Physics-Informed Neural Network for Fibre Channel Modelling in Optical Communication Systems

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Abstract: Over 95% of the data traffic is carried over optical fibre communication links. The split-step Fourier method (SSFM) has been widely employed to model the evolution of optical signals along the fibre channels in optical communication systems. However, the split-step Fourier method requires very high computational resources, especially for ultra-long-haul and wideband communication systems. Meanwhile, deep learning techniques can be applied to investigate the evolution of optical signals along the fibre links, where the nonlinear Schrödinger equation (NLSE) can be solved directly using neural networks to avoid the huge complexity of the split-step Fourier simulations. In this work, we will discuss the application of neural networks in modelling the evolution of different types of optical pulses along fibre transmission channels.

1. Introduction

Optical fiber communication systems were first deployed five decades ago and since then, the information capacity carried by a single-mode optical fiber has grown exponentially to the extent of reaching an astonishing increase of 10,000 times by the mid-2000s, and only slowing down thereafter [1]. Optical communications are undoubtedly unrivaled as the principal enabling technology and the foundation of today's global telecommunications networks for the transfer of very large amounts of data across long distances and substantial bandwidths with minimal latency. However, optical fibers which are responsible for carrying approximately 95% of all internet traffic are relatively expensive and as a result, it becomes difficult to study such large-scale optical systems or networks in lab or field experiments. Hence, modeling optical fibers becomes very important for assessing the performance of long-haul wideband optical communication systems and networks. The split-step Fourier technique (SSFM), which is predominantly used to simulate the combination of dispersion and fiber nonlinearities in optical pulse propagation in fibers, can be a laborious computational procedure, especially when long transmission distances and very large bandwidths are taken into account [2–4]. On the other hand, since their introduction, deep neural networks (DNNs) have both revolutionized and enhanced the achievable results and potentials of machine learning techniques employed in various fields of study, fiber optics being no exception. Most recently, physics-informed neural networks (PINNs) were developed to estimate nonlinear partial differential equation solutions by a non-data driven approach [5]. PINNs have been employed recently for a number of fiber modeling tasks. Wang [6] et al. investigated the uses of PINNs in optical communications, using them to solve the paraxial Helmholtz equation which describes the distribution of the electric field in fiber and to simulate the wideband spectrum evolution for C+L systems. Fang [7] et al. investigated high-order NLSE parameter discovery utilizing PINNs and data-driven femtosecond optical soliton excitations. The authors of this paper have investigated the use of the PINN scheme to model the propagation of the peregrine soliton pulse in an optical fiber. And in this paper, we report the use of PINNs to model the dynamical evolution of 3 different types of optical pulses propagating in a single-mode fiber under the impact of fiber nonlinearity and chromatic dispersion.

2. Physics-Based Models

Given a complex-valued baseband signal $x(t)$, transmitted through an optical fiber of length L, The signal will propagate following the nonlinear Schrödinger equation (NLSE), which is a nonlinear variant of the Schrödinger equation [8]. The NLSE is given as

$$\frac{\partial A}{\partial z} = \frac{\alpha}{2}A - i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial^2 \tau} + i\gamma|A|^2 A, \tag{1}$$

where γ is the nonlinear Kerr parameter, α is the loss parameter, β_2 is the chromatic dispersion and $A(z=0,\tau) = x(t)$. The signal received after propagating along the length L is denoted by $y(t) = A(z=L,\tau)$. The NLSE characterizes the evolution of optical pulses in a fiber and takes into account physical effects such as cross-phase modulation (XPM) and self-phase modulation (SPM). The NLSE also takes into account the impact of fiber losses, chromatic dispersion, third-order dispersion (TOD), and fiber non-linearity as well as their combined effects. The optical pulse envelope propagates at the group velocity v_g. The value of β_2 can either be negative or positive depending on the wavelength, whether it is above or below the zero-dispersion wavelength λ_D of the fiber. In typical silica fibers, β_2 is positive and ranges from 50 ps^{2}km^{-1} in the visible to very nearly 20 ps^{2}km^{-1} as the wavelengths approach 1550 nm, and β_2 is often negative in the anomalous dispersion domain. The NLSE (1) is written in terms of τ where the variable $\tau = t - \frac{z}{v_g}$, which represents pulse propagation in a time frame moving at the speed of the signal group. Generally, Eq. (1) does not have an analytical solution and is solved numerically. The SSFM which is implemented through block-wise processing of sampled waveforms is a popular choice amongst numerical solvers for this purpose. The signals $y(t)$ and $x(t)$ are sampled at $t = k/f_s$ and are collected into the vectors $y = (y_1,\ldots,y_n)^T$ and $x = (x_1,\ldots,x_n)^T$ respectively for n samples. To derive the SSFM, it is important to represent the NLSE in its time-discretized form,

$$\frac{dA(z)}{dz} = -\frac{\alpha}{2}A(z) + \mathbf{P}A(z) + i\gamma\mathbf{R}(A(z)), \tag{2}$$

where $\mathbf{R} : \mathbb{C}^n \to \mathbb{C}^n$ is the element-wise application of $R(x) = x|x|^2$, $\mathbf{P} = \mathbf{F}^{-1}\mathrm{diag}(H_1,\ldots.,H_n)\mathbf{F}$, $H_k = i\frac{\beta_2}{2}\omega_k^2$, and $\mathbf{F}$ is the n x n discrete Fourier transform (DFT) matrix. $\omega_k = 2\pi f_k$ is the k-th DFT angular frequency (i.e., $f_k/f_s = (k-1-n)/n$ if $k \geq n/2$ and $f_k/f_s = (k-1)/n$ if $k < n/2$). The fiber is then conceptually divided into N segments of lengths $\delta_1,\ldots,\delta_N$ such that $\sum_{i=1}^{N}\delta_i = L$. It is then assumed that for a short step δ_i, the effects of the linear and nonlinear terms can be isolated. Alternating between the linear and nonlinear operator for $z = \delta_i$ results in the SSFM [8–10].

3. PINNs for Optical Communications

3.1. Principle of PINNs

The solution, the derivative of the solution with respect to time, and the nonlinear function, respectively, are all denoted by the letters u, u_t, and $\mathcal{N}$, with the differential equation defined as $u_t + \mathcal{N}[u] = 0$. We use the PINN technique, where the output of the NN is an approximation of the solution and the nonlinear terms, to solve the equation. To this end, we create a model $f := u_t + \mathcal{N}[u]$ and, by minimising the mean squared error loss, learn the parameters that are shared between the NNs, $f(t,x)$ and $u(t,x)$ [5, 11].

$$MSE = \frac{1}{N_u}\sum_{i=1}^{N_u}|u({t_u}^i,{x_u}^i) - u^i|^2 + \frac{1}{N_f}\sum_{i=1}^{N_f}|f({t_f}^i,{x_f}^i)|^2, \tag{3}$$

Where $\{{t_f}^i,{x_f}^i\}$ are the collocation points for $f(t,x)$ and $\{{t_u}^i,{x_u}^i,u^i\}_{i=1}^{N_u}$ are the training data for u. The first loss term's goal is to satisfy the network u, while the physics-based regularisation term is the second loss term. To guarantee that the NN's solution resolves the underlying differential equations, this term is minimised. Figure 1 shows the architecture of the PINN network.

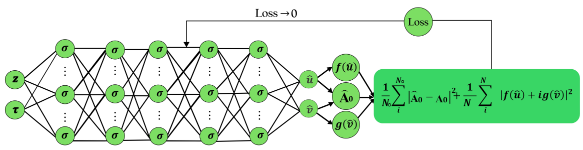

Fig. 1. Architecture of Physics-Informed Neural Networks

3.2. PINNs for fiber optics

In the fiber-optics domain, we can write the normalized distance, time, real and imaginary components of a complex envelope $E(z,\tau) = u(z,\tau) + iv(z,\tau)$ as z, τ, u and v respectively. Where $E(z,\tau)$ is the complex envelope of a slowly evolving optical field. In contrast to the traditional approach with neural networks (NNs), a PINN is essentially a PDE solver that maps from (z,τ) to $[u(z,\tau) + v(z,\tau)]$ pair, unlike other data-driven methods that would normally only fit a given pulse to ending pulse. This network mapping structure gives an insight into how the optical field $E(z,t)$ evolves in the (z,t) plane. In other words, any randomly selected point (z',τ') in the (z,t) plane satisfies the $NLSE(u',v') = 0$. The normalised NLSE, represented as $E_z + \frac{i}{2}E_{\tau\tau} - i|E|^2E = 0$, may be divided into its real and imaginary components, denoted by the symbols $f(u) : u_z - \frac{1}{2}v_{\tau\tau} + (u^2+v^2)v$ and $g(v) : v_z + \frac{1}{2}u_{\tau\tau} + (u^2+v^2)u$, respectively. Where τ, z, g, and f are the time, normalised distance, and the governing functions for $v(z,\tau)$ and $u(z,\tau)$ respectively [2, 11, 12]. The relation $NLSE(u,v) = 0$ is satisfied by solving this. The function $E(z,\tau)$ is represented by a four-layer, 100-neuron deep neural network with a tanh activation function. 15,000 discrete points have been sampled to constrain the NLSE, and they have been set as input along with 50 points from the input pulse, which is utilised to make sure the initial pulse loss term is close to zero.

4. Results and Further Discussion

To reveal the pulse development throughout the length of the fiber, a [-5T_0, 5T_0] $\times$ [0,z] 2D modeling region was constructed. The Gaussian, *sinc* and *sech* pulses were propagated through the fiber to test PINN's versatility in characterizing different pulse types and shapes. The results for this task are shown in Figures 2, 3 and 4, which show the evolution of the complex optical pulse envelope through a 10 km single-mode fiber. The PINN

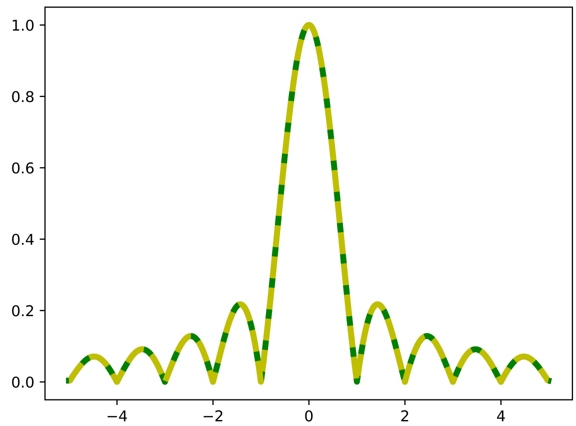

Fig. 2. Time-domain pulse profile showing the received pulse at z = 10km for the *sinc* pulse

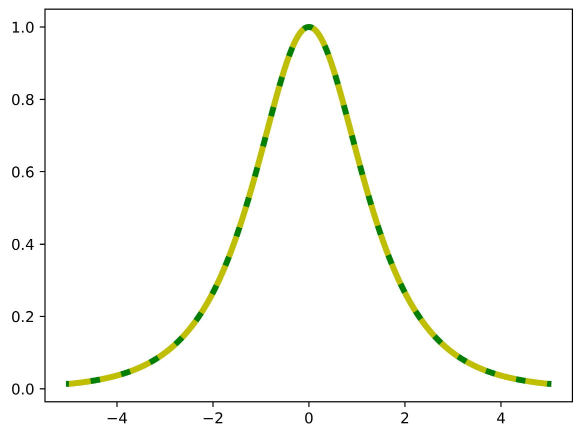

Fig. 3. Time-domain profile showing the received optical pulse at z = 10km for the *sech* pulse

based predicted solutions (green solid line) are compared with the exact solutions (yellow dashed line), which are obtained by solving the NLSE using the SSFM. We utilised this to check the algorithm's accuracy, and the prediction error was found to be 4.3×10^{-3}, 5.13×10^{-3} and 7.02×10^{-3} for the evolution of the *sinc*, *sech* and Gaussian pulses respectively. With only the initial conditions and input, the PINN algorithm is able to characterize pulse evolution with a good degree of accuracy. In this paper, the performance of PINN is measured by the accuracy of the method. We use the relative L2 norm to evaluate the error. To compute this, we use Python's Numpy Norm package. When $np.linalg.norm()$ is called, it computes the L2 norm. The L2 norm, which may be regarded as the vector's length in Euclidean space, is the square root of the sum of squared elements.

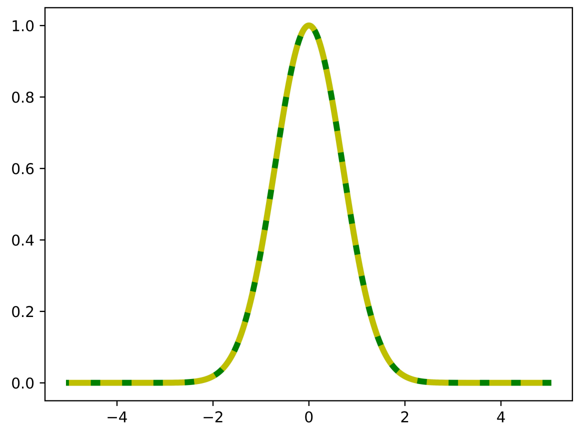

Fig. 4. Temporal profile of the Gaussain pulse showing the received optical pulse at z = 10km

5. Conclusions

In this work, the NLSE is solved using a PINN model, a method that gives more insight into the underlying physical process from the perspective of deep learning. Unlike the typical data-driven approach employed by traditional NN methods to solve the same problem, the PINN method is able to describe the entire process of pulse propagation with very little prior information (only the initial pulse is needed). The results of the PINN modeling scheme show the potential of this method as a fast and accurate alternative to the SSFM without the computational burden of the latter. Our results show that PINN is able to both accurately perceive and predict pulse propagation in a fiber and is able to characterize other physical effects in fiber-optic transmission system modeling.

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