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Network Externalities and Market Dominance*

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Abstract

We propose a duopoly model for “new economy” markets with an S-shaped Beckerian demand curve—which we micro-found and refine. Because of network externalities, firms compete not for a marginal consumer but for a substantial block of consumers all at once. This leads to a type of limit pricing—from within rather than from outside the market—where the losing firm captures a positive market share even when it does not supplant its rival, and where competition is modulated by consumer beliefs. We characterize how technologies and “consumer impulses” (e.g. past sales or defaults) affect competition, and derive policy recommendations.

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1 Introduction

In his classic analysis of monopolistic restaurant pricing, Becker (1991) introduced a novel way to think about network externalities: an S-shaped inverse demand curve that first falls, then rises, and then falls again, with different portions accessible to the monopoly depending on its “popularity”—as determined for instance by past sales and marketing efforts. While somewhat informal, Becker’s analysis offered various insights about the opportunities and challenges facing the monopolist, including a need to manage consumer expectations about each other’s behavior and a need for even the most successful firms to stay on their feet to avoid a sudden reversal of popularity.

In this paper we formalize and extend Becker’s approach to obtain insights on duopoly competition in “new-economy markets” where network externalities are large. These markets have been of great interest to both academics and practitioners as many of today’s largest firms, including the five largest publicly-traded companies (Apple, Amazon, Google/Alphabet, Microsoft, and Facebook), operate in these markets, and the externalities may have nontrivial effects.

So that the model is on firm ground, we micro-found the Beckerian demand curve and use an equilibrium-refinement concept that builds upon notions of platform focality (Caillaud and Jullien (2003) and Jullien (2011)) and k-level reasoning (see Stahl and Wilson (1994), Nagel (1995), Ho et al. (1998), Crawford (2003), Crawford and Iriberry (2007), and especially the introspective equilibrium concept of Kets and Sandroni (2021)). This refinement, which applies Kets and Sandroni (2021) to the S-shaped demand curve, and which we term “introspective equilibrium for network externalities,” allows us to formally study the consequences of a firm’s popularity. Because of network externalities, firms may compete not for the *marginal consumer*, but for the *market itself* (i.e. for a large block of consumers). When this occurs, a specific type of limit pricing arises—from within rather than from outside the market—where the losing firm captures a positive market share (a “consolation prize”) even when it does not supplant its rival, and where competition is modulated by consumer beliefs.

The introspective focality concept allows consumer “impulses” to affect their be-

havior. In our setting, impulses capture consumers’ initial inclination to buy from a given firm—driven for instance by a default, an ad campaign, or a firms’ past sales—and consumers use them to form increasingly accurate conjectures about the behavior of peers until these conjectures converge to what they (correctly) believe will be the actual level of total demand. This notion of focality is both tractable and flexible enough to accommodate heterogeneous consumers.¹

Our main comparative statics concern a scenario where competition is sufficiently intense that the dominant firm must set a price just low enough not to surrender the market. In this scenario, only the dominant firm is interested in investing to raise its quality or lower its costs (e.g. by purchasing startups) and yet such improvements have no impact on consumer surplus. This opens the door for government action—such as monitoring the acquisition of startups or mandating technology sharing—to avoid further entrenchment of the dominant firm and to ensure that consumers benefit from technological advances. We also show that a change in consumers’ impulses in favor of the dominant firm leads to higher prices, lower consumer surplus, and lower total surplus, which may also justify regulatory action. Finally, we show that an increase in network externalities has both pro-competitive and anti-competitive effects, and thus an ambiguous overall effect on prices and consumer surplus.

Related Literature. Our paper contributes to the large literature on network externalities spawned by the classic models of Katz and Shapiro (1985) and Becker (1991). A major theme in this literature, which was only superficially explored in those classic models, and where our contribution lies, is how consumer beliefs affect equilibrium outcomes.

The more systematic study of beliefs, starting with Caillaud and Jullien (2003), has centered around the idea of “focality” (the degree to which consumers are biased in favor of a given firm) and has explored the extent to which this bias grants market power and allows inefficient incumbents to survive. Jullien (2011) uses notions of focality to study the value of divide-and-conquer strategies in multi-sided markets; Halaburda and Yehezkel (2019) consider a dynamic model of platform competition where a firm’s current focality—modeled there as all or nothing—depends on past

¹In the special case where consumers are homogeneous, this refinement coincides with that of Halaburda et al. (2020).

sales; Halaburda et al. (2020) extend this framework to the case where focality can take intermediate values and thus firms can enjoy a partial belief advantage; and Markovich and Yehezkel (2021) explore how a firm can rely on a user group to cement its dominance.² Our innovation relative to this work is that we introduce a notion of focality that is flexible enough to accommodate a heterogeneous set of consumers, rather than a single consumer type. This leads to richer market structures than “winner-take-all”. Indeed, in our model, the losing firm captures a positive market share even when it does not overtake its rival.³

Biglaiser and Crémer (2020), like us, allow for more than one consumer type (in their case, two types), and thus are also able to study competition from *within* the market, where the losing firm captures a positive market share. Rather than a belief-based approach to equilibrium selection, they consider a platform migration protocol based on the idea that consumers only switch platform if it is in their individual interest to do so given any migration that has already occurred. Our belief-based refinement follows a similar logic, though rather than observing the reactions of others before making their decisions, consumers base their decisions on conjectures alone. This allows us to model such influences as advertising and defaults. Our model also differs in that it allows for a richer (continuous) set of consumers and for quality differences across competing firms, as needed for our policy implications.

Armstrong (2006) allows for heterogeneous consumers as well. There, however, beliefs play no role because an assumption of sufficient consumer heterogeneity relative to network externalities—together with a Hotelling specification with a uniform density of types—implies a single equilibrium for any given set of prices. Competition is therefore for the marginal consumer only. Our methodology offers a possible path toward combining Armstrong-style models with models in the style of Caillaud and Jullien (2003), which do allow for multiplicity, but with a single consumer type.

Also related is the literature on switching costs, beginning with Von Weizsäcker (1984) and Klemperer (1987) (see also Klemperer (1995), Fudenberg and Tirole

²Ambrus and Argenziano (2009) instead use coalitional rationalizability to select equilibria. They restrict the amount of coordination failure across consumers and show that multiple asymmetric networks can exist in equilibrium.

³Argenziano and Gilboa (2012) consider dynamics in an abstract model without firms or pricing decisions.

(2000), and Farrell and Klemperer (2007)). In our model, there is an incumbency advantage despite the absence of switching costs. Moreover, transitions can occur very quickly, may involve quantity overshooting, and are triggered by changes in impulses as well as prices.⁴

Our paper relates as well to a growing applied literature on the new economy. For instance, Gompers and Lerner (1999) show that a sizeable share of R&D investment is done by new-economy startups. Gans et al. (2002) study incumbents’ expropriation of startups’ intellectual capital and its impact on incentives to innovate. There is also an emerging debate in law and economics on appropriate anti-trust policy in the new economy. The so-called “New Brandeisian Movement” (see Khan (2017)) argues that there is too much focus on short-run consumer welfare, which misses the possibility that a firm may raise prices after building up a network (for the classic welfarist approach, see Kovacic and Shapiro (2000)).⁵

2 Model

Consider a market composed of either a single seller (firm 1) or two sellers (firms 1 and 2), each with constant (perhaps zero) marginal costs, and a unit mass of consumers, each having unit demand, with types $z \in [\underline{z}, \bar{z}]$ distributed according to c.d.f. F and density f . N denotes the number of firms.

When there is a single seller, consumer z ’s demand is $D_z(p_1, Q_1) \in \{0, 1\}$, where p_1 is price and Q_1 total sales. Network externalities are reflected in the assumption that demand depends upon total sales. Aggregate demand at price p_1 and quantity Q_1 is:

$$D(p_1, Q_1) := \int_z D_z(p_1, Q_1) dF(z).$$

⁴A broader literature on platforms, initiated by Rochet and Tirole (2003), analyzes markets that are multi-sided and involve externalities within and between sides (e.g. Rochet and Tirole (2006), Armstrong (2006), and Weyl (2010)).

⁵See Edelman (2015, June 21, 2017) and Edelman and Geradin (2016) for examples of nefarious practices used to harness network externalities.

For any given price p_1 , an equilibrium quantity Q_1^D satisfies:

$$Q_1^D = D(p_1, Q_1^D). \quad (1)$$

This equation may in principle admit more than one solution as the presence of network effects may lead consumers to coordinate on a higher or lower level of demand. We will shortly address this equilibrium multiplicity by introducing a simple equilibrium refinement.

When instead there are two sellers, assuming consumers prefer to consume from one of the firms rather than not at all, consumer z 's demand for firm 1 is $D_z(p_1 - p_2, Q_1) \in \{0, 1\}$, where $p_1 - p_2$ is the difference in firms' prices, and consumer z 's demand for firm 2 is $1 - D_z(p_1 - p_2, Q_1)$.⁶ Aggregate demand for firm 1 at prices p_1 , p_2 and quantity Q_1 is:

$$D(p_1 - p_2, Q_1) := \int_z D_z(p_1 - p_2, Q_1) dF(z),$$

and aggregate demand for firm 2 is $1 - D(p_1 - p_2, Q_1)$. The equilibrium quantities Q_1^D and Q_2^D satisfy:

$$Q_1^D = D(p_1 - p_2, Q_1) = 1 - Q_2^D. \quad (2)$$

This equation may admit multiple solutions as well.

We adopt the convention that $p_2 = Q_2 = 0$ when there is only one firm and assume throughout that $D(p_1 - p_2, Q_1)$ is continuous, decreasing in $p_1 - p_2$, and strictly increasing in Q_1 whenever $Q_1 < 1$. For convenience, let $p := p_1 - p_2$ denote firm 1's relative price.

The *inverse demand curve* (or more compactly, *demand curve*) for firm 1, denoted $P(Q_1)$, satisfies for all Q_1 :

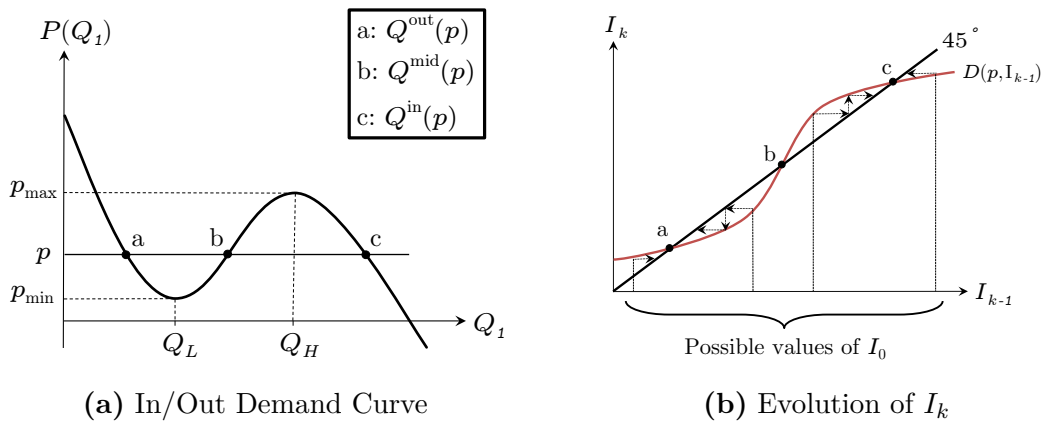
$$Q_1 = D(P(Q_1), Q_1), \quad (3)$$

where $P(Q_1)$ measures the relative price p needed for firm 1's demand to equal Q_1 , provided such price exists.

⁶We shall assume throughout that the consumers' outside options are immaterial for their decisions. In our micro-foundation below, a sufficient condition for this is that the intrinsic quality of the firms' products is sufficiently high.

In/Out Demand. In order to capture markets where firms seek to attract large swaths of consumers *en masse*, and where more than one firm is active at once, we shall assume that the demand curve $P(Q_1)$ has an “In/Out” shape as shown in Figure 1(a).⁷ This shape was first suggested by Becker (1991) in his classic (monopolistic) restaurant model. Loosely speaking, when overall consumption is low, the network effect is weak and demand has standard negative slope; when consumption exceeds a first threshold, Q_L , the network externality becomes sufficiently strong that marginal value grows with consumption; and when total consumption exceeds a second threshold, Q_H , the externality is mostly exhausted and demand again has negative slope. When there are two firms, firm 2’s inverse demand curve (i.e. $p_2 - p_1$ plotted against Q_2) is In/Out whenever firm 1’s inverse demand has that shape too.

Figure 1



While the In/Out shape may in principle seem arbitrary, it is actually easy to micro-found (something Becker (1991) did not address). Two possibilities follow:

Micro-foundation #1: Suppose consumer z buys firm 1’s product if and only if

$$\mu_1 - \mu_2 + z + \alpha(Q_1 - Q_2) \geq p_1 - p_2, \quad (4)$$

where μ_i is the *intrinsic quality* of firm i ’s product, z is a horizontal preference toward firm 1, and the parameter $\alpha > 0$ measures the strength of the network effect. When

⁷That is, between 0 and Q_L , P is strictly decreasing and weakly convex; between Q_L and Q_H , P is strictly increasing; between Q_H and 1, P is strictly decreasing and weakly concave; finally, $p_{\max} < P(0)$ and $p_{\min} > P(1)$.

there is only one firm, $\mu_2 = Q_2 = 0$. Under this formulation, demand is In/Out whenever:

1. f is single-peaked—that is, maximal at some intermediate value of z and strictly monotone elsewhere.
2. The network externality is large—specifically, $\frac{1}{N\alpha}$ is below the peak of f .
3. The tails of f are thin—specifically, $\frac{1}{N\alpha}$ is above both $f(\underline{z})$ and $f(\bar{z})$.

Intuitively, these conditions capture a scenario where most consumers are “non-partisan“ in the sense that they have only a weak intrinsic preference for one firm over the other, but partisan consumers also exist. The initial negative slope arises because the density of types is at first relatively low, and so even as a price drop attracts additional consumers, these are not sufficiently numerous to produce, at the margin, a sufficiently strong network effect. Then, as sales continue to grow and we approach the denser middle part of the type distribution, those new consumers are sufficiently numerous that their network effect exceeds the negative price effect, and hence the slope turns positive. Finally, the slope turns negative once the density of types again falls.⁸

Micro-foundation #2: A similar micro-foundation is possible for the case of two firms if we assume that consumer z buys firm 1’s product if and only if

$$\mu_1 - \mu_2 + z(Q_1 - Q_2) \geq p_1 - p_2, \tag{5}$$

where $z \geq 0$ now represents the extent to which a consumer cares about the network effect. In this case, demand is guaranteed to be In/Out whenever f is symmetric, single-peaked, and its mass is sufficiently concentrated around its peak. The intuition is similar to that of the first micro-foundation: in regions of the type space where

⁸More formally, letting \hat{z} denote the marginal type who is indifferent between the two firms (or when there is only one firm, indifferent between firm 1 and not consuming), we obtain $1 - Q_1 = F(\hat{z}) = F(-(\mu_1 - \mu_2) - \alpha(Q_1 - Q_2) + (p_1 - p_2))$. Upon rearranging terms, $P(Q_1) = (\mu_1 - \mu_2) + \alpha(Q_1 - Q_2) + F^{-1}(1 - Q_1)$. Given that $Q_1 - Q_2 = 2Q_1 - 1$ when there are two firms and $Q_1 - Q_2 = Q_1$ when there is a single firm, we find that $P'(Q_1) = N\alpha - \frac{1}{f(F^{-1}(1 - Q_1))}$, from which the claim immediately follows.

consumers are sparse, the negative price effect dominates the positive network effect; in regions where consumers are dense, the opposite happens. The difference is that as more consumers join a firm, the marginal consumer likes the firm’s product more both because the network becomes larger and because their own marginal value for the network is greater than that of their infra-marginal peers.⁹

In what follows we shall rely extensively on the first micro-foundation because it allows us to easily manipulate the shape of the demand curve (by changing the distribution f of consumers) and brings transparency to the analysis. Similar results would arise in the more general case where demand takes the form $D(p - \mu, \alpha Q)$ provided this function is increasing in both arguments and the resulting inverse demand curve is In/Out with a sufficiently pronounced increasing portion (i.e. both $Q_H - Q_L$ and $p_{\max} - p_{\min}$ are sufficiently large).

2.1 Multiple Equilibria and Equilibrium Selection

As illustrated in Figure 1(a), if firm 1’s relative price p is strictly between p_{\min} and p_{\max} , demand intersects price three times; hence there are three equilibria in the continuation game where consumers choose quantity: $Q^{out}(p)$, $Q^{mid}(p)$, and $Q^{in}(p)$, with firm 2 dominant in the first one and firm 1 dominant in the third. Similarly, if p is equal to either p_{\min} or p_{\max} , there are two equilibria, with one firm dominant in each one.

We shall address this multiplicity using a refinement concept based upon “level- k reasoning” (see Crawford et al. (2013) for a survey). Specifically, we apply Kets and Sandroni (2021)’s “introspective equilibrium” to the In/Out demand curve: each player (in our case, each consumer) starts with an exogenously-given “impulse” that determines how they react at level 0; then, at each level $k > 0$, consumers form a best response to the belief that other consumers are acting at level $k - 1$. An introspective equilibrium is the limit of this reasoning as $k \rightarrow \infty$.

We formally define an introspective equilibrium for a market with network externalities as follows:

⁹For a formal proof see the appendix.

Definition 1 (Introspective Equilibrium for Network Externalities).

Let $I_0 \in [0, 1]$ denote the consumers' impulse to consume from firm 1. An introspective equilibrium, denoted Q_1^* , is constructed as follows:

1. Consumption at level k , denoted I_k , is obtained by letting each consumer best-respond to the relative price p and to the belief that other consumers are acting at level $k - 1$:

$$I_k := D(p, I_{k-1}).$$

2. An introspective equilibrium is the limit as $k \rightarrow \infty$:

$$Q_1^* := \lim_{k \rightarrow \infty} I_k.$$

The impulse may be understood to arise from a combination of factors, not explicitly modeled, such as:

- Advertising.
- The use of defaults or nudges—e.g. a specific search engine being the default on a smartphone.
- A firm's past sales (or even its past success in related markets) and more generally its reputation.¹⁰
- The actions of “influencers,” broadly defined as economic agents with the power to change expectations.¹¹

Proposition 1 shows how the equilibrium quantity depends on the impulse.

¹⁰Even though our model is static, it has implications for the steady state of a dynamic environment in which the dominant firm does not change over time.

¹¹Large consumers (or blocks of small consumers) who happen to move first, as for instance in Akerlof and Holden (2019) and Markovich and Yehezkel (2021), may have a similar impact. See also Corsetti et al. (2004) for an analysis of the impact of large players in coordination games within a global-games environment.

Proposition 1. *Suppose demand is In/Out. When $p_{\min} \leq p \leq p_{\max}$, the introspective equilibrium depends upon the impulse I_0 as follows:*

$$Q_1^*(p) = \begin{cases} Q^{in}(p), & \text{if } I_0 > Q^{mid}(p). \\ Q^{mid}(p), & \text{if } I_0 = Q^{mid}(p). \\ Q^{out}(p), & \text{if } I_0 < Q^{mid}(p). \end{cases}$$

When instead $p > p_{\max}$ or $p < p_{\min}$, the introspective equilibrium corresponds to the unique equilibrium quantity regardless of impulse.

To establish this result, we begin with the case where p takes an intermediate value ($p_{\min} \leq p \leq p_{\max}$). Figure 1(b) shows level- k consumption $I_k := D(p, I_{k-1})$ as a function of I_{k-1} given p , together with the evolution of I_k for any given impulse I_0 .¹² Observe that whenever I_0 is strictly greater (respectively smaller) than $Q^{mid}(p)$, I_k converges to $Q^{in}(p)$ (respectively $Q^{out}(p)$) as k goes to infinity; moreover, if $I_0 = Q^{mid}(p)$, $I_k = I_0$ for all k and hence I_k converges to $Q^{mid}(p)$, as desired.

Now, suppose $p > p_{\max}$ or $p < p_{\min}$. In this case, I_k intersects the 45° line only once (as $P(I_{k-1})$ intersects p only once); this point of intersection corresponds to the unique Nash equilibrium. It is easy to see that, regardless of whether I_0 is above or below the point of intersection, I_k converges to the point of intersection in the limit. Hence, the introspective equilibrium is the same as the Nash equilibrium. **Q.E.D.**

For a simple intuition, observe that so long as $P(Q_1)$ exceeds p (i.e. relative willingness to pay firm 1 exceeds relative price) additional consumers will join firm 1; the opposite happens when $P(Q_1)$ is below p .

Corollary 1. *Suppose demand is In/Out. Upon applying the introspective equilibrium refinement, firm 1 faces one of three demand curves:*

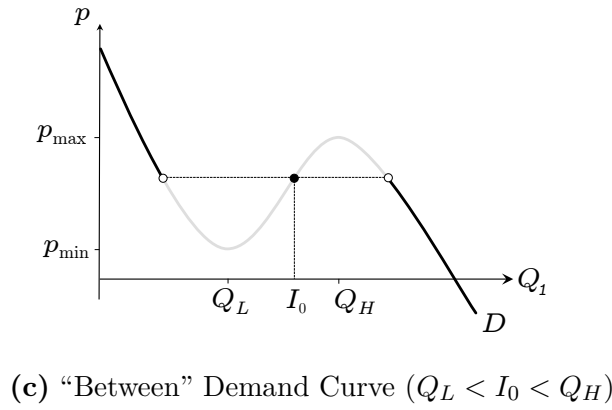
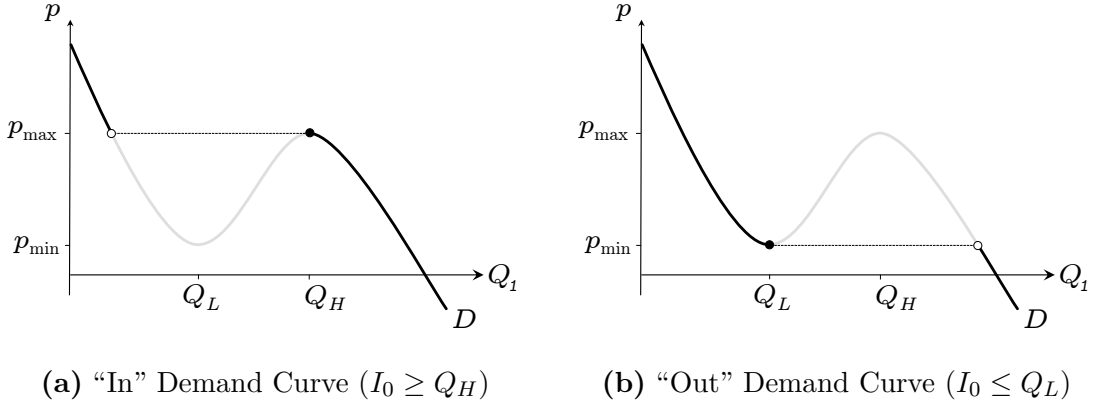
1. The “In” demand curve when $I_0 \geq Q_H$ (Figure 2a).

¹²To see why $D(p, I_{k-1})$ has the shape shown in the figure, observe that $I_{k-1} = D(P(I_{k-1}), I_{k-1})$ (per the definition of the demand curve $P(\cdot)$). Therefore, whenever $P(I_{k-1}) = p$, which occurs at the three Nash equilibria $Q^{out}(p)$, $Q^{mid}(p)$, and $Q^{in}(p)$ in the first panel, $D(p, I_{k-1})$ is equal to I_{k-1} , i.e. it intersects the 45° line. Moreover, since $D(\cdot, I_{k-1})$ is a decreasing function, whenever $P(I_{k-1})$ is above p , which occurs between $Q^{mid}(p)$ and $Q^{in}(p)$, $D(p, I_{k-1})$ is greater than I_{k-1} , i.e. is above the 45° line. Finally, whenever $P(I_{k-1})$ is below p , $D(p, I_{k-1})$ is lower than I_{k-1} . (Since $D(p, I_{k-1})$ is continuous in I_{k-1} , so is I_k .)

2. The “Out” demand curve when $I_0 \leq Q_L$ (Figure 2b).
3. The “Between” demand curve otherwise (Figure 2c).

Each of these curves has a weakly negative slope.

Figure 2



That this result follows from Proposition 1 can be seen from the fact that, in the event of three equilibria (i.e. $p_{min} < p < p_{max}$), an impulse of at least Q_H guarantees that the impulse exceeds $Q^{mid}(p)$, and hence firm 1 ends “in”; similarly, an impulse no greater than Q_L guarantees that the impulse is below $Q^{mid}(p)$, and therefore firm 1 ends “out.” In the event of two equilibria ($p = p_{min}$ or $p = p_{max}$), a similar reasoning applies.

When the impulse is such that firm 1 faces an in (resp. out) demand curve we shall say it is “in” (resp. “out”). Otherwise, we shall say that firm 1 is “between.”

Notice that when there are two firms, firm 2 faces an out demand curve when firm 1 faces an in demand curve (and vice-versa).¹³

3 Monopoly

The monopoly case serves as a simple benchmark for understanding the impact of the impulse:

- Since demand is weakly increasing in the impulse, so are the monopolist's profits.
- Because demand is discontinuous over the region spanning Q_L to Q_H , the monopolist never selects a quantity in this region: it either sells a “low” quantity ($Q_1 \leq Q_L$) or a “high” quantity ($Q_1 \geq Q_H$). Moreover, since an increase in the impulse extends the “high” portion of the demand curve and truncates the “low” portion, the monopolist is increasingly inclined to sell a high quantity as the impulse increases.
- Since a higher impulse allows an “in” firm to raise its price, but may also tempt an “out” or “between” firm to go for high sales by lowering price, its impact on price is ex-ante ambiguous.

4 Duopoly

We now bring a competitor into the model. To ensure that a pure strategy equilibrium in the pricing game exists, we assume a sequential Bertrand-Stackelberg timing where firm 1 is the price leader (sets price first) and firm 2 is the follower.¹⁴ For ease of exposition, we assume that $Q_H = 1 - Q_L$, which ensures that both firms' demand

¹³Observe that the discontinuities in the “out” and “mid” curves may imply that an optimal price within the high-quantity region (above Q_H) does not exist. To overcome this problem one may assume prices lie on a finite, but very fine, grid.

¹⁴If instead pricing was simultaneous, competition for the dominant market position would generate a type of “all-pay contest” that may not admit pure strategy outcomes.

curves are upward-sloping between Q_L and Q_H . This assumption holds under both micro-foundations when the type distribution f is symmetric.

The presence of network externalities means that firms may end up competing “for the market” (i.e., for a large block of consumers at once) rather than for a marginal consumer alone. When this occurs, the model gives rise to a form of limit pricing—from within rather than from outside the market—where the losing firm captures a positive market share (a “consolation prize”) even when it does not supplant its rival, and where competition is modulated by consumer beliefs.

The following lemma describes the limit-pricing aspect of the model.

Lemma 1. *There exist a threshold, which we call p^{win} , such that firm 1 wins the market (i.e. serves at least Q_H consumers) if $p_1 \leq p^{win}$; otherwise, firm 2 wins the market.*

To derive this result, fix p_1 and let $\Pi_2^L(p_1)$ denote firm 2’s maximum profit conditional on selling less than Q_L ; similarly, let $\Pi_2^H(p_1)$ denote firm 2’s maximum profit conditional on selling more than Q_H .¹⁵ Since $\Pi_2^H(p_1)$ exceeds $\Pi_2^L(p_1)$ when p_1 is very high, and vice versa when p_1 is very low, it suffices to show that as p_1 grows, $\Pi_2^L(p_1)$ grows less than $\Pi_2^H(p_1)$. To this end, observe that as p_1 grows, $\Pi_2^H(p_1)$ increases at a rate no less than Q_H (as an in firm serves at least Q_H consumers and firm 1 can choose to raise its in price one-to-one in response to a higher p_1 while keeping its sales unchanged), and $\Pi_2^L(p_1)$ increases at a rate no greater than Q_L (as firm 2 can at most capture the added marginal willingness to pay of all Q_L consumers). **Q.E.D.**

We shall call the inequality $p_1 \leq p^{win}$ the *win-the-market constraint* (or WIN for short). A simple intuition for this result is that a higher p_1 shifts firm 2’s demand vertically, which means firm 2 is more likely to expand; moreover, constant marginal costs imply that firm 2 will never choose to sell an intermediate quantity between Q_L and Q_H (as would be possible if it faced a “between” demand curve).

Observe that when firm 1 wins the market with a slack WIN constraint (p_1 strictly less than p^{win}), competition between the firms is over a marginal consumer as in a standard Hotelling-style model. By contrast, when firm 1 wins the market with a

¹⁵Because marginal costs are constant, firm 2 will never select the “mid” position.

binding WIN constraint ($p_1 = p^{win}$), competition is “over the market:” firms fight for the dominant market position.

To obtain further results, we impose a regularity condition on the “low” portion of the demand curve (i.e. the portion to the left of Q_L).¹⁶

Assumption 1. *The optimal price and quantity for firm 2 conditional on selling no more than Q_L are both increasing in p_1 .*

The following result gives conditions under which firm 1 wins the market while charging exactly p^{win} .

Proposition 2. *Suppose Assumption 1 holds and firms face an In/Out demand curve that admits the first micro-foundation. Then:*

1. *For any given impulse, firm 1 wins the market (i.e. sells at least Q_H) if and only if the difference in the firms’ intrinsic quality $\mu_1 - \mu_2$ is above a threshold, which itself is weakly decreasing in the impulse I_0 .*
2. *Provided firm 1 wins the market, $p_1 = p^{win}$ whenever the type distribution f has sufficient mass concentrated around its peak.*

Part 1 follows from the fact a higher relative quality ($\mu_1 - \mu_2$) raises the minimum price differential needed for firm 1 to win the market, and thus makes it more attractive for this firm to do so; a higher impulse helps firm 1 for a similar reason.

For part 2, observe that when the mass of f is highly concentrated around its peak—i.e. there is little taste differentiation amongst the bulk of consumers—the vast majority of them will end up with firm 1 when it wins. Hence, if this firm were to lower its price below p^{win} , the minimum needed to win, it would face a large infra-marginal loss (over at least Q_H consumers) while attracting very few additional consumers, as there are very few left.¹⁷ **Q.E.D.**

¹⁶This assumption holds, for instance, if the relevant portion of the demand curve is sufficiently close to linear.

¹⁷Owing to Assumption 1, an ϵ reduction in price causes firm 1 to gain at least $Q_H\epsilon$ and to lose at most $\epsilon|Q'_1(p_1 - p_2)|p_1 + O(\epsilon^2) = \epsilon/(-\alpha + 1/f(z'))p_1 + O(\epsilon^2) < \epsilon/(-\alpha + 1/f(z'))(\bar{z} + \mu_1 + \alpha) + O(\epsilon^2)$ (where z' is the marginal type). When the mass of f is sufficiently concentrated close to zero, $f(z')$ approaches zero and Q_H remains high; hence, the gain exceeds the loss.

While our model is static, firms in reality may interact repeatedly and not set prices once-and-for-all. A more realistic pricing game may therefore be one with many pricing periods such that in any given period the firm with higher past sales—by virtue of its success—is the price leader and holds the “in” position (i.e. has a high impulse). Proposition 2, as well as our comparative statics in the next section, apply equally to the steady state of an infinite-horizon model of this type.¹⁸ This provides a rationale for focusing on the Bertrand-Stackelberg environment in which the price leader wins the market.

We now derive some comparative statics that will be the backbone of our policy discussion. Our focus is on the case where firm 1 (the price leader) wins the market, which matches the steady state noted above.¹⁹

4.1 Comparative Statics

Our first result concerns the impact of the firms’ intrinsic quality on equilibrium quantities and payoffs.

Proposition 3. *Suppose firms face an In/Out demand curve that admits micro-foundation #1. Suppose further that firm 1 wins the market and the WIN constraint binds. Then:*

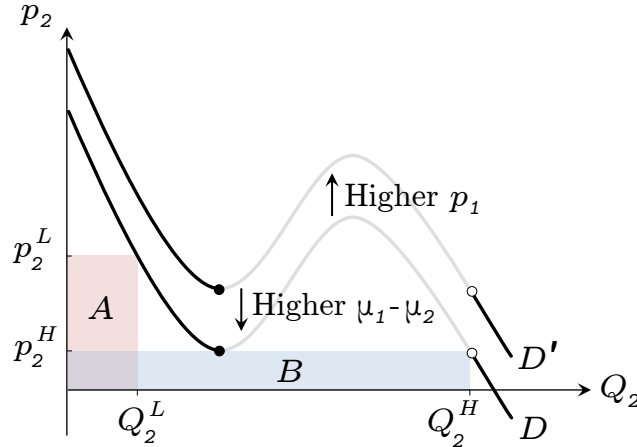
1. *Consumer surplus is independent of μ_1 and increasing in μ_2 .*
2. *Firm 1’s profits are increasing in $\mu_1 - \mu_2$; firm 2’s profits are independent of $\mu_1 - \mu_2$.*
3. *The equilibrium levels of Q_1 and Q_2 are unchanging in $\mu_1 - \mu_2$.*

Figure 3, which displays firm 2’s demand curve for a given p_1 , provides the key observation for this result. A binding WIN constraint means that firm 1 sets p_1 such

¹⁸What changes in such a model relative to the static model is that firms obtain a continuation payoff that may depend on who wins the market today. This potentially lowers the value of p_{win} needed to keep firm 2 out, but otherwise leaves our results unaffected.

¹⁹Also of interest is understanding the conditions under which firm 2 displaces firm 1, and the dynamics that result from it. To be tractable, however, such analysis is likely to require a more specialized model, which is beyond the scope of this paper.

Figure 3: The WIN Constraint



The WIN Constraint is satisfied when area A is weakly greater than area B . The figure is drawn for the case where firms have zero marginal costs.

that area A (firm 2’s maximum “low-sales” profits) and area B (its maximum “high-sales” profits) are equated.²⁰ A greater $\mu_1 - \mu_2$ shifts the demand curve vertically; hence, to keep the WIN constraint binding, firm 1 raises its price one-to-one, which leaves both p_2 and the equilibrium quantities unaffected. Consequently, an increase in μ_1 has the sole effect of raising firm 1’s profits, whereas an increase in μ_2 raises the surplus of all consumers one-to-one, with firm 1 consumers benefiting from a reduction in p_1 and firm 2 consumers directly benefiting from the higher μ_2 . **Q.E.D.**

This result has the paradoxical implication that only μ_2 benefits consumers, and yet, conditional on not winning the market, firm 2 has no reason to invest in a higher μ_2 . We shall return to this observation in Section 5.

Changes in the firms’ marginal costs have a similar impact, but with all signs reversed; that is, an increase in the marginal cost of firm i is analogous to a reduction in μ_i .²¹ It follows that only firm 2’s marginal costs impact consumers (with lower

²⁰In the figure, firm 2’s maximum high-sales profits occur at a price p_2^H right at the threshold for firm 2 to sell a high quantity, but depending on the shape of demand, p_2^H could potentially be lower than the threshold.

²¹To see why, let MC_i denote firm i ’s marginal cost and redefine variables so that $\tilde{p}_i := p_i - MC_i$ takes the place of p_i and $\tilde{\mu}_i := \mu_i - MC_i$ takes the place of μ_i and, upon this change of variables, firms have zero marginal costs. It follows that an increase in MC_1 shrinks \tilde{p}_1 one-to-one and has no impact on p_1 or p_2 , and thus merely lowers the profits of firm 1; whereas an increase in MC_2

costs helping all consumers one-to-one, like an increase in μ_2), and yet only firm 1 gains from reducing its costs.

Next, we consider the impact of the impulse I_0 .

Proposition 4. *Suppose Assumption 1 holds and firms face an In/Out demand curve that admits micro-foundation #1. Suppose further that firm 1 wins the market and the WIN constraint binds. Then, an increase in I_0 :*

1. *Weakly lowers consumer surplus and total surplus.*
2. *Weakly raises the prices and profits of both firms.*
3. *Weakly lowers Q_1 .*

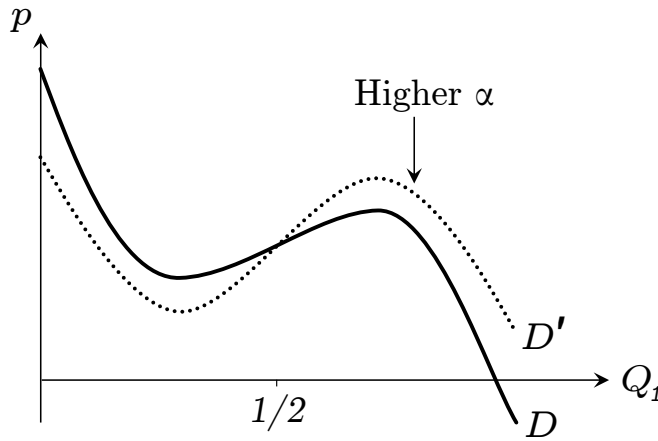
Moreover, whenever the WIN constraint binds and $I_0 < Q_H$ (so that firm 1 faces a demand curve worse than the “in” curve), all of the above changes are strict.

To obtain this result, notice that when the impulse is sufficiently high (at least equal to Q_H), firm 1 enjoys an “in” demand curve (the best possible one) and so a further increase in impulse has no effect; when instead the impulse is lower than Q_H (and WIN binds), an increase in impulse lowers, for any given p_1 , the threshold price p_2 needed for firm 2 to go in. This extends the “high” portion of firm 2’s demand curve (the portion to the right of Q_H) and shrinks its “low” portion (the portion to the left of Q_L), and hence firm 1 is able to raise p_1 while still winning the market. Assumption 1 implies that firm 2 reacts by raising both p_2 and its own sales. Firm 1’s profits rise because it faces a less severe WIN constraint. Firm 2’s profits also rise because it maximizes subject to a higher p_1 (and hence a demand curve that is better along the “out” portion). While both firms benefit from this change, the larger of the two networks (that of firm 1) falls, which damages overall surplus. Because prices rise and network externalities fall, consumer surplus falls as well. **Q.E.D.**

Our final comparative static concerns the impact of α . A natural conjecture is that a higher externality allows the winning firm to set a higher price, leading the losing firm to raise its price as well. It turns out, however, that a higher α has an ambiguous effect on prices, and therefore profits.

raises p_1 one-to-one and has no impact on \tilde{p}_2 , and thus hurts all consumers.

Figure 4: An Increase in Network Externalities (α)



Proposition 5. *Suppose Assumption 1 holds and firms face an In/Out demand curve that admits micro-foundation #1. Suppose further that firm 1 wins the market and the WIN constraint binds. Then, an increase in α :*

1. *Reduces prices when the impulse is in firm 2's favor ($I_0 \leq 1/2$).*
2. *Has an ambiguous effect on prices when the impulse is in firm 1's favor ($I_0 > 1/2$).*

To understand this result, consider Figure 4, which shows that an increase in α rotates demand counter-clockwise. The rotation increases demand on the high portion of both firms' demand curves ($Q \geq Q_H$) and decreases demand on their low portions ($Q \leq Q_H$). Other things equal, this makes it harder for firm 1 to satisfy WIN because firm 2 is more inclined to go for high demand (a pro-competitive force). At the same time, depending on the impulse, the rotation may lower the threshold price for firm 2 to achieve high demand, making it *easier* for firm 1 to satisfy WIN (an anti-competitive effect). Either effect may dominate. The case where there is no ambiguity is when firm 1's impulse is sufficiently low ($I_0 \leq 1/2$): in this case an increase in α *raises* the threshold price for firm 2 to achieve high demand, and hence a higher α is guaranteed to lower prices. (For a formal proof, see the Appendix.)

Proposition 5 contrasts with Armstrong (2006), where competition over the marginal consumer ensures that network effects decrease both prices and profits. Hence, an

increase in α is unambiguously pro-competitive.

5 Policy Implications

Here we consider what are perhaps the most salient policy implications of our model.

Acquisition of Startups. One way in which firms may improve their quality (or lower their costs) is by acquiring startups. In the decade between 2008 and 2017, Google/Alphabet made 166 acquisitions, Amazon 51, Facebook 63, Ebay 31, Twitter 54, and Apple 66. Consider a few of the startups acquired by Apple: PA Semi (purchased in 2008) has been instrumental to the development of Apple’s low-power processors; Siri (purchased in 2010) was used to create Apple’s virtual personal assistant; C3 Technologies (purchased in 2011) is one of several startups acquired to improve mapping features; and PrimeSense (purchased in 2013) powers the facial recognition features of the iPhone and iPad.

Our model suggests that these acquisitions may fail to benefit consumers and, moreover, can easily entrench the position of dominant firms—i.e. increase their ability to fend off future competitors. To illustrate, suppose we add an initial period to the duopoly model where firms 1 and 2 bid in a second-price auction for a startup that improves the winning firm’s quality by $\Delta\mu$ units. In a scenario where the WIN constraint binds and where firm 2 does not attempt to overtake firm 1, firm 1’s profits are increasing in its quality while firm 2’s are not; hence, firm 1 will place a positive bid and firm 2 will bid 0. Firm 1 therefore acquires the startup at zero cost and because firm 1’s price grows one-to-one with its quality, the acquisition does not benefit consumers. Moreover, going forward, it will be easier for firm 1 to fend off a new rival, or even a newly strengthened firm 2.²²

By the same token, it may be beneficial to force the dominant firm to share some of its technological innovations with its competitor. After all, in the case of a binding WIN constraint, consumers do not benefit from the dominant firm’s

²²A related literature considers the incentives of market leaders to invest in new technologies, perhaps through the acquisition of startups or patents, to cement their market power. For example, Gilbert and Newbery (1982) and Cunningham et al. (2021) argue that the leader may seek innovations that it does not actually use, purely as a way of keeping rivals at bay.

innovations unless they also benefit their rival. With this type of intervention the regulator would of course need to balance the competing goals of providing incentives for innovation (which favors less sharing) and making sure consumers benefit from it.

Monitoring negative externalities. The scenario of a binding WIN constraint creates a particular risk of damage from negative externalities. Suppose firms are able to raise, at a private cost, the consumers' willingness to pay for their product (raise μ_i in the first micro foundation), but in the process they create a negative externality on all consumers, regardless of their purchase decisions. Proposition 3 tell us that in this scenario only firm 1 would be willing to raise its quality, that it has a strong incentive to do so, and that firm 1 captures all value it creates in the process (before accounting for externalities): for every dollar μ_1 grows, profits grow by Q_1 dollars while consumer surplus (gross of any externality) remains unchanged. This is problematic for consumers because they bear the full impact of the externality without any compensating benefit.

This result may shed light on some of Facebook's alleged practices and their potential consequences. Francis Haugen, a whistleblower, claimed that the company has sought to increase consumer engagement by means of a change in its algorithm that, in effect, promotes anger and polarization.²³ If we interpret higher engagement as a greater willingness to pay and anger and polarization as causing a negative externality on all of society, the above analysis applies. This provides a rationale for government to attempt to mitigate the externality.

Impulse Synergies. Because the impulse may be influenced by subtle factors like defaults and brand recognition, a firm can potentially use its success in one market to create a high impulse for itself in a second, unrelated one. Amazon, for instance, presumably benefited from its initial success in the book business to establish favorable consumer beliefs for other markets as well, such as apparel. Such "impulse synergies" may partly explain why Amazon entered the shoe market and acquired Zappos, the online shoe retailer.

²³Zubrow, Keith, "Facebook whistleblower says company incentivizes 'angry, polarizing, divisive content,'" CBS News, OCTOBER 4, 2021: www.cbsnews.com/news/facebook-whistleblower-frances-haugen-60-minutes-polarizing-divisive-content/ (accessed 5/5/22).

Recall from Proposition 4 that in the scenario of a binding WIN constraint an increase in a dominant firm’s impulse leads to higher prices, and lower consumer and total surplus. This suggests that regulators should take a critical view of mergers or expansions even in seemingly unrelated businesses.

6 Conclusion

We have proposed a “new economy” duopoly model that relies on an S-shaped Beckerian demand curve. We have taken a belief-based approach to dealing with the resulting multiplicity of equilibria in demand and have obtained policy-relevant comparative statics for a scenario where the dominant firm barely keeps its rival at bay—a form of limit pricing with the rival threatening the dominant firm from within rather than outside the market. Possible directions for future work include, variously, a dynamic scenario where the dominant market position changes hands over time, competition among more than two rivals, and the possibility of multi-sided demand.

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7 Appendix

7.1 Micro-foundation #2

To see why demand is In/Out under the second micro-foundation, observe that when $Q_1 \geq Q_2$, consumers above the cutoff $\hat{z} = \frac{1}{Q_1 - Q_2}(-(\mu_1 - \mu_2) + (p_1 - p_2))$ consume from firm 1; when $Q_1 < Q_2$, consumers below that cutoff consume from firm 1. Hence, $Q_1 = 1 - F(\hat{z})$ when $Q_1 \geq Q_2$ and $Q_1 = F(\hat{z})$ when $Q_1 < Q_2$, from which it follows that $P(Q_1) = (\mu_1 - \mu_2) + (2Q_1 - 1)F^{-1}(\min(Q_1, 1 - Q_1))$ when $N = 2$. Differentiating, we find that the slope of demand is $2F^{-1}(\min(Q_1, 1 - Q_1)) - |2Q_1 - 1| \frac{1}{f(F^{-1}(\min(Q_1, 1 - Q_1)))}$. Since both the first and second terms of this expression have an upside-down U-shape with peak at $Q_1 = 1/2$, the slope also has an upside-down U-shape with peak at $Q_1 = 1/2$. At the peak ($Q_1 = 1/2$), the slope is positive since the first term is positive at $Q_1 = 1/2$ and the second term is equal to zero. Provided the distribution's mass is sufficiently concentrated, the slope is negative at high and low values of Q_1 since the second term is highly negative. It immediately follow that demand has an In/Out shape.

7.2 Proof of Proposition 5

Under micro-foundation #1, $P(Q_1) = (\mu_1 - \mu_2) + \alpha(Q_1 - Q_2) + F^{-1}(1 - Q_1)$. Rearranging terms, and substituting $1 - Q_2$ for Q_1 , we obtain the following inverse demand curve for firm 2: $p_2(Q_2) = p_1 + (\mu_2 - \mu_1) + \alpha(2Q_2 - 1) - F^{-1}(Q_2)$. Observe that an increase in α raises the curve for $Q_2 > 1/2$, lowers the curve for $Q_2 < 1/2$, and leaves it unchanged at $Q_2 = 1/2$. Since $Q_L < 1/2 < Q_H$, an increase in α causes demand to fall on the low portion of firm 2's demand curve ($Q_2 \leq Q_L$) and rise on the high portion ($Q_2 \geq Q_H$), as shown in Figure 4. Notice also that the slope of the inverse demand curve is: $2\alpha - \frac{1}{f(F^{-1}(Q_2))}$. Therefore, an increase in α increases the slope everywhere.

An increase in α has two effects on firm 1's WIN constraint. First, demand falls on the low portion of firm 2's demand curve and rises on the high portion. Other things equal, this makes firm 2 more inclined to go for high demand, and so makes it

harder for firm 1 to satisfy WIN. Second, depending upon the impulse, an increase in α may raise or lower the threshold price for firm 2 to achieve high demand. This second effect makes WIN harder to satisfy if the threshold rises and easier to satisfy if the threshold falls. The threshold price is $p_2(1 - I_0)$, which is rising in α when $I_0 < 1/2$, falling in α when $I_0 > 1/2$, and unchanging in α when $I_0 = 1/2$.

If $I_0 \leq 1/2$, both effects go in the same direction. So, an increase in α makes the WIN constraint harder to meet, and hence causes p_1 to fall. If, on the other hand, $I_0 > 1/2$, the effects go in opposite directions so a change in α may cause p_1 to rise or fall.

Let us now consider what happens to p_2 in the case where p_1 falls. Firm 2's optimal price is on the low portion of its demand curve given that WIN holds. The rise in α and fall in p_1 cause the low portion of firm 2's demand curve to shift in two ways: (1) there is a downward shift in the level of demand (due both to the rise in α and the fall in p_1), (2) the slope rises (due to the rise in α). By Assumption 1, we know that the level effect causes p_2 to fall. The slope effect means that marginal revenue is greater at any quantity and hence quantity rises and price p_2 falls. So, both the level and slope effects cause a drop in p_2 . Therefore, when $I_0 \leq 1/2$, both p_1 and p_2 fall when α rises. Q.E.D.