

Rules for making: kinematic design, shape and structure

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Abstract

The thesis examines the role of making rules, within the creative exploration of kinematic design spaces. As a process of searching within a conceptual space, creative exploration can be described using rules. When applied to design, this model for creativity affords the application of computational techniques.

In shape grammars, shape rules for 'seeing' and 'doing' apply a descriptive approach to the visual recognition, composition and modification of pictorial representations. This formalism can provide generative specifications and reveal the synthetic reasoning underlying iterative trajectories of design development. Making rules extend this approach to the tactile-visual representations of physical models and prototypes. When instantiating design representations within the material world, actions to construct and alter descriptions are grounded in material algebras.

This thesis has a focus in Kinematics, where physical models provide a synthetic alternative to analytic techniques for modelling motions. In this context, making rules describe how to construct designs, make alterations, and manipulate models. Kinematic connections afford variable spatial relations between kinematic parts, and rules for physically manipulating models elicit their motions.

Single closed-loop kinematic chains with full cycle mobility provide case studies for experimenting with making rules in design exploration, using both physical models and abstract geometric descriptions. An existing design creates a point of entry, where rules then afford the exploration of a surrounding kinematic design space. Applying alterations and transformations to physical models can identify the boundaries within which kinematic properties are preserved.

The experimental cases inform theoretical development of exploratory making, with special reference to the variable spatial relations in kinematic designs and the integration of visual and tactile sensing. The main conclusion is that: as making rules construct models, rules are abstracted into schema by comparing properties of similar designs. The schema explain the results of exploration, initiating new explorations and new designs.

1 Introduction and background

“Designing is not a search for the optimum solution to the given problem, but an exploratory process.”

(Cross, 2011, p8.)

In this thesis, we consider the notion of design as an exploratory process. We further supplement this description by examining another complementary metaphor— of creativity as a process of search, within a conceptual space of possibilities (Boden, 2003; Wiggins, 2001, 2006a). This metaphorical model raises all sorts of questions: what kind of spaces, landscapes or topologies are being traversed and explored? How do designers move around within them? How might the trajectories of their design activities across this unseen terrain be described, documented or recorded?

Rather than occurring within metaphorical, conceptual spaces, design exploration is often situated primarily within the real material world, where constructed descriptions leave trails of physical evidence that render design reasoning tangible (Schön, 1992; Patterson, 2014). Design often both begins with and returns to these material making activities (Knight & Vardouli, 2015). Making seems to possess its own inherent logic, where material calculations differ according to context, and also according to the particular algebras of tools and materials available to the maker (Ingold, 2010; Knight & Stiny, 2015).

Ranging across sketches, models and prototypes, throughout conceptual development, exploration and evaluation, material making forms a core part of design processes (Goldschmidt & Porter, 2004). Creative design exploration might consider numerous design properties and behaviours, from shape, colour, and composition, to load-bearing capacity, structural stability, or kinematic motions (Bucciarelli, 1998). Making may also include material exploration in its own right, to consider the limits of process potential, for a particular combination of media (Crawford, 2010; Korn, 2017; McCullough, 1998; Pye, 1968;

Sennett, 2009). Examining these boundaries extends the capability of what can be instantiated materially.

Certain design properties that cannot be specified directly are afforded via complex interactions, which material representations can help to reveal. Such is the case for the field of kinematics, where both shape and kinematic structure affect motions (Phillips, 2007; Shapiro & Voelcker, 1989). Kinematic design, which is concerned primarily with the motions between parts within kinematic assemblies, also necessitates its own particular logic; in this case calculating is concerned with shape and structure, or the geometries and topologies of parts and their connections. In this field, the material construction of physical models provides an important means for examining the kinematic motions of designs, and also, when design issues arise, for explaining any impedances to motions. Analytical mathematics provides one route to explaining underlying relationships and constraints (Hunt, 1978; Rooney, 2006). Systematic material exploration of design limitations via model-making provides another.

This thesis considers how formal descriptions of making can support physical generation, applying computational techniques for the material exploration of kinematic design spaces. We consider how making rules might describe the making actions afforded by certain combinations of tools and materials, to support generative computation within a material space. We then consider how these material algebras might be formally described. We examine how these rules or actions might support design exploration, in both real and conceptual spaces, within this particular field of design.

1.1 Making rules for seeing and doing

Over the past forty years, actions and observations, for constructing and examining material design representations, have been expressed as formal computations. Shape rules are formulated to reflect the visual compositions they act upon. These largely mimic the material characteristics of drawings made with ink upon paper, and support the description of visual calculations (Stiny, 1989; 2006; 2007). Sets of these rules define shape grammars for generating spatial arrangements of shapes, directed firstly towards abstract patterns and

creative compositions (Stiny & Gips, 1972; Stiny, 1975; 1977). Their application to design representations began initially with pictorial and schematic design drawings, within architectural domains (March & Earl, 1977; Stiny & Mitchell, 1978; König & Eizenberg, 1981). Subsequent focus has included designs for gardens (Stiny & Michell, 1980), furniture (Knight, 1980; Barros et al., 2015), product design (Agarwal & Cagan, 1998; Prats et al., 2006) and vehicle design (Pugliese & Cagan, 2002), applications in engineering synthesis (Cagan, 2009), and also in the generation of designs for digital fabrication (Shea et al., 2010; Wang & Duarte, 2002; Sass, 2007; Knight & Sass, 2008). Beyond design, grammars have also been using more widely, for instance, to model the motions of the human skeleton, generating sequences of yoga poses (Piedade Ferreira et al., 2011).

For calculating with shapes, the interplay between seeing and doing is examined comprehensively by Stiny (2006). More recently, shape rules as explicit descriptors of designer actions and observations have also been considered, largely for unstructured shape representations such as sketching (Prats et al., 2009; Paterson, 2014). Shape identity rules have also been applied to support visual reinterpretation within digital drawing tools using eye tracking technology (Jowers et al., 2010; 2011).

A recent refocusing of the shape rule formalism has been articulated by Knight and Stiny (2015), to encompass a wider range of making activities, including craft-based making practices (Knight, 2018), material design properties (McLachlan & Jowers, 2014; Gürsoy & Özkar 2015) and three dimensional haptic-visual representations with physical models and prototypes (Harrison et al., 2015). The extension of ideas underlying the shape rule formalism, towards making rules for modelling a wider range of creative encounters with physical materials, requires a generalisation of 'seeing' rules towards multi-sense perception, and 'doing' rules defined within shape algebras towards more general transformations within various material algebras of stuff and tools. It has been proposed that making rules for constructing engineering assemblies and describing certain manufacturing processes might be defined through the direct extension of established shape grammar formulations (Krstic, 2019).

These rules for design draw attention to the creative nature of the exploration of design possibilities through making and construction, and help to formalise the modelling of

exploratory trajectories within associated design spaces (see Woodbury, 1991; 2006). However, rules for making are not necessarily confined only to the generation, description or construction of design shapes and assemblies. Creative calculation requires both seeing and doing, and identity rules which recognise shapes or parts can allow for radical reinterpretation (Stiny, 1996). New configurational descriptions can offer new ways of understanding assembled artefacts, suggesting how they might be differently constructed from alternate sets of parts. This ‘backwards’ looking affords alternative derivations of designs, using different rules and configurations (Stiny, 1994; Charidis, 2019). In this sense, ‘looking backwards’ can explain how a design came into being, and also provide alternative, explanatory, synthetic descriptions. This thesis considers further the explanatory potential for making rules, and experiments on physical models employ rule applications to investigate properties within kinematic design spaces.

1.2 Creative exploration and explanation

Questions about how a design ‘works’ or functions are conventionally answered through mathematical abstractions, rather than direct apprehension or experiment with the design at hand. Through their practical application, rules for making offer an alternative explanatory route, and have the potential to be directly useful to designers engaged in creative design activities.

In general, there are two distinct ways in which the properties of a design proposition might be examined. A first approach employs an abstract model to effectively predict or simulate the behaviour of the design proposition. In a second approach, the design is instead physically constructed so that a physical model or prototype can be directly tested, to determine its properties and behaviour.

These two modes of simulation and physical test often proceed hand in hand during engineering design and product development (Tahera et al., 2017). Physical test is framed in terms of meeting requirements; it is known how a design should behave, and a test can confirm this. Testing can also have a wider remit, however, namely to determine what further design changes are feasible without compromising performance against

requirements. This experimental interrogation of a design proposition may be guided by both physical models and simulation. Testing, in this sense, employs making not just to determine properties- it also helps to explain how and why these properties arise. In this sense, the making and testing of physical prototypes, within engineering practice, serves both to generate and explore a design space surrounding an initial design proposition.

For the case of kinematic designs, reasoning about motions using mathematical techniques, applied to abstract models, provides an analytical form of design verification that can guide the development of design concepts. However, these analytical descriptions alone offer limited opportunity to appreciate the qualities of motions themselves, and physical models therefore play an important role in visualisation, and testing of secondary functional aspects.

In this thesis we consider how formal rules for describing material making processes can afford an alternative, generative approach, for both exploring and explaining design properties. Engineering activities of testing and simulation are most usually undertaken to validate an already well-developed design proposition. Here, an existing design is shown to provide a starting point, for a generative method which employs making schema, to open up new spaces of design possibility for material exploration.

1.3 Kinematic designs

Kinematic designs offer an appropriate category of design to formulate the wider role of rules for making, in processes of design. Examining how the motion of a kinematic mechanism is affected by the geometry of links and joints requires considerable analysis and simulation. This field requires significant departures from conventional rules for seeing and doing with shapes. Spatial relations, which lie at the core of shape rule formulation, are now variable rather than fixed or parametric. The corresponding rules for making kinematic designs, and their applications in generating and exploring design spaces, are developed in this thesis for the first time. Shapes rules can specify the shapes of parts within kinematic design assemblies, but more is needed, to model the kinematic connections which afford motions or variable spatial relations between kinematic parts.

The role of making rules within design exploration is considered for a certain set of kinematic configurations: these are kinematic closed-loop linkages, comprised of connected kinematic chains composed of links and joints. A particular focus considers the subset of closed-loop over-constrained linkages, and also other related sets which exhibit continuous or discontinuous full-cycle motions. These unusual motions derive from both the shapes of parts and the geometric configurations of kinematic connections or joints which they afford, and also the variable spatial relations afforded by joints themselves. Making rules for constructing these designs must therefore fall into distinct categories: those which afford and affect kinematic connections; those which define or alter the geometric shapes of kinematic parts; and those that assemble kinematic parts within designs. Further rules for 'sensing' are also required: whilst visual examination remains important, in order to directly examine motions in kinematic designs, rules which handle and manipulate models are needed, to actively elicit motions.

1.4 Contribution and organisation of the thesis

Grounded within shape grammar theory (see Stiny, 2006), this thesis makes a novel contribution to the emerging field of computational making, which itself seeks to harness the creative and generative potential of formal descriptions, within design theory and practice, and for materially situated activities more widely (Knight & Vardouili, 2015). In this thesis, our collation of existing theoretical models, of general creativity (see Boden, 2003; Wiggins, 2001; 2006a; 2006b; Ritchie, 2005; 2006; 2012), design space (Woodbury, 1991; 2006) and theory of practice (de Certeau, 1988) provides a framework which inspires the development of new generative methods for describing design exploration in practice.

From the perspective of computational making, the thesis addresses a central question:

When used for the formal description of both material making activities (using stuff and tools) and the things they create, can shape rules and schema discover a generative model for making which helps to explain, either the properties of things and designs themselves, or of the exploratory processes that discover them?

This question is further unpacked, with a particular focus towards kinematic design, in the detailed research questions of Chapter 3.

In this thesis, the main focus is on the application of shape rules for making, to the field of kinematics. One key contribution is the extension of shape grammar theory to include formal descriptions for variable spatial relations, as outlined in Chapter 6. This new type of rule provides a means for modelling motions and mobility in material assemblies, which is important both when describing the properties of kinematic models, and during the material manipulations which examine and construct them during material design exploration. This extension of the formalism affords the application of shape rules to the formal description of new domains of creative practice.

The detailed method developed, through the material experiments of Chapters 4 and 5, makes a further contribution towards the field of shape grammars and material computation, which also has wider implications for design. Rather than applying rules directly, to describe and construct designs, this practical method employs the general device of schema in a novel way. When applied to initial making rules, these making schema describe spaces of rules, which in turn explore spaces of designs. Derived from practical episodes of experimentation, a collection of schema afford the ongoing exploration of new possibilities, describing design activities without artificially constraining them to a particular space. The approach provides evidence for the exploratory and explanatory capacities of generative rules and schema within design practice. It also illuminates a potential for further practical application to formal description and material computation, both for kinematics and in other design domains.

The thesis is organised to reflect the development of rules for making kinematic designs.

Chapter 2 presents a review of literature which covers the elements of shape rules, kinematic design, creative exploration and design spaces, as well as the role of making within processes of design.

Chapter 3 describes a methodology which aims to understand how rules for making, for kinematic designs in particular, can be formulated. This involves preliminary examination, through material exploration, of certain restricted classes of closed-loop kinematic linkages, including the class of over-constrained closed loops. This synthetic analysis informs the development of making rules for kinematic designs more generally. The final pieces of this methodology entail theoretical development of these making rules, incorporating variable spatial relations, as well as the visual and tactile sensing which form an integral part of making for kinematic design.

Chapter 4 introduces making rules, and discusses how material algebras for kinematic designs might be instantiated. It then develops a first exploratory tranche of experiments and observations, with over-constrained kinematic loops and other closed-loop linkages as a focus. This poses several questions, as to the types of rules for making required to assist design, and identifies general making schema which then inform further work.

Chapter 5 applies these schema systematically, within a second set of experiments, extending the range and focus for how rules for doing and sensing can be systematically applied. These sets of experiments, and associated observations in Chapters 4 and 5, outline several aspects of rules for making that are required for kinematic design.

Chapter 6 consolidates these experimental investigations of kinematic design, through developing formal ways to describe the variable spatial relations required, in the rules for making kinematic designs.

Chapter 7 considers the implications for processes of design, of the experiments in Chapters 4 and 5, especially when combined with the formal and theoretical developments of Chapter 6. It also frames the role of rules, and strategic schema especially, for making in creative exploration in design.

Chapter 8 summarises conclusions and research contributions, through critical examination of the research questions, posed above and in Chapter 3. A critical assessment is made of the contributions of the thesis. Finally, this chapter sets out further work into rules and schema for making, both more generally, and specifically for kinematic designs.

Chapter 2 Review of literature

A hylomorphic model of making perceives form as distinct from matter and proposes that artefacts are created by imposing preconceived ideas for shapes upon the malleable materials of the physical world. Based on ideas attributed to Aristotle, this model of creative synthesis has influenced much of Western philosophy. Perhaps in part due to this influence, early models of design reasoning consider designs to be primarily mental constructs. Writing in the 15th Century, Alberti describes designing as a cerebral activity involving 'lineamenta'. These 'arrangements of lines in the mind' are perceived as a distinct and separate precursor to the practical and messy business of making buildings (Alberti, 1988; Ingold, 2013a). As recently as 1969, Herbert Simon describes design as a procedure of 'mental window-shopping' amongst an array of many possible ideas generated within the mind, with only a few favourites subsequently selected for physical instantiation (Simon, 1969).

Whilst designs are considered to emerge from the mind fully formed, questions about the mental processes that create them remain difficult to answer. However, this once dominant hylomorphic viewpoint has since fallen under criticism (Deleuze & Guattari, 2003; Simondon, 1964; Ingold, 2013a; Bryant, 2012; 2014). Recognising the limitations of mental representations, subsequent studies of design reasoning (see for instance Goldschmidt, 1994, 2004; Schön & Wiggins, 1992; Do & Gross, 1996) acknowledge the importance of external representation techniques for supporting design development. Alternative new models of design reasoning now place a greater emphasis upon the making of external representations, not only for the final realisation of a selected design concept, but throughout an iterative process of development, where design ideas are refined and negotiated using a diverse range of techniques for material representation.

Once design is perceived as an iterative activity involving external representations, the synthetic reasoning processes by which designs are created become easier to consider. These external representations themselves now provide a tangible record of the sequences of actions and alterations through which design ideas are developed.

Alberti's discussions of design activity are primarily concerned with the processes by which the cathedrals and other large buildings of the early Renaissance era were created. Ingold (2013a) argues that, rather than issuing from the mind of a single designer or master architect, these edifices are rather the patchwork-like result of a constructive collaboration between many teams of craftsmen, with communications and negotiations between different groups of tradesmen most likely centred around a physically situated practice, involving the marking out of drawings and trceries for distinct building elements upon the ground at 1:1 scale. The idea that design drawings could become distinct artefacts in their own right, rather than a mere component of the active on-site process of making buildings, developed gradually during the Renaissance era.

Produced in advance of the buildings they related to, perhaps at a remote site and also a different scale, architectural drawings eventually became regarded as complete speculative representations of proposed buildings, developed separately from, and in advance of, the practical process of on-site building. With this perceived separation of design from fabrication, the architect's role re-emerged as a distinct and superior to that of the mason. This development perhaps reinforces a hylomorphic model of making as the physical imposition of a preconceived form upon compliant materials. But it also furthers the idea that designs can be physically constructed and interrogated, in multiple material modes of representation, before they become set in stone as full-scale finished artefacts.

2.1 Design exploration

The possibility of representing, considering and developing designs using alternate media to those intended for their final fabrication opens up a material space where design exploration leaves a physical trace. Rendering visible exploratory trajectories across material landscapes, these material design descriptions physically articulate the design reasoning which they themselves support. The nature of techniques for instantiating and examining designs using various combinations of physical materials becomes of key importance, therefore, in understanding and modelling design exploration activity.

2.1.1 Representations for design exploration

Many modes of media and material combinations are employed when developing designs. Means of representation are developed by combining materials and media in distinct ways. Different modes of making can render certain types of properties easier to examine. The particular ways in which a design may be materially altered at each stage during its development will also depend on the media through which it is represented. Designers tend to favour visual and physical representations primarily, but descriptions of designs can also be abstract. Methods for describing designs may combine words, numbers, symbols, drawings and models, with other conventions and abstraction techniques (Stiny, 1981).

Woodbury and Burrow (2006) outline four facets which they consider important when selecting or developing representations for design.

Designs are intentional: designs are *intentional* in that they are inherently *about* other artefacts, even when these designs are also themselves physically constructed. Designs should be understood as *statements about* other things. Cross (2011) refers to this intentionality as ‘aboutness’. These design descriptions are likely to be constructed using different sets of materials, and therefore affording a different set of available transformations for making.

Designs are inherently partial: A design representation is unlikely to embody all the phenomena inherent in the artefact it pertains to. It can be important to acknowledge which qualities or properties of an intended artefact are explicitly represented by a particular instantiation, and conversely which remain unrepresented and invisible, yet still of importance within the design process. Designs are also partial in that they support the consideration of multiple levels of detail or resolution, sequentially or simultaneously.

Design representations also have exogenous properties: Just as some qualities of their referent artefacts may remain unrepresented by particular design representations, designs themselves, since they are constructed using different methods and materials from those which may ultimately fabricate the resultant artefact, are likely to exhibit additional,

auxiliary properties that will not be manifest in the artefact which they describe, or, are 'about'.

Good design representations inherently support change: Since perhaps their key role is to support an exploration of alternatives more readily than the eventual fabrication method of the designed artefact they refer to, it is essential that design representations are easy to edit. Woodbury and Burrow note that this relative ease of editing partial designs by addition and subtraction is a key desirable exogenous property required in good design representations. To enable these representations to be computable, a disciplined notion of change is also necessary (see Woodbury & Burrow, 2006). This highlights that the exogenous properties of representations are not merely a necessary sacrifice to an expediency of process, but rather that representations which explicitly deviate from the material logic of the intended physical objects represented by designs can be of active benefit. When developing representations to support particular design tasks, it may therefore be helpful to consciously consider which exogenous properties could be desirable.

Representations

In many design fields, visual and spatial methods are at least as significant as written or verbal communication and representation. For design activities involving visual and spatial properties, shape descriptions play an important role in supporting design development. Among these, unstructured shape descriptions- where parts within compositions are not fixed- appear to most readily support design exploration. Of all design media which afford such descriptions, 2d sketching is arguably the most prevalent. Through mark-making upon a surface, sketching affords the creation and alteration of shapes. Its central role in supporting design reasoning is explored within an extensive literature (Do & Gross, 1996; Do et al., 2000; Goldschmidt, 1994; Goldschmidt & Porter, 2004; Purcell & Gero, 2006; Prats et al., 2005).

A key property of unstructured shape descriptions is that they also support the visual emergence of new shapes perceived by designers as elements overlap or interact. Stiny (1991; 2006) demonstrates how a continuous visual merging and division of shape elements can be formally computed by their various shape algebras. Other design theorists have differently defined emergence as an active skill possessed by designers which affords their

perception of emergent shapes- rather than primarily as a property of shapes themselves (Oxman, 2002).

The changes that can be made to a design depend on the properties of the particular combination of media and materials selected for its representation. Theoretically, unstructured shape descriptions support the limitless alteration of design compositions, through the addition, subtraction and transformation of shapes. In practice, the physical properties of the materials employed in their instantiation may impart practical limitations.

However, certain material combinations, such as ink or pencil and paper, readily support the creation of unstructured shape descriptions where visual reinterpretation is freely afforded. In the material world, these two-dimensional surfaces support the making of marks and lines, and this affords a particular kind of shape emergence. Shape Grammar formally models these properties, allowing transformations made within a particular mode of representation to be formally computed. Other, more structured modes of representation may make certain types of shape change more difficult to implement, although these approaches may come with other benefits.

Designers employ a wide range of techniques for describing and exploring new ideas, and are skilled at making value judgements based on incomplete descriptions. Through a process of making incremental changes to representations of designs, designers often recognise and exploit opportunities to make design adaptations which affect the qualities, properties, or behaviours of existing design instances. To achieve this, they must predict or otherwise recognise the potential to encourage desired qualities in unresolved designs.

Goldschmidt (1994; Goldschmidt & Porter, 2004) describes how visual representations of designs are employed not only as externally encoded representations of mental models, but also as prompts which help designers to generate new ideas for design modifications and developments through visual reasoning. Through examining external representations, designers make value judgements which then inform alterations both to physical representations and to their own mental models. Since the chosen visualisation media necessarily influences how design descriptions may be constructed, and also the ease with which alterations can subsequently be made, the selection of appropriate tools and

techniques at each stage of the design process is of critical importance in design practice. Particularly at early design stages, it is critical that the chosen medium enables the rapid construction and iteration of useful descriptions of design ideas.

Physical modes of representation which allow design changes to be expediently made can support a rapid, iterative process of search and exploration. Woodbury (2006) questions whether the typical search strategy of *breadth first, depth next* which Akin (2001) finds to be a characteristic of the problem-solving techniques of architectural designers could in fact be a direct symptom, not of the types of design problems they are addressing, but rather the means of design representation (in this case pencil and paper) used to interrogate such problems. Woodbury further provides examples of cases where the use of computational techniques has helped to shift the relative cost of depth versus breadth of exploration, leading to alternative exploration styles.

Woodbury (2006) notes that the accessibility of new designs is a direct function of the available operations at hand, within a particular type of design representation. He notes also that recorded patterns, tracing the sequences of operations which construct known designs, can be repeated and redeployed as starting points from which to create new trajectories, potentially affording access to new design possibilities.

Many techniques for design representation make use of shape descriptions, but these are only one mode of representation used in design. Descriptions of both shape and structure can themselves become abstract. Some design methodologies avoid the use of shape-based descriptions entirely: the Function-Behaviour-Structure model, for instance, generates a shape-free structural description which contains no geometric information (Umeda et al., 1990; Gero, 1990). Shape-free abstractions, such as graph-based methods for describing structure, may also provide useful insights and support synthesis (Shea & Starling 2003; Schmidt et al., 2000; Helms et al., 2009). In design practice, where designs are represented using complex, compound representation modes, descriptions of structure may themselves be overlaid or attached to shape descriptions (Stiny, 1990). Linking descriptions together can allow the effects of changes in one mode to be observed in another, providing a better understanding of their design implications.

Action and evaluation in design exploration

Simon (1969) proposes that evaluating the suitability of generated design concepts is of comparable importance to the task of creating them. His 'generate and test' paradigm outlines temporally distinct activities of synthesis and analysis. However, when design is perceived as an iterative process involving external representations, the evaluation of design possibilities, not only once complete designs are reached, but at various stages within that process of design development, also becomes possible to consider. Depending on the problem posed, some properties of proposed designs can be calculated or measured, determining which design option is most suitable according to various objective parameters.

Visual computation is a key aspect of design reasoning, involving direct engagement with visual representations of designs. Direct consideration of visual descriptions may prompt the designer to apply changes to those descriptions, whilst simultaneously assessing their effects on various design properties. At each interactive step, changes that are deemed successful are retained, whereas unsuccessful changes are reversed.

In his seminal paper "kinds of seeing and their function in designing", Schön (1992) studies records of studio-based episodes of design activity, where architects and architecture students develop design ideas using two-dimensional sketches. Considering how these activities inform design reasoning, he suggests a model for design as a materially situated process of action and reflection. Schön and Wiggins (1992) suggest that design exploration relies upon a *reflective conversation with the materials of the design situation*, where designers engage in an iterative process, requiring interaction with external design representations. Visual and spatial reasoning informs design moves which tentatively change aspects of design descriptions, so that the effects of these changes may then be qualitatively assessed. Actions make changes to external descriptions of designs. Reflection subsequently involves visual examination, to recognise emergent shapes within designs and consider their formal and semantic properties. Reflecting on the desirability of properties as they emerge both identifies and motivates opportunities for further actions or changes (Schön & Wiggins, 1992; Schön, 1992).

Schön (1992) suggests that designs are developed through a process of 'seeing, moving, and seeing again'. Material exploration relies on moves or actions, which materially alter

designs. The first 'seeing' notices an opportunity to apply a material transformation. Moving then applies that change materially. A second seeing considers and evaluates the various properties of the altered design, to accept or reject the alteration. This approach distinguishes between two fundamental types of design activity: those involving cognitive actions of observation, interpretation and reflection (types of seeing); and those which involve physical or material actions which make changes to designs (moves).

Upon closer consideration, from a rule-based perspective, it appears that two separate activities take place within Schön's first 'see'. Firstly, the designer recognises an opportunity to act, by applying a particular move or action. The designer then also seems to mentally simulate the effect of that alteration upon a property of interest, and anticipates that this move will have a beneficial effect. Therefore, in addition to making moves using the tangible representation, the designer also possesses at least some capacity to construct internal representations of designs, where the effects of moves can be to a certain extent predicted and tested, to anticipate their effects on certain properties. Through further considering the effects of the selected move on the material representation, Schön's second 'see' is then a reflection or evaluation, of its unanticipated effects upon various properties of interest.

Using both internal and external representations, skilled designers are able to simultaneously consider the effects of design changes on a range of design properties. The capacity for anticipation, when selecting design moves, is a key distinction between the reasoning activity of a designer, and Simon's (1969) sequential, computational approach. The 'generate and test' approach seems to correspond to the latter two steps of 'see-move-see', since the ability to first filter among available actions, according to some predictive capacity, is relinquished. Another key distinction is that the designer's goals need not be fully defined at the outset: initially, only a few properties may be considered, but other properties (both desirable and undesirable) will naturally be brought into consideration as they emerge, as designs are developed. Whereas, for generate and test, both actions to alter designs, and properties of interest for computational evaluation, must be fully prescribed at the outset.

Comparison with a formal computational approach also raises the question of where ideas for suitable moves come originally from. Schön notes that design changes are

frequently made in response to undesired emergent properties, or when a certain desired property is found to be lacking. But a more general discussion concerning the kinds of moves that might be made, or how particular moves are selected from among available options, is notably absent. From a rule-based perspective. Stiny (2011) suggests that ideas for specific moves come from classes of general schema, which can be instantiated as needed, for a particular situation.

It is instructive to compare the programmes set out by Stiny (2006) and Schön (1992) as well as their relationship to the overarching conceptualisations available in Function, Behaviour Structure (FBS) theory (Gero, 1990).

Stiny's rules for seeing and doing with shapes describe visual calculations with non-symbolic elements, of greater than zero dimensions (Stiny, 2006). Schön's first 'seeing', where the designer recognises an opportunity to apply an action, is at least in part equivalent to Stiny's identity rules, or 'useless rules' (Stiny, 1996; 2006; Jowers et al. 2011). These rules recognise shapes within designs as sites where transformations may be applied. Schön's 'moves' are also largely equivalent to the doing rules of Stiny's shape grammars (Stiny, 2006). Sets of shape rules can be combined to form grammars, which can formally define a set of designs which share certain styles or properties. Specific shape rules can also be simplified, to form more generalised schema.

But whilst the shape grammar approach supports continuous visual reinterpretation of designs, it does not explicitly consider how shape descriptions of designs may be 'about' other things, where a wider range of properties may be of interest. For shapes or marks made on paper to be construed as about designs, it appears that designers possess the ability to overlay other linked representations (see Stiny, 1990) upon these two-dimensional drawings. It may be possible to interpret marks as shapes and shape configurations in a number of ways, which Schön refers to as spatial *gestalts*. But there are further symbolic or semantic interpretations attached to interpreted shapes. In architectural practice, for instance, plan view drawings, where lines represent walls and enclosed shapes represent rooms, are an accepted mode of representation. When examining a plan or drawing in U_{12} , the experienced architect seems able to extrapolate the effects of changes into a personal mental model of the proposed building in U_{33} .

In other modes of making, observations which support reflection on design properties may require more than just visual examination. For kinematics, tactile examination is also important, and spatial interaction or direct handling is often necessary to examine three-dimensional objects in the round. Knight and Stiny propose that distinct making activities can be characterised by particular types of making and sensing, potentially involving a full spectrum of senses (Knight & Stiny, 2014; 2015).

Schön describes designing as a 'cumulative process of discovery', where practical design activity does more than develop designs- it also develops designers (See also Lawson, 2004). Part of the learning afforded through actively engaging with material design representations may be a growing awareness of the applicability of Stiny's schema, in different situated, material contexts. Design, as a conversation with materials, seems to rely on a capacity to notice new material opportunities, for constructing and altering designs, and also developing an awareness of the actions available, within a materially situated context.

2.1.2 Exploratory making

Making is the manipulation of stuff to make things. These manipulation processes can involve the use of tools. Different types and combinations of stuff can be manipulated in different ways, and different types of tools also enable different kinds of actions and transformations. General classes of transformations include shaping, dividing and joining. Particular combinations of stuff and tools afford the possibility of making particular things. Changing the combination of tools, things and stuff available for making in a given situation can alter the particular set of things that can potentially be made. In a design situation, choosing combinations of stuff and tools can affect the kinds of designs that can be instantiated, and also the transformations that then can be applied to them. Whilst making traditionally refers to shape and material-based modes of exploration, the processes of design reasoning that making supports may equally occur whilst considering and manipulating more abstract representations of designs.

In many design fields, exploratory making activities play a central role in the reasoning processes through which new designs are developed. Making appears to support the

generation of new knowledge about both the use of tools and materials, and designs themselves. Combining tools, materials and media in new ways can make new modes of making possible, and these tools and materials themselves may also be altered and modified as exploration progresses.

In the introduction to their Design Studies special issue on Computational Making, Knight and Vardouli (2015) define making activity as a process that is *time-based* (unfolding in real-time), *dynamic* (changing), *improvisational* (dealing with uncertainty, ambiguity and emergence), *contingent* (subject to chance and the unique), *situated* (within a social, cultural and physical environment), and *embodied* (engaging the (maker's) active body and sensory-motor capabilities). They emphasise that the relationship between making and design is an open question, worthy of active research: making might be construed to subsume designing, or vice versa. Knight and Stiny (2015) subsequently explain that from their own viewpoint, designing can be reframed as a particular type of making.

Physical making requires interaction with tools and materials, to create new things. In contrast to fully-automated processes such as digital fabrication, where outcomes must be fully defined in advance, making by hand involves both physical action and sensory observation and feedback, and affords the opportunity to notice and pursue new ideas and directions not conceived at the outset. In exploratory making, as in sketching, continual observation and reflection also inform actions.

Ingold (2013a) suggests that Anthropology, Archaeology, Art and Architecture all constitute arts of enquiry, each involving a specific form of making, via processes of active self-discovery. In each materially situated context, he suggests that an *education of attention* takes place, where we must first learn to see and notice things differently (Gibson, 1979). Continuous ongoing learning is then afforded through *correspondence with* the material world.

Ingold highlights how the conceptual distinction between objects (which remain static and unaltered) and materials (which may rather be transformed and manipulated readily) can influence perceived agency. From an archaeological perspective, he suggests that the idea of the finished, immutable, manufactured object belies both its process of creation, and its

subsequent, inevitable deterioration and decay. Things can rather be seen as mere glimpses of static states, amidst the unceasing processes of time and transformation which continuously direct their material flows. As such, things themselves constitute a material record of the historic actions and processes which have shaped them to their current form.

In design activity, exploratory making can be defined as a process of applying transformations to external representations of designs. Experienced designers may rely on tacit knowledge or established schema to help recognise opportunities for design changes and anticipate their effects, in order to select useful or meaningful actions in a particular situation (Lawson, 2004). However, in a new situation where prediction or anticipation is not possible, making and testing possible changes to designs quickly helps to build knowledge about their effects on various properties, leading to an evolving understanding. Particular material modes make specific kinds of transformation possible, and also make it possible to directly observe certain sets of properties.

Many modes of media and material combinations are employed when developing designs, and these different modes of making can make certain types of properties easier to examine. When designing objects with moving parts, considering motions is a primary concern, and physical model-making seems to play a particularly important role. Static design representations afford limited opportunity to examine motions, whereas functional physical models allow them to be directly experienced.

Functional design concepts may contain numerous inconsistencies at early design stages, and constructing functional physical models necessarily requires a certain level of design resolution. Physical 3d media are in general less tolerant of ambiguities and inconsistencies than their 2d counterparts, and early iterations may first explore routes to achieving basic functionality, in order to permit the behaviour of a design concept to be properly examined. Therefore, in order to explore relevant design behaviours with greater expediency, designers often begin by examining existing physical objects with similar functions or behaviours.

When making design models, there is a key distinction between whether a change can be made to an existing model, and whether an entirely new model needs to be constructed to appreciate a change. For physical models, additive or subtractive changes to physical shapes

may be possible whilst considering an existing model. In contrast, drastic structural changes may require the construction of an entirely new model. For computational descriptions of designs, constructing entirely new descriptions may not be as costly, but it is still true that a change that can be reached through a single move or rule is more accessible than one requiring a complex restructuring.

2.1.3 Function and behaviour

Designs can be explored in response to perceived needs or functional requirements, and a particular design may fulfil several functions simultaneously. Many formal design methods attempt to clarify functional requirements before identifying or constructing possible behaviours through which to deliver desired functions. However, these approaches are only viable in areas where the relationships between behaviours and function are well understood. In more fluid, exploratory design approaches, rather than setting out to solve a particular problem directly, goals may be to discover novel designs, and designers may construct and explore structures or systems with interesting behaviours in parallel, to considering their possible functional applications.

In this thesis, since we are considering 3d designs with moving parts, we are interested in furthering the description and explanation of design reasoning for the general case of three-dimensional, material space.

The overall shape or form of a design may contribute directly or indirectly to its functional behaviour. In designs where distinct parts move and interact to achieve design functions, both the shapes of parts and the relationships and connections between them within a wider structure or configuration will affect the emergent behaviour. The materials of which parts are composed also exhibit distinct behaviours which will influence how a design behaves.

The Contact and Channel Model (C&CM) of Function developed at Karlsruhe University provides a pragmatic approach to constructing abstract descriptions of the mechanical functions of existing products. The approach dictates that all design functions are achieved

primarily by interactions between pairs of adjacent surfaces, which are referred to as Working Surface Pairs (WSPs) (Albers, Burkardt & Ohmer, 2004). The most basic WSPs in mechanical design are lower kinematic pairs, whose behaviours are well understood (Reuleaux, 1876; Hunt, 1978). When considering how parts of designs achieve certain functions, it is not necessary for both surfaces in a pair to be contained within the design, since the actions which ultimately achieve desired product functions often entail the transmission of forces or motions across the boundary of the product, and therefore only one of the surfaces in a WSP may be contained within the design itself. A detailed picture of the context of use is therefore important for understanding how the design functions. In much the same way as detailed in Shapiro and Voelcker (1989), necessary interactions between pairs of functional surfaces inform and constrain the geometries of sections of component surfaces, leaving the remaining geometry constrained only by secondary considerations. The C&CM model considers the surrounding structure which supports components and thereby affords the interaction of WSPs as a second structural level- parts of a design fulfilling this role are referred to as Channel and Support Structures (CSS).

In the wider literature of Design Theory and Synthesis, discussions of function tend to make a distinction between functions and behaviours, where functions embody design goals, and behaviours describe the particular manner in which solutions are instantiated. This behaviour is often referred to in the literature where relationships between Function, Behaviour and Structure are discussed (e.g. Umeda et al., 1990; Gero, 1990). An important distinction must therefore be made between this and *kinematic behaviour*, as a description of the geometric motions exhibited by a mechanism or its parts.

In the context of the F-B-S methodology, the term describes the potential for a particular building-block to achieve some desired energy transformation required within a wider system. In these situations, a top-down approach to design is assumed, where descriptions of suitable behaviours can be routinely derived from functional requirements, and structures which then achieve these behaviours are then readily constructed. This literature forms an important background to this current work, but these energy-based approaches are of limited use when describing the detailed *kinematic* behaviour of designs, where how structures achieve certain behaviours is not straightforward.

In kinematic design, although energy may be implicitly transferred through kinematic motion, and indeed this may be the primary functional goal of the design, it is rather the precise qualities of these kinematic motions, and exactly how they arise, that is of primary interest.

A third use of the term *behaviour* may be found in discussions of the qualitative properties of designs. As a general definition, behaviours are understood to be descriptions of the particular actions or mannerisms which a system or entity demonstrates in response to various internal or external stimuli. Here, designers may use analogies, metaphors, or even physical objects themselves to communicate and explore ideas about the possible behaviours of design concepts. A primary motivation for the thesis is how both generative descriptions and tangible physical models might be interpreted as descriptions of behaviours in various design contexts, both in the kinematic sense and more broadly. The approaches reviewed above offer a range of routes for examining the effects of particular geometries on kinematic behaviours, which could form useful tools within a generative approach to explore the effects of design changes on the behaviours of spatial mechanisms. In large measure, they are a basis for the methods developed in this thesis. These latter complement the more established methods of design and analysis by concentrating on physical making in exploring and explaining kinematic designs. In particular, the thesis extends rules for generating designs and shapes to kinematic designs especially, through the development of making rules.

2.2 Models in design

In his book “The Nature of Explanation” Kenneth Craik (1943) proposes a hypothesis for the basis of human reasoning. He suggests that human minds possess the ability to convert particular physical stimuli, (or, more precisely, the patterns of neural responses such stimuli elicit) into symbols, and these can then be used to reason about physical events which occur in the real world, predicting their outcomes before they happen. He suggests that, through repeated experience of routine events in the external world, rules of association between objects are identified and learnt. These rules then support abstract reasoning about the

likely results of interactions between certain objects, allowing real-world events to be predicted with a reasonable degree of success.

2.2.1 Models and explanation

Writing at a time when mechanical devices were just beginning to demonstrate some of their computational potential for numerical problem-solving, Craik hypothesises that similar mechanisms could perhaps also underlie human cognition, suggesting that both conscious and unconscious reasoning may be the resultant function of finely tuned arrays of mechanical (read as computational) processes.

Further, he boldly challenges philosophical propositions that defend the existence of consciousness as a distinct and separate entity that may prevail in the absence of any physical bodily material. His assertions for a reasoning process include:

- 1) Translation of external processes into words, numbers or other symbols
- 2) Arrival at other symbols by a process of 'reasoning', deduction, and inference
- 3) 'Retranslation' of these symbols, either into external processes (for example design), or at least towards a recognition of the correspondence between these symbols and external events (as in, realising that a prediction is fulfilled).

A recent and perhaps more conventional analysis of models and their material counterparts is provided by recent work of Roman Frigg and others. (Frigg & Nguyen (2018a; 2018b; Frigg & Hartmann, 2012). Models can also be related to scientific explanation as reviewed in Frigg and Hartmann (2012).

Other related work on models addresses specifically the roles and purposes of models in Design, drawing comparisons with models as analysed in the Philosophy of Science. In particular Eckert and Hillerbrand (2018) pick out the role of models as preliminary theories or as substitutes for theories (see Leplin, 1980), and the way that models can be formulated as a sort of preliminary exercise in theory. A related idea is the functionality of a false model,

which can help to answer questions and identify critical variables in more realistic and perhaps more complex models (Wimsatt, 1983).

Models can be seen as tools to explore causal relations (Woodward, 2003) which explain phenomena, or as in Craik, where an explanation comes from constructing a model, and in some sense, the model *is* the explanation. However, the explanatory power of the model can also be related to its fictional nature (Bokulich, 2009).

For kinematic or mechanical design, it appears that physical models play a significant role, in both exploration and explanation of kinematic behaviours (Lipson et al. 2005; Harrison et al., 2011; 2015).

2.2.2 Creative exploration

Creative behaviour has been likened to a process of search and exploration within a conceptual space of possibilities (Boden, 2004; Wiggins, 2001; 2006a; Ritchie, 2005; 2006; 2012). When considering how generative methods can support the creative discovery of new design possibilities, this idea— of searching within a conceptual space— provides a potentially useful model for creative exploration. Since design is a particular kind of creative behaviour, the concept can be more specifically examined and developed, as a possible model for design activity.

A search paradigm for design exploration is a particularly attractive proposition since many search tasks can be readily supported by computational methods. If the validity of a search metaphor is accepted, important questions arise concerning the nature of a conceptual space of design possibilities, as well as available modes of traversing it.

The premise has inspired inquiry from several research perspectives, from which several distinct models for creative search and exploration have developed. Here we examine and compare two distinct yet seemingly complimentary approaches. The first of these is a model for general creativity, proposed by Margaret Boden (2003).

Creativity as search

Boden's work on creative behaviour proposes that the concept of creativity can be described as a process of search, exploration and discovery. This proposition has been further explicated by Wiggins, Ritchie and others, who consider how a formal model of a search space can be derived from Boden's approach (McGregor et al., 2014; Ritchie 2005; 2006; 2012; Wiggins, 2000; 2006a; 2006b). Since these spaces are explored using transformational rules, they have also discussed how a corresponding rule space might be both defined and searched, to discover those transformations. Their work focuses primarily on creativity within musical composition (see for instance Wiggins, 2019), but in the context of shape composition, Stiny's schema for shape rules (2011) can be interpreted to define such spaces of rules, for a design context.

Boden proposes that creative behaviour can be viewed as searching within a conceptual space of possibilities, to discover new (and therefore creative) outcomes. She also proposes that rules can be used both to define these conceptual spaces, and to describe techniques for exploring them. She suggests that, since different individuals each have access only to their own personal processes of exploration, some may succeed in reaching new areas of the space which remain inaccessible to others, and thereby make new creative discoveries. She also makes a distinction between *psychological* (sometimes also called *personal*) creativity, where an individual alights on an idea or concept they had not previously encountered personally, and *historical* creativity, where an agent within a particular culture makes a discovery that is new not only to their own personal domain of awareness, but also to that of their collective society. However it is Boden's description of how creative exploration *processes* operate, rather than their relative novelty of their discoveries, which we shall consider here in more detail.

Wiggins (2001) sets out to define a more precise model of Boden's descriptive framework, and based on Boden's original conception of creative exploration as a kind of search, Wiggins and Ritchie develop a formal model for creative search (Wiggins, 2006a; Ritchie, 2005; 2006; 2012). In devising this formal (*AI*) description of Boden's descriptive hierarchies, Wiggins outlines how creative processes might be formally modelled as search procedures within a conceptual space, explaining how the sequences of actions and decisions which traverse that space to reach new creative outcomes can be described using sets of rules. The

trajectories of these creative rulesets within the search space are further guided by a second ruleset which, based upon various quantitative and qualitative criteria, evaluates design outcomes as they are encountered.

A further aspect of Boden's hierarchy is a distinction between transformative and exploratory forms of creativity, and Wiggins outlines how both forms interact when operating within a conceptual space of possibilities. Boden identifies these two distinct types of creativity as governing the demarcation and search respectively, of a *conceptual space* containing both partial and complete creative possibilities (Boden, 2003). *Exploratory* creativity concerns the exploration of a particular conceptual space, and *transformational* creativity concerns the adjustment of the rules which demark the boundaries of that space itself. Since *transformational* creativity results in a paradigm shift, affording access into a transformed search space, Boden considers this creativity type more likely to afford new discoveries, and therefore most powerful.

However, in formalising the model, Wiggins states that any given *conceptual space* C is merely a particular subset of a complete universe U , containing all complete and partial possibilities reachable by any conceivable creative technique. Whereas Boden only loosely defines definitional rules, Wiggins explicitly defines two distinct rule types: the first, R , defines a particular conceptual space as a subset of U , and the second, T , traverses this conceptual space in a particular way to identify individual design possibilities (see Figure 2.1a). Since all forms of creativity now entail searching within U , both of Boden's creativity types are shown to be merely different approaches to the same problem— namely searching the universe of all possible things to find creative solutions. Furthermore, since not all places in the conceptual space C may be reachable by a given T , transforming T whilst maintaining R is also shown to enable the discovery of new possibilities. Wiggins suggests that it is rather through transforming within both T and R simultaneously that new discoveries are made, through reaching previously inaccessible members of U .

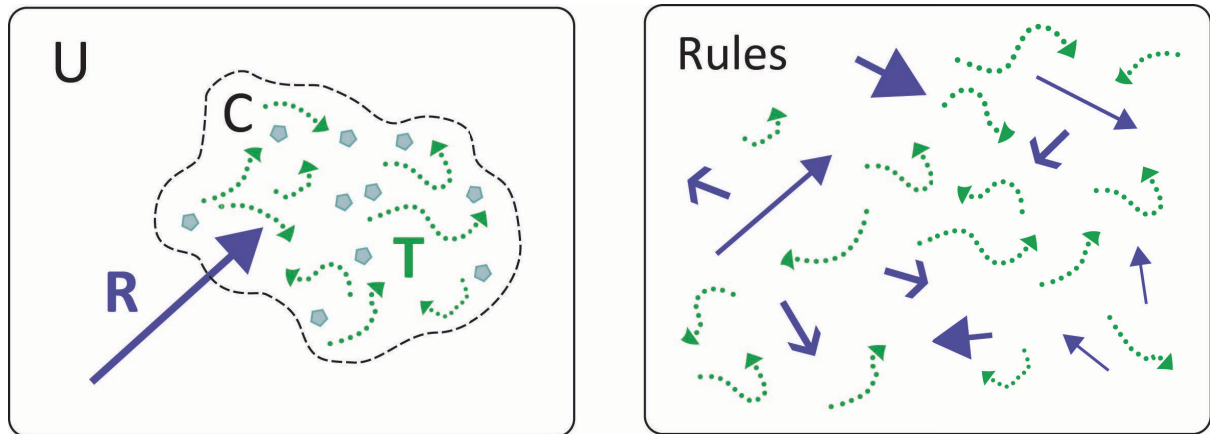


Figure 2.1: (a) Searching a universe U of creative possibilities using (b) different types of rules, which themselves theoretically exist in an abstract, searchable space (Wiggins, 2001)

Wiggins describes how transformational creativity is in fact similar to exploratory creativity, in that it involves techniques for searching a space of possibilities, but now this search is instead performed at a meta-level, where the purpose of exploration concerns identifying successful rule-sets, or, *generative* descriptions, rather than merely the direct physical descriptions of individual outcomes (Figure 2.1b).

Design spaces

The second approach we examine was developed initially by Robert Woodbury (Woodbury, 1991), and applies specifically to design activity. Whereas Wiggins explores the implications of a search paradigm for creative activities in general, Woodbury discusses the formal implications for a search-based approach to be applied specifically to design.

Shortly after Boden's book, "The Creative Mind", was first published, the desire to harness the potential of computational search within a design context also led Woodbury to propose the concept of a 'design space model', within which exploration or search for designs occurs. His model, at first based on combining ideas from geometric design and set theory, has subsequently seeded extensive research into design spaces.

Woodbury's approach to the concept of design exploration stems from a pragmatic desire to expand the role of computational tools within the design process, with a primary interest in architectural design. He (1991) outlines a formal model for search and exploration in design,

aspects of which appear to correlate well with the more general formal model that Wiggins derives from Boden's description of creative behaviour.

Inspired by Newell and Simon's problem spaces (Newell & Simon, 1973; Newell, 1980), Woodbury sets out the analogous concept of a design space (Woodbury, 1991). In order to support computational search, his early research in this area considers how formal spaces of geometric designs can be defined. His 'design space model' combines concepts from both set theory and formal language theory, as a both a motivator and testing ground for new computational search techniques (Woodbury, 1991). In this model, design becomes the task of identifying a path through a space of possible designs. His subsequent work continues to explore this idea, but eschews unstructured sets for graphs or networks, developing a more detailed conception of design spaces where nodes which represent consecutive design states are linked by arcs which record designer actions or transformations, recording the paths through the space by which new designs are reached.

Woodbury proposed that design spaces should consist of: a representation scheme; a set of constructive operators which produce well-formed outputs consistent with this scheme; an initial state for a design problem; a statement of design goals; and a set of knowledge which applies to the design problem. However, he acknowledges that systems which possess all these characteristics are not commonly encountered, or easy to construct.

Design spaces seem to be continually reinvented and discussed in the design literature. Recent conceptions of them as improvisatory during process (Charidis, 2019) rather than an explicit design world to explore accord more closely with the way that rules and schema of shape grammars (Stiny, 2006) support contingent and contextualised design exploration. Indeed, this view aligns neatly with the ways that making actions can explore designs which are made rather than found. Kinematic designs are the focus in this thesis. Complex motions arise from multiple mobile physical connections. Interactions and dependencies become difficult to understand, except in abstracted mathematical notations, in which physical sense is lost. We examine in this thesis how to recover this physical 'explanation' for design through exploratory making.

2.3 Kinematic design

At its most basic, the design of a mechanism can be described according to combinations of the relative motions of connected parts (Reuleaux, 1876). In mechanical design, kinematics is concerned with determining the precise motions of parts, within mechanisms composed of interconnected rigid components. Kinematic approaches do not consider the forces necessary to provoke and transmit motions within mechanisms, but are rather concerned with the mobilities and geometries that afford these transmissions. Of particular concern are types of connections between rigid components, and their effects on mobility.

When designing objects with moving parts, design changes which alter shape and structure can be directly applied—alterations can be made to both shapes of parts, and connections between parts. However, the primary property of interest is motion. Motion is a function of relationships between shapes of parts and the nature of the connections between them within a design. Relationships can be complex. Since motions themselves are hard to predict, when applying changes to the shape or structure of parts within a design, it can be difficult to anticipate what effects on motions might result. Motions of parts connected by rigid joints can be modelled using kinematic techniques. Mathematical techniques can be used to predict motions in designs composed of particular connection types (Hunt, 1978; Phillips, 2007; O'Rourke, 2011), but for complex designs with many parts, this analysis may be difficult. Direct experience, however, removes requirements for prediction, and functional physical models that permit interaction and manipulation can instead enable direct examination of motions.

2.3.1 Kinematic pairs

Pairs of connected parts that allow relative motions are often referred to as kinematic pairs. Kinematic pairs are classified in various ways: according to types of connection, i.e. surface, line or point; according to the type of relative motion, e.g. sliding or rolling; or according to the type of constraint applied to the pair, e.g. mechanical or due to gravity. Lower kinematic pairs can be modelled by various surfaces and sub-shapes which are shared between two parts, whereas higher pairs exhibit only point or line contact.

Reuleaux (1876) identifies six kinds of lower kinematic pairs, where contact between pairs of surfaces is maintained throughout motion (Figure 2.2). These joint types are spherical pairs or ball joints, planar pairs (which formally are spherical pairs of infinite radius), cylindrical pairs, revolute pairs or hinges (which can be considered as partially constrained cylindrical pairs), and prismatic pairs. The final example is the helical pair or screw pair, where two congruent helicoidal surfaces maintain contact throughout motion. Both revolute pairs and prismatic pairs can be viewed as special cases of the helical pair.

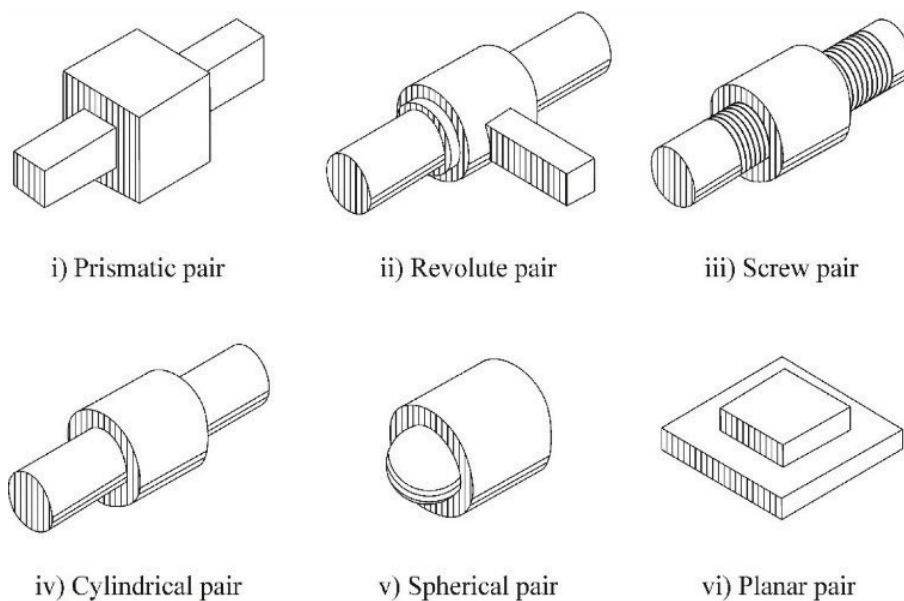


Figure 2.2 Lower kinematic pairs (Reuleaux, 1876)

Hunt (1978) describes how, since the constituent freedoms of all lower pairs can be represented by various combinations of one-dimensional rotations and translations (ie revolute and prismatic pairs), various substitutions can be made without altering the kinematic behaviour of the system. As a result, all systems composed of lower pairs can ultimately be entirely represented by helical pairs. All surfaces found in kinematic pairs are also special cases of helicoid. There are no other kinds of surface where surface-surface contact can be maintained during motion. Formal parametric descriptions of each of these

joint-types can be defined, and geometry changes must preserve relationships such that surface pairs remain congruent to preserve continuous contact. In real mechanisms, however, small variations within certain tolerances may be acceptable at contact surfaces without damaging the behaviour of the system. Since the mobilities and behaviours of lower pairs are well understood and can be formally defined, it is possible to predict mathematically the behaviour of systems composed only of these connection types.

All other types of contact between parts within kinematic systems are known as Higher Pairs, and for these, infinite geometric possibilities exist. Here, contact is between lines or points, and these are generally embedded within surfaces. Higher pairs are subject to non-holonomic constraints (McCarthy, 2000), causing the contact location to move across the surface during motion. Where components maintain a straight line of contact during motion, pairs of ruled (ie singly curved) surfaces can be described by straight lines. Pairs of surfaces can also be designed with common profiles, so that a curved contact line is maintained. Whereas lower pairs are generally closed such that surface contact is preserved, higher pairs can be less constrained, since contacts between parts constrain and guide motion to a lesser extent. Here, larger geometry changes to functional surfaces may be possible without impeding intended motions, although the ways in which changes then affect kinematic behaviours may be harder to predict. Carefully designed higher pairs can deliver complex kinematic behaviours, and precise geometries and configurations can afford particular engineering functions. Where a system contains one or more higher pairs, however, predicting overall mobilities and behaviours becomes much more complicated.

2.3.2: Kinematic chains

A kinematic chain is a series of links connected by joints, its freedom being the sum of the freedoms found at each joint (Earl, 1979). A linkage is a mechanism composed of several parts or links that are connected to one another with joints to form a closed loop. Where all joints are lower pairs, or else well-defined higher pairs such as gears, the behaviours of systems can be determined analytically. Modes of analysis vary depending on the complexity of the system, and systems restricted to sets of motions which can be represented in only 2 dimensions are significantly simpler to describe and understand. Planar linkages which exhibit movements restricted to 2 dimensions are

therefore much easier to both represent and design, and can be described using formal generative techniques, as discussed earlier. Through projecting configurations of planar linkages onto a spherical surface, spherical mechanisms with 3d motions can also be constructed (McCarthy, 2000). When considering commonly used planar or spherical linkages, methods rely on the ability to describe motions in machinery using 2-dimensional abstractions. True spatial linkages however, cannot be abstracted to 2 dimensions. The analysis of truly 3-dimensional spatial linkages therefore requires alternative, more complex approaches. Existing methods are highly analytical in character. Here however, we consider how synthetic modes, such as exploratory making of physical models, might play a complementary role. Later chapters will further consider how the actions which alter the shapes of parts and their configurations within experimental kinematic models might be formally described.

2.3.3 Mobility of mechanisms

For closed-loop planar linkages connected with revolute joints, at least 4 connected links are necessary to reliably afford motion. In 3d, generally such linkages must contain at least seven bars in order to guarantee motion with 1 degree-of-freedom. There are however some exceptions to this general condition: the set of over-constrained spatial linkages, where particular configurations with 4, 5 or 6 links exploit independent axis configurations to achieve motion (Hunt, 1978).

The standard Gruebler-Kutzbach equation, or Mobility Criterion, evaluates motions for the general case (see Phillips, 2007), but is not able to detect or account for these special instances. Rico & Ravani (2007) demonstrate why the general mobility criterion lacks the necessary detail to obtain meaningful results for over-constrained cases, and outline how these cases may be rather addressed through a more detailed analysis. Further, Hunt (1978) explains how Screw Theory provides a route for evaluating dependencies between axis configurations in spatial mechanisms, and hence the mobility of the mechanism.

Phillips explains how any instantaneous motion of a body in space can be described as a twisting motion about a particular axis, which is known as the instantaneous screw axis.

Further, any rate of change in linear or angular acceleration is the result of a force which acts along some other axis, accompanied also by a moment about that axis. For two bodies connected at a joint, equal and opposite sets of forces and therefore equal and opposite rates of twisting, or reciprocal screws, can be said to affect each body.

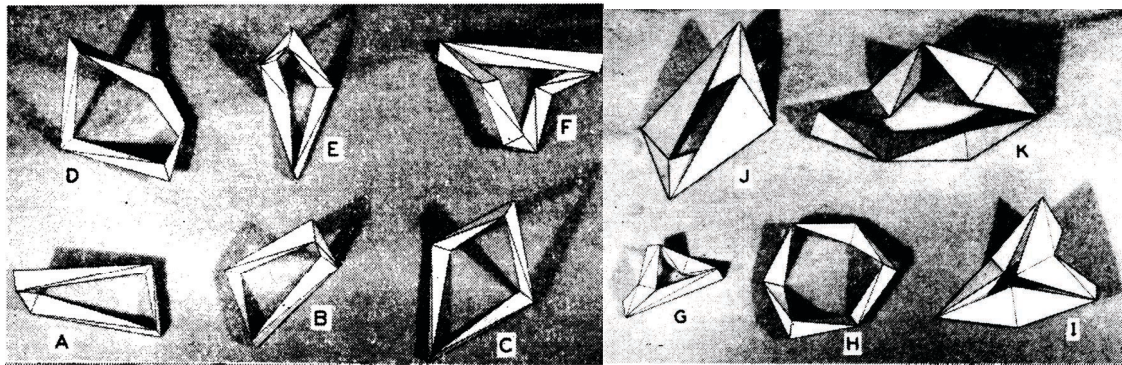


Figure 2.3 Examples of over-constrained linkages (Goldberg 1980)



Figure 2.4 3d-printed models of over-constrained linkages (Harrison et al., 2011)

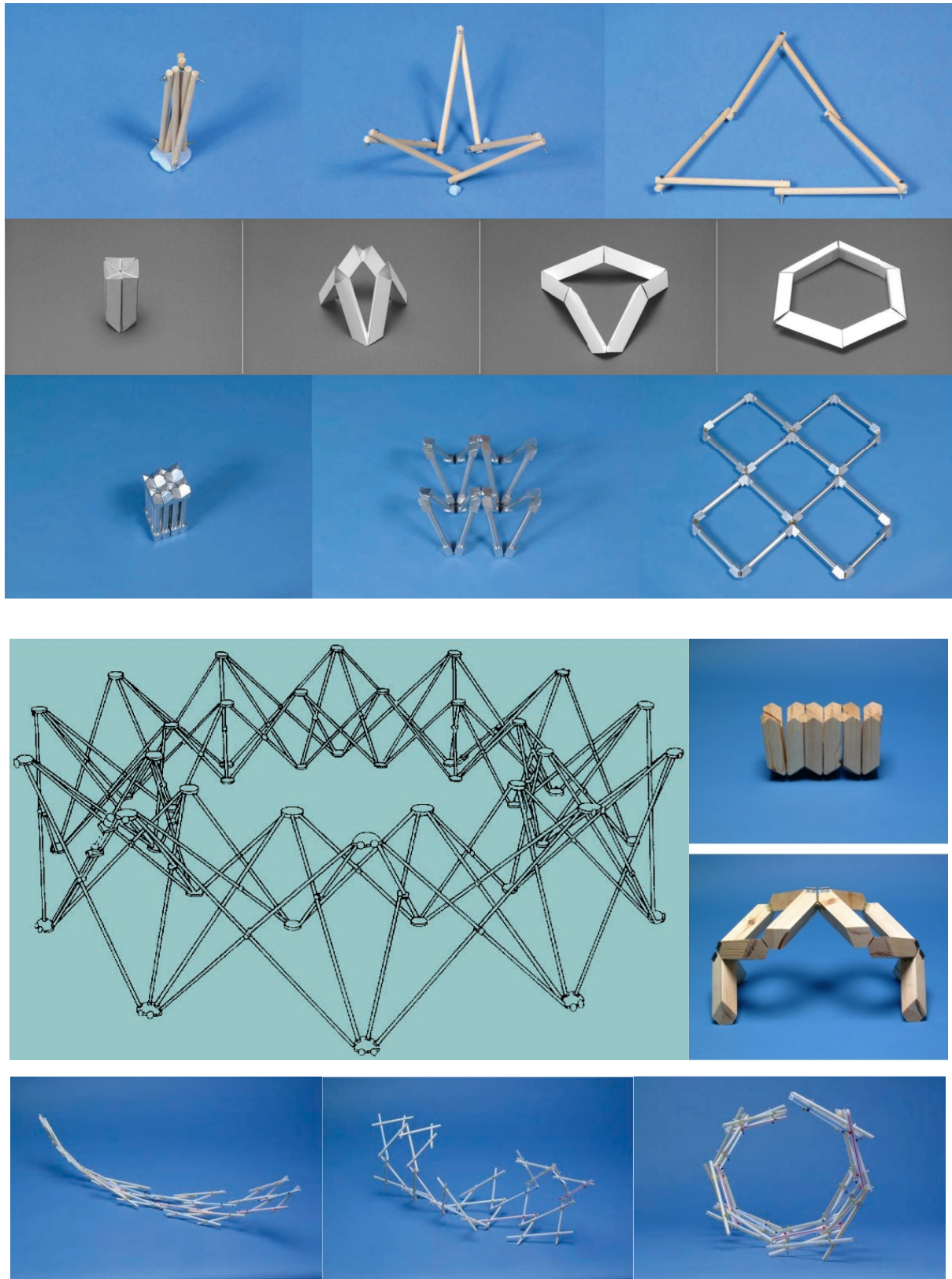


Figure 2.5 Models of linkages, and networks forming deployable structures. (Chen, 2003; You; 2007)

Mavroidis and Roth (1994) categorise all known over-constrained linkages into 4 distinct classes, namely: Symmetric mechanisms; Bennett based mechanisms; Combined special geometry mechanisms; and over-constrained mechanisms (such as in Figure 2.3). In general, over-constrained mechanisms consist of multiple loops, and have multiple degrees of freedom. However, since these multiple-loop cases are shown to be combinations of several single-loop instances, attentions of researchers have concentrated on single-loop configurations, such as those in Figure 2.4 (Goldberg 1980; Mavroidis & Roth, 1994; Harrison et al., 2011).

Chen and You demonstrate that through analysing networks of configurations of known single-loop linkages, new multi-loop configurations (Figure 2.5) with interesting functional properties can be discovered. (Chen & You, 2005; Chen, 2003).

2.3.4 Prototyping kinematic designs

In disciplines which consider the analysis or synthesis of complex objects or systems composed of several interacting parts with relative spatial motions, tangible physical models play a central role in affording an understanding of mobile behaviours. For Kinematic design, Phillips stresses the importance of descriptive drawings and physical models for constructing an understanding of the kinematic behaviours of spatial mechanisms (Phillips 2007). In functional design more generally, both physical models and existing objects are important tools for understanding and explaining design behaviours. In recent years, digital fabrication tools, and additive manufacturing in particular, have afforded designers the opportunity to rapidly convert computational representations into physical models through which to assess and verify various tangible properties or behaviours. Literature in Design Theory (see section 2.1) has considered more generally how interaction with tangible representations affords and supports visual reasoning, but the roles played by physical objects in particular, for supporting visual computation, have yet to be formally examined in detail.

When developing designs, many modes of media and material combinations may be employed, and these different modes of making can make certain types of properties easier to examine. When designing objects with moving parts, considering motions is a primary

concern, and physical model-making therefore plays an important role. Static design representations afford limited opportunity to examine motions, but functional physical models allow them to be directly experienced. When considering designs with spatial motions, 3d models can help designers explore and understand their behaviours.

Since visual examination of static descriptions often does not adequately support consideration of how connected parts within designs move, in many disciplines which involve complex 3D artefacts, the construction of physical models for the purposes of visualisation and tangible interaction constitutes an integral step in developing an understanding of their structure and behaviour. This need for interaction with a three-dimensional model becomes particularly crucial when the artefact has connected components which move and interact.

Within a generative design process, visualisation carries additional importance: the design outcomes of a generative process are constructed by deploying a specified sequence of rules, and therefore visualisation is the primary route through which the designer experiences final or intermediate designs. Therefore, since the designer only encounters the design once it has been tangibly constructed, they may not yet have a clear appreciation of the causal relationships between generative rulesets and the designs that they produce. Interacting with a physical model of a design generated with a particular rule-set affords an appreciation of the kind of physical properties these rules can be expected to produce. This may subsequently inform the adjustment of the chosen sequence of rules to realign generation with design requirements, increasing the likelihood that a generated design will possess the particular properties required by a given design situation. For all but the simplest of cases, the construction of rules is likely to be an iterative process, continually refined as an understanding of how the rules behave and interact is acquired through examining the designs they produce.

Digital fabrication affords an expedient route to make, test and iterate on physical models. For constructing an understanding of existing mechanism designs, rapid prototyping is shown to have key benefits over virtual representations, affording an intuitive appreciation of factors affecting functional performance through tangible interaction (Lipson et al. 2005). The combination of automated manufacturing techniques with generative design processes

also has significant wider potential, both in kinematics and other fields (Wang & Duarte, 2002, Shea et al., 2010). Pollack and Lipson employ 3d printing to realise physical representations of robot designs constructed using an evolutionary synthesis approach (Pollack et al. 2000). Evaluation and selection which inform development during generation are primarily executed automatically through virtual simulation. 3d printed models are then used to validate performance results derived using virtual simulation techniques. Physical models help to both visualise and validate these outcomes, but do not actively inform the generative design approach.

Sass (2007) describes the role of rapid prototyping (RP) as a visualisation tool supporting the construction of rules within a generative process which employs construction grammars for the synthesis of functional structural designs.

In robotics, physical model-making also plays a central role in design development and verification. Early experimentation stages require prototypes which provide basic functionality, and the unique features of RP offer effective routes to realising functional designs. Ebert-Uphoff et al. (2005) explore the benefits of RP within design for robotics, using RP models to demonstrate, validate, experimentally test, modify and redesign mechanism prototypes. A database of lower kinematic pairs was produced, from which prototype mechanisms were constructed.

Prototypes were found to help to provide insight into mechanism functionality, and to communicate it to others. They were also found to validate geometric and kinematic properties such as mechanical interferences, transmission characteristics, singularities and workspace. In particular, physical models help to visualise singularities, which can be predicted mathematically but can be difficult for designers to imagine, and also to check for mechanical interferences in complex assemblies. Their work also explores practical issues surrounding the RP production of rigid kinematic joints, such as the effects of geometry gaps and clearances necessary when printing ready-assembled designs' surface and process tolerances, build orientation, material compliance, assembly techniques. These technical considerations are explored by Rajagopalan and Cutkosky (1998).

Experimentation in this thesis further applies both manual and digital making techniques, within an exploratory generative process for kinematic objects. Through material design exploration, how physical objects can support visual and spatial reasoning through material computation is also explored.

In physical making, there is a key distinction between whether a change can be made to an existing model, and whether an entirely new model needs to be constructed to appreciate a change to shape or structure. For physical models, additive or subtractive changes to the shape of parts may be possible whilst considering an existing model. In contrast, drastic structural changes may require the construction of an entirely new model. For computational descriptions of designs, constructing entirely new descriptions may not be as costly, but it is still true that a change which can be reached through a single move or rule is more accessible than one requiring a complex restructuring. In the episodes of design exploration we report upon, models themselves provide a physical record of systematic material exploration, within a kinematic design space.

2.4 Rules for design

The idea of developing rules for design first emerges within early AI research. Since the term is often used to imply restrictions and constraints, the idea of defining rules for a creative application may at first seem counterintuitive. Design is generally perceived as an open-ended activity, dealing with ill-defined problems. But for the early AI community, rules were intended to explicitly describe key aspects of design activity, so that computers might generate designs automatically. Generative design descriptions use rules to define formal sets or sequences of transformations which, when applied to tangible media or materials, construct tangible descriptions of designs. They can therefore be viewed as sets of instructions for making physical artefacts, and are useful for succinctly defining synthetic processes.

Generative descriptions have the significant benefit of supporting formal computation, and the computational exploration of design alternatives may proceed far more rapidly than through conventional approaches alone. Attempts were therefore made to describe and

explain the actions and reasoning of designers, in order that design processes might be encoded within explicit instruction sets. This, however, proved a somewhat ambitious undertaking. Design activities may involve multiple complex, parallel steps, which are not always easy to detect visually, and may involve subtle actions and subjective judgements which defy ready verbal or symbolic description. Further, much of design is situation specific, and producing appropriate descriptions of the situation may form an important part of the design task itself.

Designers appear to develop designs using both external models and internal mental representations of design ideas. Since the latter are difficult to study, understanding how they operate represents a significant challenge. Actions which transform external representations, however, are somewhat easier to observe, and in some circumstances, clues may be uncovered concerning the relationships between these external models and other cognitive representations. A number of models and theories of how designers reason about designs through interacting with external representations have since been developed (Schön, 1992; Do & Gross, 1996; Purcell & Gero, 1998; Goldschmidt & Porter, 2004). This literature primarily considers design situations which employ 2d representations such as sketching. General types of rules for describing design activity have also been proposed (Stiny & Gips, 1972; Stiny, 2006; Knight 1983a; 1983b; 1983c). For some specific examples, rules have been implemented computationally, to automatically produce sets of designs (March & Earl, 1997; Stiny & Mitchell, 1978; Lipson & Pollack, 2000; Shea et al., 2010). For supporting clearly-defined tasks where precise formal descriptions can be readily developed, the use of computers in design has proved successful (Cagan & Antonnson, 2001). However, acts of observation and interpretation which seem natural to designers can still be difficult to describe and mimic computationally (Knight, 2018). When exploring new uses for formal computation in design, immediately fruitful paths appears to lie within design tasks that designers find challenging, but where precise actions and objective evaluation can be systematically supported.

For many design activities, ongoing reflection and evaluation of emerging design properties may be an informal activity following tacit, unarticulated rules. Generative instruction sets, however, afford limited opportunity to appreciate the tangible properties of the designs that they describe. To support formal computation, activities and processes which support the

iterative evaluation of designs may also require explicit description. Design actions and reflections often occur closely interlinked together, but for computational processes supported by abstract generative descriptions, opportunities to construct tangible designs and consider their properties directly need to be explicitly created, and analysis and synthesis become distinct.

Formal techniques for describing detailed design activities, which help to identify and articulate various activities and actions, are not only valuable for imitating and modelling the behaviours of designers. They also help to identify where and how design processes may be usefully assisted by computational tools. And although generative descriptions alone may not afford an immediate appreciation of the properties of the designs they describe, they can help to render visible usually unseen aspects of the synthetic processes of transformation that construct them, facilitating examination of their underlying design reasoning. Generative descriptions of making processes may also support the editing or transformation of these synthetic processes themselves, to discover new creative outcomes (Knight, 2018).

Since exploratory making is inherently a generative process, in this thesis we will consider how the steps within a making process for designs with moving parts might be described using generative rules. We also examine how formal descriptions of aspects of making can help to reveal underlying reasoning processes, and support the systematic exploration of a particular design space.

2.4.1 Shape rules and grammars

Visual computation is a key aspect of design reasoning, and involves direct engagement with visual representations of designs. Interaction with visual descriptions allows a designer to apply changes which transform and alter shapes within designs. Shape rules which both identify and transform shapes can describe these actions. Shape Grammar is a visual computation approach to generative design, which uses shape rules to construct both 2d and 3d geometric compositions (Stiny, 2006; March, 2011). Unlike many computational approaches, it employs unstructured shape-based descriptions which allow emergent

shapes perceived within designs to be readily selected and transformed. Grammars are generative descriptions comprised of shapes, rules which both identify these shapes and define possible spatial relationships between them, and labels or other operators.

Traditionally, grammars operate upon a pre-defined vocabulary of shapes, but a shape rule approach also permits the formal description of new shapes that emerge as design composition proceeds. Identity and transformation rules can be applied to select and transform parts of existing shapes within design compositions, and also to add new shapes.

Shape grammars employ sets of shape rules to define rule sequences which construct complete design compositions. A number of additional devices afford the construction of more detailed rules and schema which can be employed to numerous ends. The use of weights permits more detail to be added to rules, so that a range of distinct properties can be attributed to parts of designs (Stiny, 1992). Labels afford greater control and predictability through reducing the symmetries occurring in rules. Parametric shape rules further afford the application of transformation rules to numerous variations of existing shape vocabularies (Stiny, 1980; 2006; Tapia, 1992; Knight, 2003).

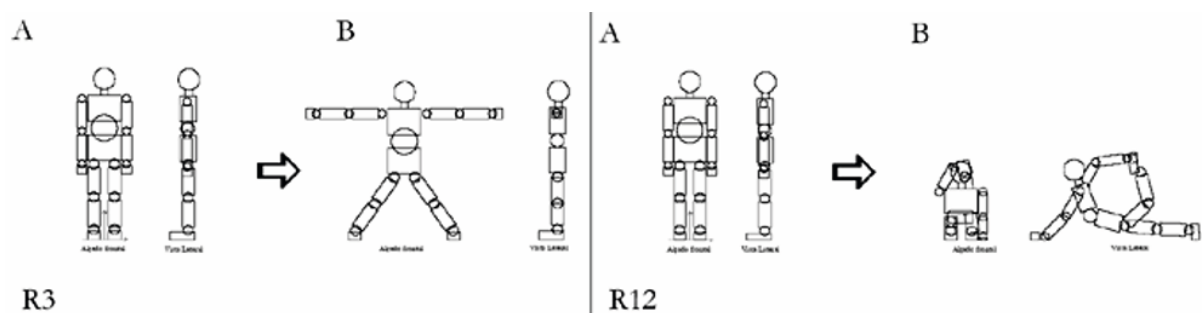
The shape grammar approach has historically seen most use for the analysis and study of sets of existing designs (e.g. Stiny & Mitchell 1978; Konig & Eizenberg, 1981; Fleming, 1987). By considering how sequences of transformations may construct the shape relationships encountered within existing geometric compositions, underlying rules can be inferred, and sets of such rules can define new shape grammars. Where larger sets of existing designs inform the definition of rules, grammars may both reconstruct both existing designs, and discover new variants.

This approach has been successfully applied to define generative descriptions for sets of designs with particular types of properties, such as distinct visual styles (Stiny & Mitchell 1978; Koning & Eizenberg, 1981; Fleming, 1987; Prats et al., 2006). Application has been largely to 2d decorative designs (Stiny & Gips, 1972; Stiny, 1977; Knight, 1980) and architectural drawings (Koning & Eizenberg 1981; Stiny & Mitchell, 1978; Flemming, 1987). In some cases, grammars may help to illuminate the design process by which the original designs were developed, but this is not necessarily the case, since there are many ways to

construct a particular design using shape rules, and not all actions will be obvious when considering the final outcome (Knight & Sass, 2010).

Although applications to date have been largely to describe visual or aesthetic properties of designs, shape grammars also have the potential to describe other design properties or characteristics. In Brown’s engineering grammars, generation explores the possible forms and features that may be produced by a set of allowable machining operations (Brown et al., 1995). Grammars producing languages of functional designs, such as simple self-supporting structures, have also been described (Mitchell, 1991).

A more recent paper proposes the use of shape grammars to describe and explore distinct aesthetic styles of movement, with specific reference to the movements of the human body within dance, yoga practice and daily routine (Piedade Ferreira et al., 2011). Connected box-like parts (Figure 2.6) represent a simple 2d mannequin, and shape rules which represent allowable motions apply transformations to these parts. Applied to the mannequin, rules describe transitions between sequences of static bodily positions or poses. Simultaneous motions of multiple body parts are not readily afforded by the approach, although composite rules could perhaps be defined, to describe sets of simultaneous transformations. However the approach offers no immediate means to describe the qualities of motions and gestures themselves— motions are rather represented by their initial and final static states. To better represent the nature of motions, a large number of infinitesimally small transformations between sequential states could be used to define sets of consecutive transformations which communicate meaningful motions of particular styles.



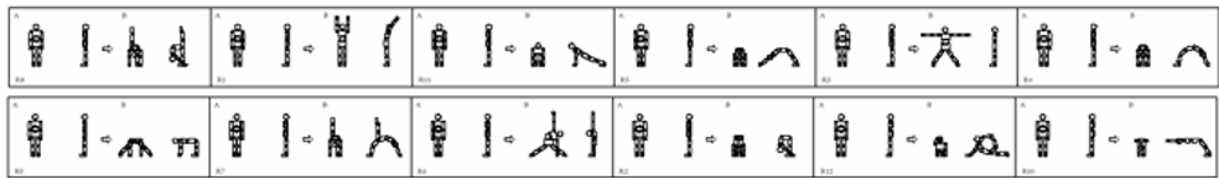


Figure 2.6 Grammars for yoga moves. (Piedade Ferreira et al., 2011)

Rather than retaining any information about the underlying structure of a composition, shape rules employs a purely visual approach: at each stage of generation, identity rules recognise and select relevant sub-shapes to which transformation rules may then be applied. The notion of embedding is central to affording the continual merging and emerging of sub-shapes within designs. Within unstructured shape descriptions, shapes can merge and recombine, allowing designs to be seen and interpreted in new ways, and creating new opportunities to apply rules which can't be predicted in advance. New relationships of interest between emergent shapes may also be noticed.

Shapes within design compositions may be perceived to be constructed from their various sub-shapes in an infinite number of ways. Alternatively, however, design compositions can also be represented definitively, using their maximal elements, which are composed from and subsume any number of touching or overlapping lesser elements (Stiny, 1994). Equally, the boundaries of these elements (3d shapes are bounded by 2d shapes, which are themselves bounded by 1d lines, themselves bounded by points of zero dimensions) can also be perceived to be composed either of a potentially infinite number of embedded parts and sub-shapes, or to be represented by their maximal descriptions (Earl, 1997; Krishnamurti & Stouffs, 2004; Krstic, 2001).

Theoretically, the ability of shape grammars to continuously reinvent the structure of a design composition renders them uniquely suited to the kinds of creative exploration that occur in the early stages of the design process, where designers exploit loosely structured, ambiguous descriptions for the rapid, iterative reinterpretation of design ideas. As shape descriptions of designs can, by definition (Stiny, 1981), allow division into parts in infinitely many ways, activities such as sketching that support the development of unstructured shape descriptions allow designs to be continually reinterpreted.

However, when representing unstructured shape descriptions computationally, the automatic recognition of sites for potential rule applications is not a trivial problem (Jowers et al., 2010; 2011). In practice therefore, this important feature of shape grammar theory proves very difficult to implement computationally for the general case, and many specific computational grammar implementations also fail to fully address this integral aspect of the approach (Krishnamurti & Yue, 2015). The majority of practical applications to have considered the potential for emergence to support design creativity have to date been restricted to relatively simple grammars, where the manual implementation of rules remains feasible. One recently developed computational approach, which demonstrates potential for recognising large numbers of emergent shapes, employs pixel-based techniques from computer vision to compare shapes within designs with shapes from a predefined vocabulary (Jowers et al., 2010; 2011). However, this method has yet to be applied to 3d design configurations, and success has yet to be demonstrated beyond a small number of rule applications (Krishnamurti & Earl, 1992).

As a means of description, however, shape grammars constitute a dramatic departure from traditional shape representations within computation, where datasets containing indivisible points define the boundaries of both 2d and 3d shapes. Within conventional CAD systems, these simple but hierarchically rigid descriptions make it difficult for the underlying structures of designs to be reinterpreted. Stiny (2006) argues that, from a shape grammar perspective, shapes of any dimension which are treated as distinct elements or immutable parts, without the potential for shapes to then merge and emerge, are essentially the same as (0-dimensional) discrete point sets, and are therefore not strictly shapes, but rather members of a predefined set (Stiny 2006:304).

Roozenburg et al. (2002) propose that future directions for shape grammar research should include its integration into 3d CAD, to form creative and educational design tools. A more recent shape grammar implementation using an open-source CAD environment affords the application of generative rules to a range of 3d primitives, but does not permit the recognition of emergent shapes (Hoisl & Shea, 2009).

2.4.2 Shape rules and schema for design exploration

Shapes have algebras that are equivalent in every important respect to drawing with pencil on paper (Stiny, 1989). It has been shown that design moves or actions which construct pictorial representations can be formally described using shape rules (Stiny and Gips, 1972). Shape rules can be used to describe and document the distinct ways in which shapes within designs are recognised and transformed: rules identify shapes and spatial relationships, and also apply shape transformations (Stiny, 2006). Shape rules have been used widely within shape grammars, for the analysis of existing designs (e.g. Flemming, 1987; Koning & Eizenberg, 1981; Stiny & Mitchell, 1978). By considering the spatial relationships encountered within these, underlying rules can be inferred. Shape Grammars can employ sets of such rules, both to reconstruct existing designs, and to discover new variants.

In addition to their use within the constraints and structures of shape grammars, Knight and Stiny (2001) also advocate an alternative, less restricted approach. Shape rules support systematic yet creative exploration, and may be employed in an open-ended way (Knight, 1992; 1995; 2005; Stiny, 1980). Since shape rules support emergence, they are well-suited to modelling open-ended design exploration activities. For design activities such as design sketching, which utilise unstructured shapes as a mode of representation, shape rules readily describe a designer's selections and actions, and in such situations, shape rules can help to record open-ended processes of creative exploration. Rules can describe design moves, and sequences of rules can help to document longer periods during which more complex design activities unfold.

Once formal descriptions of exploration processes are established, they may also be further employed, not only to describe design exploration activities, but also to explicitly guide them. In this way, trajectories afforded by particular combinations of rule applications can be re-appropriated and applied to new design situations. Further, at each stage of generation, the existing composition can be further reinterpreted, and redescribed by generative shape rules in new ways.

New rules emerge as new shapes are recognised, offering new routes for the further creative exploration of design possibilities (Knight, 2003). In this way, both visual

representations and their generative counterparts offer ongoing flexibility in interpretation. Using visual representations, a designer's ideas about the practical or functional properties of a design may inform how her drawing is initially composed, but since shape representations do not retain any history of a design's iterative development, the ready reinterpretation of the structure of parts of sub-shapes within a composition is continually afforded. In much the same way, for any given design, there can be several routes towards constructing a generative description.

To the titular question posed by his seminal paper "what rule should I use?", Stiny (2011) responds readily with "use any rules you want, whenever you want to". This perhaps prompts the further question of "where do ideas for rules come from?" (And this issue has been raised already, during our examination of Schön's 'seeing opportunities for applying moves' — where, much like Stiny's shape rules, moves alter shapes within design compositions). Stiny answers the question by presenting a set of general schema for shape.

As well as discovering and reapplying rules within a specific episode of designing, with a little flexibility shape rules may also be reapplied more generally, to entirely new situations. Shape schema provide a means for describing general classes of rules, which can be instantiated in many ways, to create specific rules for particular applications. Stiny (2011) identifies several levels of general schema for classifying types of shape transformations, including parametric variations and geometric transformations such as rotations, reflections, and translations. Schema can also be specified for joining and dividing parts, selecting shape boundaries, or changing dimensions of representation by creating shapes from their boundaries and vice versa.

Of particular importance when using shape grammars is the identity schema ($x \rightarrow x$). Since it recognises shapes but does not apply further transformations, it constitutes a special case (Stiny, 2011; 2006). These 'useless' rules at first glance do not appear to do anything, since they do not visibly transform designs. These rules however prove crucial for the practical implementation of a rule-based approach, since they allow particular shapes to be noticed within design compositions, thereby identifying opportunities for rules which actively transform shapes to be applied. They also provide a mechanism for new emergent shapes to be noticed or identified (Stiny, 1996). Further, by selecting shapes within designs, identity

rules impart structure to unstructured shape descriptions. While shape rules in general describe actions or transformations, the identity schema rather describes observations and reflections, for recognising new things, reinterpreting designs in new ways, and affording an 'education of attention' (Gibson, 1979). Ingold (2013a) would likely find them far from useless. Schema, more generally, provide a mechanism for parameterising and generalising rules, thereby extending their potential relevance.

Stiny (2011) further sets out to answer a second, closely related question: "how can I cook up rules, to match what's in art and design?" His response, put simply, is that rules found in existing creations can be generalised into schema, affording a new take on creativity and copying. Once defined, these schema can be re-instantiated repeatedly, to discover specific rules that can be usefully applied to a particular creative situation. We might therefore deduce that ideas for rules come from schema. If a designer is not already aware of them, schema can be discovered by relaxing or removing detail from rules which have proved useful before, so that they become applicable to a wider range of circumstances. Such rules might be taken from previous projects, or observed within the work of others, adding to a repertoire of meaningful schema which inform seeing and making in practice. As a corollary of this, we might therefore also also assert that ideas for schema come from rules: schema help us to copy what has worked before, and usefully reapply it towards new situations. Creativity in art and design might therefore be said to rely on an ability to move both backwards and forwards, between general schema and specific rules.

For art and design, Stiny further proposes that rules for seeing and moving within and between algebras of shapes can be broadly categorised by three distinct operator types:

1. $x \rightarrow \text{prt}(x)$ Rules for identifying and selecting parts within a shape (boolean algebra)
2. $x \rightarrow \text{t}(x)$ Rules which apply transformations (geometry)
3. $x \rightarrow \text{b}(x)$ Rules which select a shape's boundaries (topology)

These general schema, which provide useful heuristics for describing or classifying types of rules for art and design, define three specific ways of instantiating the unrestricted replacement operator $x \rightarrow y$.

Stiny (2011) presents these three schema, which incorporate key ideas regarding shape algebras, in a lattice (Figure 2.7). Here they are arranged within a hierarchy which outlines their relationship to other key operators for shape algebras (although the addition rule, $x \rightarrow x+y$, is noticeably absent here). Of these, the identity rule $x \rightarrow x$ is perhaps the most significant, since, although it itself makes no visible change to a composition, it plays a key role in recognising parts within shapes, and thereby identifying sub-shapes to which other rules can be applied. The identity schema is a subset of both the part schema, since it selects everything, and the transformation schema, using the identity matrix. Both the identity rule $x \rightarrow x$, and the erasing rule $x \rightarrow \emptyset$, are members of the schema $x \rightarrow \text{prt}(x)$.

Stiny shows how these three schema form a basis for deriving further, related and composite schema. Firstly, each rule type can be reversed to give its inverse. For transformation rules $x \rightarrow t(x)$, the idea of the inverse rule may be superfluous, since it is also a transformation. However, from inverting the other two schema, we discover rules which create wholes from their parts ($\text{prt}(x) \rightarrow x$), and rules which instantiate shapes from their boundary descriptions ($b(x) \rightarrow x$). Further, the inverse of the rule which subtracts x ($x \rightarrow \emptyset$) gives a rule which creates x from nothing ($\emptyset \rightarrow x$).

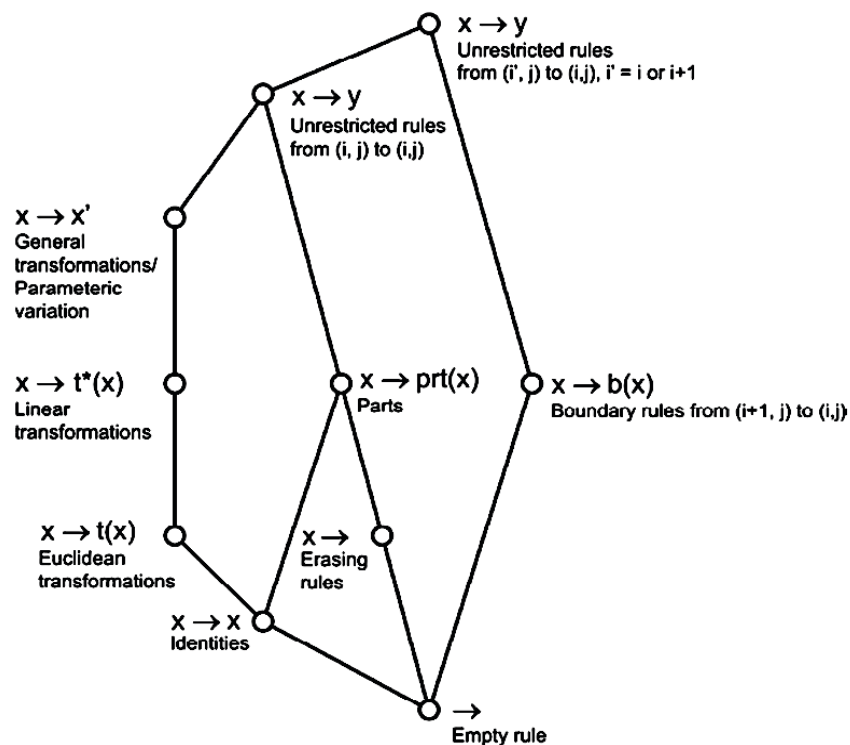


Figure 2.7 Shape schema (Stiny, 2011)

Secondly, schema and their inverses can be combined together in various ways, to mimic common design tasks and actions. Stiny explains how $\text{prt}(x) \rightarrow \text{b}(\text{t}(x))$ can be used to recognise a part (or, its approximation,) of a shape in the clouds, and then to replicate it (or, indeed to replace it– at least as far as the designers attention is concerned), by drawing the outline of that recognised shape, at a rather reduced scale (hence $\text{t}(x)$), on a sheet of paper. Conversely, the inverse of this schema might for instance provide suggestions for seeing new shapes in the clouds, prompted by descriptions of their outlines presented on paper.

Using addition rules within schema compositions can help to define rules which apply recursively, such as $x \rightarrow x + \text{t}(x)$, which can be used, for instance, to create symmetrical patterns. (Technically, these addition schemas are also members of the schema for creating wholes from their parts ($\text{prt}(x) \rightarrow x$)). Addition rules have subtraction rules ($x + y \rightarrow x$, which can be rewritten as $x \rightarrow x - y$) as their inverse. These are members of the set $x \rightarrow \text{prt}(x)$, which recognises and extracts parts or sub-shapes.

More generally, schema which contain multiple transformations or iterations can be described by $x \rightarrow x' + x''$. This generalisation might be used, for instance, to find multiple distinct but non-overlapping parts within the same shape– an operation which might in some cases be described more precisely as $\rightarrow \text{div } x$.

2.4.3 Rules for structure: graph grammars

Some design processes avoid shape-based descriptions entirely– for instance the FBS model, considered earlier, generates a shape-free structural description. In design practice, where designs are represented using complex, compound representation modes, descriptions of structure may be overlaid or attached to shape descriptions. Shape grammars can overlay further information, by adding labels and weights to shape descriptions. Graph grammars, which employ graph rules, allow structural configurations to be explored independently from shape. However, the precise interactions of structure and shape which are prevalent in kinematic designs, for example, may influence how these rules are applied.

Cagan and Mitchell (1993) have used graph grammars within computational techniques for shape optimisation. Shea and Cagan show how an assortment of complimentary methods, including graph grammars, can be combined to form an iterative process for the multi-stage generation, evaluation and selection of structural design configurations optimised towards specified engineering requirements (Shea & Cagan, 1999; Shea, 2000; Shea & Smith, 2006). Here, rules initially construct topological building blocks with known properties, but then apply further instantiation rules to explore how geometric variations of topological configurations influence various performance criteria.

Shea and others have applied a similar generative approach for both structural and mechanical design configurations (Figure 2.8), also founded primarily within the application of graph grammars (Shea & Starling 2003; Schmidt et al., 2000; Helms et al., 2009). Freudenstein and Maki (1979) also employ a graph-based approach in the preliminary stages of mechanism design. Mueller (2014) has used a graph-based approach, for the computational exploration of structural design spaces.

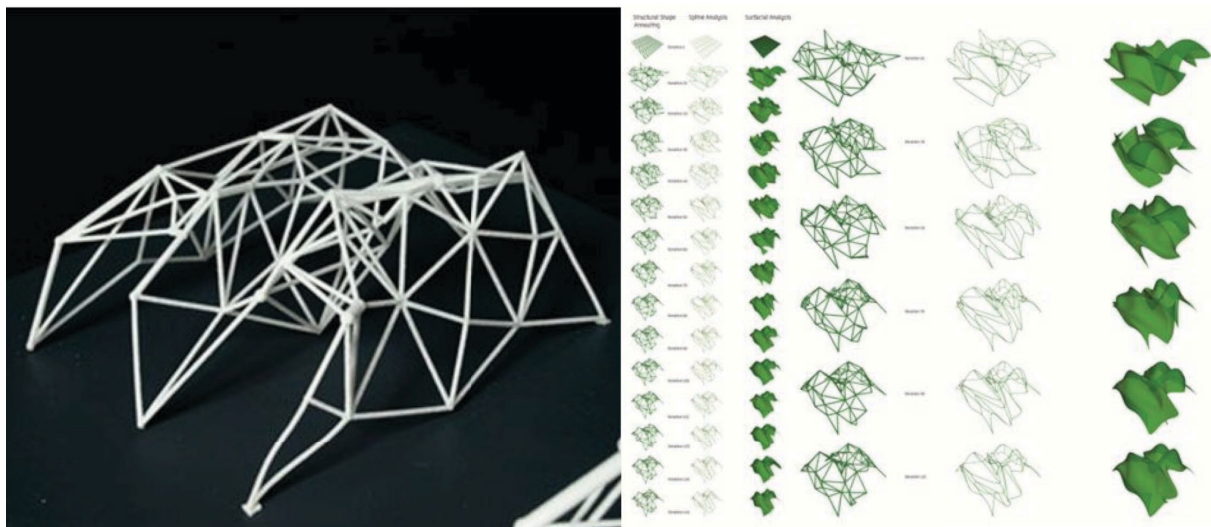


Figure 2.8 Model (a) and variations (b) generated using Shea’s EIFFORM process. (Shea, 2002; OSA et al., 2005) © Open Source Architecture (Chandler Ahrens, Eran Neuman, Aaron Sprecher), Kristina Shea, Marina Gourtovaia.

Applying generative methods for kinematics

In generative design, rather than constructing designs directly, designers create generative rules which contain all the information needed to construct descriptions of complete

designs. Computational approaches to generative design can afford the automatic construction of large populations of design variations, and these populations can be searched and examined to discover unique designs with desired properties. In engineering design, existing work in Computational Design Synthesis demonstrates a range of approaches to rule-based generation of functional designs, including the synthesis of spatial configurations for both structural and mechanical systems (Antonsson & Cagan, 2005; Shea & Cagan, 1999; Roozenburg et al., 2002). Chakrabarti et al. provide a useful review of the field to that date (Chakrabarti et al., 2011).

The application of generative methods to the design of kinematic devices has been largely to topologies through Graph Grammars (Schmidt et al., 2000; Shea & Starling, 2003). For Shape Grammars, applications to kinematics is largely unexplored, although pointers are available in research on structures (Shea & Cagan, 1999; Shea, 2000; Shea & Smith, 2006).

For their computational application for design, a key concern is the ability of generative techniques to produce designs with certain specific characteristics or behaviours. Existing approaches can be broadly divided into two categories: those which attempt to constrain the properties of all designs produced to exhibit only certain characteristics, and those which freely generate large numbers of designs, only some of which will successfully exhibit the desired characteristics.

Constrained approaches often employ compositional methods, which define building-blocks with known behaviours, connected together in a range predefined of ways. Here, prescriptive techniques ensure that all designs produced possess certain prescribed functionalities (e.g. Helms, Shea & Hoisl, 2009; Kurtoglu et al., 2005; Hsu & Woon, 1998). Unconstrained approaches provide less information about how basic units may be synthesised to form design outcomes, and use simpler units whose instantiations may vary throughout the composition.

These approaches allow for a wider variation of emergent design characteristics (Lipson & Pollack, 2000; Sims, 1994a; 1994b; Yogeve, Shapiro & Antonsson, 2010), and are described as *non-compositional* approaches (Subramanian, 1993), Here, since less can be predicted

concerning the emergent properties of designs, routes to determining whether generated designs possess successful or interesting qualities are of critical importance. These two approaches are not necessarily incompatible, but are rather the extremes of a spectrum, and different levels of constraint may be applied to different stages within a single generative process. For instance, see the discussion of Shea and Cagan (1999) above.

For all generative approaches, how to explore design spaces containing large populations of generated designs is a critical issue. Where design outcomes are unconstrained, it is essential that successful designs within the design space can be identified and selected. In methods where designs are predisposed to always possess a certain desired set of characteristics, the purpose of a generative approach is often to explore this partially-constrained design space for designs which also exhibit other secondary properties, that are less easily controlled during generation. In order to discover the most successful outcomes, generative methods rely on techniques for navigating large design spaces whilst measuring the extent to which generated designs exhibit particular characteristics. Therefore, all generative approaches to some extent require techniques for assessing the characteristics of generated designs, throughout large, unmapped design spaces.

Appropriate techniques for searching and evaluating generated designs are therefore of critical importance for many generative methods. Although generative methods have the theoretical capacity to produce very large numbers of design variations, in practice it is not often feasible to generate and compare the relative properties of all possible candidate designs. Distinct combinations of rules (generative descriptions) map to unique points in the design space, and contain all the information required to construct tangible design descriptions.

Practical techniques are required for exploring the potential design space of *generative* descriptions (i.e. rule combinations), and deciding when to actually generate complete descriptions of designs in order to examine their tangible properties. Within large design spaces, random selection of designs does not usually give meaningful results, and therefore algorithms must combine search and evaluation tasks, iteratively using evaluation results to direct future search operations.

Various algorithms exist for searching a design space, in pursuit of particular properties. Genetic or evolutionary algorithms enable a range of successful properties emerging from generative descriptions to be selected for, and then recombined within new designs (e.g. Pollack et al., 2000). Simple optimisation algorithms such as the hill-climbing approach can locate local maxima and minima, but cannot construct a full picture of the entire design space. Probabilistic methods such as simulated annealing (Cagan & Mitchell, 1993) enable a wider exploration and sampling, but still cannot guarantee that optimum solutions will be identified. It is therefore very difficult in practice for designers to obtain a complete picture of the scope and topology of the design space, and to predict either where the most successful designs may be identified, or where designs with new emergent properties might be discovered.

Further issues constrain what kinds of properties can be viably selected for. Most automatic evaluation methods are only able to make quantitative comparisons between properties of candidate designs, and therefore, in order for computational approaches to be useful, numeric data must be somehow extracted from design representations. Where discrete behaviours of building blocks are well established, engineering analysis techniques can determine various quantifiable properties of candidate design configurations.

Experimental approaches have instead used virtual simulations to test and compare measurable aspects of a candidate design's performance, evaluating the relative success of a range of emergent behaviours or characteristics against quantitative criteria (e.g. Sims 1994a). This approach can help to indirectly discover where certain design configurations possess successful emergent behaviours. Alternative methods afford the direct evaluation of generated designs by a designer, through direct experience or interaction. In this case, the number of designs that can be evaluated is greatly restricted, compared to automated techniques, and pre-selection of which designs to realise and evaluate is critical. In some cases, probabilistic methods can employ results from manual evaluation, to learn the designer's preferences and guide a wider automated search (Kurtoglu & Campbell, 2009).

As outcomes produced by generative rules become more reliable, the dependence on evaluating or verifying possible outcomes decreases. Also, as predictability increases, the size of the design space arising from a particular set of criteria decreases, and fully

predictable or deterministic rules may discover the 'one best' solution to a particular design problem. The best example of a generative approach with largely predictable outcomes lies in the well-established automatic design protocols of VLSI design, where well-defined rules describing routine design tasks construct detailed descriptions of complex systems with prescribed functionalities. However, for mechanical design, it is generally less feasible to prescribe deterministic methods for constructing designs with desired characteristics (Antonsson, 1997; Whitney, 1996), and therefore, the further development of generative design in this area is likely to be facilitated rather than through new tools and techniques for navigating unpredictable outcomes and identifying and exploiting emergent behaviours. Antonsson (1997) proposes that the development of new languages for the formal description of mechanical design behaviours might eventually afford a better understanding of the emergent properties of design configurations.

In general, limited impact has yet been made by generative methods in the field of mechanical design. Subramanian (1993) describes the problem of generating structures which realise particular kinematic behaviours as Motion Synthesis, and identifies three categories into which existing methods can be placed: these are *structural*, *behavioural* and *functional* theories of synthesis. Structural approaches systematically generate mechanism topologies and then assess generated designs to see whether desired motions are achieved, but do not explicitly direct the generation process, making these approaches computationally expensive. Behavioural theories use various strategies to construct designs with predefined input and output conditions, and may use compositional approaches to assemble pre-defined building-blocks for which input-output transformations are known.

Functional approaches first specify sets of behaviours which meet functional requirements, then generate structures with these behaviours. However, Subramanian does not find any methods for mechanism design that sit entirely in this category, and highlights the difficulties of describing or categorising mechanical design behaviours in ways that afford their adaptation and reuse in new functional applications. In the design synthesis of mechanical systems where precise kinematic motions are not considered, a range of methods have been developed which construct systems with certain desired functions or behaviours (in a *transformational* sense).

In this work, the systems considered are usually planar mechanisms, where only parametric constraints on part geometries and configurations are necessary to ensure functionality. A popular problem is the generation of optimised topologies for systems of interacting rotating gears with fixed axes. Both Schmidt et al. (2000), and Shea and Starling (Starling & Shea, 2005; Starling & Shea, 2003) explore this problem whilst developing methods for optimising the spatial configuration of planar mechanisms. Since in these mechanisms, parts exhibit only planar rotations, this work avoids the difficulties that arise when dealing more generally with combined spatial motions between parts. The required input and output conditions of these mechanisms can therefore be simply interpreted as transformations of rotational energy, and in these cases compositional approaches can provide meaningful results. However, it is unlikely that purely compositional approaches can express the complex geometries of spatial systems, except perhaps which considering specific, well-defined categories of designs which are already well understood.

Since kinematic motions are directly informed by detailed geometries and connections between parts, purely compositional approaches cannot deal with shape-based complications that arise in mechanical designs, such as physical collisions which impede intended motions. Generative approaches to mechanism design which explore the construction of more interesting kinematic behaviours tend to use purely structural approaches. In particular, Sims (1994b) (Figure 2.9), and Lipson and Pollack (2000) (Figure 2.10) both generate designs with seemingly random structures and behaviours, then select those which perform well, in the simulation of various tasks or environments. In these approaches, distinct types of structural configurations which achieve successful behaviours are indirectly selected for, without the need to explicitly categorise or describe the various kinds of behaviour that emerge.



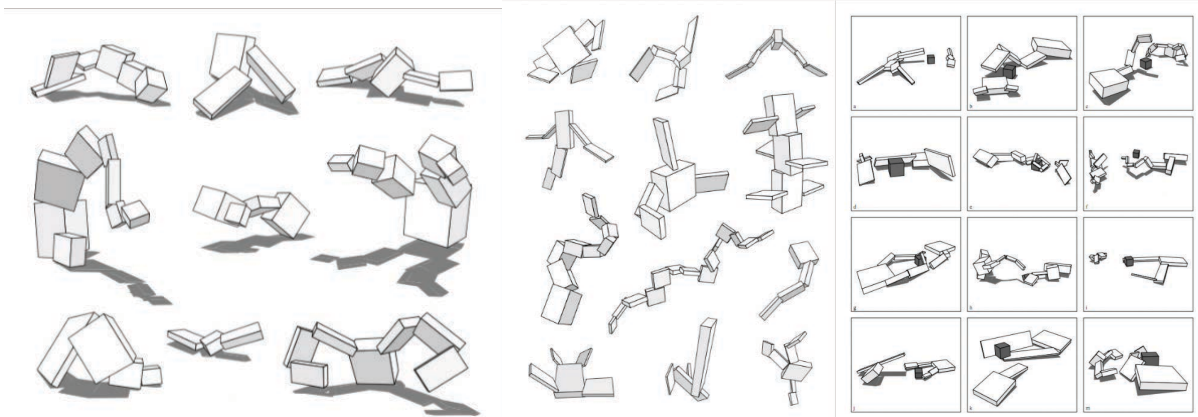


Figure 2.9 Evolved virtual creatures selected for behaviour (Sims, 1994a; 1994b; 1994c)

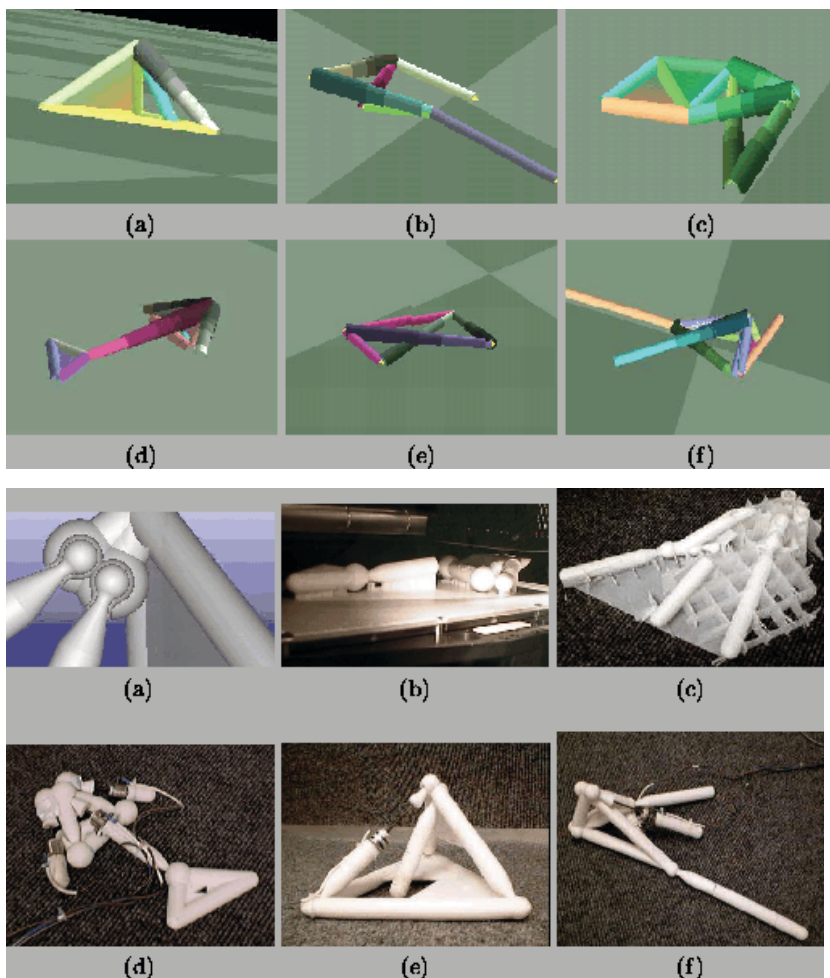


Figure 2.10 Virtual simulations and physical models for locomotive creatures (Lipson & Pollack, 2000)

In this area, little has been done to explore the potential for a generative approach to afford an exploration of how geometric variations influence the kinematic behaviours of spatial

mechanisms. Unconstrained generative approaches seem to offer potential for discovering innovative structures with new behaviours, but current computational methods do not explain how successful behaviours are achieved, or examine how the behaviours of generated designs vary across the design space.

Kinematic behaviour and AI methods

A key concern in mechanism design is finding appropriate problem representations for reasoning about kinematic behaviour. Designs with kinematic functions often rely on precise geometric descriptions, but few existing design synthesis approaches employ geometry as a primary representation. Whilst mechanisms themselves can be categorised by type according to their structure, no systematic approaches exist for describing the relationship between structure and function in mechanical design. Non-technical descriptions of mechanisms tend to refer to particular functional applications, rather than the behaviours which fulfil these functions, and this makes it difficult to recognise where behaviours of designs might be transferable to new functional applications (Subramanian, 1993).

Joskowicz and Neville (1996) propose a formal language for describing mechanism behaviours to support design automation, through describing existing mechanisms in ways that support the reuse of existing designs to meet new design requirements. However, their implementation is only demonstrated for planar fixed axis mechanisms, and includes only verbal technical descriptions, and symbolic representations of standard components which do not afford geometric descriptions. Purely symbolic representations of components dispose of all dimensional information, and so cannot help to discover the effects of geometry variations on kinematic behaviours. Faltings (1992) states that purely symbolic approaches are not useful for kinematics, since in general, multiple dimensions influence behaviour in highly interdependent, nonlinear ways.

Several authors, working in the field of AI, have attempted to formally describe how designers explore and consider ways in which interactions between parts of kinematic designs give rise to certain behaviours, in order that these problems might be systematically addressed. Reasoning about kinematic spatial motions using various design representations is an important activity for engineering designers. Forbus (1980) proposes the use of

qualitative reasoning techniques which imitate the human ability to consider only partial descriptions of motions between parts, discarding information about precise magnitudes. Qualitative descriptions of mechanisms therefore describe different kinematic states in terms of directions of motions, but not their size. Formal statements about relationships between motions can then be combined to construct partial models through which to analyse behaviours. Forbus proposes the use of envisionments, to support qualitative reasoning about a single point moving in space.

Faltings (1990a; 1990b) proposes that, rather than considering the motions of individual objects, symbolic representations of pairs of objects can construct meaningful qualitative descriptions concerning their interactions, and in this way entire mechanisms can be represented as sets of pair-wise interactions. Configuration spaces, which represent all possible combinations of possible positions for the parts within a mechanism, provide abstract descriptions of the kinematic motions between interacting parts, and can therefore be used to describe the theoretical behaviours of a mechanism without considering its physical structure.

For a mechanism composed of interacting parts, particular combinations of part positions can be represented by vectors describing the positions of parts, and a set of vectors, representing all possible combinations of positions, describes the configuration space of the mechanism, which has as many dimensions as there are dimensions of freedom within the mechanism itself. The configuration space of an entire mechanism can also be decomposed, to examine sets of interactions between pairs of parts. Faltings using configuration spaces as envisionments which represent the motions arising from physical interactions between pairs of parts with precise physical geometries. His implementation is limited to planar fixed axis mechanisms, where part geometries can be described using circular arcs and straight lines, and parts possess only one degree of freedom.

Joskowicz and Sachs (2010) also explore the use of configuration spaces in mechanical design synthesis. In general, defining the position of an object in space requires three position parameters and three orientation parameters. However, where parts are connected in predefined ways within a mechanism, descriptions of relative motions between connected parts can provide complete descriptions of possible motions. Where parts are

connected by joints with one degree of freedom, only one parameter is required to describe their relative positions.

Faltings extends the approach outlined above to explore how varying the geometries of pairs of parts then influences qualitative behaviours (Faltings, 1990a; 1990b; 1992). The extent to which design geometries may be modified without compromising design functions is an important question for designers, and other authors have also considered the potential for new methods to assist in understanding the functional aspects of mechanical design, through considering design geometries directly. Tools for understanding the effects of geometry changes on kinematic behaviours are extremely valuable within the design process.

Shapiro and Voelcker (1989) note that conventionally in the design process, design at component level concerns surface geometry and considers how forces to act over surfaces, whilst at system level, functional properties of constituent building blocks are generally expressed numerically as single linear variables. They note however that design however proceeds rather through the “*simultaneous refinement of geometry and function*” (from Alexander, 1964), and propose instead an intermediary approach which considers in greater detail the effects of topology and geometry on how parts interact within sub-assemblies.

This approach treats functional connections between interacting components within a design as energy ports whereby relevant sections of an object’s surface geometry play a central role in the transmission of forces between parts of a system. Sections of object geometry which have ports attached are therefore fully constrained by functional requirements. Connections between ports can be represented using Bond Graphs (originated by Paynter (1961)). The geometries of other sections are not fully constrained, but their forms must be restricted, for instance, to avoid impeding intended kinematic functions through material interference with other components.

In other work, for particular motions, Ilies and Shapiro (1996; 2000; 2003) consider how standard CAD operations can help to understand the maximal shapes that parts in designs can occupy without causing physical interference.

2.5 Making

Generative design descriptions use rules to define formal sets or sequences of transformations which, when applied to tangible media or materials, construct tangible descriptions of designs. They can therefore be viewed as sets of instructions for making physical artefacts, and are useful for succinctly defining synthetic processes. Since exploratory making is already inherently a generative process, we consider how the steps within a making process might be described using generative rules.

Making rules can be employed as formal descriptions of the actions and reflections which occur within exploratory making activities. In this thesis we examine both general principles for making rules, and their application as practical tools for design exploration. With a focus on making processes for kinematic designs, we explore how rules may be employed as a critical tool for supporting design exploration. Formal descriptions of aspects of making can help to reveal underlying reasoning processes, and also support the systematic exploration of a particular design space. We also examine how rules assist with understanding and explaining the interactions between shape and structure which give rise to motions.

When using physical materials to construct 3d design models, certain aspects of making and reasoning activities can be described using shape rules. We consider the extent to which shape rules can describe the exploratory making of designs with connected moving parts, how shape rules might be extended or adapted to define new types of making rule, and also what other kinds of rules may be needed.

2.5.1 Making rules

It has been shown that the moves or actions which construct pictorial representations can be formally described using shape rules (Stiny, 2006). Knight and Stiny (2015) consider how shape rules, which readily describe sketching, can be extended to describe a wider set of making activities. They suggest that shape rules can provide a basis for making rules, to describe both doing and sensing. For making in general, doing and sensing activities may occur simultaneously, making them difficult to separate and study. Making rules must support the description of processes within which actions and reflections may be closely interlinked, and occur in complex combinations. Knight and Stiny (2015) suggest that the concept of identity rules (or, *seeing* rules) for recognising shapes be expanded to create more general *sensing* rules, describing other forms of observation and reflection. They also suggest that shape rules can provide a basis for defining different types of *doing* rules, for actively transforming materials in various ways. Sets of doing and sensing rules can be employed within making grammars, to describe making activities of particular types.

Knight and Vardouli (2015) define making activity as a process which is time based, dynamic, improvisational (dealing with uncertainty, ambiguity and emergence), contingent (subject to chance), situated (socially, culturally and physically), and embodied (engaging actively with the maker's body and sensory-motor capabilities). They suggest that the study of making should be vitally concerned with the physical and material properties of made things, as well as the characteristics of the tools and technologies that making processes employ.

The use of shape rules to model aspects of making activities is not without precedent. In Brown's engineering grammars, generation explores the possible forms and features that may be produced by a set of allowable machining operations (Brown et al., 1995). The potential for grammars to describe manufacturing processes has also been considered by Shea and others (Ertelt & Shea, 2009; Shea et al., 2010).

Knight and Stiny (2015) propose that shape grammars offer a natural basis for a computational theory of making. Shape grammars employ shape rules, to describe drawing and seeing with shapes, within the making of drawings. Shape rules define calculations

within shape algebras, which dictate particular types of interaction between basic spatial elements (points, lines, planes and solids) in (non-trivially) 2 and 3 dimensions.

In the general case, rules for making appear to differ somewhat from the traditional shape rules employed by shape grammars. Knight and Stiny suggest that making in general involves doing and sensing with stuff to make things, where things are finite objects made of stuff. They propose that, just as shape rules can describe doing and seeing with basic spatial elements to make shapes, more general making rules can also be defined, and these describe doing and sensing with stuff to make things. They suggest that, rather than employing algebras for shapes, making grammars for things should calculate instead with algebras of stuff. Instead of rules for drawing and seeing (as are employed by shape grammars, within drawings composed of shape elements), each particular algebra of stuff can require its own specific kinds of doing and sensing operations. Knight (2018) claims that making grammars provide a theory of both the constructive and sensory aspects of making activity. Central to the development of making grammars, therefore, is the further development of rules for doing and sensing with stuff and things. The distinction between stuff and things, and its significance for creativity in particular, is a subject also discussed by Ingold (2010).

Knight (2018) discusses how making grammars can provide a route to directly describing the tactile making processes and craft practices by which certain artefacts are fabricated. Although shape grammars are themselves used to construct designs, they do not necessarily model the steps and techniques by which designs or artefacts are formed in practice using physical tools and materials. Although they are intended to construct shape-based descriptions of the resulting artefacts in a generative manner, the shape rules they employ do not necessarily mimic the processes by which artefacts are made originally.

For the purposes of explication, Knight and Stiny (2015) propose that, for a given making activity, making rules and sensing rules should be defined distinctly. Whereas doing rules act directly to alter stuff or things, sensing rules rather record a maker's sensory interactions, which, although they may be essential to the making process, do not themselves have lasting transformative effects upon the thing. Knight (2018) suggests that doing rules should record direct physical changes to the thing being made, whereas sensing rules are used

rather to record changes in the maker's perception of that thing. It is however acknowledged that, for many kinds of making, these doing and sensing actions will in practice occur simultaneously.

Knight points out that, since distinguishing doing from sensing within a making activity may be somewhat artificial, descriptive approaches may vary, according to the subjective perspectives of makers. She also highlights the temporal nature of making, outlining that, whereas shape grammars focus exclusively on the spatial properties of shapes, making grammars consider both the temporal and spatial quality of both processes for making things, and those things themselves. As rule-based descriptions, making grammars structure and segment both things and processes. She explains how such temporal segmentations within the making process are in some cases arbitrary. She also highlights that, as finite descriptions, grammars necessarily do not capture the full detail of all aspects of the making situation (see Suchman, 1987). The situated, time-based nature of making is also discussed further by Knight and Stiny (2015), who consider the significance of time, in the creation of Sargeant's multi-layered approach to the making of watercolour paintings.

Knight and Stiny suggest that the manner in which shape grammars describe drawings demonstrates an instantiation of a particular class of making grammar, where the particular stuff being manipulated during making is shape. Further, they propose that rules for many other types of making grammars might be developed, directly or indirectly, from shape grammars.

In her development of making grammars to describe the performative craft process of making kolam patterns, Knight (2018) implicitly advocates a pictorial approach. Kolam patterns are traditionally made using rice powder, deposited on the ground by hand, to make marks and lines. Knight notes that the properties of the materials employed, and the dexterity of the maker, will impact the gesture sequences by which patterns are constructed, and these gestures themselves may naturally provide temporal segmentations within the making process.

However, her development of a making grammar is then demonstrated using schematic seeing and drawing rules, defined pictorially in U₁₂. These proposed making rules then

operate to construct line drawings, which are themselves pictorial representations in U_{12} , of sequences of instructions describing how kolam patterns might be created in the physical world, using their traditional materials and techniques. Shape Grammars which generate complete kolam designs have been developed previously by Knight and Sass (2010), but the modular generation method those grammars employ is incompatible with the way kolam are actually made in practice. In contrast, the approach of Knight's making grammar constructs a continuous line which weaves through previously defined locations marks or 'pulli', in a manner which is faithful to the method employed when handling the craft materials themselves.

It is interesting to note the distinct and differing approaches of these two grammars, and to consider their material implications. A modular grammar such as Knight and Sass's original shape grammar might provide an appropriate making grammar if the materials in question were pre-patterned blocks or tiles, requiring appropriate arrangement and assembly. In contrast, the weaving approach of the latter, making grammar might feasibly be re-appropriated to a situation where string or rope could be woven between wooden sticks or pegs. However, it is in part the slightly abstracted, schematic, pictorial nature of these grammars, succinctly represented U_{12} , which supports us to readily reimagine their reapplication towards other situations, where physical materials interact in similar ways. The possibilities of mark-making using rice powder on a horizontal surface extend far beyond the construction of continuous lines. But when working with threads, strings or fibres, this is the only mode of making available.

Krstic (2018) agrees with Knight and Stiny that the principles on which shape grammars and their underlying algebras are based have the potential to also describe calculations undertaken with things during making. He suggests however that this endeavour may require a significant extension of the shape grammar formalism- shape rules can themselves be construed as merely members of a much more general set of making rules. Making rules may operate very differently depending on the tools and materials employed within the making situations they describe, and Krstic considers the suitability of various kinds of algebras for describing different kinds of making operation. Shape grammars have been developed primarily to capture the properties of shapes as manipulated by designers, and Krstic claims that, rather than representing shapes in a truly abstract manner, they explicitly

mimic the ways in which drawn shapes interact during the making of drawings through sketching with pencil and paper (and perhaps also eraser). Since, in many other, diverse modes of making, various algebras of things behave rather differently from the marks made by a pencil on paper during sketching, the description of a broader range of making typologies may therefore require a diverse array of algebras for their description.

Krstic points to a range of issues which may be encountered, when attempting to broaden the established shape grammar approach, to also describe the making of things more generally. A first issue highlighted is that, whereas duplicate lines, added to drawings described by shape algebras so as to overlap existing lines (in an algebra U_{12}), will merely merge and disappear, this is not the general case for all types of things. For the more general case, 'collision protecting' grammars (Krstic, 2001) may be necessary, to avoid undesirable complications where multiple instances of 'stuff' elements attempt to occupy the same physical space.

A second issue discussed is the manner in which the left hand side of a replacement rule ($\mathbf{a} \rightarrow \mathbf{b}$) within a shape grammar operates on a design \mathbf{c} , to construct \mathbf{c}' . An operation of the form $\mathbf{c}' = [\mathbf{c} - \mathbf{t}(\mathbf{a})] + \mathbf{t}(\mathbf{b})$ essentially deletes all of \mathbf{a} before subsequently adding all of \mathbf{b} – even where \mathbf{a} is a part of \mathbf{b} , and only strictly required in order to correctly position \mathbf{b} within the drawing. Krstic terms this issue 'shape flip-flopping'. When undertaking formal computations upon abstract descriptions of shapes, the momentary deletion and replacement of shape elements may be conceptually acceptable. However, for more general cases of making with real physical materials, the arbitrary removal and reattachment of recognised parts, to facilitate rule applications, quickly becomes a costly and undesirable activity. Krstic is therefore concerned that the substitutional computational formalism underlying shape rules cannot both rigorously and realistically describe the 'seeing' or, more generally, 'sensing' component of a making rule (for Krstic, the left-hand side of a shape rule $\mathbf{a} \rightarrow \mathbf{b}$ is associated with 'seeing', whereas the right-hand side is rather associated with 'doing').

It should however be further noted that, even when applying conventional shape rules to make changes to drawings with pencil and paper, this rather literal instantiation of a rule application becomes technically troublesome, since by this method, an eraser would strictly

also be required, for every rule implementation. An alternative material approach can instead involve the copying or re-tracing of complete or partial shapes, to create an entirely new drawing on every occasion a rule is applied. However, from a more philosophical perspective, $\mathbf{t(a)}$ identified within a drawing \mathbf{c} functions merely to 'see' the shape, in order to identify a location at which to apply a rule that adds further elements to the design (an operation rather of the form $\mathbf{c' = c + [t(b) - t(a)]}$). For Stiny (2006; 2011), 'seeing' is primarily associated with identity rules of the form $\mathbf{a \rightarrow a}$, whereas replacement rules of the form $\mathbf{a \rightarrow b}$ describe active changes made to add, remove or transform shapes within compositions.

A third concern regards part relations, when dealing with real physical materials or substances. Krstic considers that, for the case where the making process for a design requires the removal of material from an initial blank piece of material, it is conceptually worrisome to describe the initial blank as 'a part of' (ie $\mathbf{c \leq c'}$) a subsequent, smaller design, from which parts have in fact been removed.

Related to this is a further concern regarding realism, when parts of things are identified during rule applications: whereas for a shape rule, a shape $\mathbf{t(a)}$ identified within a shape \mathbf{c} will itself also be a shape in its own right, a $\mathbf{t(a)}$ identified by a making rule within a description of a thing may not necessarily itself describe a valid thing of the same type. For instance, for a planar 3d object in U_{33} , represented by a planar boundary description in U_{12} , a $\mathbf{t(a)}$ which discovers just a part of a boundary of \mathbf{c} will not itself describe a manifold physical object.

Motivated by these concerns, Krstic (2019) proposes both new algebras and new types of grammars for describing making activities, which go some way towards readdressing the outstanding issues raised. Compound algebras which combine descriptions of solid objects in U_{33} , and their boundary descriptions in U_{23} (or their outlines in U_{13}) are proposed. A related but more rigorous approach devises descriptions using UB_3 and UB_2 algebras (Krstic, 2014) to support the definition of grammars which afford the parallel generation of shapes and their boundaries (see Krstic, 2001). These collision protecting grammars, which guard against the addition of redundant or duplicate parts, can be further generalised to bi-conditional grammars of the form $\mathbf{a, a' \rightarrow b}$. To avoid shape flip-flopping, a second new type

of making grammar is also proposed, within which the seeing or sensing components of making rules are explicitly separated from those rule components which actively add or subtract parts, to define bi-consequential grammars, of the form $\mathbf{a} \rightarrow \mathbf{b}, \mathbf{b}'$. Krstic explains how all these new grammars can be further subsumed by quad grammars, of the form $\mathbf{a}, \mathbf{a}' \rightarrow \mathbf{b}, \mathbf{b}'$. These grammars handle boolean operations of sum and difference for things in general. They include both doing and sensing, but these elements are defined distinctly.

Krstic also draws attention to a further important consideration: he highlights the visual equivalence of a description of a solid object in U_{33} , with a related description of its boundary surface in U_{23} . He notes that these representations are indistinguishable when represented pictorially by 2d drawings on paper. However, this point can be emphasised further— in many other situations where solid objects, or comparable representations of their bounding surfaces, are similarly examined without tactile interaction, both versions will appear visually identical, to an external observer. This matter is also discussed at length by Noë (2006). A further noteworthy point Krstic highlights is that, wherever shapes are used to describe things, boundary representations in a lesser dimension provide simpler descriptions. And in many cases, these may equivalently afford the calculation of a design's engineering properties.

Krstic further highlights a key property of shape representations that does not transfer to the general case for 3d stuff, since shapes in U_{33} behave differently to 3d things in general. According to standard shape rules, shapes expressed in standard algebras that touch or overlap will become fused into a single shape, which can then itself be subsequently decomposed into parts in infinitely many ways. This is not the general case for things, however, and Krstic considers how to model assembly operations where parts remain distinct: one suggested approach employs an algebra of labelled shapes, where parts with similar labels become merged, but differently labelled parts remain distinct. An alternative approach employs set operations in a shape decomposition algebra (Krstic 2005) to partially order parts within an assembly according to explicit subset relations.

Krstic shows that algebras and grammars for making in general may require substantial variation from the original shape-based approach. The general classes of shape rules,

however, as generalised into schema by Stiny (2011), seem, perhaps in part due to their generality, to demonstrate a greater degree of immediate transferability.

2.5.2 Making rules for kinematic designs

In this thesis, since we are considering 3d designs with moving parts, we are interested in furthering the description and explanation of design reasoning through making, for the general case of three-dimensional, material space. The potential for parts within a design to move is directly related to both the composition of its connections and the shape of its parts. Since physical models embody three-dimensional descriptions of both shape and structure, they provide an expedient way to both examine and describe the motions occurring in these designs.

When designs are composed from connected moving parts, shape rules are useful, but they are not sufficient to fully encode the types of action and reflection needed to both construct and evaluate these designs, since both actions and reflections may not always be describable using shapes alone. Similarly, actions which occur within a making process may combine tools and materials in diverse ways, and shape rules may not always be sufficient to describe the kinds of transformations that are taking place.

Although applications of shape rules to date have been largely to describe visual, spatial or aesthetic properties of designs, shape grammars have been demonstrated to also have potential for modelling other design properties or characteristics. Grammars producing languages of functional designs, such as simple self-supporting structures, have been described (Mitchell, 1991; Brown et al., 1995).

Shapes in unstructured compositions merge and recombine freely, with no history of how design drawings are constructed. But for 3d designs with connected moving parts, it is necessary to specify *distinct parts*, whose separation is continuously maintained throughout relative motions. To model such behaviour using shape rules, devices such as weights are one potential route for maintaining a distinction between different shapes or parts within a design (Stiny, 1992; Krstic, 2018).

Once distinctions between parts have been established, shape rules can be useful for describing the shapes of these parts, and also for describing transformations which then alter these shapes. Even when the underlying parts and materials have potential to behave in unpredictable ways, shape rules may still be useful for describing shape transformations which can be applied to a design within a given, static state.

For kinematic designs with moving parts, joints or connections between pairs of parts impose constraints on their relative motions. In order to model motions, it is therefore *not only necessary to define distinct parts, but also the nature of the connections between them*. Graph grammars offer one route towards modelling connections between parts within a design, but they do not afford any further information about the geometry to be encoded. For three-dimensional models composed of only rigid materials, common joint types are the result of surfaces of rigid parts interacting. These connections are termed kinematic pairs. In lower kinematic pairs (Figure 2.2), connected parts share surfaces where contact is maintained throughout their motions, whereas higher pairs maintain only point or line contact between surfaces of interacting parts (Reuleaux, 1876).

Rather than using rules to describe transitions between sets of static spatial relationships between parts of designs, it might be possible to extend the shape grammar approach so that rules directly describe or specify variable spatial relationships (Figure 2.11) between these parts. This thesis will explore whether shared shapes between parts could directly define partial constraints which afford certain kinematic relationships. Surfaces, lines or points embedded in one another might describe kinematic pairs, and afford the modelling of dynamic designs, using a shape grammar approach.

Shape rules might be extended to describe sub-shape relations that afford variable spatial relationships between parts in kinematic pairs (Figure 2.11). By assigning particular qualities to the sub-shapes of interest, shape representations could provide a useful way to describe operations which add, remove or transform different types of connections between parts within designs. For lower kinematic pairs, sub-shape relations can use surface embedding to define parts of surfaces as shared sub-shapes within both parts.

Shared surfaces can be construed to both afford and constrain motions, and for lower kinematic pairs in U_{33} , these shared sub-shapes between connected parts can be used to describe their motions. For some cases, additional, equivalent yet simpler representations can also be found using sub-shapes of lower dimensions, such as shared lines and points, to model the connections which afford relative motions. Therefore, motions observed in lower kinematic pairs due shared surfaces in U_{33} can also be modelled by shape representations in alternate algebras of lower dimensions.

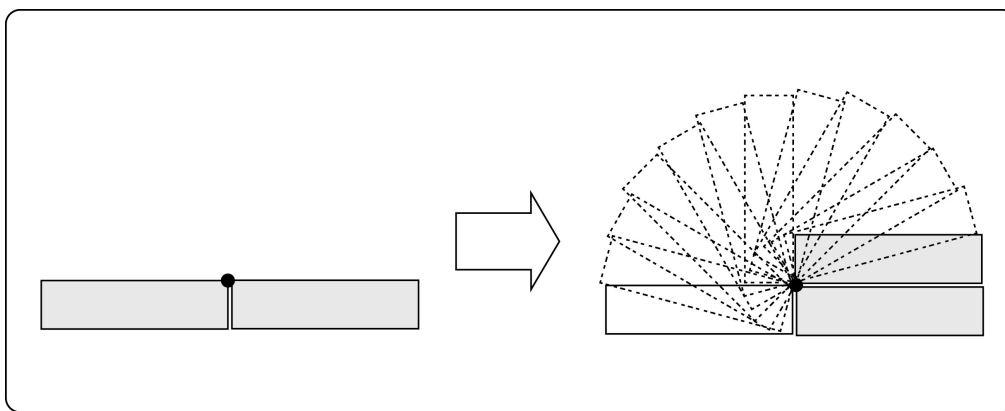


Figure 2.11 A shape rule describing a variable spatial relationship

2.5.3 Making and materials

Rules for making must attend closely to the properties of both the tools and materials employed within a making process, since particular algebras of stuff will afford particular sets of making schema.

Many materials will exhibit both elastic and plastic deformations when manipulated, and some may even deform under their own weight within designs, or as a result of how designs are constructed. Gürsoy and Özkar (2015) reflect that some combinations of shape rules may amplify the effects of material properties, with design implications that are ‘more than the sum of their parts’. In their work, shape rules guide an exploration process where

physical model-making and interaction offer a direct route for experiencing material properties. In some cases, it may also be feasible to describe and predict shape transformations due to material properties using formal material rules. Maclaclan and Jowers (2014) consider how material properties might be described within a design process using weighted shape rules.

Oxman (2010) proposes a new approach to design that prioritises the design of material properties and distributions within parts before considering the shape of the parts themselves. Her method exploits new fabrication technologies which enable parts composed of several materials to be constructed, and the exact distribution of each material throughout the design to be controlled. Hiller and Lipson (2009) have also undertaken research in this area. For designs where behaviour arising from relative motions between parts fulfils design functions, the shape, structure and materials within designs all play equal roles in determining the emergent behaviours of complex systems.

Our focus here is on primarily upon kinematic designs where rigid connected parts exhibit relative motions., Motions may also arise, however, due to the properties of materials, where elastic or plastic deformation afford flexibility and mobility. Just as exploratory manipulation may be necessary to experience and appreciate the motions of an unfamiliar kinematic model in a tactical and situated manner, similar experimental manipulation may also be required to explore the behaviour of flexible materials.

Chapter 3 Methodology

*We shall not cease from exploration
and the end of all our exploring
will be to arrive where we started
and know the place for the first time.*

(T.S. Eliot, The Four Quartets.)

*Designing is not a search for the optimum solution
to the given problem, but an exploratory process.*

(Nigel Cross, Design Thinking. p8.)

In this thesis, we adopt the metaphor of design as a form of exploration and consider spaces, terrains and geographies for these designerly excursions. In particular, we leverage the metaphor for the case of kinematic designs. Armed with the hypothesis that physical model-making activities play a significant role in reasoning about kinematic designs, we study the constructive, material actions which support exploratory design activities.

Shape rules provide generative descriptions which can both articulate synthetic actions in exploratory drawing as well forming grammars for designs in a particular set or class. Grammars can identify new designs in a corpus of possibilities, through systematic exploration and enumeration. An example is the Palladian grammar of Stiny and Mitchell (1978).

More recently, Knight and Stiny (2015) have proposed that shape rules, which readily describe the making of exploratory drawings, form a basis for the definition of general making rules for a range of creative and exploratory activities. In this thesis we concentrate on how making rules describe the synthetic actions of model-making for kinematic design exploration. We examine what kinds of design spaces these material actions traverse and the paths they take. This chapter explains the methodology employed here to answer these questions.

Things, rules and shapes

Designs are things which are about other things, and design activity is a process of reasoning about intended objects or systems which have yet to be brought into existence (Cross, 2011). Across many design domains, whatever combinations of media may be employed, this fundamental process, of constructing and transforming representations of designs, seems to play a critical part in design exploration. Here we follow the hypothesis that revealing the underlying making processes of creative design exploration can illuminate both design reasoning processes, and the properties of designs themselves.

Shape based descriptions, focussed on drawings, are important for design exploration across many design domains. It has been suggested (Stiny, 2006) that unstructured shape representations (of which 2d sketching provides prevalent example) which permit the direct manipulation of emergent shapes can support creative thinking. Unstructured modes of representation allow new ideas for design changes to be readily enacted, permitting the continual reinterpretation of shapes, and their parts, within designs.

Shape rules provide a means for describing how abstract shapes can be manipulated. For certain kinds of design activity, shape rules therefore provide a way to formally describe changes made to design representations. Sets of shape rules can define grammars used to construct well defined sets of designs. Grammars derived from a set of related designs can generate further designs, which possess similar properties to those originals (e.g. Stiny & Mitchell, 1978), thus extending the set.

Shape rules readily mimic unstructured shape representations, such as sketching and drawing. They help to formally describe not only changes made to shapes, but also shifts in seeing those shapes; namely how shape representations are perceived. Shape rules formally describe design alterations which identify and manipulate emergent shapes. They are particularly useful for encoding design activities which employ unstructured media.

However, unstructured shape descriptions are not always enough. Many design activities use models and employ modes of making which possess, or even require, inherent structure based on physical actions and material properties. This use of models seems particularly

critical when considering the design of non-planar mechanisms. Physical model-making is a means for directly examining the kinematic motions of parts.

Here we explore what might be needed to develop rule-based descriptions for a wider range of design and making activities, using kinematic designs as an exemplar. We chose this example since in this field of design, perhaps more than any other, the construction of physical models plays a critical role in enabling designers to understand their properties, especially how they move (i.e. their kinematics). These kinematic properties are very different to the visual properties of shapes within static designs, although there are high level similarities. Like shapes, the motion properties of kinematic design elements appear to behave non-trivially in combination. Further, shapes and shape rules themselves can be usefully employed to describe many aspects of the making physical models necessary for kinematic design.

Outline of method

The method has two main stages. The first (Chapters 4 and 5) is experimental and the second (Chapters 6 and 7) theoretical.

Stage 1: The first stage takes collections of existing kinematic designs and proceeds to exercises in making physical copies. Making actions (and associated observations) employed during these exercises are described using (informal) making rules. Then we consider how these same making rules might also facilitate design changes. We subsequently generalise these making rules to making grammars and schema, to describe sets of designs which exhibit similar properties of motion to the original set.

Stage 2: The second stage uses these episodes of exploration where making rules operate informally, to assess how established shape grammar theory might be extended, to permit the formal description of kinematic designs, through making schema and rules. These take account of connections between shapes, which afford (or constrain, depending on perspective) relative motions in variable spatial relations.

This broad, two stage approach is motivated in part by previous modes of inquiry within shape grammar research, but also by work on modelling creative exploration more generally

(for example Boden (2003) and Wiggins (2006a)). Taking these as a starting point, we consider how rules can support creative, exploratory search within design spaces, as well as affording and constraining the particular types of things discoverable in these design spaces. More specifically, we develop the kinds of making rules and schema necessary for defining and searching kinematic design spaces.

3.1 Search in design spaces

Imagine for a moment that you had a goal that might be more readily reached, or a problem or discomfort that might be better resolved, by the assistance of a physical object. What are the odds, that this object is already available near at hand, accessible and available for your use? If you are not accustomed to using it in this particular manner already, will its new relevance and usefulness to your current situation be immediately obvious to you?

If the artefacts in your immediate vicinity fail to reveal any easement of your dilemma, might you set off to explore more widely, travelling through new landscapes, and to new locations, until discovering a solution? Would you set off immediately, or perhaps first plan and prepare for your journey: studying maps and catalogues; asking advice from those who have undertaken similar expeditions before? What will you look for, and how will you notice, when the perfect answer presents itself?

But perhaps the artefact that will meet your specification is nowhere yet to be discovered. The landscape offers only imperfect objects. Would you accept these, as a partial answer to your problem? Perhaps they themselves contain the starting point that will lead you to a complete solution? How would you begin to alter and edit these objects? Could you take just a part of one, or combine many parts of different objects together? Weary of journeying through geographical spaces, in search of a solution, how might you turn instead to material exploration, using the various stuff available within your surroundings?

Building on Ingold's (2013) distinction between 'things' and 'stuff', we will concentrate in this thesis on how the material exploration of designs for things can be conducted by manipulating stuff. Every making exploration begins with the selection of tools and

materials, and this selection actively constructs a design situation, or material subspace, where a 'design conversation' can take place (Schön, 1983).

This approach is motivated in part by shape grammar research (Stiny, 2006) and also by the modelling of creative exploration more generally (Boden, 2003; Wiggins, 2006a). Central to both these approaches is the assumption that the creative activities undertaken by designers might be formally described using rules. In Chapter 2, we visited literature on design exploration as searching within a conceptual space. Creative search needs both a synthetic process and an evaluation function, in order to proceed in a meaningful fashion. Rules model both the creative process and the method of evaluation (Wiggins, 2006a).

A key feature highlighted by this review is that rules which describe designer actions are used to explore a 'design space'. However, whether this design space has any inherent meaning or structure, independent of the actions or moves in the search, remains an open question. In other words; is a design space implicitly defined by the rules of search? Or, does it have an explicit, objective definition.

One aim of this thesis is to establish the extent to which formal generative and computational methods, for searching conceptual design spaces, can also be carried over to the material exploration of designs, where making physical models affords a creative, synthetic approach. The focus of attention is the space of 3d spatial systems, composed of moving, connected parts. These are searched by rules which make changes to shape and structure.

Rules are more powerful, than descriptive devices for objects and processes: they also explain how a design works or behaves. Another aim of the thesis is to observe the use of physical making rules for kinematic designs, and to establish their capacity to explain the complex and often counterintuitive motions of the whole. The experiments conducted and theory developed, in this thesis, show a pattern of exploration of design space with making rules leading to generalised strategic schema, which effectively explain the space which has been explored, perhaps through delineating its boundaries. These generalised schema or explanatory models offer a jumping-off point for defining new rules, in a new round of creative making.

The explanatory power of models can be related to their constructed, fictional nature. (Bokulich, 2009; Posnik et al., 2020). Borges' short story *The library of Babel* illustrates the implications of exploring a fictional universe, the Library (Borges, 2000). This universe, it seems, contains every possible book of a particular format that can be constructed by systematically generating seemingly random strings of text of a certain length, and using a particular alphabet. The library itself is of unknown size, and is composed of an array of inter-connecting hexagonal rooms, whose walls are lined with bookshelves. There is no obvious categorisation to the books within these rooms, and the inhabitants of the library ponder the potential structure of their universe:

When it was announced that the Library contained all books, the first reaction was unbounded joy. All men felt themselves the possessors of an intact and secret treasure.....

The certainty that some bookshelf in some hexagon contained precious books, yet that those precious books were forever out of reach, was almost unbearable.

(Borges, Library of Babel, 1944; 2000)

The enumerated catalogue is worse than worthless— it is futile. The same danger lurks in exploring design possibilities, if we construct every design we alight upon. But moving away from the conceptual spaces of Boden, towards the physical spaces of models and mechanisms, usefully limits possibilities. For Wiggins' example (Wiggins, 2019) this entails, on the one hand, the difference between marks on the page construed as abstract musical notation and their conception as music, or on the other, the audible performance of that written score, enabling its sensory evaluation.

A similar situation occurs, in the ready human ability to interpret marks made on a flat sheet of paper as representations concerning 3d forms, which is commonly taken for granted. However, Ingold (2015) argues for a pause in this swift passage from marks to intended objects, with marks interpreted instead in a primary material sense, as the trajectories of gestures. A related example, given by Stiny (2011), shows how a small visual change to the shape of a written character can convert one into another. An example is the subtractive shape rule: $h \rightarrow r$, since r (for most san-serif fonts, at least) is a physical sub-shape of h . This

same shape rule within the context of a word (a very particular kind of shape composition) gives a related rule: cheating → creating; heavy with meaning.

Creativity

Boden (2003) makes a distinction between *psychological* (sometimes also called *personal*) creativity, where an individual alights upon an idea- and *historical* creativity, where a discovery is not just personal but collective. However it is Boden's description of how creative discoveries are made, rather than their relative novelty, which we consider in more detail. Wiggins (2006a) sets out to define a formal model of Boden's framework, distinguishing rules to describe elements within the space from rules to describe moves traversing the space.

Based on Boden's original conception of creative exploration as a kind of search, Wiggins (2001; 2006a) and Richie (2006; 2012) have developed a formal model for creative search (see Figure 2.1a). This builds upon Boden's idea of exploratory vs transformative creativity (adjusting rules), to examine conceptual spaces of 'design possibilities'. Wiggins shifts attention, from physical descriptions of individual designs, to instead identifying successful generative descriptions or rule sets. Rules, which explore and transform, are critical for creativity.

Wiggins describes how transformational and exploratory creativity are similar, as they both involve techniques for searching a space of possibilities. However, in transformational creativity, search is performed at a meta-level, with the purpose of identifying successful *generative* descriptions or rule-sets, rather than *physical* descriptions of individual designs (See Figure 2.1b).

Creative search also requires the ability to evaluate whether new discoveries are successful. For Wiggins and Boden, this relies on an evaluation or preference function, applied to test whether a possibility is considered desirable. In an example presented by Wiggins (2001), the historical evolution of Western musical styles is afforded by new creative techniques discovering new musical possibilities. But this ongoing search, both for new compositions and techniques for creating them, has also been accompanied by an evolving cultural

appreciation, altering what has traditionally been considered ‘musical’. This has in turn gradually extended the boundaries of the space within which discoveries can be considered ‘successful’.

For kinematic designs, motions can be tested in practice by making models. In the example developed in the thesis, material models of altered designs confirm whether the motions of the original are preserved. For over-constrained linkages, full-cycle motions have only one degree-of-freedom. Evaluation therefore requires merely a binary, ‘yes’ or ‘no’ result, and systematic experiment through making can discover the boundaries of a space containing only ‘successful’ variations on the original design (Figure 3.1b).

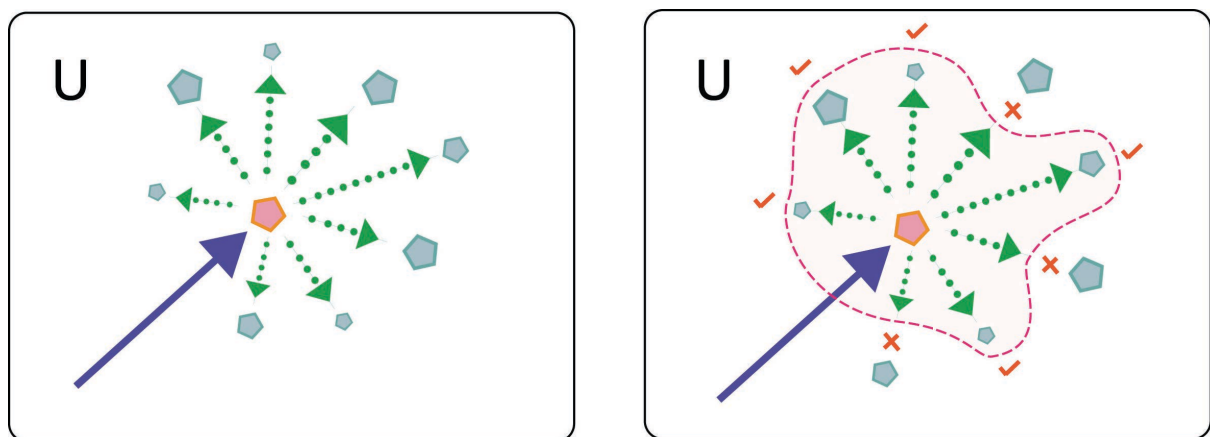


Figure 3.1 (a) Searching spaces of possibility using an existing design as a starting point, and **(b)** testing the effect of changes to kinematic designs, by examining the motions of physical models, helps to identify to the boundaries within which successful variations exist

For our case of kinematic designs, we are particularly interested in rules which transform both shape (of parts) and structure (of connections between parts). Shape rules with extensions to include variable spatial relations (developed in Chapter 6) for mobile connections are appropriate to describe these transformations. In Chapter 4 and 5 we observe what extensions are needed for the exploratory making of kinematic designs, and we begin to formalise these in Chapter 7.

Design space model

Woodbury (1991) proposes a 'design space model', within which exploration or search for designs occurs. His approach to design search and exploration stems from a pragmatic desire to expand the role of computational tools within the design process, with a primary interest in architectural design. He (1991) outlines a formal model for search and exploration in design, aspects of which appear to correlate well with the more general formal model that Wiggins derives from Boden's description of creative behaviour.

Inspired by Newell and Simon's problem spaces (Newell & Simon, 1972), Woodbury sets out the analogous concept of a design space (Woodbury, 1991). His 'design space model' combines concepts from both set theory and formal language theory, as a both a motivator and testing ground for new computational search techniques (Woodbury, 1991). In this model, design becomes the task of identifying a path through a space of possible designs. His subsequent work eschews unstructured sets for graphs or networks of design activity and focuses on the amplification of a designer's actions, through the creation of appropriate support tools.

The computational perspective of Woodbury's approach requires representations and constructive operators to be of a symbolic nature, to support automatic generation and enumeration through computational search. Consequently, to support examination of the validity of generated designs, a complete representation scheme also relies on a mapping function, which defines pairwise associations between elements in the symbolic representation space with their counterparts in a mathematical modelling space. This enables the translation of symbolic descriptions into new representations which support the mathematical calculation of properties.

Woodbury defines the modelling space as the set of all conceptually possible designs that exhibit a certain set of properties. A design space is then defined as a search space combined with a search strategy, where a search strategy is a decision-making process which keeps a record of known design states and their properties, and defines a way of moving through the search space by selecting both states, and operators to apply to them, at each

step in the design process. Later Woodbury (2006) develops ideas of design space networks where actions alter representations explicitly, with semantically relevant rewrite rules. However as he recognises, this severely limits the applicability of any system to very specific types of design problems.

It is difficult to use either of these approaches directly, to search design spaces in exploratory making. Whereas the Wiggins-Boden model provides a theoretical basis for creative activity, it is of limited practical relevance to generative approaches. In contrast, Woodbury's pursuit of practical implementation techniques highlights the difficulties of defining models which are relevant to anything but specific design domains. For these reasons, we adopt, in this thesis, the generative methods of shapes, shape rules and shape schema, and expand and adapt them for describing exploratory making. Figure 3.1 develops the model proposed by Wiggins (see Figure 2.1). Here, a design space containing complete and partial design descriptions can be traversed by rules which transform these descriptions to reach new possibilities. Rule-based descriptions of existing designs provide a starting point from which to explore a wider design space (see Figure 3.1a).

But what objects are we searching, in exploratory making? Some are physical models, and some more abstract geometric and kinematic representations. The next section considers the types of representations which will be used to record the experimental observations in Chapters 4 and 5, and further developed in the theoretical developments of Chapters 6 and 7.

3.2 Exploration and representation

Here we consider the extent to which the exploratory making of designs with moving parts can be described using shape rules, and what other kinds of rules may be needed. The capability of shape rules and grammars, for describing designs in a generative manner, has already been established (e.g. in Stiny, 2006). More recent work however, within the emerging field of computational making, has begun to consider how a formal generative approach might also be applied to the description of making processes. We evaluate these representations, for using in this thesis. We assess them as a basis for the first stage of the

research, for describing experimental observations of exploratory design making. For the second, theoretical stage, we consider their potential to formally represent creative processes of exploratory making in kinematic design.

As we have seen in Chapter 2, the work of Knight and Stiny (2015), Knight (2018) and Krstic (2019) proposes making grammars as natural extensions to shape grammars. We examine how the formalism and application of making rules might be further developed for physical making, and specifically for moving kinematic designs. We propose that structured shape representations may be developed from the Shape Grammar methodology, which will allow both shape and structural 'move' rules to be used within the same representation mode, in a formal computational context.

Shape grammars faithfully reproduce design situations where tools and materials afford the creation of unstructured shape descriptions, such as in 2d sketching and drawing. From Ingold (2013), the materials or stuff, in making drawings, are ink and paper (rather than shape), with pens or pencils as tools. Such tools and materials offer unconstrained opportunities for mark-making. These drawings may also be readily construed as visual representations of other things. But although shapes may not themselves be the actual materials of the design situation, they are a way to describe them. Further, when design drawings are developed, shape rules help to formally describe actions adding shapes to compositions and making changes, and also observations discovering emergent shapes. Although shapes cannot capture fully the nuances of actions and observations made within a material world, in many design situations they provide a useful proxy, for describing making activities in a precise, formal way.

This thesis considers how generative shape rules can describe making activities for the exploration of kinematic designs. Whereas the properties of shapes are primarily visual, and require seeing, the primary property of kinematic designs is motion, whose sensing will also involve touching and manipulation of a design. Shape rules have historically seen most use for the analysis and study of sets of existing designs (e.g. Stiny & Mitchell 1978; Konig & Eizenberg, 1981; Fleming, 1987). Knight and Stiny (2001) also advocate an alternative, more open-ended use.

We will employ rules as a means for both recording and, subsequently, motivating actions, within a design exploration process. The experiments in design exploration will begin not with a set of designs, but with a single object or model of a specific kinematic design. A set of design rules is constructed; any convenient way to construct that object. These form a starting point for the systematic making exploration of a collection of new designs, some of which exhibit similar properties to the original. During exploration we also discover new rules, and new ways to generalise rules into schema. We can then derive a collection of schema for constructing a set of new designs which reproduce the properties of the original. These schema can help to explain which spatial relationships observed in the original object are important for reproducing these properties.

Design exploration proceeds through a combination of action and evaluation. Exploration is dependent upon actions (doing) with materials- whether pencil and paper or 3d models of mechanisms- for the externalisation of ideas for sensing. In making, Knight and Stiny (2015) suggest that both doing and sensing occur simultaneously, in ways that can sometimes be difficult to separate and describe distinctly. A rule-based approach for exploration will involve rules for both doing and sensing. However, since both doing and sensing actions are only afforded through interaction with a tangible representation, there may be issues with justifying the advantage of a generative description based on this approach.

Many material modes of exploratory making also require inherent structure, such as when connections between distinct parts within designs must afford relative motions. In our experiments, we discovered the potential for detailed making rules to be encountered during exploration. Through a process of systematic experimentation using physical models, the effects of changes to shape and structure can identify boundaries beyond which motions were disrupted (and within which motions are similar). Sets of making schema are then derived, and used to define sequences of making rules for constructing larger sets, of designs each exhibiting similar motions. Further, we find that these generative descriptions of sets of designs can help to explain which relationships between shape and structure are essential for preserving motions.

The relation between schema and rules is important, for the method and results reported in this thesis. Creativity in art and design might be said to rely on an ability to move both

backwards and forwards, between general schema (for explanation and reflection) and specific rules for generating new designs. Charidis (2019) makes a cogent case for a generalised improvisation using direct interaction with objects generated by rules for an implicit description of design space. A topological basis (Haridis, 2020a; 2020b) for this implicit design space can identify continuous moves within the space. This work, although very recent, seems to support our use of rule-based shape representations for exploratory making.

Stiny (2006) uses schema, for creative exploration in art and design. Ideas for applying changes to particular designs may be derived from rules for transformations that have been successful in other design situations. More generally, particular types of rule can be described using schema, and an established and evolving repertoire of schema can be applied to new design challenges. In this thesis, we consider what kind of rules might be helpful for the exploration of kinematic designs. We set out to establish a repertoire of schema, derived from a series of experiments in Chapters 4 and 5, to support this exploration. We then develop the theoretical basis in Chapters 6 and 7, for how Stiny's schema (Stiny, 2011) for shape can be redefined more generally, as schema for making. However, we also draw attention to issues in shape representations, for 3d physical making.

Stiny's Kindergarten grammars (Stiny, 1980) appear at first glance to be firmly rooted within a world of material exploration, enumerating combinations of modular building blocks. Whilst this example demonstrates the creative potential of shape grammars, the computations do not physically manipulate the blocks themselves, but rather geometric representations which not only permit material intersection, but also the free visual interpretation of the resulting edges and surfaces.

The emphasis is on formal description of a process, which possesses many of the creative affordances of 2d sketching, but translated to a 3d world. Methods for the formal description of 3d constructions have been developed, for example by Krstic (2019). But since parts within 3d shapes are potentially unstructured and invisible sub-shapes, within solid material, extending shape representations with their associated visual 'seeing' dimensions may be problematic.

3.3 Research questions

These research questions are set in context of the adoption of shape representations for exploring 3d physical making in kinematic design. Physical model making seems to be a central activity in the development of kinematic designs. Interaction with physical models enables kinematic motions to be directly experienced. This interaction appears to play a significant role in design reasoning, but the nature of this significance is not well understood. We therefore consider how to better study and describe the processes of making by which designers construct and alter physical models to appreciate their motions. Five questions arise, the first three are addressed in stage 1 of the method (Chapter 4 and 5) and the last two in stage 2, addressing theoretical issues in Chapters 6 and 7.

Question 1: ***How can making assist kinematic design exploration?***

Question 2: ***What types of making rules are needed, to describe the exploration of kinematic design possibilities?***

Question 3: ***Can making rules discover new possibilities and identify design space boundaries?***

Question 4: ***How can making rules in a shape representation be extended to moving kinematic designs?***

Question 5: ***How can the formalisms of shape rules be extended for exploratory making where vision and touch are integral components?***

The aim of this thesis is to explore types of motion and the changes to those motions resulting from transformations to shape and structure.

To begin, we consider how we might *make copies* of a collection of existing designs, whilst describing the *making actions and observations* employed during these exercises, using (informal) making rules. With these rules for making a particular class of designs informally established, we consider how these same making rules might also facilitate design changes. We subsequently consider the potential for making grammars and schema, derived from existing designs with particular motions, to describe sets of similar designs which each exhibit similar properties of motion to the original.

An artefact which exhibits a distinctive and intriguing type of motion was selected. Model-making was then employed as a means of physical design exploration, with the goal being to develop an understanding of how the behaviour of motions in the artefact is dependent on both the shape of its component parts and the structure and types of interconnections between parts.

The programme of exploration proceeds roughly as follows: start by attempting to copy the artefact of interest. Copying requires a description. Therefore, how can we describe it? We begin by looking, examining, and manipulating. The structure is not immediately apparent from the static object: in the case of things that move, it seems that a new kind of looking is required. Looking at a single static instance of a movable object tells a limited amount about how it might transform or otherwise behave as it moves— so further interaction is required. We hold the object, and attempt to manipulate it in various ways. This object behaves in a surprising manner- its motions become possible sequentially. Merely describing the object becomes an active, tactile endeavour, which then proceeds to making. Descriptions of shape and structure afford copying, and changes to both descriptions and models support creative exploration.

From these episodes of exploration, during which we consider the operation of making rules somewhat informally, we then discuss how established shape grammar theory might be further developed to permit the formal description, not only of static shapes, but also of

connections between shapes which afford (or constrain, depending on perspective) variable spatial relations.

3.4 Review of methodology

The adopted methodology starts with experimental observation, of several sets of related kinematic designs and their properties (Chapter 4). These observations (Question 1) of our experiments, exploring kinematics across similar sets of designs, are then encapsulated within an eight-step method, which outlines the systematic examination of a class of kinematic designs. These steps use exploratory making rules acting on physical models. They are applied to a particular design in Chapter 5, and the explorations proceed by affecting physical changes upon models, to create variations of the original design, with similar motions. The outcome is a set of strategic schema (Question 2) which explain our class of designs and its motions. These explanations (strategic schema) are proposed as the route to new rules, and new designs (Question 3).

The second stage contains theoretical development in making rules, for kinematic designs in particular, and also for physical model-based design more generally. Chapter 6 focuses on kinematic design and their motions. It develops extensions to shape rules by including variable spatial relations, to describe relative motions among connected parts in kinematic designs (Question 4). Finally, in Chapter 7 the experimental observations of Chapters 4 and 5 are abstracted, to characterise making rules which correspond, albeit partially, to interacting with physical models. Making and sensing (both visual and tactile) are integrated, in tactical rules and strategic schema for design (Question 5).

Chapter 4 Rules for making in kinematic designs

The focus of the thesis is kinematic design. We consider the untapped potential for the application of generative methods to the design area of kinematic mechanisms, and explore how the lens of computational making gives insight into how kinematic designs work.

Physical models provide a crucial mode of representation for kinematic designs, since they permit motions to be directly experienced and actively examined. This direct interaction with motions appears to play a significant role in design reasoning, but the nature of this role is not well understood. Physical models of kinematic designs therefore provide a key focus for the exploration activities reported in this thesis. We consider how to describe both kinematic designs and the processes of making used by designers, to construct, evaluate and change physical models, in order to appreciate their motions.

This chapter considers how shape rules, and the ideas underlying them, might be employed to describe making, for kinematic designs. The aim is to use such rules within formal descriptions for making actions, and for systematic exploration of kinematic design spaces. We consider how this exploratory making can be described using shape rules.

Knight and Stiny (2015) propose that making involves both doing and sensing within algebras of stuff to make things. Shape rules, viewed as operations acting upon abstract spatial elements within an unstructured space, need to be reinterpreted to describe design changes through making, where material objects impart an inherent structure, to representations. In this and subsequent chapters, we consider what types of algebra are useful for making in kinematic design, and what kinds of rules can describe doing and sensing.

The motions of kinematic designs arise from interactions, between the shapes of parts, and the structure of their connections. Descriptions of shape and structure inherently contain information about a design's motions. Whereas shape and structure are design properties which can be described visually, and to some extent represented abstractly, motion is less straightforward to model, describe, or change.

In this chapter we consider how shape rules can be applied, extended and adapted, to define generative descriptions of exploratory making, for kinematic designs. Shape rules can be readily employed to describe shapes within designs. Their use in describing and constructing both shape and structure constitutes a starting point for our investigation.

We begin by revisiting key features of shape rules, with a view towards developing a shape-based approach for the formal description of making activities. In Section 4.1, we examine several exercises in model-making, and begin to uncover key aspects of making that, for kinematic designs especially, elude description using shape rules alone. Subsequently (Section 4.2), we consider the formal description of practical ways of physically constructing simple 3d shapes, using real materials.

In the core of this chapter (Sections 4.3 and 4.4), we examine a collection of kinematic artefacts, through a series of experimental making episodes. These artefacts are members of a kinematic class of closed-loop linkages, and belong to the set of kinematic designs which exhibit- relatively unusually- full-cycle motions. Our encounters with these designs begin with initial descriptions ranging from the physical to the abstract. Transforming descriptions between one mode and another plays a key role in the experimental approach. Reasoning about designs proceeds through exploratory making within a complementary range of representation modes, generating sets of equivalent descriptions, and establishing an appreciation of shapes, structures and motions.

In these episodes we begin, in our first exercise (Section 4.3), by examining an assortment of existing kinematic artefacts with unusual motions. These are considered and interrogated physically, through direct interaction with their material models. These kinematic toys take common geometric shapes as their combinatorial building blocks. However, in making and interacting with our own copies of these designs and considering how to best describe our actions, we quickly move beyond simple, combinatorial operations. For this set of toys, the emergent motions of shapes, connected together within design models to afford variable spatial relations, prove to be more than the sum of their parts. Further synthetic experiments examine the practical application of shape rules, to describe and support other aspects of experimental making. Alterations are

applied to the shapes of parts within these kinematic designs, to investigate the effects of shape transformations upon these motions.

A second exercise in exploratory making (Section 4.4) considers designs encountered initially through more abstract modes of description. Where complete descriptions of material models are not available, we find that these can be derived through material exploration. This more detailed example considers a family of mechanisms, described within a predominately mathematical literature as ‘over-constrained’ linkages. This literature’s primary vehicle for describing this class of kinematic designs involves abstract representations of geometry for spatial configurations of joints (connections), but provides no information about the shapes of parts themselves. Several stages of reasoning are therefore required, to move from these initial descriptions, towards tangible models which exhibit kinematic motions. The class is itself discovered to contain a configuration equivalent to that of a kinematic toy encountered during the first exercise. But although their underlying kinematics are equivalent, this is not immediately apparent from their overall appearances, which are visually dissimilar.

We conclude this chapter (Section 4.5) by reflecting upon how shape rules which describe the exploratory making of kinematic designs might be applied within a systematic approach, to explore conceptual spaces containing designs that exhibit particular types of motions. This approach is then developed further in Chapter 5.

4.1 Making kinematic designs

Knight and Stiny (2015) propose that making in general involves doing and sensing within algebras of stuff, to make things. They also propose that shape algebras provide a starting point for these more general algebras. Within the field of kinematic design, we consider what might constitute useful algebras for making, and also what kinds of rules for doing and sensing might be necessary. Model-making of kinematic designs can be situated within both virtual and physical design spaces. Whilst some considerations may also apply more generally, our focus here is predominantly upon making within the physical world.

We consider how formal shape rules for describing changes within abstract design compositions might be reinterpreted to describe the making actions of an imperfect, physical world. Here, the properties of non-homogeneous, discontinuous material combinations appear to non-negotiably impart structure, to materially made representations. When modelling kinematic designs, the explicit structure of connecting or otherwise interacting parts (e.g. collisions), along with the shapes of those parts themselves, affords relative motions. Therefore, to describe and construct designs with kinematic properties, it is necessary to consider both shape and structure explicitly.

Shape rules define calculations which operate within shape algebras (Figure 4.1). These algebras dictate particular types of interaction between basic spatial elements of differing dimensions (points, lines, planes and solids), in (non-trivially) 2 and 3 dimensions. Basic shape algebras are defined for a single shape element type, within a space of a given dimension. Shape rules can also transform the algebras used for description. Altering their dimensions transforms shape elements, of dimension n , into their boundary descriptions in $n-1$, or vice versa, with no inherent loss of information. Changing the dimensions of a representation space, however, can be less straightforward to achieve.

Composite algebras can also be defined using multiple types of shape elements (including shapes and their boundaries) simultaneously (Stiny, 1992). Computations within composite representations require that specific mappings between one algebra and another be defined.

$$\begin{array}{cccc}
 U_{00} & U_{01} & U_{02} & U_{03} \\
 & U_{11} & U_{12} & U_{13} \\
 & & U_{22} & U_{23} \\
 & & & U_{33}
 \end{array}$$

Figure 4.1 Shape algebras (Stiny, 2006)

In general, physical materials and artefacts behave very differently from abstract shape-based representations. Krstic (2019) suggests that, in order to adapt algebras of shape to

better represent the material artefacts of the physical world, compound algebras can combine descriptions of solid objects in U_{33} with their boundary descriptions in U_{23} (or else their outlines in U_{13}).

In this chapter, we revisit some key features which distinguish material objects from their shape representations. The primary interest in this thesis is the structure of distinct parts within kinematic designs, and the types of connections between these.

4.1.1 Parts and connections in kinematic designs

Pen-and-ink drawings create descriptions materially. Any sequential history of compositional mark-making evaporates, as rapidly as ink dries. Shape rules mimic this ambiguity of structure. More generally, however, materials in the physical world behave differently from these abstract shapes or inky marks. For many methods of making, indelible evidence of their synthetic process contributes actively to the character of the resulting artefact. But shapes themselves still remain significant within these modes of making. Shapes both readily describe physical artefacts, and afford the recognition of emergent visual features. This supports the creative reinterpretation and re-description of designs.

A key feature of shape rules is their ability to both describe and afford non-combinatorial operations. This permits the emergence of new things within designs, the perception of visually emergent shapes, and their transformation. Among available theories for generative design, this is a unique and beneficial feature; although not always fully congruent with the behaviour of material artefacts (Krstic, 2019).

Stiny's paper on 'kindergarten grammars' employs a set of children's building blocks as the basis for a seminal demonstration of the generative power of formal shape computation, as a potential alternative to creative exploration guided merely by a designer's intuition (Stiny, 1980). Shape rules are employed to create formal descriptions of spatial relations between these modular 3d shapes, permitting the systematic enumeration of many design possibilities. This exercise reveals the advantages of abstract shapes as a mode of design representation. Within abstract shape descriptions, the non-combinatorial properties of shape grammars quickly

become apparent, as shape rules create divisions within the surfaces of overlapping 3d shapes, affording the perception of newly created, emergent shapes between intersecting components.

In a related discussion, Ingold (2013b) cites architectural historian Witold Rybczynski in reminding us that the regular, modular solidity of kindergarten 'building blocks' as commonplace playthings began to shape societal consciousness only as recently as the 1850s. (Rybczynski, 1989). Ingold suggests that the current prevalence of this predominately unquestioned combinatorial worldview is a relatively modern phenomena. He explains how, in his treatise on *The Four Elements of Architecture*, Gottfried Semper rejects the idea of predefined parts and wholes in design, advocating instead that the shared Indo-European root *noc* (giving rise also to *nexus* and *necessity*) that informs the Germanic words for both knot (*Knoten*) and joint (*Nacht*), is evidence of the deep significance of techniques for the twisting, threading and knotting of linear fibres, in the genesis of early making practices for both textiles and buildings (Semper, Malgrave & Herrmann, 1989). This focus on how interacting parts come together to form connections within a wider whole, rather than on their description as individual artefacts, is a perspective that is relevant to both shapes and kinematics.

Within Stiny's Kindergarten grammars, to constrain design possibilities towards mimicking the behaviour of these tangibly rigid playthings in the solid, material world, Stiny's shape rules use labels. Other deviations from physical reality are also striking—whereas Ingold (2013b) hypothesises that the flat, even surfaces of the domestic indoor nursery were a necessary practicality for the rise of orderly and structured play activities, Stiny's explorations are conducted within a space which appears most certainly beyond the annoyance of gravity, amongst other material inconveniences. Stiny (1982) subsequently notes that the use of set grammars would provide an alternative computational basis that could model the physical behaviour of the original toy more comfortably, albeit with some loss of creative potential. Subsequent developments in shape grammars go on to consider more carefully how surfaces which define the boundaries of 3d shapes might be formally preserved (Krstic, 2019), or else how intersecting surfaces or solids with various properties might interact within different design situations (Maclachlan & Jowers, 2014).

Parts

In shape compositions, parts or sub-shapes are not fixed, and can be and continuously reinvented and rediscovered through visual engagement. A key property of shape representations is that touching or overlapping shapes within compositions merge without retaining any history of the manner in which they were constructed. When expressed within standard shape algebras, shapes which touch or overlap become instantaneously fused, yet can also be subsequently decomposed again into parts, in infinitely many ways, merely by visual re-examination (Stiny 1987; 1994; 2006). This is useful in creative design situations: parts and their relationships are not fixed. There can be a continuous reinterpretation of structure within a design composition.

However, this is not the general case for material things with readily identifiable parts, such as the rigid links in kinematic designs. For supporting the description of making for physical artefacts in general, how to use shape rules to specify distinct parts which do not automatically merge is a key question. And for the rule-based description of kinematic designs, it is of critical importance. For the kinematic designs we consider here, it is necessary not only to define distinct parts, but also the nature of the connections between them.

For constructing shape-based descriptions of physical models, several devices for distinguishing between distinct parts are readily available within existing shape grammar theory. One approach uses labels and rules which alter both shapes and labels simultaneously (Stiny, 2006; 1990; 1980). The related set grammars are a subset, where the fusing and melding of touching and interacting shapes from a specified vocabulary is no longer afforded (Stiny, 1982). A later addition to shape grammar theory defines weights, which in addition to keeping track of parts also, for specified cases, dictate outcomes when parts overlap or otherwise interact (Stiny, 1992; Knight, 1994; Knight 1989; Maclachlan & Jowers, 2014).

Krstic (2019) highlights that real, tangible artefacts in general behave differently to abstract 3d shapes in U_{33} . Shape descriptions are characterised by a lack of structure, but this feature does not translate to their realised real-world counterparts. Krstic considers how to apply shape-based methods to model assembly operations where parts remain distinct: one

suggested approach employs an algebra of labelled shapes, where parts with similar labels become merged, but differently labelled parts remain distinct. An alternative approach employs set operations within a shape decomposition algebra (Krstic, 2005) to partially order parts within an assembly according to explicit subset relations. These approaches offer a practical, schematic way to keep track of the progress of a particular type of making.

However, mechanical connections between rigid parts within assemblies usually rely primarily on shape-based interactions between those elements. Carefully specified solid geometries ensure that parts fit and stay together, in order to create and maintain direct physical connections. Even for these rigid parts, Semper's notion of weaving, twisting, and knotting appears to remain relevant: geometric or topological relations between elements can be enough to physically join parts within larger wholes, whether fleetingly or permanently. Rather than abstract descriptions and labels, it is these knots or joints, or perhaps more fundamentally, the actions of knotting and joining which create them, which hold physical assemblies together, merging parts and creating connected wholes.

Connections

The visual behaviour of the abstract shapes described by shape rules tends to correspond to drawing shapes with ink upon paper. As shape elements touch and interact they fuse visually, becoming statically connected by means of their underlying medium. Their various elements are embedded in both each other and the underlying substrate of their representation space. For abstract shape-based representations, this substrate is the 2d or 3d representation space itself. Shape compositions remain static until shape rules translate, rotate or otherwise animate and transform shapes within compositions, changing the various spatial relations between them (see for instance Piedade Ferreira et al., 2011)). To apply such transformations to alter shape elements within materially instantiated compositions, rigid connections between marks and their underlying substrates may need to be actively destroyed and recreated.

Virtual media can support the creation of design representations where shapes behave differently from mark-making in a material world. This can afford the creation of shape compositions mimicking properties of material representations. Yet many

computational platforms still rely on rigid hierarchies which may limit opportunities for creative reinterpretation. Within an object-oriented approach, visual recognition of emergent shapes within designs is not enough to afford the direct selection and transformation of those parts (Jowers et al., 2011).

In virtual environments, distinct parts can be actively rearranged far more readily than within material, pen-and-ink compositions. In three-dimensional virtual space, shape-based compositions, containing shape elements with no material weight and requiring no secondary structure, are unconstrained by physical or material restriction. This freedom can afford novel creative possibilities that would be challenging to instantiate in material space. In the physical world, 3d material compositions usually require some form of inherent secondary structure or support, to resist the gravitational forces for example. However, its absence permits the spatial relationships between distinct entities to be varied freely.

Mark-making upon a supporting surface has the benefit that an attachment to the underlying media avoids the need for any further structure, to permanently define spatial relations between shapes. But some material modes for 2d representation may make spatial relations easier to alter— for instance, where 2d shapes cut from paper are arranged together upon a table-top, many spatial arrangements can be readily created and examined. Similarly, the toy bricks of Froebel’s gifts can be arranged in patterns on the floor. But it becomes more difficult to maintain these patterns upwards, into three dimensions: to maintain stability, bricks must usually be placed with their faces in contact with the supporting surface, as contact between only corners or edges is unstable.

These transient connections or relationships between pairs of distinct artefacts may be observed, for instance, whilst they are both pinned by gravity, to the same floor or table-top, or to each other directly. For pairs of inanimate artefacts, the extent to which their mutual spatial relationships might be considered static may depend on the nature of their environment, and the relative ease with which each may be repositioned. Once parts or pieces become permanently, statically attached to one another, they may be considered to comprise a new, composite artefact.

Yet, not all actions that permanently connect parts necessarily fix them rigidly together. In this thesis, we are interested in connections which have some freedom for relative motion. Where these relationships persist, a new kind of composite artefact may be created, whose connected parts remain unmelded, still affording some form of relative motion. From a shape perspective, we will refer to these kind of connected interactions as variable spatial relationships (VSRs).

In the material world, a huge variety of physical or topological relationships between entities may be established. One might put a marble inside a jar, a hamster in a wheel, two kittens within a room, or release a flock of birds inside an aviary. One might attach an anchor to one end of a length of chain and a catamaran to the other, take a dog for a walk on leash, or use a length of rope to tether a goat to a gatepost. Forging and sequentially inter-connecting together the links of a chain gives rise to a further constrained connection and relation between parts, affording a particular type of relative motion between its successive links.

For these examples of physical and topological relationships, spatial envelopes (such as the jar and the room), which limit otherwise largely unconstrained motions, may be readily described using shape-based descriptions. Rigid assemblies (such as the chain or the hinge), whose connected parts afford relative motions with various freedoms between distinct components, may also be readily described using geometric shapes. However, although aspects of their material behaviour may be modelled mathematically, connections created by flexible material components (such as the leash or the rope), pose a slightly greater challenge when devising shape descriptions, since the shapes of these parts themselves do not remain static. We shall revisit this concern in more detail, in the following section.

Each of these examples employs either topological containment or direct material connection, to instantiate variable spatial relations between two entities. But to create such relationships, direct physical or material interaction is not always necessary. For instance, at a planetary scale, physical forces such as gravity or electromagnetism cause remote interactions between bodies, resulting in variable spatial relationships. Within virtual models, variable spatial relations may also be prescribed in an indirect manner;

rather than being actively constructed through physical or material connection or constraint, they may alternatively be defined abstractly, to obey certain geometric or mathematical rules. Variable spatial relations defined in this immaterial manner may afford further interesting routes for exploring creative possibilities.

Since our interest here is in making, our focus within this thesis remains primarily with materially constructible relationships. We are interested primarily in the strictly mechanical relationships, such as those between the rigid parts which form a revolute joint or hinge. For material compositions where direct contact between rigid parts gives rise to constrained motions, the connections themselves are amenable to description using shapes.

Kinematic connections

For three-dimensional kinematic artefacts, common connection types result from maintaining surface-to-surface contact between pairs of rigid parts. These are lower kinematic pairs (Reuleaux, 1876), of which hinges, or, revolute joints, constitute an example (Figure 4.2). Pairs of interacting surfaces must possess similar curvatures: these may be flat (planar and prismatic pairs), singly curved (cylindrical, revolute and screw pairs (and some prismatic pairs), or doubly curved (spherical pairs). These surfaces can be readily described as sub-shapes which are shared between connected parts.

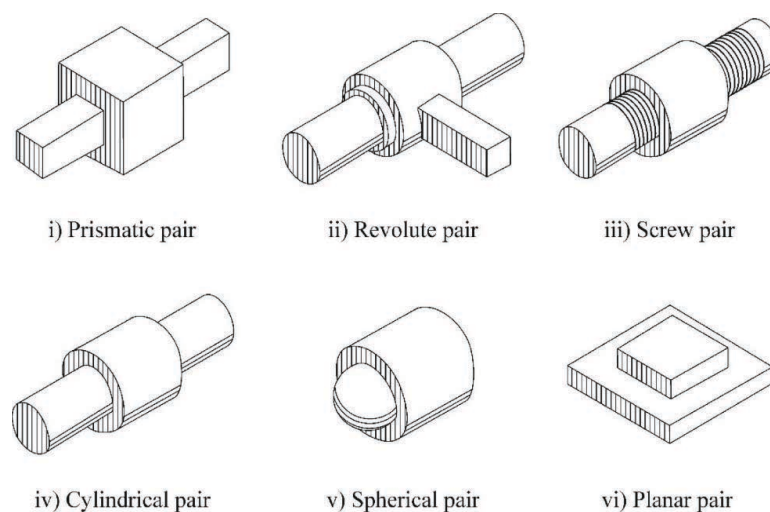


Figure 4.2 The lower kinematic pairs (Reuleaux, 1876)

Shared sub-shapes such as these provide one possible route, towards describing connections that construct variable spatial relationships within kinematic designs using only shape descriptions (Figure 4.3). By assigning particular qualities to the sub-shapes of interest, shape representations might provide a useful way to describe operations for adding, removing, or otherwise transforming different types of connections between parts. Simpler representations for these same kinematic connections can also be found using sub-shapes of lesser dimensions, such as shared lines (to represent for instance, the shared axis about which parts within a revolute joint rotate) and points (which might represent for instance the centre of rotation of a spherical joint). In some specific cases, similar kinematic behaviours can also be effectively modelled within representation spaces of reduced dimensions (see Figure 4.48). However, where simplified descriptions use connected shapes of reduced dimensions, they will not necessarily contain enough information to describe the *material* constraints required, to physically construct such connections within a physical three-dimensional space.

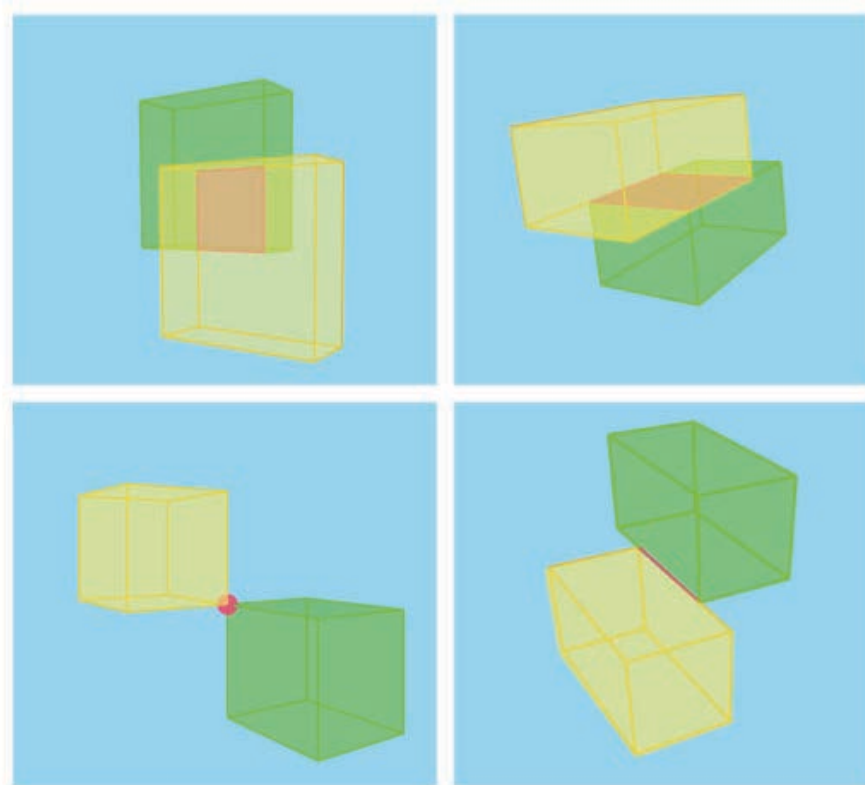


Figure 4.3 Connections which create Variable Spatial Relationships described by shared sub-shapes (shown in red)

Pairwise relationships between connected parts provide a starting point from which more complex designs can be constructed. A kinematic chain is a series of parts or links connected by joints, its freedom being the sum of those freedoms found at each joint. A linkage is a mechanism composed of several parts or links, that are connected to one another with joints, to form a closed loop. Planar linkages, which exhibit movements restricted to 2 dimensions, can be adequately represented by shape descriptions in only 2 dimensions. For closed-loop planar linkages connected with revolute joints, at least 4 connected links are necessary to reliably afford motion. For spatial linkages in 3 dimensions, generally such linkages must contain at least seven bars, in order to guarantee motion with one degree-of-freedom. However, there are some exceptions to this condition, which we shall encounter later in this chapter. The analysis of truly three-dimensional spatial linkages therefore requires complex methods which are highly analytical in character. Here we consider how synthetic material exploration modes, such as the exploratory making of physical models, might play a complementary role to established analytical techniques, for describing and considering kinematic motions.

Connection topologies within kinematic designs have been the subject of formal description and generation using Graph Grammars (Schmidt et al., 2000). These types of description do not take into account either the geometric configurations of connections, or the material shape of parts themselves. The application of shape grammars to kinematics remains largely unexplored, although pointers are available in research on structures (Shea & Cagan, 1999). Shape-based descriptions can describe both pair-wise connections between parts, and their geometric and topological configurations within designs. However, existing shape grammar theory does not explain how shapes might describe the kinds of connections between parts that give rise to variable spatial relationships in lower kinematic pairs.

4.1.2 Making things

Knight and Stiny (2015) explain how shape algebras provide a basis for making rules. We examine what these making rules might be, and also how they might operate, for the case of kinematic designs. Ingold (2013a) distinguishes between things and stuff, for affording creative agency. When artefacts and materials are perceived, not as static and immutable

objects, but rather as stuff which might be manipulated, new creative possibilities become accessible. Neither Knight and Stiny nor Ingold provide much discussion of the role of tools themselves within making processes. Maclachlan (2018) however proposes that the generative rules employed within computational design processes perform a similar role to that of physical tools within practical making activities. This would imply that, for shape grammars, shape rules themselves may be the tools by which algebras of shape are manipulated to construct compositions.

We propose that, for material making, a straightforward hierarchy exists, between stuff, things, and tools (Figure 4.4), where tools (Figure 4.5) are a special type of thing, affording the manipulation or transformation of stuff. Both things and tools are made from this stuff. Things made from stuff may themselves be complete or partial. Things may also be treated as stuff, and combined together to make new aggregate things; or else readily deconstructed into the stuff from which they are composed. Since things are made of stuff, tools may also be applied directly to things, to make alterations. Tools may be applied to alter other tools.

Access to tools, or else the facility to fabricate them, seems to be essential for creative agency— since with fewer tools, fewer things can be either created or altered, and the space of possibilities contracts. In the design process, this may apply both during the development of design ideas, due to the limited representational scope of drawing or prototyping facilities, and during fabrication directly, due to the limited scope of manufacturing capacity. Conversely, the creation of new tools opens up new spaces for creative exploration.

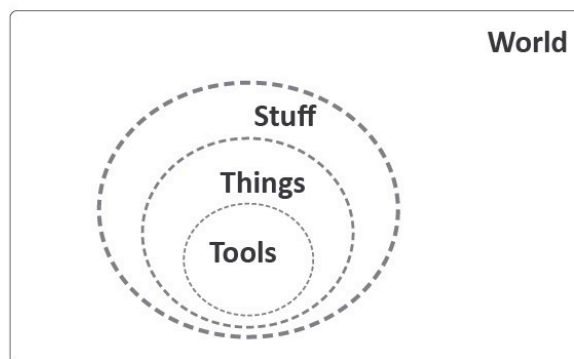


Figure 4.4 A hierarchical model for things, stuff and tools



Figure 4.5 Tools used in many of the making examples. Rubber handles (orange) augment the ruler to help repositioning

Mark-making

Since we propose to adapt shape rules to define more general making rules, a sensible starting point might be the material construction of shape compositions themselves. Shape rules can describe actions which create drawings, and also the shapes seen as they develop. But it takes more than shape rules to materially instantiate these shapes, so that they may be visually experienced. In practice, it takes pencil and paper, or some other digital or material mark-making medium. The power of shape rules depends on myriad opportunities for visual instantiation. In a material context, shapes are used to describe what is seen, and to instruct making. Examining Kolam pattern making, Knight (2018) employs shape descriptions to model actions occurring in other algebras of stuff (beyond drawing or 2d mark making), and implicitly advocates a pictorial approach to describing making actions.

We have already discussed, how various contributory materials might be co-opted into the creation of pictorial representations. The first requirement is a surface amenable to supporting mark-making. Marks may be incisions in the surface; or use another material as a pigment- creating traces across a secondary substrate (see Jarman, 1993). In each case,

manual actions which move the tip of a tool across a material surface are employed. Where the content of those marks is of consequence, shape descriptions can be helpful both to guide these gestures, and to record their visual effects. But whilst shapes can describe pictorial plans, in advance of action, or else report on them after the fact (see Suchman, 1987), they do not necessarily fully describe the many nuanced details of how tools act on materials in practice (Knight, 2018).

Material mark making may include both additive and subtractive processes, depending on the media co-opted. For the purposes of shape description, once material parts or pieces become permanently, statically attached to one another, rather than remain as separate entities they might be considered to comprise a new, composite artefact. Once materially instantiated, shape compositions themselves often become visual or symbolic representations, that are about other things.

Three-dimensional substrates do not usually afford either manual or visual access to their interiors. But numerous virtual, computational tools already provide digital substrates with enhanced 3d mark-making capabilities (Hamurcu et al., 2020). However, to support creativity using unstructured marks within a three-dimensional space, the concerns discussed by Jowers et al. (2011; 2013) regarding two-dimensional spaces for digital drawing are likely to remain significant.

In material making activities, the application of tools to materials, to alter, transform or otherwise manipulate them, plays a central role. We propose that, for the description of making processes, making rules should primarily describe, guide and model this action of tools upon materials. Many aspects of making or manufacturing activity can be described abstractly using shape-based transformations (Shea et al., 2010; Ertelt & Shea, 2009; Krstic 2019, Knight & Sass 2010; Knight et al., 2008). We review the descriptive capacity of shape rules, to describe making in a variety of contexts, and see how far this capacity extends.

Subtraction rules for rigid material

In Stiny's early developments of shape grammars for three-dimensional shapes, to illicit a demonstration of the creative potential of shape algebras as formal generative descriptions, the example of wooden kindergarten building blocks is employed. Since we now consider

instead the creative potential of making algebras more generally, perhaps we should begin with a visit to the wood-workshop, to consider how a set of these playthings might be fabricated there?

In this workshop, the major pieces of equipment perform subtractive operations: the pillar drill subtracts cylindrical shapes; the band saw and the table saw subtract extruded shapes (long and narrow cuboids or thereabouts), traced by the width of the saw blade itself, as it passes through material. As well as removing material from the workpiece, these subtractive operations can divide a piece of material into two distinct parts. Since the wooden blocks of Froebel's Gifts no. 3-6 (see Figure 4.6 and Stiny, 1980; Wright, 1957) are all either cuboids or triangular prisms, the straight-cut faces created by the saw blades are sufficient, to create them by division.

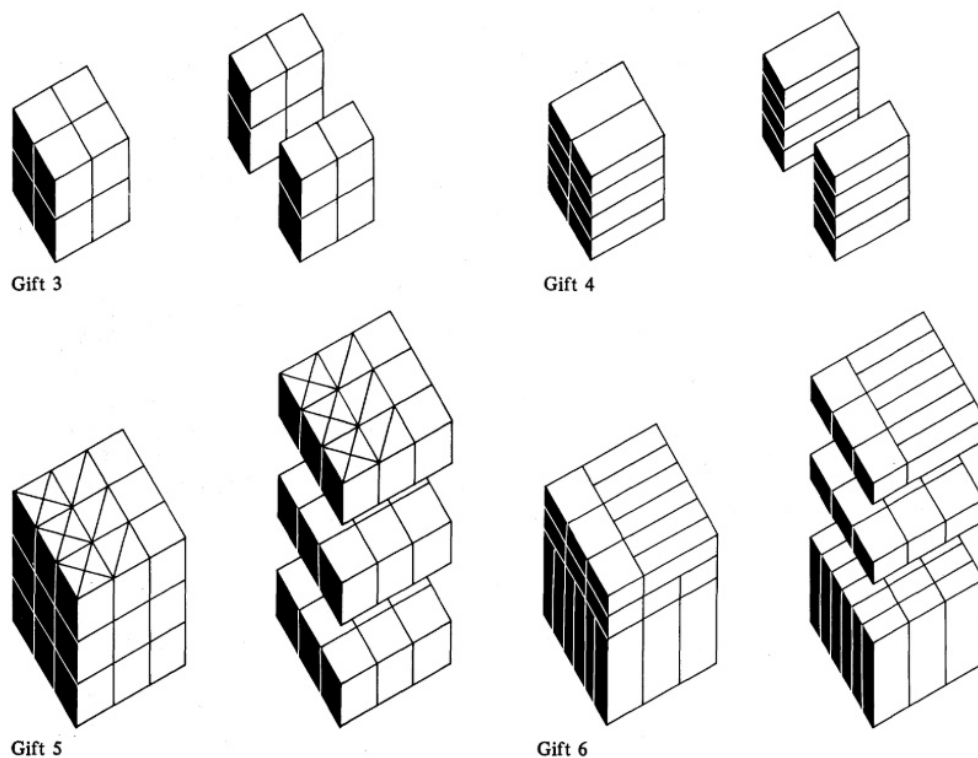


Figure 1. Froebel's building gifts.

Figure 4.6 Froebel's Kindergarten gifts (Stiny, 1980)

From a shape rule perspective, to create these blocks, a thin, planar sliver of material (describable in U_{33}) is removed by the saw blade from stock material. This shape subtraction (Figure 4.7) gives two distinct and separate solid 3d shapes, one of which will be retained as the workpiece. (Strictly, we also define a third distinct shape, which is removed and reduced to sawdust by the sawing operation). A series of mutually perpendicular cutting operations, transformed through various rotations, should be adequate to create an appropriately aligned initial shape; or else we might employ a mechanical planer, to ensure pairs of parallel surfaces, trimming our stock piece to be a suitable starting point from which to commence more precise stages of fabrication. Or else, if provided already with prismatic stock materials of an appropriately square or rectangular cross-section and of suitable dimensions, all we might need to fabricate the cube and cuboid blocks might be a single perpendicular saw-cut. With that completed block set aside, the surplus off-cut next becomes the workpiece, and in this manner a series of identical blocks can be cut from the same length of timber, merely by a series of parallel cuts of suitable spacing. Notice also that, rather than creating a series of smaller cubes, which are later assembled to construct the larger, depending on the dimensions of available stock material, the divided cube of Gift 3 may be alternatively created by beginning with the larger cube, and then dissecting it with a series of parallel and perpendicular cutting operations, to create the nine smaller cubes.

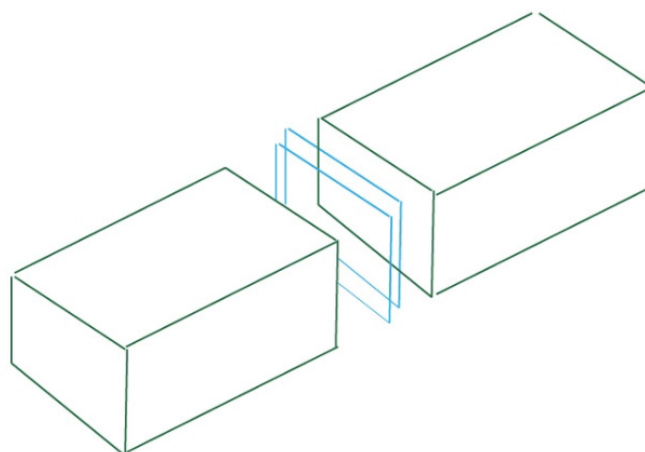


Figure 4.7 Sawing as subtraction

Creating surfaces of connection

To create assembled furniture (rather than single blocks), one must usually devise and assemble rigid joints, to combine multiple parts together. But since these wooden pieces lack the ready 'melding' of touching shapes that is peculiar to visual shape algebras, either supplementary joining materials, or particular geometries are required, to achieve rigid connections (see Sumiyoshi & Matsui, 1989). Creating these shapes using hand tools may further contribute to the creative process (see Tsukuda et al., 2015). Although Krstic (2019) has denoted how shape rules might describe the distinct parts within a rigid assembly, shape descriptions alone do not inherently afford an understanding of the capacities of particular interacting solid and surface geometries, to maintain their rigid connections. The subtractive operations of the woodworking machinery also find their counterparts in the adjacent metal workshop, with its mills and lathes. Here, shaping metal parts and machining components through subtractive operations may be only half the story, as their purpose is assembly into machines.

Deformable and flexible materials

In the metal workshop, tools also perform plastic deformations. The transformative effects of forging, hammering, and annealing malleable materials may certainly be described using shapes, but such descriptions bely their micro-structural material changes. A sheet bender, plastically deforming a metal sheet along a prescribed line and by a prescribed angle, could perhaps be used to make hollow, metallic copies of Froebel's cuboids, folded up from sheet. In a similar manner, the tube bender imparts curves of a prescribed radius to straight stock tubular extrusions. Using engineering principles, the results of these material transformations can be accurately calculated, and might therefore be modelled reliably using calculated schemas for shape.

Further tools, materials, and techniques for material transformation are available in other workshops, such as the ceramics studio. In this and other workshop operations, the ability to successfully reinterpret purely shape-based instructions into material actions may require significant, situated experience, in working within particular media. Manufacturing technologies each possess particular capacities, for materially instantiating geometric shape descriptions. The boundaries to these possibilities may sometimes be best understood

through exceeding them; with each failed episode informing an improved understanding of material potential.

When constructing and sewing fabric in the cutting room, shapes can readily be used to describe pattern pieces, which specify how reams of fabric, laid flat, may be cut and reassembled to fabricate garments. However, since they bely the complexity of skilfully handling and manipulating tools and materials in combination, simple shape instructions alone do not contain enough information, to stand by themselves as instructions for making. They are at best plans or schematics (see Suchman, 1987), requiring additional expertise for their instantiation.

Whereas making actions using tools tend to bring about permanent shape transformations, the manual manipulation of flexible or elastically deformable materials may rather bring about temporary transitions in shape. Artefacts composed of moving parts or flexible materials may refuse to conform to a single configuration, and many possible shape instantiations may lie within their ready material capability. This is a feature which designers and artists exploit. For example, Duchamp says of his piece '3 Standard Stoppages' (Figure 4.8), that

"It's a joke about the meter ...

..... If a straight horizontal thread one meter long falls from a height of one meter onto a horizontal plane, twisting as it pleases, [it] creates a new image of the unit of length."

(Moma, 2021).



Figure 4.8 Marcel Duchamp: 3 Standard Stoppages. Paris 1913-14. (Moma, 2021)

Duchamp's material quasi-experiments neatly demonstrate the inherent computational potential of certain material things, when subject to certain conditions, to create a particular set of shapes. Assisted by the gravitational forces which set it into motion, both the material properties of the string, and the canvas which breaks its fall, contribute to this physical computation.

Antoni Gaudi's hanging models for catenary arches provide another seminal example of the material computation of shape, where more precise conditions provide a deterministic set of curves, amenable to mathematical description. In this case, their material behaviour is exploited for its ability to mimic the shapes of viable structures for an alternate selection of stuff, and their materially computed shapes then provide design guidance for the construction of masonry arches. (Kilian, 2004; Burry, 2011).

Gürsoy and Özkar (2015) reflect that some combinations of shape rules may amplify the effects of material properties, with design implications that are more than the sum of their parts. Maclaclan and Jowers (2014) consider how material properties might be described within a design process using weighted shape rules. Physical model-making and interaction offers a direct route to experiencing these material properties, but in some cases, it may also be feasible to describe and predict shape transformations due to material properties using formal material rules.

Since our focus here is on designs with rigid, connected parts, of particular interest are those materials which may alternatively adopt both rigid and flexible behaviours. In its facility to create these conditions through folding and creasing, card and paper are relatively unusual. Folding, to create a crease at a particular location, differs from continuous bending. Whereas bending, for paper, relies on elastic deformation only, folding rather creates permanent deformation along a crease-line. In this way, previous actions (creasing) leave a memory in the sheet, with a preference for certain material transformations. For a previously creased sheet of card, it is difficult to subsequently create a continuous curved bend, since less force will be required to create plastic deformation along a hinge-line, than to create elastic deformation of the previously unaffected material (Coffin et al., 2012). Polypropylene sheet is another material which

affords the creation of flexible folds. For this case, since polymer chains become aligned during plastic deformation, in addition to becoming more flexible along the hinge-line, it also becomes stronger in this region (Maier & Calafut, 1998 pp18-19).

Shape descriptions may readily describe the position of fold-lines within a sheet, and through pictorial instruction, they might also provide guidance upon where folds themselves should be positioned. But, just as for kinematic connections, they less readily model the multiple shape configurations into which the creased sheet might subsequently be manipulated. These are more readily considered through direct material exploration.

4.2 From shape schema to making schema

Ideas for design changes are commonly inspired by imitating moves which have proved successful in similar design situations. General rules for transformations can be derived from specific actions undertaken within previous design episodes. These can be described using schema. An established repertoire of schema can then be applied to new design challenges, and through varied design experience, this repertoire continues to evolve (Stiny, 2011). Creativity in art and design might therefore be said to rely on an ability to move both backwards and forwards, between general schema and specific rules:

Rule → schema → rule

This approach allows specific actions from previous design activities to be reapplied in new design scenarios. We establish a repertoire of schema, to support exploration for a small subset of kinematic designs. Knight and Stiny (2015) suggest that, rather than employing algebras for shapes, making grammars for things should calculate instead with algebras of stuff. They propose that, just as shape rules can describe *doing* and *seeing* with *basic spatial elements* to make *shapes*, more general making rules can also be defined, and these describe *doing* and *sensing* with *stuff* to make *things*. Knight (2018) implicitly adopts a pictorial approach to describing material making actions, using shape descriptions to both guide and model craft-based techniques. In pursuit of a formal approach, Krstic (2019) examines the suitability of various kinds of algebras for describing different kinds of making

operation. These theoretical concerns are taken up again in Chapter 7, where we reflect on the making experiments in this and the next chapter. For now, we examine the details of the schema proposed by Stiny (2011).

4.2.1 Revisiting schema for shape

For shapes, Stiny (2011) identifies several levels of general schema, within which all shape rules can be classified. Rules replace one shape with another ($x \rightarrow y$). Transformation rules alter shapes in various ways: general transformations ($x \rightarrow x'$) include parametric variation, and more specifically linear transformations ($x \rightarrow t^*(x)$), of which Euclidean transformations ($x \rightarrow t(x)$) such as rotations, reflections and translations, are a subset. Other rules may replace shapes with only select parts ($x \rightarrow \text{prt}(x)$), or convert shapes into their boundaries ($x \rightarrow b(x)$), reducing them to a lesser dimension. Erasing rules ($x \rightarrow$) which delete shapes are a subset of part rules. Rules can also be combined (e.g. $x \rightarrow \text{prt}(b(x'))$) or reversed: shapes can be created from nothing ($\rightarrow x$); boundaries can be filled in ($b(x) \rightarrow x$); and shapes may even be interpreted as parts of larger shapes, which can then be added into the design ($\text{prt}(x) \rightarrow x$). Shapes can also be divided ($\text{div}(x)$) by adding transformations together ($x \rightarrow x' + x''$). And the inverse ($\text{div}(x) \rightarrow x$, or $x' + x'' \rightarrow x$) might be used to join shapes together. These are summarised in table 4.1.

Table 4.1 List of shape schema

1)	$x \rightarrow y$	unrestricted rules
2)	$x \rightarrow x + y$	addition rules
3)	$x \rightarrow x - y$	subtraction rules
4)	$x \rightarrow x$	identity rules
5)	$x \rightarrow$	erasing rules
6)	$\rightarrow x$	initial, or 'blank sheet of paper' rules
7)	$x \rightarrow t(x)$	transformation rules (Euclidean)
8)	$x \rightarrow t^*(x)$	transformation rules (linear)
9)	$x \rightarrow x'$	transformation rules (general / parametric)

- | | | |
|-----|------------------------------------|---|
| 10) | $x \rightarrow x + t(x)$ | rules for recursion (e.g. for symmetry, pattern and fractals) |
| 11) | $x \rightarrow \text{prt}(x)$ | rules for selecting parts |
| 12) | $x \rightarrow \text{prt}^{-1}(x)$ | rules which create wholes from their parts |
| 13) | $x \rightarrow x' + x''$ | combined transformation rules |
| 14) | $x \rightarrow \text{div}(x)$ | dividing rules, which split a shape into multiple parts |
| 15) | $x \rightarrow b(x)$ | rules which select a shape's boundaries |
| 16) | $x \rightarrow b^{-1}(x)$ | rules which instantiate a shape from its boundaries |

Many of these schema can be readily redeployed, to describe material making activities. For instance, sawing a workpiece into two distinct parts, by removing a thin sliver of material, might be described alternatively by:

$$x \rightarrow \text{part}_1(x) + \text{part}_3(x) \quad \text{or,}$$

$$x \rightarrow x - \text{part}_2(x),$$

where $\text{part}_2(x)$ is the material removed by the sawing action.

Cutting with a scalpel blade leaves no material residue, and is an example of:

$$x \rightarrow \text{div}(x)$$

Joining is:

$$\text{div}(x) \rightarrow x, \quad \text{or}$$

$$\text{part}_1(x) + \text{part}_2(x) \rightarrow x$$

and seems to be of particular importance for material making. The addition rule, however, may require further investigation, since the ready melding of shape algebras is rarely available in the material world. Instead, joining creates a connection. Such rules might refer to assembly operations, where precise geometric interactions are enough, to create rigid connections. Alternatively (just as for the case of dividing, which may be subtractive), some additional joining medium may be required, such as adhesive or bolts. Connections may be

rigid, or mobile- as in kinematic designs. The operations of casting or photography might be described by:

$$\mathbf{x} \rightarrow \mathbf{b}^{-1}(\mathbf{x})$$

for creating a shape in n dimensions from its boundaries in $n-1$.

We note the absence, of the 'blank sheet rule':

$$\rightarrow \mathbf{x}$$

for making.

4.2.2 Making 3d shapes

Rather than operations acting upon abstract spatial elements, formal shape rules for describing design changes can be reinterpreted to describe making. To this end, we begin simply, by considering various ways in which familiar, modular 3d shapes- such as the cuboids and triangular prisms of Stiny's kindergarten grammars- might be practically constructed using material algebras.

Froebel's Kindergarten gifts were themselves intended to provide a situated education in spatial reasoning. For older children, Froebel also devised a series of *occupations*. These were more open-ended endeavours, using consumable materials, and the resulting constructions were generally retained intact. Taken altogether, Froebel's educational system was intended for use far beyond the Kindergarten, and to provide a thorough grounding in abstract and creative spatial reasoning concerning solids, surfaces, lines and points (Rogers, 2016). Froebel himself enumerates six original gifts. All, apart from Gift 1, are composed of solid wooden blocks (Froebel, 1895). But Ingold (2015) might be pleased to note that Gift 1, the very first received by the child, consists instead of coloured balls of yarn. Whilst Froebel's explicitly numbered Kindergarten Gifts 1-6 are presented already instantiated as solid artefacts (see for example Gift 3, in Figure 4.9), in the more complex occupations intended for older children, the familiar

shapes of the gifts are themselves constructed by the student, using a range of both shape and material algebras (Rogers, 2016: Kraus-Boelte & Kraus, 1877; 1892).

Whilst the solid wooden cubes might be readily described by a shape algebra in U_{33} , Froebel's exercises also include the fabrication of these same shapes, as hollow paper boxes in U_{23} . A further occupation, (Figure 4.10) known as 'pea work', employs an algebra of wooden sticks and dried peas softened by soaking, to create wireframe models. (Left to dry, the peas contract around the sticks, to create permanent rigid connections). The technique is practiced first by the fabrication of simple planar shapes, representable in U_{12} . The geometry of these flat shapes becomes increasingly advanced, before they are extruded upwards out of the plane, in the creation of wireframe models. These revisit the familiar 3d geometries of the original cuboids and prisms, but now described in U_{13} , as instantiated by a material algebra of peas and sticks. The final occupation, involving modelling with clay, returns full-circle, back to U_{33} . These same solid cubes, presented as ready-mades in the kindergarten, are now themselves created and divided, using a cutting wire, to discover gifts 3 and 6.

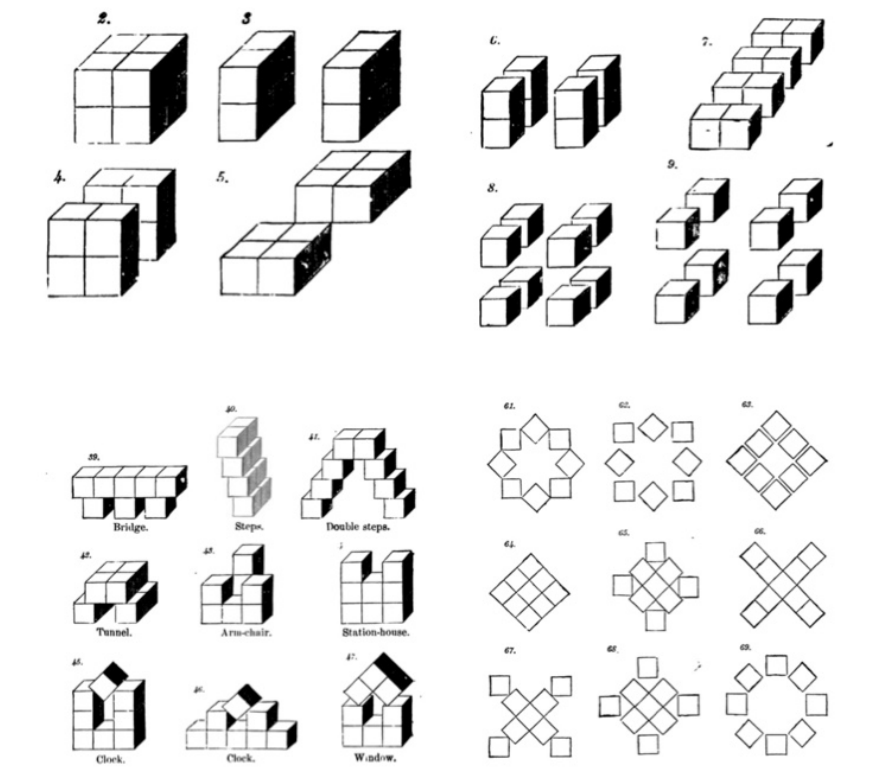
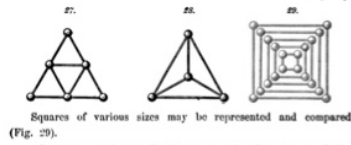
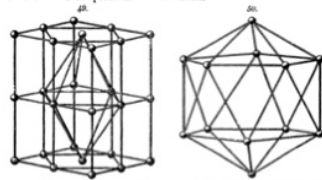
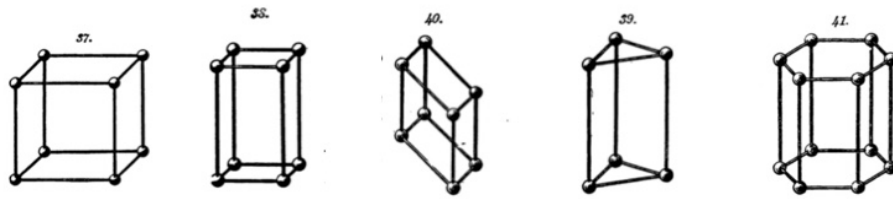


Figure 4.9 Froebel's third gift (Kraus-Boelte & Kraus, 1877)



Squares of various sizes may be represented and compared (Fig. 29).

All the geometrical outline-forms may thus be represented, for instance:

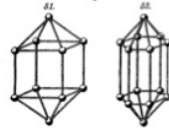
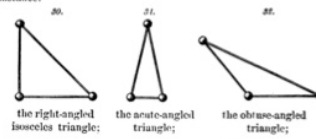
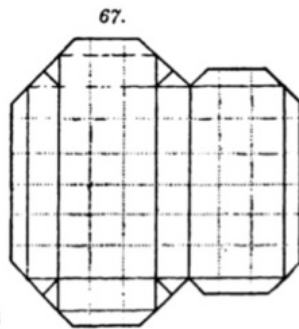
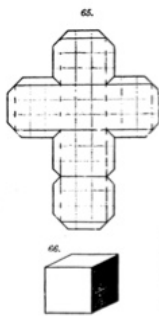


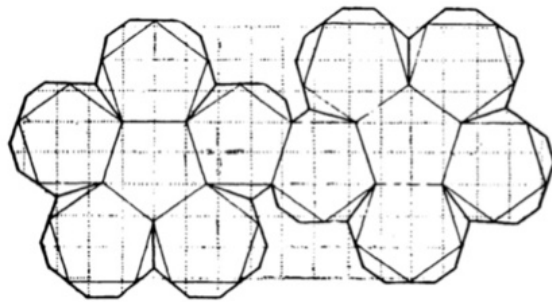
Fig. 50, the icosahedron, with its twenty equilateral outline surfaces, as well as the dodecahedron can likewise be produced. Prisms and pyramids may be joined to make one form (Figs. 51 and 52). And further, from the oblong sides of the prisms, pyra-



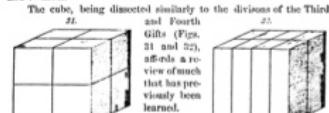
the right-angled isosceles triangle; the acute-angled triangle; the obtuse-angled triangle;



18
..1



Dividing this circle into two equal parts, the diameter is obtained, and also the semi-circle (Fig. 35).
Dividing the circle twice at right angles, produces the quadrant (Fig. 36).
These quarter circles may be divided once more, producing one eighth of a circle.
The circle may also be divided into thirds, sixths and twelfths.



The cube, being dissected similarly to the divisions of the Third and Fourth Gifts (Figs. 31 and 32), affords a review of much that has previously been learned.

The cube may also be divided by a diagonal cut, first into halves, by a further division into quarters, as found in the Fifth Gift.

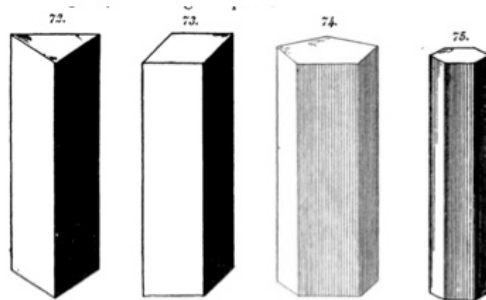


Figure 4.10 Froebel's ninth (Pea-work), tenth (Cardboard-modelling) and eleventh (Modelling in clay) 'Occupations'. (Kraus-Boelte & Kraus,1892)

Looking ahead, to the practical episodes of material making, later in the chapter; we will repeatedly use paper or cardboard modelling, as an expedient U_{23} material algebra, involving cutting, scoring, folding, and joining paper, to create surface models of 3d forms. Even paper folding models have 'hinges', or revolute kinematic connections. We examine making for a range of folding models.

4.2.3 Making shapes from nets: experimental folding from 2d to 3d

Scoring a sheet, at its simplest, divides it into 2 connected parts, with a hinge-like connection between them. The shapes of the divided parts can be described using shape rules, but for describing this new connection, more is needed. The scored line simultaneously divides the piece into two distinct parts, whilst also creating between them a lasting but flexible connection, that affords the potential for relative motion between these newly defined parts.

In terms of shape description, this fabrication process begins with linear marks in drawings, U_{12} . Upon scoring and articulating the folds, however, an entirely new workspace is subsequently accessed; the workpiece now inhabits the space U_{23} , where many subsequent actions for manipulation and joining may now take place.

In order to create closed 3d shapes from sheet materials, some calculation is usually required before cutting. The use of 2d nets affords the description of 3d shapes as closed three-dimensional surfaces, composed from cut and folded two-dimensional sheet materials. At its simplest, this mode of material making is a commonplace practice, with which most schoolchildren become familiar. The method employs drawing, cutting, scoring, folding and joining, using simple and accessible tools and materials.

Many three-dimensional shapes with flat faces and singly-curved surfaces can be created in this manner. For regular polyhedra, possessing sets of similar faces and a high degree of symmetry, the creation of their nets can be relatively straightforward. For less regular shapes, detailed reasoning may be required, to develop a valid making description.

A significant number of the models constructed during the experimental making episodes, described later in this chapter, were fabricated in this manner. This construction technique

employed several different types of making operations. The process of creating each 3d model began with drawing, to create a valid 2d net. These drawings were then transferred onto paper or lightweight card, and followed by further operations of cutting, scoring, folding and joining.

In these exploratory making episodes, constructing 2d drawings looks very much like conventional shape grammar operations, and can readily be described as such. A little more is needed, however, to describe subsequent stages of making, where cutting tools convert the marks made in drawings into cut and scored lines. Manual manipulation transforms 2d nets out of the plane, by folding and deforming them about their scored lines. For nets for closed polyhedra, manipulation brings pairs of similar edges into alignment, to then be glued in fixed connection, creating manifold 3d forms.

When creating any net for a 3d shape, there are usually two distinct ways in which adjacent faces, connected along a shared edge of the 3d shape, can be specified. One is along fold lines within the net, and the other by the glued or taped connection of pairs of free edges. We examine making nets for cubes and parallelepipeds.

Nets for cubes (and perhaps also tetrahedra) may already be familiar motifs. Most commonly used is a 'cross-motif' formation, although there are several equivalent nets (Figure 4.11). From one valid net, others can be generated geometrically, without the need to actually construct a 3d representation. A cube has 11 distinct, equivalent nets.

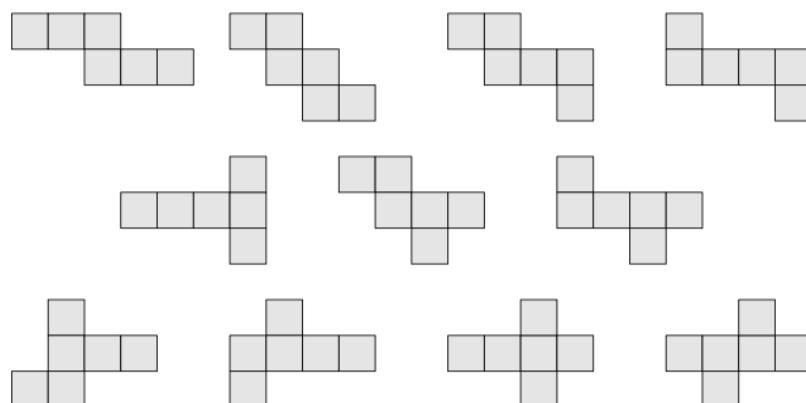


Figure 4.11 Nets for a cube

The process of finding a net for a new 3d shape may use descriptions in multiple algebras simultaneously. It may be either constructive (by reasoning in two-dimensions, about incidences between corners, edges and faces between faces) or deconstructive (by defining cuts between adjacent faces upon the surface of a 3d model). Once a valid net is obtained, it can be further transformed to find equivalent configurations (Figure 4.12). These multiple routes to making inform construction.

- A net can be begun by drawing any single face, and then aligning the edges of four adjacent faces along each of its four edges.
- Where any two of these newly added faces share a corner, along with the initial face they must necessarily form part of a 'corner triple', defining that corner. These initially unpaired edges will later *become paired*, since they are brought into alignment upon the articulation of the net about its scored lines upon folding, and can be permanently attached there using secondary connections. Therefore, these pairs of compatible edges can be conceptually labelled as 'accounted for', within the unfolded 2d layout. Functional gluing tabs may also be assigned to them, affording their permanent connection during a later stage of fabrication.
- The only sites which now remain, as candidates for the possible addition of further new faces, are those external edges of the net which remain unlabelled. For the square, only one of the six faces now remains to be attached, but there are 4 potential vacant sites. Selecting each of these 4 possible sites in turn completes four distinct versions of the 'cross-motif' configuration net— each with a different orientation but identical under rotation.
- Further reasoning about the nature of secondary connections may be applied recursively, until the secondary connections between all external edges are appreciated. All remaining unpaired, external edges, which must variously become paired upon folding, can be identified with their particular counterparts. With all faces added, and all external edges shown to have a corresponding partner, the net for the manifold 3d object is now completed.
- Once a complete net is constructed, Information about adjacencies and connections can be used to rearrange the net layout, discovering further valid net configurations.

To create new connections that align new edge pairs within the 2d layout, existing edge pair connections are broken (and to support further reconfiguration, the newly exposed pair of external edges must be accordingly labelled).

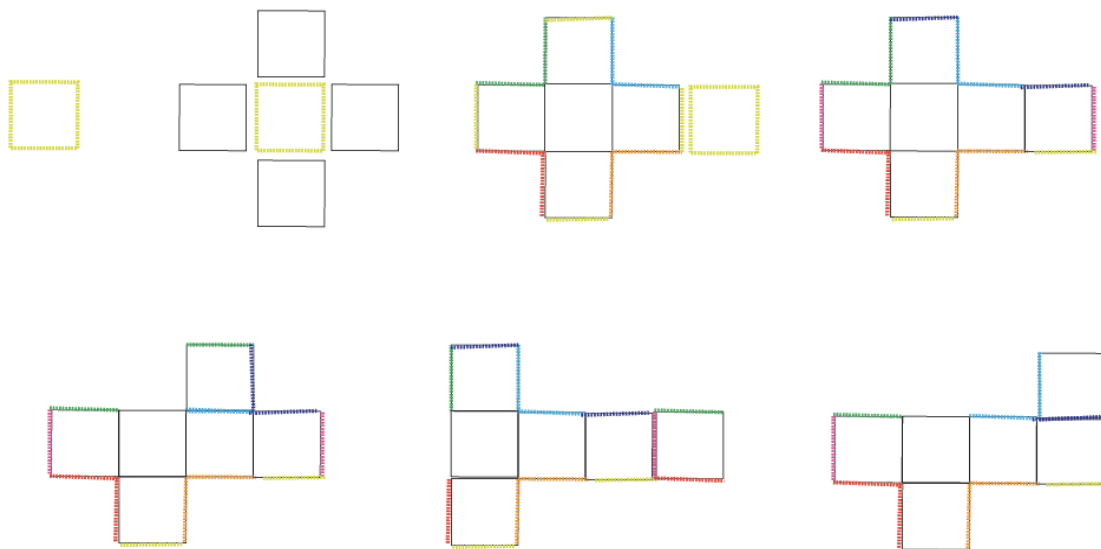


Figure 4.12 Constructing and reconfiguring nets for cubes

These steps might readily be described using labelled shape rules. The relocation of labelled shapes shown here (Figure 4.12) is largely achieved by rotation, in order to bring similarly labelled (i.e. coloured) edges newly into alignment. It is important to note however, that for representations in U_{13} , shape rules alone do not recognise square faces as permanent parts. In drawings in U_{12} , shape rules which instead rotate only the 3 external edges of a square face require a further supplementary rule, which adds a new external line to complete the net boundary. Because this additional line represents a newly exposed edge, this feature might in fact be advantageous. However, if all labels or colours were removed from the drawing, purely shape based transformations, describing alterations between one configuration and another, could be specified to instead apply *translations* to sets of 3 external lines, to largely achieve the same effect but without the need for an additional rule. (This approach works for the cube since, due to the inherent symmetry of its square faces,

all its edges are equivalent, so no rotation is fundamentally necessary. But this shortcut would not apply for the general case of a net for any 3d shape).

Alternatively, a shape representation in U_{23} could better preserve squares as constant shapes. But it is important to note, that the need to recognise and preserve both distinct parts, and the particular connections or relationships between them, is not inherently supported by conventional shape grammars. The physical models under consideration here feature distinct parts with particular types of connections and interdependencies, which are preserved under transformations. However, during any phases of making which employ unstructured descriptions, the potential opportunity to take advantage of embedding and emergence may arise.

Further exercises in net-making consider other 3d shapes. Every face of a parallelepiped is a parallelogram. Since pairs of opposing faces are identical in shape, there are 3 distinct faces. A parallelepiped is defined by any three non-zero vectors which emanate from a local origin (Figure 4.13).

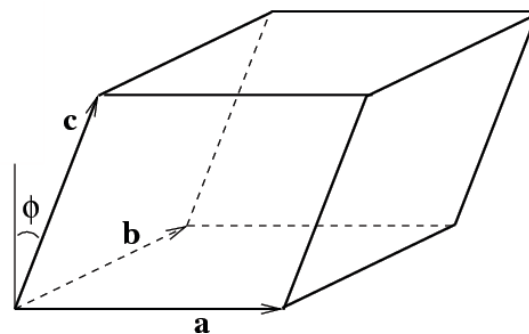


Figure 4.13 A parallelepiped defined by vectors

Initially, a parallelepiped was created with one face specified as a square, and the other two faces as identical rhombi. For this less familiar shape, an alternative approach is used to generate a net, by experimentally making models— some of which will not work, as these help to identify valid rules. For all parallelepipeds:

- Pairs of opposing faces have identical shapes
- Pairs of opposing faces cannot be adjacent within the net.

This seems to be enough, to begin assembling faces through a constructive approach (as was applied earlier for cubes) to create a 'cross-motif' net (Figure 4.14).



Figure 4.14 Constructing a 'cross-motif' net for a parallelepiped

Due to this new lack of equivalence among faces, for parallelepipeds, there initially appear to be several possible orientations in which faces may be assembled together to create a net (following the approach derived previously for a cube), and their equivalence is not immediately obvious. However, here we see that what appeared at first to be potentially different configurations are in fact identical once rotated, since for this particular parallelepiped, both pairs of rhombus-shaped faces are identical (Figure 4.15).

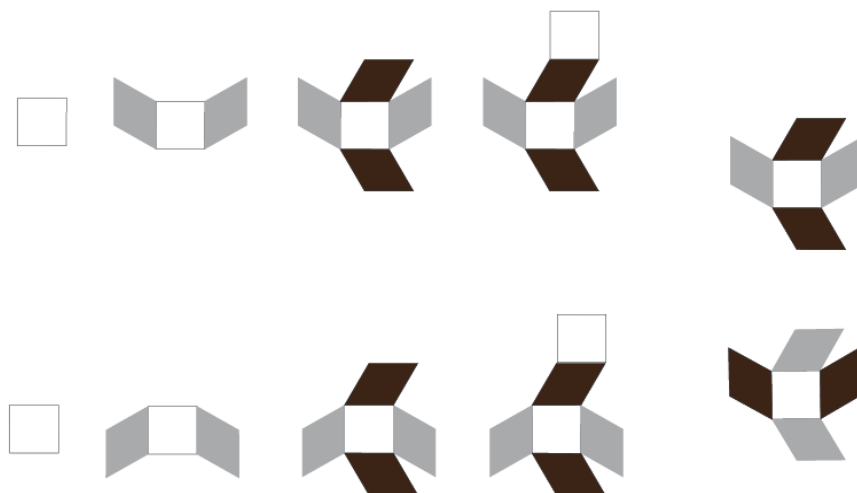


Figure 4.15 Alternative ways of assembling faces, to create nets for a parallelepiped

The diagram below (Figure 4.16) shows however that the ‘cross-motif’ net now has several visually distinct instances, since the parallelepiped’s faces possess fewer symmetries, and therefore have several distinct arrangements.

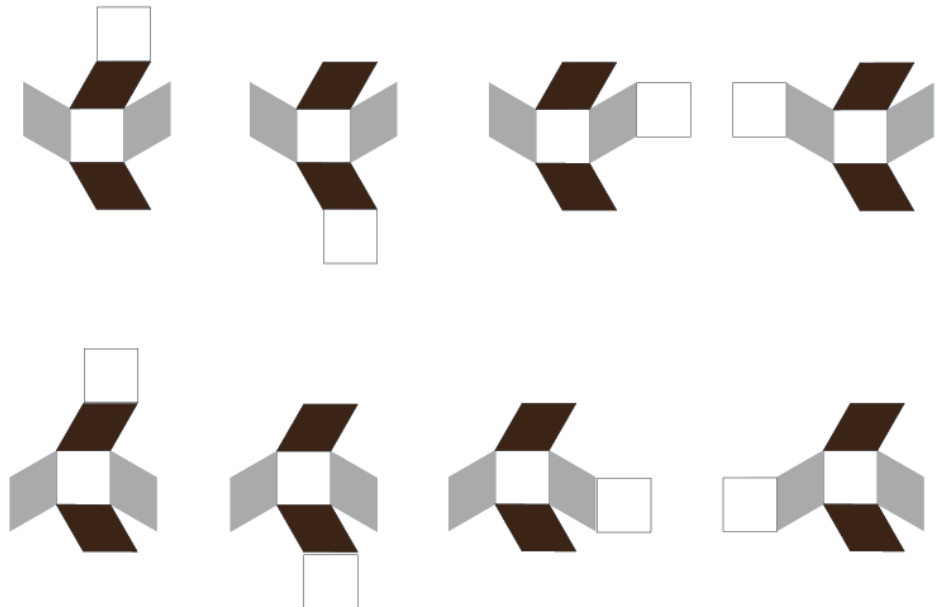


Figure 4.16 Alternatives parallelepiped net configurations

Just as for the cube, understanding edge connections now allows us to alter this net, to discover further valid configurations (Figure 4.17). These now deviate entirely from the cross-motif above. (Here, only 5 faces are shown, since (as for the cube’s 5-shape cross-motif) the final square can be added to a 5-shape net-base along any edge that remains unlabelled).

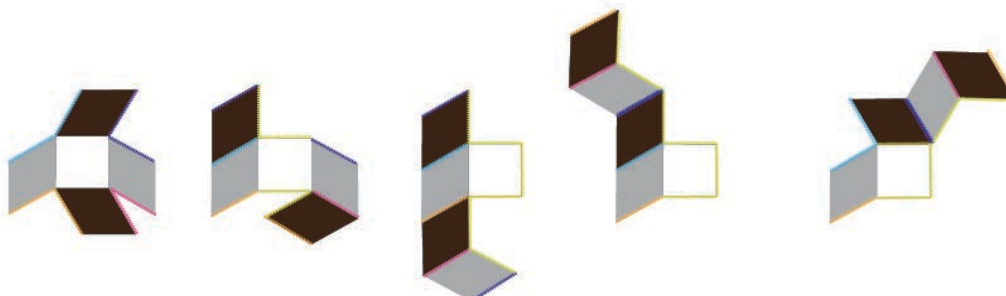


Figure 4.17 Generating equivalent valid net configurations for a parallelepiped net

With a general method for constructing the parallelepiped established, a further question, asked through exploratory making, was the extent to which the angles of the rhomboid faces might be altered, whilst still maintaining the ability to create a closed 3d shape. It was found that there was no obstacle to reducing the skew angle, to create a shape which more closely resembled a cube (Figure 4.18).

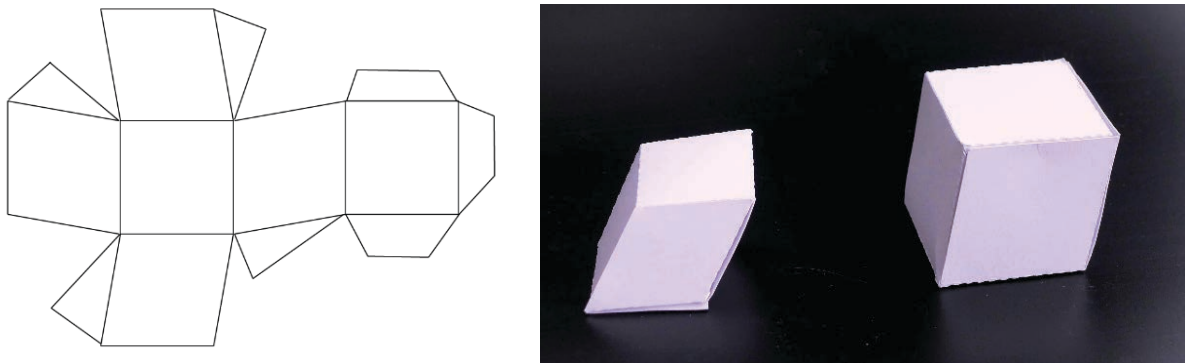


Figure 4.18 A new design, with reduced skew angle

However, upon attempting to create a more distorted form, it was noticed that there is a limit to what variation can be achieved, within the ‘cross-motif’ layout, due to altered faces intersecting each other within the planar layout (Figure 4.19). To see if this was the only practical issue hindering successful construction, a different net configuration was attempted. In this making experiment, it then became more obvious, that this geometric variation exceeds the valid limits of construction for more fundamental reasons.

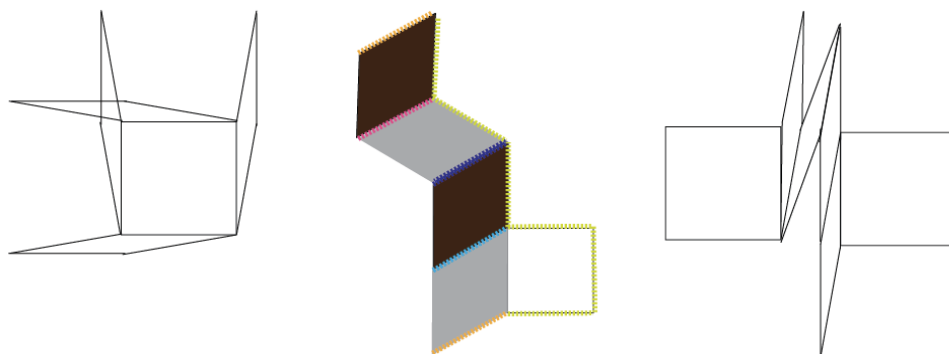


Figure 4.19 Interference between faces encountered within the standard ‘cross-motif’ net

Once the impossibility of physical assembly, due to these more fundamental concerns, was realised, further net configurations were devised, rendering fully-visible the topological issues with particular geometries. Further sequences of exploratory making sought the limits of validity. Models which *do not* work prove to be demonstrative of fundamental geometric limitations, within the design space. By marking out boundaries in this space, they explain why certain geometries work and others don't. Figure 4.20 shows an assortment of valid designs.

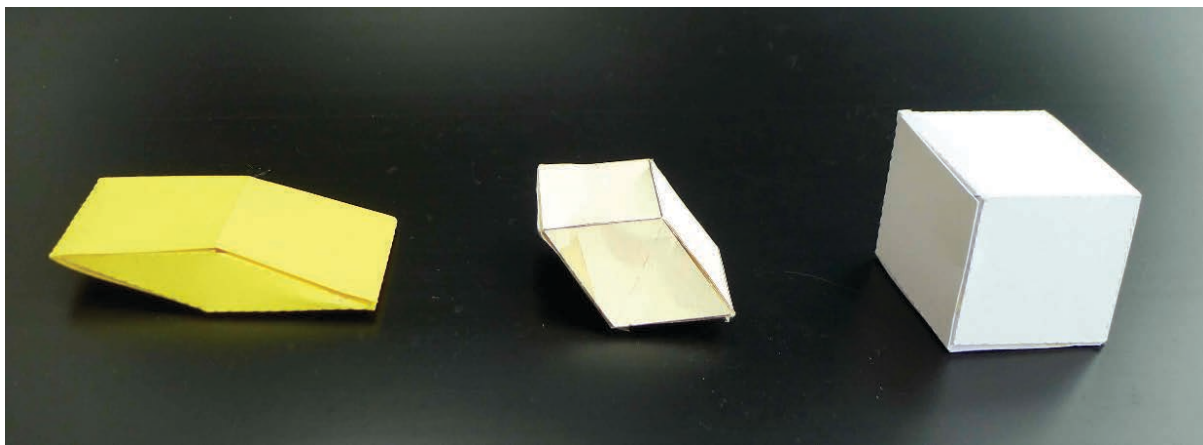


Figure 4.20 Models for parallelepipeds with a range of skew angles

Nets and models for parallelepipeds of zero volume help to clearly identify the limits within which the remaining faces for square-based parallelepipeds can be successfully defined. Pairs of faces need not be similar, but they must be feasible for construction within the 'cross-motif' configuration, and, if they are not to be of zero volume, edges of adjacent faces must not meet within the flat formation. Further exploratory making might confirm whether these, and perhaps other additional rules, also apply for the specification of non-square instances of parallelepipeds.

4.2.4 Rules for dividing shapes in U_{23}

Further constructions are developed, which use these parallelepiped shapes in the making of kinematic designs. These make a planar cut, to divide the parallelepiped into two parts,

creating a pair of separate, closed, 3d shapes (Figure 4.21). To calculate the nets for these divided shapes, marks are made on the surfaces of existing models, which inform alterations to the drawings for nets. In addition to cuts which bisect their existing faces, the division of the 3d shape, into 2 separate but manifold parts, also requires the insertion of 2 separate instances of a new 2d surface- one for each new part. These new faces define the plane of division, within each half of the newly divided net. Geometric reasoning moves between the constructed model and the unfolded net, to calculate the shape of the new cut face that should be added.

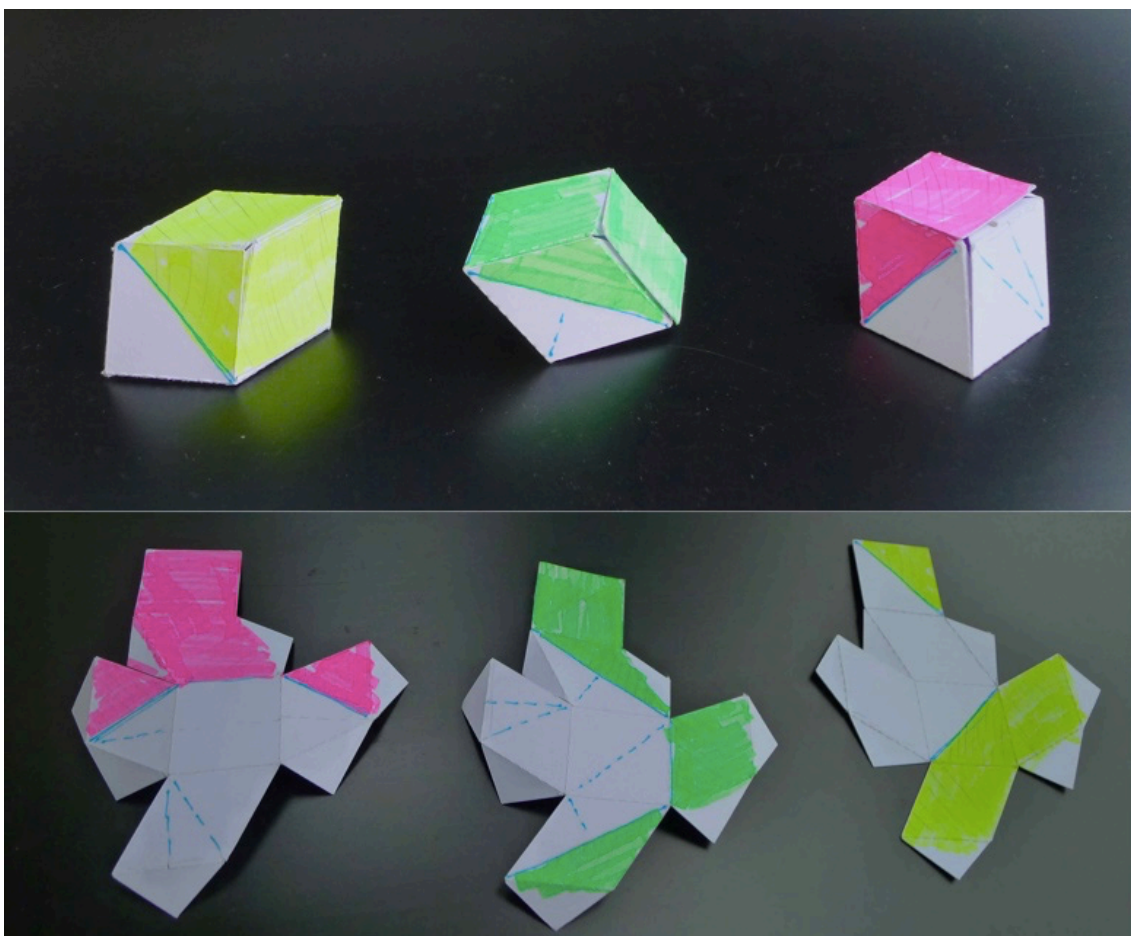


Figure 4.21 Marking both assembled models and unfolded nets informs geometric calculations for dividing the 3d shape

This making episode, begun from the starting point of the square parallelepiped net employed previously, considers how part divisions might be created. Divisions were marked upon the surface of the assembled 3d shape, specifying a particular division into 2 equal

parts. The net was then subsequently unfolded, and marks made on models interpreted into the 2d layout drawings.

Note that, although it is fairly straightforward to reason between the assembled 3d object and its 2d net, physically unfolding the model to examine it directly in its net formation requires no additional effort or abstraction whatsoever- it merely relies on copying. These division lines (Figure 4.22) both pass along both edges and bisected faces. Using the now established rules for net reconfiguration in the 2d plane, unattached segments could then be reattached to appropriate parts of the 2 divided nets. This was shown to create two distinct nets, for the two distinct parts, which are a mirror image of each other.

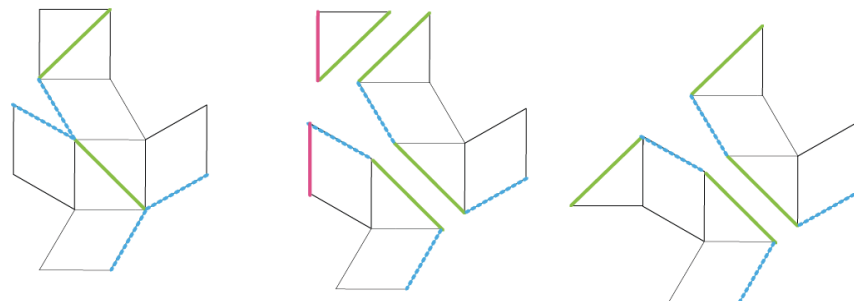


Figure 4.22 Adding division lines within nets, and rearranging

These rules, for dividing 3d shapes in U_{23} into composite 3d parts, afford the further possibility of reconnecting these sub-shapes along various edges of the cut face, to create a hinged connection— a variable spatial relation (Figure 4.23).

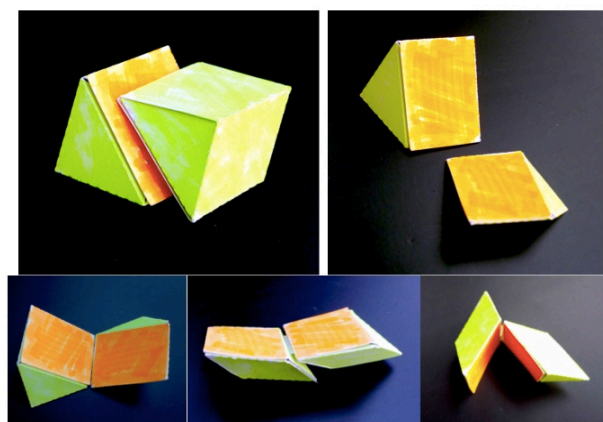


Figure 4.23 Bisecting the parallelepiped into two distinct parts, then reconnecting along one of four cut edges with a flexible hinged joint

Further variations were explored through making, including bisection rules which apply recursively, to divide and re-join parts using variable spatial relations. This creates a chain of 3d parts, connected by hinges along their edges of bisection. The capability of making rules, to provide new shapes and connected components, with potential for use in kinematic designs was confirmed.

4.2.5 Discussion – new rules for making

Under closer consideration, the use of 2d nets as a technique for making 3d models reveals itself as a salient example of both the usefulness and limitations of shape rules, for describing making activities. Shapes readily describe 2d drawings, but the seemingly simple actions employed here, of scoring and folding a 2d sheet to create a dynamic hinge, are somewhat harder to describe in this way. The manner in which folding enables a transmutation, from a 2d plane-based representation, into a new 3d space, is less readily captured by conventional shape rules. Although such shape rules might adequately describe, for instance, the piecemeal assembly of a polyhedron, face-by-face, within a 3d representation mode (i.e. by a shape rule that attaches each new face via a static spatial relationship to an existing face), shape rules alone cannot fully embody the *variable* spatial relations afforded by the paper's scored folds. So whilst an operation which plastically deforms a sheet of rigid material, about a fold line by a particular angle, might perhaps be crudely described using a static shape transformation, an operation which rather *embeds the affordance of folding*, within the model itself, requires something more.

These apparently simple actions, of scoring and folding along a line, simultaneously define both a division and dynamic connection, between two newly distinct parts. Further, these dynamic connections, now embedded within the material model itself, afford a further class of making actions- those of active manipulation. For this particular mode of material making, manipulating models whose material structure inherently *affords certain motions* seems to be a crucial activity. Altering (either experimentally or systematically) the spatial relations between parts allows the discovery of new and desirable relationships between particular elements.

When model-making with nets, the ability to manipulate a model within a 3d space, to bring about new alignments (such that secondary connections can be duly added between newly adjacent faces) seems to be a critical activity within the making process. Further, the addition of these secondary connections, only made possible through the articulation of parts, ultimately inhibits the possibility of any further motion between faces, and creates a static object. However, although completed 3d shapes that are constructed using nets may not in their final forms exhibit any kinematic behaviours, the role of variable spatial relations in affording their fabrication appears to be of central significance.

4.3 Making kinematic playthings: rules for shape and structure

Here, we report on a series of experiments in exploratory making, which consider a selection of kinematic playthings, each with interesting properties of motion. The artefacts we visit initially are commonplace in the playrooms of East Germany, and much like Froebel's gifts, take common three-dimensional geometric shapes as their combinatorial building blocks.

We have previously noted, how the creasing capability of card and paper affords an expedient material fabrication technique, for the folding and manipulation of nets for 3d shapes up out of their planar sheets. We have also considered in detail, some of the peculiarities of this process. For kinematic paper playthings, this material capability for selective flexibility has been further exploited on a more permanent basis, within the completed kinematic artefacts. For these movable models, folded paper edges create flexible joints between aligned 3d shapes, affording hinge-like connections between their parts, within kinematic designs.

4.3.1 Creating kinematic connections and closed loops

In the previous section, we have considered in detail how making rules might construct closed 3d shapes, by cutting, scoring, and manipulating sheet materials. Additionally, for these parts constructed from planar sheet in U_{23} , we have seen how computations

made within the initial 2d representation mode can afford the transformation of these rigid parts themselves. We have also considered how division rules might work, to alter and re-shape these parts, and to divide them into smaller, composite components. Here we consider how flexible connections might be made between 3d shapes, joining them together with flexible joints and affording kinematic connections. A simple approach to the construction of these connected parts subsumes multiple connected shapes together within a connected net- or else uses additional paper strips, to join kinematic shapes together.

For the collection of kinematic toys considered next, rigid parts are joined pairwise along their edges, creating hinge-like connections that afford variable spatial relations. Kinematic chains of parts can also themselves be re-connected together, to form closed loop linkages with highly constrained overall motions.

These flexible connections of paper models are somewhat fragile, however. More robust models combine multiple materials: fabric tape connections provide a robust hinge. Flexible point-point connections may likewise be constructed, by a string of zero length between the corners of two parts. Where a hinge-like relationship is required, two of these point connections, spaced along a hinge line, might equally instantiate the desired kinematic connection.

These flexible point and line connections might readily be described as shapes which are shared, between the two distinct parts (ie, shared surfaces, lines, or points, depending upon the mode of material instantiation). However, there exists yet no device within existing shape grammar theory, to model the variable spatial relations afforded by this type of connection.

'Magic cube' and kaleidocycle

The 'Magic cube', Pfeiffer cube, or 'un-folding cube' (Jenkins & Bear, 2006) is a common example of a kinematic plaything, which is claimed variously as an example of a 'kaleidocycle', 'invertible polyhedra', or 'metamorph' (Byrnes, 2008). As a children's plaything, it is encountered more widely than most East German kindergarten models.

Reminiscent of Froebel's blocks, it consists of 8 small cubes (Figures 4.24 and 4.25), connected to each other along various edge-pairs, so that a larger cube is constructed. But when manipulated, this larger cube then unfolds about its various axes, as the smaller cubes move relative to each other, and in this manner it moves through several different spatial configurations. Notably, interference between parts repeatedly limits these motions. But from these blocked configurations, a second pair of axes are then brought into alignment, enabling a further phase of motion. This process of realignment occurs sequentially, wherever a limit to motion is reached, and the parts transform, through various configuration states, before returning eventually to the initial state. We refer to this motion as 'discrete full-cycle motion', since in contrast to the motions exhibited by most of other the playthings we shall examine, its full-cycle motion is composed of several distinct steps.

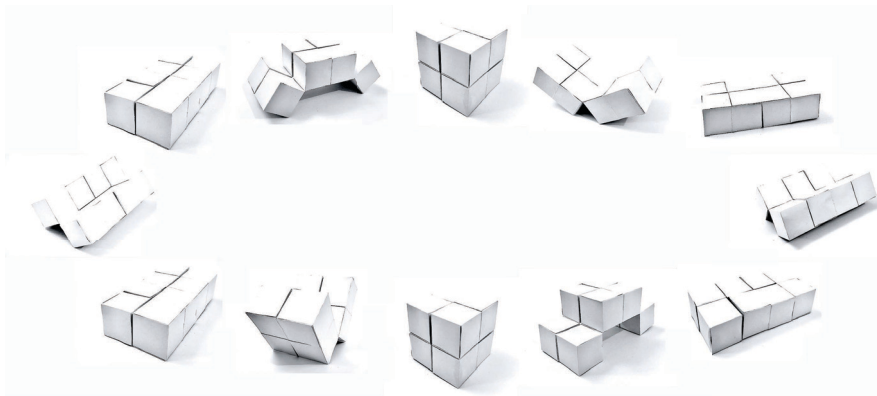


Figure 4.24 'Magic cube' toy (see Kulturata, 2006; also Langscheid & Langscheid, 2021)

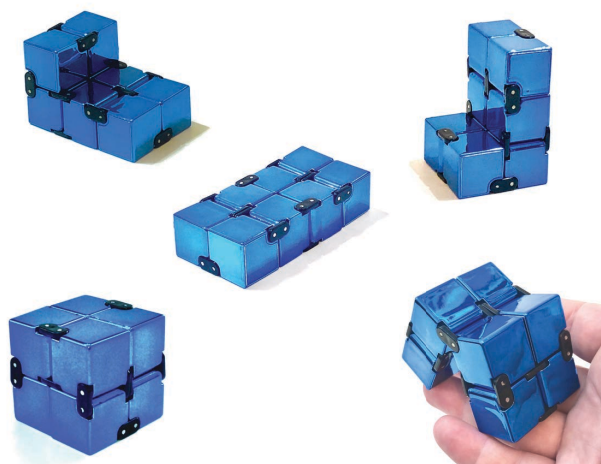


Figure 4.25 A commercially available 'magic cube' toy, with rigid hinged construction.

Rather than cubes, the *kaleidocycle* toy (Figure 4.26) is formed by a closed ring of connected tetrahedra. Its parts are able to rotate continuously, uninterrupted by collisions, through a complete cycle of motion with a single degree of freedom. This continuous, cyclic motion passes repeatedly through the model's initial position. In general mobility theory, seven bars are needed, for any kinematic mobility within a closed loop, so this design of only six parts constitutes a special case. The tetrahedral shapes of its connected parts meet precisely along their edge lines, during this cycle, but no collisions are encountered during motion.

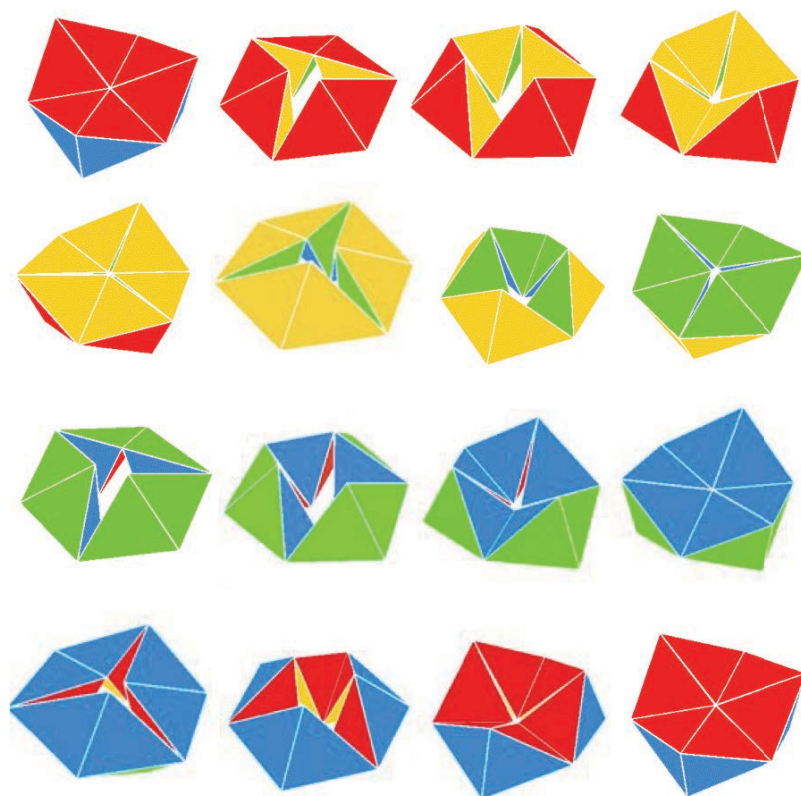


Figure 4.26 The full-cycle motion of a tetrahedral kaleidocycle (Kids Science Lab, 2014)

4.3.2 Rules for dividing and subtracting

A second, related toy is also composed of a ring of six tetrahedral links (Figure 4.27). However, in this second kaleidocycle, the shape of the connected parts of the linkage has been altered, and they no longer meet precisely at the centre.



Figure 4.27 A second kaleidocycle

Beyond its full-cycle motion, this model exhibits a further interesting property, of ‘expanding and contracting’, during cycles. Therefore this model has a somewhat different appearance, during its cycle of motion; it transforms visually between ‘closed’ and ‘open’ configurations. It was noticed too that its most compact configuration can fit precisely within its expanded configuration: the smaller form could be inserted inside the larger, where it fitted neatly into the available void. It was subsequently discovered too, that these two instances could both still continue to move through their full motion sequences, in this assembled configuration. This suggests an interesting schema: for adding certain linkages together either part-wise or in their entirety (Figure 4.28). Through experimenting with several instances of the model simultaneously, it was further discovered that four identical linkages could be successfully stacked together. This example therefore provides us not only with making rules for adding and subtracting from the shapes of kinematic parts, but also for stacking and assembling multiple linkages together, without causing their motions to be restricted.

Further, these assembled models indicate the theoretical existence of maximal shapes, for the parts within kinematic designs, from which material may be removed but not added to. (A related interesting question is whether, in order to avoid an area of collision for cases of material interference, matter might be subtracted from one part then reattached to another). Once discovered, maximal parts may then be subtracted from at whim: just so long as enough material remains, to retain a rigid connection between its pair of hinges. For the case of the kaleidocycle, the maximal lengths of hinges need not necessarily be preserved either. (In this case, a complete division of the

entire maximal linkage is afforded by a sequence of perpendicular, planar cuts, that split parts both perpendicular and to parallel to their hinge-lines. This divides of the entire kinematic loop into several distinct kinematic artefacts, each still capable of full-cycle motions. The surfaces describing cuts or divisions between 'nesting' linkages, need not necessarily be restricted to flat planes either. Surfaces of division could instead be curved; assuming that assembly and disassembly from the original configuration was still afforded.)

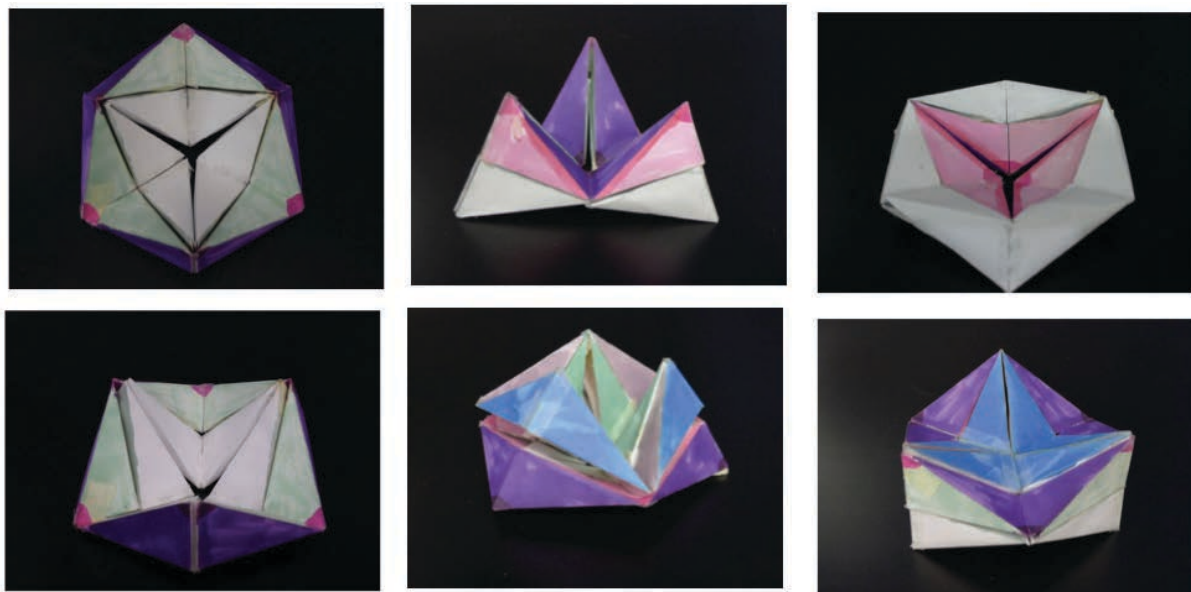


Figure 4.28 Assembling several instances of the linkage model to reveal the maximal shape of its parts

From this elaborate division schema, a far simpler proposition also holds: for a link or part within a kinematic chain, maintaining a rigid relation between its pair of hinge-lines is essential- but significant material can otherwise be removed from the shape of each part, without compromising the design. A main condition is that a continuous portion of each hinged connection be maintained, and also that the pair of hinges within a part must remain rigidly related to each other, without geometric transformation.

Note that here, through the experimental action of stacking several linkage instances together, we also discover the (potentially more useful) inverse schema, for dividing

maximal designs into multiple, smaller (and potentially stackable) kinematic linkages. A somewhat simpler subset of this schema for division is a subtractive rule, which merely removes material from the maximal linkage, to discover new design possibilities. Our next example applies this subtractive schema to the maximal tetrahedral linkage.

Tetrahedral ‘snowflakes’

Using the first kaleidocycle model as a starting point, material was cut away at the corners of each tetrahedra, to create a ‘snowflake’ design (Figure 4.29). This model deviates from our established approach, of that ensuring shapes constructed in U_{23} form closed surfaces. Further variations on the design cut away additional material, to create a different surface pattern for every snowflake created.

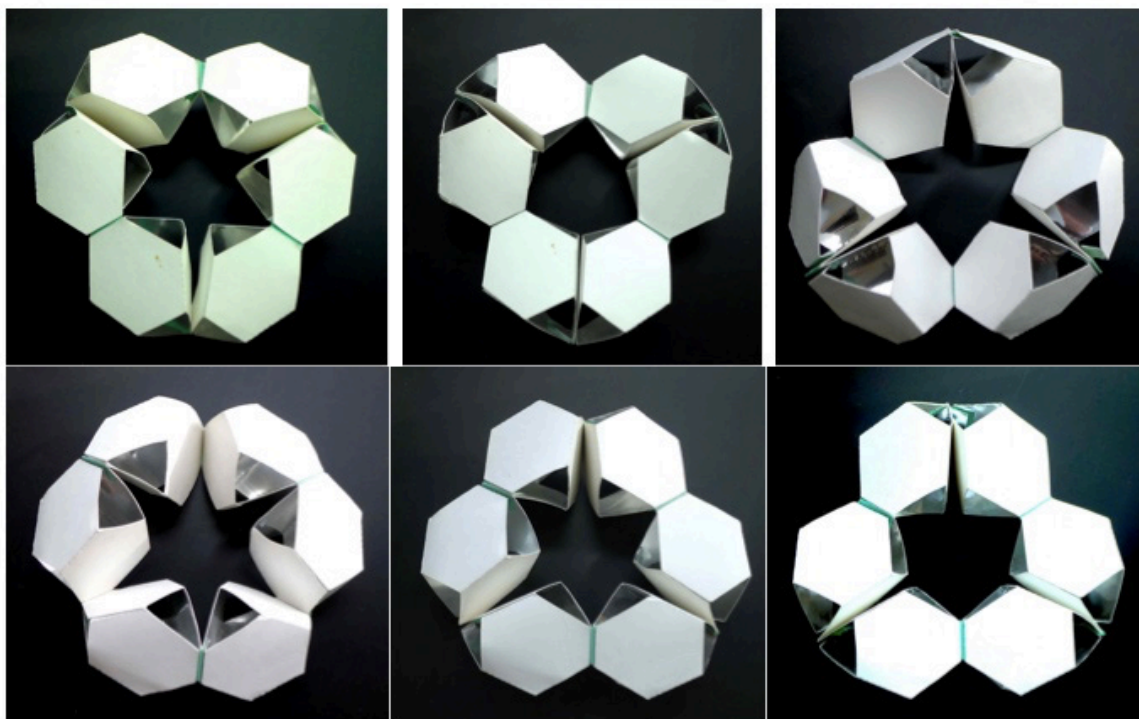


Fig 4.29 A tetrahedral ‘snowflake’ kaleidocycle

Yoshimoto cube

A second, more advanced version of the Schneider cube toy is the Yoshimoto cube (Figure 4.30), also referred to as the Starcube, Slothouber Cube, Yoshimoto 1 and Shinsei Mystery. This provides a found example, of our division schema for dividing entire linkages. At first encounter, this plaything appears visually identical to the magic cube. However, it has in fact already been divided into two complete kinematic artefacts, which stack together to create the magic cube (Figure 4.25). Here, each smaller cube is divided into two halves, and any affected hinges replicated to maintain connectivity. The result is two identical mirror versions, that stack together as the original cube. This variation on the magic cube is a reminder that, when considering designs where the maximal shape of parts is already established, an alternative starting point for exploratory reasoning is to instead consider the extent to which these parts might be subsequently *reduced or divided*, whilst maintaining a desired kinematic behaviour. Where parts can be divided in two without compromising hinge connectivity, it appears that strategic divisions can be used to create multiple, stackable linkages.



Figure 4.30 A commercially available toy based on the Yoshimoto cube (see Grand Illusions, 2018)

Other divisions have also been discovered, of the magic cube's eight component cubes, into identical halves. These are shown below in Figure 4.32. This range of models go some way towards indicating that the cube is the maximal shape. Significant material can therefore be removed from its parts, whilst still preserving the structure of its hinge

connections. Further, if divisions are made appropriately, multiple viable linkages can be retained.

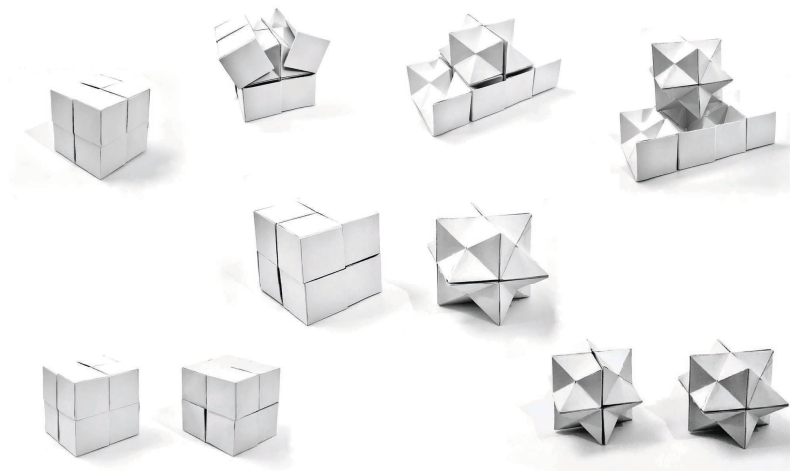


Figure 4.31 Unfolding the Yoshimoto cube (see Kulturata, 2006)

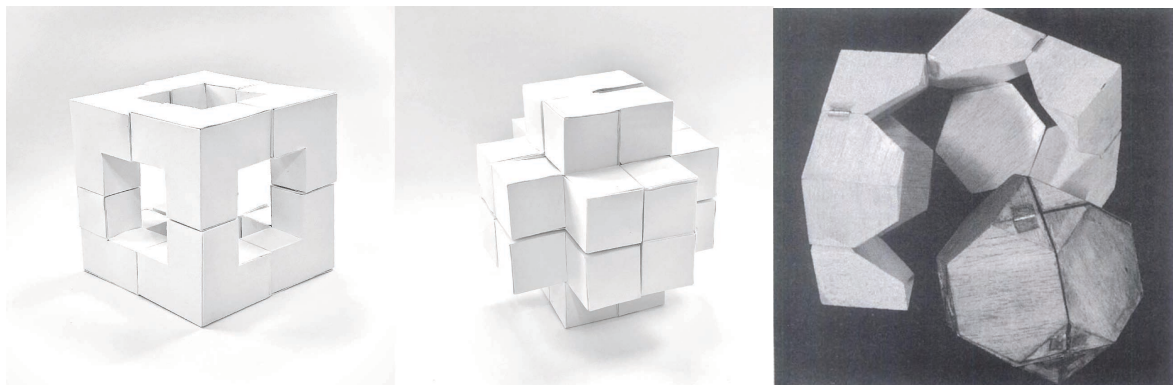


Figure 4.32 Further divisions of the magic cube: **(a)** the ‘64 cube’ (Kulturata, 2006) & **(b)** a hinged cube transforms to a truncated octahedron (Kappraff, 2001). Photo: Kappraff, © World Scientific 2001.

The Schatz cube, and other invertible polyhedra

Rather than combinatorial constructions, Froebel’s gifts (as considered by Stiny, 1980) can themselves alternatively be considered as divisions of an initial whole into smaller parts- through various operations of cutting and division. Another family of playthings takes this idea somewhat further. A collection of kinematic artefacts, described as

‘invertible polyhedra’, are discussed to date only by a largely German-language literature. The concept of ‘inverting’ 3d shapes was first proposed by Paul Schatz in 1918. His method involves the division of a 3d shape along its various planes of symmetry, to be followed by a process of re-joining the resulting parts, to create closed loop linkages with interesting kinematic behaviours.

The ‘invertible cube’, or ‘Schatz cube’ (Figures 4.33 and 4.34) was discovered by Schatz using this method (Langscheid & Langscheid, 2021). It is created by removing two precisely defined segments, from opposing corners of a solid cube. This operation leaves behind a set of solid parts which are then connected together with hinged connections, in a closed loop. This tetrahedral linkage is composed of six parts, and exhibits full-cycle motion with one degree of freedom. It is identical to the second kaleidocycle (Figure 4.27).

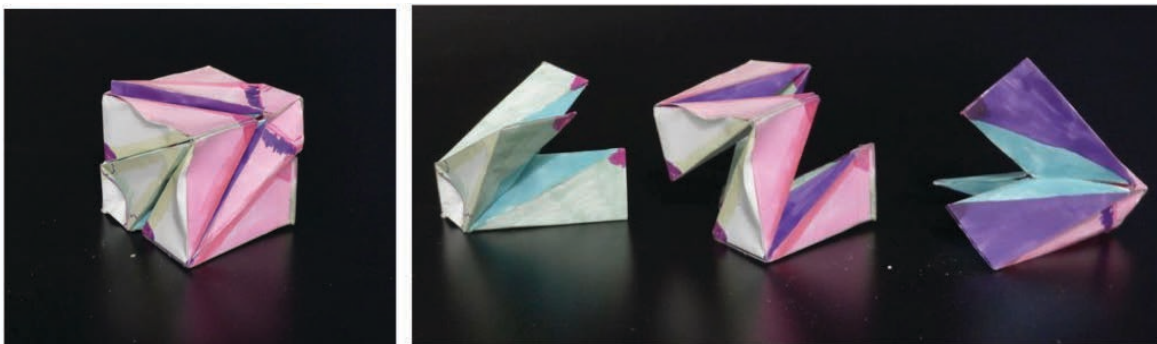


Figure 4.33 Paper model of an ‘invertible cube’ or ‘Schatz cube’



Figure 4.34 Paper model of an ‘invertible cube’ or ‘Schatz cube’ in various motion states

4.3.3. Rules for transforming shape and structure

By now, schema have been discovered for adding and subtracting from the shapes of parts, to find their maximal shapes, as well as schema for the division of maximal linkages through their hinge lines, to create sets of stackable linkages.

The next question posed to experimental making practice was whether the cube geometry, of the maximal shape of the parts of the magic cube, must be preserved– or whether this geometry might be altered. It is worth noting that, at the start of this exploratory making, very little ‘intuition’ was possessed, about what the answer to this question might be. A first step was to alter the shape of the parts, to give a cuboid of square cross-section in place of the original cube (Figure 4.35 and 4.36). This alteration was found to be acceptable.

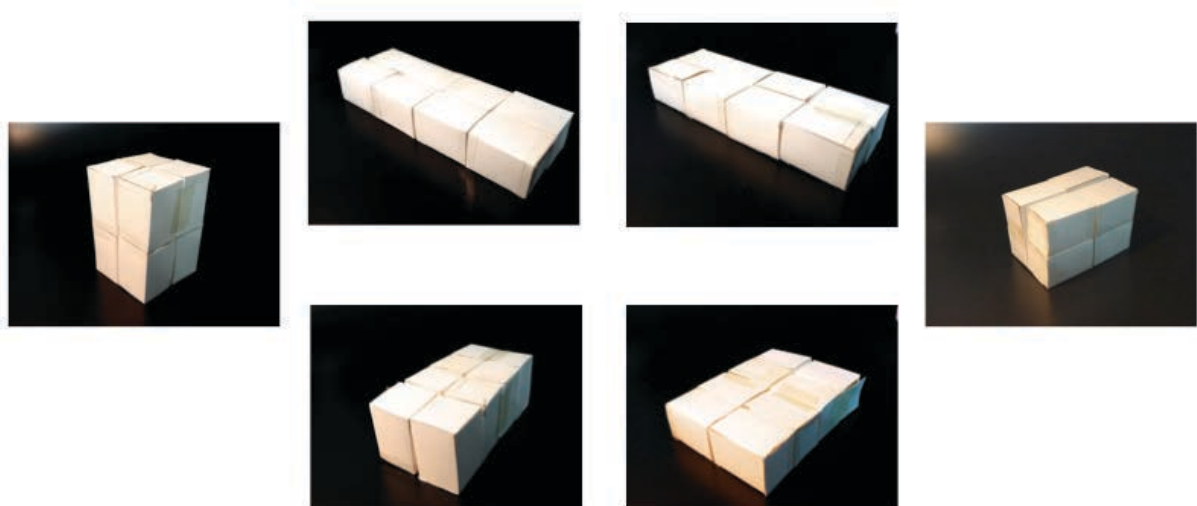


Figure 4.35 Model-making to vary the shape of parts from cubes to cuboids with square cross-section

Subsequent model-making then explored whether the maximal shape of the parts could be altered to any cuboid shape. This was also found to be acceptable, and these newly constructed models successfully exhibited the ‘discrete full-cycle motion’ of the original toy.

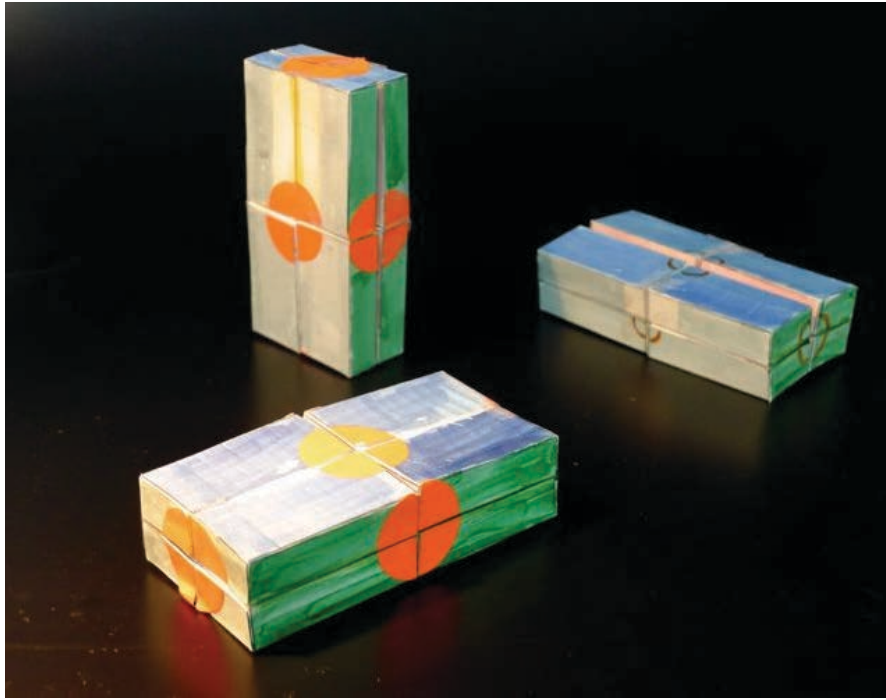


Figure 4.36 Model-making to vary the shape of parts to cuboids of general shape

In these new model variants, a further interesting new property was encountered. Due to the destruction of symmetry, caused by varying the shape of the cube's faces in three different ways, there became 3 possible ways of assembling the parts together, to create the whole (Figure 4.37). Although appearing similar in their initial state, these three alternate versions began to look very different when their appearance was considered throughout their motion cycle.

Subsequent work, to further vary the cuboids, was however found to have limits. Any variation which maintains the perpendicular edges, of the cube or cuboid parts, seems to be permissible (since it affords axis re-alignment, and therefore preserves full-cycle motion). But a change which affects this, by replacing the cube or cuboid parts with parallelepipeds, was found to destroy its character of 'discrete full-cycle motion'.

This unsuccessful model helps to demonstrate that, for the magic cube, the perpendicular arrangement of the hinge-lines is an essential feature. Notably, as the particular geometry of the chosen parallelepipeds maintained some rectangular faces, manipulation about their associated edges did successfully afford axis realignment, but not otherwise.

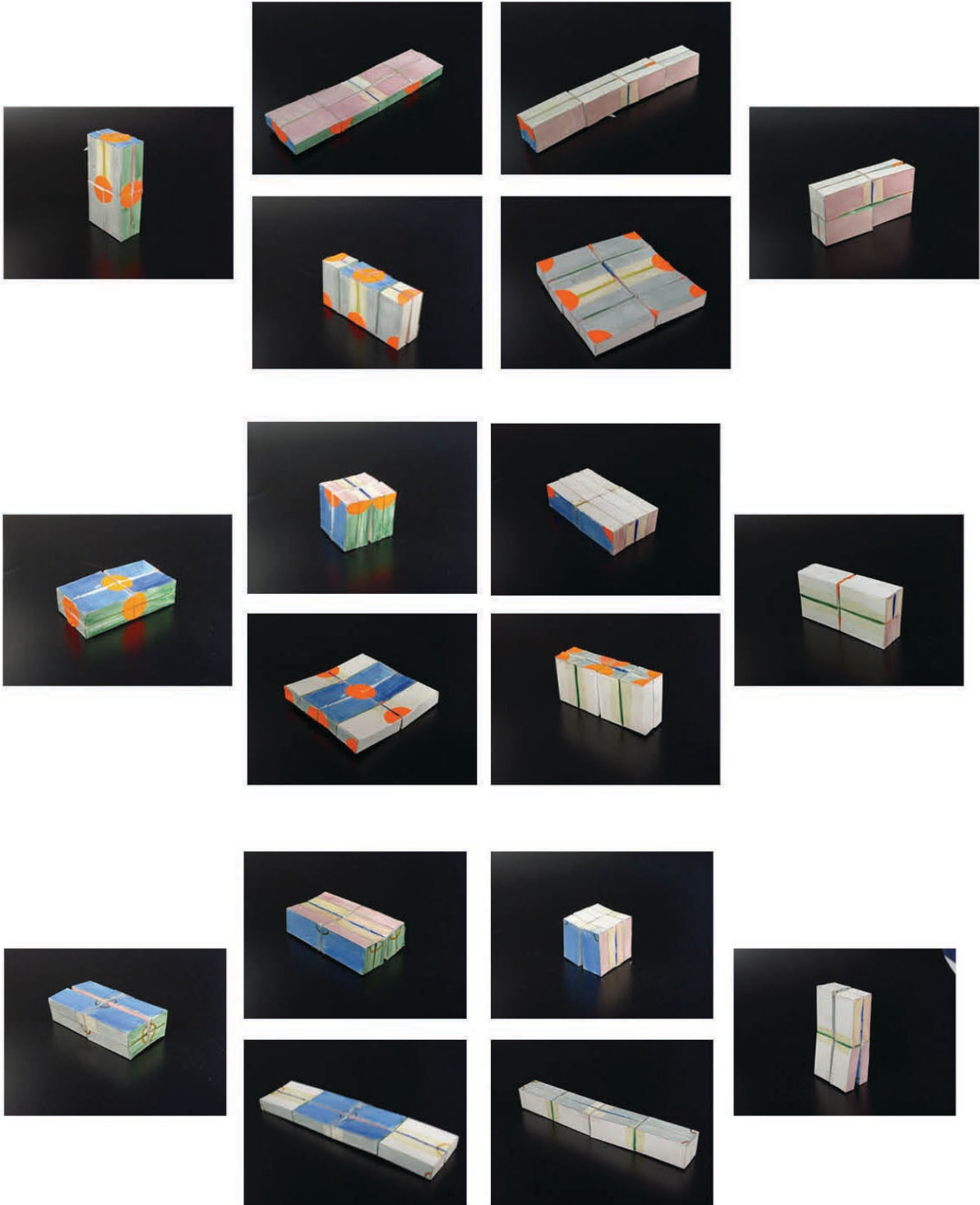


Figure 4.37 Manipulating three new instances of a generalized 'magic cuboid', through sequential states of discrete full-cycle motion

4.4 From structure to shape: reasoning through making

In a further, more detailed example of exploratory making, we encounter a related family of mechanisms with unusual kinematic behaviours, described within a predominately mathematical literature as ‘over-constrained linkages’ (Goldberg 1980). Although these closed-loop kinematic linkage configurations (Figure 4.38) are discovered to exhibit similar full-cycle motions to the toys we have so far encountered as physical models, they are initially presented using a very different mode of description. This literature’s primary descriptive vehicle for considering this class of kinematic designs involves abstract modes of representation concerned with the geometry of structure (Table 4.2) and the nature of connections, but provides no immediate information about the shapes of parts themselves. Several stages of reasoning are therefore required, to move from these initial descriptions, towards the construction of tangible physical models.

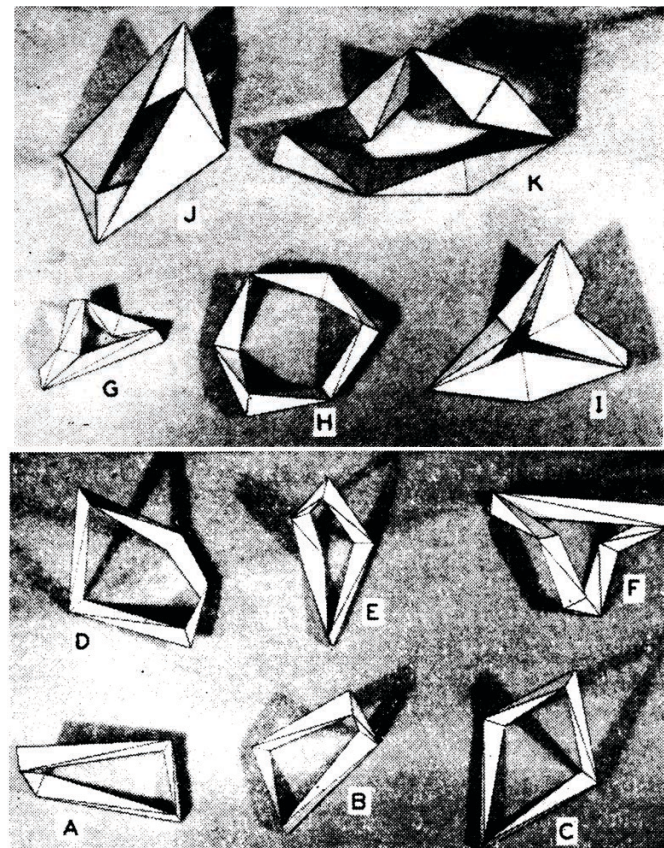


Figure 4.38 Physical models of over-constrained linkages (Goldberg 1980)

Type of Linkage		Twist and length of links					
A	Bennett four-bar	90° 2a	30° a	90° 2a	30° a		
B	Bennett four-bar	90° 2a	45° $\sqrt{2}a$	90° 2a	45° $\sqrt{2}a$		
C	Bennett four-bar	60° a	120° a	60° a	120° a		
D	Goldberg five-bar	90° 2a	60° 2a	90° 2a	30° a	30° a	
E	Goldberg five-bar	75° $(1 + \sqrt{2})a$	90° 2a	45° $\sqrt{2}a$	30° a	90° 2a	
F	Goldberg six-bar	125° $(1 + \sqrt{2} + 2 \sin 50^\circ)a$	90° 2a	50° $2a \sin 50^\circ$	30° a	45° $\sqrt{2}a$	90° 2a
G	Rigid five-bar	90° 4a	90° 4a	90° 4a	90° 4a	90° $\sqrt{97}a$	
H	Symmetric six-bar	90° a	90° a	90° a	90° a	90° a	90° a
I	Bricard rectangular six-bar $A^2 + B^2 + C^2 = a^2 + b^2 + c^2$	90° A	90° a	90° B	90° b	90° C	90° c
J	Bricard octahedral six-bar	From moveable octahedron					
K	Seven-bar	90° twists, equal length links					

Table 4.2 Over-constrained spatial loops (Goldberg, 1980)

The overall freedom of a kinematic chain or linkage is usually derived from the sum of the freedoms found at each joint or connection within that chain. For closed-loop planar linkages connected with revolute joints, at least 4 connected links are necessary to reliably afford motion, and in 3 dimensions, generally such linkages must contain at least seven bars, in order to guarantee motion. There are however some exceptions to this general condition, and the set of over-constrained spatial linkages we consider here constitute a special case. These spatial linkages consist of rigid bar elements connected with single-degree-of-freedom revolute joints, connected within mobile loop configurations composed of four, five and six bar elements. Moving with one degree of freedom, they exhibit surprising mobility due to the specific geometric configurations of the associated axes of rotation defined by their joints.

According to the standard Gruebler-Kutzbach equation, or Mobility Criterion, when rigid bars are connected with revolute joints to create a closed-loop linkage, at least seven bars should be required, to create a mobile configuration. This criterion evaluates

motions for the general case (e.g. Phillips, 2007) but is not able to detect or account for these special instances, since it considers only the topological structure of linkages, and the over-constrained spatial linkages addressed here exhibit anomalous geometric configurations. Composed of 4, 5 or 6 links connected using revolute joints, over-constrained linkages exploit particular, independent axis configurations, to achieve motion with one degree of freedom (Hunt, 1978). Rico & Ravani demonstrate why the general mobility criterion lacks the necessary detail to obtain meaningful results for over-constrained cases, and outline how these cases may rather be considered through a more detailed analysis technique (Rico & Ravani 2007). Hunt further explains how screw theory provides a route for evaluating dependencies between axis configurations in spatial mechanisms, and hence the mobility of the mechanism. Table 4.2 describes the set of kinematic linkages which constitute the over-constrained class. Although no new over-constrained linkage archetypes have been discovered for over half a century, recent work in the field of deployable structures has begun to explore the possibility of constructing mobile assemblies composed of multi-loop combinations of existing linkage archetypes, through considering the kinematic behaviours of their networks.

The kinematic motions of this class of linkages are difficult to predict without extensive mathematical analysis. Screw theory tells us about the theoretical movement of connected parts within these mechanisms, but is concerned primarily with configurations of axes of connection, rather than with the actual shapes of components (Hunt, 1978). Yet when constructing physical models in order to appreciate motions directly, these shapes are instrumental for practically realising kinematic designs, and their motions, as physical entities.

In this episode of experimental model-making, we consider linkage instances from this now well-defined class of over-constrained spatial mechanisms. This family of linkages, originating within a predominately mathematical literature, are described in our initial encounter with them only through the somewhat abstract descriptions of the geometry of their structural configurations, as described in Table 4.2. As modes of description, these constitute very different starting points to the designs for kinematic toys, which we have so far initially encountered through either physical models, or instructions for their making.

In this episode, several stages of reasoning were required, to move from these initial descriptions, to tangible models. Taking these analytic descriptions of the viable geometric configurations of rotation axes as a starting point, the physical shapes of a linkage's component bar elements are the subject of design generation and exploration. Our focus here is in establishing the design steps necessary for the practical realisation of these theoretically mobile geometries. Whilst analytically derived axis geometries in theory afford certain kinematic motions, in practice material interference between the shapes of parts, at various stages of the motion cycle, may impede and restrict those motions. Avoiding such collisions between the physical shapes of parts is therefore essential for the mechanism's theoretical full cycle continuous motion to be physically realised. In order to directly experience and examine the motions of over constrained linkages, a process of synthetic exploration and evaluation of theoretical motions is afforded and assisted by CAD and rapid prototyping. The role of both physical and virtual models within this process is described below.



Figure 4.39 3D printed models of 4 bar Bennett linkage, showing 3 possible instantiations (two of which exhibit successful full cycle motion)

4.4.1 Prototyping over-constrained linkages

In contrast to the multi-material making algebras employed within previous exploration episodes- where we used flexible materials, to expediently model variable spatial relations between connected kinematic parts- here our exploration makes use of substantially rigid materials only, and is therefore more readily described by conventional shape algebras alone. Since physical models afford the direct examination of motions, ultimately our endeavour is to discover the additional information needed to convert the initially provided descriptions, of axis geometries in U_{13} , into designs for working physical models described in U_{33} . Design exploration is conducted both within the largely procedural shape-based algebra of a virtual CAD environment (affording various operations and transformations, which operate within algebras of U_{03} , U_{13} and U_{23} primarily), and the physical, three-dimensional algebras of physical models in U_{33+} , afforded through the additive manufacturing of solid shapes from ABS, with Fused Deposition Modelling technology (FDM)(see Figure 4.39). To model revolute joints from rigid materials in 3 dimensions, here pin-jointed connections were fabricated. These prescribe pairs of interacting cylindrical surfaces with similar curvatures, which maintain contact whilst affording motions according to certain variable spatial relationships. To maximise the geometric precision of physical models, bar components with cylindrical holes were fabricated individually, and cylindrical steel pins were then added to connect each pair of parts, thereby assembling a functional model of the closed loop linkage. For descriptive purposes however, physical models can be considered to be composed from substantially rigid materials with homogeneous properties. Their separate component parts can each be described within a shape algebra of either their 3d surfaces in U_{23} , or their solid shapes in U_{33} . However, in order to create a description of the complete, assembled linkage, shape descriptions alone do not readily afford the permanent distinction between component parts that is essential for modelling motions.

Design work begins within a virtual CAD environment, where models were developed to explore the motions of a selection of the linkage archetypes described in Table 4.2. The initial axis descriptions given in Table 4.2 provide a scaffold or framework in U_{13} , around which the material shape of physical components can be experimentally sketched and

modelled in U_{23} . To construct its specific axis configuration, each linkage component is formally specified by two parameters, as shown in figure 4.40: (i) the length of the centreline which marks the shortest perpendicular distance between its two axes of rotation; and (ii) the twist about this centreline between its two axes. A third possible parameter defines the offset between the centrelines of adjacent bars, but this is zero for all linkages defined in Table 4.2.

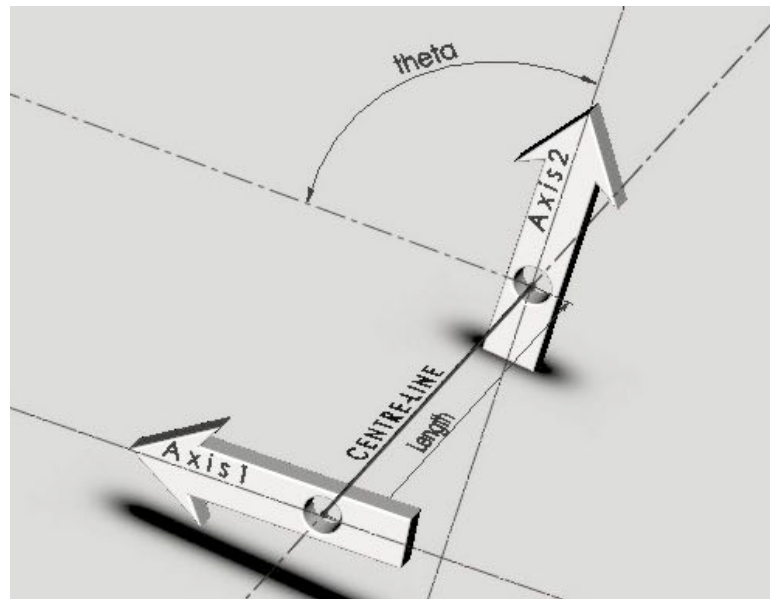


Figure 4.40 Twists and offsets

To evaluate whether design models successfully achieve theoretical motions, both virtual and physical 3D visualisation tools proved essential. Manipulation of virtual models alone can provide a useful insight into kinematic behaviour, and can also enable theoretical motions to be simulated and experienced to at least some extent, using immaterial axis descriptions in U_{13} alone. But because the parameters which enable movement in over-constrained mechanisms can be extremely precise, rounding errors can render the modelling and simulation of these theoretically improbable motions problematic using standard computational techniques. In general, therefore, it was discovered that these complex kinematic dependencies are most effectively realised and examined through physical model-making. Initially, each linkage component was constructed as a straight, twisted bar placed directly along the component's theoretical centreline. In the majority of cases, these virtual models demonstrated theoretical

kinematic properties successfully. But without exception, when examining these initial designs within a virtual environment, material collision between the bar components was encountered at various points within the linkage's cycle of motion. It thus became clear that designing the physical shape of linkage components was not necessarily a straightforward problem.

Further exploration demonstrated however that this design task could be approached somewhat systematically. Here, exploration was limited initially to one particular linkage design; namely a four-bar Bennett linkage (see Table 4.2). This linkage instance is anti-symmetric, and thus composed of two distinct pairs of identical bar components, with twists of 30 and 90 degrees respectively. Our approach here was to transform the 3d shape of each linkage component such that placing material at any potential collision sites was avoided, yet whilst still maintaining a rigid physical connection between the component's two axes of rotation. Since it had now been discovered that placing material along the component's theoretical centreline tended to lead to collisions, a new design tactic instead defined the shape of each component, by connecting its two axes of rotation using a component with a lofted surface defined by an underlying spline curve. This curve could then be further transformed as necessary, to alter the overall 3d shape. This new, curved bar shape had the effect of largely avoiding placing material at the centreline, with the exception of the end connection sites.

Because, due both to the lack of any offset between the theoretical centrelines of adjacent components, and to the manner in which revolute connections are physically instantiated for these models (i.e. with two connecting components making direct physical contact with both each other and a shared cylindrical connecting pin, at the point where each component's theoretical centreline intersects their shared axis of rotation), two distinct topological connection formats were found to be possible, between the two distinct rotation axes defined within each bar component. For each bar, this affords two distinct connection topologies: a C-shaped bar, which remains on only one side of the centreline; or an S-shaped bar, which traverses the centreline once, and whose contact surfaces are on opposing sides of that centreline (see Figure 4.41). To construct a complete linkage, sets of bar components must be chosen with appropriate topologies such that a closed loop can be practically assembled. The design

initially created using the lofted spline curve approach consisted of all C-shaped bars (C-C-C-C), but it was found that this design still encountered collisions between components, which subsequently restricted the motion cycle of both physical and virtual models (figure 4.42). The design was then further reconfigured, with one pair of C-shaped bars being replaced instead by S-shaped bars (C-S-C-S). This new configuration was found able to move through a full cycle of motion without encountering any collisions (see Figure 4.43a). Physical models of these designs were then fabricated: for the initial C-C-C-C linkage, to examine the nature of its collisions in closer, material detail; and also for a new C-S-C-S linkage design. (Further additional four-bar linkage formats are also theoretically possible- namely SSSS and CSSC- see figure 4.41). It was found that interaction with physical models immediately enabled a far greater appreciation of the nature of this linkage's kinematic behaviour.

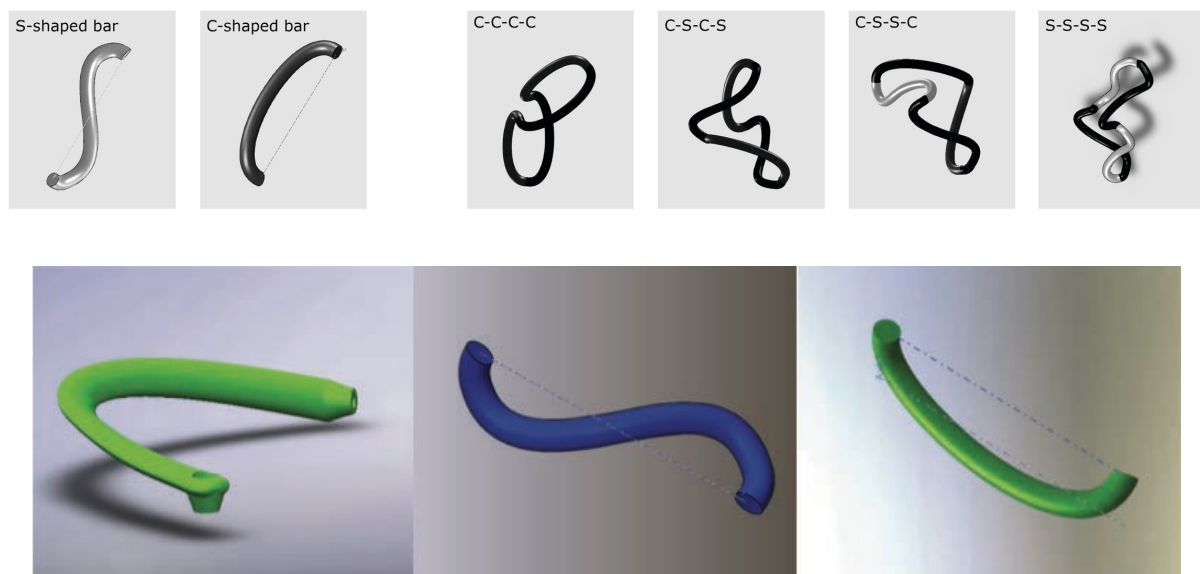


Figure 4.41 The same 4-bar linkage realised using a variety of bar shapes

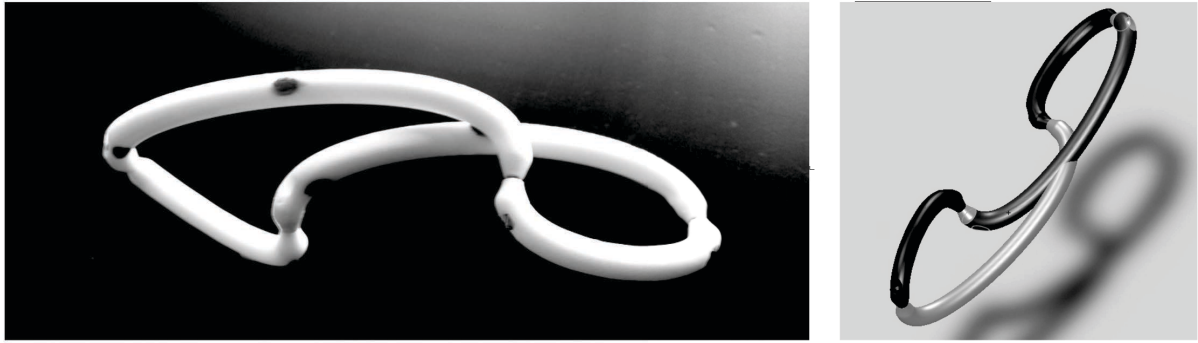


Figure 4.42 Interference of C-C-C-C configuration in physical and virtual models

The further transformation of the shapes of bar components was also explored, with the intent of removing collisions from the first C-C-C-C design, through iteratively altering the shape of the bars by parametrically transforming their underlying spline curve, but without resorting to changing the overall topology of the connection format. Known collision sites were systematically examined using both physical and virtual models (Figure 4.42), and the shapes of any colliding elements were subsequently adjusted to avoid placing physical material here. These new shapes were then tested again, for collisions through the linkage's full cycle of motion, using virtual CAD models. It was found that some alterations might move the site of a collision along the bar, but fail to eliminate it completely. Other alterations, which successfully removed one collision, were then liable to cause collisions at new sites. Through an iterative approach of transforming and testing in the virtual model, it was found for some cases that collision sites could be moved incrementally towards the ends of the affected bars, and eventually eliminated from the design completely. It was additionally noted that, when collisions were encountered between a pair of identical bar components within a linkage, through an appreciation of the underlying symmetries this component's shape could be reconfigured to simultaneously avoid both collision sites, without any need to create separate shape instances for this interacting, similar pair.

A physical model of the improved C-C-C-C design was then produced (Figure 4.43b), and the experience of interacting with these models was compared qualitatively to that of simulating motions through manipulating the virtual models. One notable effect was to gain an immediate sense of the combinations and directions of forces necessary to make the linkage move: to bring about a full cycle of motion through manipulation in a

continuous fashion, a continuously changing set of forces must be applied. This phenomenon is not readily apparent from interacting with a virtual model, which must generally be observed from a fixed angle throughout the motion cycle, and therefore perceived through a certain 2D viewing window, describing a planar projection of the 3D scene. Using a conventional CAD environment, it is therefore difficult to apply forces substantially perpendicular to this viewing plane. Interacting with physical models, however, provides an immediate sense of the complexity and continuity of motion throughout its full cycle (see Figure 4.43). It is also interesting to observe that, although both models are defined by the same axis configuration, each constructed model version exhibits its own distinctive character of motion.

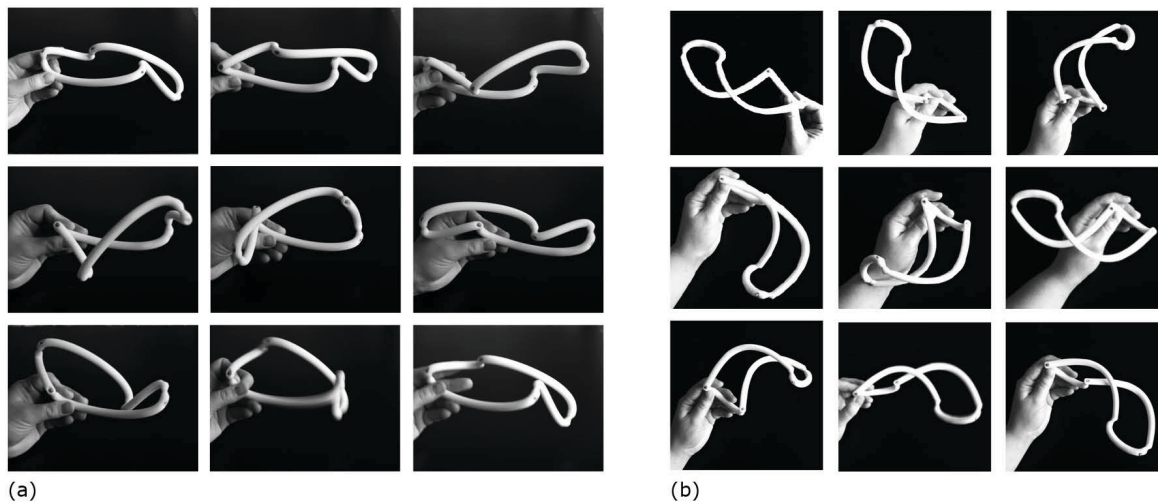


Figure 4.43 Range of motion of two successful mechanism configurations

A further inspection of Table 4.2 reveals that bar components which pertain to certain particular axis geometries occur within several distinct linkage archetypes. It was thereby discovered that parts initially fabricated for constructing the ‘Bennett 4-bar’ described above could also be re-appropriated, to construct a model of a ‘symmetric 6 bar’ linkage. Less material interference issues appear to arise for this particular linkage archetype, and the original c-shape bars were found to successfully construct a working physical model. However, in order to create a valid connection topology by which to construct this linkage in physical space without any material interference, it was found that a mirrored instance of the c-shaped bar was also required. It was further noted

that several equivalent topological configurations are equally valid, for instantiating this linkage design (Figure 4.44).

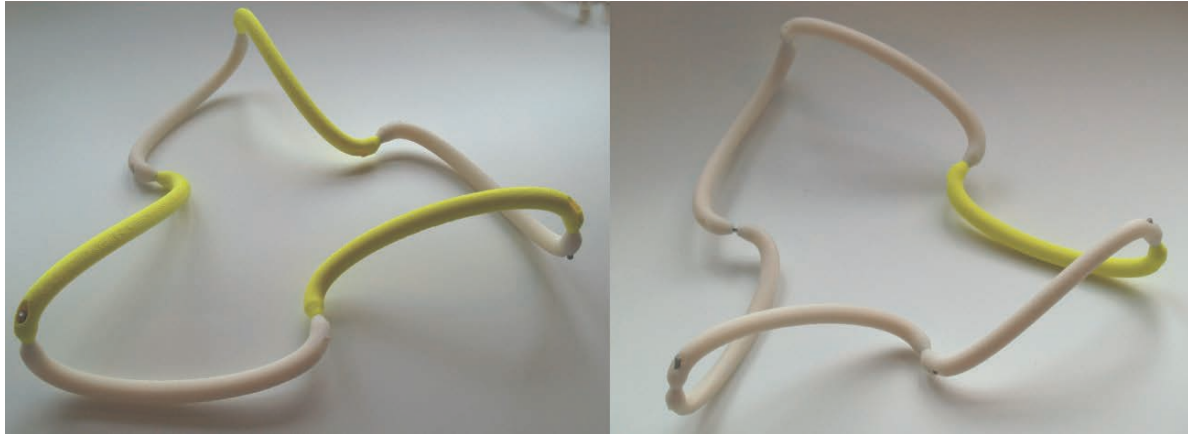


Figure 4.44 Two possible topologies for a symmetric-six-bar over-constrained linkage—the yellow components are a mirrored instance of the white components

It was also noted that, in order to fabricate the components for a ‘Goldberg 5 bar’ linkage (Figure 4.45), only one additional bar need be specified, since 4 of the bars in this linkage are similar to those which compose the Bennett 4 bar we fabricated originally. In this case, collisions were however encountered, when an assembly of these parts was tested in a virtual environment. However for this case, rather than transforming the underlying spline curve, subtractive operations could instead be applied, to successfully remove material causing collisions, without compromising the integrity of the bar component. A physical model was then fabricated (Figure 4.46). This model was found to exhibit motions to a reasonable extent, but minor interference was encountered at two points within the motion cycle. However, due to the composite material flexibility of both the structure of the linkage and the material of the components, it was found that, by forcing components slightly past their sticking points, full cycle motions could still be achieved (Figure 4.47). But, for this partially functional physical model, the continuous, fluid nature of full-cycle motions experienced when manipulating previous models was not achieved.



Figure 4.45 Goldberg Five-bar linkage



Figure 4.46 Bars from previously fabricated models were reused, with sites of interference cut away



Figure 4.47 Material deformation affords the flexibility to avoid a collision

4.4.2 Observations from the experiment

This episode of design exploration demonstrates that, for kinematic linkages, practical design issues can be readily addressed through an experimental, material approach. Though both virtual and physical making, the method employed here iteratively combines both the transformation and evaluation of designs, in order to address practical design issues surrounding kinematic mechanisms in an experimental yet somewhat systematic way. When interference is encountered between components, their shapes are iteratively adjusted and then re-tested, until the colliding material is removed. Both virtual and physical models play a role in supporting this evaluation. The practical model-making approach reported here is not necessarily a feasible way to explore and consider a large number of designs— but some progress has been made towards establishing a reliable method for moving between abstract geometric descriptions in U_{13} (perhaps really U_{14} , due to the time-based nature of motions) already known theoretically to describe mobile configurations but which can be examined only within a somewhat abstract virtual space, and physical models in U_{33} (or, U_{34}) through which motions can be directly experienced and examined.

This iterative process is situated within the context of a particular design episode, so more generally applicable formal descriptions of the method cannot immediately be prescribed in detail, since in new situations, different kinds of issues may also arise. However, the design protocol developed here establishes a basic iterative method for discovering valid component shapes, which has application more widely, for the making of both over-constrained linkage archetypes, and kinematic designs in general.

Since the only initial description available was that of the underlying geometry of structural connections, with no further information provided pertaining to the possible shape of parts (other than the images Goldberg provides in Figure 4.38, indicating that these physical models are indeed a viable proposition in the physical world), a key initial question was how to attach physical material to this geometric description, in order to construct a viable working model, affording a direct appreciation of the motions of these unusual linkage archetypes. Once this was achieved, a second question was then the extent to which the shape of parts could be varied, without creating interference

between parts with relative motions at any points during their motion cycle. A key discovery is that, for fixed connection geometries, the shape of parts within a linkage can be varied in two distinct ways: firstly, merely the shape of a link between its two fixed axes of connection can be varied, but without affecting the manner in which it connects to adjacent links; or secondly, the connection topologies of all the links with the linkage can be varied simultaneously. To physically instantiate a physical model of a closed loop linkage design, several viable link connection topologies may be viable. Since the linkages considered here contain only a small number of links, viable topologies can be systematically enumerated (although it should be noted however that it may not necessarily be possible to physically instantiate all of these configurations without material collision).

Here our primary endeavour has been to discover at least one physical instantiation for each linkage archetype, to enable its motions to be directly examined. Further transformation of the shapes of links, however, to discover further viable designs which move without collisions, would also be beneficial, and could ultimately support both a better understanding of the motions of the linkage, and a more strategic approach towards exploiting that linkage archetype within design applications. Once one valid design for the shape of links is discovered, for a particular linkage, further systematic exploration of the viable shapes of links could be conducted systematically, through either physical making or digital simulation. Here, as well as rules for parametrically transforming the shapes of parts, we have also used subtraction rules, for removing colliding material directly. A reverse approach could also be applied, using addition rules to experimentally add more material to the shapes of links, and then subsequently test whether these new additions give rise to new collisions. For each valid connection topology, systematic exploration through making could afford the discovery of a 'maximal shape' for each link within the linkage. These maximal link shapes would then define the space within which the shapes of components can successfully exist without collisions: for particular design applications, non-essential material could then be removed from these, in desirable ways. It should however be noted that: just as in cases of material interference, colliding material may in cases be removed from either colliding link, adding material to the shape of one link has an immediate effect on the

maximum viable envelopes for the shapes of the remaining links with which it interacts during motion. Therefore, for a given linkage with a particular link topology, more than one set of maximal links may potentially exist.

A further route for extended exploration is also suggested in this episode: since instances of particular geometric bar configurations are found to appear within several different over-constrained linkage archetypes, a viable approach to search for new linkage archetypes might be to recombine predefined link shapes in new ways, to see if new mobile linkage configurations might be discovered.

Preliminary exploration of this approach was undertaken within a computational model, to examine whether other viable combinations of the component parts used to construct these linkages might also be discovered. The method was validated by constructing the linkage archetypes discussed here, within the virtual search space. Further work could consider ways that a computational search approach might be employed to explore spaces containing multi-loop linkage configurations.

The differing roles played in this episode, by a range of representations that employ distinct algebras for making, becomes apparent. Geometric spatial representations in U_{13} provide a starting point, and more abstract, topological descriptions also play a role, in understanding the structure of parts within a design. With a CAD environment, shapes of parts are constructed through a combination of transformations which operate both sequentially and simultaneously in U_{03} , U_{12} , U_{13} , and U_{23} , to create designs in U_{33} which can be digitally fabricated, and also to model their motions. In order to describe the simulations of spatial motions within a virtual environment, we might perhaps describe the necessary algebras as U_{14} and U_{24} (since the additional dimension of time is added, to the U_3 space, within which spatial designs move). We propose that virtual modelling or simulation of motions rather occurs in algebras of U_{14} , U_{24} , and U_{34} , since time adds a further dimension. Exploration in the physical world also requires more than shape, and the dimension of time is certainly essential, to afford the experience of motions. However, these kinematic 3d models do not inherently make their motions visible- they rather embed the potential, for particular motions to be elicited through manipulation. Therefore, rather than the dimension of time, it is rather

the nature of the connections between parts within designs that constitutes a key feature which cannot be represented in U_{33} alone. Variable spatial relations, defined by connections between parts, require new kinds of algebras to support their description. Situated, time-based material interaction with models is also essential, to enable motions to be actively examined.

A key feature of design exploration within this episode appears to be the manner in which new design representations (such as, for instance, descriptions of topology) are discovered during processes of making. These new modes of description then afford new kinds of insights, and construct new avenues for exploration. The manner in which material models afford a more refined appreciation of motions, through providing a more precise awareness of the types of manipulations necessary to elect these motions from models, it's also a noteworthy feature.

4.4.3 Comparison of kinematic designs

Comparison of the shape, structure, and motions of models is made possible through exploratory making. The 'symmetric six bar', invertible cube' and 'tetrahedral kaleidocycle' begin to reveal themselves, not merely as a collection of related toys, but rather as three instances within a design space of wider possibilities. This space undoubtedly contains many other possible objects composed of six connected parts, which also exhibit continuous full-cycle motion with one degree of freedom. But it is also itself a subset of a wider space of kinematic possibilities, and therefore, if attainable, an understanding the nature of its boundaries seems worthy of further pursuit.

There are other echoes too, between these different modes of description. The East German models are not the only examples of closed-loop linkages described with reference to polyhedra. In the Table 4.2, several linkages, namely those discovered by Bricard, are also described in reference to classes of polyhedra. For instance, Bricard's 'octahedral six-bar' is created by removing 2 faces from an octahedron, affording

motion with one degree of freedom to the remaining faces, which form a closed loop or linkage (Goldberg 1942; 1978).

Given the near exclusive focus upon abstract descriptions, within the mathematical literature for over constrained linkages, it is interesting to discover the existence of several physical instantiations of them beyond this literature. Outside of this mathematical world, these 'kinematic curiosities' seem perhaps to have been discovered simultaneously by several different groups, who refer to them by an assortment of names.

In these different making scenarios, the selection of these different, yet equivalent, shape-based descriptions of kinematic connections, have been informed by the selection of different material algebras for making. But they have also then informed the available making actions subsequently afforded. These different approaches used so far, for describing the axis geometries of kinematic connections, demonstrate two ways of instantiating a revolute joint (folds and pins) which are very different in approach. Both assume that material must be placed at the join site, but instantiate these connections in different ways.

For the first type of connection, describable by a hinge line or axis, the straight edges of a pair of parts are connected with a flexible material that plastically deforms with minimal force. For this connection type, the axis of connection also delineates a material boundary, for the material shapes of parts.

The second approach employs instead solid, rigid 3d shapes, describable in U_{33} . It uses a rigid cylindrical pin, to connect together parts with cylindrical holes. In shape terms, it is describable by a cylindrical surface. Here however, we note that, due to the relative scale of the connecting pin, the most significant location where direct contact occurs between two parts is rather their facing surfaces— which, rather than modelled by the axis of rotation to which they are perpendicular, can alternately be modelled by their actual surfaces of contact. For this case, contact occurs on 2 flat, circular rings, described in U_{22} (or also describable by their boundaries in U_{12}). In practice however, in physical, material U_{34} space, the definition of such surfaces is not enough to maintain

their ongoing connection— and the role of the circular pin (and lugs which hold it in place) is primarily to achieve this.

We note that, since here, the contacting shapes which afford the necessary variable spatial relation can be defined by rings or discs in a plane perpendicular to the axis of rotation, rather than with direct reference to the axis of rotation itself, an alternative approach to model making could employ hollow rings of exaggerated dimensions, rather than solid pins.

This shows the beginnings, of a catalogue of options, for materially instantiating revolute joints led by a variety of shape descriptions, and warranting further consideration. Moving between descriptions of connective shapes in various algebras appears to be a useful technique for supporting reasoning, for making material models of kinematic designs.

In this case, a pair of cylindrical surfaces in U_{23} (Figure 4.48) can be modelled in lower shape algebras, in various ways— for instance as a pair of embedded circles, in U_{12} . Alternate shape descriptions can also be systematically derived. Firstly, in a 3d plane representation, perpendicularly intersecting the axis of rotation, two points embedded in each other (U_{02}) can describe the desired rotation; as can a ring or circle in U_{22} , or their boundaries in U_{12} . Describing the embedding of any of these shapes, within 2 distinct parts to construct a variable spatial relation, has the effect of modelling a revolute joint within a 2d representation. If these representations are transferred directly to a U_3 space, those in U_{13} or U_{23} maintain enough information to continue to model relations which afford rotations in the desired manner. In U_3 , however, representations employing embedded points are not enough for adequately restricting motions, as desired. But, two distinct planar representations in U_{02} , placed at different locations along the desired rotation axis in a U_3 space, could, if both these pairs of points were both embedded in each other and imbedded in a pair of connected parts respectively, construct desired motions.

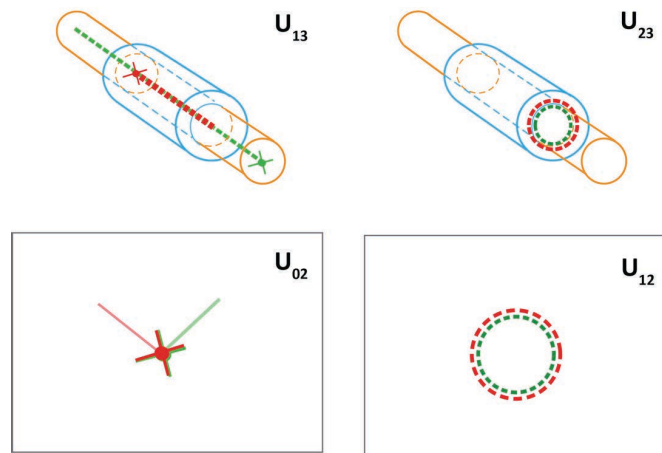


Figure 4.48 Shape descriptions of elements of connection, in assorted shape algebras U_{ij}

For the case of the revolute surfaces discussed here, it appears to be possible to transform descriptions of kinematic connections, which are in general modelled by interacting 3d surfaces, into simpler descriptions in U_{03} , U_{12} , and U_{22} .

4.5 Discussion and observations

Physical models provide a crucial mode of representation for kinematic designs, since they permit motions to be actively examined. Physical model-making is the focus of the experimental exploration activities reported upon in this chapter. We have seen practical examples of making rules at work in exploring spatial and kinematic designs.

We have initially considered the shapes of parts, and the structure of their kinematic connections, as separate concerns. But these making experiments have highlighted that kinematic structure itself also has geometric and shape aspects. Alterations to the shape of parts, that do not affect the structure or geometries of their connections, can often be made without compromising motions, so long as material interference between parts is avoided. However, some transformations to the shapes of parts may also affect their structural geometries of connection— for instance by altering the alignment of axis relations. Transformations to connection geometries and topologies

appear to have wider-reaching effects than alterations to shapes, both upon the motions of designs, and on the shape and material opportunities available for their instantiation.

4.5.1 Making schema for kinematic designs

During these experiments, we uncover two distinct approaches, to using schema within exploratory making. The first type of making schema we encounter (let's call them type 1 schema) comprise of practical material algebras, employed here for materially instantiating designs which have been first abstractly described, according to specifications for shape and structure. In this context, making rules are needed, to instruct the fabrication of both the shapes of kinematic parts, and the kinematic connections between them. The material algebras employed here have included both modelling from folded card and paper, supported by digital drawing, and 3d printing for rigid interconnecting parts, supported by CAD modelling. For kinematic making, each of these algebras provides its own peculiar material and geometric solutions, for instantiating both the shapes of parts, and the kinematic connections between them. For the examples considered here, as well as descriptions of shapes and their pair-wise connections, topological descriptions of kinematic structure are also necessary, to inform the assembly of connected parts within kinematic closed loops. To summarise, these initial, type 1 material schema, for fabricating kinematic designs, have included:

- Schema for fabricating parts
- Schema for joining parts together with kinematic connections
- Schema for assembling parts within designs, according to particular topologies of kinematic structure.

A second type of schema (type 2) for exploratory making is also encountered. Rather than fabricating models directly, these suggest ways in which already valid kinematic model designs might be further altered. Here, type 2 schema include:

- Rules for changing the shapes of parts, using material rules for addition or subtraction
- Rules for transforming both the shapes of parts and their kinematic structure (altering connection geometries)
- Rules for altering connection topologies within designs for closed loop linkage
- Division schema, which dissect entire linkages, to create a set of stackable, working linkages.

Some of these alterations have been applied directly to completed models, (see for instance, the snowflake model in Figure 4.28, where tetrahedral corners are removed. However, due to the limits of certain material algebras for directly affording change, these transformations are more usually applied instead within more abstract, largely shape-based descriptions, of kinematic shape and structure— before entirely new models are then instantiated, using the original type 1 schema. In some examples, alterations were made at intermediate stages of fabrication. (For instance, for the case of the Bricard linkages (Table 4.2), subtraction rules were applied to alter the shape of parts within the CAD model, rather than to the final, printed version).

This persistent role of shape-based descriptions, within the material exploration of kinematic designs, suggests a potentially good fit for a formal shape grammar approach within these methods. However, these abstract, unstructured shape descriptions do not contain any inherent information, about the material composition of the parts they may be describing. And as we have already discussed, the established formal approach contains limited facility for modelling either these kinematic connections themselves, or the potential motions they afford between connected parts.

However, for designs with rigid moving parts, shape rules are useful for describing the shapes of parts, and also for describing transformations which then alter these shapes. Even when the underlying materials behave in less predictable ways, shape rules may still be useful, for describing operations which alter shapes within a given state. For this type of making, it is generally necessary to specify distinct parts: devices such as weights have potential to support this approach for shape grammars (Stiny, 1992). How

kinematic connections might be described within a formal shape-based approach is an open question, which we shall consider further in Chapter 6.

4.5.2 Crossing the boundaries: which rules ‘work’ and which don’t?

In our discussions of nets for parallelepipeds, we find that models of non-viable configurations- rather than being failures or dead-ends, in making terms- are if anything rather more instructive than the successful, working models. Successful models certainly help to illustrate the scope for variation within these boundaries. But whereas the boundaries which limit the scope for variation might be described using formal or analytical methods– unsuccessful models, whose specifications deviate beyond these boundaries, help to synthetically explain the reasons for these limitations, through material demonstration. Further examples of unsuccessful models are where collisions impede full cycle mobility, giving limits of shape change, for kinematic parts.

4.5.3 Schema for manipulating models

In order to experience motions directly, kinematic designs require active manipulation, applying forces to articulate connections between parts. Active manipulation of physical models (Figure 4.49) is therefore an important activity, for appreciating what the motions of a design actually are, and for understanding how parts within designs move.



Figure 4.49 Manipulating 3d-printed models of over-constrained linkages (Harrison et al. (2015) has a video link)

As in sketching, continual observation and reflection also informs exploratory making. Physical models provide a crucial mode of representation for kinematic designs, since they permit motions to be actively examined. Physical model-making of kinematic designs therefore provided a key focus of the experimental exploration activities we have reported upon in this chapter.

Whilst tactile interaction to afford rotation may be necessary, to fully appreciate static 3d designs, designs with moving parts require more– active manipulation, applying forces to articulate the connections between parts, is needed to experience motions directly. (In making, similar exploratory manipulation may also be necessary to experience and become familiar with the inherent behaviours of flexible materials, which deform in particular ways, when particular actions or forces are applied).

We initially proposed to employ shape rules as a basis for defining making rules. When using physical materials to construct 3d models of designs, shape rules can capture certain aspects of these reasoning activities– but what is noticed or recognised may not always be describable using shapes alone. Stiny and Knight (2015) suggest that the concept of identity rules, for recognising shapes, could be expanded to create more general ‘sensing’ rules, describing other forms of observation and reflection. But action and reflection don’t always happen sequentially, and making and sensing actions may occur in parallel, closely interlinked and in complex combinations, making them difficult to separate and study.

In the context of kinematic model-making, the motions of connected parts within designs are an essential aspect of the inherent identity of the kinematic artefact. Experiencing these motions, however, requires not just visual observation, but also active interaction and engagement, to physically manipulate the model.

We therefore identify a further type of schema, necessary for the material exploration of kinematic design spaces. These (type 3) schema are necessary to describe and model interactions and manipulations of models, to actively illicit and experience their motions. Kinematic configurations inherently possess the potential for motion. But in practice, it is

these manipulation rules which determine the overall motions afforded by the combined effects of the variable spatial relationships, defined by connections between kinematic parts.

In design exploration, since the inherent motions of designs may not yet be well understood, manipulation itself may be exploratory. Understanding precisely what the possible motions of designs with moving parts *can be* is not always straightforward, and therefore, just as creating new designs requires a process of creative exploration, understanding and evaluating, what the nature of motions of these designs actually are, similarly requires exploration.

Interaction with unfamiliar physical objects is both tactical and situated, and therefore cannot be fully specified in advance. Through a process of experimentally applying forces, particular sequences of actions which successfully articulate a design's full range of motions may come to be understood. These can then be routinely and systematically applied, to enact the full range of motions of that design.

For unusual or complex designs with many degrees of freedom, full and systematic exploration, in order to experience and understand a design's full range of motions, may not be trivial. In further design exploration, such practiced manipulation sequences may then be experimentally applied to new designs, to test whether motion sequences encountered in previous design iterations are preserved within new models.

Because manipulation rules are situated, and act directly upon models, it is difficult to define them out of context. Further, as the motions they elicit from models may be complex, it may not always be possible to succinctly describe manipulations, in terms of the variable spatial relations they enact. Here, rather than attempting to describe the manipulation rules required for our episodes of design exploration in full detail, we consider that, from a set of all possible manipulations, a particular sub-set is discovered to successfully elicit the motions of a given artefact. Subsequently, these can be routinely applied, to test the behaviour of new design iterations.

4.5.4 Summary of findings

We conclude this chapter by reflecting upon how shape rules that describe the exploratory making of kinematic designs might be applied within a systematic approach, to explore conceptual spaces of kinematic designs. This method for exploratory making is applied to a type of kinematic design that is significantly different from the examples of over-constrained mechanisms considered here. Developed further in Chapter 5, the method takes forward the collection of schema derived through the relatively free-ranging experiments in exploratory making, completed in this chapter.

Through these making episodes, we have discovered several general schema, for altering kinematic designs. These include:

- Schema for adding to the shapes of kinematic parts
- Schema for subtracting from the shapes of parts
- Schema for transforming both the shapes of parts and their kinematic structure
- Schema for dividing linkages, into two or more functional linkages, bisecting their hinge lines to maintain connections.

Applied to one class of design, the method is described in Chapter 5, with the focus on practical techniques for exploratory making. More wide-ranging observations include the following, and will be used as the basis for a systematic method, to explore a kinematic design through making.

- There is a concept of maximal shapes, for parts within kinematic designs, to maintain their motion.
- A range of material algebras have been used for the construction of kinematic models, including cutting and folding card and paper, and the 3d printing of rigid connecting parts.

- The choice, both of material algebra, and of shape descriptions for kinematic connections, may influence the resulting topologies of connections within kinematic designs for spatial linkages.
- For the closed loop kinematic designs considered here, rules which alter the alignment of hinge-lines are not reliably successful. They lead to significant alteration of degradation of motion.
- Manual manipulation and interaction with the physical models is critical, for exploring the potential motions in a kinematic design. Shape descriptions of multiple configurations during the motion cycle can provide a pragmatic means to describe this experience.
- There is an important role for shape-based descriptions, throughout these processes of making, which seem to rely on an ability to readily transform designs between different representations, employing different material modes of making.
- Physical models which 'don't work' can be potentially more instructive than successful ones, for positively identifying the boundaries to design spaces within which kinematic properties are preserved under transformations. The explanatory potential of this phenomenon is considered further in the following chapter.

Chapter 5: Searching design spaces with making schema

Inspired and informed by the various experiments and explorations in Chapter 4, we now apply making schema to investigate one type of kinematic design in detail. Whereas our previous explorations have sought to consider descriptions for making, here making schema are applied within a systematic exercise in design exploration, centred around a specific kinematic design.

First, we outline a method for systematic design exploration using making schema. The approach sequentially alters an existing artefact, in order to access and explore the wider design spaces that contain it. The objective of the method is to develop rule-based descriptions of the wider design space regions surrounding an existing design, and also to identify the boundaries containing new design instances with similar properties. This exploratory, non-linear approach contrasts with design processes described by a sequence of actions and reflections along a single trajectory.

We subsequently report on an episode of systematic exploration of kinematic designs through making, which puts this method into practice. From an existing kinematic artefact as a starting point, we examine how rule-based exploration can construct related instances within a surrounding kinematic design space. Some of these design variants exhibit similar characteristics— for example, similar motions. Manipulation schema are applied to observe and evaluate the motions of these new design variants. Evaluating helps to establish the boundaries within which desired kinematic characteristics are preserved.

The kinematic design selected for study has been chosen for its unusual kinematic behaviour. Making rules are applied to this design, transforming both its shape and structure, before considering the effects of these changes upon motions. These rule applications create variations on the initial design, which becomes an entry point for exploring a surrounding region of a kinematic design space.

From a design perspective, the objective of this exercise is to identify new design variants which still preserve the motions of the original, through experimentally varying the shapes and structures of physical models. The aim of the method is to identify which relationships between shape and structure encountered in the initial design are essential to preserving its

motions, and conversely which design parameters may be varied freely. Once these essential relationships are identified, rule-based descriptions of designs can be actively articulated: these new schema both describe how to construct a set of related designs with similar kinematic motions, and succinctly explain the extent to which certain design aspects can be varied without affecting kinematic behaviours.

From a kinematic perspective, the objective of the method is to examine the types of spaces which may contain kinematic designs, to consider their underlying nature and structure. We also consider the extent to which these structures may hold some explanatory power, for describing which design properties affect the boundaries to specific types of kinematic motions. From a computational perspective, this exercise considers the potential generative and explanatory power of making schema, within a practical design context.

At each stage in this episode of design exploration, the actions, observations, interactions, and reflections that occur within making are observed and recorded. Each design change or alteration constructs a new physical model. The resulting collection of models provides a lasting physical record of the design activity undertaken. We also reflect upon how other aspects of making activity, employed within this episode but not yet explicitly encompassed within the general making schema outlined in the previous chapter, might be formally described by new kinds of making rules.

First, a method for using making schema in design exploration, derived in part from the wide range of experimental activities in Chapter 4, is proposed in Section 5.1. This method is presented as sequential steps; although its application in an exploratory, experimental mode will be far from sequential. Never-the-less, for the purposes of this presentation we emphasise these steps, in order to draw out the distinct types of activity involved in exploratory making. The method is applied in Section 5.2, with an existing kinematic design as starting point, and divided into the eight (artificially) discrete steps of the method. Section 5.3 examines how design explorations lead to strategic making schema which can themselves lead to new rules, and hence to new making and new designs. Section 5.4 focusses on modelling motions, and the possibilities for exploratory making to explain motion behaviour.

5.1 A method for exploratory making

Various types of schema for making and altering kinematic designs were described in Chapter 4. These include:

Type 1: material making schema for fabricating valid kinematic designs:

- Rules for fabricating and shaping kinematic parts
- Rules for creating kinematic connections
- Rules for connecting kinematic parts within designs, according to a prescribed kinematic structure.

Type 2: transformation schema, to alter existing kinematic designs, including:

- Rules for adding to the shapes of kinematic parts
- Rules for subtracting from the shapes of parts
- Rules for transforming both the shapes of parts and their kinematic structure (altering connection geometries)
- Rules for altering connection topologies for closed loop linkages
- Division schema which dissect entire linkages through their hinge-lines, creating a set of stackable, working linkages.

These rules can be applied either directly, to existing models, or indirectly, to change geometric descriptions of kinematic design configurations that are subsequently fabricated as physical models using material making schema.

Type 3: supplementary schema:

- rules for eliciting the motions of kinematic artefacts, through direct interaction and manipulation.

The method builds upon the material making and transformation schema, encountered in the experiments of Chapter 4 for models of kinematic designs. Transformations which

preserved motions in kinematic models were identified, through the comparison of complementary designs and models. In the proposed method, however, we concentrate on how rule-based instructions for making specific designs can themselves inspire ideas for transformations; through sequentially removing precision from material rules, and relaxing descriptions to discover more general schema. These new schema (denoted as type 4 schema) are then used to construct new kinematic design instances. The method has the following steps:

Step 1. Select a starting point:

An existing design or artefact is selected as a starting point. For the case of kinematics, designs appropriate for the exploratory method are those that are not already well-known and well-explored by conventional analysis techniques. Those kinematic designs with distinctive, unusual motions are also advantageous, because the presence or absence of distinctive motion characteristics will be more immediately apparent. To avoid explorations becoming artificially limited by expectations about an object's function, designs that have no familiar functional applications are preferred. As in our previous exploratory making activities, this practical inquiry begins by selecting an existing kinematic toy or plaything.

Step 2. Manipulate physical models:

Physical models of designs with moving parts afford the direct interrogation of motions through manipulation. This step examines and describes how connected parts within the original kinematic artefact move, to gain an appreciation of the artefact's motions. Certain variable spatial relationships are inherently afforded by particular designs, but these must be elicited and observed through direct interaction. The motion characteristics of the design can be described by configuration states into which the physical model of the kinematic design can be manipulated.

Step 3. Describe a design in terms of shape and structure:

This step examines the existing model, in order to create pictorial descriptions of shape and structure, during its motion. The shape and structure of a kinematic design includes both the shapes of kinematic parts, and the kinematic connections between them. Pictorial descriptions can be readily used to visually describe the shape configurations of parts within kinematic designs, as observed between motions, in their various static states. Schematic

pictorial descriptions can also help to describe and consider the structure of kinematic connections.

Step 4. Material making rules for motions:

A critical task in this synthetic, exploratory process is the selection of an appropriate material schema, with which to both describe and construct a working copy of the chosen artefact. Certain tool and material combinations afford the fabrication of specific connection types more readily than others, so appropriate material algebras must be selected. For hinged designs, for example, sheet materials which can be readily cut and folded afford the expedient construction of physical models that exhibit appropriate variable spatial relationships between parts. More sophisticated connection types may require more advanced tools, materials, and fabrication processes.

During the fabrication of a first copy, actions of seeing and doing are recorded. These material making actions then define a specific set of making rules, detailing the precise actions and manipulations required for shaping and connecting materials to construct the physical model.

Step 5. Abstract the making rules:

Subsequently, these making rules are systematically varied. Removing detail, from the descriptions of the material making rules for shape and structure of the original design, can abstract more general making schema. Combinations of these new rule variations can then construct physical models of new designs related to the original artefact.

Step 6. Evaluate the motions of new designs:

Once fabricated, new models can be manipulated using the actions discovered when interacting with the original artefact, so that the relative success of a modified design in reproducing these motions can be observed. Evaluating the properties of each design variation develops a more precise understanding, of the scope for transformations to shape and structure, within design constraints on kinematic behaviour. Some modified kinematic designs will exhibit restricted motions, perhaps through transformations of shape causing collisions, or transformations of connections causing reduced mobility. Identifying subsets

of making schema that reliably reproduce the motions of the original artefact can then help to explain how a design works.

Step 7. Situated transformations:

Whilst the original making rules provide a starting point from which to derive new rule variations and construct new designs, new physical models themselves may prompt additional, situated ideas for changes. An exploratory (type 4) schema is derived by removing detail from the set of material making rules (type 1 schema) for the original design. But in this step, a situated, tactical approach explores further, by applying transformational (type 2) schema to act directly upon models.

Step 8. Discover strategic schema:

Through systematically constructing model variations, subsets of the exploratory making schema that create new objects which successfully reproduce the motions of the original design are identified. These subsets may then be used to consider a wider range of designs.

5.2 Applying the method to explore kinematic shape and structure

The method above is applied to an existing kinematic artefact, presenting details of a practical exercise in exploratory making.

Step 1: Select an existing object as a starting point for design exploration

The chosen starting point for exploration is a mechanical toy that exhibits an unusual sequence of motions, which fulfil no immediate function beyond their ability to entertain. The selection of a design with no immediately familiar application helps to avoid exploration becoming artificially limited by functional expectations.

Composed of two connected layers of flat, hinged panels which form a rectangular sheet, the toy exhibits an unusual sequence of motions. The panels can be manipulated into four

distinct static states. When manipulated continuously, the toy transforms through the four states and returns to the first state, and so on, in a continuous cycle. Its behaviour is rather reminiscent of model 1, the ‘magic cube’ considered in Chapter 4, although its configuration is different, and more unusual.

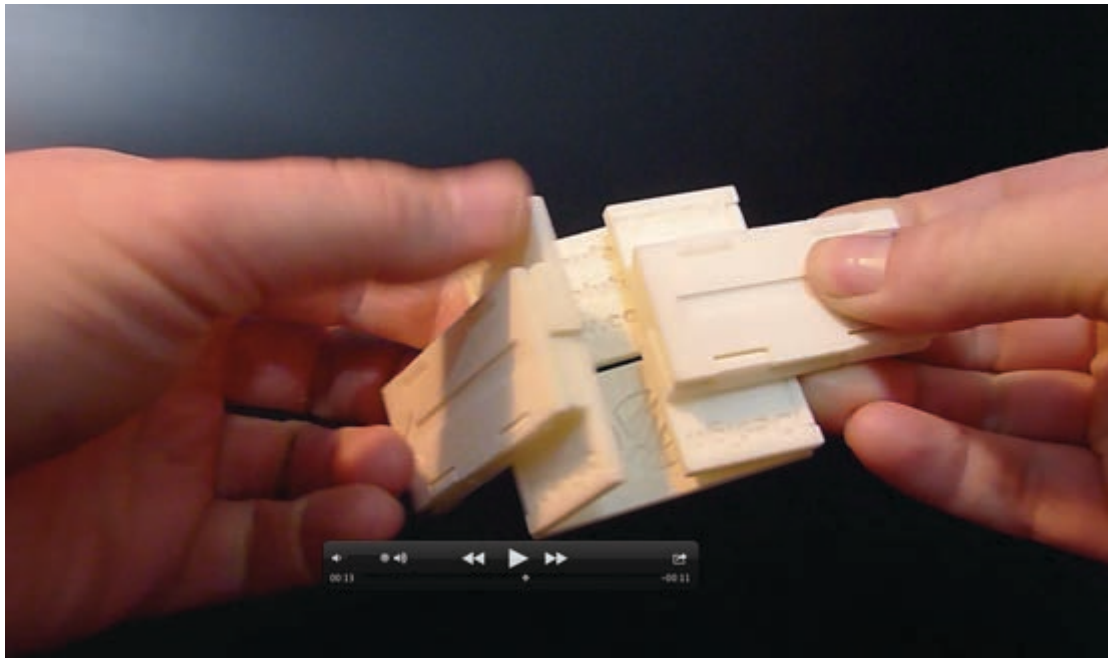


Figure 5.1 Manipulating the mechanical toy through four consecutive states, demonstrating full-cycle mobility (Harrison et al., 2015 has a video link)

This toy design has been used as a sample model by a Selective Laser Sintering equipment manufacturer. A 3d-printable copy is available (Thingiverse, 2013), but we have not yet discovered the designer of the original. However, there is a patent for a similar design- with an identical sequence of motions but composed of only one layer, and intended to be folded from a single sheet of card (Byrnes, 2001; 2004). We expect that the two-layer version may be an intentional variation, in order to achieve the same interesting motion sequence as the protected design, but without infringing on the shape and structural descriptions specified in the patent.

Step 2: Discover motions through manipulation

The toy's hinged panels can be manipulated into four distinct states. When manipulated continuously, it transforms between these states consecutively in a continuous cycle. Figure 5.2 illustrates the sequence of unfolding actions through which the toy eventually returns to its initial state. This type of motion sequence is referred to as full-cycle mobility. Designs with full-cycle mobility may move in a continuous or non-continuous manner and have one or more degrees of freedom. This design exhibits non-continuous motions, since symmetric pairs of hinged panels can rotate through a maximum of 180 degrees before shape interference limits motion. However, this interference then brings about the alignment of a second pair of hinges, and the next phase of motion becomes possible.

This non-continuous, full-cycle behaviour is relatively uncommon. It is therefore not yet anticipated, upon first encounter with the object. However, after brief exploratory manipulation, playing with the toy, its motions become recognisable. After some practice and rehearsal, the sequence of actions required to cycle through its states repetitively becomes familiar, although the toy continues to maintain interest.

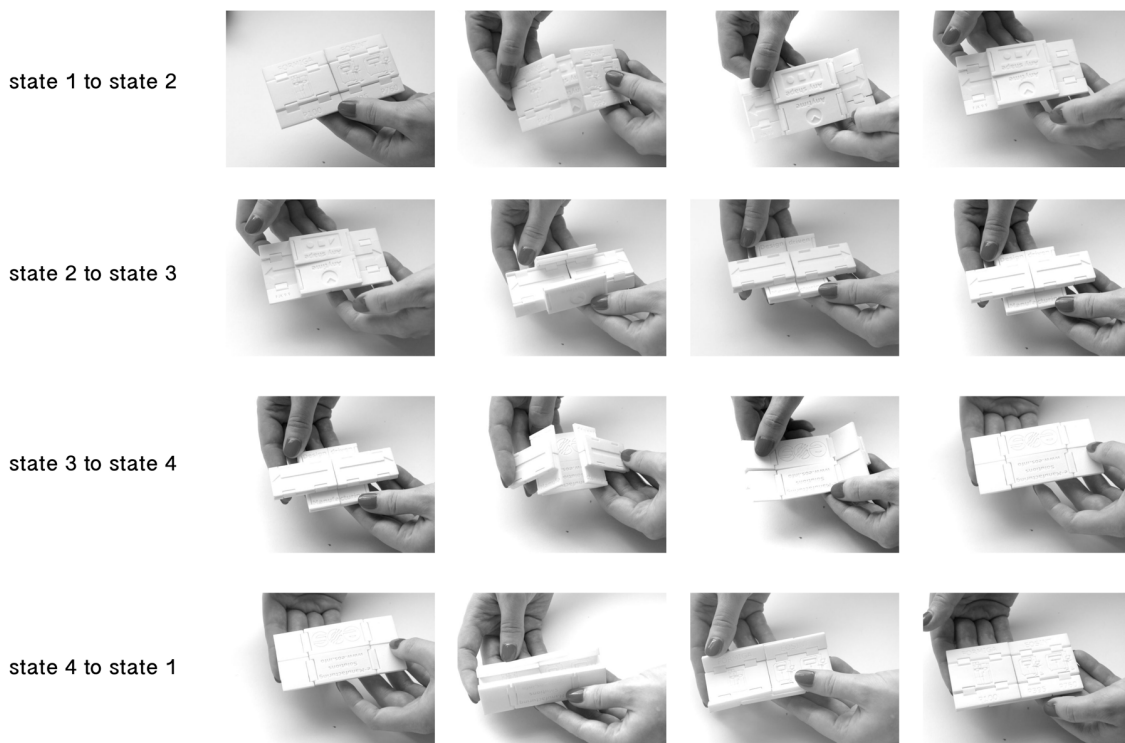


Figure 5.2 Motion sequence between the four states (which can also occur in reverse)

Let's suppose that, prior to encountering the toy, individuals (for this study- both the author, and several others informally presented with the artefact) already possess general techniques for examining and experimentally manipulating objects with moving parts. We could describe these tactical techniques by the general schema M of manipulations. Experimentation then helps to recognise which manipulations from this schema are successful in eliciting motions from the toy. Let's call this smaller, reliable set of manipulations M'. In order to directly observe the toy's full range of motions, this set of actions must be successfully discovered. When making new models, this practiced sequence can then be reapplied, to test whether a new design exhibits similar motions. However, if M' fails, we can always return to M, to explore whether alternative manipulations M'' of the model elicits other motions.

The effects of the manipulation sequence upon shape and structure can be described by examining the toy in its four possible states from different perspectives, and then creating drawings as shown in Figure 5.3. Within later stages of exploration, this sequence of actions was subsequently used also to test whether new physical models successfully re-instantiated the motions of the original design.

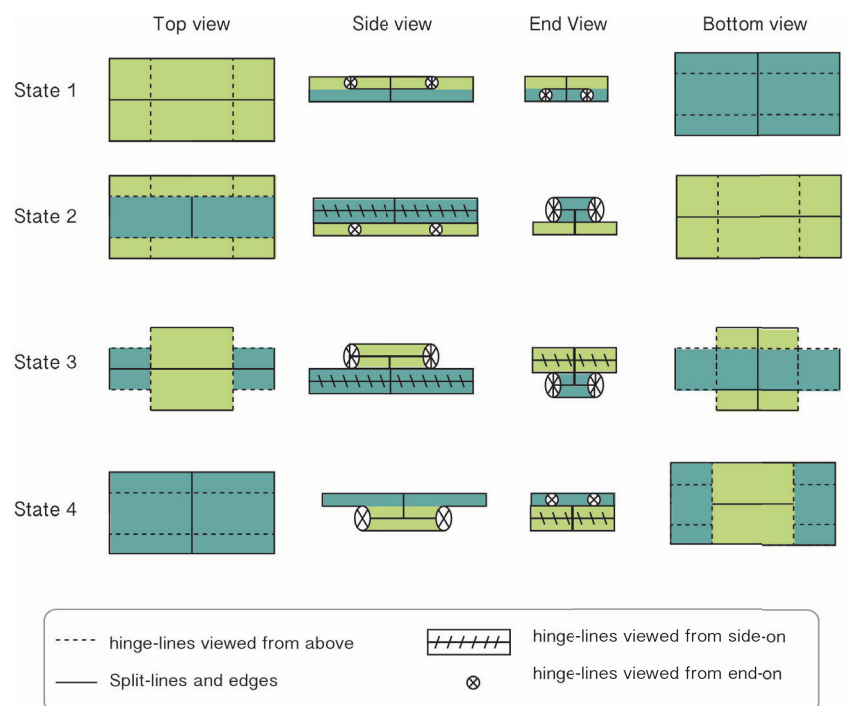


Figure 5.3 Drawings of the design in its four distinct states

Step 3: Describe the design using shapes

Although the toy's unusual motions have been physically experienced, they are still not readily described either verbally or schematically. However, handling the toy whilst at rest in each state, to view it from various angles, enables it to be described using pictorial, shape-based descriptions. In order to sequentially observe the artefact in its multiple states and from multiple viewpoints, it is also strictly necessary to handle and manipulate the artefact. In addition to the manipulation schema M , suppose we have a general schema H for rotating and examining static 3d objects. With these in place, the shape identity schema $x \rightarrow x$, in U_{33} , for recognising 3d shapes, may be applied to the model. The schema

$$x \rightarrow b(b(x))$$

then creates line drawings of the 3d object from various angles. In order to recognise its hinge-lines, a little more is needed. Let's assume for now that a general identity schema for noticing can also recognise common connection types within kinematic structures, and describe them schematically in drawings using labels or weights (Stiny, 1992).

Whilst considering multiple states helps to describe and examine motions, the toy's shape and structure (that is, its parts and connections) are most simply represented by pictorial descriptions of its initial state, where it resembles a flat sheet (Figure 5.4). Drawings of each face in this configuration are relatively simple to produce, and embody multiple spatial relationships— although the extent to which each can be varied to produce new designs is not yet known.

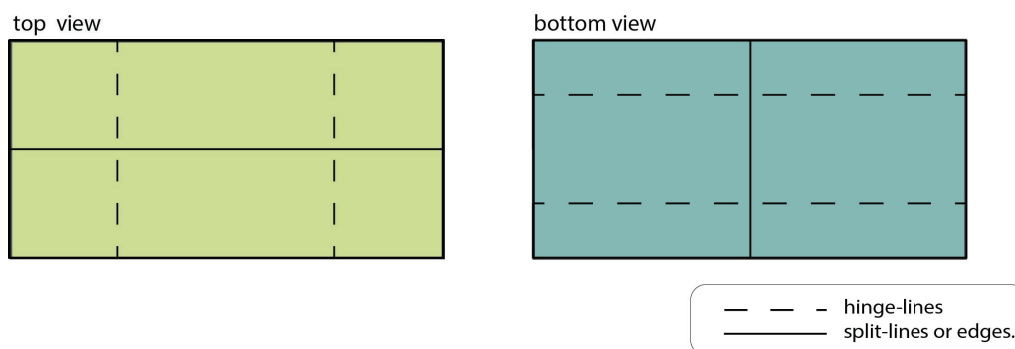


Figure 5.4 Spatial relationships in the original design

These drawings of each face embody multiple spatial relationships. Table 5.1 outlines a list of relationships which can be observed between elements within the original design. In making rules for producing precise copies, these relationships must be preserved.

Initial spatial Relationships	Secondary relationships
The overall shape of the design is a rectangular sheet, twice as long as wide.	The overall shape is a rectangle. The edges of the overall shape are straight.
The hinge-lines on a given face are parallel to the edges of sheet, and therefore also to each other.	Hinge-lines within pairs on each face must be parallel to each other
The hinge- lines are positioned 1/4 and 3/4 along the face.	Pairs of hinge-lines are positioned symmetrically about a centreline. The hinge-lines are spaced apart by half the panel's full width/length.
Hinge pair orientation and spacing is identical but perpendicular on each face.	Pairs of Hinge-lines must be perpendicular to those on the opposing face
The split-line must be perpendicular to the hinge-lines.	The split-line is a straight line. The split line on one face is perpendicular to the split line on the opposing face.
The design is composed of 2 full sheets of material.	

Table 5.1 Spatial relationships of the original design

Step 4: Tools, materials and rules for making

Using the relationships for shape and structure identified in step 3, we developed material making rules, to construct a working copy of the design. For this activity, we gathered

together common model-making materials, to make and alter new models. Since the design is composed of flat, hinged panels, to create each layer, foam-board was cut into shapes and taped to selectively re-join panels along cut-lines, instantiating hinges. Our tools included a pencil, a ruler and a scalpel (Figure 4.5).

Shape rules, with conventions for describing hinge-lines, could then be employed as pictorial instructions, describing rules 1, 2 and 3 on shaping and joining (Figure 5.5). This practical, schematic approach does not begin to describe the detail of the complex yet familiar, situated activity, of cutting using a blade, or the precise actions involved in attaching tape. However, this contextual knowledge can be readily accessed within the situated application of pictorial or schematic rules, as plans for guiding making (Suchman, 1987).

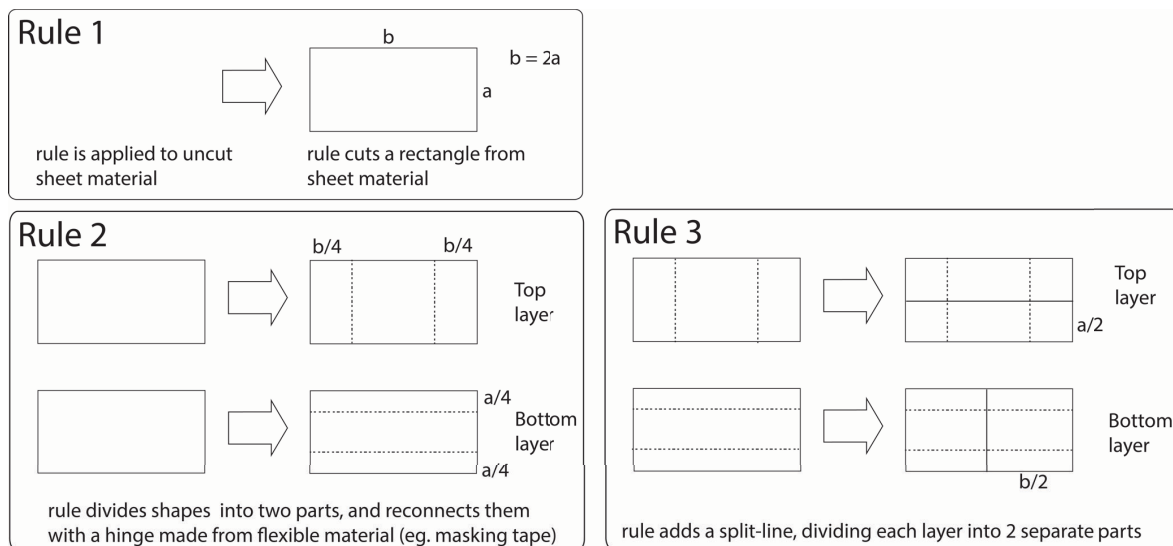


Figure 5.5 Rules for making

Rules 1, 2 and 3, together describe how parts for each panel are cut and joined together. However, when considering the original artefact, external examination alone (see step 3 for describing shape and structure) does not readily identify how the two panel layers should become connected.

Situated experimentation yielded this information. Manipulating the two assembled sections, to establish which parts must move and therefore must *not* be attached, was conducted in parallel with the tentative application of adhesive at various points, to test whether the model could still move as expected. Figure 5.6 demonstrates schematically a

reasoning process which determines an additional rule for where adhesive can be applied; although tactics of trying and testing also discovered the same result. Figure 5.7 describes this additional making rule, which is needed for the complete assembly of a working model. Manipulation through the four states confirms that this model reproduces the motions of the original toy.

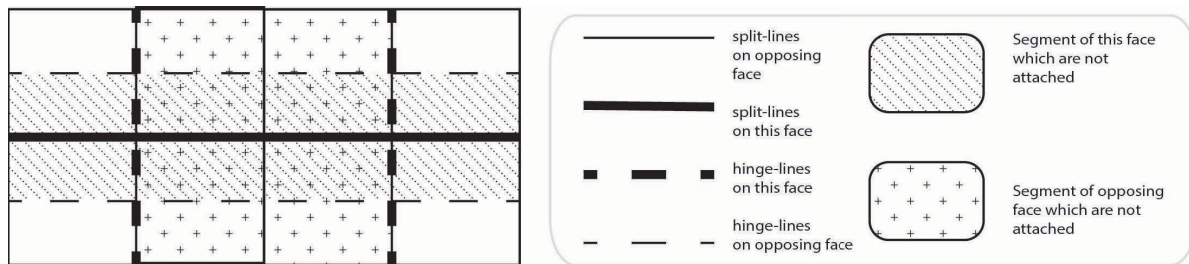


Figure 5.6 A reasoning schematic to derive Rule 4 (for joining the two panel layers)

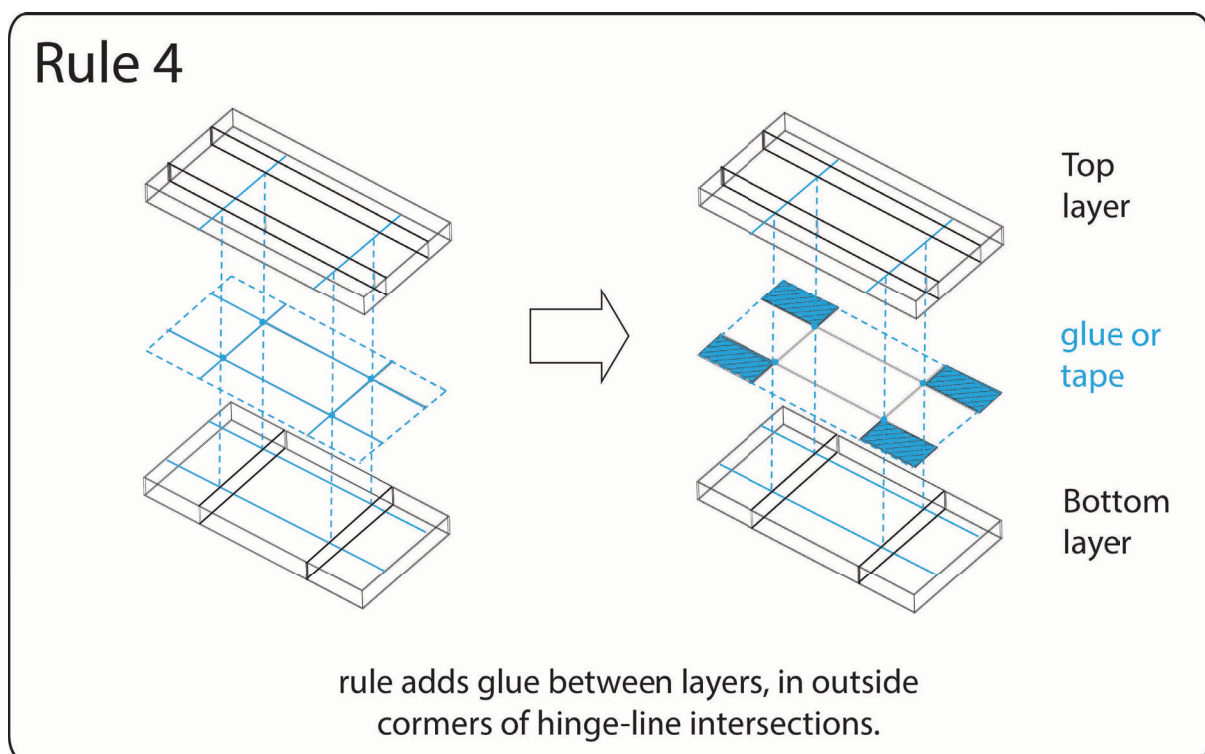


Figure 5.7 Additional rule for making

Step 5: Abstracting making rules: from tactics to strategies

With a method for completing models established, we then considered how rules might be generalised, simplified and abstracted into more general schema, through exploring how the relationships underlying the original making rules might be varied. For each variation, a new physical model tests the effects on motions.

After fabrication, each new model was manipulated, using the same sequence of actions as was discovered to elicit the motions of the original toy. When a new model can be successfully manipulated through all four states sequentially, the rule variations that constructed it are deemed to preserve those spatial relationships of the original design essential for achieving its full-cycle motions. The right-hand column in Table 5.1 outlines how this new knowledge of successful variations allows the descriptions of spatial relationships observed in the original design to be relaxed, creating less precise descriptions that define wider categories of objects, and ultimately identifying the essential relationships for achieving motions.

Making rules 1-4, as initially prescribed, instantiate the spatial relationships between shape and structure defined in the left-hand column of Table 5.1. By relaxing these conditions, more general schemas are created. We considered how rules might be simplified and abstracted into more general schema, from which new variants on the original making rules can be defined.

Step 6: Evaluating motions of new designs

By varying the original rules, through these tactical, material schema, new models explore and test which aspects of the original design are critical for preserving motions. For each variation, we constructed a new physical model. Figure 5.8 shows the full set of models constructed (Models 1-14). Note that at the outset, the author possessed limited intuition, as to which alterations might be successful, and endeavoured to explore a wide range of possible variations systematically.

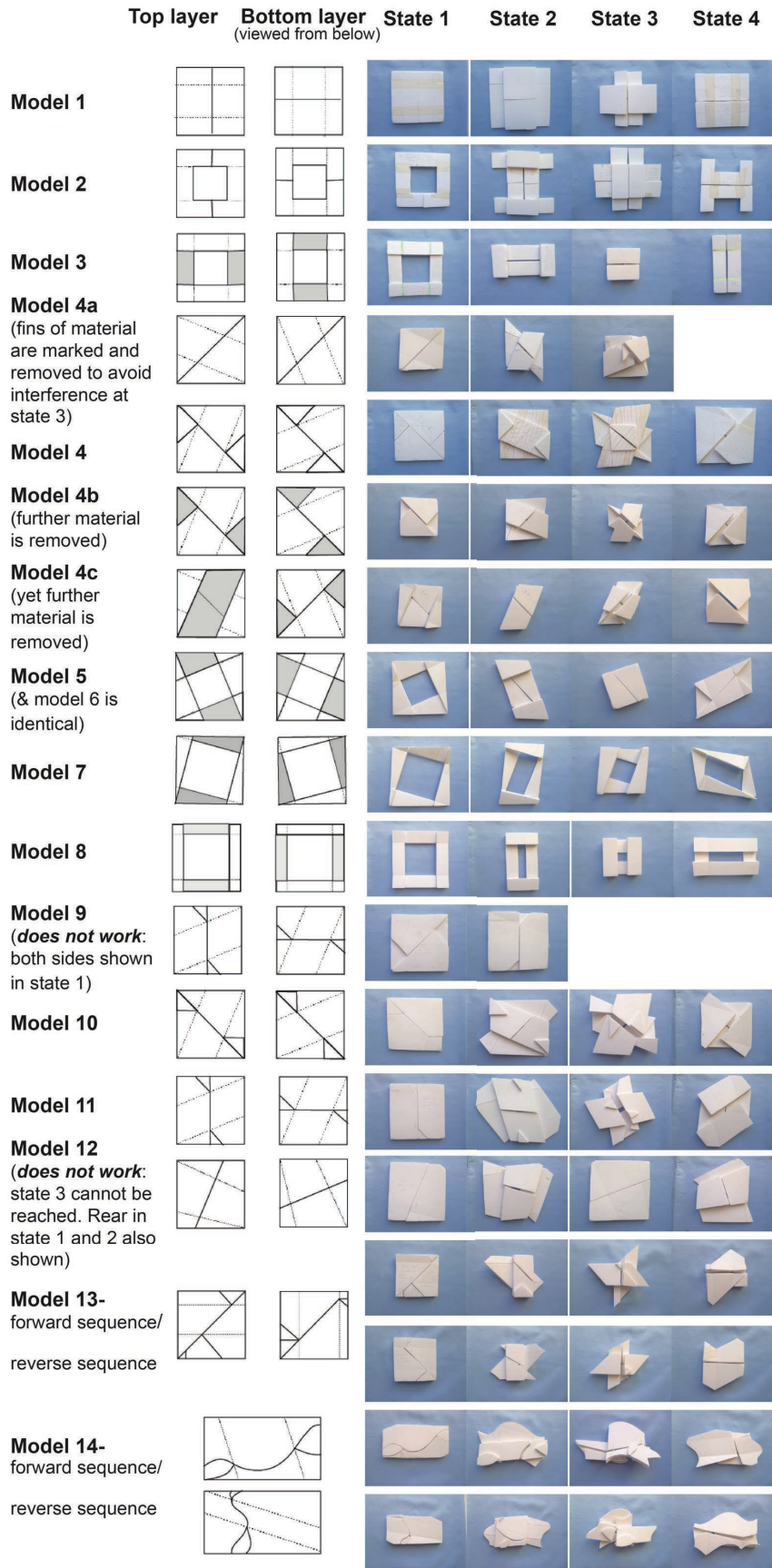


Figure 5.8 Experimental models, to test design variations

The first variation to the making rules generalises the proportions of the rectangle in Rule 1 to allow parametric variations. Note that when the overall shape is set to a square, both faces become identical, and the schematics for Rules 2 and 3 become simplified (See Figure 5.9). In this case, for assembly, one layer must merely be rotated through 90° relative to the other. With this parametric variation afforded, Rule 1 evolves into Schema 1 (Figure 5.9), which constructs both square and rectangular panels of all proportions (for example Model 14). Schema 1'' is further abstracted, to allow for non-rectangular shapes (Figure 5.10).

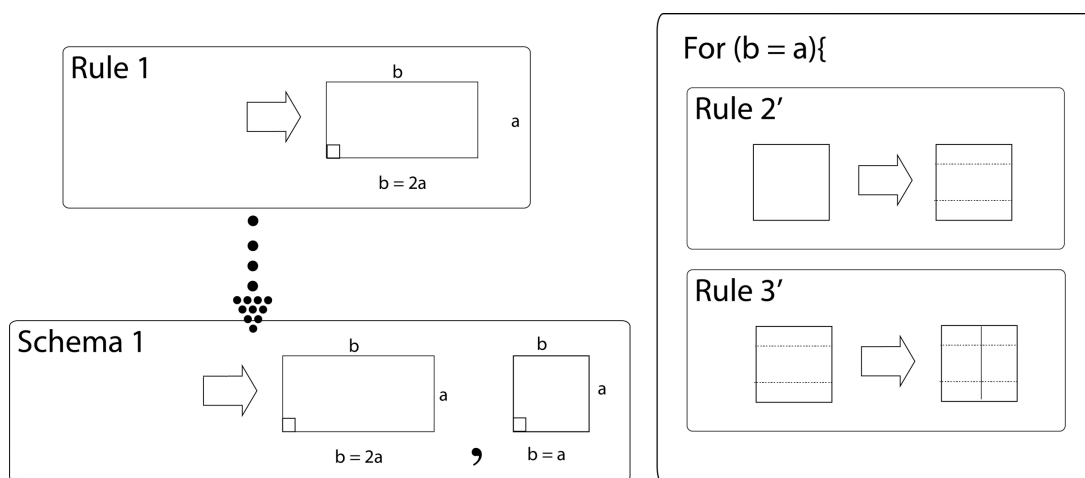


Figure 5.9 From Rule 1 to Schema 1

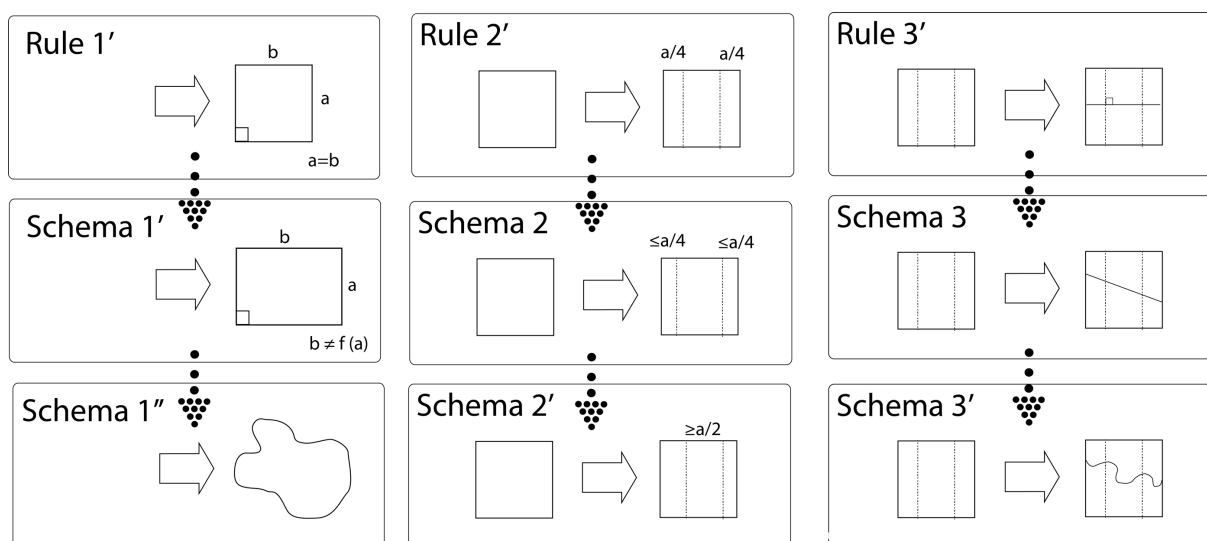


Figure 5.10 From making rules to making schema

A second variation tests whether the pairs of hinge-lines created by Rule 2 must be located as precisely as originally specified, or whether a more general schema (Schema 2, Figure 5.10) would also work. In the original design, outer edges meet precisely along a centreline upon folding inwards. Avoiding overlapping and associated interference requires that the spacing between the hinge lines must be at least half the width of the face, but a variation that relaxed this rule so that hinges might also be further apart, was tested and found to be successful (Models 7 & 8). Further variation also discovered that hinge-lines need not be symmetrically placed, deriving Schema 2' (Figure 5.8; Models 13 and 14).

It was also considered how Rule 3 (Figure 5.5) might be simplified. Figure 5.10 shows its generalisation into schema: removing the requirement that the split-line be perpendicular to the hinge-lines on that face (Models 4 and onwards; Schema 3); removing the requirement that the split-line must be a straight line (Schema 3'; Models 13 and 14); and also testing whether split-lines must be placed similarly on both faces. Models where the split-line is no longer perpendicular to the hinge-lines on that face are found to function well.

However, a pragmatic requirement is that the split-line still intersects both hinge-lines. Model 9 tested whether orientation between the pairs of hinge-lines within the 2 layers could also be varied, but found this to be a fundamental requirement of the design.

Step 7: Situated transformations – extending the rules for exploration

In order to derive the additional Rule 4 (Figure 5.7), we noted the need for situated, tactical experimentation. This need, for a situated approach to conceive of new making rules, continued to occur throughout our process of exploration, in reaction to partial or imperfect models. This became particularly important when rule variations initially appeared to create unsuccessful models that could not be manipulated through the full four-state motion cycle.

Further variations explored whether hinge lines must lie parallel to the panel edges (Figure 5.11, Rule 2' and Model 4), or parallel to each other (Figure 5.11, Rule 2'' and Model 12).

Whilst the latter variation is found to be fundamentally unsuccessful, the former can be instantiated successfully, but only when further, situated adjustments are made directly to models. Models 4a-4c provide the following example.

When hinge alignment within the panel is varied, external panel edges meet at an altered centreline upon folding. The greatest possible deviation would orient this centreline along the 45 degree diagonal. To achieve this, the hinge lines were rotated by half this angle (Models 4, 5, 10, 11). Note that, to preserve the perpendicularity of hinge alignment between the faces during this variation, the layout of one face must be a mirror image of the other, rotated by 90, rather than an absolute copy, since this new panel version possesses less symmetry than the square, perpendicular version of earlier models.

When Rule 2'' is applied (Figure 5.11) the resulting model (Model 4a) can be manipulated into states 2 and 4, but material interference initially prevents state 3 being reached. However, here a new idea for a situated rule was identified: when examining the design in states 2 and 4, its footprint was noticed to be larger than its square shape boundary in State 1. A tactic of trimming sections that protruded beyond this original square footprint was tried. This change resulted in a new model that could then reach all four states successfully (Model 4b). It additionally was found that the sections removed by trimming, if glued onto the opposite layer, could remain attached there without affording further interference.

It was noticed later that this procedure could be further refined, and slightly less material could be removed (and optionally reattached), while still yielding a working design (Model 10). Interfering material could be marked precisely, directly on the model in states 2 and 4 respectively, and these marked sections could then be removed. It was found latterly that this trimming approach could be applied successfully to complete any design variation where material interference was initially encountered. Since this occurs for a significant subset of the designs produced here, this new schema, Schema 5 (Figure 5.11), becomes an important addition to the making sequence, allowing a new subset of designs to be constructed.

Note that the original making rules assemble the design exclusively in State 1. But applying Schema 5 relies on subsequent manipulations $M(x)$ (Figure 5.11), to then transform the

model into states 2 and 4, where these marking and trimming operations can be applied. Actions $M(x)$, for manipulating designs between states, are therefore required not only for testing the success of completed designs, but also as part of the making process itself. Although conceived through situated experimentation, once practised and found to be a reliable, these schema for situated alteration could be defined precisely, and applied systematically.

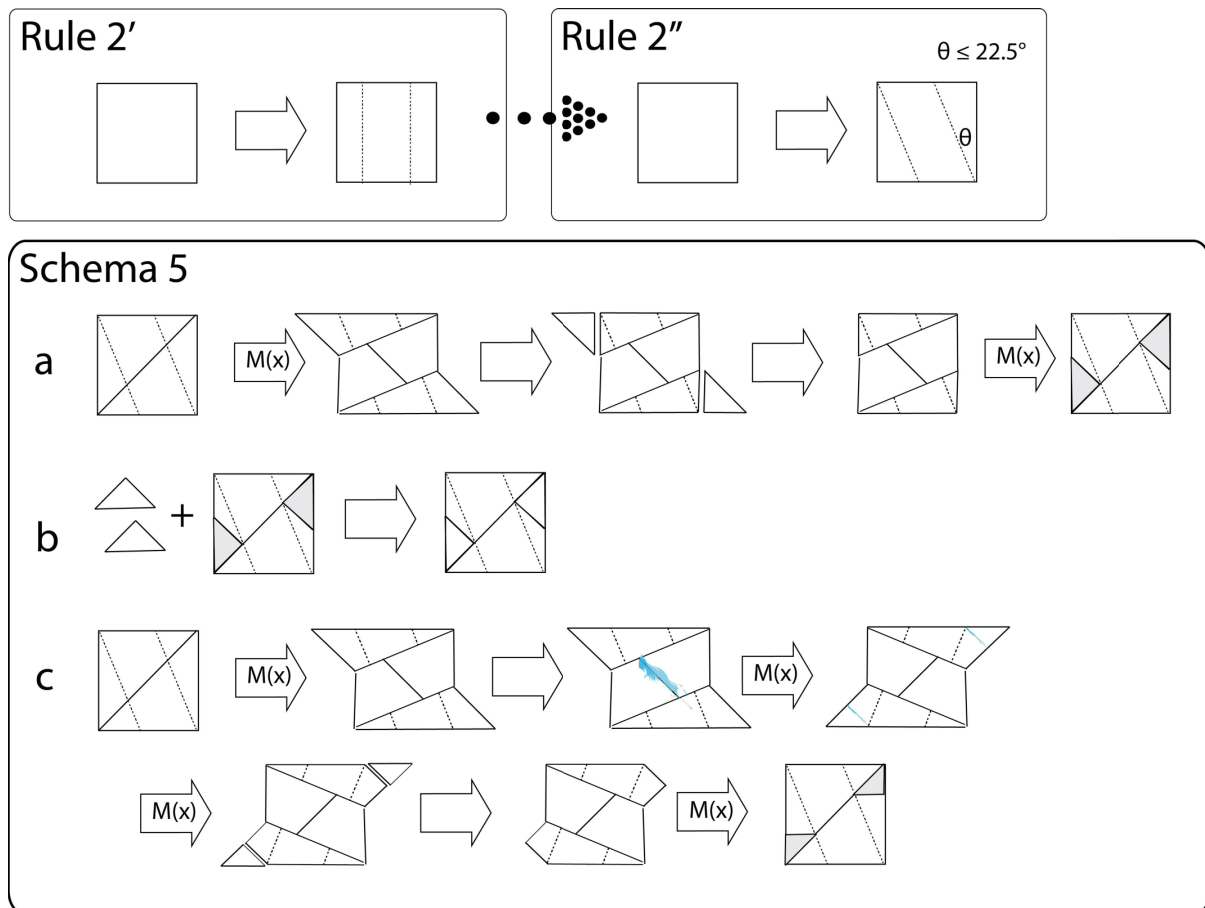


Figure 5.11 Successful application of rule 2'' requires an additional 'tactical' schema, to remove and optionally reattach interfering material

With this role of rules for situated alterations established, further situated tactics were subsequently uncovered. Another example of subtraction rules (Models 2 & 3) considered how successful models might be altered by selectively removing material. Rule 4 (Figure 5.7) indicates that panel material lying outside of the hinge-lines plays a role in connectivity between the two layers, conversely suggesting that the material lying within these hinge-lines might be less essential. Through experimentally removing this material, it was

ascertained that this was indeed the case (Model 2) and so a corresponding schema could be defined. (Looking ahead, this is presented as Schema 6 in Figure 5.15.) Further, considering the resulting model in its intermediate states indicated that further portions of material also played no role in essential connectivity. Removing these parts leads to a pared-down model (Model 3), whose footprint changes dramatically upon transforming between states. These subtractive or reducing rules were also found to be applicable to other model variants, producing a subset of designs that possess the additional property of expanding and collapsing between states (Models 5, 7, 8). These pared-down designs can also be constructed from a single sheet of thin material. As such, they therefore appear to fall directly within the subset of designs protected by the patent referred to earlier (5.2; Step 1).

Note that the resulting designs do not require full sheets of material in their construction. Thus, they could also be constructed more efficiently from their necessary parts directly using appropriate making schema (Schema 7; looking forward to Figure 5.17).

Table 5.2 Essential spatial relationships discovered through exploration

<i>Essential spatial relationships</i>	
Hinge-lines within pairs on each face must be parallel to each other	Pairs of hinge-lines must be perpendicular to those on the opposing face

After making several models, it was discovered that many of the relationships inherent in the original design were inessential for preserving its motions. Significantly, the only relationships which proved essential (Table 5.2) were the relative orientations of the hinge-lines. These define variable spatial relationships between the design's parts. Models 9 & 12, (Figure 5.8) where hinge orientations were altered, were found to lack the full-cycle mobility

of the original design. This was not due to material interference, which can be fixed using Schema 5 (Figure 5.1), but due to a structural misalignment of the underlying connections.

Step 8: Strategic schema for making

This importance of hinge orientation indicates that Schema 2, which defines hinge geometries, is perhaps the most critical step in the making process. Placing Schema 2 at the beginning of the making sequence therefore helps to define a more strategic approach to constructing new design variations.

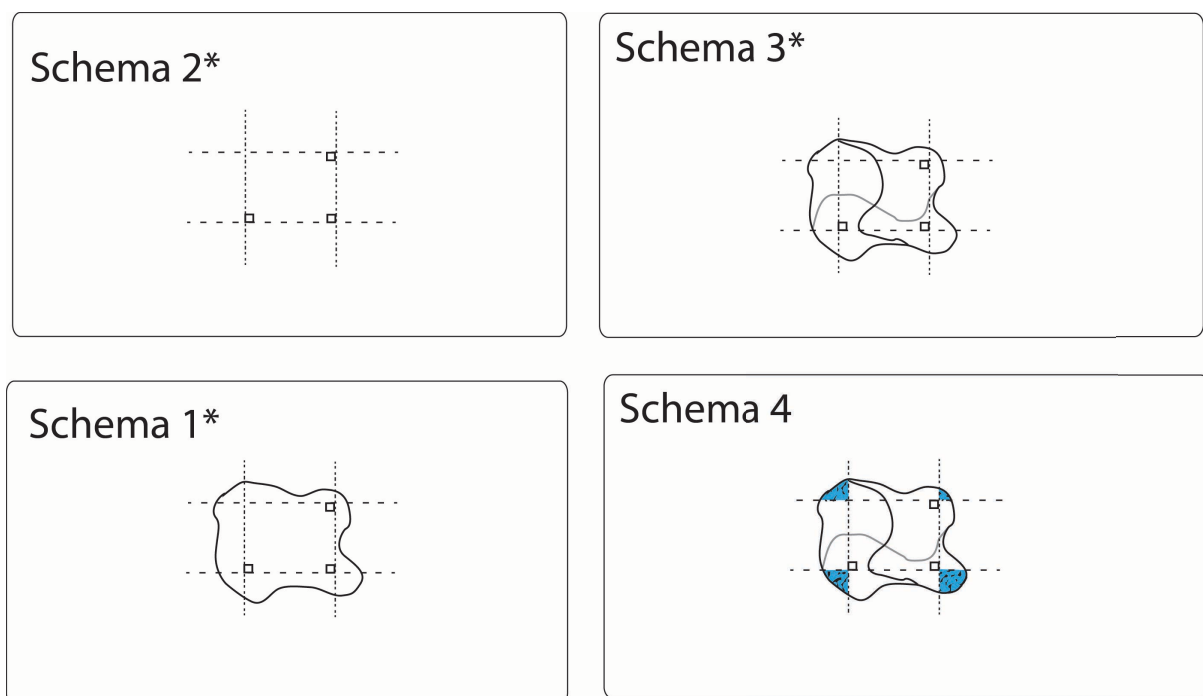


Figure 5.12 New strategic making schema

In a new, strategic approach to model making, an evolved Schema 2* (Figure 5.12) initially defines relations between hinges for both layers. Other design aspects are then constrained only by their locations relative to these hinge lines. With these defined, the remaining panel edges may otherwise form any closed shape. To ensure practical construction, a corresponding Schema 1* must merely place material outside of these hinge-lines and their intersections. A Schema 3* adds a split-line or curve to each face, which must bisect both

hinges. As before, Schema 4 is required for gluing, and an additional situated schema (Figure 5.11, Schema 5) is sometimes necessary for removing material interference. These new, strategic schema (outlined pictorially in Figure 5.13) permit a more direct, strategic approach to constructing further designs.



Figure 5.13 Creative designs developed using the evolved making schema

With the allowable limits to variations of shape and structure now clearly defined, creative exploration can focus on other design properties, such as colour, composition, and communication (Figure 5.13). Formally defining the manipulations and alterations that

occur within Schema 5 can also support further creative exploration using digital drawing tools. Since hinge-lines, until instantiated, are not material features but rather abstract structural concepts, visualisation tools to help to consider and predict the interactions between Schema 1-3 for a range of design options become useful (Figure 5.14). The interplay between exploration in physical making and in other representations, including digital tools, is the subject of further work.

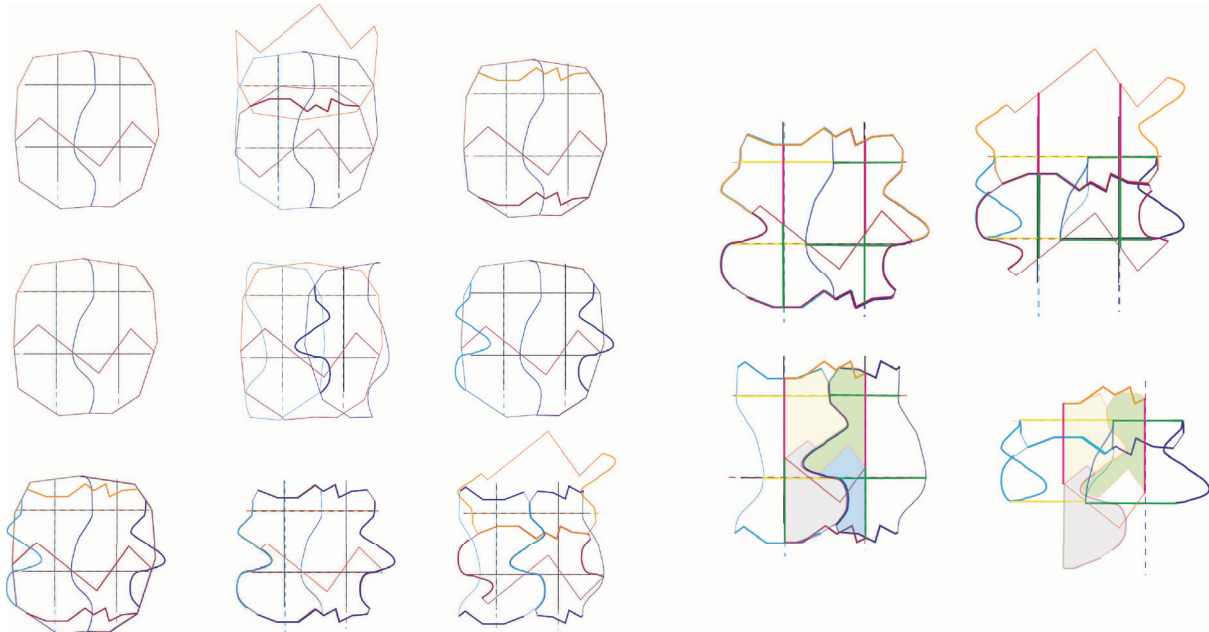


Figure 5.14 Drawing tools to consider schema interactions in multiple states

In this detailed example, exploration of multiple variations to a specific kinematic design has been demonstrated using making rules and schema. Each making intervention prompts further making rules. Necessary for making in practice, these rules and schema also lead to an explanation of the motions and configurations discoverable for this class of kinematic designs. Combined with explanation embedded in strategic making rules, this process of exploration prompts a cycle of creativity, where new rules and new designs are conceived and constructed.

5.3 Discovering new rules

Further ideas for design changes came from considering these new, strategic schema.

Schema 4 demonstrates that material which lies outside of the hinge-lines appears to play a role in connectivity between the layers. This indicates that material lying inside the hinges might be less essential. Through removing this material, we found that this was indeed the case. Further, considering the resulting model in its intermediate states demonstrates that further portions of material also play no role in essential connectivity. Removing these 'fins' led to a paired down model, which changes its footprint dramatically, during transformations between states. These 'reducing rules' (Schema 6 in Figure 5.15) can also be applied to other model variants, to produce a new subset of designs which 'expand' and 'collapse' between states. Models are shown in Figure 5.16.

Notice that the 'deployable' property of these designs is reminiscent of the reduced kaleidocycle design of model 3, in Chapter 4. For the designs here too, it is found that a model instance in its 'collapsed' state (Figure 5.16, state 3) fits neatly inside the central void in a second model instance, in its 'expanded' state (Figure 5.16, state 1). Multiple instances of these reduced designs can therefore be combined together, without compromising the motions of this assembly, to create a 'maximal' design. The type 3 division schema, for dividing linkages through their kinematic connections to create sets of working, stackable linkages, seems to also apply to this design.

Whilst the original design was composed of 2 full sheets, these 'reduced' designs appear rather to be composed of triangular or rectangular strips. This suggests that, rather than applying the original rule sequence and then removing much of the material, they could instead be constructed from their component parts (i.e. triangles and rectangles) in an alternate, additive way.

Schema 6

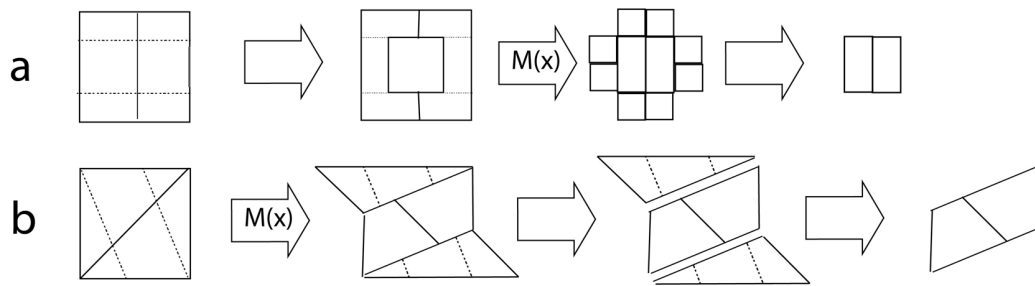


Figure 5.15 Removing material to create 'reduced' designs

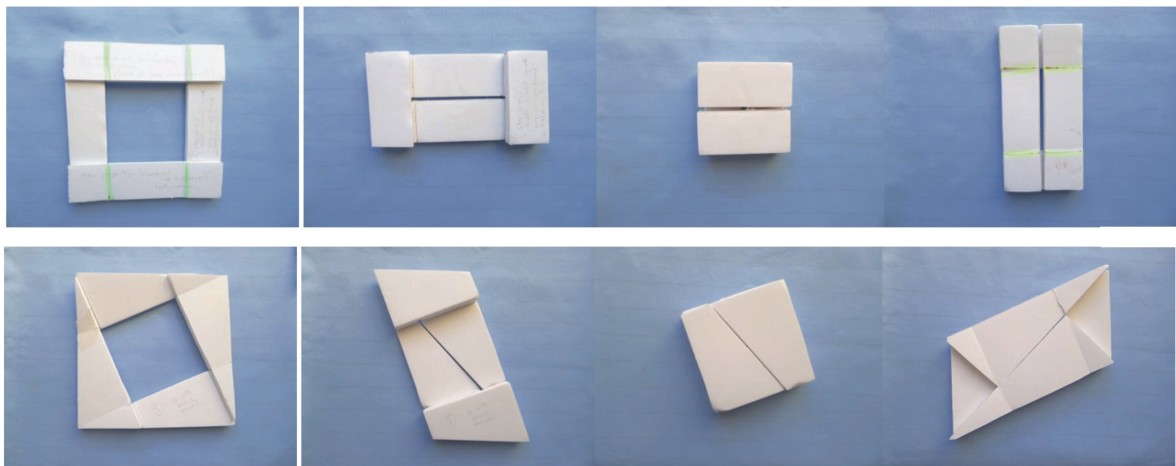


Figure 5.16 Models of the two 'reduced' designs in Figure 5.15, shown in states 1-4

Schema 7

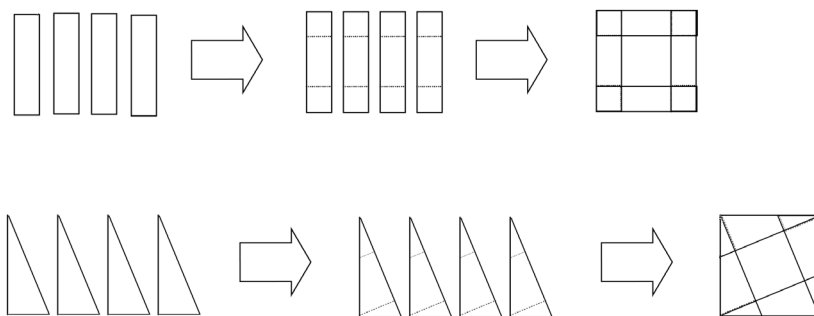


Figure 5.17 Rules to assemble wholes from parts

5.4 Modelling motions

The original design with which we began possessed unusual and distinctive full cycle motions. The presence or absence, of these full-cycle sequences in new design models, could be therefore be readily evaluated through interaction. In our example, ‘unsuccessful’ designs were those where full-cycle motion sequences were no longer possible. The binary nature of this test makes it far easier to apply, than detection of the subtler effects of design changes on more complex, or less-constrained, kinematic behaviours.

Direct techniques and language for describing detailed motions in designs are not commonly available, and description in this chapter focused primarily on shape and structure. But for physical models which embody allowable design variations, practised sequences of physical manipulations provided a way to elicit expected motions. This manipulation of physical models, to examine motions, was at first tactical and exploratory in nature, but a practised sequence of manipulations later became familiar and routine.

The importance of tangible models, for affording opportunities and ideas for new transformations and modifications, also became apparent: these new directions could not have been readily conceived through considering abstract descriptions alone. Further, our collection of experimental physical models also seems to play a useful role in communicating. As tangible representations of motions, they provide an expedient route for disseminating the explanations of relationships between shape, structure and motion, uncovered by our generative, material approach.

This example highlights the importance of physical interaction for the manipulation of kinematic designs, as well as the presence of physical models themselves, for direct examination. Manipulation develops an understanding of motions, tests the success of design modifications, and itself plays a role within the synthetic steps of the making process.

Exploring a kinematic space and explaining using models that don't work

This detail-rich activity of situated making might be studied via a range of methods. The systematic, rule-based approach to exploration employed here appears to yield interesting insights, into aspects of the underlying design spaces. The exercise set out to determine which aspects of the original artefact could be varied freely, and conversely which must be preserved. This led us to discover strategic generative schema for making, helping to *explain* which aspects of shape and structure found in the original design are important for preserving its motions within new model variations.

When employing this systematic, synthetic approach to exploration, an ability to anticipate or predict the effects of changes became less essential. Where there is limited intuition about the possible results of a design change, most was learnt from making, and direct experience of its effects. Unsuccessful models also appear to contribute significantly to explanation, since they identify where the boundaries which contain valid designs have been exceeded.

Significantly, models which embodied unsuccessful changes proved far more useful for positively identifying essential relationships, than successful models. Usually seen as 'mistakes', the unsuccessful results of design exploration actually appear to play a critical role in both identifying and explaining essential relationships. Alterations which affected the spatial relationships between hinge-lines were discovered to exceed allowable limits for variations. This helped the precise boundaries within which motions are preserved to be positively identified, allowing reliable schema to be developed.

Discoveries from the experiments of the previous chapter, concerning fundamental relationships between shape and structure for kinematic designs, appear to be encountered and reinforced in this detailed example. Fundamental alterations to the structure of connections within designs led generally to wider-reaching affects, than changes which altered only the shapes of the kinematic parts.

Changing descriptions

How descriptions are developed when observing designs appears to be critical in determining how exploration proceeds. Various descriptions of the original object helped to derive making rules, and to motivate ideas for applying changes to initial rule sequences. Additionally, direct consideration of new models helps to recognise opportunities for alternative descriptions, with new shapes and making rules. This motivates proposals for variation, leading to new making schema. For a given description, it may be possible to perform an exhaustive search, to derive an associated generative description that explains essential relationships between shape and structure. However, translating to new ways of describing repeatedly opens up further opportunities for alteration and exploration.

Tactics to strategies

Physical manipulations and associated exploratory making are a situated set of actions and evaluations. Situated behaviours in other fields have helped to provide a framework for considering the tactical manner in which appropriate manipulation sequences are discovered, through material interaction. Because anticipating the effects of changes to shape and structure is difficult in kinematic design, tactical exploration assumes a particularly important role.

Exploration, where the effects of actions are initially not predictable, seems to generate design knowledge, which can then inform a more strategic approach. There appears to be a spectrum between experimental, tactical explorations, and fully informed and reliable, pre-planned processes, which can operate deterministically without feedback. Tactics, since they are situated and respond flexibly, may be more difficult to formally describe than strategies. Within situated practice, their capacity for variation is potentially unlimited. In the example described in this chapter, opportunities and ideas for new design variations were recognised through a range of descriptions.

New variations of making rule sequences were at first tested experimentally, but if found successful could then be repeatedly deployed. The tactics for varying spatial relations within

making rules can be described by general schema for the translation, rotation, and parametric variation of shape elements. These constitute the (type 2) general transformation schema. Further types of making schema are encountered in this example. Those acting on connecting elements, such as joints or hinge-lines, are especially important in kinematic design, and should also be included in a toolkit of schema for design.

We initially speculated that these making activities may be tactical and situated in nature, and therefore difficult to specify in advance. These activities had in common a need to observe and respond to material aspects of the design situation. However, the investigations reported here have led to a distinction between tactical schema and situated rules.

Our tactical schema for design exploration responded to opportunities to vary elements of the existing design description embodied in our initial making rules. They were considered first as abstract ideas for variation, which were then translated into specific rules, as appropriate in the situation. Varying one making rule can also have implications for the other rules in the sequence. Crucially however, the effects of these variations on motions is not generally predictable in advance. In contrast, situated rules appear to have reliable results, but whose actions include detailed situated knowledge which is difficult to specify within an abstract format. Schema 5 above (Figure 5.11), which combines making rules in sequences with direct manipulations, seems difficult to describe formally. This is because techniques for describing the configurational transformations that manipulations afford are not well-developed. However, once these motions are understood through manipulation, shape rules prescribing variable spatial relationships might accurately specify these motions as distinct steps within the making process. In the next chapter, we shall develop possible descriptions of these variable spatial relations.

However, material computation via physical model-making alternatively affords a reliable, situated approach to formal rule-based descriptions. More generally, relying at some level on situated knowledge, to instantiate schema into rules through material action and interaction, provides an expedient route to constructing designs, which negates the need to formally encode potentially limitless contextual information.

5.5 Conclusions

This exercise in exploratory making goes some way towards demonstrating a possible role of a generative approach, in developing understanding of the relationships between shape, structure, and motion in kinematic designs. For the case of the single design considered here, practical design reasoning using exploratory model-making successfully derived sets of making schema that constructed larger sets of designs with similar properties. Further, these schema also possess an explanatory function, illuminating how or why the original design works. Formal synthetic descriptions of classes of designs through making rules encode design knowledge in a useful way, enabling others to access the knowledge about designs which is developed through exploration.

Significantly, models that embodied unsuccessful changes were often more useful for positively identifying essential relationships than successful models. Usually seen as abortive experiments, these unsuccessful results of design exploration played a critical role in both identifying and explaining essential relationships. In our study, alterations that affected spatial relationships between hinge-lines were discovered to exceed allowable limits for variations. This discovery assisted the positive recognition of boundaries within which motions are preserved, thereby allowing reliable generative schema to be developed. These schema, in turn, helped to explain which aspects of shape and structure found in the original design were important for preserving its motions in new design variations.

We suggest that using these making schema, to communicate the results design exploration, presents opportunities to 'learn from the mistakes of others'. However, to date, collections of experimental physical models themselves appear to be the most expedient route to disseminating explanations about relationships between shape, structure and motion. Digital fabrication of collections of model variations could support wider sharing in this manner. However, some of the understanding developed through model-making may derive not from the models themselves, but from the reasoning processes and rules underlying exploration activity. The time-based nature of exploratory making allows active anticipation of success or failure, for each physical change to a model variation, which may also play a role in developing a working understanding of designs.

For designs with moving parts, physical manipulation of both completed and partial designs is an inherent part of making. Since designs have limited relative motions, exploration or experimentation may be necessary to discover how best to manipulate a design through its full range of motions.

Formally describing how tactical, exploratory manipulation helps to develop an appreciation of motions within design reasoning remains problematic, and direct examination of situated material practice remains an effective route to this appreciation. General techniques for describing this direct experience of interacting with kinematic designs are not generally available. However, these could be useful for both recognising and categorizing types of designs. Formal approaches to describing motions could also support their more precise description, which could perhaps assist in the protection of intellectual property concerning a design's motions, which currently relies on descriptions of shape and structure to indirectly specify behaviours. The method outlined, in the single example examined in this chapter, offers a starting point for analysis of other designs, and the generality of the method means that the approach can be applied and tested widely. We expect that the method is transferable most easily to other types of kinematic design with closed-loop linkages, where the overall motion of the design is highly constrained.

Chapter 6 Variable spatial relations in kinematic design

Designing kinematic mechanisms can be a challenging problem, because their underlying motions are typically not incorporated intuitively within common techniques for design representation.

In previous chapters, we have seen how physical models provide an expedient route for affording a direct appreciation of these motions, through physical interaction and manipulation. In Chapter 5, where models are constrained by the variable spatial relations of their kinematic connections, interactions to manipulate models through their motion states are essential not only for experiencing motions, but also as an integral part of the situated making actions needed to fabricate models themselves. In this chapter, we consider how formal descriptions for the motions of kinematic designs might be developed for use within a shape-based generative method.

Kinematic grammars build upon the shape grammar (Stiny, 2006) and making grammar (Knight & Stiny, 2015; Krstic, 2019) formalisms, to afford a more intuitive, visual approach. Instead of static shape algebras, these grammars operate upon kinematic shapes: these are composed of multiple pre-defined shapes or parts, between which variable spatial relationships afford relative motions.

This chapter introduces how variable spatial relations can be formally described through shared shape elements; and how kinematic shape rules can both generate and explore motions in mechanisms.

The shape grammar formalism readily affords the description of visual exploration within design compositions. It supports reinterpretation, and the recognition of emergent forms. The computational mechanism of shape rules employs abstract shapes which model the pictorial representations used during design activities (Stiny, 2006). Shape grammars have also been applied to describe and support creative design processes (Prats et al., 2009).

Shape grammars have been extended to making grammars (Knight & Stiny, 2015), where the aim is to formally describe the physical manipulations of materials and objects, in order to represent making processes encountered within arts, crafts, and manufacturing. Consideration of ‘things’ made from ‘stuff’ introduces new constraints, to ensure that shapes mimic the behaviour of physical objects in physical space– for example to take account of collisions (Krstic, 2019). This chapter is concerned with a subclass of physical objects; mechanisms with moving parts (Harrison et al., 2015). It explores the constraints that arise when shapes are used to represent and explore mechanisms in kinematic grammars.

A variety of well-proven methods exist for designing mechanisms, e.g. (Tsai, 2000), but the underlying kinematics involved are typically not incorporated intuitively into common techniques for design representation. In some instances, linked static representations (such as series of images) may communicate the combined effects of the possible motions of parts within a design. Alternatively, physical or virtual models can be used to test motion– through either simulation or material interaction. But in general, exploration depends on a designer’s ability to apply understanding of potential motions between parts, to independently predict and model (mentally or otherwise) their combined effects within a designed object.

Building on shape grammars, kinematic grammars aim to provide a formalism which will enable a visually intuitive approach for modelling and exploring the types of mechanisms described in Chapters 4 and 5. In abstract terms, the motion of mechanisms can be modelled according to connected objects that move relative to each other. Consequently, kinematic grammars incorporate shapes with explicit parts, connected together through variable spatial relations. These kinematic grammars are introduced with reference to a specific class of mechanisms with lower kinematic pairs (Reuleaux, 1876). Section 6.1 reviews the lower kinematic pairs (previously described in Chapter 2); Section 6.2 considers the concept of kinematic shapes as models of physical mechanisms, such as those in Chapters 4 and 5; Section 6.3 examines kinematic shapes as physical entities- as things made of stuff. Section 6.4 explores types of rule in kinematic grammars; and, Section 6.5 discusses variable spatial relations as the basis of rules in kinematic grammars.

6.1 Mechanisms in Motion

At its most basic, the design of a mechanism can be described according to combinations of the relative motions of connected parts (Reuleaux, 1876). The pairs of parts that give rise to motions are often referred to as kinematic pairs. These are subject to certain spatial conditions. Firstly, one of the parts within the pair must be fixed with respect to the local spatial neighbourhood. Since the motion between parts is relative, which of the two parts is considered fixed is of no consequence, and temporally, that part needs to be fixed only for the duration of the motion. Secondly, the geometry of the two parts must restrict their relative motion in some way. As a result of this, the fixed part determines an envelope of motion for the moving part. In order to ensure motion, the shared geometry of the connected parts must have the same curvature. This means that their shared geometry must either be a point, or have constant curvature; i.e. be either rectilinear, circular, or a helical combination.

Kinematic pairs are classified in various ways: according to types of connection- i.e. surface, line or point contact; according to type of relative motion- e.g. sliding or rolling; or according to the type of constraint applied to the pair- e.g. mechanical or due to gravity. Here, the focus is on a particular classification of kinematic pairs, referred to as lower pairs. Identified according to a surface connection, these are differentiated from higher pairs, where connection is rather a point or a line— e.g. the connection between a cam and its follower. In total, there are six lower kinematic pairs, as illustrated in Figure 6.1. These lower pairs enumerate spatial restrictions on motion, resulting in pairs of parts with relative motions of varying degrees of freedom (DoF):

- **Prismatic pair (slider)**: the axes of the two parts are aligned, allowing translation along the axes and no rotation. This results in one DoF. (Figure 6.1i)
- **Revolute pair (hinged joint)**: the axes of the two parts are aligned, allowing rotation about the axes and no translation. This results in one DoF. (Figure 6.1ii)
- **Screw pair**: the axes of the two parts are aligned, allowing a combination of translation and rotation relative to the axes. This results in one DoF. (Figure 6.1iii)

- **Cylindrical pair:** the axes of the two parts are aligned, allowing independent translation and rotation relative to the axes. This results in two DoF. (Figure 6.1iv)
- **Spherical pair (ball joint):** the spherical centres of the two parts are aligned, allowing rotation about three axes and no translation. This results in three DoF. (Figure 6.1v)
- **Planar pair:** the surfaces of the two parts are in contact, allowing translation in two directions and rotation about one axis, perpendicular to the surfaces in contact. This results in three DoF. (Figure 6.1vi)

In the design of a mechanism, kinematic pairs can be combined in chains to create models with complicated motions (Tsai, 2000). The connections within these chain models are often abstracted as graphs or hypergraphs of links and nodes. These can be used to determine the potential motion of a mechanism, based on connections, but without any consideration of their geometry (Berge, 1973). Consequently, when a design is realised as a physical model, complications can arise when material geometry interacts or collides, during the motion of parts.

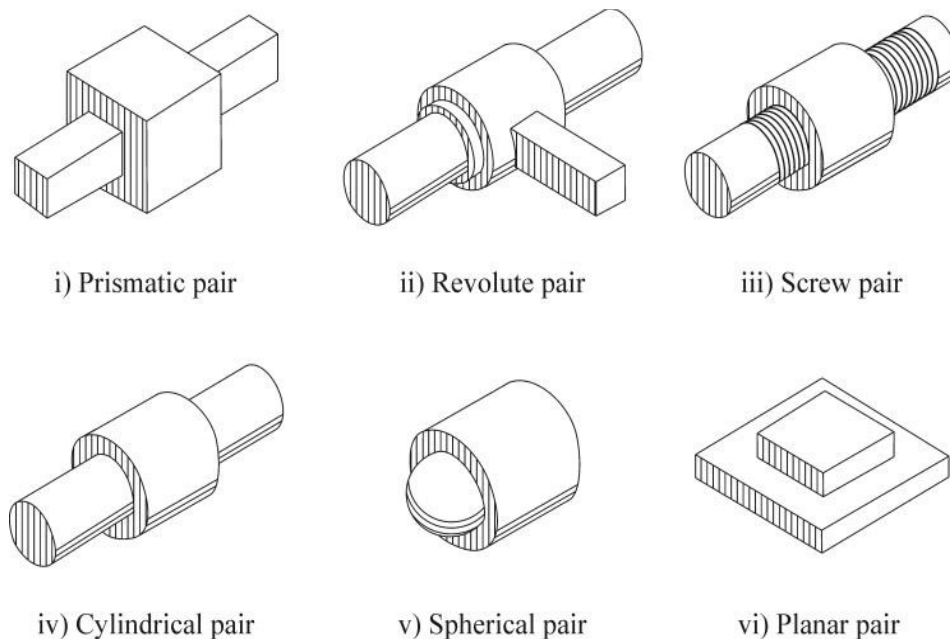


Figure 6.1 Examples of the six lower kinematic pairs

The spatial nature of kinematic pairs implies that mechanisms can be readily described as shapes in shape computations, and there are certain benefits in doing so. Shapes can provide a model of a mechanism that includes geometry as well as the connections of parts, whilst retaining a level of abstraction that can support creative exploration. Designers are primarily concerned with modelling physically realisable designs. However, real motions are not necessarily easy to describe, using shape computation. Conversely, more abstract notions of motion, which could not be achieved in the physical world, can give interesting results when modelled virtually, and therefore should not be precluded from investigation. Chapter 5 demonstrated that shape computations can be used to design and explore mechanisms in a designer-friendly way which is visually intuitive. In this chapter, kinematic shapes, describing lower kinematic pairs, are used to model mechanisms and their motion.

6.2 Kinematic Shapes

In a shape grammar, shape rules are used to generate designs through consideration of shapes and the spatial relations between shapes and/or parts of shapes (Stiny, 2006). Together, any two shapes (or parts of a shape) define a spatial relation. For example, all of the shape compositions in Figure 6.2 are composed of the same three parts: a small square, a larger square, and a point located at their shared vertex. But because of the distinct spatial relation between the two squares, each of these eight instances is different. Shape grammars often make use of such relations, through applications of shape rules which produce repetition of form and arrangement to generate visually cohesive patterns, or designs consistent with a particular style (Prats et al., 2006).

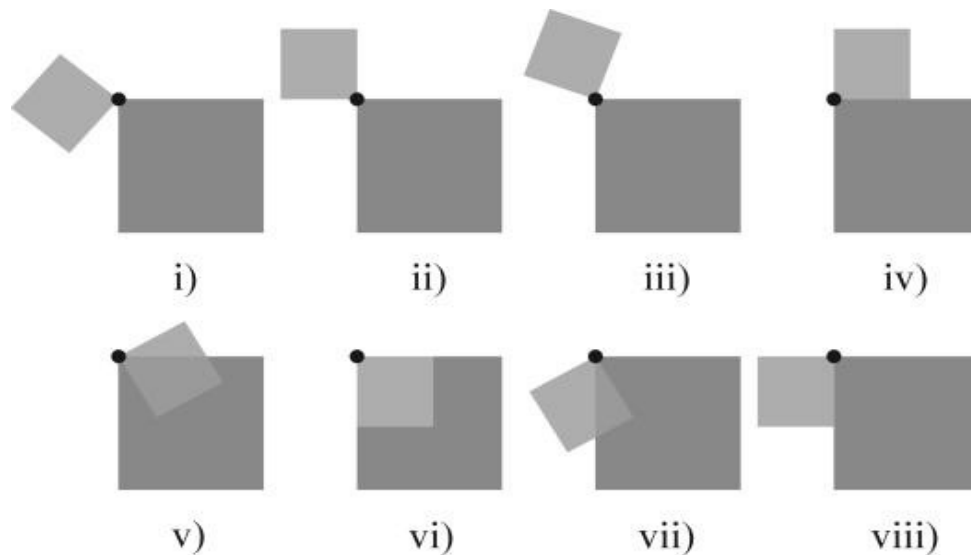


Figure 6.2 Examples of spatial relations

The spatial relations used in a shape grammar are typically fixed or, for parametric shape grammars, are instantiated during the application of a shape rule. Spatial relations within shape compositions can be changed via the application of shape rules. However, these rules cannot accurately describe the behaviour of mechanisms, such as the lower pairs illustrated in Figure 6.1, where spatial relations between parts change continuously according to their motion. Therefore, to support formal exploration of mechanisms via shape computation, it is necessary to consider their motions according to *variable spatial relations (VSRs)* between parts— i.e. the continuously changing relations between parts that are in motion.

In kinematic design, VSRs result from the well-defined motions of parts, and a *kinematic shape* is a shape which includes one or more VSRs between its parts. For example, in the eight different shapes in Figure 6.2, the spatial relations between the small and large squares vary according to the rotation of the small square about the point. These eight shapes can be recognised as instantiations of a single kinematic shape, in which there is a VSR between the two squares defined according to a rotation of the small square. This motion is not a consequence of transformations realised during the application of shape rules— it is instead an implicit property of the kinematic shape. When considered as kinematic shapes, all eight of the shapes in Figure 6.2 are equivalent, and comparison with Figure 6.1 reveals that they are a 2d equivalent of a revolute pair (Figure 6.1ii). This example

highlights the key features of kinematic shapes; they include connected parts that are in relative motion.

Shapes in Motion

Shape algebras (Stiny, 2006) provide a framework suitable for exploring the motions of shapes, as summarised in Table 6.1. The algebras are denoted U_{ij} , where i is the dimension of the shape elements used to construct a shape, j is the dimension of the embedding space, and $i \leq j$. Motion of a shape is defined according to a reference shape, which is a shape element of dimension k , where $k < j$. The lower dimensional embedding spaces, defined by points and lines, are more restrictive with respect to motion than the higher dimensional spaces of planes and volumes.

Table 6.1 Shape motion in algebras U_{ij}

Algebra	Space	Motion	Reference	DoF
U_{00}	Point	-	-	-
U_{i1}	Line	Translation	Point	1
U_{i2}	Plane	Rotation	Point	1
		Translation	Point	2
		Translation	Line	1
U_{i3}	Volume	Rotation	Point	3
		Rotation	Line	1
		Translation	Point	3
		Translation	Line	1
		Translation	Plane	2

In the algebra U_{00} , the embedding space is a single point, and no motion is possible. While in algebras U_{i1} , the embedding space is a straight line, shapes are composed of points or lines, and the only possible motion is translation, with one degree of freedom (DoF), which is defined relative to a point. Algebras U_{i2} are familiar to designers who work with sketches to develop design concepts. Their embedding space is a plane, shapes are composed of points,

lines or planes, and motion is composed of rotations and translations. Rotation is defined relative to a point, with one DoF, and translation is defined either relative to a point, with two DoFs, or relative to a line with one DoF.

Algebras U_3 are analogous to physical space, or the 3d space within a CAD system. Here the embedding space is a volume, shapes are composed of volumes, planes, lines or points and, as with U_{i2} , motion is composed of translations and rotations. Rotation is defined relative either to a point, with three DoFs, or to a line, with one DoF, while translation is defined either relative to a point, with three DoFs, or relative to a plane, with two DoFs. As an example, Figure 6.3 illustrates moving shapes in the algebra U_{22} , where 2d planar shapes are arranged in a plane. Representing motion within a static image can be difficult, and Figure 6.3 adopts a convention of using arrows, to indicate the motion of the squares. In Figure 6.3i, a square is rotated around a reference point; in Figure 6.3ii, a square is translated relative to a reference point; and in Figure 6.3iii a square is translated relative to a reference line.

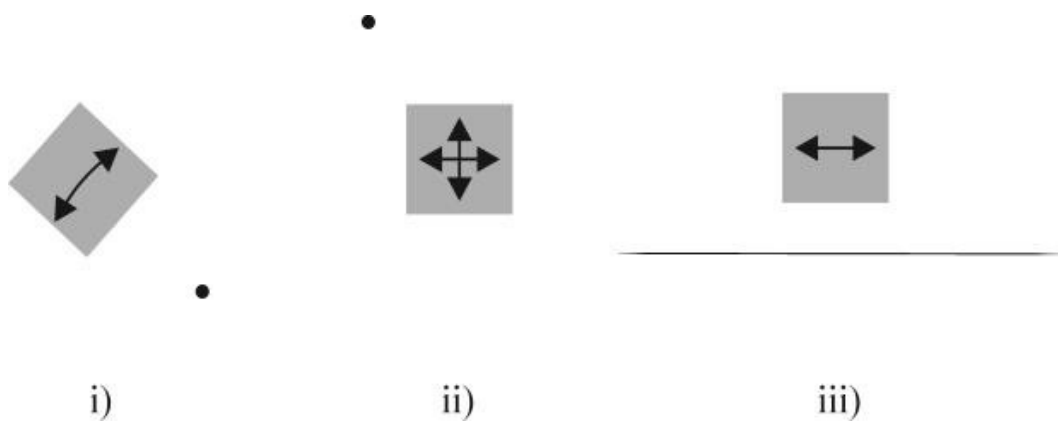


Figure 6.3 Moving shapes in U_{22}

By considering the relative motions and VSRs of connected shapes, shapes composed of connected moving parts- i.e. kinematic shapes- can be formalised in algebras U_{ij} . The simplest kinematic shapes are described by a triple of shapes, $\{s, \alpha, e\}$, where s represents a static part, α represents a moving part, and e represents a reference shape, as enumerated

in Table 6.1. The existence of multiple parts, which move relative to one another in a partially constrained way, distinguishes the kinematic shape from a set of unconnected shapes in motion. For a kinematic shape in motion, the motion of α , the moving part, is not defined relative to s , the static part, but rather relative to a distinct reference shape e . For example, the kinematic shape in Figure 6.2 is composed of two squares and a third reference shape. The rotation of the small square is not defined relative to the large square, but instead defined relative to the reference point, which acts as the reference shape e . In general, the motion of a moving part α is defined relative to e , the reference shape, which is a shape element, of dimension k , in an algebra U_{ij} , $k < j$. The VSR therefore defines the spatial relationship between α and e , and $VSR(\alpha, e)$ is a shape given by an instantiation of the motion of α relative to e . A simple kinematic shape is therefore given by $s + VSR(\alpha, e)$. For connected kinematic shapes, such as the shape illustrated in Figure 6.2, the VSR can be determined by considering the connectivity of the parts s and α . (Conversely, where the VSR between two connected shapes is known, its reference shape may be deduced.)

Shapes with connected parts

Shapes are connected when they touch, and a shape is said to be a *connected shape* when each part touches some other part (Stiny, 2006). For example, Figure 6.4 illustrates different connected shapes in U_{22} , composed of two squares, labelled x and y . In Figure 6.4i, the two squares are connected because x is a sub-shape of y ; in Figure 6.4ii, they are connected because they overlap; and in Figures 6.4iii-vi they are connected because they touch, either at their edges or at their vertices.

Shape connectivity can be defined in terms of the recursive embedding relation applied to parts, boundaries of parts, boundaries of the boundaries of parts, etc. (Stiny, 2006). The boundary of a shape in an algebra U_{ij} is a shape in an algebra $U_{(i-1)j}$, and the operator $b^i(S)$ formalises this recursive relation between boundaries b , and shapes S , with integer $i \geq 0$ and $b^0(S) = S$. For example, a shape S in U_{33} is composed of volume shape elements and has a boundary $b(S)$ composed of planes in U_{23} . This in turn has a boundary $b^2(S)$ composed of lines in U_{13} , which in turn has a boundary $b^3(S)$ composed of points in U_{13} . For all the

connected shapes in Figure 6.4, x and y contain parts that share a boundary (an edge), or a boundary of a boundary (a vertex).

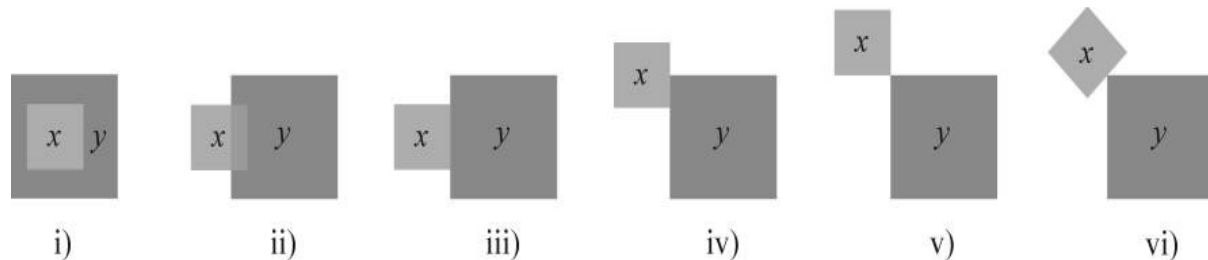


Figure 6.4 Examples of connected shapes in U_{22}

Generalising this example, using the boundary operator and sub-shape relation \leq , two shapes, x and y , can be defined as connected if there are shapes z and z' such that $z \leq b^i(x)$ and $z' \leq b^j(y)$, and $b^k(z).b^l(z')$ is not the empty shape, with integers $i, j, k, l \geq 0$. This definition can be applied to the connected shapes in U_{22} illustrated in Figure 6.4 as follows, although there may be multiple possible choices of z and z' in each case:

- in Figure 6.4i, x is embedded in y ; z and z' are both in U_{22} , $z \leq x$ and $z' \leq y$ so that $z.z'$ is not the empty shape
- in Figure 6.4ii, x and y overlap; z and z' are both in U_{22} , $z \leq x$ and $z' \leq y$ so that $z.z'$ is not the empty shape
- in Figure 6.4iii, x and y share part of their boundary; z and z' are both in U_{12} , $z \leq b(x)$ and $z' \leq b(y)$ so that $z.z'$ is not the empty shape
- in Figure 6.4iv, x and y share part of their boundary; z and z' are both in U_{12} , $z \leq b(x)$ and $z' \leq b(y)$ so that $z.z'$ is not the empty shape
- in Figure 6.4v, x and y share a vertex; z and z' are both in U_{02} , $z \leq b^2(x)$ and $z' \leq b^2(y)$ so that $z.z'$ is not the empty shape
- in Figure 6.4vi, an edge of x touches a vertex of y ; z is in U_{12} , z' is in U_{01} , $z \leq b(x)$ and $z' \leq b^2(y)$ so that $b(z).b^0(z')$ is not the empty shape.

This intuitive definition of shape connectivity captures the idea that shapes are connected if their parts touch. It also applies to shapes in composite algebras, which are composed of spatial elements of different dimensions.

In shape grammars, the connectivity between parts of a shape is temporary and changing, depending on the application of rules that dynamically alter the structure of a shape. In kinematic shapes, the connectivity between parts also changes, but not according to rule applications— instead it varies according to different instantiations of the VSR, given by $VSR(\alpha, e)$. For example in Figure 6.2, as the small square rotates about the point, the connectivity of the two squares changes: in Figure 6.2iii, the two squares are connected due to the shared vertex; but in Figures 6.2iv & viii, they are connected due to a shared boundary; while in Figures 6.2v & vii, they are connected due to a shared part; and in Figure 6.2vi, they are connected because the small square is embedded in the large square. If retained, these various connections then have implications with respect to the potential motion of the small square, as illustrated in Figure 6.5.

In these examples, the connectivity of the small and large squares is explicitly identified by the connecting shape elements (drawn in black), and arrows are used to indicate the motion of the small square according to the VSR. Figure 6.5i combines all the shapes from Figure 6.2 within a single kinematic shape; the two squares are connected at a shared vertex, and the VSR is defined by the rotation of the small square about this point. This kinematic shape is a U_{22} equivalent of a revolute pair (Figure 6.1ii). In Figure 6.5ii, the two squares are connected at a shared edge, and the VSR is defined by the horizontal translation of the small square parallel to this edge. The kinematic shape is a U_{22} equivalent of a planar pair (Figure 6.1vi). In Figure 6.5iii, the two squares overlap, and are connected by a shared sub-shape. Consequently, they are locked in position and the small square cannot move. In these three examples, the spatial relations of the two squares are instantiations of the kinematic shape illustrated in Figure 6.2, but different interpretations of their connectivity within that kinematic shape give rise to different possible motions. Ambiguity about how connectivity of shape is interpreted can be reduced, by explicitly including the connecting shape element as part of the shape (for example, these are drawn in black in Figure 6.5).

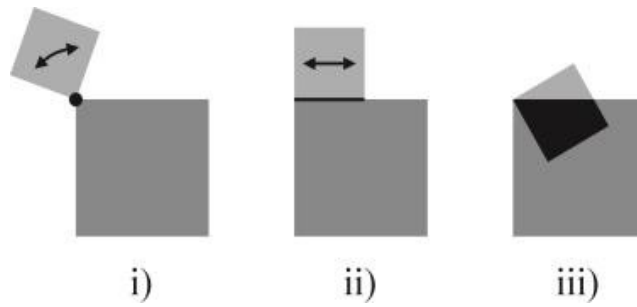


Figure 6.5 Motion of connected shapes

For these examples, the connecting elements also act as a reference shape, defining the motion of the small square. This approach is potentially of benefit, since when the connecting shape element and the reference shape are different, a restriction of potential motion can result. This is illustrated in Figure 6.6: in 6.6i, rotation of the small square about the reference point identified at its centre is restricted, due to the connectivity of the two squares (as specified by the black line on the shared boundary). The connectivity of these two squares is such that only vertical translation of the small square is possible— as illustrated in Figure 6.6ii, where the connecting shape is also the reference shape. Alternatively, if a rotating part is required, this connectivity issue can be resolved by changing the geometry of the parts— as illustrated in Figure 6.6iii, where the small square is replaced with a circle. Since the boundary of the circle is invariant under rotation, the specified motion is no longer restricted by the connectivity of the parts. The kinematic shape in Figure 6.6iii is a U_{22} equivalent of a spherical pair (Figure 6.1v). Its connecting shape element also adequately defines the VSR. However, in more abstract compositions where moving shapes remain spatially unconnected, reference shapes may still be required, to specify the VSRs between otherwise unconnected shapes.

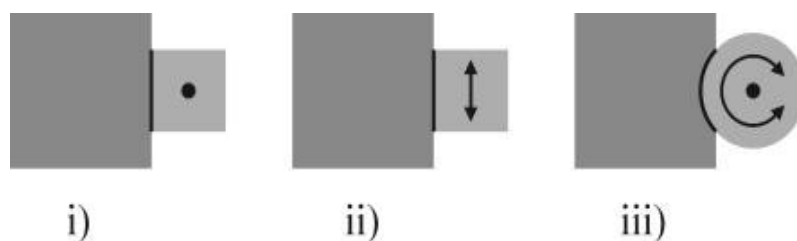


Figure 6.6 Exploring the consequences of shape connectivity

This explicit inclusion of connecting shapes allows kinematic shapes to behave similarly to physical objects- whose motion is defined not according to abstract concepts such as reference shapes, but rather according to material interactions between parts. This gives rise to some conflict, however, between the behaviour expected of the shapes in a shape grammar, and the expected behaviour of a mechanism. For example in Figure 6.2; as the small square rotates, it overlaps the large square, so that there are parts of both occupying the same region of the embedding space. Since shape grammars are a visual formalism, when this situation arises it is common for overlapping shapes to merge, forming a single shape element. For physical mechanisms however, it not possible for material parts to occupy the same region of physical space, and connected parts in motion must instead remain distinct. However, by recognising that physical mechanisms behave as things made of material stuff- which can modelled within making grammars- this issue can be resolved.

6.3 Kinematic shapes as things made of stuff

Making grammars (Knight & Stiny, 2015; Krstic, 2019) apply the computational framework of shape grammars to physical objects, to model processes of making that take place in the arts, crafts, and manufacturing. To support this, shape algebras are extended to include spatial or material *stuff*, which is the composite matter of physical *things*. Making grammars incorporate actions applied to stuff as consequences of both active doing, by manipulating physical materials, and also of sensing their properties- for instance by seeing or touching. How these grammars and their schema work for sensing, as well as making, will be examined further next in Chapter 7. Examples include knotting of strings in Incan khipu, and painting with watercolours (Knight & Stiny, 2015). However, as discussed in the previous section 6.1, physical objects exhibit different behaviours to shapes, and these must be taken into consideration when exploring how computations for making grammars might work within shape algebras. Krstic (2019) identifies several key factors that distinguish the things and stuff described by making grammars from the shapes and parts used in shape grammars. These concern the equivalence of representations within different algebras, and also the treatment of boundaries.

Stuff and things are by definition three-dimensional, and are therefore most naturally represented by shapes in algebras U_{33} , where volumes are arranged in three-dimensional space. For the purposes of illustration, it is also useful to consider their two-dimensional equivalents in algebras U_{22} , where planar shapes are arranged in 2d space. In design, it is common to use boundaries as a representation for a shapes– for example in 3d modelling using CAD, surface models in U_2 are common representations for U_{33} objects. However, representing the making of material things in an algebra U_{ij} , where $i < j$, can give rise to conceptual inconsistencies. This is illustrated in Figure 6.7, where a shape rule $a \rightarrow b$ is represented in three different algebras, which are visually similar but conceptually distinct.

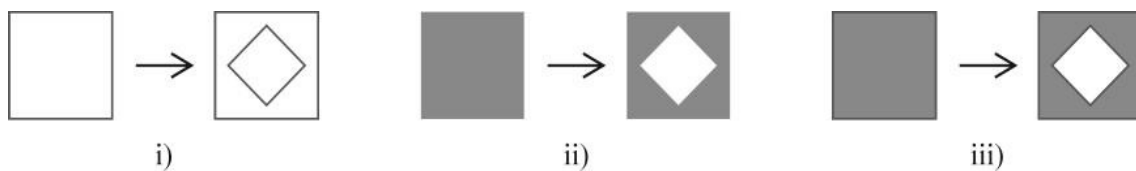


Figure 6.7 Examples of shape rules in **i) U_{12} , ii) U_{22} and iii) $U_{22} \times U_{12}$**

The rule described in Figure 6.7i, where the shapes a and b are both in U_{12} , is a typical of a shape grammar for design applications. It is an addition rule, with a small square (composed of four lines) added to a larger square identified by the left-hand side of the rule, and the partial order of the shapes is $a < b$. The rule in Figure 6.7ii, where the shapes a and b are both in U_{22} , is rather a subtraction rule, which subtracts a planar square from the larger square identified by the left-hand side of the rule. The partial order of the shapes is $a > b$. This subtractive rule could be from a making grammar– for cutting a hole in a sheet of material, for example. Note that these two rules are related by the boundary function $b(S)$: the shapes in Figure 6.7i describe the boundaries of the shapes in Figure 6.7ii. Because of this, and despite the visual similarity of the two rules, they perform opposite functions; one adds a square, whereas the other subtracts a square (see Krstic, 2019).

This example illustrates that the logic of shape rules for boundaries (e.g. the U_{12} rule in Figure 6.7i) does not reflect the material logic of making (which in this example is better

modelled by the U_{22} rule in Figure 6.7ii). However, shape rules in a U_{ii} algebra (such as U_{22}), where shapes and the embedding space are of the same dimension, are also problematic, since these unconstrained rules can be applied to shapes in U_{ii} in infinitely many ways. Because of this, Krstic (2019) suggests that in making grammars, rules should include both shapes and their boundaries, as illustrated in Figure 6.7iii. This rule includes shapes from the composite algebra $U_{22} \times U_{12}$, and works as expected for a making grammar, because the inclusion of boundaries provides the context to ensure that the rule is applied correctly.

Shapes in U_{ii} algebras, together with their boundaries, form a sub-algebra of $U_{ii} \times U_{(i-1)i}$ denoted UB_i , which contains ordered pairs of both shapes and their boundaries (Krstic, 2001). These algebras are weaker than shape algebras U_{ij} , because they lack Boolean operations of sum, product and difference, and partial order is defined component-wise, for shapes and their boundaries. But they are useful for modelling things in making grammars, because they preserve the boundaries of shapes, which streamlines rule application (Krstic, 2019).

The UB_i algebras are closed under symmetric difference, and this can be used in the application of shape rules in a shape grammar. The symmetric difference of two shapes, x and y , is composed of the parts that are in either of the shapes, but not in their intersection. It is given by:

$$x \oplus y = (x - y) + (y - x), \quad \text{or equivalently,}$$

$$x \oplus y = (x + y) - (x \cdot y).$$

For the boundaries of shapes in U_{ii} algebras, symmetric difference is distributive, so that

$$b(x \oplus y) = b(x) \oplus b(y), \quad \text{where } x \text{ and } y \text{ are in } U_{ii}.$$

For shapes in U_{ij} , it can replace sum, when shapes are disjoint, or difference, when one shape is embedded in the other:

$$x \oplus y = (x + y) \text{ if } (x \cdot y) = 0, \text{ and } x \oplus y = y - x \text{ if } x \leq y. \quad (\text{Earl, 1997})$$

Using symmetric difference, a shape rule $a \rightarrow b$ can be applied to shape c under a transformation t when $a \leq c$ and $[c - t(a)] \cdot t(b) = 0$, to give

$$c' = [c \oplus t(a)] \oplus t(b).$$

(Krstic, 2019)

The sub-shape condition $a \leq c$ ensures that the first instance of the symmetric difference results in a subtraction of $t(a)$ from c , while the discrete condition $[c - t(a)] \cdot t(b) = 0$ ensures that the second instance results in the addition of $t(b)$ to $c - t(a)$. A further condition, on the boundaries of the shapes $b(t(a)) \cdot b(c) \neq 0$, can be applied to provide registration for the transformation t , to restrict the application of rules.

The discrete condition $[c - t(a)] \cdot t(b) = 0$ has the additional benefit of giving shapes the behaviour of physical objects during rule application, by avoiding collisions between parts. It ensures that the shape b on the right-hand side of the rule does not collide with the shape that remains after subtracting the shape a from the left-hand side of the rule. For kinematic shapes, this mechanism for collision protection is useful, and should be applied continuously to moving parts. To achieve this, a VSR condition should also be included, so that in a simple kinematic shape composed of a triple of shapes $\{s, \alpha, e\}$, the static part s and the moving part α should always be discrete, i.e.

$$s \cdot VSR(\alpha, e) = 0.$$

This will ensure that the parts of a kinematic shape do not overlap as a result of its motion.

6.4 Kinematic rules for kinematic grammars

Inclusion of kinematic shapes in shape or making grammars requires a mechanism for distinguishing between the parts of shapes that are in motion, and those that remain static.

For this purpose, one of two opposing philosophical approaches can be adopted.

Conceptually, it can either be assumed that all parts are by default either free to move or actively in motion— or alternatively, that they are all by default fixed and static. For marks or shapes drawn on a sheet of paper, the latter is the general case.

In the first approach, all the parts of a kinematic shape are free to move around the embedding space, except for those parts which are explicitly prescribed as being static. In

the second approach, the parts of a kinematic shape instead are fixed relative to the embedding space, unless they have been explicitly defined as being in motion relative to specified reference shapes. In terms of physical intuition, either approach may be equally valid depending on context: in general, material things tend to be free to move or be moved, unless they become actively constrained in some way; whereas, for mechanism design, it is common to assume that parts are static unless their motion has been actively specified.

For the kinematic rules developed in this chapter, the latter approach has been adopted, and the notation used in the examples identifies parts of a kinematic shape that are actively in motion. Symbolically, their moving parts are represented with a Greek letter, while schematically they are represented using a lighter shade of grey, and labelled with an arrow to indicate the resulting motion. This simple notation is useful for the exploration presented here, but there is perhaps benefit in exploring alternative representations too, for example using colour grammars (Knight, 1989) or weights (Stiny, 1992) to support the further exploration of languages of kinematic designs. MacLachlan's recent research employs these formal devices to define making rules for jewellery. (MacLachlan & Jowers, 2016; MacLachlan, 2018).

In a kinematic grammar, motion can be introduced to a static shape by applying kinematic shape rules which take the form:

$$a \rightarrow b + VSR(\alpha, e).$$

Here, a and b are static shapes in UB_i , and α is a moving shape, also in UB_i . e is a shape element in U_{ki} ($k < i$) and acts as both a connecting shape element for b and α , as well as a reference shape for the motion of α . $VSR(\alpha, e)$ is a UB_i shape given by an instantiation of the motion of α relative to e . Figure 6.8 illustrates an example of a kinematic shape rule (Figure 6.8i) and its application to a static shape (Figure 6.8ii) to produce a kinematic shape (Figure 6.8iii).

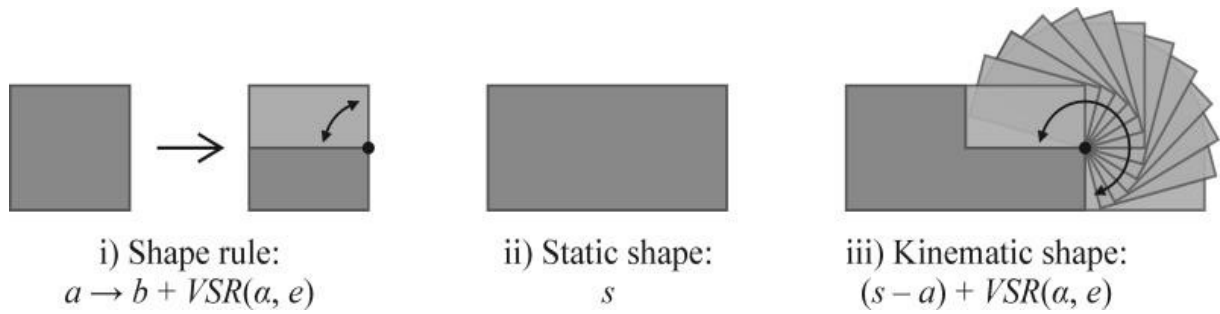


Figure 6.8 Example of a kinematic shape rule and its application to a static shape.

In Figure 6.8iii, the motion of the kinematic shape is illustrated by the inclusion of multiple overlapping instantiations of the moving part, all of which observe the collision protection condition $s \cdot VSR(\alpha, e) = 0$. The resulting kinematic shape is a two-dimensional equivalent of a revolute pair (Figure 6.1ii). In this example, the logic of rule application follows the shape grammar formalism, and since $a \cdot b \neq 0$, the rule proceeds by recognising and replacing the shape $a - b$ with $VSR(\alpha, e)$. Alternatively, if $a \cdot b = 0$, then the rule would proceed by replacing a with b , and adding a moving part.

Kinematic shapes can also be combined into chains, to model mechanisms with more complicated motions, as already presented in Chapters 4 and 5. In a kinematic grammar, this can be achieved by applying kinematic shape rules which take the form:

$$VSR(\alpha, e) \rightarrow VSR(\beta, e) + VSR(\gamma, f)$$

Here, α , β and γ are moving shapes in UB_i . The element e is a shape element in U_{ki} ($k < i$), and is the reference shape for the motion of α and β . The element f is both a connecting shape element for β and γ , and also the reference shape for the motion of β . The variable spatial relations $VSR(\alpha, e)$, $VSR(\beta, e)$ and $VSR(\gamma, f)$ are UB_i shapes, given by instantiations of the motion of the moving shapes α , β and γ relative to e , e and f , respectively. Figure 6.9 illustrates an example of a kinematic shape rule (Figure 6.9i) and its application to the kinematic shape in Figure 6.8iii. In this example, $\alpha \cdot \beta = 0$, and the rule proceeds by adding the second moving part, modelled by $VSR(\gamma, f)$. Alternatively, if $\beta < \alpha$, then $VSR(\alpha, e)$ is replaced with two moving parts, modelled by $VSR(\beta, e)$ and $VSR(\gamma, f)$. As a result of applying

the rule, the kinematic shape in Figure 6.9ii has two parts in motion, both of which rotate about a connecting point. The result is a kinematic shape that is a two-dimensional equivalent of two revolute pairs (see Figure 6.1ii) combined in sequence.

The motion of the kinematic chain is too complicated to be illustrated according to the method used in Figure 6.8. Instead, in Figure 6.9iii it is illustrated according to the envelopes of motion of the two moving parts. These define a sub-space of the embedding space and are represented as shaded regions. For both moving parts, the motion is restricted according to the connectivity of the parts, and also according to the collision protection conditions $s \cdot VSR(\beta, e) = 0$, and $VSR(\beta, e) \cdot VSR(\gamma, f) = 0$; where s is the stationary part of the shape, and β and γ are the moving parts.

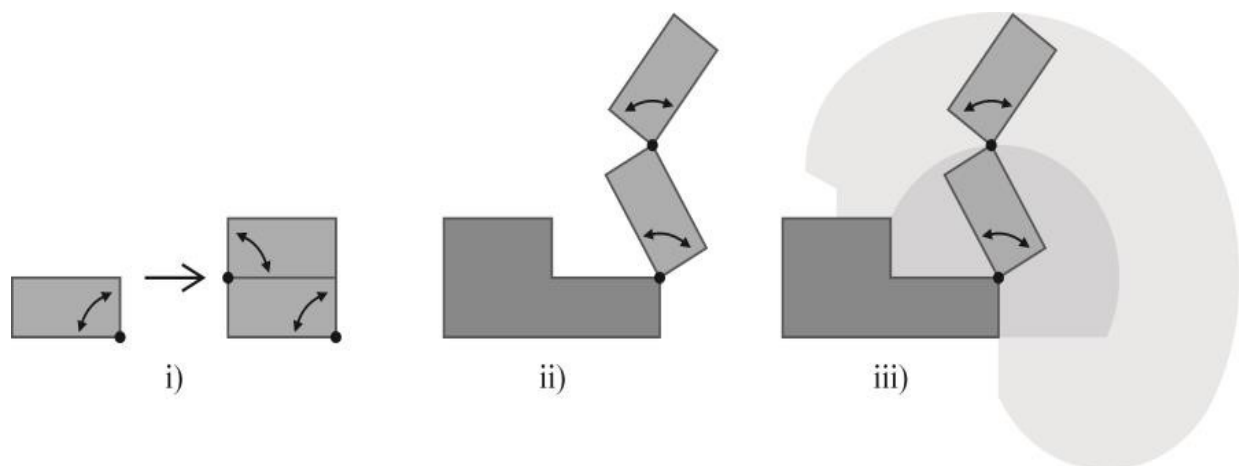


Figure 6.9 Example of a kinematic shape rule and its application to a kinematic shape

Application of kinematic shape rules requires a mechanism for recognising embedded parts of a shape. This is complicated by VSRs, since spatial relations between moving parts are dynamic, and cannot be used to provide registration for determining where shape rules can be applied. For example, the kinematic shape rule illustrated in Figure 6.10i is of the form $a + VSR(\alpha, e) \rightarrow b$, and its application requires recognition of both a static part and a moving part.

Rules of this form can be used to merge connected moving parts in a kinematic shape, and include: static shapes a and b , both in UB_i ; a moving shape α , also in UB_i ; a shape element e in U_{k_i} ($k < i$); and a $VSR(\alpha, e)$, which is a UB_i shape given by an instantiation of the motion of α relative to e . In applying the rule, recognition of a (the static shape on the left-hand side of the rule) follows the logic of rule application from shape grammars, where rule $a \rightarrow b$ applied to a shape c proceeds by first identifying a transformation t such that $t(a)$ is an embedded part of c , $t(a) \leq c$. The rule is then applied, by removing the transformed instance of the shape a , and replacing it with a similarly transformed instance of the shape b . In practice, identification of the transformation t is implemented by considering how distinct elements of a , such as vertices, are transformed, and ensuring that all distinct elements of $t(a)$ are embedded in c (Krishnamurti, 1980).

For the moving shape on the left-hand side of the rule, this approach does not work, because the spatial relation between its distinct elements is changing according to the motion of α relative to e . An alternative approach must therefore be devised for recognising the moving parts of a shape. For this purpose, the invariants of the motion can be employed to identify the VSR between distinct elements, and the reference of motion. For example, to apply the rule in Figure 6.10i to the kinematic shape in Figure 6.10ii requires that the static and moving parts of the shape on the left-hand side of the rule are recognised as parts. Figure 6.10iii illustrates the distinct elements of the shape in Figure 6.10ii that are used to support this recognition. The motion of the moving part is a rotation about the connecting point, and consequently the distance from this point is invariant. In Figure 6.10iii, this is illustrated by dashed lines, which are of equal length for the two instantiations of the motion. These provide enough information to determine how the shape rule can be applied, and the result is the kinematic shape in Figure 6.8ii.

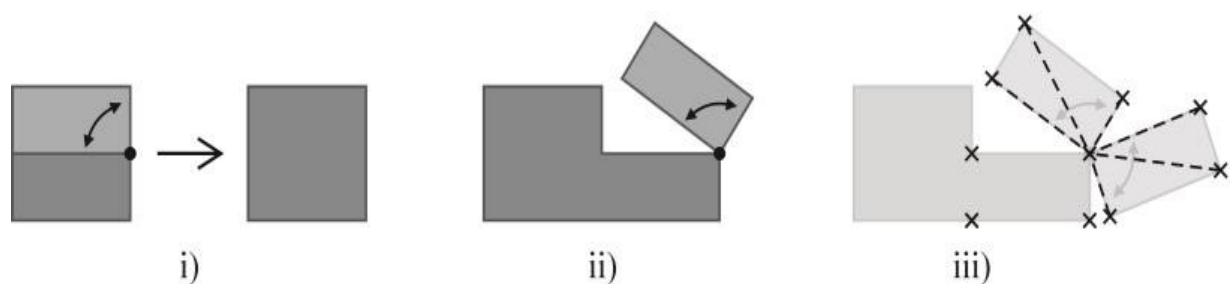


Figure 6.10 Recognising parts of a kinematic shape

6.5 Discussion

This chapter has explored how the shape grammar formalism can be applied to the problem of designing mechanisms with moving parts. With reference to the lower kinematic pairs (Figure 6.1), kinematic shapes were introduced as connected shapes composed of static parts, moving parts, and shape elements used as reference for motion. In essence, each kinematic shape represents an infinite number of static shapes, each of which is given by an instantiation of the moving part. Despite this, it is still possible to recognise kinematic shapes and their moving parts for the purpose of rule application in a kinematic grammar, by considering the invariants of motion.

A variety of kinematic shapes were introduced as illustrations, and these were identified to be two-dimensional equivalents of the revolute pair (Figure 6.5i), the spherical pair (Figure 6.6iii) and the planar pair (Figure 6.6ii). The other lower kinematic pairs- the prismatic pair, the screw pair and the cylindrical pair- do not have formal 2d equivalents, because this would violate the collision condition, which ensures that parts of shapes do not occupy the same region of an embedding space. For visual shapes, examples such as concurrent lines, overlapping planes, or intersecting volumes are common; but as models of mechanisms, kinematic shapes should behave as physical things composed of spatial stuff, and material collisions between moving parts should be avoided.

The prismatic pair, the screw pair, and the cylindrical pair can be represented however as three-dimensional shapes, in an UB_3 algebra, as illustrated in Figure 6.11. For each of these

kinematic shapes, the moving cuboid is connected to the static shape by a shared surface, and the motion of the cuboid is with reference to a line that is parallel to this surface. If the collision condition is adhered to, then motion is defined and constrained both by the reference line and by the geometry of the static and moving parts. As a result: in the prismatic pair, only translation parallel to the line is possible; in the screw pair, only a screw rotation with dependent motions about and parallel to the line is possible; and in the cylindrical pair two independent motions are possible- rotation about the line, and translation parallel to the line.

To be of use in the design of mechanisms, kinematic grammars should give some indication of the resulting behaviour of kinematic shapes- i.e. the extent of the motion of the moving parts. Representing motion in a static image can be difficult. Here, various approaches have been employed to give some insight- including the use of arrows (Figure 6.5), inclusion of multiple instantiations of moving parts (Figure 6.8iii), and inclusion of envelopes of motion of moving parts (Figure 6.9iii). Envelopes of motion are perhaps the most expressive of these, and as regions of space, can themselves be analysed and modelled using shape arithmetic.

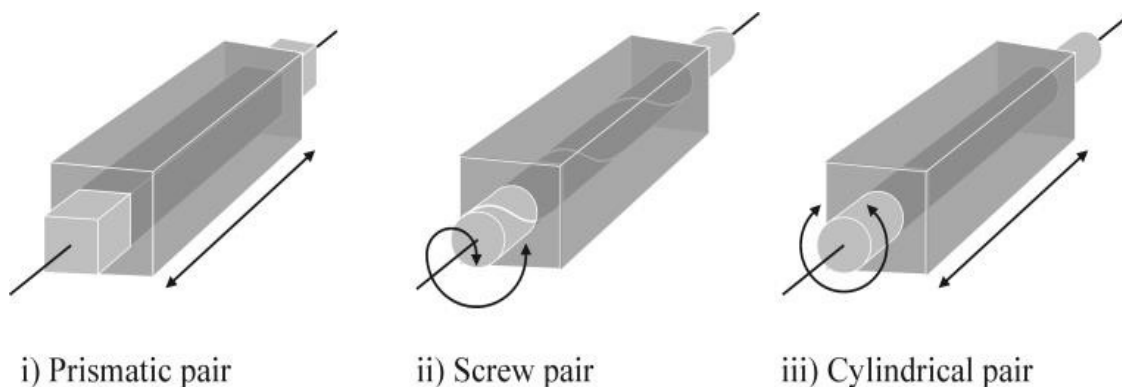


Figure 6.11 Modelling kinematic pairs in U_{33}

The use of the lower kinematic pairs as a reference for defining kinematic shapes has resulted in certain restrictions. Because we are concerned primarily with the properties of material things, only the motions of connected shapes have been considered here. Consequently, the motions enumerated in Table 6.1 are not all applicable. For example, in a two-dimensional embedding space, the moving parts of a connected kinematic shape can rotate about a point or translate parallel to a line, but cannot translate according to a point.

A more general consideration of kinematic shapes could take account of all these possible motions.

Kinematic grammars have been developed here as a variation of making grammars- where shapes adhere to physical constraints- but with the new inclusion of VSRs to account for moving parts. As a result, kinematic grammars do not readily support the visual emergence that typifies shape grammar applications, and much of the richness of the shape formalism has been lost. But this is perhaps true for all making grammars– since it is not obvious how visual emergence can work in a U_{33} algebra, where from any given viewpoint only part of a shape is visible, and only the boundaries (i.e. surfaces) of a part can be seen. This issue with making grammars is examined further next, in Chapter 7. However, there is still scope for reinterpretation of shape structure via shape rule applications, and emergence can also arise in the material behaviours of shapes in motion (Gürsoy & Özkar, 2015).

Kinematic connections between parts permit relative motions, and kinematic behaviours (i.e. composite motions) emerge in kinematic designs, as indicated in Figure 6.9iii. Further, for material things whose material deformations further permit relative motions, elastic, plastic or even auxetic behaviours may also emerge. Consequently, rule sequences which add together multiple kinematic shapes, with interacting VSRs, can compose designs exhibiting emergent kinematic behaviours that may be complex and surprising.

This treatment of variable spatial relations, and their place in grammars for kinematic design, is preliminary and exploratory. It serves to indicate ways to formalise the making rules which have been applied in the kinematic designs described in Chapter 5, when exploring possibilities and explaining behaviour.

More generally, there remains considerable work in examining how kinematic shapes can be included in shape computation, to formalise motions of physical materials and objects. Of particular interest is whether an experimental making approach, using kinematic rules and schema, can yield interesting kinematic designs previously only found through the application of analytical techniques.

Chapter 7 Making rules for doing and sensing

At the start of this thesis, we noted Knight and Stiny's proposal (Knight & Stiny, 2015) that making rules can describe both doing and sensing with algebras of material 'stuff', in the process of making things. This chapter analyses the relation between doing and sensing in processes of physical making in design. In choosing particular avenues for further embodiment, envisioning of design possibilities is as critical a component as their physical construction in experimentation. However, this thesis concentrates on the physical actions of rules, and how these actions can inform the process of design, through exploring new possibilities and simultaneously generating their properties. In this thesis, these are largely the motion properties of kinematic designs. Chapters 4 and 5 detailed experiments where these properties are revealed through applying making rules to physical models to explain kinematic behaviour. Rules identify and transform shapes using spatial relations, including the variable spatial relations developed in Chapter 6.

Chapters 4 and 5 developed evidence for the thesis that making rules can support both the exploration of possible designs, and also explanations about how they behave. In particular in Chapter 5, using a suite of making rules, we have analysed in detail a class of kinematic designs, their motions and design modifications. These rules assist in exploring kinematic designs through local modifications, and in explaining their motion by observing how changes in shape, geometry and configurations affect how the design moves.

The analysis of the kinematic designs in chapters 4 and 5 of the thesis has focused on the value of shape computations for creating pictorial descriptions of making actions. These shape computations incorporate active, 'doing' rules which describe operations occurring within material algebras, defined by various assortments of things, material stuff and tools. For aspects of kinematic designs where shape algebras alone cannot fully encode these 'doing' actions, in Chapter 6 we have developed a mechanism of Variable Spatial Relations (VSRs), to extend the existing shape grammar formalism.

In the thesis so far, the emphasis has been on describing the ‘doing’ aspects of making. In this chapter, we now turn attention to how the ‘sensing’ actions which necessarily occur within making might also be schematically described. These sensing actions are a natural part of the shape computation framework, in the sense that they identify the parts of interest for generation. They extend this ‘seeing’ aspect to perception in general, and further to explanatory understanding. We put forward a refinement of Knight and Stiny’s definition of making rules: one that helps to explicate the capacity of shape algebras to describe both doing and sensing, in supporting generative descriptions.

To this end, we begin in Section 7.1 by reviewing making schema, including the observations from Chapter 5, that visual and tactile interactions with kinematic models drive the interplay between tactics and strategic schema, underpinning creativity in exploratory making. Section 7.2 recaps our current conception of material making rules and schema. We then consider, in Section 7.3, how making rules move between representations. These moves are driven by sensing (Section 7.4), and Section 7.5 examines schema for seeing and touching within exploratory making.

7.1 Tactics and strategies

Suchman (1987) explains how plans (abstract, verbal descriptions of activities) may be used to predict and structure interactions in real-world situations before they occur, or to report on them afterwards. They omit the detail necessary to describe the richness of the full situation. De Certeau (1988) outlines a related approach for describing activities and interactions occurring in everyday life. He outlines a thesis of tactics and strategies. Strategies can be implemented, top-down, by those with ownership of a given space, whereas tactics provide a contingent approach to operating in a space which is not owned, understood or predictable. Although de Certeau is concerned primarily with political ownership of urban spaces, these concepts can be helpfully translated to spaces of material interaction for making and design. We interpret ‘ownership’ of a design space to imply a situation which is well-understood, where the effects of actions can be predicted. In such a situation, generative schema can be deployed unsupervised to create new design variants.

Strategies are pre-defined sequences of actions that can be implemented 'blindly', without ongoing observation and reflection. Tactics operate in the opposite scenario, where circumstances are not yet owned or understood, and outcomes must be experienced and negotiated rather than predicted. They continually respond to the materials of the situation, noticing opportunities for action, exploiting them, and observing the consequences. With the repetition of experience, tactics found to be repeatedly successful can eventually become strategies to be relied upon.

But even when reliable sequences of making actions become practised and polished by the rhythms of repeated production, how can they be described? Even when the maker has developed a practised intimacy with the activity and all its opportunities for error, communicating verbally the detailed nuances of making is often not straightforward. Making seems best demonstrated through practice, in a language of tools and materials, which together afford particular motions, operations and sensations. As Suchman highlights, these situated activities, when described out of context, become simplified into abstract schemes lacking the contextual information which defines how activities unfold in practice.

From this point of view of situated actions, documenting or describing the moves and actions by which designers explore and develop an understanding of unfamiliar territories of making is a contingent exercise. Juxtaposing Suchman's plans with De Certeau's strategies can be illuminating for the exploratory making activities being considered in this thesis. The two however are not consistent. De Certeau's strategies and tactics operate in different spheres (different political groups), while for situated activities such as making, strategies and tactics rather operate simultaneously, but at different levels of confidence, to describe practice. A strategy describes what can be planned in advance. Tactics respond to observation of the materials of practice, and are more difficult to specify in an abstract, dematerialised sense. The representations of making in this thesis, which apply shape rules and schema to describe and document design exploration, lie on a spectrum between strategies and tactics.

7.2 Material making rules

Applying shape rules to material objects can be viewed as a generalisation of their actions on drawings and pictorial representations. Several examples of these have been presented in the experiments on kinematic design described in Chapters 4 and 5. The series of design developments in Chapter 5, exploring variations on a particular type of kinematic loop, demonstrate rules generating variations prompted by material and kinematic properties. Such rules also serve as ways to explain how shape, configurations and motions are related in the kinematic design.

Developed in Chapter 6, as a counterpart to the fixed spatial relationships which underpin conventional shape rules, variable spatial relationships provide the means to generate kinematic designs. However, the core idea is that making rules act on material in multiple and diverse ways, including design and craft actions to: cut, tear, divide, bend, fold, deform, melt, mould, cast, forge, weld, solder, braze, glue, join, connect, interweave, knot, sew, assemble, interlink, constrain, surround, draw, paint, print, dye, and so forth. Two key observations are that:

- Specific material algebras, of things, stuff and tools, afford their own specific forms of creative computation
- Doing actions are often communicated using pictorial or symbolic descriptions— shapes are useful for this.

Some of these types of making rules are more formally discussed below, in ways analogous to the drawing and pictorial representation applications of shape grammars.

Blank sheet making rules

Shape grammars assume the material capability to add shapes:

→ x

These pull something from nothing. But in fact this involves a schema for a particular type of shape (within a particular type of algebra— for example a polygon, curve or straight line from the algebras U_{22} , or U_{12} , or perhaps a polyhedra in U_{13} , U_{23} , or U_{33}), combined with some schema for media or materials, which allows lines, planes or solids to be practically

constructed. Whilst these materials afford the representation of the particular types of algebras for shapes, they will also have their own individual algebras, in terms of how parts are aggregated and decomposed.

Cut-and paste making schema

One schema, employed extensively in the design experiments in Chapter 5, takes part of one thing and combines it with part of another thing, to create a whole new thing. It can happen within all sorts of different algebras, and when used with material things, can generate, repurpose or 'up-cycle' designs. Parts of material objects made for one application are used as parts for a new application: a simple example might elucidate.



Figure 7.1 Picasso's *Bull's head* (WSJ, 2011) © Succession Picasso/DACS, London, 2022

Picasso (1942) provides an exemplary demonstration of this schema, to reinterpret multiple parts of a found object, to see and create something new:

$$\text{Part}_1(x) + \text{Part}_2(x) \rightarrow y$$

The final sculpture is actually a bronze cast of the original found object, so a casting operation is also required. In material terms, the bronze sculpture is instantiated by creating a mould, M, into which the bronze, B is poured, before the mould is later removed.

$$y + M - y + B - M \rightarrow B$$

In shape terms, a series of boolean operations, á la Whiteread (2013), are required.

A further example considers the up-cycling or repurposing umbrellas. The underlying mechanisms are deployable structures which may be used in repetition, in the schema:

$$x + x + x \dots \rightarrow y$$

to create an folding array, select a range of colours, and include a binary open/close shape rule for their kinematic unfurling. Designs are abstracted from types of motions, using sets of making schema. One architectural project for a deployable ‘portable wall’ uses multiple instances in a tessellating assembly schema, to create a modular array (Schumacher, 2010). Another art and design application is ‘Parasol Carousel’ (2016), which exploits symmetries and rotations in the schema:

$$x \rightarrow x + t(x) + t_2(x) + \dots$$



Figure 7.2 A repurposing of umbrellas (Parasol Carousel, 2016) © Parasol Carousel. Photo: © Terry Rook, Glance Image.

Looking vs Making

Rules can apply for 'looking', to recognise parts within a design, or they can apply to make an active change. However, the identity rules needed to 'see' parts embedded within the 3d objects employed in physical making are more complex, and arguably, these rely on more imagination. Does this sort of 'seeing' not require more imagination, than the perception of a readily visible planar shape, composed of lines in U_{12} , or polygons in U_{22} ? The division of a line, into two parts, requires merely cutting at a point. But cuts to divide two-dimensional shapes may be any straight or curved lines within the algebra U_{12} , and divisions in three-dimensions are in general complex surfaces in U_{23} . This may explain why, for elements of higher dimensions, visual emergence is generally less prevalent.

In physical making, there is also a key distinction between whether a change can be made to an existing model, and whether an entirely new model needs to be constructed, to appreciate a change to shape or structure. For physical models, additive or subtractive changes to the shape of parts may be possible whilst considering an existing model. In contrast, structural changes to spatial relationships among component shapes may require the construction of another model.

Stiny (2011) implies that 'seeing in a new way' may be enough, to support creative discovery. He explains how

$$\mathbf{part(x) \rightarrow b(t(x))}$$

can be used to recognise a part (or, an approximation) of a shape, in the randomly shaped clouds– and then to replicate it (or, indeed to replace it– at least as far as the designers attention is concerned), by drawing the outline of that recognised shape, transferring to paper and possibly also rescaling (hence $t(x)$). Conversely, the inverse of this schema provides suggestions for seeing new shapes in the clouds, prompted by descriptions of their outlines presented on paper. From the perspective of making rules, such schema can be interpreted in two ways– they may be either (rather ambitious) instructions, for the fabrication of clouds (doing rules), or, (more modestly) a manual for seeing and discovering new shapes and images within the clouds which are already there and awaiting discovery (seeing/sensing rules).

For looking at 3d shapes, boundary rules and their inverses can be useful to support reinterpretation of the object, by considering its surfaces, edges or corners, without removing attention from the 3d artefact. However, these rules can also be active; that is, they could be used for instance to add coloured areas, or lines and other marks, upon the surfaces of objects, in order to leave a visible trace of reasoning processes. In terms of making, the rule:

$$\mathbf{b(x) \rightarrow x}$$

can also be used, for instance, to 'join up the dots' within a drawing, or for 'colouring in', between the lines, to define planar shapes where the analogous 3d rule is filling a hollow mould.

For 2d shapes, the blank sheet rule:

$$\rightarrow \mathbf{x}$$

might create something from nothing. In terms of making. However, nothing really comes from nothing in a physical world of making. Such a rule assumes the availability of suitable materials to support a synthetic action- perhaps making do 'on the fly', improvising with whatever is found to hand. Further, since the rule for an instance of x already incorporates the 'idea for an x', the biggest jump is perhaps the instantiation of a specific rule from a certain schema, to match the circumstances at hand. Selecting an appropriate rule instantiation will require creative effort: materially enacting a selected rule requires an associated physical effort.

Copying

In general, in order to apply a rule:

$$\mathbf{x \rightarrow y}$$

to a drawing, there are two viable approaches in practice. If x is not a sub-shape of y, then it will be necessary to remove x from a drawing D, before then adding y. This can be described by

$$D \rightarrow D - x$$

followed by

$$D - x \rightarrow D - x + y$$

For a drawing using pen and ink, however, erasing may not be viable. Therefore, the actual mechanism for creating the altered design may require the tracing or copying of the entire drawing, to a new sheet of paper. In this case, the rules required would be:

$$\rightarrow D - x$$

followed, as before, by

$$D - x \rightarrow D - x + y$$

Stiny develops the argument that creativity = recursion + embedding (Stiny, 2011), where embedding enables wide ranging copying without preconception. Trying schema and rules is creative. Stiny proposes that creating starts with copying. Where making is concerned, copying identifies a way of describing and constructing something, which may differ from the way the original was produced, since there are many ways in which shapes can be formed, whether from lines, planes or solids. Copying necessitates new making rules, which then form new generative descriptions for making.

Inverting schema

Stiny (2011) identifies types of schema (Figure 7.1) for art and design, and then demonstrates how each schema type can be reversed to give its inverse. One schema acts on another schema to invert it. More generally, transforming schema facilitates creative exploration. Where descriptions of rules and schema are visual, basic transformations such as inverting schema seems to have considerable power, giving a new lattice of schema. (Further, in shape terms, all that is needed to apply the inversion rule to an existing schema is a rule for reflecting the arrow within its symbolic definition: $\rightarrow \rightarrow \leftarrow$).

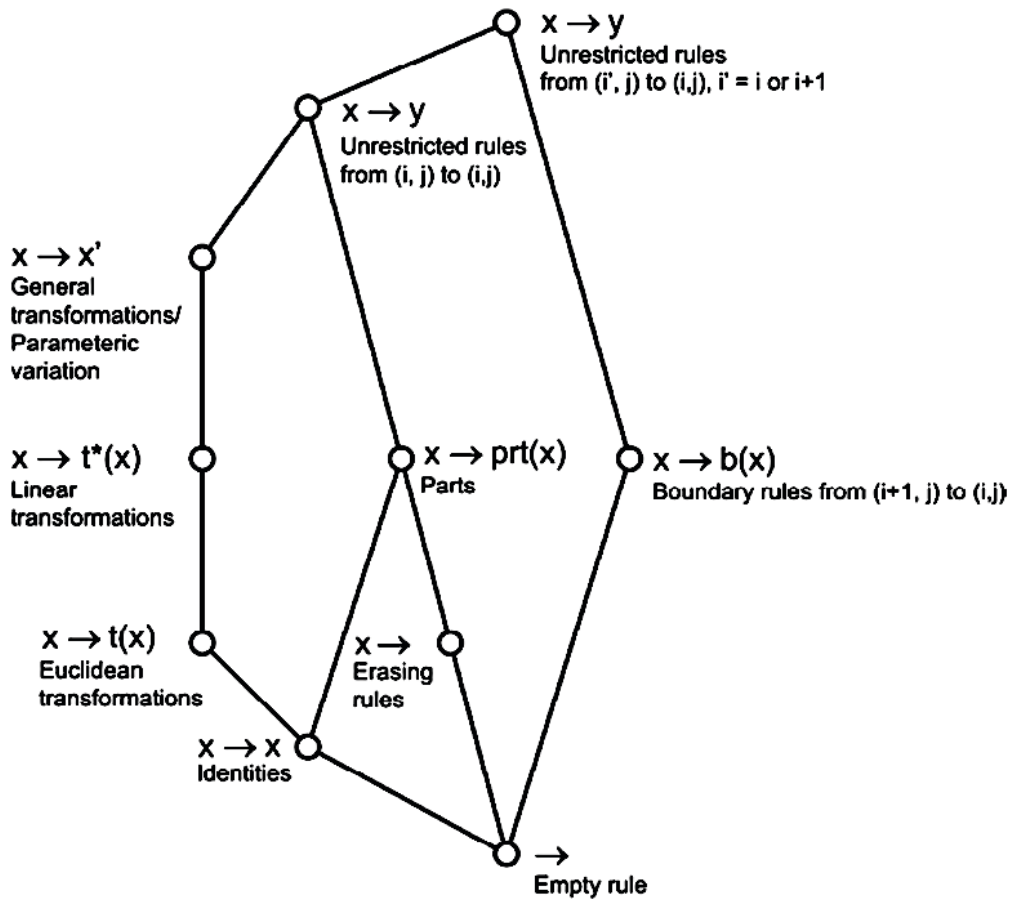


Figure 7.3 Lattice of schema, from Stiny (2011)

Making schema

The experiments described in Chapters 4 and 5 indicate how repertoires of meaningful schema can be discovered and collated, through examining past projects. Wiggins' model for computational creativity (Wiggins, 2001; 2006a; reviewed by Xiao et al., 2019) suggests that searching for new rules, within a space of rules, may be as important for creativity as using rules to move between design states.

Ice-ray grammars (Stiny 2006; Stiny 1977) provide an early example of how making grammars can differ from shape grammars. Rather than acting directly on the material of these wooden Chinese window grilles, the shape rule approach in the schema:

$$x \rightarrow \text{div } x$$

acts on the negative spaces within the window frame, represented as 2d planar shapes. Although $\text{div}(x)$ describes an operation which adds a new bar, it is instantiated as a subset of

$$x \rightarrow x' + x''$$

where x' is a general parametric transformation. Rather than physically dividing the shape in two, this schema transforms the original shape in two different ways, before placing them adjacently. This method works for line drawing representations in U_{12} , and although suggesting the physical action of adding a cross member, does not exactly correspond. We note that

$$x \rightarrow \text{div}(x)$$

(or its inverse) corresponds nicely to the physical operation of laser cutting the outlines of an ice ray design.

Shapes can be divided into parts anywhere, but once shapes touch, they merge visually, and the parts from which they are composed effectively disappear. Additional boundary elements, which also merge, preserve visual separation. For physical making, parts as well as their boundaries will generally remain separate. The kinematic designs analysed in Chapters 4 and 5 present an example where adjacency at a moving joint preserves separation of parts.

Krstic (2019) further notes that transformation rules, of the form

$$A \rightarrow g(a)$$

can describe making operations such as bending, twisting, folding or knotting. Transformation rules can also be used for repositioning objects relative to one another (while preserving distinctness of parts) through translation and reorientation in assembly operations. Krstic describes a 'blank paper start', allowing an initial shape to be placed anywhere on a new sheet of paper. This fixes it in relation to the sheet. Further shape rules can reposition shapes relative to one another. However, in general, since all shapes are fixed in relation to the page, they also maintain fixed spatial relationships to each other. This is

not the case for the experimental making of kinematic designs examined in Chapters 4 and 5, where the role of variable spatial relations (Chapter 6) in making rules is critical.

Other features of making grammars include an emphasis on continuity rather than modularity. Shape Grammars by Knight and Sass (2010), which generate a complete kolam design, use a modular generation which is incompatible with how kolam are made in practice. In contrast, the approach of Knight's making grammar (Knight, 2018) constructs a continuous line which weaves through previously defined location marks or 'pulli'. A modular grammar might provide an appropriate making grammar if the materials were pre-patterned blocks or tiles, requiring arrangement and assembly. In contrast, the weaving approach of the making grammar can feasibly be re-appropriated to a situation where string or rope is woven between wooden sticks or pegs. It is in part the schematic, pictorial nature of these grammars, succinctly represented in U_{12} , which supports their reapplication in other situations, where physical materials interact in similar ways.

Seeing and sensing during making are critical, to the exploration and explanation in Chapters 4 and 5, which depend on such reapplication of schematic and pictorial grammars to specific physical interactions. Before examining sensing in making (in section 7.3) we take a closer look at one of the other salient features of our experiments in Chapters 4 and 5 – namely the way that making rules move frequently between representations.

7.3 Making rules: moving between representations

In Chapters 4 and 5, reasoning about the kinematic design proceeds through exploratory making in range of representations. Equivalent descriptions of kinematic designs help to establish an appreciation of their shapes, structures and motions. Transforming between these descriptions plays a key role in the experimental approach.

Knight and Stiny's schema for creating drawings from clouds is an example of moving between representations (Knight & Stiny, 2015). The clouds are left where they are, while a drawing appears on a sheet of paper, where it becomes the main subject of the designer's attention. As far as 'seeing' is concerned, the clouds themselves have been removed from

the equation, in favour of the newly constructed image. New drawings or descriptions are freely constructed. Although the final composition will conceal the stages of its construction, the series of drawings will constitute a record of rule applications. The identity rule $x \rightarrow x$ copies and creates a new representation; the starting point for further action. During making, the distinction between copying and recognition, in the construction of representations, is particularly significant. A schema such as:

$$x \rightarrow x + t(x)$$

can add to an existing object or construct a new one. The rule:

$$\rightarrow x$$

when making with materials, has as its prerequisites both the availability of appropriate tools and materials (e.g. paper and pencil), and the idea for an action (described here within the rule itself).

Krstic (2019) highlights the visual equivalence of a description of a solid object in U_{33} , with a description of its boundary surface in U_{23} . However, visual equivalence in physical making is complemented by tactile interaction (Noë, 2006). For the kinematic designs in this thesis, their connections within lower kinematic pairs can be modelled by surfaces which are shared between connected parts. For some cases, further, equivalent representations can also be found using sub-shapes of lower dimensions, such as shared axis lines or points, to model the connections which afford relative motions. Corresponding rules act on these boundary descriptions to add, remove or transform connections between parts.

Stiny (1991) suggests that, when multiple modes of representation are employed to support the development of designs, linked computations are necessary, within composite descriptions in Cartesian products of the separate representations. Making rules, including those for copying examined in 7.1, show the possibility of moving between descriptions by copying to render new aspects visible, and to present transformed design properties. Descriptions of material models can also be derived through material exploration. This is the case for both the over-constrained linkage experiments in Chapter 4, and for the exploration of new mechanisms in Chapter 5.

The types of description and representation through which making rules move are wide-ranging. Knight (2017), in describing performative craft processes, implicitly advocates the use of pictorial representations. Krstic (2019) indicates that the key benefits of making grammars may lie in their potential to create precise formal descriptions of the sequences of actions which occur in practice during physical making processes. These making rules create and transform descriptions which at all stages accurately mimic the properties of things, and the way they behave during making. In contrast, shape grammars construct designs which may implicitly be representations of other things, but shape rules are only required to handle the properties of shapes themselves.

Moving between representations has a long history in art and design. Alberti suggests that human creativity may have begun through the seeing of likenesses (for instance faces or animals) in natural objects, and then by subsequently editing— through the adding or removal of details, to further reveal and encourage those likenesses. He suggests that, from this starting point, by gradually learning how to edit found objects, humans eventually developed to skills to synthesise completed likenesses from no starting point other than the raw materials to hand, (ie, the blank slate, or, pen and paper). Duchamp suggests that artists have no need even to copy— they may merely *select* completed objects and transform them. Placing the object on a plinth:

$$X \rightarrow t_1(x)$$

and perhaps reorienting it:

$$t_1(x) \rightarrow t_2(x)$$

can remove functional connotations. Interactions with the situated artwork become primarily visual, with tactile interactions strongly discouraged, thereby limiting further transition between descriptions, except for the visual one, from object to shadows and projections (Duchamp, 1964). A further example from art and design is Rachel Whiteread's transformations from 3d objects (U_{33}) to 3d surfaces (U_{23}) and back to 3d solids (U_{33}) (Whiteread, 1993).



Figure 7.4 (a) Duchamp's *Fountain*. (Tate, 2021) © Association Marcel Duchamp / ADAGP, Paris, and DACS, London 2023. Photo: © Tate. **(b)** Whiteread's *House* (Whiteread, 1991) © Rachel Whiteread. All Rights Reserved, DACS 2022. Photo: © John Davies, 2022

7.4 Rules for sensing in physical making

The features of making rules and associated algebraic structures, which have been reviewed in the previous two sections, focused on aspects observed in the experiments with kinematic designs in Chapters 4 and 5. These material algebras; of things, stuff, and tools, and the manipulations and transformations they afford, readily describe the 'doing' of making. However, as Knight and Stiny (2015) emphasise, the 'sensing' actions which play a critical role within making activities can often be difficult to consider separately from the 'doing' actions they support.

In general, sensing can be visual or tactile, or include other senses, such as hearing, smell or taste. In at least some circumstances, sensing is clearly distinguishable from aspects of 'doing'. For example, the physical skill involved in playing an instrument is not a prerequisite for listening to the music it creates. Nor is an awareness of ingredients or recipe needed, to appreciate the smell or taste of food. Tactile awareness may be important for the manipulation of tools, but detailed knowledge of a production process is not usually a prerequisite for handling the manufactured artefact or crafted workpiece. While visual or pictorial descriptions are a convenient way to describe many transformative aspects of making processes, lack of vision itself does not necessarily preclude the capacity to make.

This begins to suggest that describing the sensory actions involved in material making may require different types of rules. ‘Doing’ actions have been described as rules within algebras of things, stuff, and tools. These rules are by definition well-suited to describing the material properties of making outcomes, but may not necessarily correlate precisely, with those capabilities afforded to the maker by her available sensory equipment. Describing the sensing actions that occur within making may therefore require additional rules.

Sensing within making

A simplistic approach to examining the role of sensing actions within making might separately consider each of the five senses, and the qualities they detect. When considering or making kinematic designs, smell and taste seem perhaps the least essential. Crafted mechanical artefacts may indeed have sonorous or even musical qualities (see for instance Crawford’s conversations with organ makers in Crawford (Crawford, 2015)), but more generally, hearing too might perhaps be sacrificed, in preference for visual and tactile interaction. Both touch and vision remain, then, as essential sensory tools for considering kinematic designs. Each of these capacities might perhaps be separately addressed, and their various roles in the making process separately considered.

Noë (2006) highlights that the perceiving and interpreting of objects and situations depends heavily on a capacity to both move through and actively engage with our surroundings, rather than passively observing from a single static viewpoint. He argues that perception depends upon active, skilful investigation, to establish a sensory-motor picture of the world. During tactile interaction, sense data is not statically received, but rather traced out in spatial trajectories over material surfaces that reach out and manipulate artefacts. Noë argues that vision itself is also ‘touch-like’, being similarly dependent upon the dynamic bodily trajectories by which the eye collects data. Our meaningful interpretation of visual data relies on the bodily skill of proprioception, calibrating how visual stimuli vary in response to the eyes’ motions, a turn of the head, or the body’s movements through space. This dynamic path of eye’s gaze constructs a visual field. Whereas sounds and scents can

often be experienced involuntarily, both touch and vision require an active direction of their attention.

The Shape Grammar approach adopted here, to describe physical actions as making rules, is grounded primarily in visual reasoning. We therefore begin our analysis by considering in more detail the sensory role of vision alone. We then consider how tactile interaction might both independently collate sensory information, as well as playing a supporting sensory role to vision, within making. A third area of consideration is how a sensory-motor perspective on making might further inform how Variable Spatial Relations (VSR) are represented within this context.

Seeing as Sensing

Receptors in the eye detect both colour and motion (Gregory, 1970), and the eye and brain construct multiple interpretations from this information. Noë (2006) suggests that the visual system consists not only of the eye and the brain, but also incorporates the entire musculoskeletal system. Once perception is considered to be the result of the engagement of an active body, able to move at will within its physical environment to continually discover new details within its surroundings, the special importance of an internally constructed representational model of the external world becomes problematic and can be thrown into question.

The importance of active physical involvement in sensing, during creative activity, is also recognised by Ingold (2013). He advocates a situated, bodily approach to archaeological practice. He discusses how the physical practice of visually framing one's surroundings can dramatically alter one's experience of an environment. He reports upon a field experiment, conducted with a group of students, where, in order to mimic the experience of inhabiting a certain style of dwelling likely to have once stood at an archaeological site, sticks were tied and assembled together to make frames resembling those which might once have once formed doorways, separating the inside of a dwelling from its surrounding landscape. The group subsequently contemplated how, through this active, material framing of particular views, an experience of separation was achieved between inside and outside spaces. By

viewing it through the frame, they came to considered themselves as architecturally separated from their wider environment, rather than immersed within it. Perhaps then architecture, rather than photography, provides the original technological metaphor, normalising the separation of the active body from its surroundings? On the other hand, both metaphors use the idea of the frame to transform visual experience.

In the terminology of shape rules and schema, these visual frames select part of what can be seen. The rule looks like:

$$\mathbf{W} \rightarrow \mathbf{part(W)}$$

Where a $\mathbf{part(W)}$ is selected by a frame.

The architectural frame still permits some interactive visual correspondence with the external 3d environment, with W and $\mathbf{part(W)}$ both remaining in U_{33} . The photograph instead creates a projection:

$$\mathbf{W} \rightarrow \mathbf{t(part(W))}$$

where $t(x)$ is a transformation $t: U_{33} \rightarrow U_{22}$, which maps visual data from a 3d scene in U_{33} , to a 2d snapshot in U_{22} .

The creation of the photograph therefore results in a notably more complex shape operation, which alters the dimensions of the underlying space. It imprints a compositional description, of the perceived 3d world, but one reduced to a static image on a 2d surface. This 2d image describes a 'snapshot' of the 3d world. Using pen and ink to make a drawing, transcribing the outlines of visible shapes and coloured regions onto paper, one could also similarly create a 2d visual description of a portion of one's perceived surroundings in U_{12} , from a particular viewpoint:

$$\mathbf{W} \rightarrow \mathbf{b(t(part(W)))}$$

where $b(x)$ is a boundary operation which replaces coloured 2d regions with their outlines.

But whereas photographic techniques afford the capturing of 'snapshot' images automatically; significant skill, practice, attention, and mastery of the materials may be

required, to construct a suitably faithful impression of a 3d environment by hand. (Those who have learnt to test the accuracy of their drawings, by using a pencil to take measurements of angles and proportions, will appreciate the extent to which observational drawing by hand relies on a technical mapping operation, which requires a very particular and precise type of visual examination.) However, once a photographic image has been created (and the operation $t(\text{part}(W))$ performed), to then merely trace its outlines by hand, (an operation of $x \rightarrow x + b(x)$) is rather more straightforward route, to a similar end result. More generally, a transformation that alters the dimensions of the underlying space, rather than the dimensions of the elements deployed for description within that space, might be expected to be a more onerous operation.

Although themselves not an accurate description of visual processes, once created, these constructed, compositional 2d approaches, to describing the visual sensing of 3d objects, are particularly suitable for applying shape rules. Within the framed image, 2d shape rules can recognise what is seen and identify new things. For representing design activities, shape rules have primarily described both doing and seeing or sensing actions in the pictorial context of 2d compositions and drawings.

Our daily experience, however, is usually informed by far more than shape. Imagine that all colour was suddenly bleached from the surroundings, all translucency solidified, and all surface texture rendered uniform. In all but the most clinical of environments, so much detail would be altered or discarded that the nature of the space would be radically altered. The world more usually presents itself not as a flat or framed composition, but as a 3d landscape in the round, extending to the horizon. In this context, recognising and describing the shapes of common 3d objects is an aptitude to be skilfully acquired by the unframed visual system. Even in the absence of a material or technological frame, merely focusing the gaze momentarily, in a particular direction, constitutes a framing or selection operation similar to

$W \rightarrow \text{part}(W)$

To then tune one's attention specifically towards a particular object X embedded in that part(W), as a recognised solid 3d shape, might require only the shape algebra U_{33} , to filter out 'superfluous' detail on surfaces and edges:

$$W \rightarrow [\text{part}(W)] \rightarrow X$$

where both W and X are in U_{33} .

Such schema begin to provide formal descriptions for simple modes of 3d shape perception, which seem to be central both for recognising familiar objects in everyday life, and within design activities. We note that cases, such as the physical models used to examine kinematic designs in Chapters 4 and 5, where 3d shapes form the main or only means by which an object is considered or encountered, seem to be uncommon. Suitable schema for such situations are now examined.

7.5 Seeing and touching– surfaces and boundaries

"Our vision does not penetrate the surface of things."

(H D Thoreau, 1854).

The 'snapshot conception' of perception (where a sequence of snapshot images is construed to build up an internal model of the external world) creates a tendency for the visual system to be considered independently from the body, as a distinct device which receives and interprets a continuous sequence of 2d images. Noë (2006) argues there is potentially no need for a separate, internal representation, since we can readily move around within the world itself, to access information as necessary. In this situated state, we see, sense and interpret our surroundings in manner which is both skilled and specific.

Noë (2006) questions why, when facing a tomato on a table, we are not usually overtly concerned as to whether or not the opposing face of the tomato is currently intact. He also

discusses our contentment to see only parts of a cat, though the gaps in a fence, without urgently doubting the presence of those parts which remain concealed. He proposes that, in both these cases, since vision itself is active, our ready ability to investigate, by moving to further inspect the entity in question, calms our actual desire to do so.

Experiences of viewing cats or tomatoes differ from those of transparent objects, such as a glass of water, which can provide reasonable reassurance, from a single viewpoint, that they are wholly present and intact. How light passes through them may convey further information about their relative solidity or liquidity. But for opaque objects, all that is readily available for visual examination is the object's exterior surface. Further, when considering opaque 3d objects, we necessarily see only incomplete parts of these surfaces, at any one time. Observing an object from a particular angle creates a horizon line, automatically dividing it into two or more parts— the seen and the unseen. Moving the eye relative to the object continually relocates this line of division, in a familiar manner which can sequentially reveal the object's entire surface. Situated in the world, we are inherently comfortable with encountering its artefacts in this partial, sequential, active manner. How these horizon lines move, as the eye moves relative to various objects, and how what is seen changes accordingly, is a significant feature of all our worldly encounters. As we move relative to objects, we expect the parts of them we see to change, in very specific ways. It appears that we quickly come to associate certain collations of coherently associated views as relating to a single, familiar entity. Once a type of entity or artefact has been adequately examined and understood, seeing just the visible portion of its surface can immediately bring about awareness and recognition of the whole.

How might we describe this type of recognition, using formal shape rules? By merely focusing our gaze in a particular direction, we are electing to observe just a small part of our surrounding environment;

W → part (W)]

Unless our focus falls upon transparent or translucent materials, we will visually experience only the exterior surfaces of objects in view. By maintaining a static viewpoint, we further limit ourselves, to seeing only incomplete parts of these exterior surfaces. But frequently

this gives us enough information to recognise and identify familiar entities, whether tomatoes or apples, teapots or elephants, with hardly a notion of incompleteness.

Within an observed world W , visual observation may lead to particular shapes or coloured regions being recognised and identified as familiar, nameable entities. For instance, noticing the coincidence that:

$$W \rightarrow \text{part}_1(W) = \text{part}(b(X)) \rightarrow X$$

one might recognise a familiar 3d object X , to momentarily become the focus of attention. Other objects (Y and Z) might themselves present coincidence with different parts of W (part_2 and part_3):

$$\text{part}_2(W) = \text{part}(b(Y)), \quad \text{part}_3(W) = \text{part}(b(Z))$$

leading to the recognition of multiple objects that may be present and visible within one's immediate surroundings.

On occasion, the same part might coincide with different objects ambiguously:

$$W \rightarrow \text{part}_2(W) = \text{part}(b(X)) = \text{part}(b(Y))$$

requiring further contextual detail, to afford a distinction.

When operating visually, in U_{33} , the identity rule, to recognise a familiar artefact within the environment, is not limited to $X \rightarrow X$, but now extends to:

$$\text{part}(b(X)) \rightarrow X$$

This recognition of the visible portion, of a 3d surface, describable in U_{23} , enables a solid object in U_{33} to be identified. Since the entirety of the object's surface is not usually visually accessible simultaneously, X itself seems to be the result of the synthesised collation of many partial visual experiences of X :

$$\Sigma \text{part}(b(X))$$

as seen from multiple viewpoints.

It seems, therefore, that whilst we tend to believe our everyday experiences and interactions to be in correspondence with a material world containing solid, material artefacts, our *visual* experience of this world is more usually constrained merely to an awareness of its opaque 3d surfaces. But without further comprehension of their internal composition (or the ready freedom to interrogate it), we still often feel equipped, either to make the assumption of a world of homogeneously solid objects, or else to happily accept these opaque artefacts as 'black boxes', whose interiors have no meaningful bearing on our experience.

Opportunities to 'see' in U_{33} are relatively rare. 3d forms composed of transparent materials come closest to affording this experience. Perhaps this is why the effect of light on water, diamonds, clouds, or curls of smoke, continue to retain their visual intrigue? And yet, in familiar environments, we *do* seem able to leverage our previous material experiences, to generate reliable hypotheses about what might lie beyond the surface of things.

These collated, multiple views can arise from observer movement, or else by active movement of the artefact itself. For the visual perception of motions in kinematic models, as in Chapters 4 & 5, relative motions between connected parts are actively solicited through tactile interactions. For these kinematic designs of articulated mobile assemblies, three modes come into play: moving the design through its states or cycles; observing sequences of configurations; and moving observer viewpoint. Next, we examine these tactile interventions for both moving models and sensing objects.

Visual examination alone still provides us with limited information about an object's interior— it might be solid or hollow, heavy or light, empty or three-quarters full of elephants. For an unfamiliar object, other sensory modes of investigation might be required, to interrogate this further. If X is a pebble, previous tactile encounters with similar objects may lead us to expect that it be weighty, and its surface cooling to the touch. When Y is a tomato, our understanding may include not only previous visual observations of the object's exterior, but also the tactile experience of cutting through that surface with a knife, or perhaps with teeth, and this leading to further visual, tactile, olfactory and gustatory

examination of its heterogeneous interior. Where Z is a cat, we do not merely recognise him by his static 3d shape, but rather identify him as an entity equally liable to appear in a numerous variety of fluid contortions, and perhaps witnessed in active transition between these. (See Fardin's article, about whether cats are liquid, which won the IgNoble Physics prize in 2017 (Fardin, 2014)). It seems likely therefore that, in design activities, whose goal is often the creation of novel and unfamiliar artefacts, more than visual examination may be necessary, to become familiar with the physical qualities of a new material composition. However, just as we rarely question whether the exterior surface of either a cat or a tomato is fully manifold, an absence of available information about an object's interior also rarely causes us to doubt its material viability.

Noë (2006) proposes that vision itself can be 'touch-like'. Both touch and vision are similarly constrained to examining, for the most part, surfaces. Both require the active direction of their attention, via physical action. Visual inspections require relative motion between the eye and the object, while tactile explorations can require both surface touch (moving trajectories over parts of the object's surface) and object manipulation. In the case of kinematic designs, these manipulations include articulated sequences of motion. Note that the touch trajectories on a surface $b(X)$ of an object X might be represented by a schema:

$$b(X) \rightarrow \text{part}(b(\text{part}(b(X))))$$

To experience a larger entity may require perambulatory activity to circumnavigate it. For a smaller artefact which can be held and physically manipulated, a more complete understanding of the object in-the-round can be readily constructed, by observing visually whilst moving the object itself. As well as information about shape, tactile interactions with 3d surfaces can collect data about surface texture. And, for artefacts whose connected parts possess a capacity for relative motions, direct tactile interactions which apply exploratory forces can help to reveal and literally 'sense' these motions. Can these touch interactions, which alter the way we perceive objects, be described by shape schema? There are changes in perception, but no changes in the object itself. In terms of material schema, rules which select parts, or parts of parts, and also change these selections, can represent sensing and tactile interactions. Other schema, to manipulate a kinematic design through cycles of motion, also vary spatial relations between selected (rigid) parts.

Whereas doing rules act directly to alter stuff or things, sensing rules rather record a maker's sensory interactions, which, although they may be essential to the making process, do not themselves have a lasting transformative effect upon the thing. The manipulations of a kinematic design through its cycle of movement include both tactile (manipulation) and visual (seeing and the results of manipulation) sensing. Knight (2017) suggests that doing rules should record direct physical changes to the thing being made, whereas sensing rules are used to record changes in a maker's perception of that thing. I propose sensing does more. Sensing rules help to grasp things, in two distinct literal senses. The first grasps things visually or physically, to allow a doing rule to be applied. The second grasps in the sense of affording a new understanding, about an object or design's properties.

A first step to altering a design is to appreciate and describe what is already there. This involves selecting a suitable representation for description. For example, understanding how designs are composed, by recognising parts, leads directly to a description of parts in decompositions, suggesting where new parts can be added (according to spatial relations specified by rules) or designs otherwise altered. This corresponds to the see_1 -move- see_2 iteration identified by Schön (1983).

' See_1 ' sees an opportunity to make a change, and uses an identity schema to recognise or identify a part to alter. It also simultaneously selects a relevant active schema to apply to it. (This might involve, for instance, imagining it extended or transformed in some way.)

' See_2 ' highlights that material design representations can be seen differently, through a particular contextual 'frame' (i.e. that of the architect, the engineer, etc). These 'frames' could include for instance structural or material efficiencies, manufacturability or manufacturing cost, design usability or aesthetics. This 'second seeing', or, reflection, involves a form of reverse mapping, from a design representation constructed within a particular shape algebra on paper, for instance (and accompanied perhaps by various symbols), into a description of those shapes X , as a projection into the complex, situated, nature, of the real-world design application, situated in W ;

$$X \rightarrow \text{part}^{-1}(W)$$

This second mapping is from a shape algebra, back to a complex material and perhaps even social algebra.

‘See₁’ yields emergent shapes (and associated patterns) which enable creativity . In the case of design sketching, shape schema are directly applied. For general exploratory making, seeing emergent algebras affords a similar form of interpretation, but requires a ‘filling in’:

part(X) → X

While only dealing with seeing a part, ‘See₁’ is enough to give the idea for something in which part(X) is embedded (part(X) → X). Exploratory making then proceeds to gather material, or fabricate tools, to test possible embeddings.

A well-rehearsed example of ‘seeing’ from literature is provided by St Exupery’s Little Prince (St Exupery, 1943). His first drawing, while readily construed as a ‘hat’, described either in outline in U₁₂, or revealing a hat shape in U₂₂, is then explained by a second drawing in U₂₂ to actually conceal a very different interpretation, of an elephant being digested by a boa constrictor; presenting part(X) → X as a frightening, creative possibility.

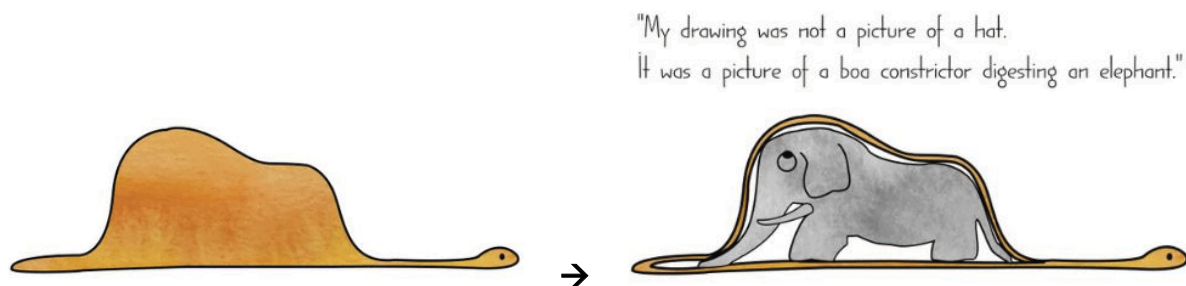


Figure 7.5 Drawings from the Little Prince (St Exupery, 1943)

Although more than shape is sensed in the material world, for making activities where transformations alter the shapes of material things, shapes are often the main focus of attention. Shape descriptions seem therefore to be of central importance for design. Familiar objects may be recognised by a variety of visual features, including shape, colour and surface texture. But in design exploration, where objects are often newly constructed

and unfamiliar, the first point of reference is often a shape description. The ability to infer a variety of shape descriptions from a visual scene is a skill that requires moving between descriptions- such as recognising familiar objects from perceived boundaries between visual fields, for example. The same also applies, when either creating or interpreting descriptive outline drawings of visual scenes, where colour fields are replaced by their outlines. This might be described by a visual transformation:

$$U_{33} \rightarrow U_{13}$$

from a visual scene in U_{33} , to a pictorial representation in U_{13} .

7.6 Conclusion

In this chapter, we have considered how distinct activities of doing and sensing might act and interact, within making activities. Algebras of construction, employing physical materials and tools to make things, provide an expedient route to describing the sequences of physical transformations that occur within a making process.

Sensory algebras describe aspects of the external world readily accessible to sensory perception. Depending on the skills of the observer, sensory perception affords the further attachment of multiple descriptions, to observed objects and artefacts. Details perceived within an environment may provoke the recall and use of a particular algebra, for their description and interpretation. That mode of description and interpretation may itself then subsequently guide how further sensory information is processed. For instance, once marks on a surface are recognised as characters belonging to an alphabet, their symbolic and linguistic meaning is brought into focus, whilst perhaps momentarily obscuring awareness of other visible qualities, such colour, surface texture and curvature. These alphabet characters can be readily described by shape schema. Learning to recognise, distinguish and recreate this set of shapes is central to early education. Developing familiarity with sonic and semantic interpretations of the various sequences of shapes in the alphabet A, across one or more spoken or written languages, is for many an ongoing project. But whilst the literal meaning implied by these markings might be initial focus of attention, comprehension of

this symbolic content does not obscure the simultaneous awareness of the colour of the paint or ink, with which those marks were made. Ingold (2010) argues that the quality of the dynamic gesture by which marks are created can communicate as much as the literal content, of a handwritten note. A graphic designer might be so preoccupied with the selection of typeface, and the spacing of its layout on the page, that the semantic meaning of its symbolic content is momentarily obscured.

A key point, however, is that the material object remains available, for continual reinterpretation via new modes of perception, informed or filtered by the development of new algebras for description. Once separate symbolic or abstract descriptions have been constructed, these may themselves be transformed further, overlaying meanings onto the object that are not immediately accessible via sensory perception directly.

Shape algebras and associated schema for identifying and transforming operate in multiple dimensions, and can describe both drawings and the 3d shapes of physical objects.

Depending on the situation, visual and tactile algebras of description may also be used to sense these shapes. Retained knowledge about an artefact's construction may lead to making algebras themselves being used as modes of description. Schema, operating on shape descriptions and depending on visual or tactile sensory inputs, may also provide clues about how an artefact has been made, how it behaves, and how it moves— all critical features of the kinematic designs examined in this thesis.

8 Conclusions and future work

In the thesis, we have considered the potential for material making rules to both explore and explain relationships between shape, structure and motions, in kinematic design configurations. Material calculations have informed an exploration of the wider design spaces surrounding kinematic artefacts with unusual motion behaviours. We have seen how descriptions of shape and structure can afford both the copying of kinematic instances, and the creation or discovery of novel designs configurations. Further theoretical development has investigated how the variable spatial relationships afforded by kinematic connections might be formally modelled via generative rule sets, and has also considered formal descriptions for sensing, both for making kinematic models, and for three-dimensional material exploits more generally.

In this outline conclusion we include three things. First, some reflections are made about the progress of the research and its direction. Second, taking account of the wide range of experimental cases and theoretical strands presented here, we extract a summary of main research contribution. Third, we revisit the research questions from Chapters 1 & 3; and fourth, we look from this contribution towards future work.

8.1 Reflection

In our initial stage of research, a practical, experimental approach was employed, for considering how making rules might support the material exploration of kinematic design spaces. In Chapter 4, we began to consider how making rules might operate in general, and also how they might apply more specifically, for the exploratory making of kinematic designs. Initial excursions in material exploration helped to identify rules and schema for making, to describe, construct and transform kinematic models. This led to the development of a method for the systematic discovery and application of making schema, which we then applied in Chapter 5, in an episode of experiment focused upon a particular kinematic artefact.

In Chapter 6, we subsequently considered how the shape grammar formalism might be extended to include the description of variable spatial relationships, which are essential for both describing and constructing kinematic designs. In Chapter 7, we begin to extend this formal approach to schema for making in general, and to their rules for sensing in particular.

Stiny (2011) explains that rules found in existing creations can be generalised into schema, affording a new take on creativity and copying. Throughout the thesis, we have considered how material making rules for kinematics can describe, copy and create kinematic designs. We have considered how making rules might model aspects of both doing and sensing, for the exploration of kinematic designs. We have encountered some of the material algebras that describe and construct kinematic designs, and have considered the extent to which shape rules can support their description. We have also considered how the shape grammar formalism might be extended to include variable spatial relations, which are needed both to describe designs themselves, and also to model actions for manipulating models.

8.2 Contribution

The work reported here draws upon the groundwork set down in a special issue on Computational Making of the journal of Design Studies (see Knight & Vardouli, 2015). Chapter 5 of this thesis is largely the contribution prepared for this invited special issue, following a successful workshop of the same name at DCC the previous year, which instigated a wider dialogue across multiple creative domains, and where preliminary versions of this work were well received. The wide scope of rules for making in general is artificially restricted within the thesis, through our particular attention to kinematic designs, and to also to their particular subsets of relatively unusual closed-loop linkages with full-cycle motions. However, the general method we have validated here is potentially more widely applicable. As are contributions to formal descriptions for variable spatial relations, since these are fundamental not just when moving kinematic models, but also more widely, when manipulating materials within making processes. Certain insights into sensing are also afforded through preliminary discussion of a formal descriptive approach, which may also have wider implication. Further, the application here of general making schema to a

particular domain of investigation illustrates the need for the detailed, context-dependent instantiation of making rules.

Boden (2003) and Wiggins (2006a) define exploratory creativity as a process of search and exploration, using rules, within a conceptual universe of possibilities. In this thesis, we have considered how making rules and schema might support the systematic yet creative exploration of kinematic design spaces, to discover new designs, and also to explain the behaviour of the original. We have shown that existing kinematic designs provide a useful entry point into wider spaces of possibility, and that making rules which both describe and construct their shape, structure and motions can then be varied and abstracted to discover general schema, enabling the exploration of a locality surrounding the original design. Making new models and experiencing their motions has helped to identify the limits to allowable alteration whilst still preserving motions. Unsuccessful models which fail to exhibit motions positively recognise and explain the boundaries to allowable change.

Whereas exploratory creativity involves searching spaces of designs using rules, Wiggins suggests that transformational creativity rather searches spaces containing rules themselves. In the context of shape composition, Stiny's schema for shape rules (2011) can be interpreted to define such spaces of rules, for a design context. In this work, schema have afforded an ability to search within spaces of rules, which in turn search spaces of kinematic designs. We have also seen how schema may themselves be transformed, to discover further rules. A key example from Stiny (2011) is the inversion schema, which itself acts on other schema, to discover their inverses.

Our approach to discovering schema in Chapter 4 involves the comparison of related kinematic models to identify general transformations, and in Chapter 5 is primarily through the removal of detail from more precise making rules, to discover generalised sets. Beginning by identifying sequences of making rules which successfully copy the design, we then apply the generalisation:

rule* → *schema

to define general sets of making rules derived from the original sequence. These tactical schemas generate variations on the original design that inspired them. After applying new

rule variations to construct and test design models, a further assertion identifies the subset of the tactical schema which generate only valid designs: that is, the strategic schema. This makes possible a further step of:

schema → rule**

to generate new designs which successfully mimic the properties of the original.

It seems that schema, when applied in this manner, help to answer the question of ‘what rule should I use?’, for the generation of valid kinematic designs. They therefore help to explain the essential relationships and parameters that successfully construct motions.

8.3 Research questions revisited

For each research question posed in Chapter 3, we provide a summary of results and indicate where the thesis addresses each.

Question 1: ***How can making assist kinematic design exploration?***

The literature review indicates possibilities. The experimental cases in Chapter 4 show that physical models are essential to appreciating motions. Making operations from the studio workshop certainly have a place, especially folding and bending to create 3d elements, and taping along edges for hinges. In addition, digital fabrication provides new ways to explore complex component shapes which present freeform surfaces. For examining possible element shapes that allow full cycle mobility, this kind of computational making is invaluable. Most importantly perhaps, these models allow the ready exploration of motions through physical manipulation.

Question 2: ***What types of making rules are needed, to describe the exploration of kinematic design possibilities?***

Shape rules can represent several aspects of making. Transformations between different representations, and their associated shape rules, can assist in design exploration. Thus, making rules in different dimensions and algebras are needed: from drawing, or cutting and folding sheet, to 3d assembly rules for realising mobile connections. A particular class of rules are division rules. In making, these are applied either directly to materials (e.g. cutting and sawing), or indirectly through transfers to more abstract representations, and then back again to the physical making. For instance, dividing a polyhedron by creating two separate sub-polyhedra (as surfaces in a 3d boundary representation), and then making each of those subdivisions separately, is itself a powerful sequence of making rules, employing multiple modes of description (see Section 4.2.4). This example also serves to illustrate a further class of rules, which move between representations: polyhedral nets, defined by 2d drawings, create 3d forms upon scoring and folding. For kinematic models, the making rules presented in the experiments of Chapter 5 show that, in addition to modifying shape elements, making rules can also alter the configurations of their mobile connections. Making rules act on both shape and structure, in exploring kinematic designs.

Question 3: ***Can making rules discover new possibilities and identify design space boundaries?***

These making rules (outlined above in answer to Question 2) modify kinematic designs with the intention of maintaining certain types and characters of motion. Some rules will give changes which are unsuccessful, in the sense that motions are not preserved. These are particularly important in exploratory making, since they indicate where the boundaries of a design space have been exceeded. The experimental case in Chapter 5 also highlights a further, design critical use of rules. By generalising specific rules

to define strategic schema, it is possible to characterise an explicit space of related designs, lying within these boundaries. When strategic making rules are then applied materially to make new models, constraints in their application reflect these boundaries, to ensure only valid designs are created.

In Chapter 5, schema are shown to do two things. Firstly they serve to explain the motions of the set of kinematic designs generated by the making rules. Secondly, they act as a launching point to formulate new making rules, exploring new designs. Combining shape rules and making schema shows scope for creative exploration through making.

Question 4: ***How can making rules in a shape representation be extended to moving kinematic designs?***

When making rules are formulated as shape rules in the conventional sense, one limitation is that the spatial relations upon which they are based remain static. If these shape rules are to become realistic making rules for kinematic design, their formalism needs extension. This is done theoretically in Chapter 6, where routes to incorporating variable spatial relations into shape rules are investigated. These are feasible extensions, based on the requirements identified in the experimental cases of Chapters 4 and 5. However, further work is required to determine how the formalism of these variable spatial relations works in practical exploratory making.

Question 5: ***How can the formalism of shape rules be extended for exploratory making where vision and touch are integral components?***

Shape rules and schema are tailored to visual explorations. Chapter 7 analyses how rules and schema for making also require sensing actions to be integrated with making and manipulation, through devices of constructing new visual parts and transforming between representations. The essential mechanisms of touch, in implementing the rules of making, are outlined –

whether touch tracing across a surface, or physically manipulating a kinematic chain. However, substantive developments are necessarily the subject of further research.

In addition to these detailed research questions, posed in Chapter 3, we also revisit the initial, high-level question outlined in our introduction, in Chapter 1.4.

When used for the formal description of both material making activities (using stuff and tools) and the things they create, can shape rules and making schema discover a generative model for making which helps to explain, either the properties of things and designs themselves, or of the exploratory processes that discover them?

For the case of kinematic design, evidence is found for both the generative and explanatory capacities of making rules and schema, at two distinct levels. Rules and schema illuminate the properties of discovered design instances, as well as material processes of design exploration.

The strategic schema developed in Chapter 5 identify a wider space of possibility surrounding an existing design, whose boundaries also explain its properties. Through exploratory making, for kinematic designs, these schema reveal relationships between shape, structure and motion.

When used in the formal description of making processes, rules also render visible the multiple types of transformation occurring in design exploration. We have seen in Chapters 4 & 5, how making for kinematics involves an interplay of description, construction and material interaction. The making of shape descriptions can be an inherently generative activity, and Chapters 4 & 5 demonstrate the general utility of shape rules for making, to describe, identify, copy and alter designs, within an assortment of descriptive and material algebras. To be tangibly experienced, shape descriptions themselves must be materially constructed or otherwise visually instantiated. Conversely, sensing, through both sight and

touch, can actively construct shape descriptions, multiply overlaid upon material models. Multiple descriptions, embedded simultaneously in material models, enable designs to be reinterpreted, transformed, and translated to new spaces of creative exploration.

When making for design, shape algebras seem to afford a conceptual link, between actions and reflections occurring across both sensory and material algebras. Shapes seem to be important algebras of description, for both guiding and recording making actions, in material algebras of construction. Their dual descriptive and generative capacities enable designs to be continuously reinterpreted– both and perceptually and materially.

In Chapter 4, schema describe types of general operation in material making processes, and also transformations relevant for kinematics. The method outlined in Chapter 5 also demonstrates how schema work in design exploration to discover new rules. Further, the general toolkit of making schema, collated through experiment in Chapter 4, perhaps suggests a formal analogue for the situated acquisition of design expertise, as traditionally amassed through ongoing experience in creative studio practice.

8.4 Further work

The discovery of further classes of schema which, instead of directly searching design spaces, alter other schema to rather search spaces of rules, has interesting potential for supporting creativity, across the spectrum of domains where rule-based descriptions of reasoning are appropriate.

How general making schema can both be identified, and also subsequently applied to derive their situated instantiation into the specific rulesets appropriate for a particular making scenario, provides another fertile area for future research exploration.

In the context of making more widely, making rules and schema, operating within both material and sensory algebras, have demonstrated usefulness for the identification, description and explanation of properties such as motion. This approach has been

rigorously applied only to designs possessing a particular, highly constrained class of kinematic motions, and its applicability more widely, both within kinematics, and other design fields, has yet to be investigated.

A further key focus has been the value in design both for multiple complimentary representations or descriptions, constructed within a range of material algebras, and also for multiple interpretations of these descriptions themselves. Here, material models have provided a route for both describing and explaining motions. Our preliminary consideration of formal descriptions for the sensing of motions have helped to illuminate why complete descriptions for kinematic motions might prove elusive. However, this emphasises the potential value of new techniques for modelling, representing, and describing motions, and this is an area worthy of further work. Beyond kinematic modelling, novel descriptions for motions could support their creative consideration within both design and engineering application more widely, and also perhaps across further fields, such as dance and performance, animation and cinematography, zoology, geography and archaeology.

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