Probabilistic Modeling of Sequential Effects in Human Behavior: Theory and Practical Applications

by

Matthew H. Wilder

B.A., Middlebury College, 2001

M.S., Computer Science, University of Colorado at Boulder, 2009

M.S., Applied Mathematics, University of Colorado at Boulder, 2011

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Michael C. Mozer

Matt Jones

Tim Curran

Nikolaus Correll

Date

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Probabilistic Modeling of Sequential Effects in Human Behavior: Theory and Practical Applications

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Whether driving a car, making critical medical decisions in the ER, answering questions in a marketing survey, or selecting shots in a basketball game, people make decisions and actions that are biased by the sequence of recent experience. These sequential effects are ubiquitous in human behavior and have been demonstrated in a wide range of experimental paradigms. This dissertation begins with a synthesis of the vast computational modeling landscape in this domain. Building upon one of the core principles revealed in this synthesis, I explore how sequential effects in simple choice tasks reflect an individual's attempt to optimize behavior in an ever-changing world. A Bayesian model is proposed which asserts that humans are sensitive to multiple environmental regularities and adapt their behavior according to expectations derived from these sensitivities. Through analyses of two experiments that question how far into the past these sensitivities extend, I demonstrate that events far in the past can exert an observable bias on behavior. This finding is surprising given the prevailing perspective in the literature that sequential effects are relatively ephemeral, fading after roughly 4-6 intervening events. To accommodate this new perspective, a hierarchical generalization of the model is presented that allows for long-range sensitivities exhibiting power decay. Given an expanded understanding of the mechanisms underlying sequential effects, the final chapter focuses on how this understanding can be put to practical use. I address sequential biases in judgment tasks and develop techniques for removing the biases from a sequence of responses. By *decontaminating* the responses using a novel hierarchical Bayesian model that exploits knowledge of sequential effects, a set of new responses is obtained that is more representative of the individual's true opinions. For each question or stimulus that is judged, the goal is to uncover what the individual would have responded in the absence of any sequential context. Given the growing interest in collecting human judgments and using them to predict individual

preferences (e.g., Netflix, Amazon), the ability to effectively decontaminate sequences of judgments is of significant value because it produces more reflective estimates of an individual's internal state with fewer total judgments required.

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Chapter 1

Introduction

Our world is structured such that we experience a steady stream of sensations and continually formulate decisions and actions in response to them. Understanding this input/output relationship is the primary goal of psychology. In the laboratory, researchers often study this relationship by exposing individuals to a collection of sensations, requiring them to take some action, and recording properties of their behavior. A universal characteristic of these experiments is that behavior exhibits variability, even across multiple presentations of the exact same sensations. Typically variability is removed in experimental analyses by averaging across common trials. Rather than discarding it with the blunt tool of the average, we relish behavioral variability in this work and use it to uncover hidden aspects of cognition and to extract more reliable measurements of human behavior.

A large portion of behavioral variability can be attributed to the fact that human decisions and actions are deeply influenced by the context of the moment. The study of sequential effects recognizes this and aims to develop a more complete understanding of how behavior is modulated by the current context (i.e., the sequence of sensations, decisions, and actions that precede the behavior under scrutiny). In this dissertation, we investigate new theoretical perspectives on human behavior through computational modeling of sequential effects. Furthermore, we turn the problem inside-out and ask how a computational understanding of sequential effects can improve conclusions drawn from sequentially contaminated data.

Sequential effects have long received attention in the experimental literature, though they are acknowledged and addressed in only a small minority of studies. Despite the fact that sequential effects are often ignored, experimental evidence suggests that they are present in nearly all aspects of cognition and are even ubiquitous across more rudimentary components of our nervous system, such as motor control.

One early example of a sequential effect is the gambler's fallacy identified by Jarvik (1951) in which after a long sequence of repeated events, individuals have a bias towards expecting a reversal of the sequence even if the events are independent and equally probable. This bias resulted in a huge win by a casino in Monte Carlo one evening in August, 1913, when one of the roulette wheels logged a remarkable 15 blacks in a row, sparking a rush of bets on red, and then continued to a record streak of 26 consecutive blacks, breaking the bank of many individuals who kept doubling down their bets believing that there was no possible chance the streak could continue.

Sequential effects have been demonstrated in a wide range of experimental paradigms, including simple choice tasks (e.g., Bertelson, 1961; Hyman, 1953; Remington, 1969), probability matching (e.g., Estes, 1957), absolute judgments (e.g., Garner, 1953; Ward and Lockheed, 1970), magnitude estimation (e.g., Jesteadt, Luce, and Green, 1977; Ward, 1973), categorization (e.g., Petzold, 1981; Treisman and Williams, 1984), visual search (e.g., Chun and Jiang, 1998; Maljkovic and Nakayama, 1994), and language production (Bock and Griffin, 2000).

Beyond these somewhat contrived laboratory tasks, sequential effects have been identified in significant real world situations. For example, recent braking or acceleration actions of automobile drivers can explain variability in response latencies of up to 100 ms, potentially the difference between a collision and a near miss (Doshi, Tran, Wilder, Mozer, and Trivedi, 2012). Professional basketball players' choices of shot location have been shown to depend directly on recent attempts and successes (Neiman and Loewenstein, 2011). A bias of the recent past on decision-making has been demonstrated in legal reasoning and jury evidence interpretation (e.g., Furnham, 1986; Hogarth and Einhorn, 1992), clinical assessments (Mumma and Wilson, 2006) and financial decisions (e.g., Johnson and Tellis, 2005; Vlaev, Chater, and Stewart, 2007).

Furthermore, sequential effects are present in motor control (Scheidt, Dingwell, and Mussa-Ivaldi, 2001) and pain sensation (Link, Kos, Wager, and Mozer, 2011), suggesting that dependencies on the recent past are not just a property of high-level cognition. In fact, other animals have been shown to exhibit sequential dependencies. For example, rats adjust their behavior according to the sequence of recent rewards (Gallistel, Mark, King, and Latham, 2001), the foraging behavior of starlings is biased by recent experience (Cuthill, Kacelnik, Krebs, Haccou, and Iwasa, 1990), and the flower choice of bumblebees seeking nectar is influenced by the few recent flower visits (Real, 1991).

In the following chapters, we explore sequential effects from a computational perspective. This work begins with an overview of the literature in Chapter 2. Instead of just surveying the many existing models of sequential effects, we synthesize and organize this broad field according to several underlying computational principles. These computational principles serve as a springboard into the novel research we present. In Chapter 3, we develop a Bayesian model that explains response time in simple choice tasks by assuming that individuals are optimizing their behavior in an environment that can be characterized by the sequence of recent stimulus values and an abstraction where trials are encoded as a repetition or alternation of the previous trial. Building on this perspective, in Chapter 4, we provide psychological evidence for the dissociation of sequential effects driven by stimulus properties and those driven by a repetition/alternation abstraction of the stimuli sequence. Chapter 5 presents a thorough investigation of the persistence of incidental experience and reports the surprising finding that events far in the past can still exert an influence on behavior. Generalizing a version of the model explored in Chapter 3, we offer a theoretical justification for why the influence of incidental experience should be so persistent. In Chapter 6, we study several methods for *decontaminating* or removing sequential effects from human judgments. We present a novel hierarchical Bayesian model that produces a meaningful improvement in the quality of what can be inferred from an individual's sequence of contaminated judgments. Finally, Chapter 7 recapitulates the contributions of this work and suggests potential directions for future research.

Chapter 2

Computational Principles Underlying Models of Sequential Effects[1]

The diversity of situations where sequential effects arise has led to a proliferation of theoretical models that seek to explain them. However, in many cases, researchers view these effects as a quirk of their specific domain and account for them by making subtle tweaks to their domain specific models in a way that obscures the common principles underlying sequential effects. Our goal in this chapter is to survey the wide swath of theoretical and computational models and whittle this disparate collection down to a few central explanations for why sequential effects occur. By extracting the core principles from this scattered literature, we hope to clarify the role of sequential effects in behavior and provide guidance for interpreting sequential effects across all domains.

In synthesizing these models, we find that in most cases, sequential effects can be understood as an efficient solution to the challenges imposed by a dynamic environment under the constraints of a limited cognitive architecture. This perspective highlights the importance of understanding and recognizing sequential effects in cognition. Instead of taking the common view that these effects are a suboptimal idiosyncrasy of the brain, we encourage researchers to explore how sequential effects can further enrich our understanding of all aspects of behavior. For example, our analysis finds a deep connection between sequential effects, adaptation, learning, memory, and the statistics of the environment. In fact, sequential effects may be a manifestation of the brain's attempt to keep up with continually changing environmental conditions and may serve as the initial stage in developing more complex adaptive behavior.

[1] The synthesis and organization provided here was moulded out of many discussions with Michael Mozer and Matt Jones.

2.1 Neural Inertia

Most early explanations for sequential effects have a strong mechanistic feel, attributing them to residual neural activity that yields a simple form of priming. The general idea is that the previous trial or sequence of trials produces a residual trace of neural activity or an implicit short-term memory that influences subsequent behavior. Figure 2.1 depicts a caricature of a neural inertia model. In the early literature, residual neural activity is often associated with a facilitation effect where performance is enhanced on repetition trials—where the current stimulus is the same as the previous stimulus (Bertelson, 1961; Hyman, 1953). This priming is not just limited to the relationship between the current and previous trial but can extend several trials into the past with greater facilitation after a string of repetitions (Remington, 1969). This is closely linked to the notion of expectancy in early theoretical accounts, where participants are assumed to adjust behavior according to a loose implicit expectation about what will occur next formed from recent experience. Falmagne (1965) presents an early formalization of this concept in a model for a multi-choice reaction time task. Though presented via a slightly more complex mathematical framework, the model essentially implements a decaying memory trace for each potential stimulus and predicts response time (RT) for a given stimulus to be a function of an individual's level of preparation, defined as the strength of the trace for that stimulus. Typically, expectancy refers to a positive bias towards recent experience. However, the effect can also be negative, as in Jarvik (1951), though the gambler's fallacy most likely results from explicit memory rather than implicit short-term memory.

The general theoretical perspective underlying these neural inertia accounts is that the cognitive system has a built in expectation for successive events to be similar. In this way, sequential effects can be viewed as a simple heuristic to improve performance in situations that do in fact exhibit positive autocorrelation from one trial to the next.

Much of the subsequent theoretical work on choice RT tasks targets subtleties in the experimental data such as how behavior changes with the response-stimulus interval (RSI), the stimulus/response mappings, different baserate probabilities, sequential autocorrelation structure, the

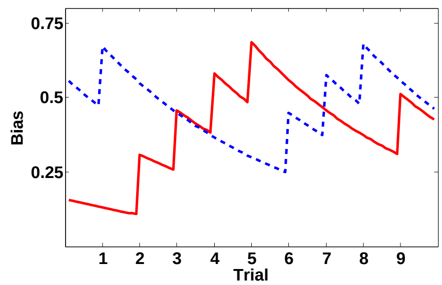


Figure 2.1: An example of neural inertia. If the level of bias affects behavior in some way, there will be spillover of neural activity from previous trials that will yield sequential effects in behavior.

number of stimuli or choices, and effects of practice (Hale, 1969; Kirby, 1976; Laming, 1968; Soetens, Deboeck, Boer, and Hueting, 1984; Soetens, Boer, and Hueting, 1985; Vervaeck and Boer, 1980). The theories put forth appear diverse on the surface, with conflicting perspectives on the presence and role of facilitation and expectancy. However, in most cases the sequential effects still result from some form of residual neural activity occurring within the mechanisms proposed. Much of the debate focuses not on whether neural inertia is present, but rather what is the time course of decay and how the different mechanisms combine.

For example, Vervaeck and Boer (1980) explain sequential effects by a variable state of excitation or inhibition of two processing pathways, one for each choice. When a pathway is used on a trial, it experiences an increase in excitation that leads to faster responses when that pathway is used on subsequent trials. Further, they propose that the state of the unused pathway is differentially affected by repetitions and alternations in a way that depends on RSI and practice. Soetens et al. (1985) conclude that facilitation and expectancy are completely different mechanisms that predict different patterns of sequential effects. The extent to which each mechanism affects performance depends importantly on experimental design, especially RSI and response compatibility. The authors explain facilitation via rapidly decaying memory traces but are less clear as to how expectancy is developed. It is suggested that expectancy loosely corresponds to the local impression from a run of binary stimuli which could naturally be derived from a decaying short-term memory of recent trials.

A more recent class of models that seek to explain sequential effects is based on the leaky, competing accumulator choice model of Usher and McClelland (2001). These models share the common property that they supplement the accumulator model with additional traces that track properties of the sequence and offset the decision dynamics from one trial to the next. Cho et al. (2002) demonstrate that individuals maintain multiple biases relevant to different properties of the sequence. These biases, which are implemented via simple decaying memory traces, preferentially contribute to the accumulation rate of each decision unit in a way that depends in part on the identity of the previous trial. A. D. Jones, Cho, Nystrom, Cohen, and Braver (2002) add conflict monitoring to the model of Cho et al. to capture sequential effects in RT that can be attributed to higher-level issues of cognitive control (e.g., speedup and slowdown as the properties of the task change). The model is identical to Cho et al. except that an additional strategic priming bias is added to the accumulation equation. Strategic priming is defined as an exponentially decaying trace of conflict between competing decision units over past trials. Gao, Wong-Lin, Holmes, Simen, and Cohen (2009) extend this model further to account for modulations due to varied RSI, but the mechanisms of their model are still rooted in simple decaying memory traces.

Two models that are similar to the accumulator model of Usher and McClelland (2001) are the random walk decision model of Laming (1968) and the diffusion model of Ratcliff, VanZandt, and McKoon (1999). Sequential effects are addressed in the context of both of these models, though neither makes strong claims about the specific mechanisms that drive the sequential effects. Because no strong claim is made about sequential effects, it is not completely fair to classify them under the neural inertia category. However, the most natural way to give a mechanistic explanation of sequential effects in these models is to incorporate a bias variable that exhibits neural inertia and affects the starting point of the random walk or diffusion process.

Laming's (1968) random walk model for two-choice (and multi-choice) tasks proposes a decision trace that follows a random walk through the decision space until it reaches one of the absorbing boundaries, at which point the decision is made. The increments of the random walk are driven by a segmented stream of input values independently sampled from a distribution that is determined by the stimulus present. On average the random walk gravitates toward the correct response, but the variation in input values can slow the decision process or even lead to an incorrect response. The model hypothesizes that the appropriate starting point for the random walk is dependent upon the relative proportion of the two stimuli. However, Laming suggests that sequential effects may arise because participants sub-optimally estimate the response probabilities from the recent sequence. Such an estimate could easily be derived from a decaying memory trace of previous trials and could be used to produce variations in the starting point of the random walk.

Ratcliff et al. (1999) proposes a diffusion model for decision processing that is in essence the continuous-time counterpart to Laming's (1968) random walk model. Ratcliff et al. demonstrates that variability in drift rates and response boundaries are required for the model to produce sequential effects. However a theoretical explanation for what may cause this variability is not given, though again, as suggested above, this variability could result from residual activity.

Though the choice RT domain has been one of the most active for studying sequential effects, the concept of neural inertia has served to explain sequential effects in many other domains as well. For example in the judgment literature, sequential effects such as response assimilation have been explained as a bias driven by an implicit memory of recent events. In their judgment model for absolute identification, Petrov and Anderson (2005) maintain a decaying memory trace for each anchor or response that represents the degree of activation. The probability of selecting a particular anchor on the next trial increases with the strength of its activation trace. Brown, Marley, Donkin, and Heathcote (2008) capture sequential effects in their model of absolute identification via a contrast effect resulting from residual activation in the perceptual scale and a decisional bias towards recent stimuli resulting from residual activation present in the decision units of a ballistic accumulator. In the visual search domain, Maljkovic and Nakayama (1994, 1996) propose that RT is affected by priming of the attention-driving feature (color or spatial frequency) and the target position implemented in a decaying short-term memory trace.

2.2 Incremental Learning

Under the neural inertia explanation, the mechanisms responsible for sequential effects are hardwired. Though this is a simple, efficient solution to improving performance when successive events tend to be correlated, it offers no sensitivity to actual performance. An alternative perspective that suggests more engagement with the individual's trial-to-trial performance portrays sequential effects as a consequence of incremental learning processes. As individuals engage in a stream of events, they incrementally adapt behavior in such a way as to reduce future errors or increase future rewards. From this perspective, sequential effects are dependent upon the sequence of stimuli and responses as in the neural inertia explanation. However, the effects also depend on the appropriateness of past responses. The key property of these models is that the process of long-term learning also indirectly produces short-term sequential effects. Figure 2.2 displays a simple network architecture in which incremental learning might be used to update weights following each trial according to the error difference between the network output on a trial and the correct response.

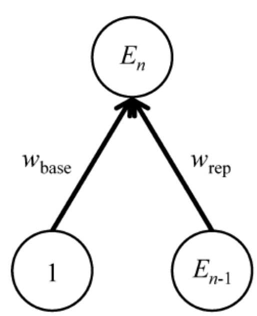


Figure 2.2: A simple model in which the predicted event at time n, E_n, depends on the previous event and a constant cue. Following the trial, an error signal is obtained by computing the difference between the predicted event outcome and the true outcome. This error signal is used to update weights in an incremental fashion.

The connection between sequential effects and theories of incremental learning has long been recognized. Statistical learning theory (Estes, 1950; Estes and Burke, 1953) hypothesizes a response strategy where response probabilities are updated on a trial-to-trial basis according to the reinforcing events. Because response probabilities fluctuate with each trial presentation, this model predicts simple sequential effects in event prediction paradigms (Estes, 1957; Estes and Straughan, 1954) and in probabilistic discrimination learning paradigms (Estes and Burke, 1955). Taking the analysis a step further, N. H. Anderson (1959) derives relationships between parameters of similar incremental learning models and observable sequential effects demonstrating the importance of sequential effects in understanding and testing general models of learning.

Incremental learning is also critical to classical conditioning theory in which the association between a stimulus and an outcome strengthens in a way proportional to how unexpected the outcome was given the stimulus (Rescorla and Wagner, 1972). In connectionist models, this corresponds to a delta-learning rule or error-correction learning and is employed in several models for categorization (e.g., Gluck and Bower, 1988; Kruschke, 1992; Nosofsky, Kruschke, and McKinley, 1992). These models focus on explaining the time course of learning during an experimental session or even across sessions. However, because error-driven learning occurs on every trial, these models naturally exhibit behavior that varies as a function of the sequence of recent stimuli. M. Jones and Sieck (2003) demonstrate that this class of models accurately capture the positive recency effects found in typical categorization experiments but that the models fail in producing the effects found in experiments that manipulate the correlation between outcomes on successive trials. By expanding the ALCOVE model of Kruschke (1992) to include an additional simple statistic that encodes aggregate information about the recent sequence of trials, M. Jones and Sieck (2003) account for the extra variation in the data that results from sequential autocorrelation.

In the motor control literature, movement error has been accounted for using an autoregressive model that includes past errors. Scheidt et al. (2001) obtain better fits to human movement error in a reaching task with artificially imposed forces by accounting for sequential effects in the data via a regression model that includes current force, previous force, and previous error. Similarly, Wong and Shelhamer (2011) account for the effect of the saccade errors in a repetitive eye-movement task using a simple autoregressive moving average (ARMA) model, though they give evidence for a longer-range learning process as well. Unlike in the connectionist models where weights are adjusted in accordance to the strength of the error signal, in these models, the next behavior depends directly on the previous error. Still these models can be viewed as a coarse form of incremental learning where behavior is continually adjusted according to recent performance, not just recent experiences.

The concept of incremental learning is also embodied in reinforcement learning (Sutton and Barto, 1998) where internal state/action pairs are updated after each experience according to the reward associated with the experience. Because decisions are biased towards recently rewarded actions, these models capture the types of sequential effects in which individuals tend to repeat previously successful actions. For example, reinforcement learning models have been successfully explained sequential effects in shot choice by basketball players (Neiman and Loewenstein, 2011) and attention allocation in a visual cueing paradigm (Chukoskie, Snider, Mozer, Krauzlis, and Sejnowski, 2013).

2.3 Prediction in a Dynamic Environment

Theories for incremental learning have been popular in part because they represent a simple way for the brain to obtain approximate solutions to complex optimization problems under the constraints of limited memory. Often, models based on incremental learning are proposed for these reasons and the sequential effects they produce are viewed as an inconsequential byproduct of the machinery. Similarly, under the neural inertia account, sequential effects are cast as a secondary byproduct of cognitive mechanisms. Generally these mechanisms are portrayed as suboptimal because they can lead to irrational behavior in certain settings. The classic example is a two-alternative forced-choice (2AFC) task where there is no correlation between successive stimuli and the stimuli occur with equal frequency. Participants who explicitly know that the stimuli are completely random still exhibit a bias toward one stimulus or the other depending on the recent history of trials (e.g., Cho et al., 2002). This has led many researchers to view sequential effects as a cognitive deficiency. However, if successive events tend to be positively correlated—as seems to be the case in the majority of real-world situations—sequential effects could reflect an intelligent means for adjusting behavior.

Rather than characterizing sequential effects as suboptimal, idiosyncratic behavior, an alternative perspective that has recently gained traction in the literature explains sequential effects as the reflection of a cognitive system designed to adapt to a dynamic environment (i.e., one in which the properties or statistics regularly change). The models that embody this perspective typically begin with basic assumptions about how the statistics of the environment change and then demonstrate that sequential effects result from optimal or approximately optimal behavior given these assumptions. While the neural inertia and incremental learning explanations fall into the implementation or algorithmic analysis levels of Marr (1982), the present explanation addresses sequential effects from a computational level of analysis. Here we are less interested in the mechanisms that produce sequential effects and instead seek to understand their critical function or purpose. Figure 2.3 gives a simple depiction of adaptation to the environment in a 2AFC task.

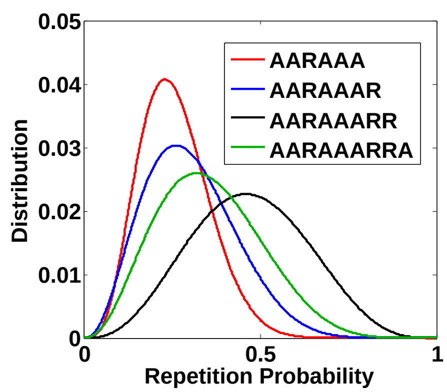


Figure 2.3: An example of adaptation to the statistics of the environment. Here an individual is assumed to maintain a probability distribution over the probability that the next trial will be a repetition in a 2AFC task. After the sequence AARAAA, where R is a repetition and A is an alternation, there is more density in the lower repetition probabilities. After two Rs, the densities shift toward greater values, but with the final A, the distribution shifts back slightly. Statistics are continually updated as trials occur and a bias is placed on more recent trials such that the model will be flexible to changes in the environmental statistics.

The perspective that behavior reveals underlying assumptions of environmental nonstationarity has been developed in the literature through the years. In considering the seemingly suboptimal behavior of probability matching, where participants roughly match the frequency of their responses to the outcome probabilities instead of always predicting the most probable response, Flood (1954) suggested that this behavior is appropriate if the participants expect outcome probabilities to change over time. Nissen and Bullemer (1987) propose that implicit learning of environmental statistical regularities allows for more accurate prediction of events in the near future leading to faster, potentially more accurate behavior. Further, J. R. Anderson and Schooler (1960) provided an ecological justification for the functional property of memory decay by comparing it to the empirical need probabilities for information in a variety of real-world domains. This sort of normative argument—in which behavior is cast as optimal given a set of assumptions about the environment—is compelling because it justifies aspects of cognition according to the contexts in which they are meant to function. As M. Jones and Sieck (2003) pointed out, the domain of sequential effects in decision-making was in need of such normatively inspired theories.

Recently there has been a proliferation of models that explicitly take this normative approach. Yu and Cohen (2009) explain the classic RT pattern in 2AFC with their Dynamic Belief Model (DBM). The fundamental assumption in the DBM is that the repetition probability in a sequence of trials follows changepoint dynamics—i.e., the probability of the current stimulus/response being the same as the previous is fixed for a period of time until it is randomly resampled. A rational agent operating under these assumptions produces sequential effects that closely match human data. In Chapter 3 (and in Wilder, Jones, and Mozer, 2010), we extend the DBM exhibiting a better fit to experimental data under the assumption that sequences result from the combination of the repetition probability component of DBM and a baserate stimulus probability also participant to changepoint dynamics. Gokaydin, Ma-Wyatt, Navarro, and Perfors (2011) demonstrate that only baserate probabilities need to be tracked to account for participant RTs in a 3AFC. Though their model is not specifically based on normative principles about the change dynamics, the model predictions are consistent with what would result from the baserate component in Wilder et al. (2010) if it were extended to a three choice task. Though the assumption of abrupt changepoint dynamics has been adopted in these models, it is equally feasible that the environmental statistics change gradually according to random walk dynamics instead. In many cases, these two different assumptions predict the same pattern of sequential effects though there should be testable differences. In several multiple cue probability learning experiments where the cue-criterion is nonstationarity, Speekenbrink and Shanks (2010) demonstrate that participants adapt predictions to both gradually changing dynamics and abrupt changepoint dynamics. The authors find some success in modeling the data using a Bayesian linear filter model which essentially uses a Kalman filter to track a random walk in cue criteria, but the model does not conclusively perform better than other existing models suitable for the domain.

In addition to estimating simple sequence statistics like the baserate and repetition probability, individuals have been shown to change behavior according to more abstract properties of the sequence. For example, in difficulty manipulations that intermix easy tasks with similar hard tasks, participants' RT and accuracy vary depending on the difficulty of recent trials (Taylor and Lupker, 2010). M. Mozer, Kinoshita, and Shettel (2007) propose that these sequential effects arise because individuals estimate task difficulty from recent experience. The model modulates performance on the current trial by taking a weighted average between a current accuracy trace and a historical accuracy trace in a hidden Markov model that is a generalization of a basic accumulation model. M. Jones, Mozer, and Kinoshita (2009) explain the same data with a rationally motivated model founded on the assumption that participants use the recent history to estimate the drift rate for the correct response—i.e., task difficulty—in a multi-response diffusion model. In a study of difficulty manipulation in a categorization task, Brown and Steyvers (2005) use a changepoint detection model to capture the shift in participant behavior that occurs when there is a shift in the discriminability of the two categories (i.e., how much overlap there is between the continuous stimuli from each category). Steyvers and Brown (2006) take the changepoint detection hypothesis a step further in an experiment where participants predicted the next item in a sequence generated using changepoint dynamics. The authors show that participants' behavior corresponded closely to a Bayesian optimal model performing changepoint inference, though it was necessary to parameterize the model sub-optimally to reproduce individual participant's tendency to over or under react to real changes. Interestingly, the ability for tracking environmental changepoints seems to be fundamental for most animals—even rats have been shown to perform close to Bayesian optimal detection of changes in reward rates (Gallistel et al., 2001).

The difficulty manipulations described above require the individual to monitor the properties of the task. Participants are not told when the difficulty changes and may not even be explicitly aware of the changes, but their behavior reveals that they are tracking task difficulty. In a similar paradigm that studies cognitive control, participants are given several different tasks to perform and are explicitly aware of task switches. Even still, sequential effects are commonly found in trials following task switches because participants are not able to change their behavior instantaneously and confuse the previously relevant strategy with the appropriate strategy for the current task. Reynolds and Mozer (2009) take a similar normative approach toward explaining this sort of cognitive control. Specifically, they obtain close fits to human data in a task-switching paradigm by modeling control as a dynamical inference process over the current task. Essentially they characterize cognitive control as an adaptive process that anticipates and responds to changes in the constraints of the environment.

The current explanation of sequential effects as adaptation to changing environmental statistics is not completely at odds with the previous two explanations. In fact, neural inertia and incremental learning can be viewed as implementation- and algorithmic-level solutions to the computational level problem of adapting to a dynamic environment. Yu and Cohen (2009) demonstrate that the DBM is well approximated by an exponentially decaying sum of past trials. This exponentially decaying sum could be implemented in the brain via exponentially decaying residual activation. Incremental learning methods also produce a similar exponentially decaying dependence on past trials. It may well be that our cognitive system is designed to be flexible to a dynamic environment, but lacking the ability to represent the complete statistics of the environment and compute the correct behavior given those statistics, the brain resorts to simpler solutions to produce behavior that is still flexible and adaptive. Nonetheless, the computational perspective that sequential effects reflect an attempt to optimally adapt to changing statistics offers important guidance in understanding why sequential effects are so ubiquitous in behavior.

2.4 Sensitivity Adjustment

There is a class of models, mostly in the judgment domain, that address sequential effects from a different angle. These models share the common property that sequential effects result from continual adjustments made to the sensitivity of perceptual and decisional processing. Though sensitivity adjustment can be viewed as a form of adaptation—i.e., the sensitivity is being adjusted in a way that is more appropriate for the type of events likely to occur—this perspective has different implications for how sequential effects manifest.

Perhaps the most familiar example of sensitivity adjustment is found in the visual system's high sensitivity to changes in intensity across a broad range of luminance levels. To achieve this sensitivity, many adaptive adjustments occur within the visual pathway as the illumination conditions of the environment change (Ver-Hoeve, 2007). In the judgment domain, a classic theory of sensitivity adjustment is the range-frequency model of Parducci (1965). The model assumes two principles that combine to direct how the range of categories is used. The range principle assumes category boundaries that depend only on the extreme stimulus values and the number of categories. The frequency principle assumes that each category is used for a fixed proportion of the judgments. Whereas the range principle is independent of the stimulus distribution (apart from the extreme values), the frequency principle hypothesizes that the category boundaries shift according to the stimulus distribution. Though sequential effects are not the focus of the model, the frequency principle naturally implies the presence of sequential effects due to changes in sensitivity across the range under the assumption that the boundaries depend more strongly on the recent stimulus distribution because of memory constraints. Figure 2.4 provides a simplified example how the sensitivity to three different categories might shift after several trials in a categorization task.

From an adaptation perspective, as suggested by Treisman and Williams (1984), the frequency principle serves to maximize the information transmitted by category responses given the current environmental statistics. Similarly, Wainwright (1999) proposes that adaptation in the visual system is driven by the goal to maximize information transmission. At a computational level of analysis, sequential effects that result from range adjustment can be understood as a strategy designed to improve the efficiency of perceptual and decisional processing. Maximizing the information transmitted is one such way in which this efficiency can be improved. Another way to improve efficiency of processing is to incorporate sensitivities to statistics of which types of events are more likely.

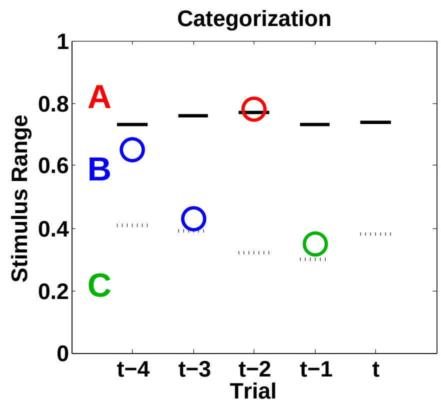


Figure 2.4: Shifts in category boundaries following a sequence of stimuli. After two presentations of category B, the sensitivity or preference for category B increases. However, with presentations of an A and a C on the subsequent trials, the sensitivity to B decreases again.

Treisman and Williams (1984) build both of these properties into their model for binary categorization. They capture short-term shifts in category boundaries using two competing components—a tracking system and a stabilizing system—the second of which closely parallels the frequency principle. The tracking system moves boundaries such that recently used categories correspond to a larger range of stimuli. The authors motivate this strategy with the observation that the external world changes: if an object has recently been detected then it is more likely to be detected in the near future, and thus the boundary for detection should be lowered to be less restrictive. The fault in the tracking system is that following a series of positive detections, the boundary could be set so low that all discrimination is lost. The stabilizing system counteracts this force by shifting the boundaries such that each category is used with roughly equal probability, i.e., following the frequency principle. In contrast to Parducci (1965), both components of the Treisman and Williams (1984) model adjust sensitivity in response to environmental properties. Advocating the adaptive properties of sensitivity adjustment, Treisman and Williams make the strong claim that "sequential dependencies arise from and reflect the operation of a system that attempts to place criteria at those positions that are optimal at any given moment and to keep them there, and that this same system is at work in all the tasks in which dependencies have been observed." (Treisman and Williams, 1984, p. 93)

Instead of categorizing a stimulus according to a set of boundaries, judgments can be modeled via a set of prototypes to which the stimulus is compared (e.g., Petrov and Anderson, 2005; Smith and Minda, 1998). A shift in the prototype locations can be mapped to an equivalent shift in criteria. However, when feedback is given, prototype locations are defined completely by the preceding stimulus/feedback pairs and thus shifts of the type produced by the frequency principle cannot be produced unless there is a concomitant shift in the feedback distribution. Nonetheless, prototype models have successfully accounted for sequential effects in a variety of identification and categorization experiments. Petrov and Anderson (2005) compute a selection probability for each category as a function of the similarity between the stimulus and the prototype (or anchor as they refer to it) and a category bias that depends on recent usage of the category. The selected anchor shifts after every trial to incorporate the new stimulus value in a way that discounts the previous anchor location exponentially. In this way, the sensitivity of decisional processing is continuously being adjusted to fit the stimulus/response mapping appropriate for the environment. In the absence of feedback, the model has the additional benefit that it adjusts the prototypes to produce balanced category responses under skewed stimulus/response distributions in a way consistent with the Parducci's frequency principle.

One criticism of criterion and prototype models is that they require a fairly large working memory when the set of potential responses or categories is large. Relative judgment theories present a popular alternative that avoids these memory constraints. The basic premise of relative judgment models is that people use the stimulus/response/feedback of the previous trial as a basis for assessing the current stimulus. The sensitivity of processing is continually being adjusted according to the previous stimulus and how it compares to the current stimulus. In some cases, relative judgment strategies also propose trial-to-trial adjustments in the scaling parameter that map a psychophysical difference between stimuli onto the decisional range. In these models, sequential effects generally occur because judgments are based on the memory or perception of the previous stimulus which is often inaccurate because of contamination due to stimuli farther in the past.

Though originally proposed to explain behavior in a magnitude estimation task, where continuous response values are assigned to the stimuli, relative judgment theories are equally applicable to behavior in absolute identification and categorization tasks. Luce and Green (1974) proposed that responses are chosen such that the ratio of the current response to the previous response is proportional to the ratio of the internal representations of the current stimulus and an unfaithful representation of the previous stimulus. The key assumption in the model that produces sequential effects is that the representation of the stimulus from the previous trial is not equal to the stimulus value used to generate the response on the previous trial—i.e., the act of responding to a stimulus changes the representation of the stimulus. DeCarlo and Cross (1990) generalize the response ratio hypothesis by modeling the response as a transformation of the stimulus sensation that depends on a weighted combination of the ratio of the previous stimulus and response and the ratio of a fixed reference stimulus and response. Responses are also affected by an autocorrelated error process. Furthermore, the model assumes that the perception of a stimulus is affected by the context of previous stimuli in such a way that can produce response assimilation or contrast effects depending on parameterization.

Stewart, Brown, and Chater (2005) present a relative judgment model for absolute identification that pushes the relative judgment concept further by assuming that the fundamental currency for perception and decision is the stimulus difference rather than just the pure stimulus. From this perspective, all processing is inherently relative because it depends on the relationship between the previous stimulus and the current stimulus. In their model, this difference probabilistically maps to a response above or below the previous feedback depending on the sign of the difference. The mapping depends on a fixed scaling parameter and a fixed variance noise signal that is distributed over the potential responses. The model also assumes that the stimulus difference on the current trial is confused with or contaminated by the differences on previous trials. This is similar to the context that affects stimulus perception in DeCarlo and Cross (1990), and is primarily responsible for the sequential effects produced by the model because it leads to a contamination in the range of processing (see Appendix A for a derivation that illustrates the similarity between Stewart, Chater, and Brown, 2005 and DeCarlo and Cross, 1990).

In addition to shifting the range of processing, relative judgment strategies also impose strong constraints on the range that is relevant for the current trial (e.g., if the current stimulus is perceived as larger than the previous one, only responses greater than the previous feedback will be considered). These constraints generate a shift in sensitivity across the decisional range. Petzold and Haubensak (2001) propose another relative judgment model that places even more constraints on the acceptable response range. In their model, the current stimulus is judged relative to the two previous stimuli and the extreme ends of the range which are assumed to be stored in memory. The appropriate range of potential responses is bounded above and below the current stimulus by the ends of the spectrum or the recent stimuli depending on how the current stimulus compares to the previous two. For example, if the bounds of the range are 1 and 10, the previous two stimuli were 5 and 7, and the current stimulus is perceived smaller than both the previous two, then the range of responses will be 1 to 5. Once the sub-range is determined, the response is selected based on the ratio of the difference between the stimulus and an endpoint and the difference between the sub-range endpoints. The model produces sequential effects because the response depends both on the similarity between the current stimulus and the previous stimuli and the relative location of the stimuli.

The aforementioned models place a greater emphasis on explaining sequential effects that result from adjustments made to sensitivities in decisional processing. However, adjustments made to the perceptual sensitivity can also result in sequential effects. In some cases, it is difficult to separate perceptual from decisional effects because stimuli and responses are confounded in most experiments. Jesteadt et al. (1977) further develop the response ratio hypothesis of Luce and Green (1974) and explore the weights of the previous stimulus and response in a regression analysis. They obtain a positive coefficient for the previous response, suggesting assimilation to the previous response, and a negative coefficient for the previous stimulus, suggesting stimulus contrast. This result opens the possibility that perceptual and decisional effects may compete with each other. However, DeCarlo and Cross (1990) have shown that caution must be taken when analyzing the coefficients in a regression equation because different theoretical perspectives lead to different interpretations of the parameters. Nonetheless, perceptual contrast appears to be present in many judgment tasks. Petzold (1981) base their model for categorization on perceptual contrast and a guessing strategy that produces assimilation to the previous response. The model proposes that the trace of the previous stimulus serves as an internal standard for judgment. Helson (1964) has suggested that encoding stimuli using such a relative strategy may serve to expand the representational range of a finite neural system and that perceptual contrast effects may be a consequence of this relative strategy.

Recently, stronger evidence has been provided for perceptual contrast in categorization and identification paradigms via experimental designs that avoid the confound between stimuli and responses (M. Jones, 2009; M. Jones, Love, and Maddox, 2006; M. Jones and Sieck, 2003). By imposing overlapping probabilistic categories or identification labels, it is possible to isolate the effect of the previous stimulus on behavior by considering all trials where the previous feedback label is inconsistent with the previous stimulus. (M. Jones et al., 2006) propose a mathematical model for quantifying the perceptual contrast in experimental data. Though the model does not take a strong theoretical stance, the perceptual component is consistent with the relative judgment model of DeCarlo and Cross (1990) in which the perceptual range used to assess the current stimulus is skewed by the context of stimuli that were recently presented.

The observation that sequential effects result from adjustments in both perceptual sensitivity and decisional sensitivity suggests a greater level of complexity in the experimental data and implies that greater care must be taken in assessing models of sequential effects. Specifically, models that attempt to capture the aggregate sequential effects present in the data may be misguided because they fail to recognize the possibility that the effects are produced by multiple mechanisms operating in concert. Recognizing this division also opens the door for interpreting sequential effects via multiple high-level explanations that play out in different stages of the processing. In fact, this is the stance presented in (M. Jones et al., 2006) and (M. Jones, 2009) in which sequential effects are explained as arising from perceptual sensitivity adjustment combined with the notion of decisional generalization. This notion of generalization is the final high-level explanation we present for sequential effects.

2.5 Generalization

In classical conditioning, stimulus generalization refers to the tendency for the conditioned stimulus to evoke similar responses after the response has been conditioned and for similar stimuli to evoke similar responses. The degree to which knowledge about one stimulus will generalize to another stimulus has been shown to depend on the similarity between them, with the probability of generalization decreasing exponentially with psychological distance (Shepard, 1957, 1987). In judgment tasks, where participants categorize, identify, or make estimates about a stimulus and are conditioned through feedback, stimulus generalization can have a significant impact on behavior. Typically, stimulus/response associations are assumed to be relatively constant following a learning period. However, if generalization has a greater dependence on recently conditioned stimulus/response pairs, we would observe sequential effects in the data. Typically, generalization produces an assimilation effect where the response for the current stimulus is biased towards recent responses to similar stimuli. Though when recent stimuli are highly dissimilar to the current stimulus, there is some evidence that the effect is contrastive (M. Jones et al., 2006). Figure 2.5 portrays a simple generalization process in action.

The concept of generalization is strongly embodied in exemplar models (e.g., Medin and Schaffer, 1978; Nosofsky, 1986) in which decisions result from direct comparisons between the current stimulus and previous stimuli. In their simplest form, exemplar models do not produce

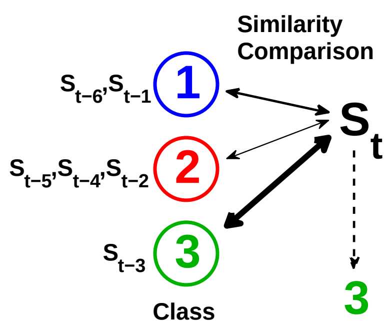


Figure 2.5: Generalization to past trials. Here previous trials are stored in memory and used as exemplars for three classes. A new stimulus, St, is compared to the exemplars and the class label chosen corresponds to the exemplar that best generalizes to the current stimulus.

sequential effects because stimuli that occurred far in the past have an equal affect on behavior as recent stimuli. However, when the models are modified so that more recent exemplars have a greater likelihood of being recalled or a stronger memory trace, the current response is biased toward recent responses (Nosofsky et al., 1992; Nosofsky and Palmeri, 1997). Prototype models (e.g., Petrov and Anderson, 2005; Smith and Minda, 1998) are also consistent with the concept of generalization if the role of prototypes matches the role of exemplars above (i.e., judgments are based on the degree to which the different prototypes generalize to the current stimulus). For example, in Petrov and Anderson's (2005) absolute identification model, an anchor (prototype) selection process essentially measures how well each anchor location generalizes to the current stimulus and probabilistically chooses a response. In fact, the prototypes themselves represent a slightly different form of generalization in which knowledge from the collection of past experiences is generalized into a single representative for the class. As new experiences are encountered, they are continually integrated into the anchor estimates in a way that places more weight on recent experiences and consequently results in sequential dependencies similar to those in the exemplar models.

Recent models of category learning have taken an even stronger stance on the relationship between generalization and sequential dependencies (M. Jones, 2009; M. Jones et al., 2006; M. Jones and Sieck, 2003). M. Jones et al. (2006) demonstrate the dependence of decisional recency on similarity and posit that the effect is a direct byproduct of generalization. Furthermore, they present a mathematical model for quantifying the strength of generalization by measuring the decisional recency effect. M. Jones (2009) expands these ideas in a model for absolute identification based on the assertion that decisional processes are modulated by similarity-based generalization. In this model, generalization is implemented using reinforcement learning—an incremental learning strategy discussed above. In fact, reinforcement learning is deeply tied to generalization throughout the conditioning literature. Here the computational-level explanation for the sequential effects is that generalization of recent experience affects behavior. However, this perspective is compatible with an algorithmic-level explanation that posits incremental learning processes.

Generalizing in a way that is biased towards recent experience can also be viewed as an adaptive strategy. M. Jones et al. (2006) suggest that this sort of generalization is well suited for a changing environment. Similarly, Petrov and Anderson (2005) hint that the continual updating of anchor locations helps maintain their utility as the environmental statistics shift

2.6 Discussion

We have argued that the broad, diverse literature on sequential effects can be unified under a small set of theoretical accounts for their existence. Rather than viewing these effects as idiosyncratic behavioral modulations unique to each different domain of study, sequential effects can be thought of as reflecting a few high-level computational goals of the cognitive system. This integration brings coherence to sequential effects phenomena and serves researchers in interpreting sequential effects in a way that is consistent and unified across the myriad domains in which they occur.

In our first computational-level explanation, we understand sequential effects by taking the perspective that individuals are trying to form expectations about what stimulus will occur next. Further, we hypothesize that the individuals assume that the environment is changing and thus base these expectations on the sequence of previous stimuli with greater weight placed on recent stimuli. The models that embody this perspective begin with assumptions about the environment and then show that behavior is optimal or near optimal given these assumptions. This perspective naturally extends to the real world where individuals are continually inundated with a barrage of sensations and are required to produce appropriate actions. By preparing ourselves for the situations that are likely to occur next, we can improve the efficiency of our behavior. If expectations are formed with greater emphasis placed on recent experience, our behavior will be more adaptive to the frequent changes in the statistics that govern our environment.

The second computational-level explanation for sequential effects characterizes them as resulting from adjustments made to the sensitivity of perceptual and decisional processing. Rather than forming direct expectations about what events might occur next, here individuals change how they process stimuli according to the stimuli and responses recently experienced. These adjustments can be interpreted as a way to improve the efficiency of stimulus processing and response formulation, for example, maximizing the information transmitted by responses, expanding the representational range, or leveraging knowledge about the likelihood of different events.

In our third computational-level explanation, we cast sequential effects as a consequence of generalization. Generalization refers to the process of applying previous knowledge or experience to the current situation. For example, in a judgment study, one may generalize their knowledge with past stimulus/response pairs to form a judgment for the current stimulus. If individuals have a stronger preference to generalize using more recent experiences, sequential effects will appear in the behavioral data. Typically generalization is dependent on the similarity between the current stimulus and the generalizing stimulus. This same dependence on similarity is found in some sequential effects studies, further supporting the presence of generalization.

In addition to the three computational-level explanations for sequential effects, we observed two lower-level explanations present in a broad collection of theoretical accounts. In many models, sequential effects are explained as the result of neural inertia in the cognitive system. The basic priming that results from these mechanisms can explain many types of sequential effects. However, because it is a simple implementation-level account, this explanation fails to provide strong claims about the underlying purpose of sequential effects. Similarly, sequential effects have been explained at an algorithmic level as arising from incremental learning processes in the brain. Again sequential effects are accounted for mechanistically, but there is no strong theoretical stance on why they occur. There is still value in these lower-level explanations for sequential effects because in most cases they are consistent with the three primary computational-level explanations we propose. Though it is important to understand how sequential effects are produced by actual mechanisms in the brain and algorithmic processes, we believe there is significant utility in understanding and classifying sequential effects at a computational level because this level strongly addresses the questions of why sequential effects occur and why they are important.

Other salient distinctions can be made between models of sequential effects that are interesting and informative. However, we have avoided addressing these up to this point so as to not muddle our categorization. For example, many models differ in the duration of memory of past experience and the persistence of the sequential effects. The majority of models posit that sequential effects are very short lived. However, some recent models propose that implicit memory of the past can last over a longer duration or across many intervening events thus resulting in temporally extended sequential effects. Exemplar models are the extreme example of long-lasting memories—i.e., in theory all events are remembered forever. Though, as mentioned earlier, to produce sequential effects, these models must place greater weight on recent trials. Typically, the weights decay exponentially and rapidly go to zero resulting in only short-lasting sequential effects, but a weighting that falls off more slowly would reasonably produce longer-lasting effects. Petrov and Anderson (2005) capture sequential effects in their model in part by maintaining an activation level for each anchor that increases the availability of that anchor on future trials. The activation of an anchor roughly exhibits power function decay dependent on the time elapsed since past uses of that anchor. With its characteristic heavy tail, power function decay results in a longer-lasting memory of the sequence of responses or anchors selected. Wong and Shelhamer (2011) similarly propose power function decay over past trials and demonstrate correlations in behavior that extend out to almost 100 intervening trials. In Chapter 5, we offer further evidence for longer-lasting sequential effects in a 2AFC task and a motor control task. Building upon theories that assume a dynamic environment, we propose the Hierarchical Dynamic Belief Model (HDBM) that relaxes unnatural assumptions in the DBM and results in an integration of past trials that more closely resembles a power function weighting.

Another distinction between models is the degree of abstraction used for representing trials or events. The simplest representation is the actual trial identity (e.g., a specific digit between 1 and 6 as in Falmagne, 1965; or light 2 and light 3 in Remington, 1969). Alternatively, trials can be represented in how they relate to the previous trial. This is most sensible in 2AFC tasks where each trial is either a repetition or an alternation (e.g., Hale, 1969, studies the effect of runs of repetitions and alternations). Depending on which abstraction is used in the analysis, the pattern of sequential effects can appear very different. Cho et al. (2002) demonstrate the value of considering multiple abstractions by modeling sequential effects via the combination of a mechanism that tracks the actual trial identities and a mechanism that tracks the repetition/alternation sequence. Even more complex abstractions can be responsible for sequential effects as in models that capture dynamic changes in task difficulty (e.g., Jones et al., 2009; Mozer et al., 2007). There is less evidence of representational abstraction influencing the type of effects observed in judgment studies. However, Stewart et al. (2005) propose that the currency used in processing a stimulus is its difference relative to the previous trial rather than its pure psychophysical magnitude. Perhaps there are other levels of abstract representations that influence effects in judgment tasks, but have yet gone unnoticed. For example, how might task difficulty modulations change the sequential effects observed in an identification or categorization task? Brown and Steyvers (2005) suggest that there will be an effect, but they only consider changes in difficulty that occur across blocks of trials, not within blocks.

The difference between the neural inertia explanation and the incremental learning explanation hints at another distinction that can be made between models. Unlike in the neural inertia account where only the sequence of previous trials affects behavior, with incremental learning, modulations in behavior are based on both the sequence of past trials and the individual's performance on those trials. This distinction can be cast as a difference between unsupervised learning and supervised learning strategies. Models that utilize unsupervised learning learn from the environment without feedback. Neural inertia and short-term priming models have this characteristic as do the more complex models that perform Bayesian inference over the environmental statistics assuming nonstationarity. Supervised learning is present in incremental learning models and can be present in generalization models that utilize reinforcement learning (e.g., Jones, 2009). Most models in the sensitivity adjustment explanation category can be viewed as unsupervised learning models (e.g., the tracking and stabilizing systems in Treisman and Williams, 1984, are unsupervised). Nonetheless, it is feasible that adjustments to sensitivity could be made in a supervised manor.

Sequential effects have been attributed to another factor that we have not yet discussed. Specifically, correlations in successive responses can arise from latent variables in the cognitive system that influence behavior but change over longer timescales. For example, in a task where the participant simply presses a button each time a single stimulus appears, it is likely that the participant's attention will slowly drift throughout the experiment resulting in nearby trials to have similar characteristics (i.e., faster when attention is high and slower when attention is low). In the motor control literature, Kording, Tenenbaum, and Shadmehr (2007) propose a model that uses a Kalman filter to capture the influence on behavior of multiple latent variables which drift at varying timescales. Similarly, DeCarlo and Cross (1990) incorporate an autocorrelated error process into their model for sequential effects in judgement. The authors explicitly discuss how this error process can capture the effects of slowly drifting latent variables. Gilden, Thornton, and Mallon (1995) have demonstrated the presence of 1/f noise in many aspects of cognition (i.e., long-range correlations in behavior that result from some property of the internal state). Though this type of response autocorrelation is often classified as a sequential effect, it not consistent with other types of sequential effects because behavior is not modulated by the exact sequence of recent trials. Nonetheless, it is important to identify this response autocorrelation and be aware of how it can change the appearance of other sequential effects.

In this chapter, we have demonstrated the widespread presence of sequential effects in cognition and the lack of organization that exists in the large collection of models that seek to explain these effects. Our goal has been to introduce a theoretical framework that synthesizes this disparate literature and highlights a few high-level principles that explain sequential effects across all domains of cognition. In the process, we have observed that most sequential effects reflect an adaptive process in the brain. This is most evident in our first computational-level explanation in which sequential effects result from the attempt to form expectations of future events under the assumptions of a dynamic environment. By studying sequential effects through the lens of the models that fall in this category, it is possible to expose what individuals are adapting to and how they go about that adaptation. In the other two computational-level explanations, an adaptive process is suggested, but the claims about the specifics of adaptation are weaker. However, by founding these sorts of models more directly on normative principles regarding adaptation to the environment, it may be possible to learn more about the dynamics of cognitive adaptation. We believe this synthesis will provide a useful framework for understanding sequential effects in all domains. We encourage researchers to pay closer attention to sequential effects and give greater consideration to what insight these effects might offer with respect to the role of adaptation in their specific domain.

Chapter 3

Sequential Effects Reflect Parallel Learning of Multiple Environmental Regularities[1]

The synthesis in Chapter 2, presents a compelling argument for the perspective that sequential effects reflect an individual's attempt to optimally adapt to a dynamic environment. One classic domain where this account is successful is in *two-alternative forced-choice* (2AFC) tasks (e.g, Jentzsch and Sommer, 2002; Hale, 1969; Soetens et al., 1985; Cho et al., 2002). Specifically, the Dynamic Belief Model (DBM) (Yu and Cohen, 2009) explains sequential effects in 2AFC tasks as a rational consequence of a dynamic internal representation that tracks the repetition rate of the trial sequence and predicts whether the upcoming trial will be a repetition or an alternation of the previous trial. However, experimental results suggest that stimulus baserates also influence sequential effects. In this chapter, we propose a model that learns both baserates and repetition rates, each according to the basic principles of the DBM but under a unified inferential framework.

3.1 Background

In 2AFC tasks, participants are shown one of two different stimuli, which we denote as X and Y, and are instructed to respond as quickly as possible by mapping the stimulus to a corresponding response, say pressing the left button for X and the right button for Y. Response time (RT) is recorded, and the task is repeated several hundred or thousand times. To measure sequential effects, the RT is conditioned on the recent trial history. (In 2AFC tasks, stimuli and responses are

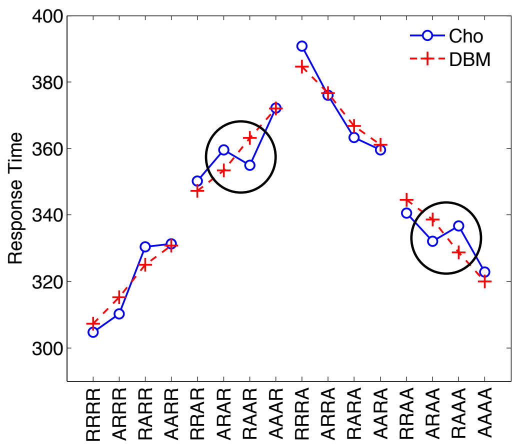
[1] Most of this chapter is an adaptation of the published work of Wilder, Jones, and Mozer (2010)

confounded; as a result, it is common to refer to the 'trial' instead of the 'stimulus' or 'response'. In this chapter, 'trial' will be synonymous with the stimulus-response pair.) Consider a sequence such as $XYYXX$, where the rightmost symbol is the current trial (X), and the symbols to the left are successively earlier trials. Such a four-back trial history can be represented in a manner that focuses not on the trial identities, but on whether trials are repeated or alternated. With R and A denoting repetitions and alternations, respectively, the trial sequence $XYYXX$ can be encoded as $ARAR$. Note that this R/A encoding collapses across isomorphic sequences $XYYXX$ and $YXXYY$.

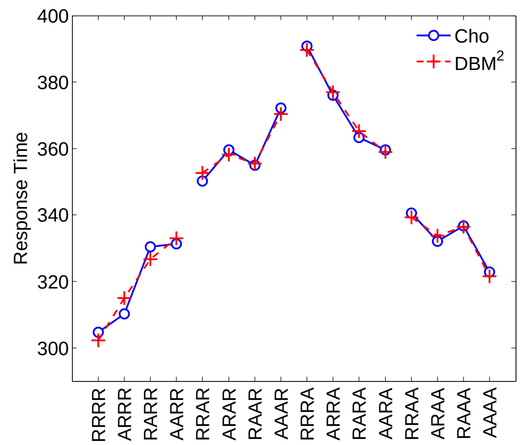
The small blue circles in Figure 3.1a show the RTs from Cho et al. (2002) conditioned on the recent trial history. Along the abscissa in Figure 3.1a are all four-back sequence histories ordered according to the R/A encoding. The left half of the graph represents cases where the current trial is a repetition of the previous, and the right half represents cases where the current trial is an alternation. The general pattern we see in the data is a triangular shape that can be understood by comparing the two extreme points on each half, $RRRR$ vs. $AAAR$ and $RRRA$ vs. $AAAA$. It seems logical that the response to the current trial in $RRRR$ will be significantly faster than in $AAAR$ ($RT_{RRRR} < RT_{AAAR}$) because in the $RRRR$ case, the current trial matches the expectation built up over the past few trials whereas in the $AAAR$ case, the current trial violates the expectation of an alternation. The same argument applies to $RRRA$ vs. $AAAA$, leading to the intuition that $RT_{RRRA} > RT_{AAAA}$. The trial histories are ordered along the abscissa so that the left half is monotonically increasing and the right half is monotonically decreasing following the same line of intuition, i.e., many recent repetitions to many recent alternations.

3.2 Toward A Rational Model Of Sequential Effects

Many models have been proposed to capture sequential effects in 2AFC (e.g., Laming, 1969; Soetens, Boer, and Hueting, 1985; Cho et al., 2002). Other models have interpreted sequential effects as adaptation to the statistical structure of a dynamic environment (e.g., M. Jones and Sieck, 2003; M. Mozer, Kinoshita, and Shettel, 2007). In this same vein, with the DBM, Yu and



a)



b)

Figure 3.1: a) DBM fit to the behavioral data from Cho et al. (2002). Predictions within each of the four groups are monotonically increasing or decreasing. Thus the model is unable to account for the two circled relationships. This fit accounts for 95.8% of the variance in the data. ($p_0 = \text{Beta}(2.6155, 2.4547), \alpha = 0.4899$) b) The fit to the same data obtained from DBM^2 in which probability estimates are derived from both first-degree and second-degree trial statistics. 99.2% of the data variance is explained by this fit. ($\alpha = 0.3427$, $w = 0.4763$)

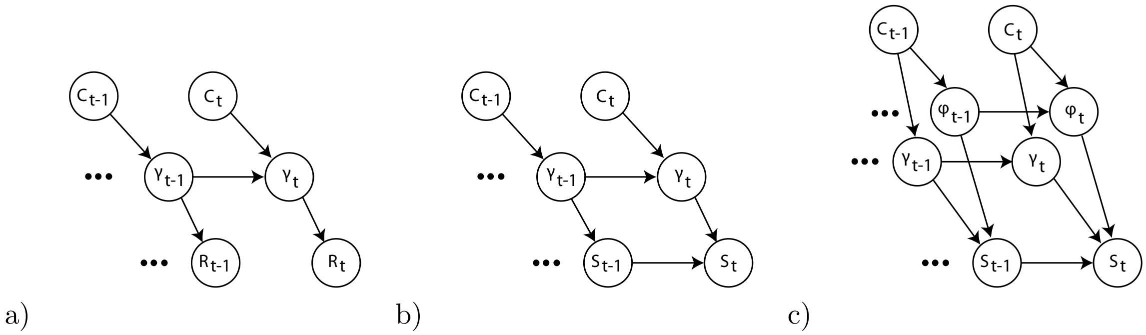


Figure 3.2: Three graphical models that capture sequential dependencies. a) Dynamic Belief Model (DBM) of Yu and Cohen (2009). b) A reformulation of DBM in which the output variable, S_t, is the actual stimulus identity instead of the repetition/alternation representation used in DBM. c) Our proposed Dynamic Belief Mixture Model (DBM^2). Models are explained in more detail in the text.

Cohen (2009) have suggested a rational explanation for sequential effects such as those observed in Cho et al. (2002). The key contribution of their work is that it provides a rational justification for sequential effects that have been previously viewed as resulting from low-level brain mechanisms such as residual neural activation.

The DBM describes performance in 2AFC tasks as Bayesian inference over whether the next trial in the sequence will be a repetition or an alternation of the previous trial, conditioned on the trial history. If R_t is the Bernoulli random variable that denotes whether trial t is a repetition ($R_t = 1$) or alternation ($R_t = 0$) of the previous trial, DBM determines $P(R_t|\vec{R}_{t-1})$, where $\vec{R}_{t-1}$ denotes the trial sequence preceding trial t, i.e., $\vec{R}_{t-1} = (R_1, R_2, ..., R_{t-1})$.

DBM assumes a generative model, shown in Figure 3.2a, in which $R_t = 1$ with probability γ_t and $R_t = 0$ with probability $1 - \gamma_t$. The random variable γ_t describes a characteristic of the environment. According to the generative model, the environment is nonstationary and γ_t can either retain the same value as on trial $t-1$ or it can change. Specifically, C_t denotes whether the environment has changed between $t-1$ and t ($C_t = 1$) or not ($C_t = 0$). C_t is a Bernoulli random variable with success probability α. If the environment does not change, $\gamma_t = \gamma_{t-1}$. If the environment changes, γ_t is drawn from a prior distribution, which we refer to as the *reset prior* denoted by $p_0(\gamma) \sim Beta(a, b)$.

Before each trial t of a 2AFC task, DBM computes the probability of the upcoming stimulus conditioned on the trial history. The model assumes that the perceptual and motor system is tuned based on this expectation, so that RT will be a linearly decreasing function of the probability assigned to the event that actually occurs, i.e. of $P(R_t = R|\vec{R}_{t-1})$ on repetition trials and of $P(R_t = A|\vec{R}_{t-1}) = 1 - P(R_t = R|\vec{R}_{t-1})$ on alternation trials.

The red plusses in Figure 3.1 show DBM's fit to the data from Cho et al. (2002). DBM has five free parameters that were optimized to fit the data. The parameters are: the change probability, α; the imaginary counts of the reset prior, a and b; and two additional parameters to map model probabilities to RTs via an affine transform.

3.2.1 Intuiting DBM Predictions

Another contribution of Yu and Cohen (2009) is the mathematical demonstration that DBM is approximately equivalent to an exponential filter over trial histories. That is, the probability that the current stimulus is a repetition is a weighted sum of past observations, with repetitions being scored as 1 and alternations as 0, and with weights decaying exponentially as a function of lag. The exponential filter gives insight into how DBM probabilities will vary as a function of trial history. Consider two 4-back trial histories: an alternation followed by two repetitions ($ARR-$) and two alternations followed by a repetition ($AAR-$), where the $-$ indicates that the current trial type is unknown. An exponential filter predicts that $ARR-$ will always create a stronger expectation for an R on the current trial than $AAR-$ will, because the former includes an additional past repetition. Thus, if the current trial is in fact a repetition, the model predicts a faster RT for $ARR-$ compared to $AAR-$ (i.e., $RT_{ARRR} < RT_{AARR}$). Conversely, if the current trial is an alternation, the model predicts $RT_{ARRA} > RT_{AARA}$. Similarly, if two sequences with the same number of Rs and As are compared, for example $RAR-$ and $ARR-$, the model predicts $RT_{RARR} > RT_{ARRR}$ and $RT_{RARA} < RT_{ARRA}$ because more recent trials have a stronger influence.

Comparing the exponential filter predictions for adjacent sequences in Figure 3.1 yields the expectation that the RTs will be monotonically increasing in the left two groups of four and monotonically decreasing in the two right groups. The data are divided into groups of 4 because the relationships between histories like $AARR$ and $RRAR$ depend on the specific parameters of the exponential filter, which determine whether one recent A will outweigh two earlier As. It is clear in Figure 3.1 that the DBM predictions follow this pattern.

3.2.2 What's Missing in DBM

DBM offers an impressive fit to the overall pattern of the behavioral data. Circled in Figure 3.1, however, we see two significant pairs of sequence histories for which the monotonicity prediction does not hold. These are reliable aspects of the data and are not measurement error. Consider the circle on the left, in which $RT_{ARAR} > RT_{RAAR}$ for the human data. Because DBM functions approximately as an exponential filter, and the repetition in the trial history is more recent for $ARAR$ than for $RAAR$, DBM predicts $RT_{ARAR} < RT_{RAAR}$. An exponential filter, and thus DBM, is unable to account for this deviation in the data.

To understand this mismatch, we consider an alternative representation of the trial history: the first-degree sequence, i.e., the sequence of actual stimulus values. The two R/A sequences $ARAR$ and $RAAR$ correspond to stimulus sequences $XYYXX$ and $XXYXX$. If we consider an exponential filter on the actual stimulus sequence, we obtain the opposite prediction from that of DBM: $RT_{XYYXX} > RT_{XXYXX}$ because there are more recent occurrences of X in the latter sequence. The other circled data in Figure 3.1a correspond to an analogous situation. Again, DBM also makes a prediction inconsistent with the data, that $RT_{ARAA} > RT_{RAAA}$, whereas an exponential filter on stimulus values predicts the opposite outcome—$RT_{XYYXY} < RT_{XXYXY}$. Of course this analysis leads to predictions for other pairs of points where DBM is consistent with the data and a stimulus based exponential filter is inconsistent. Nevertheless, the variations in the data suggest that more importance should be given to the actual stimulus values.

In general, we can divide the sequential effects observed in the data into two classes: first- and second-degree effects. First-degree sequential effects result from the priming of specific stimulus or response values. We refer to this as a first-degree effect because it depends only on the stimulus values rather than a higher-degree representation such as the repetition/alternation nature of a trial. These effects correspond to the estimation of the baserate of each stimulus or response value. They are observed in a wide range of experimental paradigms and are referred to as stimulus priming or response priming. The effects captured by DBM, i.e. the triangular pattern in RT data, can be thought of as a second-degree effect because it reflects learning of the correlation structure between the current trial and the previous trial. In second-degree effects, the actual stimulus value is irrelevant and all that matters is whether the stimulus was a repetition of the previous trial. As DBM proposes, these effects essentially arise from an attempt to estimate the repetition rate of the sequence.

DBM naturally produces second-degree sequential effects because it abstracts over the stimulus level of description: observations in the model are R and A instead of the actual stimuli X and Y. Because of this abstraction, DBM is inherently unable to exhibit first-degree effects. To gain an understanding of how first-degree effects could be integrated into this type of Bayesian framework, we reformulate the DBM architecture. Figure 3.2b shows an equivalent depiction of DBM in which the generative process on trial t produces the actual stimulus value, denoted S_t. S_t is conditioned on both the repetition probability, γ_t, and the previous stimulus value, S_{t-1}. Under this formulation, $S_t = S_{t-1}$ with probability γ_t, and S_t equals the opposite of S_{t-1} (i.e., XY or YX) with probability $1 - \gamma_t$.

An additional benefit of this reformulated architecture is that it can represent first-degree effects if we switch the meaning of γ. In particular, we can treat γ as the probability of the stimulus taking on a specific value (X or Y) instead of the probability of a repetition. S_t is then simply a draw from a Bernoulli process with rate γ. Note that for modeling a first-degree effect with this architecture, the conditional dependence of S_t on S_{t-1} becomes unnecessary. The nonstationarity of the environment, as represented by the change variable C, behaves in the same way regardless of whether we use the model to represent first- or second-degree structure.

3.3 Dynamic Belief Mixture Model (DBM^2)

The complex contributions of first- and second-degree effects to the full pattern of observed sequential effects suggest the need for a model with more explanatory power than DBM. It seems clear that individuals are performing a more sophisticated inference about the statistics of the environment than proposed by DBM. We have shown that the DBM architecture can be reformulated to generate first-degree effects by having it infer the baserate instead of the repetition rate of the sequence, but the empirical data suggest both mechanisms are present simultaneously. Thus the challenge is to merge these two effects into one model that performs joint inference over both environmental statistics.

Here we propose a Bayesian model that captures both first- and second-degree effects, building on the basic principles of DBM. According to this new model, which we call the Dynamic Belief Mixture Model (DBM^2), the learner assumes that the stimulus on a given trial is probabilistically affected by two factors: the random variable ϕ, which represents the sequence baserate, and the random variable γ, which represents the repetition rate. The combination of these two factors is governed by a mixture weight w that represents the relative weight of the ϕ component. As in DBM, the environment is assumed to be nonstationary, meaning that on each trial, with probability α, ϕ and γ are jointly resampled from the reset prior, $p_0(\phi, \gamma)$, which is uniform over $[0, 1]^2$. Figure 3.2c shows the graphical architecture for this model. This architecture is an extension of our reformulation of the DBM architecture in Figure 3.2b. Importantly, the observed variable, S, is the actual stimulus value instead of the repetition/alternation representation used in DBM. This architecture allows for explicit representation of the baserate, through the direct influence of ϕ_t on the physical stimulus value S_t, as well as representation of the repetition rate through the joint influence of γ_t and the previous stimulus S_{t-1} on S_t. Formally, we express the probability of S_t given ϕ, γ, and S_{t-1} as shown in Equation 3.1.

$$P(S_t = X|\phi_t, \gamma_t, S_{t-1} = X) = w\phi_t + (1-w)\gamma_t$$
$$P(S_t = X|\phi_t, \gamma_t, S_{t-1} = Y) = w\phi_t + (1-w)(1-\gamma_t) \tag{3.1}$$

DBM^2 operates by maintaining the iterative prior over ϕ and γ, $p(\phi_t, \gamma_t|\vec{S}_{t-1})$. After each observation, the joint posterior, $p(\phi_t, \gamma_t|\vec{S}_t)$, is computed using Bayes' Rule from the iterative prior and the likelihood of the most recent observation, as shown in Equation 3.2.

$$p(\phi_t, \gamma_t|\vec{S}_t) \propto P(S_t|\phi_t, \gamma_t, S_{t-1})p(\phi_t, \gamma_t|\vec{S}_{t-1}). \tag{3.2}$$

The iterative prior for the next trial is then a mixture of the posterior from the current trial, weighted by $1-\alpha$, and the reset prior, weighted by α (the probability of change in ϕ and γ).

$$p(\phi_{t+1}, \gamma_{t+1}|\vec{S}_t) = (1-\alpha)p(\phi_t, \gamma_t|\vec{S}_t) + \alpha p_0(\phi_{t+1}, \gamma_{t+1}). \tag{3.3}$$

The model generates predictions, $P(S_t|\vec{S}_{t-1})$, by integrating Equation 3.1 over the iterative prior on ϕ_t and γ_t. In our simulations, we maintain a discrete approximation to the continuous joint iterative prior with the interval [0,1] divided into 100 equally spaced sections. Expectations are computed by summing over the discrete probability mass function.

Figure 3.1b shows that DBM^2 provides an excellent fit to the Cho et al. data, explaining the combination of both first- and second-degree effects. To account for the overall advantage of repetition trials over alternation trials in the data, a repetition bias had to be built into the reset prior in DBM. In DBM^2, the first-degree component naturally introduces an advantage for repetition trials. This occurs because the estimate of ϕ_t is shifted toward the value of the previous stimulus, S_{t-1}, thus leading to a greater expectation that the same value will appear on the current trial. This fact eliminates the need for a nonuniform reset prior in DBM^2. We use a uniform reset prior in all DBM^2 simulations, thus allowing the model to operate with only four free parameters: α, w, and the two parameters for the affine transform from model probabilities to RTs.

The nonuniform reset prior in DBM allows it to be biased either for repetition or alternation. This flexibility is important in a model, because different experiments show different biases, and the biases are difficult to predict. For example, the Jentzsch and Sommer experiment showed little bias, but a replication we performed—with the same stimuli and same responses—obtained a strong alternation bias. It is our hunch that the bias should not be cast as part of the computational theory (specifically, the prior); rather, the bias reflects attentional and perceptual mechanisms at play, which can introduce varying degrees of an alternation bias. Specifically, four classic effects have been reported in the literature that make it difficult for individuals to process the same stimulus two times in a row at a short lag: attentional blink Raymond, Shapiro, and Arnell (1992), inhibition of return Posner and Cohen (1984), repetition blindness Kanwisher (1987), and the Ranschburg effect Jahnke (1969). For example, with repetition blindness, processing of an item is impaired if it occurs within 500 ms of another instance of the same item in a rapid serial stream; this condition is often satisfied with 2AFC. In support of our view that fast-acting secondary mechanisms are at play in 2AFC, Jentzsch and Sommer (Experiment 2) found that using a very short lag between each response and the next stimulus modulated sequential effects in a difficult-to-interpret manner. Explaining this finding via a rational theory would be challenging. To allow for various patterns of bias across experiments, we introduced an additional parameter to our model, an offset specifically for repetition trials, which can serve as a means of removing the influence of the effects listed above. This parameter plays much the same role as DBM's priors. Although it is not as elegant, we believe it is more correct, because the bias should be considered as part of the neural implementation, not the computational theory.

3.4 Other Tests of DBM^2

With its ability to represent both first- and second-degree effects, DBM^2 offers a robust model for a range of sequential effects. In Figure 3.3a, we see that DBM^2 provides a close fit to the data from Experiment 1 of Jentzsch and Sommer (2002). The general design of this 2AFC task is similar to the design in Cho et al. (2002) though some details vary. Notably we see a slight advantage on alternation trials, as opposed to the repetition bias seen in Cho et al.

Surprisingly, DBM^2 is able to account for the sequential effects in other binary decision tasks

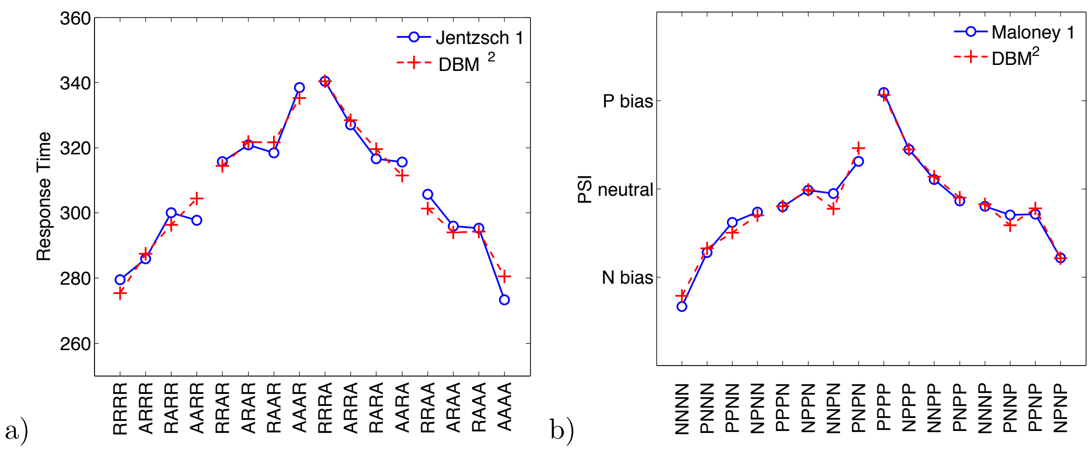


Figure 3.3: DBM^2 fits for the behavioral data from a) Jentzsch and Sommer (2002) Experiment 1 which accounts for 96.5% of the data variance ($\alpha = 0.2828$, $w = 0.3950$) and b) Maloney et al. (2005) Experiment 1 which accounts for 97.7% of the data variance. ($\alpha = 0.0283$, $w = 0.3591$)

that do not fit into the 2AFC paradigm. In Experiment 1 of Maloney et al. (2005), participants observed an apparent rotation of two points on a circle and reported whether the direction of rotation was positive (P) or negative (N)—i.e., clockwise or counterclockwise rotation respectively. The stimuli were constructed so that the direction of motion was ambiguous, but a particular variable related to the angle of motion could be manipulated to make participants more likely to perceive one direction or the other. Psychophysical techniques were used to estimate the Point of Subjective Indifference (PSI), the angle at which the observer was equally likely to make either response. PSI measures the participant's bias toward perceiving a positive as opposed to a negative rotation. Maloney et. al. found that this bias in perceiving rotation was influenced by the recent trial history. Figure 3.3b shows the data for this experiment rearranged to be consistent with the R/A orderings used elsewhere (the sequences on the abscissa show the physical stimulus values, ending with Trial $t-1$). The bias, conditioned on the 4-back trial history, follows a similar pattern to that seen with RTs in Cho et al. (2002) and Jentzsch and Sommer (2002).

In modeling Experiment 1, we assumed that PSI reflects the participant's probabilistic expectation about the upcoming stimulus. Before each trial, we computed the model's probability that the next stimulus would be P, and then converted this probability to the PSI bias measure using an affine transform similar to our RT transform. Figure 3.3b shows the close fit DBM^2 obtains for the experimental data.

To assess the value of DBM^2, we also fit DBM to these two experiments. Table 3.1 shows the comparison between DBM and DBM^2 for both datasets as well as Cho et al. The percentage of variance explained by the models is used as a measure for comparison. Across all three experiments, DBM^2 captures a greater proportion of the variance in the data.

Table 3.1: A comparison between the % of data variance explained by DBM and DBM^2.

	Cho	**Jentzsch 1**	**Maloney 1**
DBM	95.8	95.5	96.1
DBM^2	99.2	96.5	97.7

3.5 Discussion

Sequential effects reflect updating of environmental statistics under the assumption of a non-stationary environment. These statistics can be of various sorts, from simple first-degree statistics concerning the frequency of different stimuli and responses to higher-degree statistics concerning temporal patterns such as repetitions and alternations. Clearly, these various statistics need to be integrated to predict future environmental states, and DBM^2 suggests a generative model for this combination that is consistent with behavioral data.

Another model has attempted to simultaneously explain the detailed pattern of sequential effects in 2AFC, and in doing so, to accomodate first- and second-degree sequential effects, even though the model did not explicitly name them as such. Cho et al. (2002) presented a leaky, stochastic, nonlinear accumulator model of decision making based on Usher and McClelland (2001). Accumulator activation is biased by detectors that observe baserates and alternations in the sequence. Although this model explains the patterns of data, it was impossible to determine whether the fit came about through the accumulator dynamics, the representation of environment statistics, or some interaction between them. DBM^2 suggests that the structure of the environment itself is sufficient to explain sequential effects, without recourse to internal processing mechanisms.

Chapter 4

Dissociating the Two Components of DBM^2 in the Brain[1]

The Dynamic Belief Mixture Model (DBM^2)—developed in the previous chapter—proposes that individuals in binary choice tasks track both the baserate and the repetition rate of the sequence. On the surface, this may seem an unnecessary addition of complexity to the model. However, in this chapter we present a strong argument for the inclusion of both components through varied experimental analyses. Furthermore, we suggest that the origin of two components can be dissociated, with first-degree sensitivities predominantly a characteristic of the response system and second-degree sensitivities a characteristic of perceptual processing.

4.1 EEG Evidence for First- and Second-degree Predictions

One such line of evidence for the psychological separability of the two mechanisms of DBM^2 comes from Jentzsch and Sommer (2002), who used electroencephalogram (EEG) recordings to provide additional insight into the mechanisms involved in the 2AFC task. The EEG was used to record participants' lateralized readiness potential (LRP) during performance of the task. LRP essentially provides a way to identify the moment of response selection—a negative spike in the LRP signal in motor cortex reflects initiation of a response command in the corresponding hand. Jentzsch and Sommer present two different ways of analyzing the LRP data: stimulus-locked LRP (S-LRP) and response-locked LRP (LRP-R). The S-LRP interval measures the time from stimulus onset to response activation on each trial. The LRP-R interval measures the time elapsed between

[1] Part of this chapter is adapted from the published work of Wilder, Jones, and Mozer (2010). The movement studies result from a collaboration with Alaa A. Ahmed.

response activation and the actual response. Together, these two measures provide a way to divide the total response time (RT) into a perceptual-processing stage and a response-execution stage.

Interestingly, the S-LRP and LRP-R data exhibit different patterns of sequential effects when conditioned on the 4-back trial histories, as shown in Figure 4.1. DBM^2 offers a natural explanation for the different patterns observed in the two stages of processing, because they align well with the division between first- and second-degree sequential effects. In the S-LRP data, the pattern is predominantly second-degree, i.e. RT on repetition trials increases as more alternations appear in the recent history, and RT on alternation trials shows the opposite dependence. In contrast, the LRP-R results exhibit an effect that is mostly first-degree (which could be easily seen if the histories were reordered under an X/Y representation). Thus we can model the LRP data by extracting the separate contributions of ϕ and γ in Equation 3.1. We use the γ component (i.e., the second term on the RHS of Eq. 3.1) to predict the S-LRP results and the ϕ component (i.e., the first term on the RHS of Eq. 3.1) to predict the LRP-R results. This decomposition is consistent with the model of overall RT, because the sum of these components provides the model's RT prediction, just as the sum of the S-LRP and LRP-R measures equals the participant's actual RT (up to an additive constant explained below).

Figure 4.1 shows the model fits to the LRP data. The parameters of the model were constrained to be the same as those used for fitting the behavioral results shown in Figure 3.3a. To convert the probabilities in DBM^2 to durations, we used the same scaling factor used to fit the behavioral data but allowed for new offsets for the R and A groups for both S-LRP and LRP-R. The offset terms need to be free because the difference in procedures for estimating S-LRP and LRP-R (i.e., aligning trials on the stimulus vs. the response) allows the sum of S-LRP and LRP-R to differ from total RT by an additive constant related to the random variability in RT across trials. Other than these offset terms, the fits to the LRP measures constitute parameter-free predictions of EEG data from behavioral data.

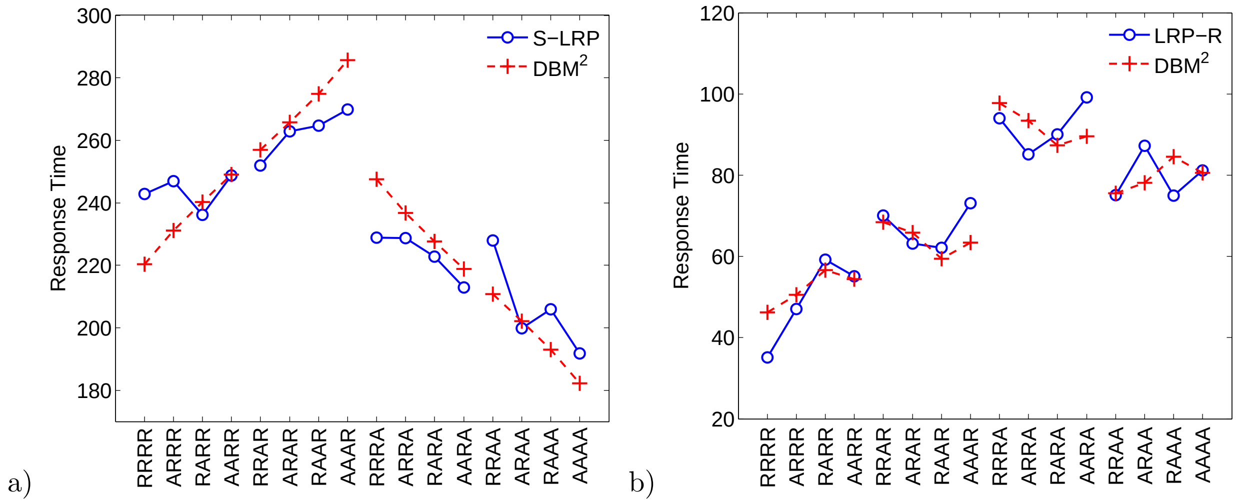


Figure 4.1: DBM^2 fits to the S-LRP (a) and LRP-R (b) results of Jentzsch and Sommer (2002) Experiment 1. Model parameters are the same as those used for the behavioral fit shown in Figure 3.3a, except for offset parameters. DBM^2 explains 73.4% of the variance in the S-LRP data and 87.0% of the variance in the LRP-R data.

4.2 Eliminating First-degree Effects

Evidence for the dissociation of first- and second-degree effects has also been found in behavioral data. In the second experiment reported in Maloney et al. (2005),[2] participants only responded on every fourth trial. The goal of this manipulation was to test whether the sequential effect occurred in the absence of prior responses. Each ambiguous test stimulus followed three stimuli for which the direction of rotation was unambiguous and to which the participant made no response. The responses to the test stimuli were grouped according to the 3-back stimulus history, and a PSI value was computed for each of the eight histories to measure a participant's bias toward perceiving positive (P) vs. negative (N) rotation. The results are shown in Figure 4.2. As in Figure 3.3b, the histories on the abscissa show the physical stimulus values, ending with Trial $t-1$, and the arrangement of these histories is consistent with the R/A orderings used elsewhere in this chapter and the previous chapter.

The DBM^2 explanation for Jentzsch and Sommer's EEG results indicates that first-degree sequential effects arise in response processing and second-degree effects arise in stimulus processing.

[2] See Chapter 3 for an explanation of the general experimental setup.

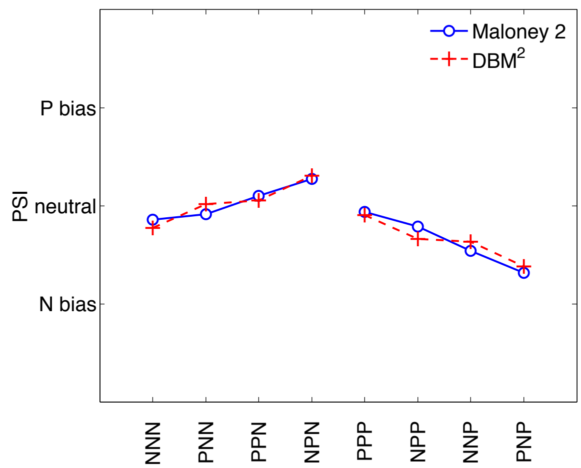


Figure 4.2: Behavioral results and DBM^2 fits for Experiment 2 of Maloney et al. (2005). When responses are only given every fourth trial, the sequential effects pattern becomes dominated by second-degree effects.The model fit explains 91.9% of the variance in the data ($\alpha = 0.0283$, $w = 0$).

Therefore, the model predicts that, in the absence of prior responses, sequential effects will follow a pure second-degree pattern. The results of Maloney et al.'s Experiment 2 confirm this prediction. Just as in the S-LRP data of Jentzsch and Sommer (2002), the first-degree effects have mostly disappeared, and the data are well explained by a pure second-degree effect (i.e., a stronger bias for alternation when there are more alternations in the history, and vice versa). We simulated this experiment with DBM^2 using the same value of the change parameter (α) from the fit of Maloney et al.'s Experiment 1. Additionally, we set the mixture parameter, w, to 0, which removes the first-degree component of the model. For this experiment we used different affine transformation values than in Experiment 1 because the modifications in the experimental design led to a generally weaker sequential effect, which we speculate to have been due to lesser engagement by participants when fewer responses were needed. Figure 4.2 shows the fit obtained by DBM^2, which explains 91.9% data variance. Again this is essentially a parameter-free fit of the data.

4.3 Manipulating First- and Second-degree Effects in Motor Control[3]

In the second experiment of Maloney et al. (2005), first-degree effects were eliminated by removing a large portion of responses. But is it possible to eliminate second-degree sequential effects by removing the perceptual aspect of a task? Though it seems challenging to remove the stimulus from an experiment, we posited that it may be possible within the domain of motor control. Specifically, by replacing a visual stimulus with a force "stimulus" that acts on the response system, sequential effects could result without providing any additional perceptual information.

In a series of three motor control experiments with a sequential structure similar to 2AFC, we manipulated the relative amounts of first- and second-degree effects by changing the design according to our hypothesis that first-degree effects are due to response processing and second-degree effects are due to perceptual processing. In all experiments, participants held the handle of a robotic manipulandum (Figure 4.3a) and made out-and-back movements from a home circle to a target circle—displayed on the monitor above. In Experiment 1, participants received no visual stimulus but experienced a perturbation force perpendicular to their direction of movement. In Experiment 3, participants were given a visual stimulus identifying the direction they should move the handle of the manipulandum and no forces were applied during the movement. Experiment 2 was designed to be a combination of 1 and 3. Participants made out-and-back movements that were perturbed by forces, but on each trial, a 100% reliable visual cue was given for the direction of the perturbation force.

4.3.1 Methods for Experiments 1, 2, and 3

Participants

Fifty-eight right-handed young adults participated for monetary compensation—20 in Experiment 1, and 19 in each of Experiments 2 and 3. Participants gave informed consent in accordance with the University of Colorado's Institutional Review Board.

[3] The experiments presented in this section represent a close collaboration with Alaa A. Ahmed and were all run in her lab.

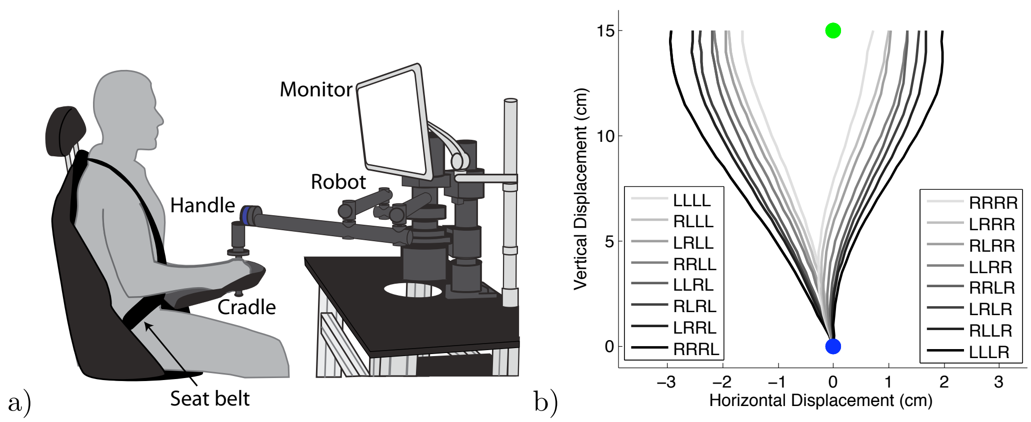


Figure 4.3: a) Experimental setup for motor control task. Participants are seated in a chair and use the handle of the robotic manipulandum to make out and back movements. Forces can be applied by the manipulandum and visual display of the movement is given on the monitor above. b) Mean trajectories for Experiment 1 from blue dot to green dot for different sequences of right (R) and left (L) perturbations (current trial at right end of label). As hypothesized, sequential dependencies here result from the history of right and left forces rather than the repetition/alternation sequences.

Stimuli and Apparatus

Participants sat in a chair with full back support and made horizontal planar reaching movements while grasping the handle of a robotic arm (Interactive Motion Technologies, Shoulder-Elbow Robot 2). The handle position, handle velocity, and robot-generated force were recorded at 20 Hz.

For Experiments 1 and 2, the task involved making rapid 15cm out-and-back movements along the midline of the transverse plane. Visual feedback of a cursor representing hand position and the home and target circles was presented on an LCD screen in front of the participants (Figure 4.3a). Once participants had centered the cursor within the home circle, the target appeared, and an audio cue signaled the trial onset. On each trial, a perturbing force was applied perpendicular to the desired direction of movement. The force increased linearly as a function of distance from the home circle over the first 5cm (1 N/cm) and remained fixed at 5N for the remaining 10cm. No forces were applied on the return. Participants received warning messages if trial duration exceeded 1.4 seconds. For Experiment 1, there was no other visual information. However, for Experiment 2, at the onset of the trial, an arrow that indicated the direction of the perturbation force was displayed on the computer monitor. For Experiment 3, two circles always appeared on the upper half of the screen, one to the right and one to the left. Before a trial began, the participant had to keep the cursor within the home circle. At the trial onset, an arrow appeared pointing to the right or to the left and the participant was instructed to move to the correct circle and then back to the home circle. No forces were present at any point during this experiment, i.e., the manipulandum was used purely for recording the participants' trajectories.

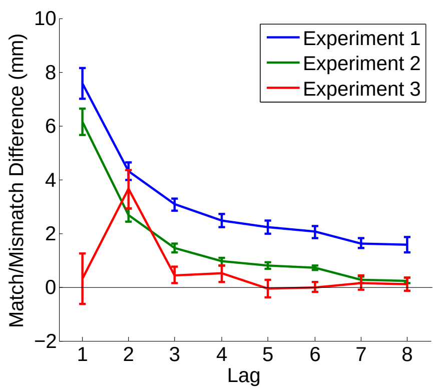
Procedure

For Experiment 1, two versions of the task were run, identical except for the control of the stimulus sequences, with 10 participants each. In version 1, 10 introductory null trials with no forces were followed by 490 force trials with the force direction randomly selected with equal probability. Participants were given a 30 second break after every 100 trials. In version 2, participants completed a total of 1106 trials with 10 introductory null trials and 30 second rests every 137 trials. The 9 trials following each rest were excluded from analyses. Local stimulus histories of right and left trials were controlled to a depth of 9 trials so that each of the 512 (2^9) trial sequences occurred exactly twice. Because there were no interactions between versions 1 and 2, data from the two versions are collapsed in the analyses. For Experiments 2 and 3, the sequences for all subjects were designed according to the same constraints in version 2 of Experiment 1 (of course for experiment 3, the sequences dictated arrow directions rather than perturbation forces). In all analyses, the deflection measures for right and left trials were normalized—for each participant—to have the same mean and standard deviation, thus eliminating imbalances due to structural constraints of the arm.

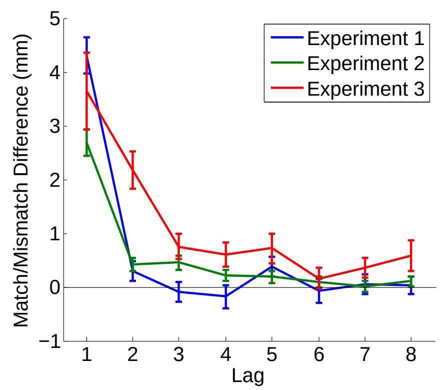
4.3.2 Results

Individual trial movement trajectories in Experiment 1 were affected by the recent trial sequence: participants compensated for the current perturbation more accurately when it was consistent with the recency-weighted sequence of prior perturbations (Figure 4.3b). Consistent with our hypothesis, the sequential effects appeared to be predominantly first-degree as shown by the ordering in Figure 4.3b which is by actual stimulus value right (R) or left (L) force, not the repetition/alternation sequence. Sequential effects were also found in the movement trajectories of Experiments 2 and 3. For the purpose of analysis, the accuracy of the trajectory on a given trial was quantified as the maximum horizontal deviation of the trajectory from the desired straight-line path (which was vertical in Experiments 1 and 2 and diagonal in Experiment 3). However, other deflection measures—e.g., initial angle, mean deviation, area under deflection curve—gave similar results.

To assess the amount of first- and second-degree effects in the data from the three experiments, it is useful to study lag profiles derived from the data. The lag profile isolates the effect of the trial ℓ trials in the past by computing the difference between mean RT when the current trial does not match the lag-ℓ trial (we call this a mismatch) and mean RT when those trials do match. If the trials are encoded by the actual stimulus value, a first-degree lag profile is obtained and if they are encoded using repetitions and alternations, a second-degree lag profile is obtained. In general, nonzeros values in the lag profile indicate the presence of a first- or second-degree effect. However, there are a few caveats: 1) the first-degree lag 1 value may be non-zero because of a general repetition or alternation bias that does not extend father back than one trial; and 2) the first-degree lag 2 value is always the same as the second-degree lag 1 value in a 2AFC design because the sequences that produce a first-degree match at lag 2 (i.e., XXX, XYX, YXY, YYY) exactly correspond to those that produce a lag 1 second degree match (i.e, RR, AA, AA, RR). Because of caveat #1, we are careful to interpret the lag 1 value on the first-degree lag profile and because of caveat #2 we must recognize that the first-degree lag 2 value and the second-degree lag 1 value cannot be used to distinguish the type of effect present because they are completely confounded.



a)



b)

Figure 4.4: a) First-degree lag profiles for the three movement experiments. b) Second-degree lag profiles for the experiments. The error bars display the standard error across participants.

Figure 4.4a and 4.4b depict the first- and second-degree lag profiles (respectively) for the three experiments. Experiment 1 shows a strong first-degree sequential effect that appears to extend many trials back but exhibits no second-degree effect (i.e., all lags—other than lag 1 which we must ignore—are within a standard error of zero). The opposite result is obtained in Experiment 3, with a non-existent or very weak first-degree effect and a strong second-degree effect that appears to extend at least five trials into the past. The lag profiles for Experiment 2 reside in the middle with a strong first-degree effect and marginal second-degree effect. It is not surprising that the first-degree effect is still dominant in this experiment because the perturbation force was much more salient than the arrow cue and exerted an effect on movement throughout the whole trial. In fact, in our experimental design, the arrow cue was not as visually salient as it could have been. We posit that with a stronger visual cue, we might elicit an even greater second-degree sequential effect. Nonetheless, these three experiments demonstrate nicely the trade-off between first- and second-degree effects and their relationship to the response and perceptual processing systems.

The lag profiles in Figure 4.4 are computed using the maximum deviation from a straight line trajectory throughout the trial. While this single measure makes it easy to assess the nature of the sequential effects at a glance, it discards a large amount of the trajectory data. To visualize how the sequential effects evolve on average throughout the course of a trial, each lag value is traced as a function of the distance traveled along the y-axis for the first-degree (Figure 4.5) and second-degree (Figure 4.6) lag profiles. The lag values are computed by differencing the average match and mismatch deviations in the x-axis for the given y. In the figures, the brightest blue line represents the lag 1 value and the colors fade to red which is lag 8. It should be noted that all trial sequences were adjusted to all begin at the origin to make latter points along the trajectory more comparable. Before translating the trajectories, we verified that there were no reliable sequential effects in the starting position. Because of this adjustment, all lag values are zero for the y position that corresponds to the home circle (left end of plots).

In the first-degree lag profiles for Experiment 1 and 2 in Figure 4.5, the magnitude of the match/mismatch difference grows throughout the trial. This is not surprising given that the perturbation force is pushing the hand farther from the straight line all the way until the target is reached. Nonetheless, the sequential effect is apparent from early in the trial. The first-degree lag profile for Experiment 3 confirms the lack of a first-degree effect throughout the whole trial with the only two non-zero lines corresponding to lag 1 and 2 which cannot be used to confirm a first-degree effect. The lag 1 line in this plot is still of interest as it suggests that error is smaller for alternations than for repetitions early in a trial. However, the effect disappears or perhaps even reverses by the end of a trial.

In the second-degree lag profiles for Experiment 3 in Figure 4.6, a consistent second-degree

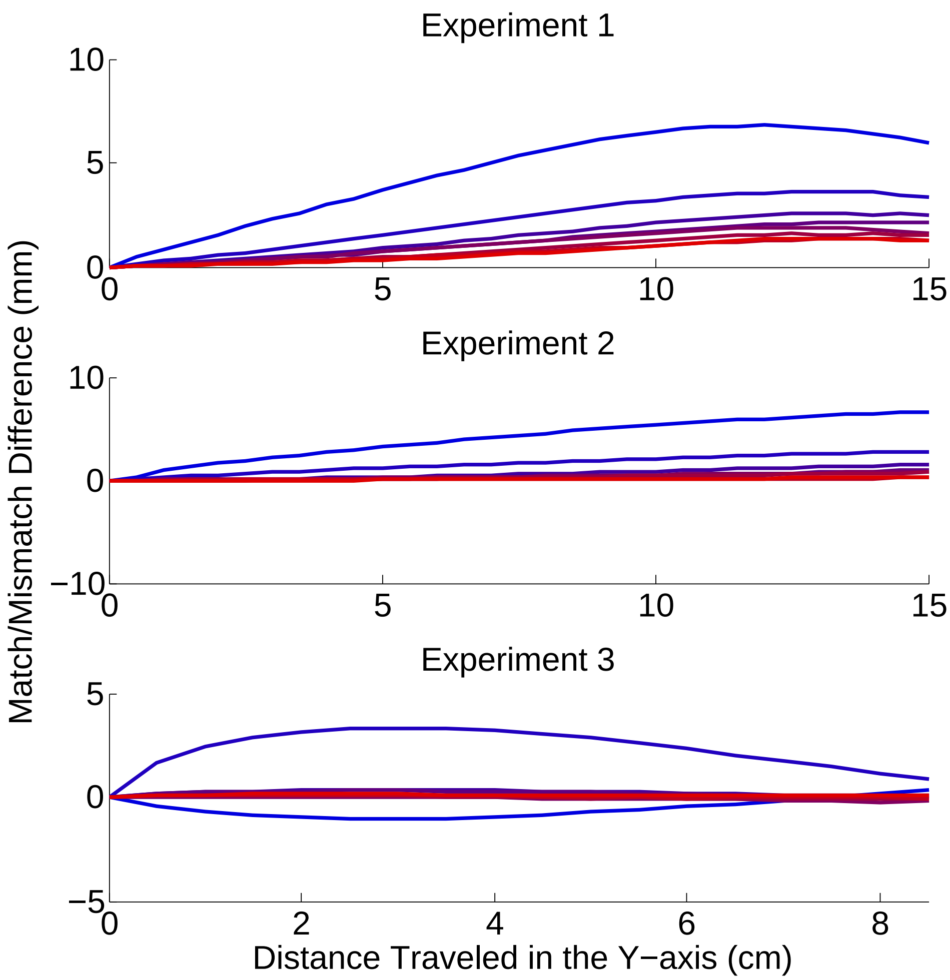


Figure 4.5: First-degree lag profile in x-axis deviation as a function of y position. The bright blue line corresponds to lag 1 and the colors fade to red which is lag 8. For the bottom plot (Experiment 3), where it is difficult to distinguish, the bottom blue line is the lag 1 line and the upper blue line is the lag 2 line.

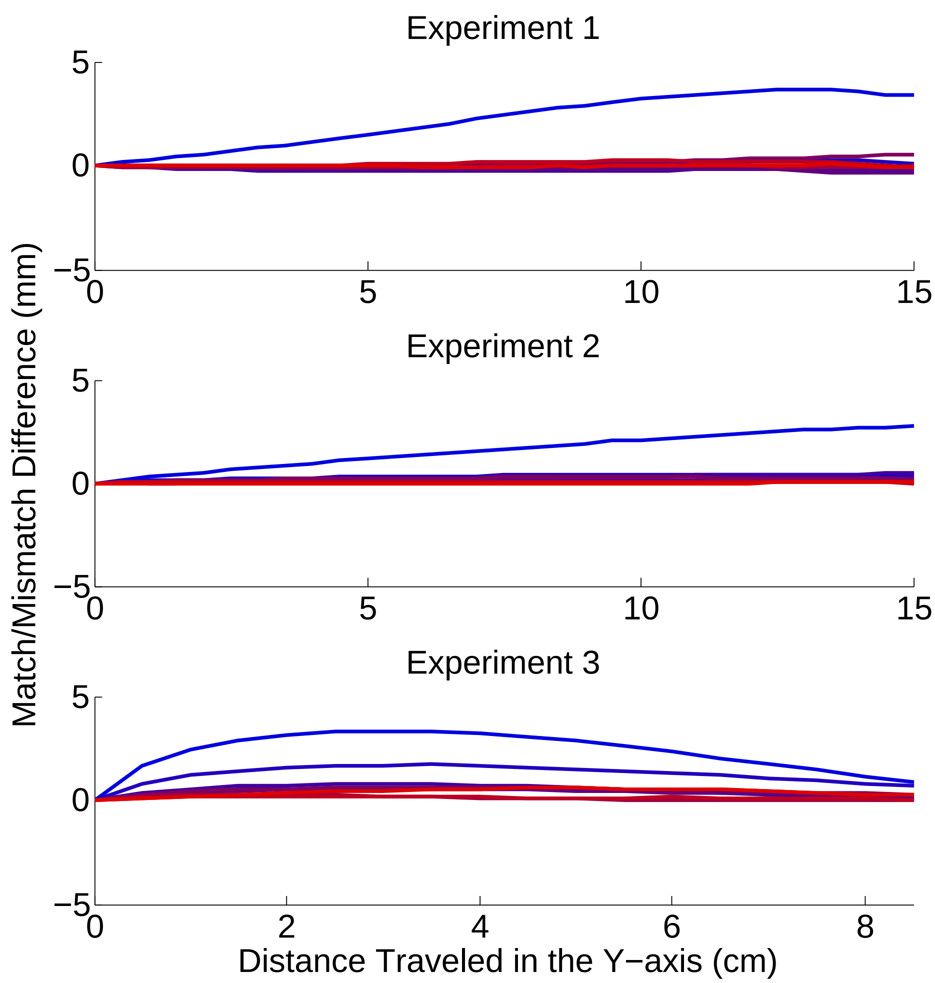


Figure 4.6: Second-degree lag profile in x-axis deviation as a function of y position. The bright blue line corresponds to lag 1 and the colors fade to red which is lag 8.

effect is shown throughout the course of a trial. The effect appears largest in the early half of the trial, likely due to the fact that the trajectory is corrected as the participant nears the target. The lag lines for Experiment 1 and 2 are more difficult to interpret because the second-degree effect is small or negligible. Despite this, the lines for Experiment 2 show a more consistent gradient from blue to red than the lines for Experiment 1 suggesting that a second-degree pattern is more reliable across the duration of the trial. It is reasonable to expect a second-degree effect that is strong at the start of Experiment 2—when the visual stimulus is strong and the perturbation force is weak—that fades away as the perturbation force produces an overriding first-degree effect. Though this is not revealed in the data, it is possible that a more salient visual stimulus might elicit such an effect. More sensitive tests would likely be needed to test this hypothesis as the current analysis only provides a crude depiction of the time course of the effects.

These three experiments were designed as a behavioral validation of the dissociation between first- and second-degree effects. Just as in Experiment 2 of Maloney et al. (2005) where first-degree effects are eliminated by removing responses, we show with Experiment 1 that second-degree effects can be eliminated by removing perceptual aspects of the task. Though a force is still a perceptual event, we view it as qualitatively different than a visual or auditory stimulus because it acts directly on the response system. To verify that the observed first-degree effects did not just result from changing the experimental paradigm, we demonstrate that second-degree effects arise in a motor control task with visual stimuli. Furthermore, with Experiment 2, we demonstrate that it is possible to manipulate the relative amounts of first- and second-degree effects by balancing the strength of perceptual stimuli with the factors that affect the response system.

4.4 Are Two Components Sufficient?

Having presented strong evidence for the combination of first- and second-degree effects in simple choice tasks, we now ask if these two components are sufficient for capturing variability in behavior. On the surface it may appear that two component models such as DBM^2 do very well—e.g., in the previous chapter, DBM^2 explained 99.2%, 96.5%, and 97.7% of the variance across the three experiments studied. However, there is a dirty secret in this method of reporting model performance that has been prevalent in the sequential effects literature. The models are fit to the mean RTs for the 16 different 4-back trial contexts. Because each point represents an average across many trials with the same sequential context, it is possible that a huge amount of behavioral variability is being averaged away. Ideally, models should predict an RT for each trial conditioned on the sequence that precedes the trial. When this approach is taken, the percent of trial-to-trial variability that is explained by the model becomes shockingly low. When a two-component model with exponential decay[4] was fit to the trail-to-trial data from Jentzsch and Sommer (2002) Experiment 1, the variance accounted for averaged across ten participants was a meager 12.35%. Figure 4.7 depicts the often large per-trial residual RT values produced when the two-component model was fit to RTs from one representative participant.

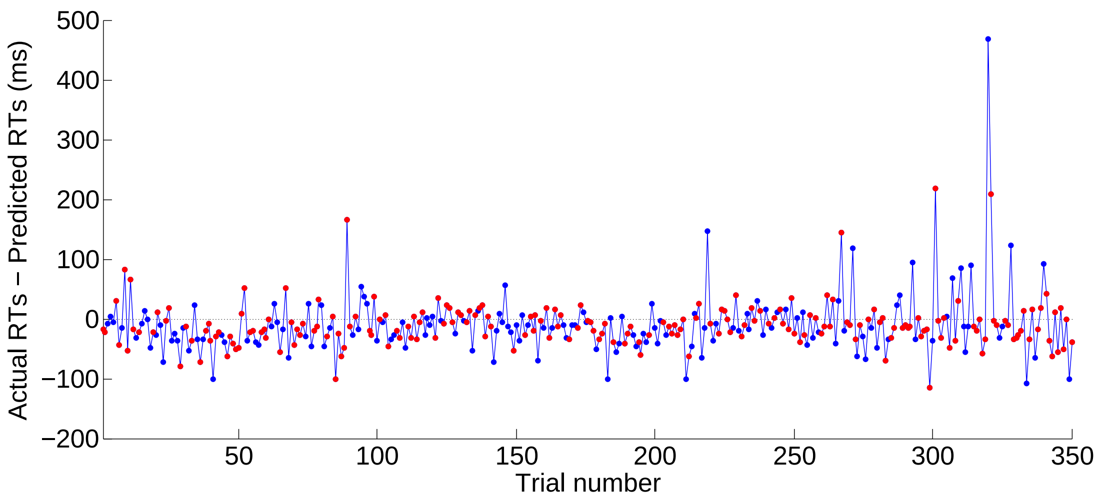


Figure 4.7: Residual response time (RT) for a representative participant in Jentzsch and Sommer (2002) Experiment 1. The residuals correspond to a simple two-component model fit to the trial-to-trial RT data. Red and blue dots represent the actual stimulus type on the trial (above or below fixation line).

With roughly 87.5% of behavioral variability still unaccounted for, there is a lot of room for improvement with 2AFC models. Though it is possible that a large portion of this variability is truly random, independent white noise, some of this variability should be explainable. For

[4] This provides a very close approximation to DBM^2 and is used in this analysis for simplicity.

example, it may be that individuals drift through different levels of attention during the course of the experiment. When they are particularly attentive, RTs may be faster than when they are more distracted. There are many other possible explanations that can be devised to explain this variability.

With the hopes of accounting for a larger portion of the variability, we developed an extension of the two-component model that included a latent state variable that could represent something like attention. We adapted the two-component model to fit the general Kalman filter framework by creating a three-dimensional state space that includes a first-degree bias, a second-degree bias, and a latent state. See Appendix B for the full mathematical specification of the model. The model yields a significant improvement in variability explained over the simple two-component exponential model (15.02% compared to 12.35%). A reasonable portion of this improvement can be attributed to the latent state as the Kalman model with only two dimensions explained 13.46% of the variance. Although the Kalman filter model offers an improvement, there remains a large amount of unexplained variability in the trial-to-trial data.

4.5 Discussion

In this chapter we have provided strong evidence for the dissociation of the two components in DBM^2. Our approach highlights the power of modeling simultaneously at the levels of rational analysis and psychological mechanism. The details of the behavioral data (i.e. the systematic discrepancies from DBM) point to an improved rational analysis and an elaborated generative model (DBM^2) that is grounded in both first- and second-degree sequential statistics. In turn, the conceptual organization of the new rational model suggest a psychological architecture (i.e., separate representation of baserates and repetition rates) that was borne out in further data. The details of these latter findings now turn back to further inform the rational model. Specifically, the fits to Jentzsch and Sommer's EEG data and to Maloney et al.'s intermittent-response experiment suggest that the statistics individuals track are differentially tied to the stimuli and responses in the task. That is, rather than learning statistics of the abstract trial sequence, individuals learn the baserates (i.e., marginal probabilities) of responses and the repetition rates (i.e., transition probabilities) of stimulus sequences. Furthermore, with an understanding of this division, we were able to design a series of three experiments that manipulated the level of first- and second-degree effects present.

Though the theoretical perspective of DBM^2 combined with the experimental results presented in this chapter increase our understanding of the sources of variability in 2AFC tasks, there is still much to be understood about human behavior in these tasks. We demonstrate that by representing latent states such as attention, even more variability in RT can be explained. It is possible that the Kalman filter framework we developed could be improved more by adding multiple latent variables and perhaps representing multiple timescales of decay. However, it is likely that other insights will be needed uncover the full nature of behavioral variability in 2AFC. In either case, we will not accept the tenet that the remaining 85% of the variability is unexplainable noise.

Chapter 5

The Persistent Impact of Incidental Experience[1]

A common characteristic of the sequential effects models presented thus far is that the weighting over the sequence of past trials decays exponentially. From this perspective, sequential effects are viewed as rapidly decaying perturbations of behavior with no long-term consequences. In this chapter, we challenge this traditional perspective in a new study designed to probe the impact of more distant experience and through a reanalysis of Experiment 1 in the previous chapter.

Sequential dependencies arise naturally from psychological and neurobiological models of incremental learning, including error correction methods (Rescorla and Wagner, 1972), reinforcement learning (Sutton and Barto, 1998) and Hebbian learning (Hebb, 1949). These models yield an exponentially discounted influence of past trials, which explains the inverted-V pattern common to many 2AFC experiments (as in Figure 5.1a). Similarly, models from optimal control theory for tracking nonstationary environments, such as the Kalman filter (Kalman, 1960), also produce exponential decay. These models are all appealing because the past trial history is captured by a single state variable (or sufficient statistic) that can be maintained and updated from trial to trial.

Models that produce exponential decay of past trials predict sequential dependencies to operate only on short timescales. Moreover, analyses of sequential dependencies have focused on the short timescale, and the design of experiments has not been well suited to measuring longer-range effects. However, several studies hint at the possibility that a single experience can have an influence on behavior persisting minutes (e.g., Link, Kos, Wager, and Mozer, 2011; Wong

[1] This chapter is an adapted version of the work by Wilder, Jones, Ahmed, Curran, and Mozer (2013) accepted for publication in Psychonomic Bulletin and Review.

and Shelhamer, 2011) or even a day (Ward and Lockheed, 1970), consistent with an alternative theoretical perspective in which each experience is stored in long-term memory, and behavior is guided by the cumulative impact of these memories (e.g., Kasif, Salzberg, Waltz, Rachlin, and Aha, 1998; Stanfill and Waltz, 1986).

Instead of an exponential discounting of the past, long-term memory is typically characterized as following a "power law of forgetting" (J. R. Anderson et al., 1960; Rubin and Wenzel, 1996; Wixted and Carpenter, 2007; Wixted and Ebbeson, 1997). Power functions are qualitatively different from exponential functions because they can produce a single curve that exhibits both rapid decay of the most recent trials (a strong short-term recency effect) and slow decay of far-back trials (a long-range residual effect). With exponential decay, long-term effects are vanishingly small, at least with decay rates in the range needed to explain short-term recency.

Our investigation explored the persistence of incidental experience, both in terms of the scope of its influence and the nature of its decay. We began by reanalyzing trial-to-trial data from a typical 2AFC experiment (Jentzsch and Sommer, 2002). We compared two models of sequential effects that assume that participants form an expectation for the next trial using an average of previous trials that is weighted either exponentially or according to a power function (see Appendix C for mathematical details of our modeling approach).[2] Response time (RT) was predicted to be fast when the expectation matched the actual trial and slow when it did not. Throughout the chapter, each model was fit to the specific trial history of individual participants by minimizing the mean squared error across all trials. Both models had a single theoretically relevant free parameter for determining the relative weighting of past trials.

The analysis used in previous investigations, in which RTs are conditioned on the four-back sequence (Figure 5.1a) , does not gauge the persistence of experience or facilitate discrimination of the two models. Thus, to examine the influence of past trials more closely, we studied how model fits vary as a function of the number of past trials used to form each expectation (the context

[2] The model we use here is purely a single degree model (second-degree for analyzing the new experimental data and first-degree for reanalyzing the data from Experiment 1 in the previous chapter). We felt it was reasonable to make this simplification because in both cases effects due to one of the degrees dominated the other.

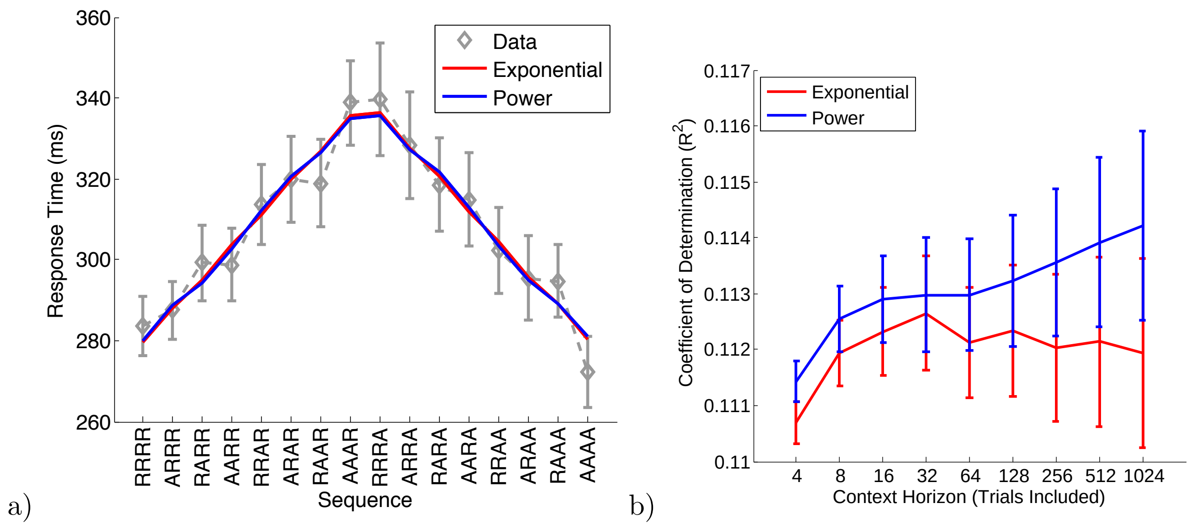


Figure 5.1: Reanalysis of a representative sequential effects study (Jentzsch and Sommer, 2002, Expt. 1) a) Mean RTs for current trial type—repetition (R) or alternation (A)—as a function of sequence history (current trial at top of label). Error bars here and elsewhere for behavioral data indicate standard error. Exponential (red) and power (blue) models—with full context horizon—fit to per-participant trial-to-trial data and averaged across participants. b) Accumulative prediction error (R^2) as a function of context horizon. Error bars indicate standard error of R^2 difference between models (Loftus and Masson, 1994), thus aiding in comparing models but not horizons.

horizon). Out-of-sample fits were obtained for each context horizon by iteratively computing a prediction for each trial using a model that was fit to the preceding trials and constrained to have the desired context horizon. All models have one free parameter regardless of horizon size. Accumulative prediction error (Wagenmakers, Grunwald, and Steyvers, 2006) is computed from the out-of-sample fits (Figure 5.1b). For an error measure, we use the coefficient of determination (R^2) which is derived from the sum of squared residuals error measure recommended in Wagenmakers et al. (2006). Increasing the horizon beyond four trials back yields reliable improvements in fit: across models that use 4 to 1024 past trials, there is a significant main effect of horizon on R^2 (F(8,72)=3.28, p=.003), but despite the appearance of a better fit for the power model, the interaction between horizon and model was not reliable (F(8,72)=1.033, p=.42).

The Jentzsch and Sommer (2002) study was limited because higher-order sequence statistics were not controlled—introducing an additional source of variability—and because distinguishing predictions of the two models is difficult when sequences have no structure. The latter point is due to the fact that, when the two trial types—repetition and alternation—occur with equal probability, their influence tends to cancel out, regardless of how strongly individual trials are weighted.

5.1 Autocorrelation in the Sequence Structure

We therefore conducted a 2AFC study with a biased sequence structure in two opposing conditions, one in which 2/3 of the trials were repetitions of the preceding trial and one in which 2/3 of the trials were alternations of the preceding trial—positive and negative autocorrelation, respectively.

5.1.1 Methods for Experiment 4

Participants

Twenty-eight young adults (age 21.5 ± 2.9 yrs, 9 female, 19 male) participated for monetary compensation. Each participant performed two sessions, one each in the Positive and Negative conditions. Sessions were spaced by 2-7 days, and order was counterbalanced between participants.

One participant was removed from the analysis because of an error recording responses during one block. Participants gave informed consent in accordance with the University of Colorado's Institutional Review Board.

Stimuli and Apparatus

The participant's task was to respond to the location of a white dot, 5 mm in diameter, presented 11 mm above or 12 mm below a 4 mm horizontal white fixation line visible throughout the task. Responses were made using a button box, oriented vertically so as to be spatially compatible with target locations. The left and right index fingers were assigned to the two buttons, with the assignment counterbalanced across participants and fixed across sessions for each participant. Stimulus duration was 100 ms. A 700 ms response-to-stimulus interval followed each response. Reaction time was recorded at 1000 HZ.

Procedure

Each session consisted of 3402 experimental trials divided into 14 blocks of 243 trials. Within each block, local stimulus histories were controlled to a depth of six trials, and the frequency of each of the 64 (26) different trial sequences was exactly as dictated by the repetition rate for the condition (1/3 and 2/3 for the Negative and Positive conditions, respectively). The actual stimulus identities (above or below the fixation line) were equally probable. Participants were given rest breaks roughly every 116 trials, and additional practice and post-rest contextual lead-in trials were inserted into the sequence for a total of 3744 trials.

5.1.2 Results

As expected, RTs were modulated by the short-term context (Figure 5.2a). However, behavior also depended on the autocorrelation structure: RTs for repetition trials (left side of Figure 5.2a) were faster in the positive condition than the negative condition and vice versa for alternation trials. The difference due to autocorrelation structure when conditioned on the immediate context indicates that the influence of the past extends beyond four trials back. Although one cannot determine how far back from Figure 5.2a, a preference for the power model emerges when fits to the per-participant trial-to-trial data are aggregated according to the four-back sequence history. R^2 between model and data across the 32 histories (16 in each condition) was greater for the power model for 25 of the 27 participants (mean R^2 across participants: .798 vs. .730; paired t-test, t(26)=7.45, p<.001). R^2 values reported for the means of the four-back sequence histories are higher than those for the individual trial data—e.g., Figure 5.1a—because some sources of variability are averaged out.

Support for a long-range sequential effect is obtained by examining the accumulative prediction error values for the two models with varied context horizons (Figure 5.2b). There is a significant main effect of horizon (F(8,208)=85.1, p<.001) and an interaction between model type and horizon (F(7,182)=62.3, p<.001). The exponential model fit improves reliably as more trials are included out to 32 trials (comparing 32 vs. 16, t(26)=4.12, p=.0003) but no further (1024 vs. 32, t(26)=0.36, p=.72). In contrast, the power model fit improves out to 1024 trials (1024 vs. 512, t(26)=2.84, p=.0086). Behavior in this task is clearly affected by a long history of prior experience.

Further support for power over exponential decay is obtained by studying the lag profile derived from the data, plotted on a log-log scale in Figure 5.2c. Because the exponential and power models both predict a lag profile that matches the decay function, this analysis offers another means of differentiating the models. The empirical lag profile appears linear in log-log coordinates suggesting power decay. We fit individual participant lag profiles to both power and exponential functions and obtained a better fit for the power function (mean R^2 across participants: .878 vs. .855; t(26)=2.17, p=.039).

Even though both the power and exponential functions have a single free parameter, one could argue that the power function fits better because it has more flexibility. To rule out this possibility, we compare out-of-sample fits using leave-one-out cross validation. The power fit is consistently better than the exponential fit across lags and across participants: mean absolute deviation between the empirical and predicted lag values is smaller for the power function (F(1,26)=10.47, p=.003). For 9 of the 10 lags, the mean absolute deviation is smaller for the power function. Furthermore, we compared fits for individual participants using an extension of the likelihood ratio test that is

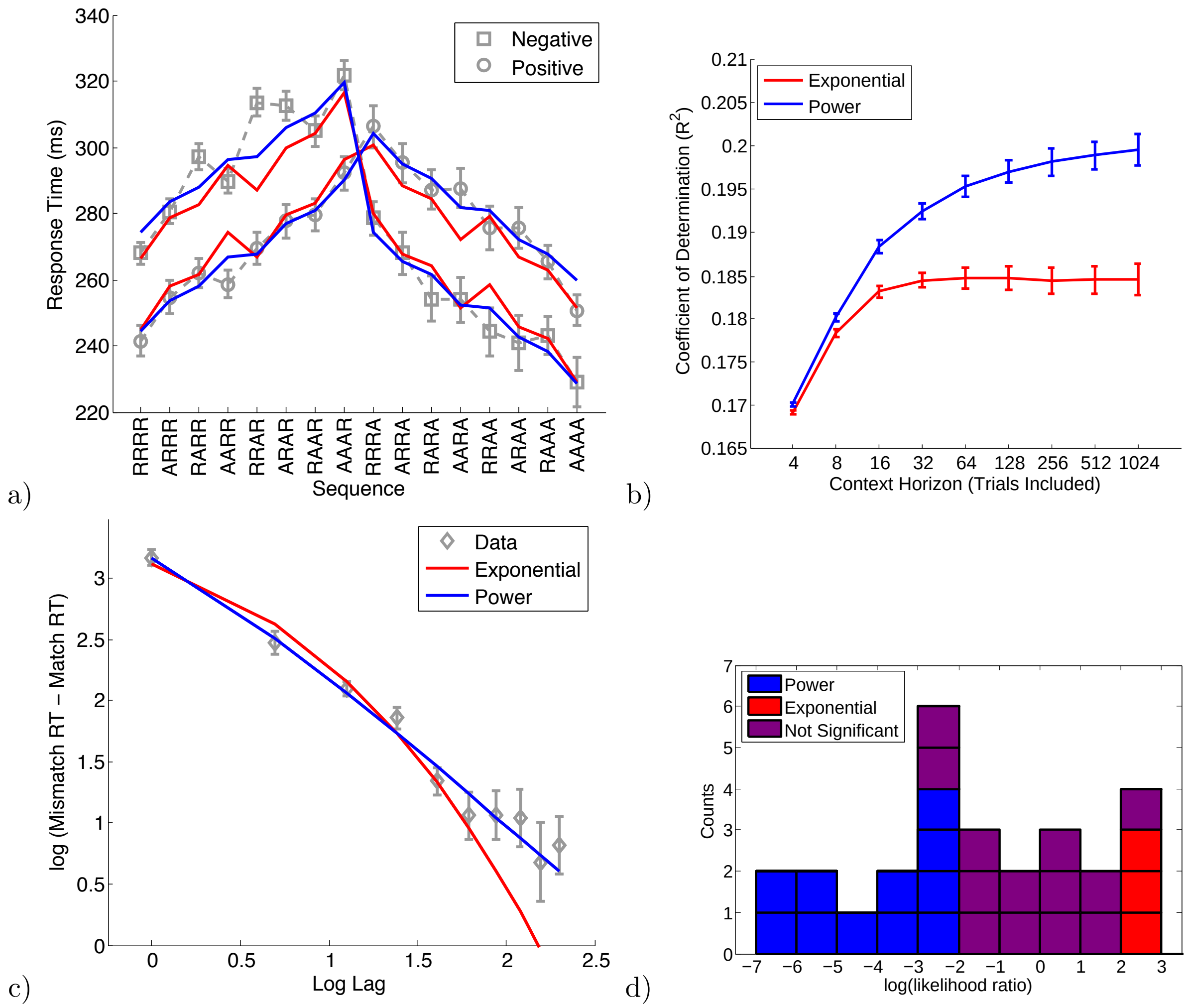


Figure 5.2: Results for Experiment 4. a) Mean RTs for positive and negative autocorrelation conditions as a function of sequence history. Exponential and power modelswith full context horizonfit to per-participant trial-to-trial data and averaged across participants. b) Accumulative prediction error (R^2) as a function of horizon. Error bars as in Figure 5.1. c) Lag profile averaged across conditions and participants in log-log coordinates. Mean of exponential and power function fits to per-participant lag profiles. d) Histogram of the log-likelihood ratios for individual participant fits (negative supports power model and positive supports exponential model). Significance determined by the Vuong's closeness test.

appropriate for non-nested models (Vuong, 1989). Figure 5.2d histograms the log-likelihood ratios across participants. A preference for the power model is evidenced by both the larger number of significantly negative ratios according to the Vuong test (11 blue vs. 3 red boxes) and the larger total number of negative ratios (18 vs. 9).

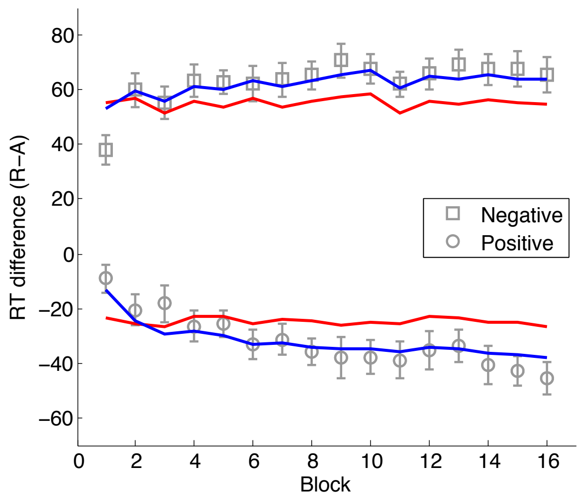


Figure 5.3: Difference in mean RT for repetition and alternation trials by block (234 trials) for each autocorrelation condition. Red line corresponds to the exponential model fit and the blue line corresponds to the power model fit.

If incidental experience has a long-lasting influence, a cumulative effect of trial statistics across the entire course of the experiment might be observable. Figure 5.3 reveals a preference for repetitions in the positive condition that increases as the experiment progresses, and a preference for alternations in the negative condition. When superimposed over Figure 5.3, predictions derived from power model fits capture the long-range effect of condition. In contrast, the trajectory from the exponential model fits is roughly flat because the model cannot benefit from integrating beyond about 64 past trials.

The power model is appealing because it is capable of explaining effects across a range of timescales, from the variation due to the immediate four-back context to the bias that grows over the hour-long duration of the sequence in each autocorrelation condition.

5.2 Persistent Sequential Dependencies in Motor Control

Although we have argued for a unified explanation of short- and long-term adaptation via the power model, there is an alternative, though somewhat less parsimonious, possibility that the two timescales reflect distinct mechanisms. For instance, in Experiment 4, sequence structure might have been detected by participants, leading to explicit learning and deliberate biasing of behavior. We thus aimed to strengthen our account by demonstrating the persistence of incidental experience in the absence of sequential structural regularity.

However, as our reanalysis of the Jentzsch and Sommer (2002) data revealed, it was difficult to uncover long-range effects when the sequence history was balanced and response latency was the dependent variable. We conjectured that response latency may not be a terribly sensitive measure because speedy responses are a secondary consideration in the performance of 2AFC; responding correctly is the participants' primary goal. Consequently, RTs may be more susceptible to perturbation by task-unrelated factors. A task whose behavioral measures are better aligned with the participants' primary goals might be more effective in exposing a persistent influence of incidental experience, despite the previously described cancellation of far-back effects that results from balanced sequences.

Given the success in Experiments 1 through 3 in eliciting sequential effects in movement trajectories—which are a more direct measure of behavior because they reflect planning processes—it seemed suitable to explore the the depth of sequential effects in the motor control domain. Long-term motor adaptation has been observed when systematic and consistent perturbations were applied to the control system (e.g., Hoppand and Fuchs, 2004; Robinson, Sotedjo, and Noto, 2006; Shadmehr and Mussa-Ivaldi, 1994). Some support for the persistent influence of incidental experience is found in an eye movement task in which error-based adaptation was observed extending back nearly one hundred trials and decaying according to a power function (Wong and Shelhamer, 2011). However, in this task, correlations could be attributed to endogenous variation rather than exogenous effects of the target sequence because target timing and position were completely predictable on every trial. Though ignored in many motor control studies, short-term sequential dependencies have been demonstrated in reaching tasks where straight-line arm movements were disrupted by variable perpendicular perturbation forces (Fine and Thoroughman, 2006; Scheidt et al., 2001).

In examining the lag profiles from Experiments 1 through 3 (Figure 4.4), Experiment 1 displayed the most consistent sequential effect. Moreover, further back lags appeared to be non-zero suggesting a more persistence effect. For these reason, we examined the data from Experiment 1 using the methods developed above.

5.2.1 Results

The persistence of past experience is revealed by analyzing accumulative prediction error as a function of context horizon (Figure 5.4a). We find support for the hypothesis that sequential effects extend back more than 32 trials (one-tailed t-test for 64 vs. 32, exponential: t(19)=1.93, p=.035; power: t(19)=1.86, p=.040). Because the exponential and power models differ primarily in the weights they assign to far-back trials, we expected that the balanced sequences in this experiment would make it difficult to compare the two models directly. Despite this limitation, evidence for power decay over exponential decay is found in the near linear trend of the lag profile in log-log coordinates (Figure 5.4b). Per-participant fits to the lag profile values are reliably better for a power function than an exponential function (mean R^2 across participants: .891 vs. .835; t(19)=4.98, p<.001).

Using leave-one-out cross validation, the power fit is significantly better than the exponential fit across lags and across participants: mean absolute deviation between the empirical and predicted lag values is smaller for the power function (F(1,19)=15.26, p=.001). Additionally, for 9 of the 10 lags, the mean absolute deviation is smaller for the power function. Figure 5.4d shows a strong preference for the power model according to Vuong's closeness test (Vuong, 1989) with more significantly negative log-likelihood ratios (12 blue vs. 0 red) and a larger total number of negative ratios (17 vs. 3).

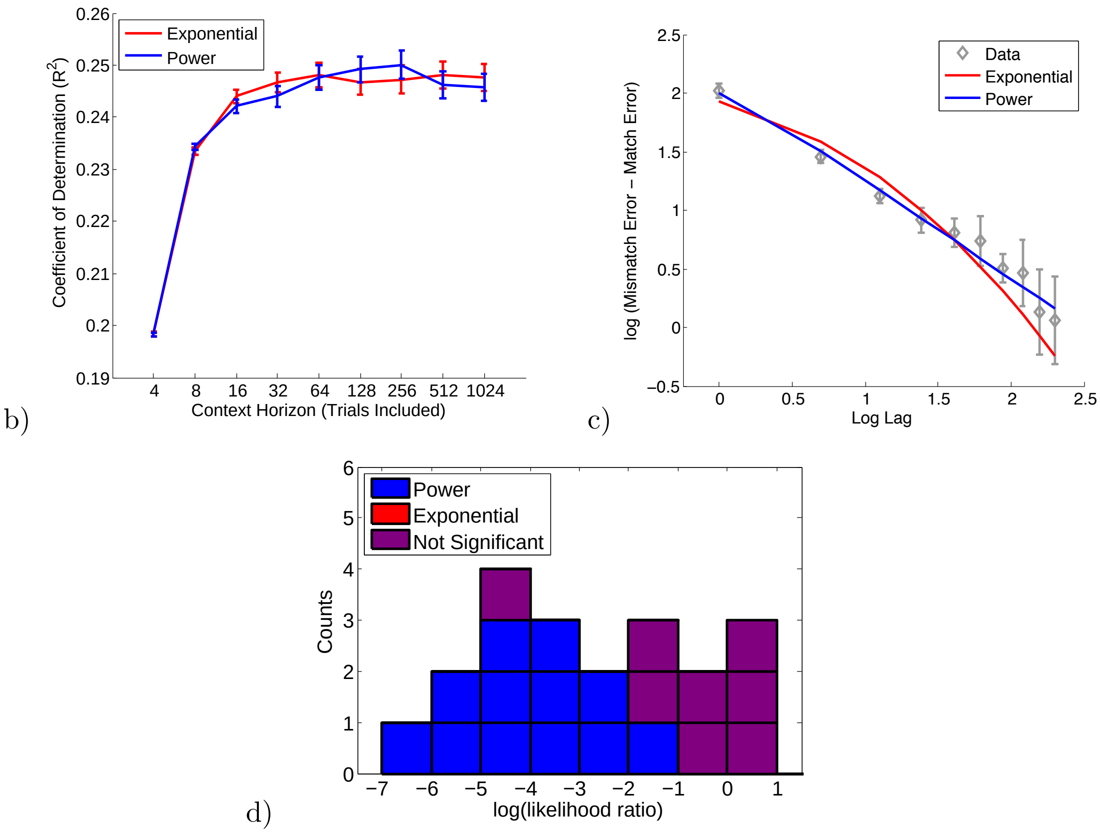


Figure 5.4: a) Accumulative prediction error (R^2) as a function of context horizon. Error bars as in Figure 5.1. b) The lag profile in log-log coordinates with mean exponential and power function fits. d) Histogram of the log-likelihood ratios for individual participant fits (negative supports power model and positive supports exponential model). Significance determined by the Vuong's closeness test.

5.3 A Normative Account of Long-Range Effects

Many theoretical accounts characterize sequential dependencies as a by-product of adaptation to the statistical structure of a dynamic environment (e.g., DBM^2 in Chapter 3, Jones and Sieck, 2003; Mozer, Kinoshita, and Shettel, 2007; Yu and Cohen, 2009). These accounts suppose that statistics of the environment are tracked over time—statistics such as relative stimulus frequency or the magnitude and direction of perturbing forces. The statistics represent not only a summary of the past, but an expectation for the future, facilitating tuning of perceptuo-motor control to perform optimally in the anticipated environment.

If environments have temporal nonstationarity, more recent experience is most indicative of what an individual will experience next. Specific theoretical formulations lead to specific characterizations of how past experiences should optimally be combined to predict future events. Yu and Cohen's (2009) Dynamic Belief Model (DBM) explains sequential effects as a consequence of optimal Bayesian inference in an environment whose characteristics are stationary for an interval of time until they are redrawn from a reset distribution at abrupt changepoints distributed in time according to a Bernoulli process. The DBM assumptions lead to predictions about behavior that are consistent with an exponentially decaying lag profile. Consequently, the model fails to produce long-range effects of experience.

We propose an extension of the DBM, called the Hierarchical Dynamic Belief Model or HDBM (Figure 5.5a), that yields roughly a power function lag profile and consequently outperforms the DBM when fit to the entire experimental data in one pass (Figure 5.5b; Experiment 4: $t(26)=7.69$, $p<.0001$, Experiment 1: $t(19)=3.87$, $p=.0010$).[3] The HDBM relaxes a seemingly unnatural assumption in the DBM: that environmental statistics have a time-invariant probability of change. For example, it would seem that the dynamics of change during a four-hour plane flight are not the same as those during the half hour it takes to deplane, walk through the terminal, collect bags, catch a taxi, and check into a hotel. The HDBM avoids this restrictive assumption by taking a

[3] We define HDBM as an extension of DBM as a proof of concept. However, it would be easy to integrate the two components of DBM^2 into the HDBM framework.

hierarchical Bayesian approach in which the underlying generative model is a non-homogeneous Bernoulli process, i.e., a process with a fluctuating changepoint probability that is driven by a separate Markov process (see Appendix C for the full model specification). Because the HDBM models a spectrum of environments—ranging from rapidly changing to stable—its expectations of the future reflect strong short-term recency as well as long-range dependencies.

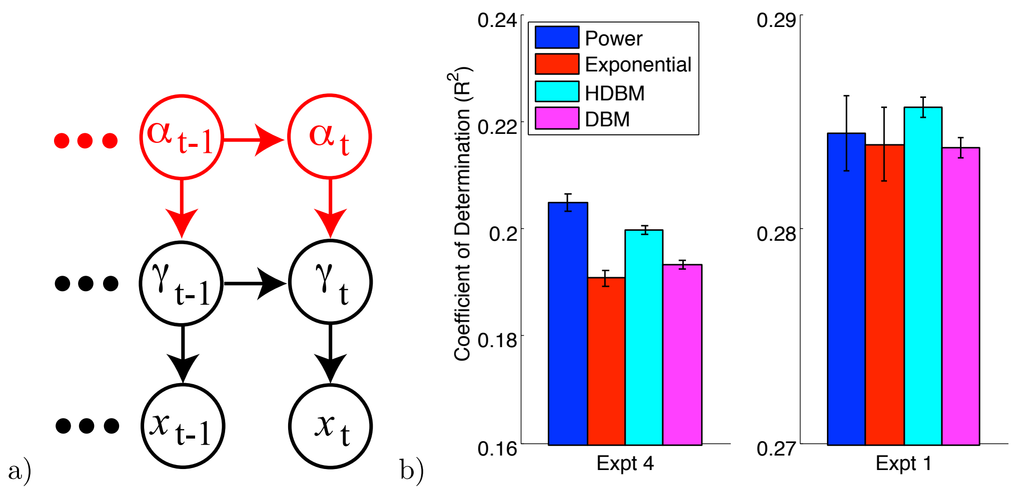


Figure 5.5: a) The graphical model for the Hierarchical Dynamic Belief Model (HDBM). x_t is the trial type at time t, γ_t is the parameter of the Bernoulli process generating x_t, and α_t is the change probability. The original DBM (Yu and Cohen, 2009) consists of only the black part of the graph, with α constant. b) Comparison of model performance for Experiment 4 and 1. Error bars for the power and exponential models—and similarly for the HDBM and DBM models—represent the standard error of the R^2 difference between the two models across participants.

The success of the HDBM in fitting the data suggests a normative explanation for the long-range influence of incidental experience on behavior. Under the assumptions of the HDBM, the mind optimally adapts to a complex dynamic environment in which even seemingly irrelevant experiences that occur far in the past offer predictive information about upcoming environmental states and task demands. Specifically, the expected relevance of a past experience to the current moment falls off according to an approximate power function.

As previously mentioned, human forgetting of explicit (declarative) knowledge in long-term memory is often characterized in terms of power decay. This decay function has been cast as rational via the observation that in diverse domains—newspaper articles, parental speech, and electronic mail—the empirical probability of needing access to a specific piece of information is well fit by a power function of time (Anderson and Schooler, 1991). The present analyses of the DBM and HDBM indicate that this observation is not well explained by nonstationarity with a fixed change probability, but that introducing variable change rates offers the basis for a normative explanation. Thus power decay serves as an informative connection between sequential effects, long-term memory, and the statistical structure of the environment.

5.4 Discussion

Contrary to the prevailing assumption that variations in experience produce only fleeting perturbations in behavior, we have argued that incidental priming yields enduring modulations of behavior. Modeling indicates that past experience is integrated to anticipate the future using a weighting that is strongly recency based but also has a heavy tail, consistent with power but not exponential discounting. Power discounting can be characterized as optimal adaptation to the statistics of an environment with second-order nonstationarity.

To perform optimal prediction in nonstationary environments with changepoint dynamics, the complete history of experience must be maintained (Adams and MacKay, 2006). Consequently, our results are consistent with the perspective that as individuals interact with their world, they continually log their experiences, forming a library of memory traces that is called on to adapt behavior to an environment that can change on timescales ranging from seconds to months. Alternatively, a good approximation to optimal prediction can be achieved by combining across several exponentially decaying sequence statistics that span a range of timescales (e.g., Kording et al., 2007; M. C. Mozer, Pashler, Cepeda, Lindsey, and Vul, 2009; Sikstrom, 1999, 2002; Staddon, Chelaru, and Higa, 2002; Wixted, 2007). Indeed, Mozer et al. (2009) and Murre and Chessa (2011) demonstrate mathematically that power functions emerge when an infinite collection of exponential functions are averaged together assuming certain constraints on the distribution of decay rates. Our work suggests the necessity of combining across multiple timescales ranging from just a few trials to hundreds of trials to the entire duration of the experiment. The presence of power decay, regardless of the precise mechanisms that produce it, suggests that sequential dependencies in rapid decision making are best understood as a memory phenomenon akin to human long-term declarative memory rather than as a byproduct of short-term incremental learning.

The perspective that sequential effects reflect memory storage and updating offers a novel interpretation of the continual and often long-range (Gilden et al., 1995) fluctuations observed in human behavior and cognition. Far from being internal noise in the system, trial-to-trial variability in choice, response latency, and movement reflect an adaptive process in which individuals exploit their extensive experience to respond optimally to a dynamic world.

Chapter 6

Decontaminating Human Judgments[1]

The work presented thus far primarily seeks to explain sequential effects in response time and other simple measures of trial performance. As we saw in Chapter 2 that provided an overview of sequential effects and a synthesis of theoretical perspectives, recent context has also been shown to bias actual responses in magnitude estimation, absolute identification, and categorization tasks. Beyond simple experimental tasks, sequential effects have been shown to contaminate judgments in domains such as legal reasoning and jury evidence interpretation (e.g., Furnham, 1986; Hogarth and Einhorn, 1992), clinical assessments (Mumma and Wilson, 2006), and financial decisions (Johnson and Tellis, 2005; Vlaev et al. 2007).

Given the widespread prevalence of sequential effects in judgments, it is natural to question how these effects bias conclusions that are drawn from these judgments. In an experimental environment, items are often judged many times and biases due to sequential effects are removed through averaging. However, in many situations there may only be one or two judgments per item and averaging may be inadequate. Consider for example a marketing research company that is assessing 20 potential additions to Pepsi's beverage lineup. This company will enlist a large number of people to taste-test the different concoctions and rate each one. Even with the presentation order randomized across individuals, we expect significant sequential effects in the ratings of each individual that could be large enough to skew the final outcome of the study. To avoid this undesired consequence, the company could have the individuals rate each item multiple times.

[1] The work described in this chapter represents a collaboration with Michael Mozer and Matt Jones.

Unfortunately, such an approach would likely increase the cost of the study significantly.

As an alternative to collecting more data and averaging over it, it would be valuable to leverage our knowledge of sequential effects to remove the biases from a sequence of judgments. In *decontaminating* the ratings of individuals, we seek to extract the pure responses that they would have given in the absence on any sequential context. By using decontaminated judgments instead of traditional averages, we expect greater fidelity in the outcome of the survey or experiment and less time, effort, and cost in obtaining the data. In this chapter we seek to explore methods of decontamination and assess their value in varying experimental contexts. With a growing interest in collecting human judgments and using them to predict individual preferences (e.g., Netflix, Amazon), the ability to effectively decontaminate sequences of judgments has the potential to be of great value.

6.1 Problem Formulation

The primary goal of any decontamination method is to infer from the sequences of responses a latent impression for each item judged. We use the term *impression* to represent the internal state of an individual, though the term *sensation* might be more appropriate in a purely perceptual judgment task and the term *evaluation* might be preferred in domains requiring high-level cognitive processing. In this work, we assume that the stimulus-to-impression mapping is veridical and that contamination due to sequential effects occurs in the impression-to-response mapping. Thus the impression reflects an individual's pure reaction to the stimulus. One might argue that sequential effects can also contaminate the perceptual process in a way that renders the concept of a fixed impression to a stimulus meaningless. However, most practical judgment tasks are administered with the goal of determining an individual's raw internal impression of an item. Because this process is dependent on the existence of fixed impressions, we carry this assumption into our decontamination methods.

Without decontamination, the natural method for obtaining a participant's impression for a given item is to average across all responses they have given for that item. We refer to this approach at the Response Mean (RM) model and use it as a baseline for comparison when assessing decontamination algorithms. Naturally, the estimated impressions will be more accurate as the number of repetitions of each item grows. However, in many real-world situations, there is a significant cost in obtaining extra judgments. Thus the goal of decontamination is to uncover impressions that are strongly representative of an individual's internal state with the fewest number of repeated judgments.

6.2 A Linear Sequential Effects Model

In the preceding chapters, we have developed models that seek to provide a rational explanation for sequential effects. From a theoretical perspective, these models contribute significantly to our understanding of how and why humans modulate their behavior according to recent experience. These models provide a computational-level explanation, but there are no doubt mechanistic models that produce the same behavior. From a practical perspective with the challenge of decontamination at hand, we want to employ a model that is flexible enough to capture the majority of different sequential effects that occur and are less concerned about the theoretical implications of the model we choose.

In the domain of simple choice tasks, we have modeled response time as depending on the sequential structure of the preceding stimuli. The rational models we have developed are well approximated by a weighted sum over past stimuli and/or the repetition/alternation encoding of the past stimuli. In domains where there are more than two choices, there is less support for including the repetition/alternation encoding as it is not as well defined (Gokaydin et al., 2011). Thus we might be tempted to consider a simple model that includes a weighting of past stimuli. However, these models are used to predict an individual's response time, whereas in a decontamination application, predictions of the individual's actual responses are desired.

Fortunately there is a rich literature on judgment models (some of which has been reviewed in Chapter 2). One common finding across the models is that the previous sequence of stimuli and responses *both* exert a bias on the current response. We define a general model of sequential dependencies in judgments that draws inspiration from the model presented in DeCarlo and Cross (1990). DeCarlo and Cross (1990) define the current response as a weighted sum of the current stimulus and previous stimuli and responses. The weights in DeCarlo and Cross (1990) are constrained by theoretical assumptions, but for the purpose of decontamination, we remove these constraints. For a history length h, we model the response on trial t as

$$R_t = I(S_t) + \sum_{j=1}^{h} \beta_j I(S_{t-j}) + \sum_{k=1}^{h} \beta_{h+k} R_{t-k}, \tag{6.1}$$

with weights, $\beta \in \mathbb{R}^{2h}$, that can take on any values. Because the stimulus values, S, are often unknown in more complex judgment tasks, we use instead the impression triggered by the stimulus, $I(S)$. This model is mechanistically consistent with many judgment models in the literature. However, it is incapable of capturing nonlinearities such as memory based anchoring (Petrov and Anderson, 2005), capacity limitation (Stewart et al., 2005), and generalization that depends on the similarity between successive items (Jones et al., 2006). Despite these limitations, we expect this model to provide a reasonable foundation for the decontamination methods we will explore.

6.3 Supervised Decontamination

As an initial foray into decontamination, we (M. C. Mozer et al., 2010) explored decontaminating judgments in a simple gap detection task where participants judge the distance between two points as one of 10 possible lengths. In this absolute identification task, there were exactly 10 stimuli, and feedback was only given for the first 10 trials. Participants performed a total of 180 judgments with each item appearing exactly 18 times, and items were blocked in groups of 10 trials with all items appearing in each block. We explored a collection of models designed to infer the impression[2] on each trial from the response on that trial and the preceding responses and impressions. The space of models included the cartesian product of three model types (*regression*, *lookup*, and *hybrid*) and three inference techniques (CRF, *simple*, *oracle*). The *regression* model simply expressed the current stimulus as a linear combination of current and previous responses and the

[2] The term *sensation* was used in M. C. Mozer et al. (2010) instead of *impression*, though for consistency in this context, we use the term *impression*.

previous impressions (similar to Equation 6.1). The *lookup* model consisted of a table of impression values conditioned on the current response and previous responses and impressions. The *hybrid* model combined these two methods to complement the regression model with the non-linearities encoded in the lookup table. Inference over the set of unknown impressions was performed using a linear-chain conditional random field (CRF) with one of the three model types built into the CRF. Two other inference methods, *simple* and *oracle*, were implemented for the purpose of comparison. The *simple* method removed all impression terms from the regression and lookup tables such that performing inference was reduced to solving a simple least-squares regression problem. Similarly, the *oracle* method assumed that the previous impressions were known thus also reducing inference to solving a regression problem (i.e., the previous impressions are set to be the actual gap distances (1-10) on the previous trials). The *oracle* method was used to achieve an upper bound on model performance if the previous impressions are inferred perfectly. However, it is not a valid method on its own because it assumes knowledge of the impressions which are unknown in the decontamination problem.

The models were trained via a supervised procedure using the known ground-truth impressions for a subset of the participants. Performance was assessed by computing the root mean squared error (RMSE) between the ground-truth impressions and the model-estimated impressions over the set of validation participants (i.e., those not used in training) for 100 different splits of the data. The CRF method of inference outperformed the *simple* method and achieved results closer to the *oracle* method. The *hybrid* model type offered the least RMSE of the three model types. Overall, the decontamination method resulted in an RMSE improvement of about 20% over the baseline model, however, roughly 15% of that improvement can be attributed to simple decompression in the response scale and debiasing of individual response tendencies.

6.4 Unsupervised Decontamination

Though the methods developed in Mozer et al. (2010) successfully recovered impressions closer to the ground-truth values, the approach is significantly limited by its dependence on ground-truth data for training. In most real-world applications, ground truth data is unobtainable or may not even exist. For example, in a movie rating task, an individual's impression for a given movie will likely differ from the impressions of others because of personal preferences. Though in theory it is possible to obtain ground-truth data by averaging over many repetitions of a given item, this can be very costly and the data may not apply to other individuals.

A more practical approach to decontamination is to treat the problem as an unsupervised learning problem (i.e., infer the true impression for each item using only the responses given by the participant, perhaps with multiple repetitions of each item). M. C. Mozer, Link, and Pashler (2011) present a method for decontamination in this spirit called the Iterated Impression Inference (I^3) algorithm. The I^3 algorithm is founded on a simple regression model where the current response is given by Equation 6.1. Because the dependent variable is the current response, the regression problem is unsupervised in the sense that ground-truth impressions need not be known. This differs from the regression approach in Mozer et al. (2010) where the dependent variable was the current impression. Of course the unknown impressions still exist in the model and thus must be inferred in some way. As the name suggests, the I^3 algorithm takes an iterative approach and can be described via the following simple steps:

(1) Initialize the impressions to the RM solution.

(2) Solve the least-squares regression problem for the weights.

(3) Fix the weights to the maximum likelihood solution and rearrange the regression problem so that the impressions are the unknown values.

(4) Solve for the impressions in this new regression problem with a regularizer that ensures that the values do not drift far from the RM solution.

(5) With these new impressions, go back to step 2.

Because mean squared error (MSE) is strictly non-increasing in steps 2 and 4, the algorithm is guaranteed to converge on a local optimum in the search space. The algorithm is terminated when MSE on the training data converges (i.e., when the change in MSE within epsilon of zero).

The preferred approach for evaluating any decontamination method is to compare the inferred impressions to the true impressions. However, when the true impressions are unknown, which is often the case, an alternative is to use the inferred impressions to predict future responses. In the RM model, this amounts to computing the mean response for each item across the training data for a participant and using those values as the predicted responses in the test set. With a sequential effects model like I^3, impressions are again inferred from the training data, but the whole model is used to predict the responses in the test set—rather than just the raw inferred impression values—because we expect those responses also to be contaminated by sequential effects.

I^3 is used to decontaminate the judgments from three different experiments: the gap detection task evaluated in Mozer et al. (2010) and two new judgment tasks involving rating movie posters and judging moral actions. We explain these experiments in more detail because they will also be used to evaluate the model presented in this chapter.[3] In the movie rating task, 120 participants rated 50 different movie posters on a 1-10 scale, where 1 means “would never watch this movie” and 10 means “can’t wait to see it” (see Figure 6.1 for example movie posters). The rating task should not be confused with the more typical rating task of indicating enjoyment for a previously viewed film; this sort of task might be used by film marketers who attempt to design advertisements to have broad appeal. The participants rated each movie poster four times for a total of 200 judgments throughout the experiment. For the moral judgment experiment, 50 participants rated 25 different actions on a moral scale from 1 (not bad at all) to 10 (extremely bad). Again, each action was rated 4 different times for a total of 100 judgments. In both experiments, trials were blocked such that all items were rated before the next repetition.

Figure 6.1: Several examples of movie posters rated in the movie experiment. For each item participants would rate how interested they would be in watching the film on a scale from 1 to 10.

[3] Comprehensive details of the experiments can be found in Mozer et al. (2011).

Mozer et al. (2011) report a significant percent reduction in MSE for I^3 over the RM model (roughly 5% in the movie ratings and 4.5% in the morality judgments). For the gap detection task, the MSE for the inferred impressions compared to the ground-truth data was roughly 10% lower for I^3 than RM when the model was trained on a small number of trials containing in the range of two to four repetitions per item. The difference between the two models decreased as more repetitions of each item were included in training.

6.5 Judging Appropriate Tax Rates

To supplement the gap detection, movie rating, and morality judgment experiments described above, we conducted a new judgment experiment to further validate the decontamination methods presented in this chapter. The experiment, described below, asked participants to assign appropriate tax rates to different scenarios in which money is acquired.

6.5.1 Methods for Tax Experiment[4]

Participants

One hundred participants were enlisted using Mechanical Turk and were given a small amount of monetary compensation. Each participant answered 64 questions in one continuous session. The experiment was approved by the University of Colorado's Institutional Review Board.

Stimuli and Apparatus

Participants responded to 64 questions regarding appropriate tax rates for a specific situation by selecting a percentage value between 0 and 100. For example, "Mary inherited $100,000 from her grandmother this year. What percent of this amount should she pay in tax?" Participants were given instructions at the beginning of the task to choose a value between 0 and 100 on the sliding bar that appeared below the question. Each response was followed by a short delay and then the next question until the experiment terminated.

Procedure

[4] I am indebted to Robert Lindsey for programming and running this experiment on Mechanical Turk.

Eight unique question types were used in the experiment with two different dollar amounts. Thus there were 16 different questions each repeated four times for a total of 64 questions. The questions were divided into four blocks such that all questions appeared before the next repeat. The sequences were controlled to avoid the same question type appearing twice within five consecutive trials. Participants were divided into two conditions. In the first condition, the sliding bar was fixed to lock into percentages at 5% increments. In the second condition, the sliding bar was set to allow continuous responses between 0 and 100. The goal of this manipulation was to test whether finer resolution in the response scale improves the ability to decontaminate the responses. However, this manipulation did not offer any improvement in the decontamination process, so we collapse across conditions in the analyses that follow.

6.6 Bayesian Impression Inference (BI^2)

Though I^3 offers an improvement over the baseline RM model, there are two main constraints that hinder its performance. First, the weights are constrained to be the same for all participants. This corresponds to the psychological assumption that all individuals weight their recent experience in exactly the same way. Second, the impressions for each item are assumed to be independent (both within participants and across participants). Under this assumption, there are no similarities across items and the preferences of one individual are completely unrelated to the preferences of all other individuals. Of course this is unrealistic; for example in the domain of movie ratings, similar movies are likely to induce similar impressions and a group of individuals might have highly correlated impressions because the individuals have similar taste in movies.

When mapped onto a Bayesian framework, the I^3 model roughly corresponds to the graphical model depicted in Figure 6.2. Using the sequential effects model in Equation 6.1 as a foundation for the Bayesian model, the response on trial t is defined to have the following distribution,

$$R_t \sim \mathrm{N}(I(S_t) + \sum_{j=1}^{h} \beta_j I(S_{t-j}) + \sum_{k=1}^{h} \beta_{h+k} R_{t-k}, \sigma_R^2).$$

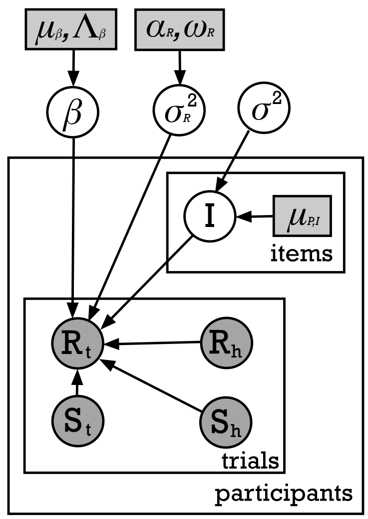


Figure 6.2: A Bayesian model that matches the assumptions and structure of I^3. See text for a description of architecture and variables.

The variables β and σ_R^2, which are shared across all participants, are distributed as follows:

$$\beta \sim \mathrm{N}(\mu_\beta, \Lambda_\beta) \text{ and } \sigma_R^2 \sim \mathrm{IG}(\alpha_R, \omega_R),$$

with μ_β, Λ_β, α_R, and ω_R fixed to produce relatively uninformative Normal and Inverse Gamma (IG) priors which are the standard priors used in Bayesian linear regression. The impression for each participant and item is distributed as

$$I \sim \mathrm{N}(\mu_{P,I}, \sigma^2),$$

with $\mu_{P,I}$ given by the RM solution from the training data and σ^2 shared for all participants and items. Using the RM solution as the mean of the prior is equivalent to the regularization in I^3 that keeps the impressions from straying too far from the RM solution (with σ^2 closely related to the ridge parameter in ridge regression). Using the training data to set the prior is unconventional because priors generally represent information believed before any data is viewed. This constraint will be removed in the models we develop but is included here to recreate I^3 as closely as possible in a Bayesian framework.

From a Bayesian perspective, it is somewhat straightforward how to relax the limiting constraints of I^3. First, we desire each individual to have a unique set of weights over past trials. In the graphical model, this is accomplished by moving the β random variable into the participants plate. This yields a form of Bayesian linear regression where the weights are constrained by a prior which in this context represents information about how a population of individuals tends to weight previous trials. Similarly, it is reasonable to assume that the variance of the response noise (σ_R^2) varies from one participant to the next, so this random variable can also be moved into the participants plate.

One challenge with moving the weights into the participants panel is that the mean and covariance of the prior over these weights needs to be specified. This prior is important because it encodes information about how sequential effects manifest in populations of participants. Though the mean and covariance could be fit to the data, we take the more principled approach of adding a hierarchical prior over the weights prior. This hyperprior can have very loose constraints and essentially allows the model to learn the appropriate mean and covariance for the prior over the weights.

On the impression side, it is less obvious how to modify the model to be sensitive to correlations across items and participants. One potential solution is inspired by Item Response Theory (IRT) which in the unidimensional case posits that each item is characterized by difficulty and each individual is characterized by ability (Lord, 1980). In the graphical model, there is a contribution value for each item (C_I) and participant (C_P) that are summed to form the mean of the prior for each participant/item impression. The variance of the prior (σ^2) remains fixed across all participants and items. This model is depicted in Figure 6.3a, though it is shown with the added complexity of hierarchical priors that will be explained shortly.

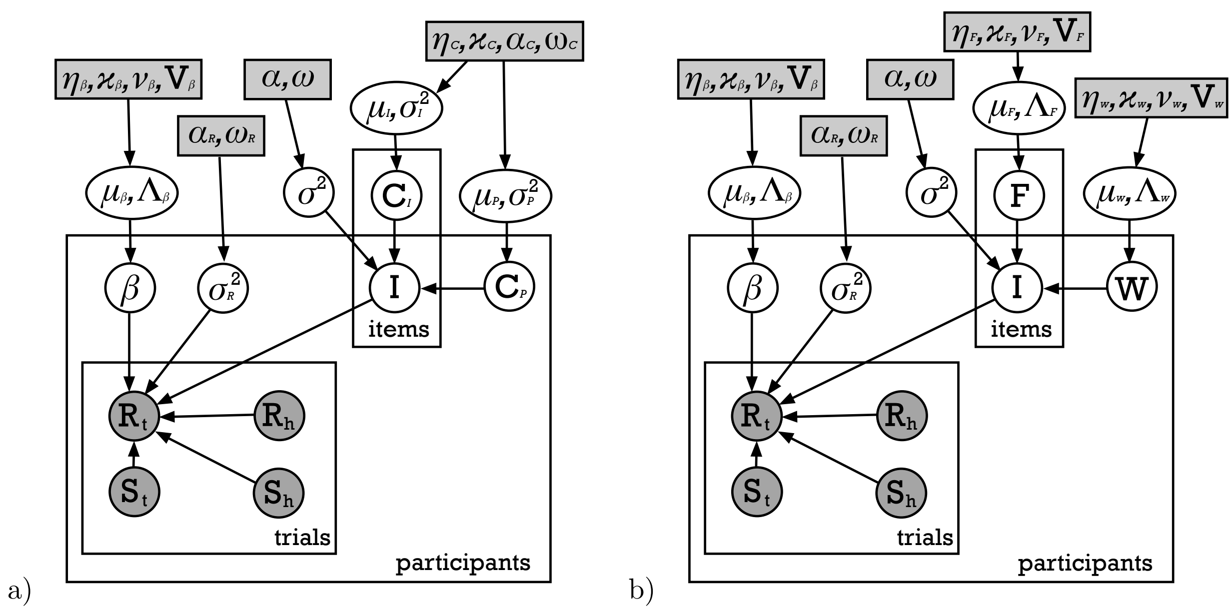


Figure 6.3: The graphical models for the IRT and MDS versions of BI^2. See text for a description of the architecture and variables.

One limitation of the IRT approach is that each item contribution is the same for all participants (i.e., there is no notion of item preference that varies across participants). It would be preferable to have a prior for each participant/item pair that captures individual preferences. The research on multidimensional scaling (MDS) (Torgerson, 1958) and factor analysis (Gorsuch, 1983) provide guidance for an alternative characterization of the priors over impressions. In MDS, items are represented in a higher dimensional space in such a way that distances between items, as defined by similarity, are preserved. When determining the prior over the impression for a specific participant/item pair, the latent multidimensional representation for the item can be scaled by the individual's preferences over the latent dimensions or factors. The graphical model corresponding to this approach, Figure 6.3b, contains an item-specific variable ($\mathrm{F} \in \mathbb{R}^d$) representing the d-dimensional factors and a participant-specific set of weights over the factor space ($\mathrm{W} \in \mathbb{R}^d$). The mean of the prior over the impression for a given participant/item pair is given by the inner product of the corresponding F and W.

A hierarchical approach is also applied on the impression side of the model by placing a hyperprior over the priors for the item contribution and participant contribution in the IRT version or the priors for the factors and weights in the MDS version. Though we could have also placed hyperpriors over the variance variables, we kept these priors fixed and parameterized them to allow for a wide range of reasonable variances. We call this full model Bayesian Impression Inference (BI^2) and refer to the two versions using the acronyms IRT and MDS. The full graphical models for these two versions are depicted in Figure 6.3.

6.6.1 Mathematical Specification of BI^2

The foundation of both the IRT and MDS versions of BI^2 remains Equation 6.1 that expresses the current response as a function of the current impression, the previous impressions, and the previous responses,

$$R_t \sim \mathrm{N}(I(S_t) + \sum_{j=1}^{h} \beta_j I(S_{t-j}) + \sum_{k=1}^{h} \beta_{h+k} R_{t-k}, \sigma_R^2).$$

However, the β values are now assumed to be different for each participant but drawn from a Normal prior distribution shared across participants, with the prior distribution itself drawn from a Normal Wishart (NW) hyperprior which is the conjugate prior for the multivariate normal and provides a sensible prior that can be parameterized to be relatively uninformative:[5]

$$\beta \sim \mathrm{N}(\mu_\beta, \Lambda_\beta^{-1})$$

$$(\mu_\beta, \Lambda_\beta) \sim \mathrm{NW}(\eta_\beta, \kappa_\beta, \nu_\beta, \mathrm{V}_\beta).$$

The variance of response noise (σ_R^2) and the variance of the impression priors (σ^2) are defined to be drawn from Inverse Gamma priors which again are chosen because of conjugacy,

$$\sigma_R^2 \sim \mathrm{IG}(\alpha_R, \omega_R) \text{ and } \sigma^2 \sim \mathrm{IG}(\alpha, \omega).$$

The preceding specification applies to both the IRT and MDS versions of the model as they share the same assumptions about sequential effects but differ with respect to how they treat impressions.

In the IRT version, the prior over the impression for a specific participant/item pair (indexed as P, I) is as follows:

$$I_{P,I} \sim \mathrm{N}(\mathrm{C}_P + \mathrm{C}_I, \sigma^2).$$

The contribution terms C_P and C_I each have their own Normal prior and these two priors share a common Normal Inverse-Gamma (NIG) hyperprior which is the univariate version of the Normal Wishart prior used as the hyper prior over the weights:

$$\mathrm{C}_P \sim \mathrm{N}(\mu_P, \sigma_P^2)$$

$$\mathrm{C}_I \sim \mathrm{N}(\mu_I, \sigma_I^2)$$

$$(\mu_P, \sigma_P^2) \text{ and } (\mu_I, \sigma_I^2) \sim \mathrm{NIG}(\eta_C, \kappa_C, \alpha_C, \omega_C).$$

The MDS version of BI^2 uses the following prior over the impression for a participant/item pair:

$$I_{P,I} \sim \mathrm{N}(\mathrm{F} \cdot \mathrm{W}^T, \sigma^2).$$

F and W each have a d-dimensional Normal prior which in turn has its own conjugate Normal Wishart hyperprior:

$$\mathrm{F} \sim \mathrm{N}(\mu_F, \Lambda_F^{-1})$$

[5] Throughout this specification, we parameterize the Normal distribution and the Normal Wishart using a precision matrix (inverse covariance matrix) instead of the covariance matrix because it simplifies derivations.

$$\mathrm{W} \sim \mathrm{N}(\mu_W, \Lambda_W^{-1})$$

$$(\mu_F, \Lambda_F) \sim \mathrm{NW}(\eta_F, \kappa_F, \nu_F, \mathrm{V}_F)$$

$$(\mu_W, \Lambda_W) \sim \mathrm{NW}(\eta_W, \kappa_W, \nu_W, \mathrm{V}_W).$$

6.7 Simulation Details

BI^2 was evaluated across the four experiments and compared to I^3 and RM. Before presenting results, it is necessary to explain how inference was carried out in BI^2, how the model was parameterized, and what methods were used for evaluating model performance.

6.7.1 Inference in BI^2

Given the mathematical specification for the two versions of the model, it is possible to express the posterior distribution of each variable conditioned on the other variables in the model as a well known distribution (see Appendix D for these derivations). For this reason, we used Gibbs sampling to perform inference in the model. The shaded variables in the graphical model—including the sequence of responses and the hyperparameters—are treated as observed variables, and the goal is to infer the distribution of all other variables in the model. From the perspective of decontamination, the inferred impressions (I) are of greatest interest. However, because we will assess the model by predicting response sequences in the test data, we are also interested in the β values inferred for each participant. After running Gibbs sampling, we have many samples for β and the impressions. In theory it is desirable to take the mode of these samples which is the maximum a posterior (MAP) estimate. However, for convenience and because we have no reason to believe these distributions are multi-modal, we obtain our final estimate for β and the impressions by computing the mean of the samples.

6.7.2 Model Parameterization

Apart from the model parameters displayed in the graphical model, there are several other parameters required in the simulation. An important model parameter is h, the number of previous impressions and responses used in the regression model. In all our simulations, cross validation was used to find the best fitting h and regularization parameter for the I^3 algorithm, for each set of experimental data.[6] These parameters were used to obtain the I^3 fits which are used for comparison. The best fitting h value for each experiment was carried over to BI^2. The values of h for the different experiments were 13 for the movie poster ratings, 7 for the morality judgments, 18 for the gap detection, and 5 for the tax questions. These values appear to be related to the amount of data available to constrain the model (200, 100, 180, and 64 trials respectively). For the MDS version of BI^2, we defined the dimension (d) of the factor space to be four. This was arbitrarily chosen to allow a flexible representation without increasing the model complexity too much. However, model performance was not very sensitive to this parameter. For Gibbs sampling, we used 2000 iterations with a burn-in of 100 iterations and thinned the samples to every 10th iteration. These values were selected by visualizing the dynamics of the model across iterations and observing roughly how long it took for the effect of initialization to vanish and how much correlation there was between successive iterations. Model performance was not greatly affected by other choices for these values. Finally, the hyperparameters for the model were chosen to yield very general, uninformative priors. The goal was to place as few constraints as possible in the hyperpriors so that the priors were free to find the most appropriate values given the training data. Appendix D presents a more specific description of each hyperparameter and the value assigned to it for the simulation.

6.7.3 Evaluation

For each experiment, the sequence of trials were divided into a training and test set. The divisions were made at the block level to control the number of repetitions of each item in the two sets. For example, in the movie experiment where there are four blocks of 50 trials, the training set might consist of trials 1-100 and the test set would then include trials 101-200. Gibbs sampling

[6] The cross validation procedure used the first two blocks in the movie and gap experiments and the first three blocks in the moral and tax experiments as the training set. The last 25 trials of the training set were used for validation in the movie and moral experiment; the last 30 were used for validation in the gap experiment; and the last 16 were used in the tax experiment.

was run on the training data to estimate β and the impressions for each participant. These values were then used to predict responses according to the model. Mean squared error (MSE) was computed to quantify how closely the predicted responses for each participant matched their actual responses. Overall model performance for an experiment was defined as the average MSE across all participants.

For most of the analyses, four different training/test divisions were used. Furthermore, each division was repeated four times to reduce some of the variability in the Gibbs sampling procedure. Thus the final MSE values reported for each experiment represent the average across 16 different runs of the model. For the movie data and the gap experiment, the training and test data were split so that each set contained half of the trials. Because there were fewer trials in the morality judgment and tax experiments, the training set consisted of three blocks of data and the test set consisted of one block. It should be noted that the training/test division serves as a means for evaluating the model and that in an applied setting, all of the data would be used to train the model.

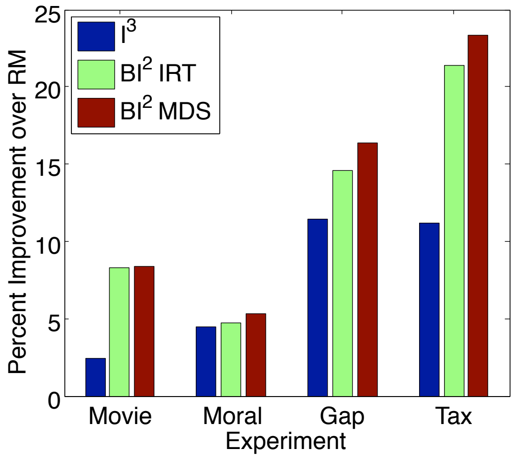
Because the sequential effects model embedded in I^3 and BI^2 includes h previous trials, the first h trials in the experiment are excluded from the training or test set (depending on which set includes the first trials). However, the trials are used in the sequence history of the subsequent trials. At the overlap of the training and test sets, h trials from the end of the first set are used for the sequence contexts of the trials that follow. For example, if $h = 9$ and trials 1-100 are used in testing and trials 101-200 are used for training, judgments for trials 1-9 will not be predicted directly and will only appear in the contexts for trials 10-18. Additionally, trials 92-100 will be used as lead-in trials for the beginning of the training set. Though this overlap may appear to unfairly improve performance on the test set, the actual information provided by these lead-in trials is minimal because the judgments for those trials do not end up as target values in the training regression equations.

6.8 Decontamination Results

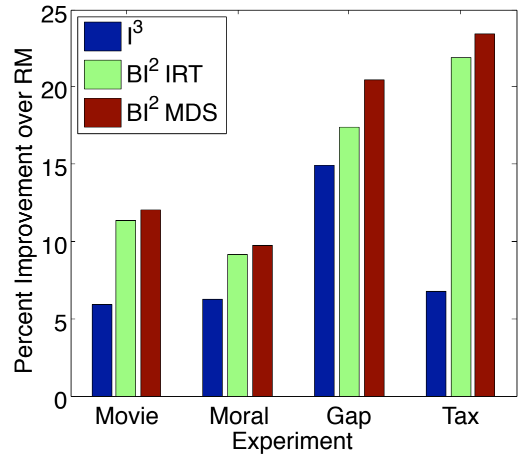
As outlined earlier in this chapter, the primary goal of decontamination is to obtain more representative estimates of an individual's true impressions to the judged items with fewer total judgments required. The representativeness of the BI^2 inferred impressions will be evaluated in comparison to the RM model and I^3 by studying MSE for the different models on the test data. The *efficiency* of the model, which can be defined as the relative improvement in MSE for a fixed number of judgments or as the reduction in judgments necessary to achieve a fixed MSE, will be examined by evaluating the model across a range of training set sizes.

Figure 6.4a displays the percent improvement in MSE over the baseline RM model for I^3 and the two versions of BI^2 across the four experiments. The MSE values (Table 6.1) were obtained by averaging across the 16 runs of the experiment (four repetitions of each of four training/test splits). Thus the values represent prediction performance equally balanced across all responses made in the experiment. It may seem strange to consider a training/test division that has non-continuous blocks, for example, training the model on blocks 2 and 4 and testing on blocks 1 and 3. For the sake of completeness and to alleviate this concern, Figure 6.4b displays the percent MSE improvement when the models are trained on the first two or three experimental blocks and tested on the last two or one blocks—recall that half the data was used for testing in the movie and gap experiments because there were more trials, but only one block was used for testing in the moral and tax experiments. For all the experiments, the models produce a meaningful improvement over the baseline RM model. Further, BI^2 yields a consistent improvement over I^3. Because the MDS version of BI^2 appears to offer a slight improvement over the IRT version, we will use this version in the rest of the analyses and simply refer to is as BI^2.

Though the BI^2 model appears superior to both I^3 and RM in Figure 6.4, the results may not tell the whole story because the training set includes at least two blocks. In many real-world situations, it is impractical or costly to repeat every item even just twice. Ideally, decontamination would be performed on a data set where each of the N items are judged exactly once. However, the



a)



b)

Figure 6.4: The percent improvement of mean squared error (MSE) over the baseline RM model for I^3 and two versions of BI^2 across the four experiments. a) Error was averaged over four repetitions of four different splits of the data into training and test sets. b) Error averaged across 4 repetitions of only the split that uses the first portion of the data for training and the last portion for testing.

Table 6.1: Mean squared error (MSE) for the RM, I^3, and BI^2 models across the full set of responses given for each experiment.

	Movie	**Moral**	**Gap**	**Tax**
RM	1.3590	0.6673	0.7534	0.5082
I^3	1.3264	0.6372	0.6671	0.4511
BI^2 IRT	1.2466	0.6356	0.6434	0.3996
BI^2 MDS	1.2449	0.6316	0.6303	0.3897

models developed here would be underconstrained with only N judgments because there $N + 2h$ unknowns in the regression equation. If h is relatively small, it may take only a few extra judgments to constrain the models. To test the efficiency of BI^2 and I^3, we reduce the flexibility of the models by setting h to 2. This makes it possible to compare the performance of the model with just four repeated items. To further reduce the model complexity of BI^2, the dimensionality of the latent factors (d) is reduced from four to two. MSE is computed for the models trained with the first block of trials plus the subsequent 4 to 48 trials at intervals of two trials (a maximum of 32 extra trials are used in the tax experiment to avoid incorporating the test data in the training data). The test data consists of the last 2 blocks for the movie and gap experiments and the last block in the moral and tax experiments (i.e., the same as used in Figure 6.4b). When MSE is computed as a function of the number of extra judgments, BI^2 shows an impressive reduction in MSE in comparison to I^3 and RM (Figure 6.5). Each datapoint in the figures represents the average MSE across 4 repetitions of model training.

The disparity in MSE is greatest with roughly 8 to 12 extra judgments and decreases as more judgments are added. This decrease is evident in Figure 6.6a which plots the percent reduction in MSE of BI^2 compared to RM—the size of the reduction decreases as more items are repeated. When only a small number of extra judgments are added, the percent improvement is dramatic ranging from roughly 10% to 30%. This reduction is likely attributed to the BI^2 model's ability to incorporate meaningful cross-participant constraints—a characteristic that is valuable in the real world because it is more common to have many participants but few repetitions of items. For the moral judgment experiment, Figure 6.5b highlights the value of BI^2 over I^3 which is less apparent in Figure 6.4b. However, Figure 6.5b reveals another attribute worth studying—the steep slope in MSE for all three models. The slope of the MSE curves hints at the value of adding more judgments, with steeper curves implying greater value. A natural question to ask when studying these plots is how many extra judgments are needed for the RM model to achieve the same MSE as BI^2 (i.e., what is the benefit of BI^2 in terms of the number of judgments it saves). This can be quantified roughly by averaging the horizontal distance between each point on the red curves

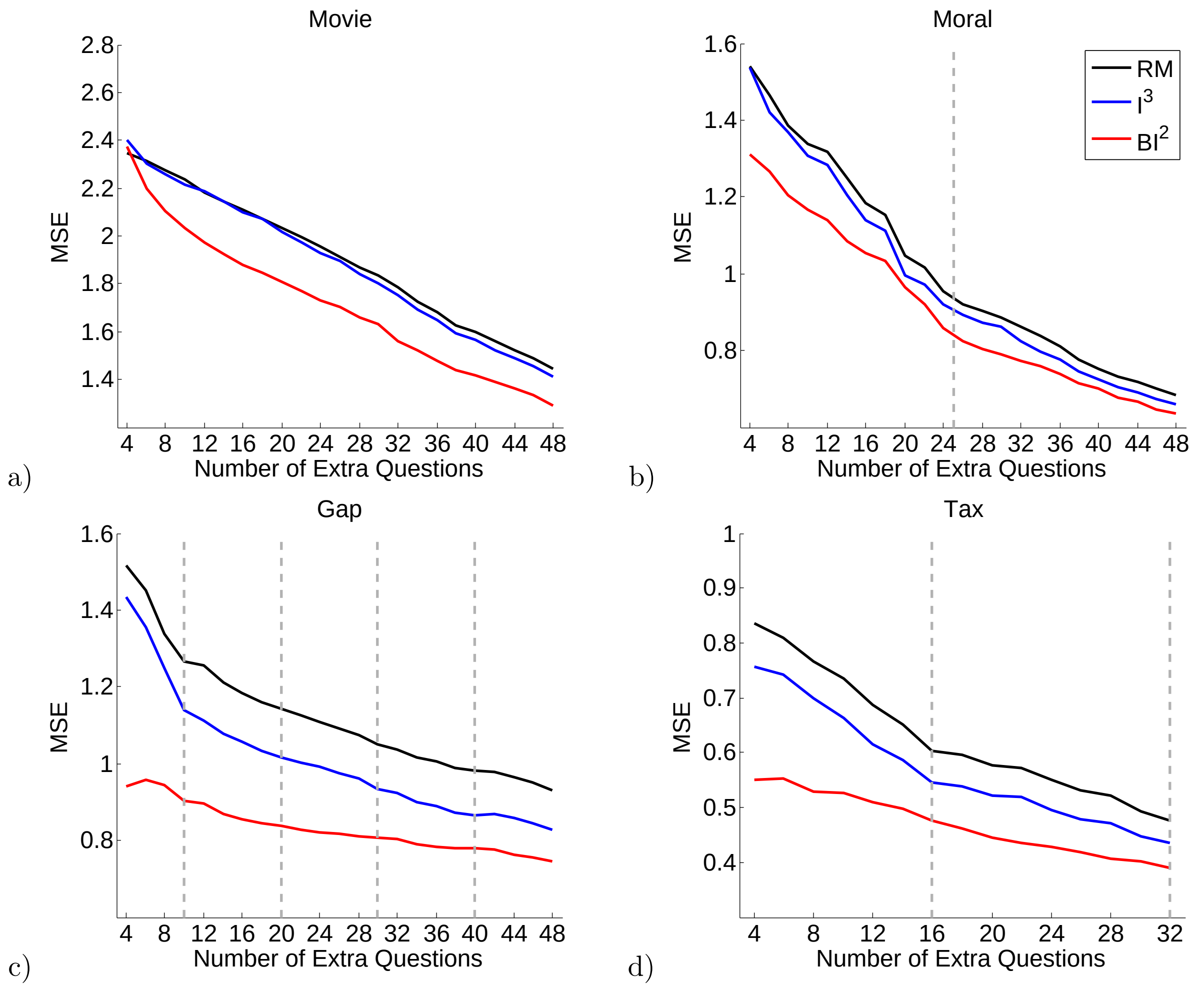


Figure 6.5: An analysis of how extra judgments improve the model MSE on the test data. Here the training set includes the first presentation of each item plus 4 to 48 extra judgments. Adding trials yields a reduction of error in all cases, but BI^2 (red line) has significantly lower MSE for each number of extra trials included across the four experiments: a) movie, b) moral, c) gap, and d) tax. The dashed gray lines in the figures indicate the points at which the full set of items has been repeated and depends on the total number of items used in each experiment (there were 50 items in the movie study and thus the far right side of the graph corresponds to almost 1 full repetition).

and the corresponding point on the black curves with the same MSE in Figure 6.5.[7] The average number of extra judgments required by RM to match BI^2 ranges from about 8 to 40 (Figure 6.6b). As shown in Figure 6.6c, these differences can correspond to relatively large differences in terms of proportion of dataset size.

One might question why percent improvement is much greater in the gap and tax experiments. For each experiment there is a plausible explanation for why BI^2 would yield such a large improvement. In the tax experiment, there was high variability in the quality of the responses provided by the participants. Some participant appeared to produce response patterns (e.g., response autocorrelation or bias toward using a very small range) that BI^2 was able to capture and use to improve response predictions. Indeed, when we post-processed the experimental data to remove suspect participants, the percent improvement decreased though BI^2 still fared best (for the full collection of training/test splits and repetitions, I^3 MSE was 5.22% lower than RM and BI^2 MSE was 7.46% lower than RM). In the gap detection task, all participants are evaluating the exact same stimuli which will induce the same impressions for all participants given the assumption that the stimulus-to-impression mapping is veridical. Thus there is significant value to be gained by sharing information across participants—exactly what BI^2 was designed to do. This same characteristic also leads to large improvements in how well the BI^2 impression match the ground-truth data as compared to I^3 and RM. After applying an affine transformation to match the inferred impressions to the whole range—which is done because some participants do not use the full range—ground-truth MSE for the BI^2 impressions is roughly 50% lower than the RM model whereas I^3 only yields a marginal improvement.[8] It is difficult to say what the value of this result is given that the model is particularly tuned to the experimental environment (i.e., it is sensitive to properties of the items that are consistent across participants). Ground-truth data would be more useful as a means to evaluate BI^2 if it contained items with impressions that varied across individuals. However, such data would be difficult to obtain because it would require averaging over many judgments for each

[7] Of course this is only done when there exists a matching MSE point on the black curve.

[8] The improvement varies depending on the size of the training set. The improvement reported was obtained when training on half the data and testing on the other half.

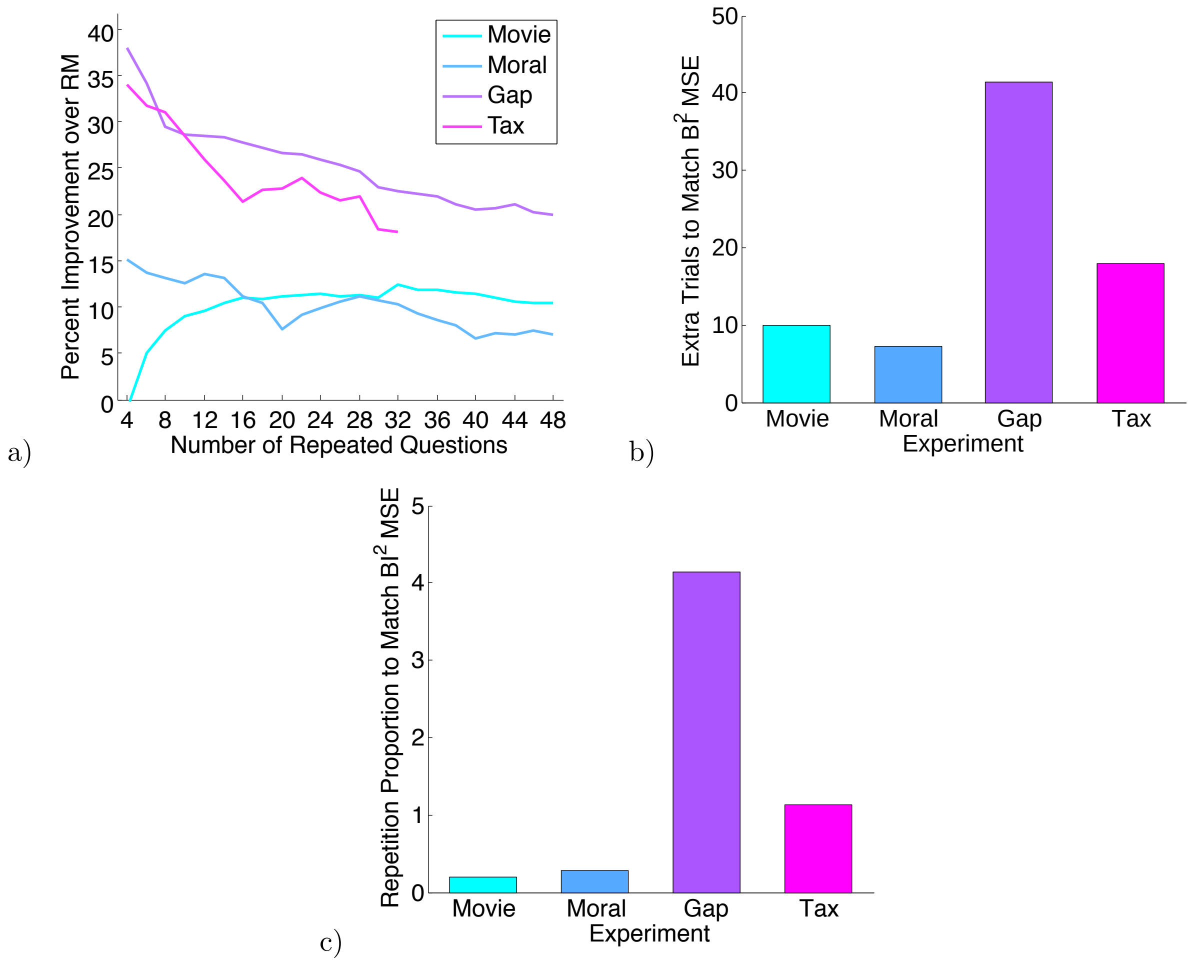


Figure 6.6: a) The percent improvement in MSE of BI^2 over the RM model as a function of the number of extra trials included in training. b) The mean number of extra trials needed in the RM model to match the MSE of BI^2 for each experiment. c) The same graph as in b) but plotted in terms of the proportion of repeated items (1 corresponds to 1 repetition of each item in the dataset).

item across various sequential contexts.

Apart from comparisons to ground-truth data and computing error for response predictions, there are other ways to evaluate the quality of the inferred impressions. For the movie poster ratings in Mozer et al. (2011), the genres of the 50 movies are known (10 from each of 5 genres) but are not used in training. Because individuals are likely to have genre preferences, we expect that impressions to movie posters within a genre should be similar.[9] Mozer et al. (2011) quantify this concept by computing the ratio of within genre variance to between genre variance for each participant's set of impressions. When trained with all 200 responses, the variance ratio is 8.5861 for the RM model; 8.3887 for the I^3, which is a 2.30% improvement over RM; and 7.4756 for BI^2, which is a 12.93% improvement over RM (both of these improvements are significant across participants using a sign test: $p = 0.033$ and $p < 0.001$ respectively). A similar approach is taken for the morality judgments. Mozer et al. (2011) hypothesize that actions should receive similar ratings across participants because of cultural norms. Interrater agreement is measured as the ratio of the variance of ratings to an item over participants to the variance of mean ratings over items. For RM, I^3, and BI^2 this ratio is 0.6923, 0.6882, and 0.6242, respectively, which correspond to a 0.59% improvement for I^3 and a 9.84% improvement for BI^2 (this improvement is significant across items according to a sign test for BI^2 ($p < 0.001$) but not for I^3). For both of these analyses, all four blocks are used to train the models. For the morality judgments, we should be wary of using this method of evaluation because BI^2 may have an implicit bias toward increasing interrater agreement as a consequence of the priors over the impressions.

A similar approach can also be applied to the tax experiment. Recall that each question type was repeated twice with two different dollar amounts that differed by a factor of ten. At the onset of the experiment, we hypothesized that individuals would exhibit a graduated tax scale that depended on the amount of money under question. If this were the case, we would expect the ratio between the responses given for the two dollar amounts to be similar across the 8 question

[9] Remember that participants are rating their desire to see a particular movie based on the poster so we do not need to worry as much that actual quality of movies within a genre may vary greatly.

types. To evaluate the inferred impressions, we determine the graduated rate for the 8 questions and compute the ratio of within participant graduated rate variance to the variance of mean per-participant graduated rates. For the RM model, this ratio is 3.018. I^3 results in a ratio of 2.7148 which is a 10.05% decrease, and BI^2 yields a ratio of 1.4589 for a 51.66% decrease. Again the improvement for BI^2 is so large because the model likely captures the abnormal behavior of some participants. If we remove questionable participants, we get 5.51% and 23.80% improvement for I^3 and BI^2 respectively.

In formulating BI^2, changes were made to the treatment of the β values and the impressions. But which of these changes is responsible for the improved performance of the model? To answer this question, four versions of the model were trained and evaluated: 1) the Bayesian replication of I^3 as shown in Figure 6.2; 2) the Bayesian I^3 with β treated as in BI^2; 3) the Bayesian I^3 with impressions treated as in BI^2; and 4) the full BI^2 model. Figure 6.7a displays the percent improvement in MSE of these four versions over the RM model across the four experiments. For all but the movie experiment, most of the gain is achieved by incorporating the MDS priors over the impressions. Changing the way β is treated in the model is most beneficial in the movie experiment, but offers a marginal improvement in the tax experiment. Further, the combination of the two treatments in the full BI^2 model yields a greater than additive improvement for the movie and tax experiment. It is not surprising that adding more informative priors over impressions results in all of the improvement for the moral and gap experiments. In these two experiments, we expect there to be strong consistency between impressions for each item across individuals because of societal norms in morality judgments and constant gap sizes in the gap experiment. Naturally the MDS component of BI^2 will represent and leverage this consistency. We suspect that the value of the two components will be more balanced in domains where impressions vary greatly from one individual to the next, as suggested by the movie experiment.

Even though the impression side of the model appears to produce the most improvement in these experiments, it should be noted that impression inference is still heavily reliant on the sequential effects encoded in the model. This is a consequence of the fact that the regression

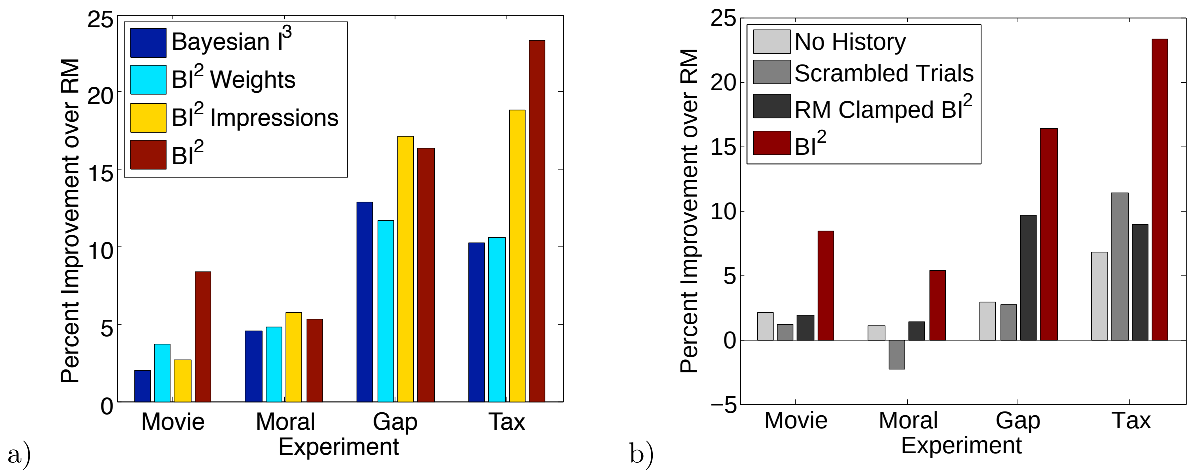


Figure 6.7: An analysis of the value of the treatment of weights and impressions in BI^2. a) Percent improvement in MSE over RM for 4 versions of the model. The first bar represents the Bayesian version of I^3. The second bar is the Bayesian version of I^3 with per-participant weights that share a prior. The third bar is the Bayesian version of I^3 but with an MDS prior over the impressions. The fourth bar is the complete BI^2 model. b) An illustration of the importance of sequential effects in the model. BI^2 with no history (light gray) and a randomly scrambled trial sequence (middle gray) is compared to the full BI^2 model. Additionally, the dark gray bar demonstrates that the improvement of BI^2 over RM is not just obtained by using the sequential context to predict responses. The improvement over RM is small for response predictions that are obtained from BI^2 model with impressions clamped to the RM solution.

equations relevant for solving for the impression of a given item include not only the trials where that item was the displayed item, but also trials where that item was in the recent history.[10] The importance of sequential effects is best displayed by removing the sequential effects from the model (i.e., setting h to 0) and comparing the results to the full model. When sequential effects are removed from the model, MSE is only slightly better than the baseline RM model (lightest gray bar in Figure 6.7b). The importance of the recent history can also be demonstrated by leaving the BI^2 model unchanged but scrambling the sequence ordering in the training and test sets (middle gray bar). The fact that performance is roughly equivalent to the RM model verifies that the trial history included in the model does not capture some other property of the data that is independent of the sequential ordering. The larger improvement in the tax experiment for these two tests confirms our suspicion that the model was capturing other regularities in the data beyond sequential effects. Nonetheless, adding sequential effects to the model still produces a dramatic improvement in this experiment.

In most of the analyses presented above, the models under scrutiny are compared to the RM predictions which have no dependence on sequential context. Our claim is that the performance of BI^2 is better because the inferred impressions are more reliable. However, it is possible that the improvement is simply a consequence of using the sequential context when predicting responses in the test data and does not reflect better inferred impressions. To dispel this possibility, the BI^2 model was run with the impressions clamped to the RM solution (i.e., response predictions were produced using the RM impressions but incorporating sequential context). Performance was only slightly better than the RM model without sequential context (dark gray bar in Figure 6.7b). For the gap experiment, the RM impressions appear to be closer in quality to the BI^2 impressions. This is not surprising given that the training data for this experiment included nine repetitions of each item which allows for most of the sequential biases to be averaged out of the RM impressions.

[10] This is a rather technical point that we do not expect the reader to completely understand given the model description above. However, the derivation for the impressions in Appendix D should help clarify this.

6.9 Discussion

In this chapter we have addressed the practical issue of how to deal with sequential effects that arise in contexts where they are undesirable. Specifically, we have examined how to improve the quality of human judgments in tasks where the sequential context biases the observed responses. Across a range of experiments in different domains, our Bayesian Impression Inference (BI^2) model yields a dramatic improvement of 6-24% over the default approach in which a participant's responses for each item are simply averaged. Furthermore, BI^2 produces a more representative estimate of an individual's internal state with fewer required judgments—an attribute that can be of great value in domains where it is expensive to collect many judgments.

Whether analyzing ratings for products on Amazon, restaurants on Yelp, movies on Netflix, hotels on TripAdvisor or studying traditional surveys meant to assess the marketability of a given set of products or perhaps just gauge something mundane such as which sports teams people like most, BI^2 has the potential to significantly improve the reliability of the conclusions drawn from these data. For example, if the 8% improvement in the movie poster rating task transferred to pure movie ratings, the BI^2 model may have produced a meaningful boost in performance for those in the $1,000,000 Netflix contest vying to achieve the prized 10% improvement in predicting individual movie preferences. It is also possible that the techniques developed here could be easily packaged into the online survey products offered by companies like SurveyMonkey or SurveyGizmo.

One beneficial attribute of BI^2 is that it requires very little customization for different domains. The only parameter that varies across experiments in our study is the length of the trial history included in the sequential effects model. As the history grows, the model becomes more flexible and has a higher risk of overfitting the training data. In our simulations, we chose history lengths that were roughly proportional to the number of judgements available for training. It is likely that this single free parameter could be removed from the model by adding a simple rule that determines the length of history as a function of data size.

Beyond the domain of online ratings and surveys, decontamination techniques such as BI^2

might prove useful in general experimental analyses. For example, in estimating psychophysical functions, decontamination could be used to remove the sequential effects known to occur in the response sequences to obtain more accurate function estimates. It is even possible that decontamination could replace the ubiquitous use of averaging in many experimental data analyses. In most typical psychological experiments, multiple responses to a given item are collected and averaged to eliminate noise. However, if some of the noise is the result of sequential biases, using a decontamination method will yield better estimates of the true desired responses than pure averaging, as shown by the consistent improvement of BI^2 over RM in this work. The potential improvement to be gained by using decontamination on the data is directly related to the number of repetitions of each item. As more repetitions are collected, the utility of decontamination over averaging declines. A more thorough investigation of the value of decontamination in this domain would of course need to be performed before adopting these methods in real experimental analyses. Nonetheless, this is an interesting avenue for future exploration.

In the current work, BI^2 performs well even when only minimal extra judgments are provided. Although this is a big improvement over previously developed decontamination models, it still falls short of the optimal scenario in which only one judgment per item is required. In BI^2, several extra judgments are needed to constrain the regression model for each individual (though not shown in our results, performance deteriorated significantly and inferred impressions were less stable when fewer that 4 extra judgments were added). However, it seems possible to modify BI^2 in such a way that imposes greater constraints on the weights and impressions of each individual. For example, instead of assuming that every individual has unique regression weights, the model could assign the same regression weights to groups of individuals that exhibit similar sequential dependencies. Alternatively, an individual's sequence of judgments could be randomly supplemented with extra judgments from other individuals—perhaps ones that have similar preferences or response behavior. Randomizing this process and repeating many times could add the necessary constraints needed to make quality impression inferences without adding too much noise. The ultimate goal is to eliminate the need for extra judgments by extracting useful information from the collection of individuals who performed the task.

It is clear that methods of decontaminating human responses have the potential to be beneficial in a wide range of domains. We have developed a sophisticated decontamination model that uses a Bayesian framework to capture regularities in the sequential dependencies and impressions across collections of individuals and judged items. Though there is potential for improvements to BI^2, the model as it stands can be readily applied to improve the fidelity of human judgments in many situations.

Chapter 7

Major Contributions and Future Directions

The primary contribution of this work is two-fold. On the theoretical side, through normative modeling and experimental investigation, we elucidate computational principles underlying sequential effects and cognitive mechanisms that produce them. Furthermore, we reveal a persistence of incidental experience that challenges the widely held perspective that sequential effects are ephemeral behavioral perturbations with no long-term consequences. Beyond theory, we demonstrate that knowledge of sequential effects can be leveraged in real-world problems to extract more reliable characterizations of individual preferences from sequences of judgments.

In the Dynamic Belief Mixture Model and the Hierarchical Dynamic Belief Model, sequential effects reflect adaptation in a dynamic environment that is characterized by multiple variables and exhibits nonstationary change dynamics. Through a novel experimental study and a reanalysis of existing behavioral and electrophysiological data, we confirm that individuals are sensitive to multiple environmental regularities and substantiate a cognitive dissociation of these sensitivities with the response system exhibiting sequential effects that result from direct properties of the stimuli and the perceptual system exhibiting effects that result from abstractions of the stimulus sequence. Taken as a whole, our modeling of sequential effects in 2AFC illustrates the value of seeking a normative explanation of the phenomena rooted in adaptation and considering a collection of components that may all contribute to the complex effects observed.

In our investigation of the persistence of incidental experience, we demonstrate that future events are anticipated by integrating over past experience using a weighting that is strongly recency based but also has a heavy tail, consistent with power but not exponential discounting. The presence of power decay suggests that it may be more informative to interpret sequential effects in decision making as reflecting memory processes rather than short-term incremental learning.

Expanding upon the nascent field of *decontamination*, we proposed a novel hierarchical Bayesian model that effectively removes sequential biases in judgments to obtain more representative estimates of an individual's internal state. Decontamination methods are rife for exploration and our model serves as a solid foundation for future inquiries. In fact we have identified several avenues of exploration in this realm. Specifically, it is still an open problem to perform decontamination on datasets with no repeated judgments. In our methods, we evaluate the effectiveness of decontamination by assessing prediction error on a held-out data set. However, this is not feasible in environments where individuals make a single judgment per item. Nonetheless, it would be of great value to be able to assess the efficacy of a decontamination method in each judgment domain to which it is applied. Developing validation approaches that leverage a large participant pool rather than multiple judgments per item would significantly increase the practical utility of decontamination. Another interesting avenue of investigation similar to decontamination involves developing methods for determining sequence orderings that minimize or maximize sequential effects. This concept could be taken one step further by trying to adapt the presentation order an individual is experiencing to account for sequential effects and reduce the sequential biases present in the summary data at the end of the experiment. A technique such as this could be used to obtain more representative judgements with fewer questions asked and could perhaps be tailored to continue repeating items until the estimated biases are below a specific threshold.

The trial-by-trial modeling introduced in Chapter 4 highlighted a large amount of variability in human behavior that is unexplained by current models. An intriguing, though bold, goal for the future is to explain away all the variability in response times in 2AFC. No doubt, much of this variability is unexplainable random noise, but with only about 15% of the variability currently explained, it is possible that we might still be able to account for another 50% or more. There are several promising directions that could pay off. Using the large quantity of EEG data recording during some 2AFC studies, it may be possible to obtain a better characterization of the internal state from moment to moment that results in better predictions of response time. Alternatively, exploring more complex mechanisms that include within-trial time dynamics could be fruitful. We have considered implementing a cascading diffusion process that captures—with fine-grained resolution—the dynamics of the multiple components that combine to influence behavior.

In the realm of rational models, it would be interesting to investigate the utility of the general HDBM changepoint framework in environments that are more complex than 2AFC. In the original DBM work of Yu and Cohen (2009) and our HDBM, the changepoint dynamics are applied to an underlying model that is encoded with a single variable. In our DBM^2, we demonstrate the dynamics applied to a model for an environment that includes two states (a baserate probability and a repetition rate). However, in both cases these are extremely simplistic environmental models that are completely disconnected from any realistic model for a non-laboratory environment. Nonetheless, the math of the HDBM suggests that the changepoint dynamics may be easily generalized to any underlying model. Perhaps the framework could offer more insights when applied to real-world or real-world-like situations where the environment is much more complex. Along the same lines, we have performed a derivation that suggests that the DBM is a specific instantiation of a more general, continuous-time model in which changepoints occur according to a Poisson process instead of a Bernoulli process with a prior distribution that is characterized by a gamma distribution instead of the beta distribution in DBM. It could be informative to investigate this model more, testing it on experiments with widely varying inter-trial intervals, and exploring how to map the HDBM generalization of the DBM into the continuous-time domain.

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Appendix A

Comparing DeCarlo and Cross (1990) and Stewart, Brown, and Chater (2005)

In studying the absolute identification model of Stewart, Brown, and Chater (2005), which will be referred to as SBC, I observed a close similarity to the general magnitude estimation model presented in DeCarlo and Cross (1990), which will be referred to as DC. Because someone somewhere might just be interested in this, I have included my derivations in this appendix.

To begin the comparison, I will first make a note about the notation of the two models. In SBC, R_t represents the response at time t which is a ranking on the ranking scale (note: I'll use t to index time instead of n as in SBC). In DC, R_t corresponds to a magnitude response rather than a ranking response. I will use M_t to reference this magnitude and use R_t as used in SBC. Similarly, S_t refers to the stimulus ranking in SBC and the stimulus magnitude in DC. Following the lead of SBC, I will use X_t to reference the stimulus magnitude and use S_t for the stimulus ranking. SBC points out that the magnitude and ranking have a logarithmic relationship in most absolute identification experiments. We will be more explicit about the form of this relationship soon.

The SBC model expresses the ranking response random variable, $\mathbf{R}_t$, as a function of the previous feedback, F_{t-1}, the perceptual difference between the current and previous stimulus (contaminated by previous differences), D_t^c, and a random quantity, $\rho\mathbf{Z}$:

$$\mathbf{R}_t = F_{t-1} + (1/\kappa)D_t^c + \rho\mathbf{Z},$$

where $D_t^c = A\sum_{\tau=0}^{t-2} \alpha_\tau \ln(X_{t-\tau}/X_{t-\tau-1})$ and SBC's λ is replaced with κ to avoid confusion later. For sake of the derivation, we will at first assume $\alpha_2 = 0$, $\alpha_3 = 0$, $\ldots$,[1] which yields the equation:

$$\mathbf{R}_t = F_{t-1} + (A/\kappa)(\alpha_0 \ln X_t - \alpha_0 \ln X_{t-1} + \alpha_1 \ln X_{t-1} - \alpha_1 \ln X_{t-2}) + \rho\mathbf{Z}.$$

In SBC, the magnitude of a stimulus, X_t, is related to the ranking, S_t, by the equation $X_t = I_0 r^{S_t}$, where I_0 is the smallest stimulus magnitude and r is the geometric scaling factor. Given this relationship, we can translate the response ranking, $\mathbf{R}_t$, to a response magnitude value, $\mathbf{M}_t$, as follows:

$$\mathbf{M}_t = I_0 r^{\mathbf{R}_t}$$

$$\ln \mathbf{M}_t = \ln I_0 + \mathbf{R}_t \ln r$$

$$\mathbf{R}_t = (1/\ln r)(\ln \mathbf{M}_t - \ln I_0).$$

Without access to explicit feedback, we replace F_{t-1} in the model with R_{t-1} (using the actual response, not the random variable). Performing this substitution and writing the equation in terms of magnitudes instead of rankings, we get:

$$(1/\ln r)(\ln \mathbf{M}_t - \ln I_0) =$$

$$(1/\ln r)(\ln M_{t-1} - \ln I_0) + (A/\kappa)(\alpha_0 \ln X_t - \alpha_0 \ln X_{t-1} + \alpha_1 \ln X_{t-1} - \alpha_1 \ln X_{t-2}) + \rho\mathbf{Z}$$

$$\ln \mathbf{M}_t = \ln M_{t-1} + \eta(\alpha_0 \ln X_t - \alpha_0 \ln X_{t-1} + \alpha_1 \ln X_{t-1} - \alpha_1 \ln X_{t-2}) + \rho \ln r\mathbf{Z},$$

with $\eta = (A \ln r)/\kappa$. The derivation continues with

$$\ln \mathbf{M}_t = \ln M_{t-1} + \eta\alpha_0 \ln X_t + \eta(\alpha_1 - \alpha_0) \ln X_{t-1} - \eta\alpha_1 \ln X_{t-2} + \rho \ln r\mathbf{Z}.$$

After substituting $\beta = \eta\alpha_0$ and $\lambda\gamma = \eta\alpha_1$, we have:

$$\ln \mathbf{M}_t = \ln M_{t-1} + \beta \ln X_t + (\lambda\gamma - \beta) \ln X_{t-1} - \lambda\gamma \ln X_{t-2} + \rho \ln r\mathbf{Z}.$$

The equation above is quite similar to the DC model (Equation 17a in their work), which is as follows (using the new notation for magnitudes):

$$\ln \mathbf{M}_t = \lambda \ln M_{t-1} + \beta \ln X_t + (\gamma - \lambda\beta) \ln X_{t-1} - \lambda\gamma \ln X_{t-2} + \mathbf{u}_t + K,$$

[1] The derivation should hold if we relax this assumption, however, there will be many more terms to track.

where $\mathbf{u}_t$ is independent gaussian noise and K is a constant intercept. In the SBC model, the noise term, $\rho \ln r\mathbf{Z}$, is constrained more than $\mathbf{u}_t$ such that the current response cannot be pushed below (or above) the previous response/feedback depending on whether the current perception is greater than (or less than) the previous perception. The other main differences here are that there is no coefficient for the previous response term and the one-back stimulus is constrained in a slightly different way.

The DC model is based upon two theoretical assumptions: (1) the current perception is obtained via Steven's law according to $\Psi_t = X_t^{\beta} C^{\gamma} \delta_t$ where C is the context that affects the representation and δ_t represents error in perception/memory, and (2) the judgement process is affected by immediate context and long-term context (i.e. $M_t = \Psi_t(M_{t-1}/\Psi_{t-1})^{\lambda}(M_0/\Psi_0)^{1-\lambda}\mu_t$, where μ_t is judgement error and $0 \leq \lambda \leq 1$). These two theoretical assumptions lead to DC Equation 17a when $C = X_{t-1}/X_t$ or more simply $C = X_{t-1}$ (which is what they actually use for the derivation).

Given the similarity between SBC and DC, it would be interesting to show that the SBC model also emerges from these two theoretical assumptions, perhaps with a slightly different context, C, and ignoring the different treatment of noise.

Appendix B

A Kalman Filter Two-Component Sequential Effects Model

The Kalman filter sequential effects model will be parameterized by first- and second-degree decay rates λ_1 and λ_2, $w \in [0,1]$ which is the relative weight of the first-degree component, and α and β which are the transformation parameters that map from the prediction space to actual RT values.[1] Given RT $R(k)$ on trial k, it is convenient to define an observation

$$z^{(k)} = (R(k) - \beta)/\alpha.$$

Let $s(k) \in \{-1, 1\}$ be the stimulus type on trial k and note that $s(k) \cdot s(k-1)$ is 1 for repetitions and -1 for alternations and thus serves as the second-degree encoding. Define the hidden state as

$$\mathbf{x}^{(k)} = \{x_1^{(k)}, x_2^{(k)}, x_3^{(k)}\}^T,$$

with $x_1^{(k)}$ the first-degree bias on trial k, $x_2^{(k)}$ the second-degree bias on trial k, and $x_3^{(k)}$ the the latent state (e.g., attention) on trial k which results in a positive on negative bias on RT. Consider the trial-dependent observation matrix on trial k,

$$H^{(k)} = \begin{bmatrix} w \cdot s(k) & (1-w) \cdot s(k) \cdot s(k-1) & 1 \end{bmatrix}$$

with a first-degree weight of w and a second-degree weight of $1 - w$. According to the Kalman filter model, the observation $z^{(k)}$ is given by the following equation:

$$z^{(k)} = H^{(k)}\mathbf{x}^{(k)} + v^{(k)},$$

[1] Actually, we used three transformation parameters to allow for an alternation bias which is commonly found in these sorts of experiments. However, it will be simpler to explain the model with two transformation parameters.

with $v^{(k)} \sim \mathrm{N}(0, R)$ where in this case R is a scalar variance. The state transition matrix F is given by

$$F = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and serves the purpose of decaying the state estimates. To include the current trial into the state estimate, we use the control-input model B, which is applied to the control variable $\mathbf{u}^{(k)}$:

$$\mathbf{u}^{(k)} = \{s(k), s(k) \cdot s(k-1), 0\}^T$$

$$B = \begin{bmatrix} 1-\lambda_1 & 0 & 0 \\ 0 & 1-\lambda_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

With these variables, we can then express the state update equation as

$$\mathbf{x}^{(k)} = F\mathbf{x}^{(k-1)} + B\mathbf{u}^{(k)} + w_k,$$

with process noise $w_k \sim \mathrm{N}(0, Q)$. Q is chosen to be diagonal with the first two entries equal and the third entry corresponding to variance in the latent (attention) state (the two values that parameterize Q are considered as extra model parameters to be set in the model optimization process).

With the model defined as such, inference in the Kalman filter follows by applying the predict and update phases iteratively over the trials. We begin with a state estimate $\hat{\mathbf{x}}^{(0|0)} = \{0, 0, 0\}^T$ and an estimate, $P^{(0|0)}$, for the error covariance matrix which represents state accuracy given by

$$P^{(0|0)} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}.$$

The predict phase includes updating these two values according to the following equations:

$$\hat{\mathbf{x}}^{(k|k-1)} = F\hat{\mathbf{x}}^{(k-1|k-1)} + B\mathbf{u}^{(k-1)}$$

$$P^{(k|k-1)} = FP^{(k-1|k-1)}F^T + Q.$$

For the update equations, we have

$$\tilde{y}^{(k)} = z^{(k)} - H^{(k)}\hat{\mathbf{x}}^{(k|k-1)}$$

$$S^{(k)} = H^{(k)}P^{(k|k-1)}{H^{(k)}}^T + R$$

$$K^{(k)} = P^{(k|k-1)}{H^{(k)}}^T(1/S^{(k)})$$

$$\hat{\mathbf{x}}^{(k|k)} = \hat{\mathbf{x}}^{(k|k-1)} + K^{(k)}\tilde{y}^{(k)}$$

$$P^{(k|k)} = (I - K^{(k)}H^{(k)})P^{(k|k-1)}.$$

RT for trial k is predicted using the state estimate $\hat{\mathbf{x}}^{(k|k-1)}$ that has integrated past RTs and the trial sequence including the current trial, and is given by

$$\hat{R}(k) = H^{(k)}\hat{\mathbf{x}}^{(k|k-1)}\alpha + \beta.$$

Appendix C

Model Specifications for Long-lasting Sequential Effects

Here we provide a formal description of the models used to assess the depth of sequential effects in Chapter 5. Two simple models are first presented that model sequential effects as an exponential or power function weighting of past trials. This is followed by a recap of the Dynamic Belief Model (DBM) presented in Yu and Cohen (2009) and the formal specification of our generalization, the Hierarchical Dynamic Belief Model (HDBM). Note that the notation used here is slightly different than what is used in Chapter 3, though we tried to maintain some consistency between DBM parameters. The most important change is that ϕ has a different meaning here than it does in the context of the DBM2 model.

C.1 Modeling Details

The models form an expectation for trial t based on a weighting of past trials, $w(\ell)$ for trial $t-\ell$, and yield a quantity ϕ_t reflecting the match between expectation and actual outcome:

$$\phi_t = x_t \sum_{\ell=1}^{\min(t-1,T)} w(\ell) x_{t-\ell},$$

where $w(\ell) = \lambda^\ell$ and $w(\ell) = (1+\ell)^\kappa$ for the exponential and power models, respectively, $x_t \in \{-1, 1\}$ denotes the binary type of trial t (repetition versus alternation for Experiment 4, left versus right for Experiment 1), and T is the context horizon.

To fit data, ϕ_t is converted to a response time (RT) or movement error via an affine transformation. In both experiments, an additive offset was incorporated in the transformation of repetition trials to allow for a default bias towards repetitions or alternations commonly observed in 2AFC studies. Transformation and model parameters were fit to each subject separately to minimize the mean squared error across individual trial predictions for the entire sequence of trials and were constrained to be equal for the two conditions of Experiment 4.

C.2 HDBM Mathematical Specification

The Dynamic Belief Model (DBM) assumes that individuals maintain a distribution over a single environmental statistic, γ_t, that represents the probability of a repetition vs. alternation (Experiment 4) or left vs. right (Experiment 1). The value of γ_t is inferred from the sequence history, $\mathbf{x}_{t-1}$, subject to the constraints of a fixed change probability, α. The expectation match, ϕ_t, is defined to be $P(x_t|\mathbf{x}_{t-1}, \alpha)$ which is given by $E[\gamma_t|\mathbf{x}_{t-1}]$ when x_t is a repetition and $1 - E[\gamma_t|\mathbf{x}_{t-1}]$ when x_t is an alternation. The posterior distribution over γ_t is iteratively updated:

$$p(\gamma_t|\mathbf{x}_{t-1}, \alpha) = (1-\alpha)p(\gamma_{t-1}|\mathbf{x}_{t-1}, \alpha) + \alpha\, p_\gamma, \text{ with}$$

$$p(\gamma_{t-1}|\mathbf{x}_{t-1}, \alpha) \propto P(x_{t-1}|\gamma_{t-1})p(\gamma_{t-1}|\mathbf{x}_{t-2}, \alpha),$$

where p_γ is the standard uniform. (See Yu and Cohen, 2009, for more details).

In the Hierarchical Dynamic Belief Model (HDBM), instead of assuming a fixed change probability α, we define α_t as a time-varying change probability subject to the same dynamics that govern γ_t in the DBM. Specifically, with probability η, called the 'meta change probability', α_t will be redrawn from a Beta resampling distribution, p_α, and with probability $1-\eta$, α_t will remain unchanged. In the HDBM, ϕ_t is defined as $P(x_t|\mathbf{x}_{t-1})$:

$$P(x_t|\mathbf{x}_{t-1}) = \int_0^1 P(x_t|\mathbf{x}_{t-1}, a)p(\alpha_t = a|\mathbf{x}_{t-1})\, da,$$

where $P(x_t|\mathbf{x}_{t-1}, a)$ is the DBM probability for the fixed changepoint a. The posterior distribution over α_t is recomputed iteratively:

$$p(\alpha_t = a|\mathbf{x}_{t-1}) = (1-\eta)p(\alpha_{t-1} = a|\mathbf{x}_{t-1}) + \eta\, p_\alpha(\alpha_t = a), \text{ with}$$

$$p(\alpha_{t-1} = a|\mathbf{x}_{t-1}) \propto P(x_{t-1}|\mathbf{x}_{t-2}, a)p(\alpha_{t-1} = a|\mathbf{x}_{t-2}).$$

The HDBM has three free parameters: the meta-change probability and two parameters for the resampling distribution p_α.

Appendix D

$\underline{\mathrm{BI}}^2$ Derivations and Parameterization

D.1 Deriving Conditional Probabilities in $\underline{\mathrm{BI}}^2$

To perform Gibbs sampling, the standard approach is to clamp the values of all variables but one and then resample the value for the unclamped variable conditioned on all of the other clamped variables. To implement this method, it must be possible to express the conditional distribution for all random variables in a common form that is easy to sample from. Using other Markov Chain Monte Carlo (MCMC) techniques, t is possible to possible to sample from variables that do not have a clean conditional distribution, but this requires much longer run time. Fortunately, by choosing the appropriate prior, in most cases to be conjugate, we have made it relatively straightforward to derive all of the conditional distributions.

D.1.1 Conditional Distribution over β

The problem of specifying the conditional distribution for β given all the other random variables is essentially the problem addressed by Bayesian linear regression (BLR). In most derivations of BLR, the random variable β depends on the variance of the noise. In our problem formulation, we have removed this dependence because it seems unnatural to assume that the weight an individual assigns to past trials is somehow dependent on the random noise they make in responding. The derivation we present here is for one participant, but it applies to all participants and the sampling of the β values for each participant can be performed independently because they are conditionally independent given the prior parameters μ_β and Λ_β.

To simplify the derivation and to map onto the BLR framework, we define Y as the T x 1 vector of responses across the trials for a given participant. X is defined as the T x K matrix with response and impression histories for each trial. T is the total number of trials and $K = 2h$ with h the number of previous trials included in the model. The coefficient for the current impression is fixed to be 1 so this term actually gets wrapped into Y in the implementation. The values of I are implicitly coded into X via the indexing variables S_h and S_t.

The goal is to find the distribution form and parameters for $p(\beta|Y, X, \mu_\beta, \Lambda_\beta, \sigma_R^2)$. By Bayes rule we have:

$$p(\beta|Y, X, \mu_\beta, \Lambda_\beta, \sigma_R^2) \propto p(Y|\beta, X, \mu_\beta, \Lambda_\beta, \sigma_R^2)p(\beta|\mu_\beta, \Lambda_\beta).$$

Inserting the equations for these probabilities (and shortening $p(\beta|Y, X, \mu_\beta, \Lambda_\beta, \sigma_R^2)$ to $p(\beta|*)$), we have the following derivation:

$$p(\beta|*) \propto \exp\left\{-\frac{1}{2\sigma_R^2}(Y - X\beta)^T(Y - X\beta)\right\}\exp\left\{\frac{1}{2\sigma_R^2}(\beta - \mu_\beta)^T\sigma_R^2\Lambda_\beta(\beta - \mu_\beta)\right\}$$

$$p(\beta|*) \propto \exp\left\{-\frac{1}{2\sigma_R^2}\left[(Y - X\beta)^T(Y - X\beta) + (\beta - \mu_\beta)^T\sigma_R^2\Lambda_\beta(\beta - \mu_\beta)\right]\right\}$$

$$p(\beta|*) \propto \exp\left\{-\frac{1}{2\sigma_R^2}\left[\beta^T X^T Y - Y^T X\beta + \beta^T X^T X\beta + \beta^T\sigma_R^2\Lambda_\beta\beta - \beta^T\sigma_R^2\Lambda_\beta\mu_\beta - \mu_\beta^T\sigma_R^2\Lambda_\beta\beta\right]\right\}$$

$$p(\beta|*) \propto \exp\left\{-\frac{1}{2\sigma_R^2}\left[\beta^T(X^T X + \sigma_R^2\Lambda_\beta)\beta - \beta^T(X^T Y + \sigma_R^2\Lambda_\beta\mu_\beta) - (X^T Y + \sigma_R^2\Lambda_\beta\mu_\beta)^T\beta\right]\right\}.$$

Note that when the binomials were expanded, some of the terms disappeared because they are constant with respect to β. In the current from, the terms inside the exponential are close to being the square of a binomial so we will complete the square. First we will make a tricky substitution to clean up the expression and set up the terms that will help us define the final conditional distribution. Let $\tilde{\Lambda}_\beta = X^T X + \sigma_R^2\Lambda_\beta$ and $\tilde{\mu}_\beta = \tilde{\Lambda}_\beta^{-1}(X^T Y + \sigma_R^2\Lambda_\beta\mu_\beta)$. Substituting and completing the square we have

$$p(\beta|*) \propto \exp\left\{-\frac{1}{2\sigma_R^2}\left[\beta^T\tilde{\Lambda}_\beta\beta - \beta^T\tilde{\Lambda}_\beta\tilde{\mu}_\beta - \tilde{\mu}_\beta^T\tilde{\Lambda}_\beta\beta + \tilde{\mu}_\beta^T\tilde{\Lambda}_\beta\tilde{\mu}_\beta - \tilde{\mu}_\beta^T\tilde{\Lambda}_\beta\tilde{\mu}_\beta\right]\right\}$$

$$p(\beta|*) \propto \exp\left\{-\frac{1}{2\sigma_R^2}\left[(\beta - \tilde{\mu}_\beta)^T\tilde{\Lambda}_\beta(\beta - \tilde{\mu}_\beta)\right]\right\}.$$

Finally we recognize this as the kernel for the multivariate normal distribution with mean $\tilde{\mu}_\beta$ and precision matrix $\tilde{\Lambda}_\beta/\sigma_R^2$. Thus we have

$$p(\beta|Y, X, \mu_\beta, \Lambda_\beta, \sigma_R^2) \sim \mathrm{N}(\tilde{\mu}_\beta, \sigma_R^2\tilde{\Lambda}_\beta^{-1}).$$

D.1.2 Conditional Distribution over μ_β and Λ_β

The hyperparameters μ_β and Λ_β are shared across participants so their conditional distribution will depend on the sampled β values for all participants. For J participants, consider $\beta^* = \{\beta_1, \beta_2, \ldots, \beta_J\}$ to be J independent samples from the prior over β. The prior over μ_β and Λ_β is the Normal Wishart (NW) distribution, with parameters η_β, κ_β, ν_β and V_β, which is conjugate to the multivariate normal distribution with unknown mean and precision matrix. From conjugacy, the posterior distribution over Λ given the J samples is as follows:

$$p(\mu_\beta, \Lambda_\beta|\beta^*) \sim \mathrm{NW}\left(\frac{\kappa_\beta\eta_\beta + n\bar{\beta}^*}{\kappa_\beta + n}, \kappa_\beta + n, \nu_\beta + n, \left(V_\beta^{-1} + C + \frac{\kappa_\beta n}{\kappa_\beta + n}D\right)^{-1}\right),$$

with $\bar{\beta}^*$ the sample mean, $C = \sum_{j=1}^{J}(\beta_j - \bar{\beta}^*)(\beta_j - \bar{\beta}^*)^T$, and $D = (\bar{\beta}^* - \mu_\beta)(\bar{\beta}^* - \mu_\beta)^T$.

D.1.3 Conditional Distribution over σ_R^2

Recall that a different value of σ_R^2 is used for each participant. The following update equation applies for each participant using their specific trial history. We define X and Y as in the derivation for β. The prior of σ_R^2 is chosen to be Inverse Gamma with parameters α_R and ω_R and the likelihood of Y is given σ_R^2, X, and β is normal according to the regression equation. The parameters for the posterior are given by:

$$p(\sigma_R^2|Y, X, \beta, \alpha_R, \omega_R) \propto p(Y|\sigma_R^2, X, \beta)p(\sigma_R^2|\alpha_R, \omega_R)$$

$$p(\sigma_R^2|Y, X, \beta, \alpha_R, \omega_R) \propto (\sigma_R^2)^{-n/2}\exp\left\{-\frac{1}{2\sigma_R^2}(Y - X\beta)^T(Y - X\beta)\right\}(\sigma_R^2)^{-\alpha_R - 1}\exp\left\{-\frac{\omega_R}{\sigma_R^2}\right\}$$

$$p(\sigma_R^2|Y, X, \beta, \alpha_R, \omega_R) \propto (\sigma_R^2)^{-(\alpha_R + n/2) - 1}\exp\left\{-\frac{\omega_R + \frac{1}{2}(Y - X\beta)^T(Y - X\beta)}{\sigma_R^2}\right\}$$

$$p(\sigma_R^2|Y, X, \beta, \alpha_R, \omega_R) \sim \text{IG}\left(\alpha_R + n/2, \omega_R + \frac{1}{2}(Y - X\beta)^T(Y - X\beta)\right).$$

Here n is the number of trials predicted for the participant. This is equivalent to treating the regression residuals as normally distributed variables with mean 0 and variance σ_R^2 and using the conjugate prior for a normal random variable with known mean and unknown variance.

D.1.4 Conditional Distribution over elements of I

In this situation, we want to fix the impressions for all items but one, I_i, and find the conditional distribution over I_i. For convenience, we will redefine X and Y here. Let X be a n x 1 matrix that contains the combined weight that was given to I_i on trial t over a total of n trials. In cases where item i was the displayed item on trial t, the corresponding entry in the tth row of X will be 1. If instead on trial t the item i appeared in the past h trials, then the tth row of X will contain the weight assigned to I_i on that trial. When the item was not the current item or in the history, the entry in X is 0. The inference of impressions in the model is improved considerably by the fact that we are setting up a regression equation where the impressions are constrained by not only trials where they appear but also trials where they are in the recent history. Let Y be a modified output with the tth row given by $R_t - \left(I(S_t) + \sum_{j=1}^{h} \beta_j I(S_{t-j}) + \sum_{k=1}^{h} \beta_{h+k} R_{t-k}\right)$, with $I(S_k) = 0$ if S_k is item i (i.e., do not include the terms we are trying to solve for). There are two versions of the model that we will derive in the following sections. However, for either version, we can assume that there is some mean for the prior of the impression for the given participant/item pair which we will denote μ_i (we drop the P for participant because the updates will be done independently for each participant. Also, do not confuse this with the μ_I in the prior of the item contribution in the IRT version of the model). The prior also has fixed variance σ^2 not to be confused with the response variance, σ_R^2, which will also appear in this derivation. The likelihood is given by:

$$p(Y|I_i, X, \sigma_R^2) \propto \exp\left\{-\frac{1}{2\sigma_R^2}(Y - XI_i)^T(Y - XI_i)\right\}.$$

The posterior is given by:

$$p(I_i|Y, X, \sigma_R^2, \mu_i, \sigma^2) \propto p(Y|I_i, X, \sigma_R^2)p(I_i|, \mu_i, \sigma^2)$$

$$p(I_i|Y,X,\sigma_R^2,\mu_i,\sigma^2) \propto \exp\left\{-\frac{1}{2\sigma_R^2}(Y-XI_i)^T(Y-XI_i)\right\}\exp\left\{-\frac{1}{2\sigma^2}(I_i-\mu_i)^2\right\}$$
$$p(I_i|Y,X,\sigma_R^2,\mu_i,\sigma^2) \propto \exp\left\{-\frac{1}{2\sigma_R^2}\left[(Y-XI_i)^T(Y-XI_i)+\frac{\sigma_R^2}{\sigma^2}(I_i-\mu_i)^2\right]\right\}.$$

At this point we recognize that the derivation is taking the same path as the derivation for β which it should since it is analogous. If these terms are multiplied, the substitution is performed, and the square is completed, then the new form of the normal will be recognizable. The substitution is

$$\frac{1}{\tilde{\sigma}^2} = X^TX + \frac{\sigma_R^2}{\sigma^2} \text{ and } \tilde{\mu}_i = \tilde{\sigma}^2(X^TY + \frac{\sigma_R^2}{\sigma^2}\mu_i).$$

The final posterior is given by

$$p(I_i|Y,X,\sigma_R^2,\mu_i,\sigma^2) \sim \mathrm{N}\left(\tilde{\mu}_i, \sigma_R^2\tilde{\sigma}^2\right).$$

D.1.5 Conditional Distribution over σ^2

The derivation here is similar to the one for σ_R^2 except with different residuals. With M items each with a different impression, $I_{P,I}$, with prior mean, $\mu_{P,I}$, for the J participants, the residuals are $I_{P,I} - \mu_{P,I}$ of which there are $M \cdot J$ (we incorporate the P, I subscript here because we are computing across participants). Define I^* to be the set of all $I_{P,I}$ and μ^* the same for $\mu_{P,I}$. With the Inverse Gamma distribution as the conjugate prior with parameters α and ω, the posterior distribution is given by:

$$p(\sigma^2|I^*,\mu^*,\alpha,\omega) \sim \mathrm{IG}\left(\alpha + \frac{MJ}{2}, \omega + \frac{1}{2}\sum_{P=1}^{J}\sum_{I=1}^{M}(I_{P,I}-\mu_{P,I})^2\right).$$

D.1.6 Posterior Distributions for the IRT Impression Priors

The posterior conditional distributions are relatively simple for the IRT model as they are easily derived through conjugacy. Given the impressions for a specific item, m, across all participants, $I_{P^*,m}$, the participant contributions for all participants, C_P^*, and the impression variance, σ^2, we have J observations, $o_m^* = \{(I_{1,m} - C_P^{(1)}), (I_{2,m} - C_P^{(2)}), \ldots, (I_{J,m} - C_P^{(J)})\}$ that are normally distributed with unknown mean $C_I^{(m)}$ and variance σ^2. With the normal prior over $C_I^{(m)}$ defined with mean μ_I and variance σ_I^2, the posterior distribution for $C_I^{(m)}$ is easily obtained through conjugacy:

$$p(C_I^{(m)}|o_m^*, \sigma^2, \mu_I, \sigma_I^2) \sim \mathrm{N}\left(\tilde{\sigma}_I^2\left[\frac{\mu_I}{\sigma_I^2} + \frac{O_m}{\sigma^2}\right], \tilde{\sigma}_I^2\right)$$

with $O_m = \sum_{j=1}^{J} o_m^{(j)}$ and $\tilde{\sigma}_I^2 = \left(\frac{1}{\sigma_I^2} + \frac{J}{\sigma^2}\right)^{-1}$. This distribution is used to sample each item contribution.

The distribution for contribution for participant j, $C_P^{(j)}$, is obtained analogously:

$$p(C_P^{(j)}|o_j^*, \sigma^2, \mu_P, \sigma_P^2) \sim \mathrm{N}\left(\tilde{\sigma}_P^2\left[\frac{\mu_P}{\sigma_P^2} + \frac{O_j}{\sigma^2}\right], \tilde{\sigma}_P^2\right)$$

with $o_j^* = \{(I_{j,1} - C_I^{(1)}), (I_{j,2} - C_I^{(2)}), \ldots, (I_{j,M} - C_I^{(M)})\}$, $O_j = \sum_{m=1}^{M} o_j^{(m)}$, and $\tilde{\sigma}_P^2 = \left(\frac{1}{\sigma_P^2} + \frac{M}{\sigma^2}\right)^{-1}$.

The normal prior over C_I given C_I^* and the parameters for the Normal Inverse-Gamma (NIG) hyperprior is updated through conjugacy:

$$p(\mu_I, \sigma_I^2|C_I^*, \eta_C, \kappa_C, \alpha_C, \omega_C) \sim \mathrm{NIG}\left(\frac{\kappa_C\eta_C + J\bar{C}_I^*}{\kappa_C + J}, \kappa_C + J, \alpha_C + \frac{J}{2}, \omega_C + L_I + D_I\right)$$

with sample mean $\bar{C}_I^*$, $L_I = \frac{1}{2}\sum_{m=1}^{M}(C_I^{(m)} - \bar{C}_I^*)^2$, and $D_I = \frac{\kappa_C J}{\kappa_C + J}\frac{(\bar{C}_I^* - \eta_C)^2}{2}$. Similarly, for the prior over C_P the distribution is:

$$p(\mu_P, \sigma_P^2|C_P^*, \eta_C, \kappa_C, \alpha_C, \omega_C) \sim \mathrm{NIG}\left(\frac{\kappa_C\eta_C + M\bar{C}_P^*}{\kappa_C + M}, \kappa_C + M, \alpha_C + \frac{M}{2}, \omega_C + L_P + D_P\right)$$

with sample mean $\bar{C}_P^*$, $L_P = \frac{1}{2}\sum_{j=1}^{J}(C_P^{(j)} - \bar{C}_P^*)^2$, and $D_P = \frac{\kappa_C M}{\kappa_C + M}\frac{(\bar{C}_P^* - \eta_C)^2}{2}$.

D.1.7 Posterior Distributions for the MDS Impression Priors

In the MDS version of the model, the key to determining the posterior distributions is to realize that the item factors, F, and the participant weights, W, are analogous to the β and I terms. When one of the two is known, solving for the other becomes a Bayesian linear regression problem. Let X_m be a J x d matrix with each row corresponding to the weights for a specific participant and let Y_m be a J x 1 vector containing the impressions given for item m across all participant. The posterior distribution over the factors for item m, $F^{(m)}$, is given by:

$$p(F^{(m)}|I_{P^*,m}, W, \sigma^2, \mu_F, \Lambda_F) \sim \mathrm{N}\left(\tilde{\mu}_F, \sigma_2\tilde{\Lambda}_F^{-1}\right),$$

with $\tilde{\Lambda}_F = X_m^T X_m + \sigma^2 \Lambda_F$ and $\tilde{\mu}_F = \tilde{\Lambda}_F^{-1}(X_m^T Y_m + \sigma_2 \Lambda_F \mu_F^T)$.

Similarly, for the posterior distribution over the weights for participant j, $W^{(j)}$, we define X_j to be an M x d matrix with each row corresponding to the factors for a specific item and Y_j to be an M x 1 vector containing the impressions given for participant j across all items. The posterior is then:

$$p(W^{(j)}|I_{P^*,j}, F, \sigma^2, \mu_W, \Lambda_W) \sim \mathrm{N}\left(\tilde{\mu}_W, \sigma_2 \tilde{\Lambda}_W^{-1}\right),$$

with $\tilde{\Lambda}_W = X_j^T X_j + \sigma^2 \Lambda_W$ and $\tilde{\mu}_W = \tilde{\Lambda}_W^{-1}(X_j^T Y_j + \sigma_2 \Lambda_W \mu_W^T)$.

Finally the posterior for the priors over F and W are obtained in the same way as for the priors for C_I and C_P except that each prior has its own Normal Wishart hyperprior. The posterior for the prior over F is given by:

$$p(\mu_F, \Lambda_F|F^*) \sim \mathrm{NW}\left(\frac{\kappa_F \eta_F + J\bar{F}^*}{\kappa_F + J}, \kappa_F + J, \nu_F + J, \left(V_F^{-1} + C_F + \frac{\kappa_F J}{\kappa_F + J} D_F\right)^{-1}\right),$$

with $\bar{F}^*$ the sample mean, $C_F = \sum_{j=1}^{J}(F_j - \bar{F}^*)(F_j - \bar{F}^*)^T$, and $D_F = (\bar{F}^* - \mu_F)(\bar{F}^* - \mu_F)^T$. The posterior for the prior over W is given by:

$$p(\mu_W, \Lambda_W|W^*) \sim \mathrm{NW}\left(\frac{\kappa_W \eta_W + M\bar{W}^*}{\kappa_W + M}, \kappa_W + M, \nu_W + M, \left(V_W^{-1} + C_W + \frac{\kappa_W M}{\kappa_W + M} D_W\right)^{-1}\right),$$

with $\bar{W}^*$ the sample mean, $C_W = \sum_{m=1}^{M}(W_m - \bar{W}^*)(W_m - \bar{W}^*)^T$, and $D_W = (\bar{W}^* - \mu_W)(\bar{W}^* - \mu_W)^T$.

D.2 Setting the Hyperparameters

Though there are many hyperparameters to the model, we set their values to produce general priors that have little influence on the model fit. In what follows, we describe the meaning of the parameters and the values used for them in our simulations.

For the parameters that affect the means of the hyperpriors (η_β, κ_β, and η_C, and κ_C for IRT or η_F, κ_F, η_W, and κ_W for MDS) the means, η values, were chosen to be 0 for the weights and factors, and 2.75 for each contributing component in the IRT model. Furthermore, these means were weighted by 0.0001 which essentially results in the prior being overwhelmed by the data in the posterior mean estimate (i.e., the κ values were 0.0001). For the regression variance, σ_R^2, the prior was chosen to have a shape of 2 and a mean of 1 (α_R is the shape and ω_R is related to the mean). For the prior over variance in impressions, α and ω, the shape is 2 and the mean is 2. The mean is slightly larger than for response noise because this represents variability across the range of impressions as opposed to pure noise (i.e., it allows for more of the range to be used, and it allows MDS and IRT to not be that accurate). This provides a reasonable density over the range of most likely errors (i.e., on average, the noise will be no more than +/- 1 rating 68% of the time). These values are also used for convenience with the hyperpriors over the item and participant contribution priors in the IRT model (α_C and ω_C). For the MDS model, the covariance of the factor prior, V_F, was chosen to have the identity as its mean and the covariance of the factor weights prior, V_W, was chosen to have a mean with 0.01 along the diagonal and zeros elsewhere. However, these two values were given the minimum weight possible ($\nu_F = \nu_W = d$, the dimension of the factor space) so that the prior information is overwhelmed by the data in the posterior because the number of observations is much greater than d.

The only parameter choice that seems to have a meaningful affect on the model performance is the precision matrix, V_β, and the count associated with it, ν_β, for the Normal Wishart prior over the prior for the regression weights. V_β was chosen such that the covariance matrix for the prior over the regression weights, Λ_β, was relatively small in order to enforce the constraint that weights are similar across participants. V_β was set such that its inverse, a covariance matrix, had 0.001 along the diagonal and negative correlations between response and impression term corresponding to the same trial in the past (this was something we observed in the weights and it is logical because the two terms should balance each other). The count, ν_β, was chosen to be as small as possible so that the data could still override the influence of V_β.