R_{det})}{v}$, where v is the velocity of the unmanned aircraft and $E(R_{det})$ is the expected detection radius.

Using the results derived in the previous section and Section 3.1.2, the mean interference period, $E(I_{det})$ and expected total interference time, $E(T_{ID})$, are calculated as below

$$E(I_{det}) = \frac{1}{\lambda} \tanh \left(\frac{\lambda}{v} (E(R_{int}) - E(R_{det})) \right)$$

$$E(T_{ID}) = (e^{-\frac{2\lambda E(R_{det})}{v}} - e^{-\frac{2\lambda E(R_{int})}{v}})T$$

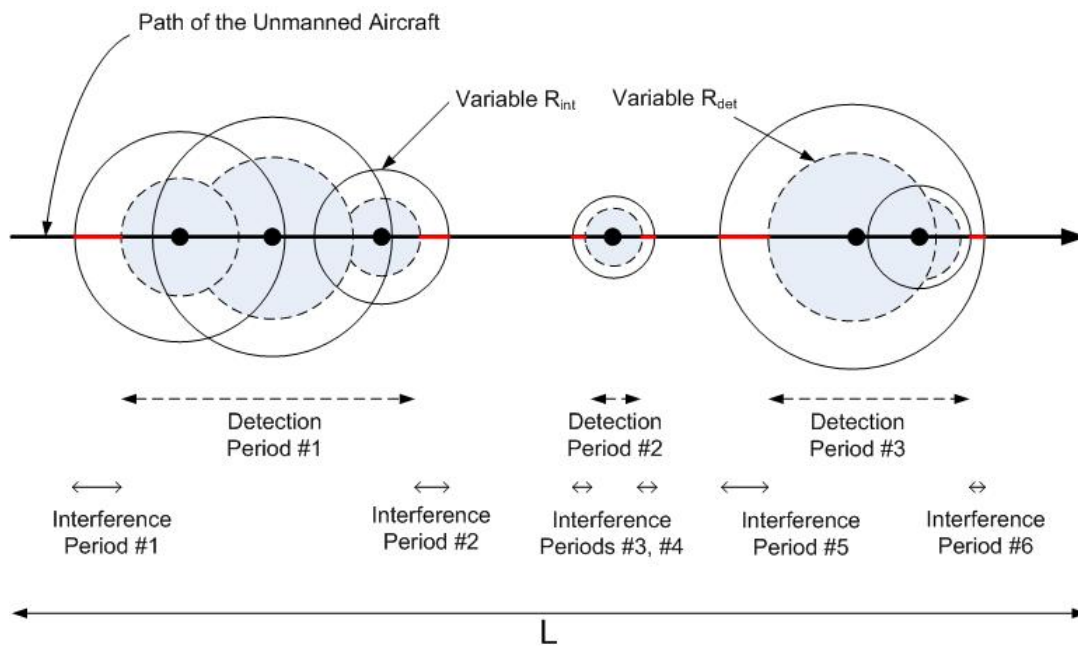


Figure 3.5: 1D scenario for primary receivers with variable interference radius R_{int} and a CR with variable detection radius R_{det}

3.2.3 Cognitive radio with variable detection capability with $R_{det} > R_{int}$

This case is similar to the one discussed in Section 3.1.3, but with variable interference and detection radii. The expected number of interference outages will be zero since $R_{det} > R_{int}$. Hence, only detection outages are present. The statistics of these detection outages can be found using the results derived in Section 3.2.1 with $E(R_{det})$ substituted for $E(R_{int})$.

3.3 Simulation and results for 1D scenario

The simulations were carried out in MATLAB. The total length to be covered by the unmanned aircraft, L , was taken to be 1000 units. The number of primary receivers, $N \sim \text{poiss}(\lambda L)$. The path of the unmanned aircraft was chosen to be the x -axis. For the 1D scenario, N uniform random variables $H = \{h_1, h_2, \dots, h_N\}$ were chosen such that $h_i \sim \text{unif}(0, L)$, where $i = 1, 2, \dots, N$ and the location of each primary receiver is given by $(h_i, 0)$, i.e. all the primary receivers were located on the x -axis. The 1D scenario was analyzed when the CR had no detection capability and when the CR could detect a primary receiver with a detection radius R_{det} . The velocity, v , of the unmanned aircraft is assumed to be constant of 1 unit so that all the distance calculations directly map to the time calculations.

To analyze the interference time distribution, we use the empirical cdf and qq-plot. The qq-plot is a probability plot that is used for graphically comparing two probability distributions [16]. Let F_A denote the cdf of the unknown distribution and F_B denote the cdf of the reference distribution. Then a qq-plot graphs $(F_A^{-1}(X), F_B^{-1}(X))$ for $X \in [0, 1]$. It has the property that if F_A and F_B are the same distribution, then the qq-plot is a straight line. If A or B is empirical data, then empirical distribution is used. For more details on the qq-plot, refer to Appendix B.

Figure 3.6 gives the mean and total interfering time for 1D scenario when the cognitive radio interferer has no detection capability. The interfering radius was 0.5 units. This corresponds to a mean interfering period per primary receiver, $E(S) = \frac{2R_{int}}{v} = 1$. It can be seen that the total interfering time also matches the analytical results and saturates for $\lambda > 7.5$. The mean interfering

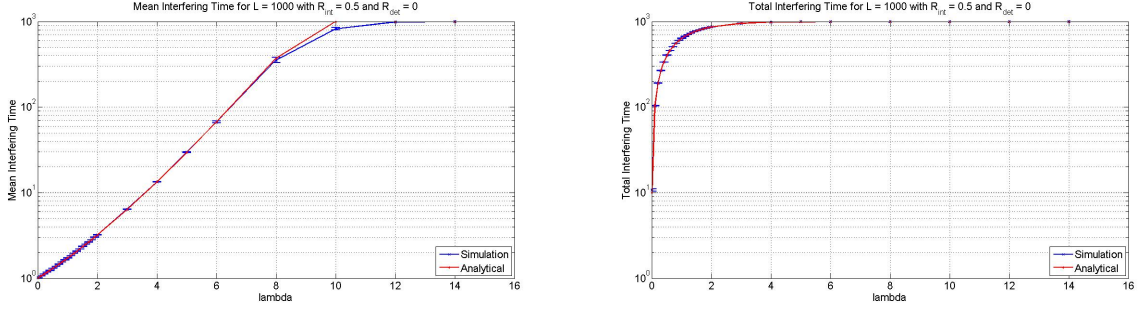


Figure 3.6: Mean and total interfering time for $L = 1000$, $R_{int} = 0.5$ and $R_{det} = 0$, 1D case - No Detection

time increases exponentially, matching with the analytical calculation, $\min\left(\frac{e^{\lambda E(S)} - 1}{\lambda}, L\right)$, for any λ before saturation.

Figure 3.7 and Figure 3.8 give the qq-plot and cdf of the interfering time. It can be seen that the interfering time follows an exponential distribution with the mean $\frac{e^{\lambda E(S)} - 1}{\lambda}$ for $2 < \lambda < 7.5$. For $\lambda > 7.5$, the interference time saturates to the total time.

Figure 3.9 gives the mean and total interfering time for 1D scenario when the cognitive radio interferer has no detection capability. The interfering radius was 1.5 units ($E(S) = 3$). With the increase in interference radius, we can note that the total interference time saturates for a lower density λ . In this case it is $\lambda = 2.5$, as compared to $\lambda = 7.5$ when $E(S) = 1$.

Figure 3.10 gives the mean and total interfering time for 1D scenario when the cognitive radio interferer has an interfering radius of 0.5 units i.e. mean interference period per primary receiver is 1 unit and a detection radius of 0.25 units, i.e. mean detection period per primary receiver is 0.5 units. The mean and the total interference time match with the analytical results. In this case, we can note that the total interference time does not saturate for any λ . It increases initially because the interference time will be more than detection time. With an increase in λ , the detection time also increases. It will be more likely that the interference period of one primary receiver overlaps with the detection period of the adjacent primary receiver. So, the total interference time decreases and gradually reaches zero for large λ . The value of f is calculated analytically using equation 3.3 to be 0.25, which means that maximum interfered time is 25% of the total excursion time. The

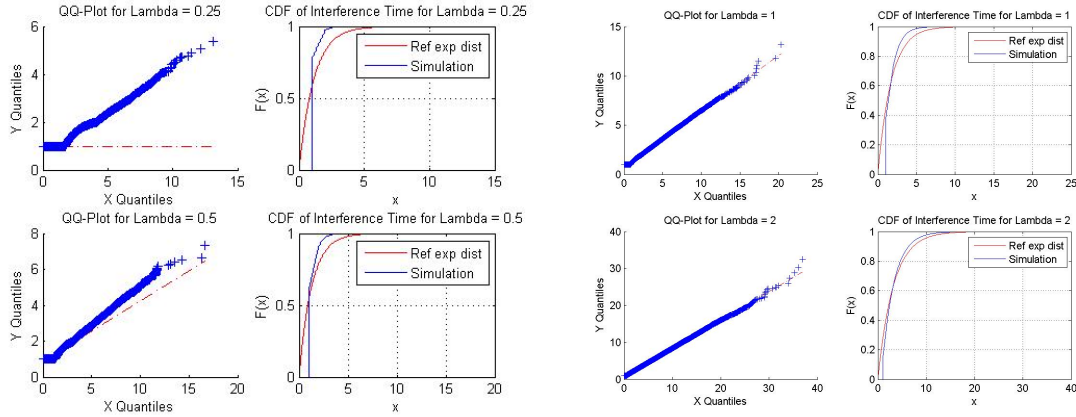


Figure 3.7: QQ-Plot and CDF plot of Interference Time for $R_{int} = 0.5$ and $R_{det} = 0$, $\lambda = 0.25, 0.5, 1, 2$, 1D case - No Detection

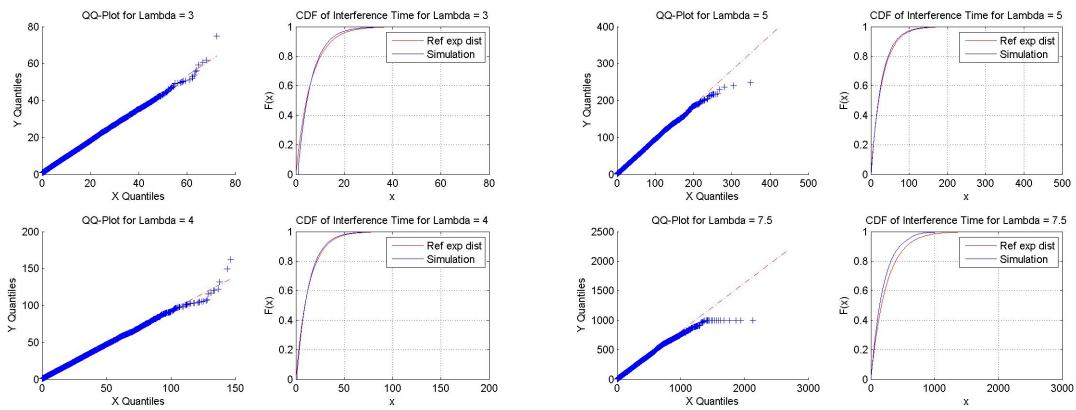


Figure 3.8: QQ-Plot and CDF plot of Interference Time for $R_{int} = 0.5$ and $R_{det} = 0$, $\lambda = 3, 4, 5, 7.5$, 1D case - No Detection

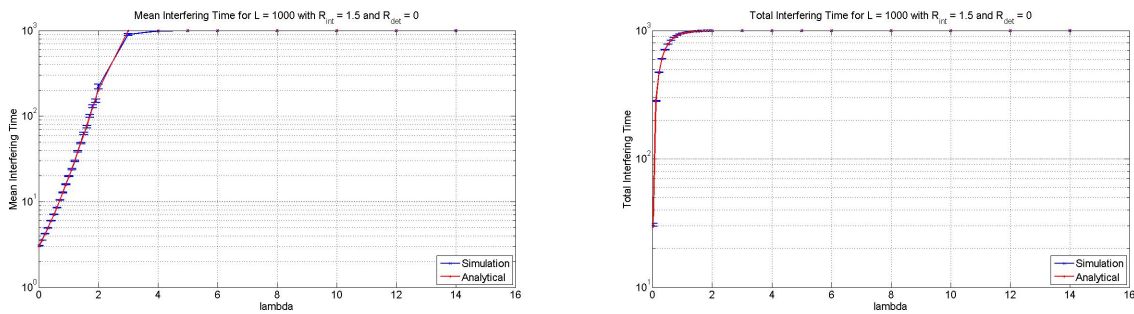


Figure 3.9: Mean and total interfering time for $L = 1000$, $R_{int} = 1.5$ and $R_{det} = 0$, 1D case - No Detection

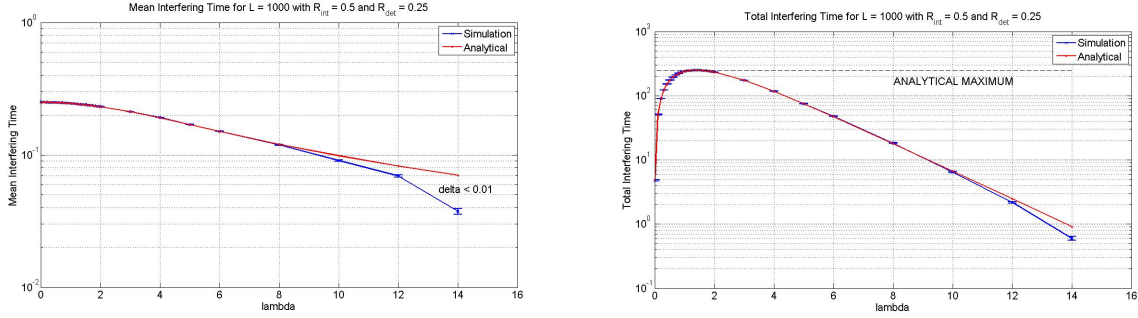


Figure 3.10: Mean and total interfering time for $L = 1000$, $R_{int} = 0.5$ and $R_{det} = 0.25$, 1D case - With Detection

total excursion time for the simulation is 1000 units. Hence, the maximum interfered time is 250 units which agrees with the analytical calculation.

Figure 3.11 gives the mean and total interfering time for 1D scenario when the cognitive radio interferer has an interfering radius of 1.5 units (mean interference period per primary receiver is 3 units) and a detection radius of 1 unit (mean detection time per primary receiver is 2 units). The results are similar to the previous case except that the total interference time reaches its maximum for a lower value of λ with the increase in interference and detection radii. The value of f is calculated analytically using equation 3.3 to be 0.1481, which means that maximum interfered time is 14.81% of the total excursion time. The maximum interfered time is 148.1 units in simulation which agrees with the analytical calculation.

Figure 3.12 gives the mean and total interfering time for 1D scenario when the cognitive radio interferer has a variable interfering radius and no detection capability. The interference radius is drawn from an uniform distribution, i.e. $R_{int} \sim \text{unif}(0, 1)$. Therefore, the mean interference radius, $E(R_{int})$, is 0.5 unit. This corresponds to a mean interference period per primary receiver, $E(S)$, of 1 units. The mean and total interference time depict the same behavior as for the fixed $R_{int} = 0.5$ units (Refer to Figure 3.6). This is because these parameters are independent of the distribution of interference period and depend only on $E(S)$.

Figure 3.13 gives the mean and total interfering time for 1D scenario when the cognitive radio interferer has a variable interfering radius and variable detection capability. The interference radius

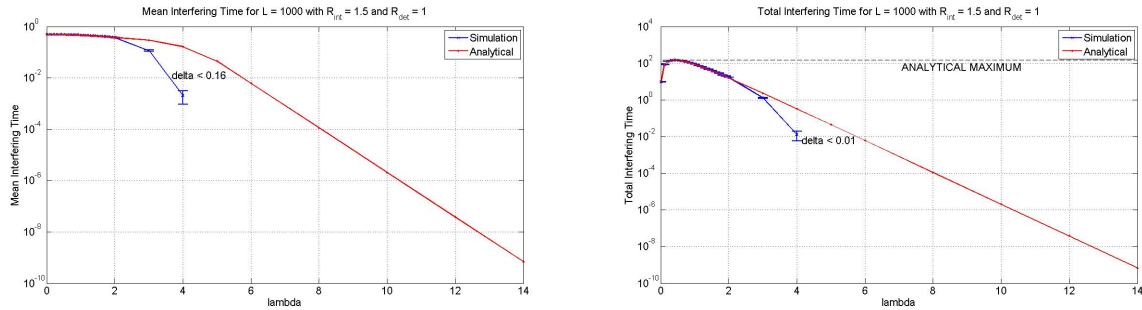


Figure 3.11: Mean and total interfering time for $L = 1000$, $R_{int} = 1.5$ and $R_{det} = 1$, 1D case - With Detection

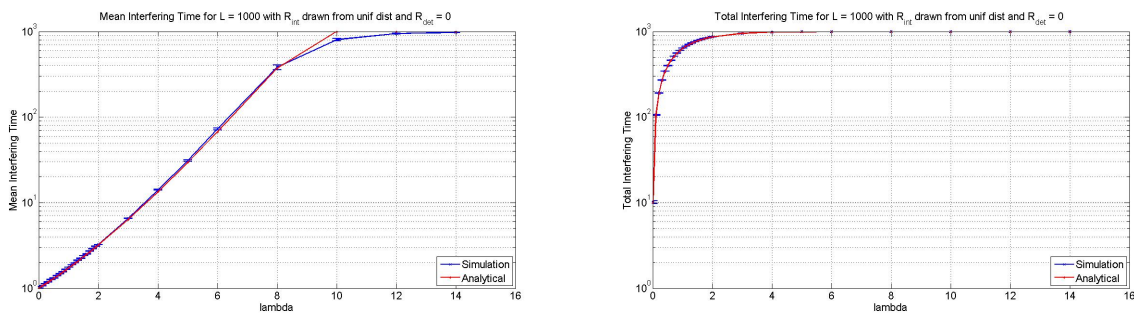


Figure 3.12: Mean and total interfering time for $L = 1000$, $R_{int} = \text{unif}(0, 2)$ and $R_{det} = 0$, 1D case - Variable R_{int} , No Detection

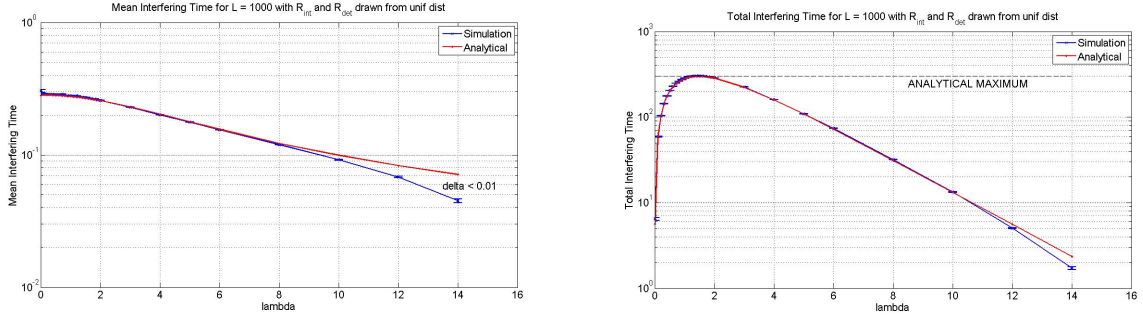


Figure 3.13: Mean and total interfering time for $L = 1000$, $R_{int} = \text{unif}(0, 2)$ and $R_{det} = \text{unif}(0, 1)$, 1D case - Variable R_{int} , Variable R_{det}

is drawn from an uniform distribution, i.e. $R_{int} \sim \text{unif}(0, 1)$. The detection radius is also drawn from an uniform distribution such that $R_{det} \sim \text{unif}(0, 0.5)$. If $R_{det} > R_{int}$, then we set $R_{det} = R_{int}$. This corresponds to a mean interference period per primary receiver, $E(S)$, of 1 unit. The mean detection period per primary receiver, $E(D)$, is calculated using MATLAB to be 0.431 units. The value of f is calculated analytically using equation 3.3 to be 0.3044, which means that maximum interfered time is 30.44% of the total excursion time. The maximum interfered time is 301 units through simulation which agrees with the analytical calculation.

3.4 Summary

This chapter described the interference time analysis for a 1D system using queueing theory. It explained the mapping between an $M/G/\infty$ queue and interference time. It is found that the statistics of interference time are independent of the distribution of R_{int} and R_{det} . Table 3.1 summarizes the results derived in this chapter for a 1D system.

Parameter	No detection	With Detection
Mean Interference Time per primary user, $E(S)$	$\frac{2E(R_{int})}{v}$	$\frac{2E(R_{int})}{v}$
Mean Detection Time per primary user, $E(S_{det})$	$\frac{2E(R_{det})}{v}$	$\frac{2E(R_{det})}{v}$
Expected Number of Detection Periods, $E(N_{det})$	0	$\lambda e^{-\frac{2\lambda E(R_{det})}{v}} T$
Expected Number of Interference Outages	$\lambda e^{-\frac{2\lambda E(R_{int})}{v}} T$	$\lambda(e^{-\frac{2\lambda E(R_{det})}{v}} + e^{-\frac{2\lambda E(R_{det})}{v}}) T$
Mean Interference Time for the entire excursion	$E(I) = \frac{e^{-\frac{2\lambda E(R_{int})}{v}} - 1}{\lambda}$	$E(I_{det}) = \frac{1}{\lambda} \tanh\left(\frac{\lambda}{v}(E(R_{int}) - E(R_{det}))\right)$
Total Interference Time for the entire excursion	$E(T_{int}) = (1 - e^{-\frac{2\lambda R_{int}}{v}}) T$	$E(T_{ID}) = (e^{-\frac{2\lambda E(R_{det})}{v}} - e^{-\frac{2\lambda E(R_{int})}{v}}) T$

Table 3.1: Results for the 1D system

Chapter 4

Interference Time Analysis for a two-dimensional system

As in Chapter 3, we again consider a cognitive radio placed on an unmanned aircraft which flies over a region consisting of primary receivers. We extend the results for interference time derived for 1D systems to 2D systems, where the receivers are distributed in 2D plane according to a Poisson's process. Again, queueing theory (Refer to Appendix C) is used as a tool to derive the statistics of interference time.

4.1 Mean interference period per primary user for a 2D system

From the previous chapter, we note the mean interference period per primary user in the 1D case, $E(S)$, is given by $E(S) = \frac{2E(R_{int})}{v}$, where $E(R_{int})$ is the expected interference radius and v is the velocity of the unmanned aircraft. This was because the primary receiver “always” lies along the path of the unmanned aircraft in the 1D case and the interference distance is the diameter of a circle with radius R_{int} . But in the 2D case, the primary receiver need not lie exactly on the path of the unmanned aircraft. A method to determine the mean interference period per primary user in a 2D system is described below.

4.1.1 CR has no detection capability

Let us consider a simple model consisting of a single primary receiver with an interference radius R_{int} . Let (X, Y) be the set of random variables that represents the x and y coordinates of the cognitive radio respectively. Let a cognitive radio (CR) interferer move along a straight line

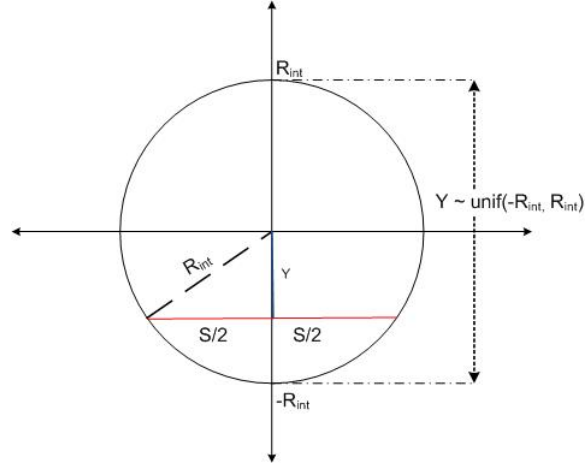


Figure 4.1: Interference Period (Chord length of a circle)

parallel to the horizontal axis of the receiver, i.e $X = 0$. The CR has no detection capability. Then the CR will interfere with the primary receiver if $-R_{int} \leq Y \leq R_{int}$. Let $Y \sim \text{unif}(-R_{int}, R_{int})$. Then, the interference distance is computed as the length of a chord of a circle with radius R_{int} whose distance from the center of the circle is $abs(Y)$. Let us denote this by S . (Refer to Figure 4.1)

From Figure 4.1, we can note that

$$S = 2\sqrt{R_{int}^2 - Y^2}$$

The cdf of the interference distance G can be derived as below

$$\begin{aligned}
G &= F_S(s) \\
&= P(S \leq s) \\
&= P(2\sqrt{R_{int}^2 - Y^2} \leq s) \\
&= P\left(Y^2 \geq \left(R_{int}^2 - \frac{s^2}{4}\right)\right) \\
&= 1 - P\left(Y^2 \leq \left(R_{int}^2 - \frac{s^2}{4}\right)\right) \\
&= 1 - P\left(-\sqrt{\left(R_{int}^2 - \frac{s^2}{4}\right)} \leq Y \leq \sqrt{\left(R_{int}^2 - \frac{s^2}{4}\right)}\right) \\
&= 1 - \frac{\sqrt{\left(R_{int}^2 - \frac{s^2}{4}\right)}}{R_{int}}
\end{aligned}$$

The mean interference distance for a single primary receiver $E(S)$ can be calculated as follows

$$\begin{aligned}
E(S) &= \int_0^{2R_{int}} (1 - F_S(s)) ds \\
&= \int_0^{2R_{int}} \left(\frac{\sqrt{\left(R_{int}^2 - \frac{s^2}{4}\right)}}{R_{int}} \right) ds \\
&= \frac{1}{2R_{int}} \int_0^{2R_{int}} \sqrt{(4R_{int}^2 - s^2)} ds \\
&= \frac{\pi R_{int}}{2}
\end{aligned}$$

Let v be the velocity of the CR. Then, the interference period, I , is given by $I = \frac{S}{v}$. Considering that the velocity of the CR remains constant throughout its flight, the mean interference period, T , can be calculated as

$$\begin{aligned}
T &= E(I) \\
&= \frac{E(S)}{v} \\
&= \frac{\pi R_{int}}{2v}
\end{aligned}$$

4.1.2 CR has detection capability

The model in this case is similar to the one considered in Section 4.1.1. The only change is that the CR can now detect a primary receiver within a detection radius R_{det} , i.e. the primary receiver falls in the detection range of the cognitive radio if $-R_{det} \leq Y \leq R_{det}$. Then, the mean detection period for a single primary receiver is given by

$$E(D) = P(-R_{det} \leq Y \leq R_{det})E(D | det = 1) + (1 - P(-R_{det} \leq Y \leq R_{det}))E(D | det = 0)$$

where $E(D | det = 1)$ is the mean detection period for a single primary receiver when it is IN the detection range and $E(D | det = 0)$ is the mean detection period for a single primary receiver when it is OUT of the detection range

Since $Y \sim \text{unif}(-R_{int}, R_{int})$, we can calculate

$$\begin{aligned} P(-R_{det} \leq Y \leq R_{det}) &= \frac{R_{det}}{R_{int}} \\ E(D | det = 1) &= \frac{\pi R_{det}}{2v} \\ E(D | det = 0) &= 0 \end{aligned}$$

Therefore,

$$E(D) = \frac{\pi R_{det}^2}{2R_{int}v}$$

Hence the mean interference period, T , for the CR can be calculated as below

$$\begin{aligned} T &= E(I) - E(D) \\ &= \frac{\pi R_{int}}{2v} - \frac{\pi R_{det}^2}{2R_{int}v} \\ &= \frac{\pi R_{int}}{2v} \left(1 - \frac{R_{det}^2}{R_{int}^2} \right) \end{aligned}$$

These calculations are used in the further parts of this chapter for the interference time analysis in 2D systems. If the locations of the primary receiver and the interferer are known, the mean interference distance can be calculated through a different approach which is discussed in Appendix A.

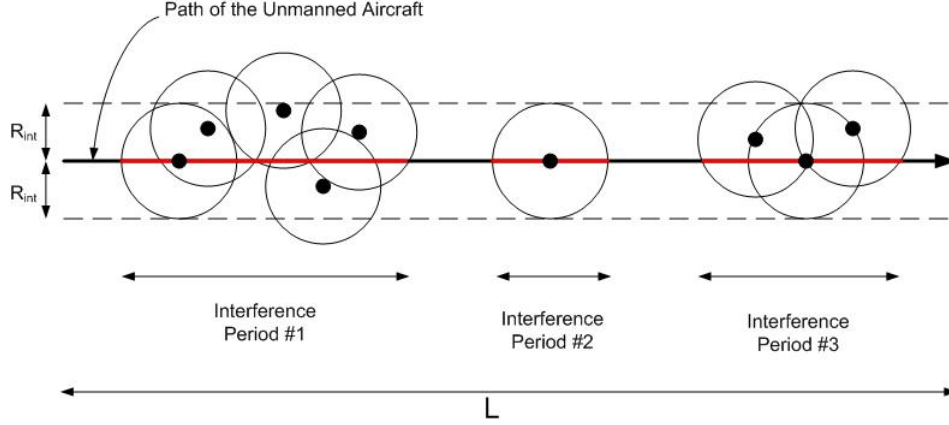


Figure 4.2: 2D scenario for a cognitive radio without detection capability

4.2 Interference Time Analysis for a 2D system with fixed interference and detection radii

4.2.1 Cognitive radio without any detection capability i.e $R_{det} = 0$

The setting of a 2D system model in this case is as shown in Figure 4.2. The path of the unmanned aircraft is a straight line with a total distance L . Assuming that the velocity of the aircraft is constant, let the total time required to cover this distance be T . The cognitive radio on the unmanned aircraft has no detection capability. Each primary receiver has an interference radius of R_{int} . The primary receivers are distributed across a 2D plane with length L and width R_{int} according to a Poisson's process with 2D density λ' . This density λ' denotes the number of primary receivers per unit area. Let λ_1 denote the number of primary receivers per unit time in 2D. Then, $\lambda_1 = \lambda' v$, where v is the velocity of the unmanned aircraft. If (X, Y) is the set of random variables that denotes the x and y coordinates of the primary receivers, then $X \sim \text{unif}(0, L)$ and $Y \sim \text{unif}(-R_{int}, R_{int})$. The equivalent 1D density calculated per unit time, λ , is given by $\lambda = \lambda_1 \cdot 2R_{int}$. Therefore, the expected number of primary receivers, $N = \lambda T$.

This model can be analyzed using an $M/G/\infty$ queue where G denotes the cdf of the interference period. Using the results for the $M/G/\infty$ queue discussed in the previous section, we know that the interference period for the entire excursion of the unmanned aircraft follows an exponential

distribution with mean

$$E(I) = \frac{e^{\lambda E(S)} - 1}{\lambda}$$

Using the results from Section 4.1.1, we have $E(S) = \frac{\pi R_{int}}{2v}$. Therefore,

$$E(I) = \frac{e^{\frac{\pi \lambda R_{int}}{2v}} - 1}{\lambda}$$

For this 2D model, the expected number of interference periods N_{int} is given by

$$N_{int} = \lambda e^{-\lambda E(S)} T$$

Using $E(S) = \frac{\pi R_{int}}{2v}$ and $E(N) = \lambda T$,

$$N_{int} = e^{-\frac{\pi \lambda R_{int}}{2v}} E(N)$$

Let the total interfering time for the entire excursion of the unmanned aircraft be T_{int} . Then, the mean total interfering time $E(T_{int})$ can be calculated as

$$E(T_{int}) = e^{-\frac{\pi \lambda R_{int}}{2v}} \cdot E(N) \cdot E(I)$$

where $N \sim \text{poiss}(\lambda T)$ and $I \sim \exp\left(\frac{e^{\frac{\pi \lambda R_{int}}{2v}} - 1}{\lambda}\right)$

$$\begin{aligned} E(T_{int}) &= e^{-\frac{\pi \lambda R_{int}}{2v}} \cdot \lambda T \cdot \frac{(e^{\frac{\pi \lambda R_{int}}{2v}} - 1)}{\lambda} \\ &= (1 - e^{-\frac{\pi \lambda R_{int}}{2v}}) T \end{aligned}$$

4.2.2 Cognitive radio with detection capability i.e. $0 < R_{det} \leq R_{int}$

The setting of a 2D system model in this case is the same as the model in the previous section. But the cognitive radio on the unmanned aircraft can now detect any primary receiver within a radius R_{det} . This is shown in Figure 4.3.

Using the results from 4.1.1 and 4.1.2, the number of interference periods, N_{int} , and the mean interference period, $E(I)$, due to interfering radius R_{int} can be calculated as below

$$\begin{aligned} N_{int} &= e^{-\frac{\pi \lambda R_{int}}{2v}} E(N) \\ E(I) &= \frac{e^{\frac{\pi \lambda R_{int}}{2v}} - 1}{\lambda} \end{aligned}$$

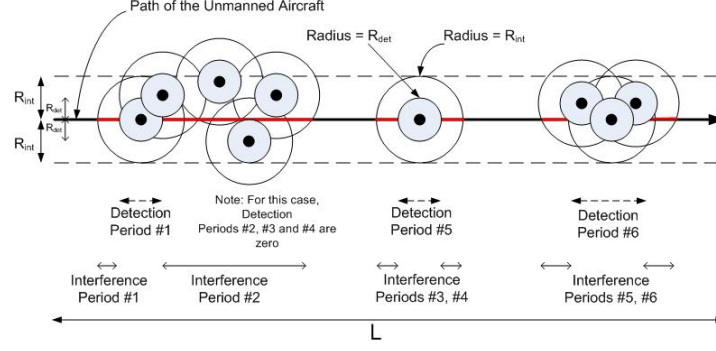


Figure 4.3: 2D scenario for a cognitive radio with detection capability R_{det}

Also, the number of detection periods for the entire excursion of the unmanned aircraft, N_{det} , and the mean detection period for the excursion, $E(D)$, can be calculated as below

$$N_{det} = e^{-\frac{\lambda \pi R_{det}^2}{2R_{int}v}} E(N)$$

$$E(D) = \frac{e^{\frac{\lambda \pi R_{det}^2}{2R_{int}v}} - 1}{\lambda}$$

As in Section 3.1.2, the mean interference period, $E(I_{det})$, and the mean of total interfering time for the entire excursion of the unmanned aircraft, $E(T_{ID})$, can be calculated as below

$$E(I_{det}) = \frac{1}{\lambda} \tanh \left(\frac{\lambda \pi R_{int}}{4v} \left(1 - \frac{R_{det}^2}{R_{int}^2} \right) \right)$$

$$E(T_{ID}) = \left(e^{-\frac{\lambda \pi R_{det}^2}{2R_{int}v}} - e^{-\frac{\pi \lambda R_{int}}{2v}} \right) T$$

4.2.3 Cognitive radio with detection capability with $R_{det} > R_{int}$

In this case, there are no interference outages. Only detection outages occur. The results are similar to those derived in Section 4.2.1 with R_{det} substituted for R_{int} .

4.3 Interference Time Analysis for a 2D system with variable interference and detection radii

4.3.1 Cognitive radio without any detection capability i.e $R_{det} = 0$

The setting of a 1D system model in this case is as shown in Figure 4.4. It is similar to the one discussed in Section 4.2.1, but the the interference radius R_{int} is not from a deterministic distribution. It can be drawn from any general distribution G satisfying the necessary conditions stated in Section 3.1.1. The primary receivers are distributed across a 2D plane with length L and width R according to a Poisson's process with 2D density λ' , where λ' is the number of primary users per unit time. If (X, Y) is the set of random variables that denotes the x and y coordinates of the primary receivers, then $X \sim \text{unif}(0, L)$ and $Y \sim \text{unif}(-R, R)$. The equivalent 1D density, λ , is given by $\lambda = \lambda' \cdot 2R \cdot v$, where v is the velocity of the unmanned aircraft. Therefore, the expected number of primary receivers, $N = \lambda T$. It should be noted that only those primary receivers which satisfy the condition $R_{int} \geq \text{abs}(Y)$ are interfered by the operation of the cognitive radio, the interference distance being $2\sqrt{R_{int}^2 - Y^2}$. Hence, the mean interference period per primary user, $E(S)$, can be calculated as below

$$E(S) = \frac{1}{v} \cdot E \left\{ P(R_{int} \geq |Y|) \cdot E \left(2\sqrt{R_{int}^2 - Y^2} \mid R_{int} \geq |Y| \right) \right\} \quad (4.1)$$

where, v is the velocity of the unmanned aircraft.

Using the results from Section 4.2.1, the mean interference period, $E(I)$ and expected total interference time $E(T_{int})$, are calculated using the following relation.

$$\begin{aligned} E(I) &= \frac{e^{\lambda E(S)} - 1}{\lambda} \\ E(T_{int}) &= (1 - e^{-\lambda E(S)})T \end{aligned}$$

where $E(S)$ is determined by equation (4.1).

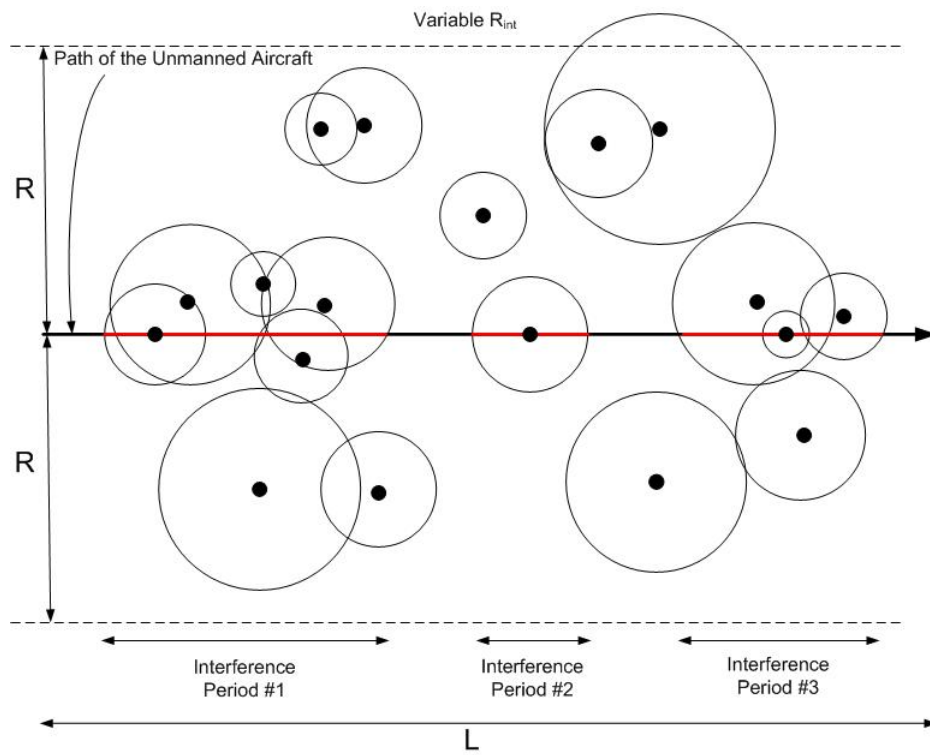


Figure 4.4: 2D scenario for primary receivers with variable interference radius R_{int} and a CR without detection

4.3.2 Cognitive radio with variable detection capability i.e $0 < R_{det} \leq R_{int}$

The 2D system for this case is similar to that discussed in previous section. It is shown in Figure 4.5. But the cognitive radio can detect a primary receiver with a detection radius R_{det} . The detection radius, R_{det} , is not a constant. It is drawn from a distribution H which satisfies the conditions discussed in Section 3.1.1. As in the previous section, the mean interference period $E(S)$ is given by

$$E(S) = \frac{1}{v} \cdot E \left\{ P(R_{int} \geq |Y|) \cdot E \left(2\sqrt{R_{int}^2 - Y^2} \mid R_{int} \geq |Y| \right) \right\} \quad (4.2)$$

where v is the velocity of the unmanned aircraft. In this case, only those primary receivers which satisfy the condition $R_{det} \geq \text{abs}(Y)$ are detected by the operation of the cognitive radio, the detection distance being $2\sqrt{R_{det}^2 - Y^2}$. Hence, the mean detection period, $E(S_{det})$, can be calculated as

$$E(S_{det}) = \frac{1}{v} \cdot E \left\{ P(R_{det} \geq |Y|) \cdot E \left(2\sqrt{R_{det}^2 - Y^2} \mid R_{det} \geq |Y| \right) \right\} \quad (4.3)$$

Since the detection uses beacons on a different channel, R_{det} is independent of R_{int} . But in those instances where $R_{int}(i) < R_{det}(i)$, we set $R_{int}(i) = R_{det}(i)$ so that we satisfy the condition $0 < R_{det} \leq R_{int}$. Therefore, we have

$$R_{int}(i) = \begin{cases} R_{det}(i), & \text{if } R_{int}(i) < R_{det}(i) \\ R_{int}(i) & \text{otherwise} \end{cases} \quad (4.4)$$

where $i = 0, 1, 2, \dots, N - 1$

Using the results from the previous section and Section 4.2.2, the mean interference period, $E(I_{det})$ and expected total interference time, $E(T_{ID})$, are calculated as below

$$\begin{aligned} E(I_{det}) &= \frac{1}{\lambda} \tanh \left(\frac{\lambda}{2} (E(S) - E(S_{det})) \right) \\ E(T_{ID}) &= (e^{-\lambda E(S_{det})} - e^{-\lambda E(S)}) T \end{aligned}$$

where $E(S)$ and $E(S_{det})$ are determined by equation (4.2) and equation (5.1) respectively. It should be noted that the new relation for R_{int} given in equation (4.4) is used the computation of $E(S)$

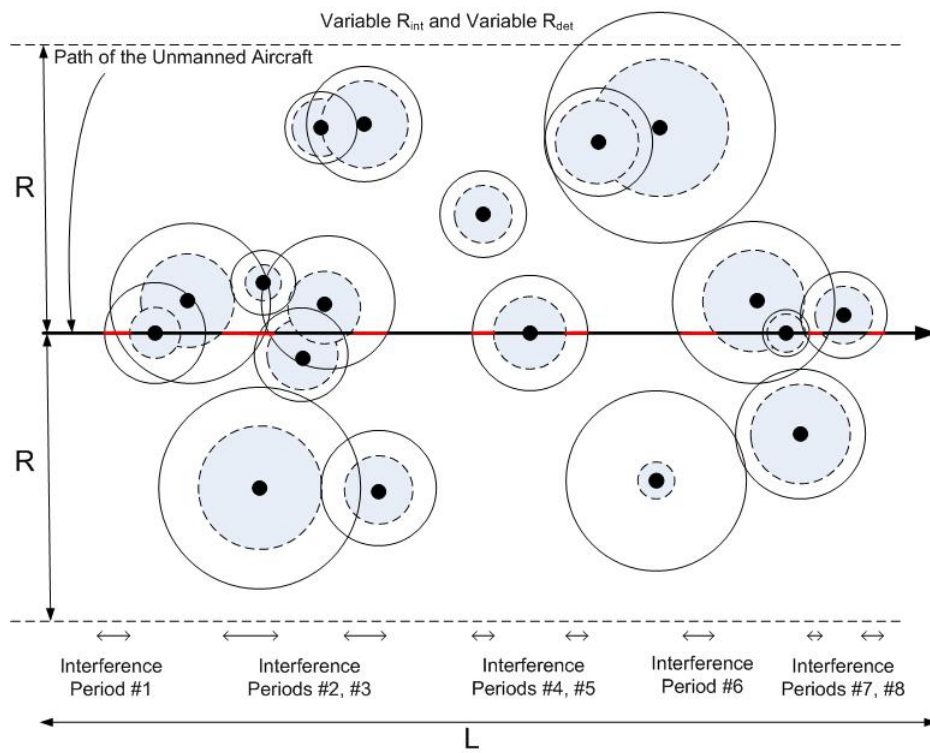


Figure 4.5: 2D scenario for primary receivers with variable interference radius R_{int} and a CR with variable detection radius R_{det}

4.3.3 Cognitive radio with variable detection capability with $R_{det} \geq R_{int}$

This case is similar to the one discussed in Section 4.2.3, but with variable interference and detection radii. In this case, for those instances where $R_{det}(i) < R_{int}(i)$, we set $R_{det}(i) = R_{int}(i)$. This will satisfy the criterion $R_{det} \geq R_{int}$ and there will be no interference outages. Hence, only detection outages are present. The statistics of these detection outages can be derived using the results are similar to those derived in Section 4.3.1 with $E(S_{det})$ substituted for $E(S)$.

4.4 Simulation and results

The simulations were carried out in MATLAB. The total length to be covered by the unmanned aircraft, L , was taken to be 1000 units. The number of primary receivers, $N \sim \text{poiss}(\lambda_1 \cdot L \cdot 2R)$, where λ_1 is the 2D-density. Let λ be the equivalent 1D-density given by $\lambda = \lambda_1 \cdot L \cdot 2R$. The path of the unmanned aircraft was chosen to be the x -axis. For the 2D scenario, two sets of N uniform random variables $H = \{h_1, h_2, \dots, h_N\}$ and $K = \{k_1, k_2, \dots, k_N\}$ were chosen such that $h_i \sim \text{unif}(0, L)$ and $k_i \sim \text{unif}(-R, R)$, where $i = 1, 2, \dots, N$ and R_{int} is the interference radius for the receiver¹. The location of each primary receiver in this case is given by (h_i, k_i) . The 2D scenario was analyzed when the CR had no detection capability and when the CR could detect a primary receiver with a detection radius R_{det} . The velocity of the unmanned aircraft is assumed to be constant of 1 unit so that all the distance calculations directly map to the time calculations.

Figure 4.6 gives the mean and total interfering time for 2D scenario when the cognitive radio interferer has no detection capability. The interfering radius was $\frac{2}{\pi}$ units. This corresponds to a mean interfering time per primary receiver, $E(S) = \frac{\pi R_{int}}{2v} = 1$. It can be seen that the total interfering time also matches the analytical results and saturates for $\lambda > 7.5$. The mean interfering time increases exponentially and agree with the analytical calculation, $\min\left(\frac{e^{\lambda E(S)} - 1}{\lambda}, L\right)$, for any λ before saturation.

Figure 4.7 and Figure 4.8 give the qq-plot and cdf of the interfering time. It can be seen that the interfering time follows an exponential distribution with the mean $\frac{e^{\lambda E(S)} - 1}{\lambda}$ for large values of

¹ $R = R_{int}$ when the interference radius is a fixed value

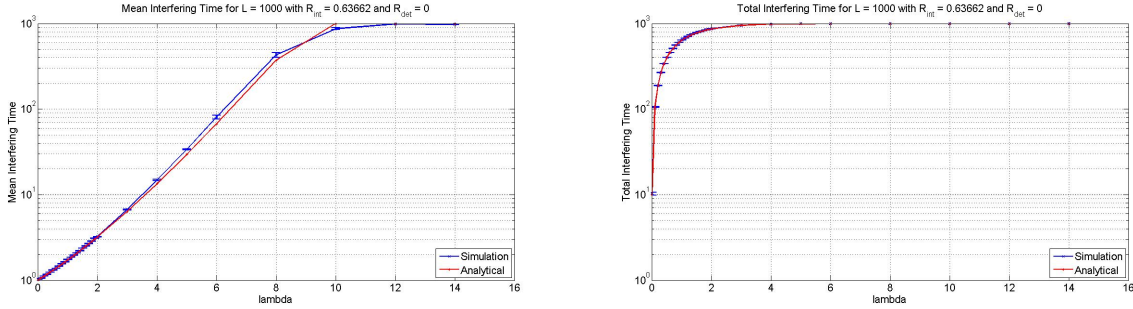


Figure 4.6: Mean and total interfering time for $L = 1000$, $R_{int} = 2/\pi$ and $R_{det} = 0$, 2D case - No Detection

λ . In this case $2 < \lambda < 7.5$, since for $\lambda > 7.5$, the interference time saturates to the total time.

Figure 4.9 gives the mean and total interfering time for 2D scenario when the cognitive radio interferer has no detection capability. The interfering radius was $\frac{4}{\pi}$ units. ($E(S) = 2$). With the increase in interference radius, we can note that the total interference time saturates for a lower density λ . In this case it is $\lambda = 2.5$, as compared to $\lambda = 7.5$ when $E(S) = 1$.

Figure 4.10 gives the mean and total interfering time for 2D scenario when the cognitive radio interferer has an interfering radius of $\frac{2}{\pi}$ units (mean interference time per primary receiver is 1 unit) and a detection radius of $\frac{1}{\pi}$ units (mean detection time per primary receiver is 0.5 units). The mean and the total interference match the analytical results. Similar to the 1D case with primary receiver detection, we can note that the total interference time does not saturate for any λ . It increases initially because the interference time will be more than detection time. With increase in λ , detection time also increases. So, the total interference time decreases and gradually reaches zero for large λ . The value of f is calculated analytically using equation (3.3) to be 0.4275, which means that maximum interfered time is 42.75% of the total excursion time. The maximum interfered time is 472.5 units through simulation which agrees with the analytical calculation.

Figure 4.11 gives the mean and total interfering time for 2D scenario when the cognitive radio interferer has an interfering radius of $\frac{4}{\pi}$ units (mean detection time per primary receiver is 2 units) and a detection radius of $\frac{3}{\pi}$ units (mean detection time per primary receiver is 1.5 units). The results are similar to the previous case except that the total interference time reaches its maximum

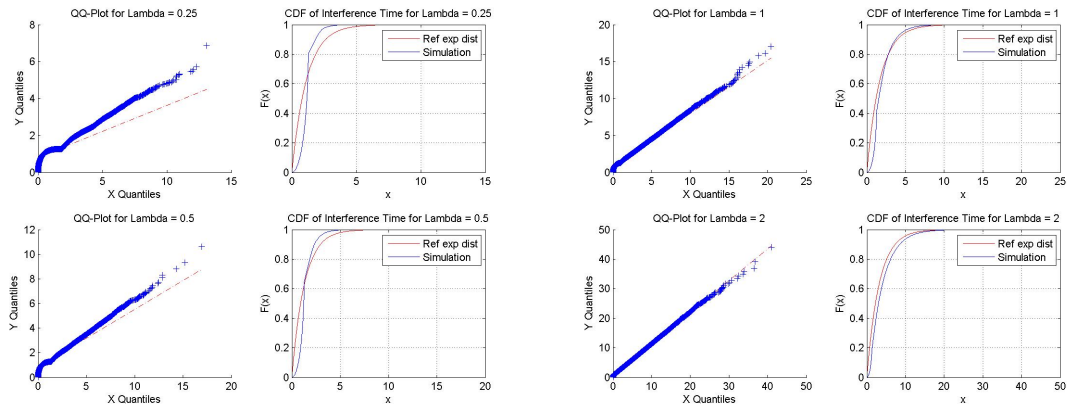


Figure 4.7: QQ-Plot and CDF plot of Interference Time for $R_{int} = 2/\pi$ and $R_{det} = 0$, $\lambda = 0.25, 0.5, 1, 2$, 2D case - No Detection

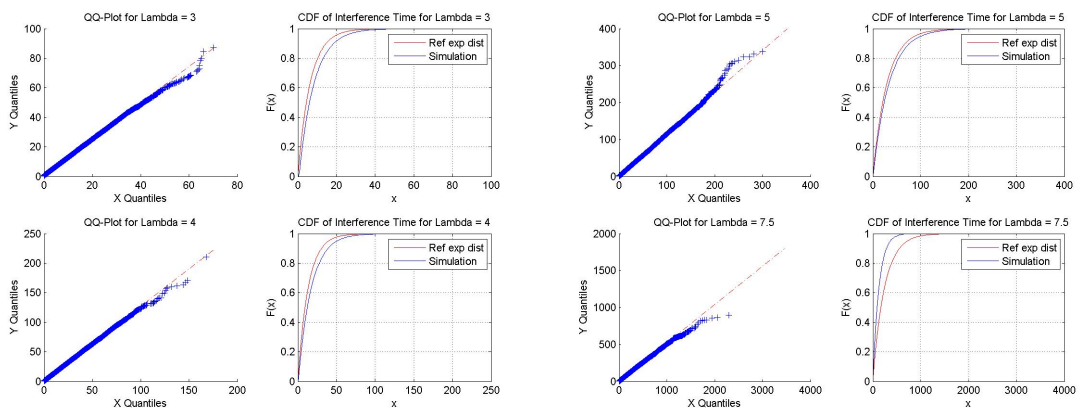


Figure 4.8: QQ-Plot and CDF plot of Interference Time for $R_{int} = 2/\pi$ and $R_{det} = 0$, $\lambda = 3, 4, 5, 7.5$, 2D case - No Detection

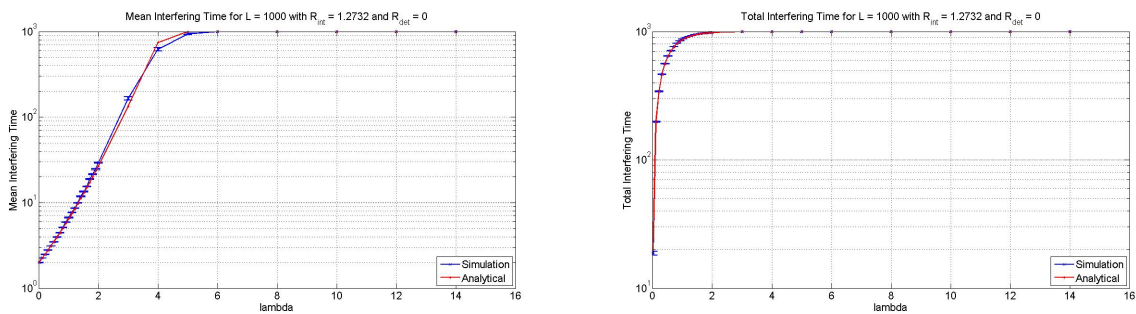


Figure 4.9: Mean and total interfering time for $L = 1000$, $R_{int} = 4/\pi$ and $R_{det} = 0$, 2D case - No Detection

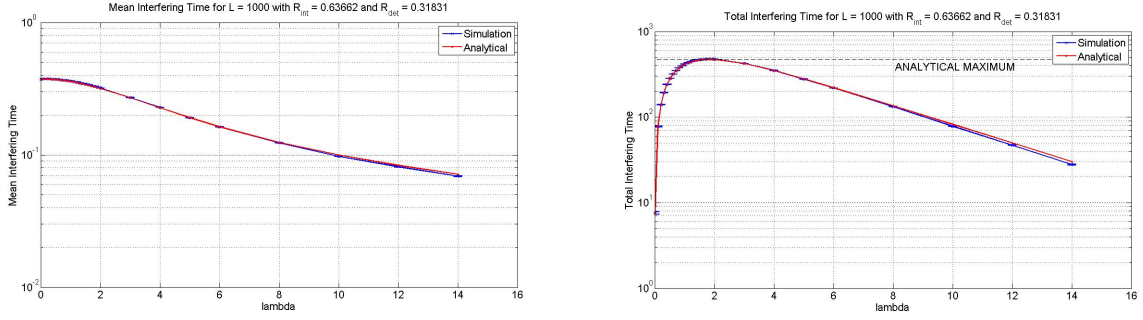


Figure 4.10: Mean and total interfering time for $L = 1000$, $R_{int} = 2/\pi$ and $R_{det} = 1/\pi$, 2D case - With Detection

for a lower value of λ with the increase in interference and detection radii. The value of f is calculated analytically using equation (3.3) to be 0.2088, which means that maximum interfered time is 20.88% of the total excursion time. The maximum interfered time is 209 units through simulation which agrees with the analytical calculation.

Figure 4.12 gives the mean and total interfering time for 1D scenario when the cognitive radio interferer has a variable interfering radius and no detection capability. The width, R , is taken to be 5 units. The interference radius is drawn from a uniform distribution, i.e. $R_{int} \sim \text{unif}(0, 2)$. The mean interference period per primary receiver, $E(S)$, given by equation (4.1) is calculated using MATLAB to be 0.4189 units and used for analytical calculations. The mean and total interference time from simulation lie within 2dB of the analytical results.

Figure 4.13 gives the mean and total interfering time for 2D scenario when the cognitive radio interferer has a variable interfering radius and variable detection capability. The width, R , is taken to be 5 units. The interference radius is drawn from a uniform distribution, i.e. $R_{int} \sim \text{unif}(0, 2)$. The detection radius is also drawn from a uniform distribution such that $R_{det} \sim \text{unif}(0, 1)$. If $R_{det} > R_{int}$, then we set $R_{det} = R_{int}$. The mean interference period and the mean detection period per primary receiver, $E(S)$ and $E(S_{det})$ given by equation (4.2) and equation (5.1) are calculated using MATLAB to be 0.4142 units and 0.0786 units respectively. These values are used in the analytical calculations. The mean and total interference time from simulation match the analytical results. The value of f is calculated analytically using equation (3.3) to be 0.549, which means

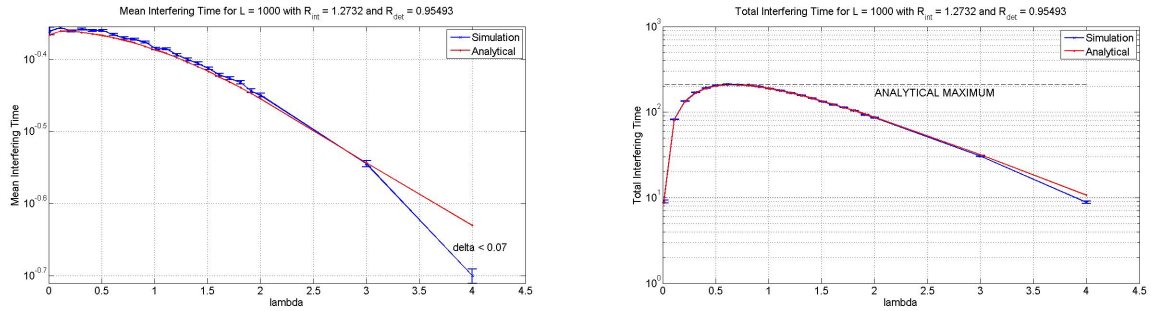


Figure 4.11: Mean and total interfering time for $L = 1000$, $R_{int} = 4/\pi$ and $R_{det} = 3/\pi$, 2D case - With Detection

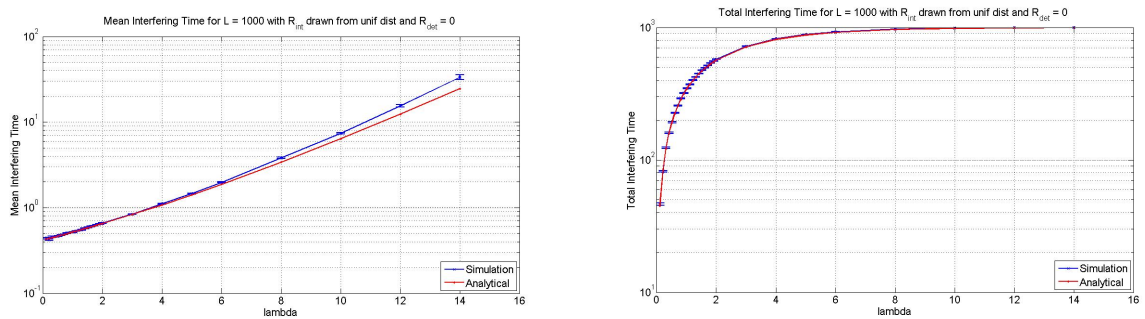


Figure 4.12: Mean and total interfering time for $L = 1000$, $R_{int} = \text{unif}(0, 2)$ and $R_{det} = 0$, 2D case - Variable R_{int} , No Detection

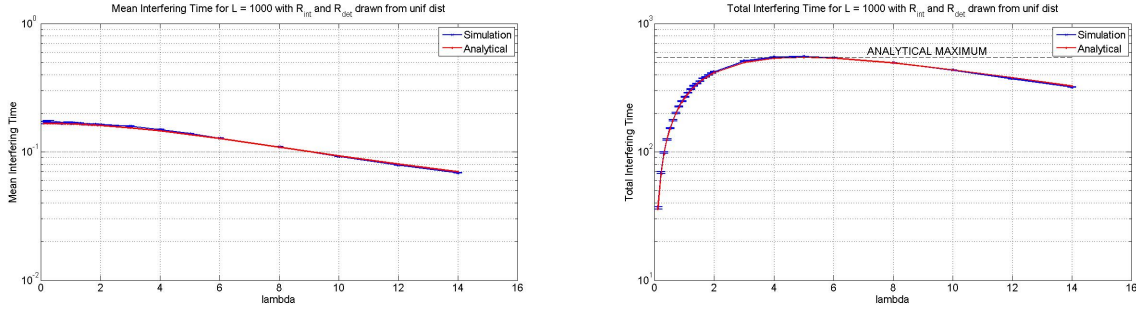


Figure 4.13: Mean and total interfering time for $L = 1000$, $R_{int} = \text{unif}(0, 2)$ and $R_{det} = \text{unif}(0, 1)$, 2D case - Variable R_{int} , Variable R_{det}

that maximum interfered time is 54.9% of the total excursion time. The maximum interfered time is 550 units through simulation which agrees with the analytical calculation.

4.5 Summary

This chapter presented the way to calculate the mean interference time, $E(S)$ and the mean detection time, $E(S_{det})$ for the 2D system. It also described the methods to extend the results for interference time derived for 1D systems to 2D systems using queueing theory. Here too, we find that the statistics of interference time are independent of the distribution of R_{int} and R_{det} . Table 4.1 and Table 4.2 summarize the results derived in this chapter for a 2D system of length L and width $2R^2$

² Note that for fixed R_{int} , $R = R_{int}$.

Parameter	No Detection
Mean Interference Time per primary user, $E(S)$	$\frac{\pi R_{int}}{2v}$, for fixed R_{int}
Mean Detection Time per primary user, $E(S_{det})$	$\frac{1}{v} \cdot E \left\{ P(R_{int} \geq Y) \cdot E \left(2\sqrt{R_{int}^2 - Y^2} \mid R_{int} \geq Y \right) \right\}$, for variable R_{int}
Expected Number of Detection Periods, $E(N_{det})$	0
Expected Number of Interference Outages	0
Mean Interference Time for the entire excursion	$\lambda e^{-\lambda E(S)} T$
Total Interference Time for the entire excursion	$E(I) = \frac{e^{\lambda E(S)} - 1}{\lambda}$ $E(T_{int}) = (1 - e^{-\lambda E(S)}) T$

Table 4.1: Results for the 2D system without detection capability for the cognitive radio

Parameter	With Detection
Mean Interference Time per primary user, $E(S)$	$\frac{\pi R_{int}}{2v}$, for fixed R_{int}
Mean Detection Time per primary user, $E(S_{det})$	$\frac{1}{v} \cdot E \left\{ P(R_{int} \geq Y) \cdot E \left(2\sqrt{R_{int}^2 - Y^2} \mid R_{int} \geq Y \right) \right\}$, for variable R_{int}
Expected Number of Detection Periods, $E(N_{det})$	$\frac{\pi R_{det}^2}{2R_{int}v}$, for fixed R_{det}
Expected Number of Interference Outages	$\frac{1}{v} \cdot E \left\{ P(R_{det} \geq Y) \cdot E \left(\sqrt{R_{det}^2 - Y^2} \mid R_{det} \geq Y \right) \right\}$, for variable R_{det}
Mean Interference Time for the entire excursion	$\lambda e^{-\lambda E(S_{det})} T$
Total Interference Time for the entire excursion	$\lambda (e^{-\lambda E(S)} + e^{-\lambda E(S_{det})}) T$
	$E(I_{det}) = \frac{1}{\lambda} \tanh \left(\frac{\lambda}{2} (E(S) - E(S_{det})) \right)$
	$E(T_{ID}) = (e^{-\lambda E(S_{det})} - e^{-\lambda E(S)}) T$

Table 4.2: Results for the 2D system with detection capability for the cognitive radio

Chapter 5

Applications

This chapter describes the application of the results to practical scenarios. It considers random placement of TV transmission stations and analyzes duration of interference for the same. It also determines the fraction of reduction in interference with improved detection capability.

Let us consider a cognitive radio placed on an unmanned aircraft flying over the United States. Let the velocity of the unmanned aircraft, v , be 450mph. We model the interference caused by the cognitive radio to the TV stations transmitting Channel 15 as shown in Figure 5.1. There are 168 stations spread over the entire region of length, $L = 2500$ miles and breadth, $W = 1500$ miles. Each station has an exclusion zone with a radius of 100 miles around it. Any radio communicating within the exclusion zone will interfere with the transmission of the TV station. Hence, the radius of the exclusion zone can be mapped to the interference radius, $R_{int} = 100$ miles. Let us say that each station is transmitting beacons to aid detection. These beacons can be detected within a detection zone whose radius is R_{det} . The total time for the excursion of the aircraft, $T = \frac{L}{v} = 5.56$ hours. The density of TV stations per unit time, λ is calculated as below

$$\begin{aligned}\lambda &= \lambda_1 W \\ &= \frac{168}{3,000,000} \cdot v \cdot W \\ \lambda &= 30.24\end{aligned}$$

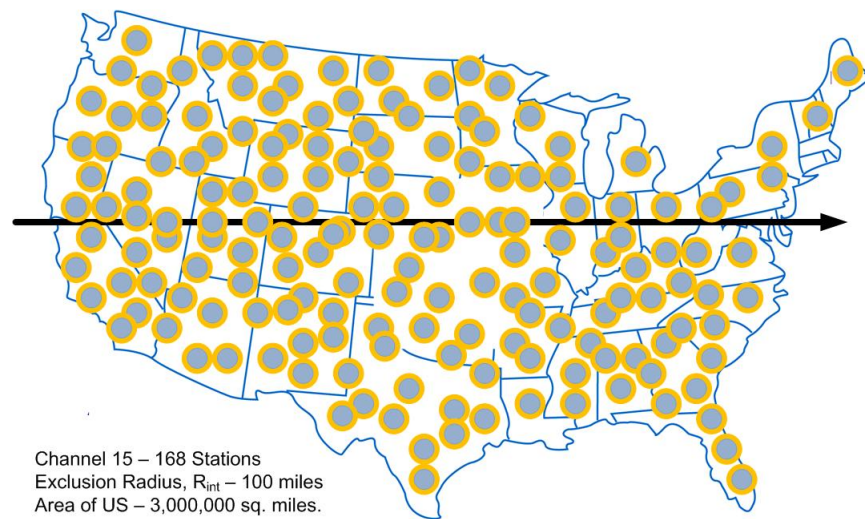


Figure 5.1: Interference Model for Channel-15 TV stations in the United States

The mean interference time per primary user, $E(S)$ is calculated using equation (4.2) as below.

$$\begin{aligned} E(S) &= \frac{1}{v} \cdot \frac{R_{int}}{(W/2)} \cdot \frac{\pi R_{int}}{2} \\ &= 0.058 \text{ hours} \\ E(S) &= 3.49 \text{ minutes} \end{aligned}$$

Similarly, the mean detection time per primary user $E(S_{det})$ is calculated using equation (4.3) as below.

$$E(S_{det}) = \frac{1}{v} \cdot \frac{R_{det}}{(W/2)} \cdot \frac{\pi R_{det}}{2} = \frac{\pi R_{det}^2}{vW} \text{ hours}$$

5.1 Case 1: Basic detection

The number of interference periods, the mean and total interference time for the entire excursion of the unmanned aircraft are listed Table 5.1.

We can note that the unmanned aircraft interferes for 82.73% of the total flight time in the absence of detection with a mean interference time of 9.54 minutes. In the presence of detection, the number of interference outages increase. But the mean interference time decreases considerably when compared to the no-detection case. Therefore, the total interference time decreases. It can be seen that the mean and total interference times decrease with an increase in detection radius.

5.2 Case 2: Extended detection

This is similar to the case discussed in Section 5.1, but the cognitive radio has an improved detection capability. This is because the detection zone of the cognitive radio extends till boundary of the interference zone along the direction of flight as shown in Figure 5.2. We can note that region within the dotted line is the detection zone and the remaining region is the interference zone for a primary user. Then, the mean detection time per primary user can be calculated as below

$$E(S_{det}) = \frac{1}{v} \cdot E \left\{ P(R_{det} \geq |Y|) \cdot E \left(R_{int} + \sqrt{R_{det}^2 - Y^2} \mid R_{det} \geq |Y| \right) \right\} \quad (5.1)$$

Parameter	No detection	$R_{det} = 25$ miles	$R_{det} = 50$ miles	$R_{det} = 75$ miles
Expected Detection Time per primary user, $E(S_{det})$	0	0.0036 hours	0.0145 hours	0.0327 hours
Expected Number of Interference Outages	29	180	137	91
Mean Interference Time for the entire excursion	9.54 minutes	80.4 seconds	69 seconds	43.56 seconds
Total Interference Time for the entire excursion	4.6 hours (82.73%)	4.01 hours (72.3%)	2.62 hours (47.12%)	1.11 hours (19.96%)
Maximum Fraction of total time that can be interfered, f	100%	77.93%	47.25%	20.88%

Table 5.1: Results for basic detection

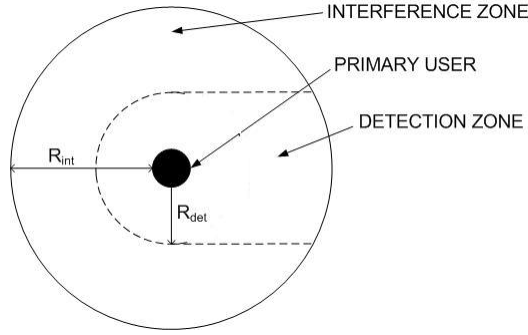


Figure 5.2: Extended Detection Zone

In this case, $E(S_{det}) = \frac{1}{v} \cdot \frac{R_{det}}{(W/2)} \cdot \left(R_{int} + \frac{\pi R_{det}}{4} \right)$.

Table 5.2 gives the results for this case.

We can note that with improved detection, for the same R_{det} , the mean and total interference time are less compared to those computed in Section 5.1. This is because the mean detection time per primary user, $E(S_{det})$, increases. So, the ratio, $r = \frac{E(S_{det})}{E(S)}$, increases which in-turn leads to a reduction in the fraction of the flight time being interfered.

Similarly, we can analyze other cases where R_{int} and R_{det} are not fixed. In any case, the results are dependent only on the mean statistics of R_{int} and R_{det} . So, if we are able to calculate the new mean due to the random nature of R_{int} and R_{det} , we can analyze the duration of interference accurately.

Parameter	No detection	$R_{det} = 25$ miles	$R_{det} = 50$ miles	$R_{det} = 75$ miles
Expected Detection Time per primary user, $E(S_{det})$	0	0.011 hours	0.0258 hours	0.044 hours
Expected Number of Interference Outages	29	149	106	73
Mean Interference Time for the entire excursion	9.54 minutes	72.72 seconds	54.04 seconds	24.89 seconds
Total Interference Time for the entire excursion	4.6 hours (82.7%)	3.017 hours (54.3%)	1.59 hours (28.6%)	30.34 minutes (9.1%)
Maximum Fraction of total time that can be interfered, f	100%	54.81%	29.13%	10.13%

Table 5.2: Results for extended detection

Chapter 6

Conclusions and Future Work

This chapter summarizes the work in this thesis. The problem of analyzing the interference time for a cognitive radio on an unmanned aircraft flying over a region of primary users is considered. We begin with a simple model consisting of one primary receiver-transmitter pair and one interferer. Using this model, we define the interference radius, R_{int} , as the boundary of the area within which a primary user might be subjected to harmful interference due to the operation of the CR interferer. We consider the case of a primary receiver that broadcasts beacons to aid detection. We define the detection radius to be the boundary of the area within which the presence of a primary user might be detected by the CR interferer. Then, we derive the dependence of R_{int} and R_{det} on radio propagation parameters like antenna height, antenna gains, shadowing, etc. The analysis is carried out for free-space pathloss and ground reflection radio propagation models. It is shown that the interference radius increases with an increase in distance of the primary receiver from the primary transmitter. Based on the detection radius, we have two cases – $R_{det} \leq R_{int}$ and $R_{det} > R_{int}$. When $R_{det} \leq R_{int}$, there is interference to the primary user in spite of detection since the CR starts detecting the primary user after entering its interference zone. We note that the total duration of interference time decreases with increase in detection capability. When $R_{det} > R_{int}$, there is no such interference to the primary user. This case is significant if the CR interferer adopts the strategy of switching-off its radio on detecting a primary receiver. It will result in communication outages with respect to the CR.

The interference and detection metrics derived are then used to analyze the statistics of

the duration of interference for systems consisting of multiple primary receivers. We show that this problem can be reduced to a queueing theory problem and analyzed using the results for a $M/G/\infty$ queue. The interference period can be mapped to the busy period of the $M/G/\infty$ queue. The statistics of busy period of the $M/G/\infty$ queue are derived in [8]. The results in [8] are used to determine the statistics of interference time. The interference time follows an exponential distribution for higher densities of primary receivers. This analysis is applied to a one dimensional system, where the primary receivers are distributed uniformly randomly on a straight line, along the path of the unmanned aircraft. It is then extended to a two dimensional system where the primary receivers are distributed uniformly randomly over an area. These 1D and 2D systems are analyzed for two cases – with detection and without detection capability for the cognitive radio. In each case, the theoretically determined statistics for the interference time are verified using MATLAB simulations. In the cases with detection, we accurately determine the fraction of the mean total time of the excursion of the unmanned aircraft that is subjected to interference. It is also shown statistics of the interference time are independent of the distribution of R_{int} and R_{det} and depend only on their mean values.

The results from this work can be used to determine the optimum setting for the cognitive radio system such that it restrains the duration of interference below tolerable limits. For instance, we can determine the optimum value for the detection threshold of the CR such that the total interference time lies within the specified limit. This work paves the path to characterize the duration of interference for random deployments like mesh networks and ad-hoc networks. The analysis of such random deployments and extension of the results to more complicated models forms the potential future work.

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Appendix A

Interference analysis for “known-location” primary receivers and cognitive radio

Let us consider a simple model with the cognitive radio (CR) interferer at a distance $R_I = z$, from the primary receiver R_x located at the origin. The CR is moving towards R_x an angle θ . We consider the following two cases — (1) the CR cannot detect the primary receiver and (2) the CR can detect a primary receiver within a distance R_{det}

A.1 CR cannot detect the primary receiver

Figure A.1 depicts the case where the CR has no primary receiver detection capability i.e. $R_{det} = 0$. The interfering distance d is the length of a chord in the circle of radius R_{int} . It can be calculated as

$$d = 2\sqrt{R_{int}^2 - z^2 \sin^2 \theta}$$

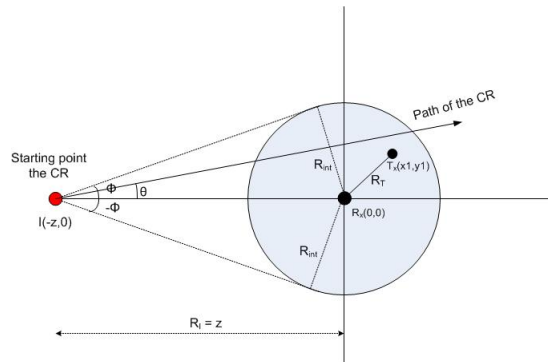


Figure A.1: Interference Distance for simple model for a CR with no detection capability ($R_{det} = 0$)

We can find the interfering distance averaged over the range of θ , for fixed values of z and R_{int} . We can note that the interfering distance is non-zero for θ between $-\phi$ to ϕ where $\phi = \sin^{-1}\left(\frac{R_{int}}{z}\right)$. So, the average interfering distance is

$$d_{avg} = \frac{1}{2\pi} \int_{-\phi}^{\phi} 2 \sqrt{R_{int}^2 - z^2 \sin^2 \theta} d\theta$$

The above integral can be solved using numerical integration.

Now let us say that CR can choose any angle between $-\pi$ and π and start moving in that direction. Let Θ be the random variable that denotes the angle of the CR. Then, $\Theta \sim \text{unif}(-\pi, \pi)$. The corresponding random variable for interfering distance would be D , which is related to Θ by $D = 2 \sqrt{R_{int}^2 - z^2 \sin^2 \Theta}$. We can find the cdf of the interfering distance as follows.

$$\begin{aligned} F_{d|z}(D | Z) &= P(D \leq d) \\ &= P\left(2 \sqrt{R_{int}^2 - z^2 \sin^2 \Theta} \leq d\right) \\ &= P\left(\sin^2 \Theta \leq \left(\frac{4R_{int}^2 - d^2}{4z}\right)\right) \\ &= \begin{cases} 1 - \left(\frac{1}{\pi}\right) \sin^{-1}\left(\sqrt{\left(\frac{4R_{int}^2 - d^2}{4z}\right)}\right) & \text{for } 0 \leq d \leq 2R_{int} \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

A.2 CR can detect a primary receiver within a distance R_{det}

Figure A.2 depicts the case where the CR can detect a primary receiver within a distance R_{det} . In this case, the interfering distance d can be calculated as below.

$$d = \begin{cases} 2 \left(\sqrt{R_{int}^2 - z^2 \sin^2 \theta} - \sqrt{R_{det}^2 - z^2 \sin^2 \theta} \right) & \text{if } |\theta| \leq \phi_1 \\ 2 \sqrt{R_{int}^2 - z^2 \sin^2 \theta} & \text{if } \phi_1 > |\theta| \leq \phi_2 \end{cases}$$

where, $\phi_1 = \sin^{-1}\left(\frac{R_{det}}{z}\right)$ and $\phi_2 = \sin^{-1}\left(\frac{R_{int}}{z}\right)$

We can find the interfering distance averaged over the range of θ , while fixing the values of

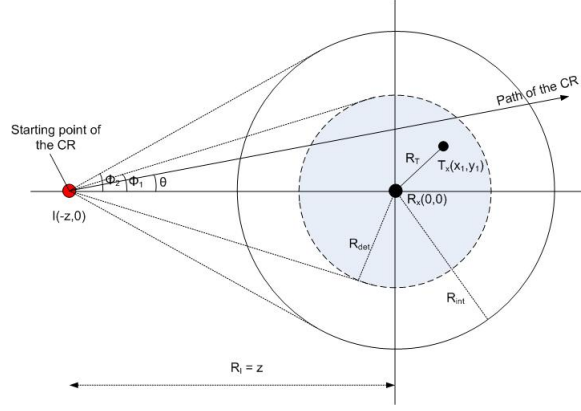


Figure A.2: Interference Distance for simple model for a CR with detection capability R_{det}

z and R_{int} . Since θ can take values between $-\pi$ and π , the average distance is

$$d_{avg} = \begin{cases} \frac{1}{2\pi} \int_{-\phi_1}^{\phi_1} 2 \left(\sqrt{R_{int}^2 - z^2 \sin^2 \theta} - \sqrt{R_{det}^2 - z^2 \sin^2 \theta} \right) d\theta & \text{if } |\theta| \leq \phi_1 \\ \frac{1}{2\pi} \int_{-\phi_2}^{-\phi_1} \left(2\sqrt{R_{int}^2 - z^2 \sin^2 \theta} \right) d\theta + \frac{1}{2\pi} \int_{\phi_1}^{\phi_2} \left(2\sqrt{R_{int}^2 - z^2 \sin^2 \theta} \right) d\theta & \text{if } \phi_1 > |\theta| \leq \phi_2 \end{cases}$$

Let us say that CR can choose any angle between $-\pi$ and π and start moving in that direction. Let Θ be the random variable that denotes the angle of the CR. Then, $\Theta \sim \text{unif}(-\pi, \pi)$. The corresponding random variable for interfering distance would be D . We can find the CD of the interfering distance as follows.

$$\begin{aligned} F_{d|z}(D | Z) &= P(D \leq d) \\ &= \begin{cases} P \left(2 \left(\sqrt{R_{int}^2 - z^2 \sin^2 \theta} - \sqrt{R_{det}^2 - z^2 \sin^2 \theta} \right) \leq d \right) & \text{if } |\theta| \leq \phi_1 \\ P \left(2\sqrt{R_{int}^2 - z^2 \sin^2 \theta} \leq d \right) & \text{if } \phi_1 > |\theta| \leq \phi_2 \end{cases} \end{aligned}$$

On simplification, we get

$$F_{d|z}(D | Z) = \left\{ \begin{array}{l} 1 - \left(\frac{1}{\pi}\right) \sin^{-1} \left(\sqrt{\left(\frac{4R_{int}^2 - d^2}{4z}\right)} \right) - \left(\frac{1}{\pi}\right) \sin^{-1} \left(\sqrt{\left(\frac{64R_{int}^2 R_{det}^2 - (4R_{int}^2 + 4R_{det}^2 - d^2)}{16d^2 z^2}\right)} \right) \\ \text{for } 2(R_{int} - R_{det}) \leq d \leq 2\sqrt{R_{int}^2 - R_{det}^2} \\ \text{and} \\ 1 - \left(\frac{1}{\pi}\right) \sin^{-1} \left(\sqrt{\left(\frac{4R_{int}^2 - d^2}{4z}\right)} \right) \\ \text{for } 0 \leq d \leq 2(R_{int} - R_{det}) \end{array} \right.$$

A.3 Simulation

The average interference distance and the cdf for interfering distance were simulated using MATLAB with $R_{int} = 5$ units. Figure A.3 shows the average interference distance and cdf of interfering distance for a CR with no detection capability. Since the interference radius and the position of the primary receiver are fixed, the angle $\phi_1 = \sin^{-1} \left(\frac{R_{int}}{z} \right)$ decreases with increase in z (distance between CR and R_x). Hence, the average interference distance also decreases.

Figure A.4 shows the average interference distance and cdf of interfering distance for a CR with a detection radius $R_{det} = 3$ units. As expected, the average interference distance reduces with the introduction of detection capability.

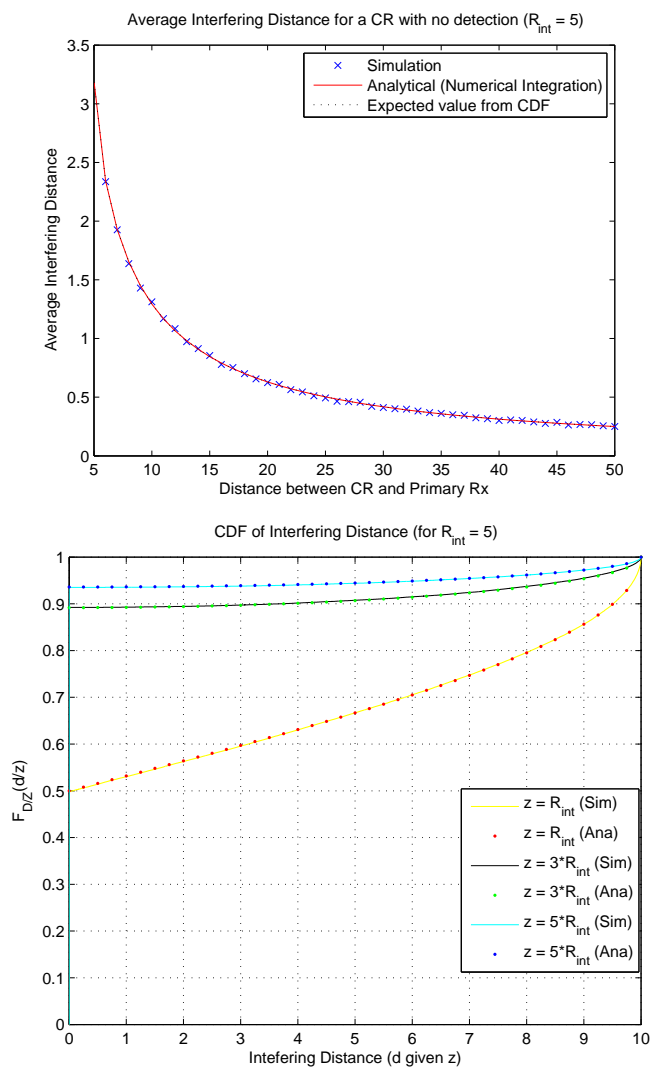


Figure A.3: Average and cdf of interfering distance for a CR with no detection capability

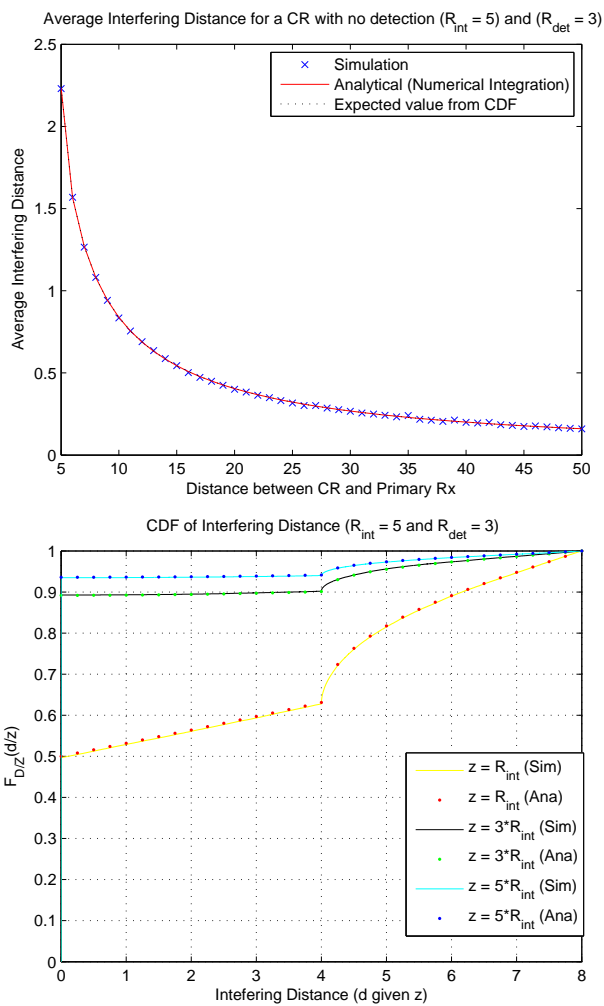


Figure A.4: Average and cdf of interfering distance for a CR with detection capability R_{det}

Appendix B

QQ-Plot

The qq-plot compares two distributions by plotting their quantiles against each other (In qq-plot, 'q' stands for quantile). Quantiles are defined as points taken at regular intervals from the cdf of a random variable. In other words, quantile is the fraction of data points below the given value. For example, if the data size is n , the 60% quantile is the point at which $0.6n$ data samples fall below this point. In a qq-plot, 'q' such quantiles from the ordered data set obtained using the first distribution are plotted against the 'q' quantiles obtained from the ordered data set of the second distribution. Let $\{X_1, X_2 \dots X_n\}$ be an ordered data set of n points. from the unknown distribution F_A and let F_B be the second distribution. The qq-plot graphs $\left(X_i, F_B^{-1}\left(\frac{i-\frac{1}{2}}{n}\right)\right)$ A 45-degree reference line is also plotted. If the two distributions are similar, the points should fall approximately along this reference line. The greater the departure from this reference line, it is more likely that the data sets belong to different distribution i.e the two distributions are dissimilar.

Appendix C

An Introduction to Queueing Theory

This thesis considers the scenario of a cognitive radio on an unmanned aircraft that might interfere with the primary users on the ground. The mean and distribution of the duration of interference outages, and the total interference duration for the entire excursion of the aircraft are the metrics of interest. These metrics can be derived using queueing theory. Queueing theory deals with the study and analysis of queues or in a more generic sense waiting lines. It is often regarded as a branch of operations research since the results derived from queueing theory are widely used to determine resource allocation and optimum service strategy for a business model. It can also be considered as a branch of applied probability theory since it uses probability and random process theory to model and analyze several processes related to queues or waiting lines. For example, queueing theory enables us to determine the average number of customers in a queue, arrival and departure rate of the customers, average or expected waiting time and service time per customer and the probability of the queue being empty or full. It is applied in numerous fields such as computer networks, telecommunications, machine plants, intelligent transport systems, traffic flow analysis, etc.

C.1 A typical Queueing system

A typical queueing system model consists of a service center with a certain number of servers and a population of customers. A new customer arriving to obtain service is serviced immediately if the service center is free (i.e. the service center is not servicing any customer). Otherwise, the

customer has to wait in a queue until he gets his chance to be serviced. A simple example would be a coffee shop with a single server servicing the customers. If a new customer enters the coffee shop and finds that there are no other people at the server, he is served immediately. But if he enters the shop and finds say 5 people in queue, he has to join the end of the queue and wait for his turn [8].

A typical queueing system model is as shown in Figure C.1. Let us consider that the customers enter the service center in a random manner. The service facility has one or more servers and each server can serve exactly one customer at a time. The service time required per customer is also modeled as a random variable following a certain distribution with a finite mean. Let us assume that there are infinite number of customers and the n^{th} customer C_n arrival at time τ_n . The difference in arrival times of consecutive customers is called the interarrival time. It is denoted by t_n and is defined as

$$t_n = \tau_n - \tau_{n-1}$$

Let us assume that these interarrival times (t_n) are i.i.d random variables, i.e. the interarrival times are independent from each other and are drawn from the same distribution with the cdf $A(t)$.

$$A(t) = P(t_n \leq t)$$

Similarly, we assume that the service times s_n corresponding to each customer C_n are also i.i.d random variables drawn from the same distribution with the cdf $B(t)$. Also, the queue has an infinite length to accommodate every customer entering the system.

A queueing system employs a service strategy to handle the customers. Some of the commonly used service strategies are

- FIFO: (First In First Out): This is the most widely used service strategy. A new customer entering the system goes to the end of the queue if he finds the system busy. Hence the customer who entered the system first will be the first one to obtain service.

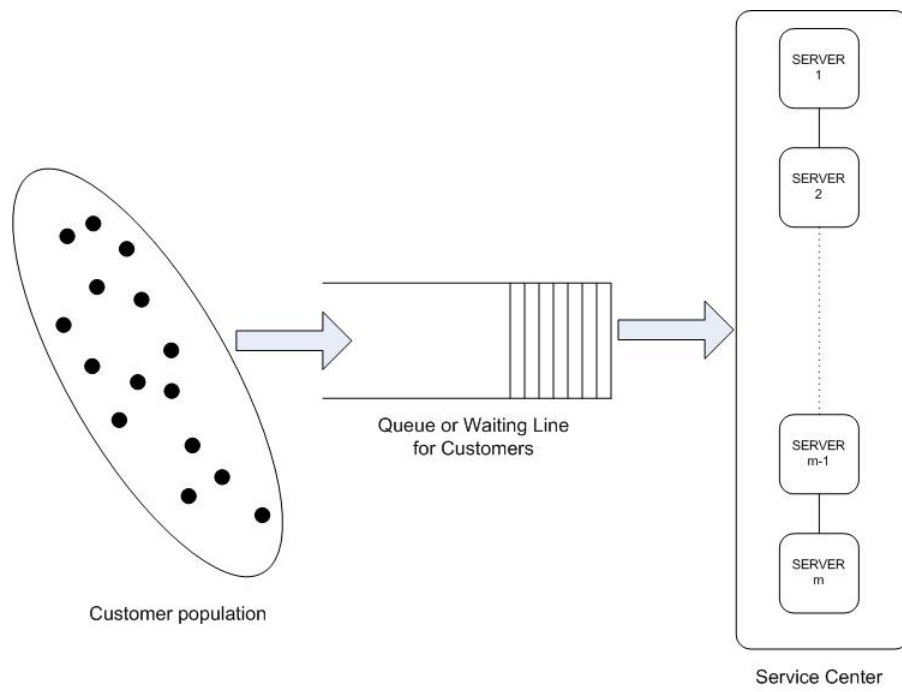


Figure C.1: Typical Queueing System Model

- LIFO: (Last In First Out): A new customer entering the system goes to the head or top of the queue if he finds the system busy. Hence the customer who entered the system most recently will be the first one to obtain service.
- Round Robin: In this case, every customer gets an equal time slice. If the service is not completed in this round, the customer will re-enter the queue from the end.
- Priority Disciplines: Here each customer is assigned with a priority and the service center serves the customers based on the decreasing order of priority. i.e the highest priority customer is served first. The priority assignment can be static or dynamic and the system can be preemptive or non-preemptive.
- Random Service: The customers are serviced randomly.

C.2 Kendall Notation

Queueing systems are characterized mainly by the distribution of interarrival time, distribution of service time and the number of servers. Kendall notation is used as the standard notation to characterize queueing systems. This notation is named after D. G. Kendall who introduced it in 1953 as a three-factor notation [12]

$$A/B/C$$

where,

A denotes the arrival process,

B denotes the distribution of the service time, and

C indicates the number of servers.

Some additional factors like K , N and D are used along with the three-factor notation and represented in the following way

$$A/B/C/K/N/D$$

where,

K denotes the maximum number of customers allowed in the system (inclusive of those being serviced),

N denotes the total size of the population from which the customers come, and

D indicates the service strategy used like FIFO, LIFO, etc.

When $K/N/D$ are omitted, it is assumed that $K = \infty$, $N = \infty$ and $D = FIFO$.

The common abbreviations used for A and B are:

- M (Markov): This indicates the underlying distribution obeys Markov property i.e. it is memoryless. Exponential distribution is the only continuous distribution that satisfies this property. So, $M(t) = 1 - e^{-\lambda t}$ and the corresponding pdf $m(t) = \lambda e^{-\lambda t}$, where $\lambda > 0$ is called the “rate”. The mean or the expected value when this distribution is used is $\frac{1}{\lambda}$.
- Ek (Erlang- k): This indicates an Erlang distribution with k phases ($k \geq 1$). We have $Ek(t) = 1 - \sum_{n=0}^{k-1} e^{-\lambda t} \frac{(\lambda t)^n}{n!}$ and the corresponding pdf is $ek(t) = \frac{\lambda^k t^{k-1} e^{-\lambda t}}{(k-1)!}$, where the “rate” $\lambda > 0$ and the mean is $\frac{k}{\lambda}$. For example, this is used to model the distribution of the arrival of telephone calls at a central office.
- D (Deterministic): This indicates that all the values are a constant and hence drawn from a “deterministic distribution”. $D(t) = \alpha$, where α is a constant. The mean is α .
- G (General): This indicates that the underlying distribution is a general distribution which varies according to applications. Usually it will have finite mean and variance.

C.3 Little’s law and some commonly used queues

C.3.1 Little’s Law

Little’s law is a simple theorem and an integral part of queueing theory. [12] It states that “The long-term average number of customers in a stable system L is equal to the product of the long-term average arrival rate, λ , and the long-term average time a customer spends in the system,

W ”

$$L = \lambda W$$

This shows that the long-term average number of customers is completely independent of the underlying distributions for the arrival process and the service times. It is only dependent on their mean or expected values. This law holds good for any $G/G/1$ queue and even for service strategies other than FIFO.

C.3.2 Commonly used queues

Some of the commonly used queue are $M/M/1$ queue, $M/M/m$ queue and the $M/G/1$ queue. Some important results corresponding to performance metrics of these queues such as the distribution and mean of the waiting time for a customer, the mean number of customers within a duration, the mean and distribution of the busy and idle times of the queue, etc. [12]

C.3.2.1 $M/M/1$ queue

This is a single server queue model and is widely used to model simple systems. Using Kendall notation, we can note the following characteristics about the queue - the interarrival time is derived from an exponential distribution with rate λ (or the arrival process is a Poisson process since a Poisson process results in exponential interarrival times), the service time is also from an exponential distribution with rate μ and there is only one server. The length of the queue and population size are infinite and the service strategy is FIFO. Such a queue can be used to model the coffee shop example discussed in the previous section with the constraint that the shop has only one employee serving all the customers and the interarrival and service times for customers are exponential (which is mostly true in many real world scenarios)

C.3.2.2 $M/M/m$ queue

This is a multi server queue model. Using Kendall notation, we can note the following characteristics about the queue - the interarrival time is derived from an exponential distribution

with rate λ , the service time is also from an exponential distribution with rate μ and there is m servers. The length of the queue and population size are infinite and the service strategy is FIFO. Again the same coffee shop example can be considered with the constraint that it now has m employees serving the customers and the interarrival and service times for customers are exponential. Such queues are used to analyze traffic flows in computer networks too.

C.3.2.3 $M/G/1$ queue

This is also a single server queue model and used to model simple systems whose service time distribution depends on their application. Using Kendall notation, we can note the following characteristics about the queue - the interarrival time is derived from an exponential distribution with rate λ , the service time is from a general distribution with mean μ and variance σ and there is only one server. The length of the queue and population size are infinite and the service strategy is FIFO.

C.3.2.4 $M/G/\infty$ queue

Using Kendall notation, we can note the following characteristics about the queue - the interarrival time is derived from an exponential distribution with rate λ , the service time is from a general distribution with mean μ and variance σ and there are infinite servers. The length of the queue and population size are also infinite and the service strategy is FIFO. Since there are infinite servers, every customer begins to get serviced immediately. In other words, the waiting time for any customer in the queue is zero.

C.4 Summary

This chapter presented a brief introduction to queueing theory, some commonly used queues and their representation. The model that we consider in this thesis can be analyzed using queueing theory. The primary receivers can be regarded as customers entering a queue from an arrival process. The interference period per primary receiver can be mapped to the service time for each

customer. The cognitive radio on the unmanned aircraft does not wait for an interference period from one primary receiver to end before it begins interfering with the next one. This is analogous to having infinite number of servers in the queueing system. If we consider the arrival process to be Markov and any general distribution for interference period, this model can be analyzed $M/G/\infty$ queue, where M denotes the Markov arrival of primary receivers and G denotes the cdf of the interference period. The results are derived in Chapter 3.