COMPUTATIONAL MECHANICS 2023
38th conference with international participation
Srní
October 23 - 25, 2023

The magic of angles

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1. Introduction

The calibration of robots and machine tools requires the measurement of position of points in space. The modern devices for such measurements are laser trackers and laser tracers. The precision of measurements of position of points is quite high. However, the calibration of robots requires to determine the position of body (robot gripper) based on measurements of several points.

The paper describes the surprising result that the subsequent determination of body position only from position of points is possible only with low accuracy. The accuracy of determination of body position can be significantly (more than 10 times) increased if there are measured not only the position of points but also some angles. The influence of measurements of angles on the determination of body position is magic. The paper describes the simulation and then resulting patented calibration device for robots.

2. Problem formulation

The industrial robots especially with translation on the 7th axis require calibration. For stationary robots the suitable device for calibration is the laser tracker. Laser tracker looses its precision for longer distances given for example by movement on the 7th axis and besides that it is quite expensive. Therefore a new calibration device was proposed (Fig. 1) [2] that should be capable to calibrate long robot workspace around the 7th axis and achieve similar accuracy as laser tracker for shorter distances.

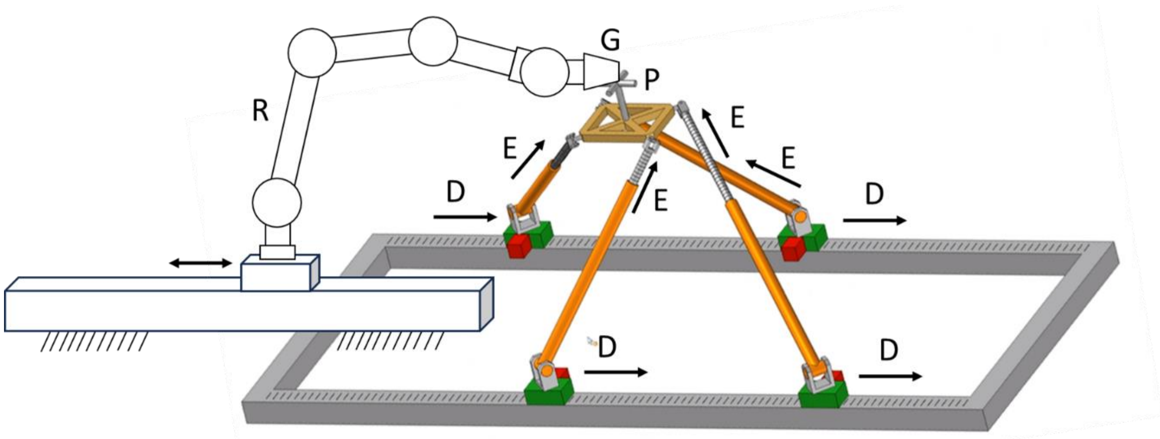

Fig. 1. Calibration concept of robot with new calibration device

Its concept is derived from the similar calibration device RedCaM (Redundant Calibration and Measuring Machine) [3, 1]. The industrial robot R using its gripper G is firmly connected with the platform P. The platform P is suspended through spherical joints on four legs with measured extensions E that are by joints with azimuth and elevation angles connected to four carriages with measured displacements D. The carriages move on two parallel sliding guides. The positions of displacements of carriages on these sliding guides are individually and independently driven by drives and measured. Such device uses redundant measurements that was successfully applied to RedCaM. The redundancy is given by usage of four legs instead of necessary minimum three legs and by usage of multiple (unlimited) positions of carriages. In each measured positions (that means the certain position of the robot gripper G and the positions of carriages D) the four displacements D and corresponding four extensions E are measured. The redundancy is quite big as multiple positions of each carriage is used. Therefore it was expected that similar accuracy would be achieved as RedCaM did. RedCaM besides self-calibration is capable to measure the position of its platform with accuracy equal to uncertainty of one single sensor, i.e. no addition of errors from sensors for determination of 6 DOFs of platform occurs.

Thus it was quite surprising that the accuracy of determination of 6 DOFs of robot gripper was quite poor. The analysis showed that the accuracy of determination of positions of centres of spherical joints between the legs and the platform P can be due to redundant measurement quite high but then the accuracy of determination of the position of the platform P (6 DOFs – 3 translations and 3 rotations) is at least 10 times lower than the accuracy of determination of positions of centres of spherical joints. The experience with laser tracers justifies that the accuracy of determination of positions of centres of spherical joints is high but the rest is mysterious.

Several modifications of the calibration device from Fig. 1 were proposed: usage of intersecting sliding guides, usage of skew sliding guides, usage of displacement of carriages not only along the sliding guides but also orthogonal side. Nothing helped.

Finally it was proposed to use the measurement of angles in azimuth, elevation or in both of them. This helped immediately and fully solved the problem. The determination of positions of platform from the positions of centres of spherical joints between the platform and four legs achieved accuracy 10 times better than previously. It was emotionally called the magic of angles. This paper is devoted to computational explanation of this reality.

3. Planar analyses

The planar variant of the calibration device is in Fig. 2. Two coordinate systems were chosen to calculate the position of the platform. The first, fixed, basic coordinate system x_1, y_1 and the second coordinate system x_p, y_p rigidly connected to the platform see Fig. 2. The carriage moves along the x_1 axis. The platform is rectangular with dimensions p by q. The centers of the rotational joints on the carriages are marked Si, the centers of the rotational joints on the platform are marked Ri. The lengths of the legs are $l_1 = \overline{S_1R_1}$ and $l_2 = \overline{S_2R_2}$.

Coordinates of the centers of rotational joints in the basic coordinate system x_1, y_1 are $S_1 = [\,x_1\,; 0\,]$, $R_1 = [\,x_{\mathrm{R1}}\,;\, y_{\mathrm{R1}}]$, $S_2 = [\,x_2\,; 0\,]$, $R_2 = [\,x_{\mathrm{R2}}\,;\, y_{\mathrm{R2}}\,]$. The essential constraints are

$$(\,x_{\mathrm{R1}} - x_1\,)^2 + y_{\mathrm{R1}}^2 = l_1^2, \tag{1}$$

$$(\,x_{\mathrm{R2}} - x_2\,)^2 + y_{\mathrm{R2}}^2 = l_2^2, \tag{2}$$

$$(\,x_{\mathrm{R2}} - x_{\mathrm{R1}}\,)^2 + (\,y_{\mathrm{R2}} - y_{\mathrm{R1}}\,)^2 = p^2. \tag{3}$$

The carriages S_1, S_2 are in multiple (at least two) different positions. The position of the platform is described by coordinates x_{1Op}, y_{1Op}, φ bounded by transformation equations

$$\mathbf{T}_{1\mathrm{p}} = \mathbf{T}_{\mathrm{x}}(x_{1\mathrm{0p}})\, \mathbf{T}_{\mathrm{y}}(y_{1\mathrm{0p}})\, \mathbf{T}_{\varphi}(\varphi)$$

$$\mathbf{r}_{1\mathrm{R}i} = \mathbf{T}_{1\mathrm{p}}\mathbf{r}_{p\mathrm{R}i}, \quad i = 1,2 \tag{4}$$

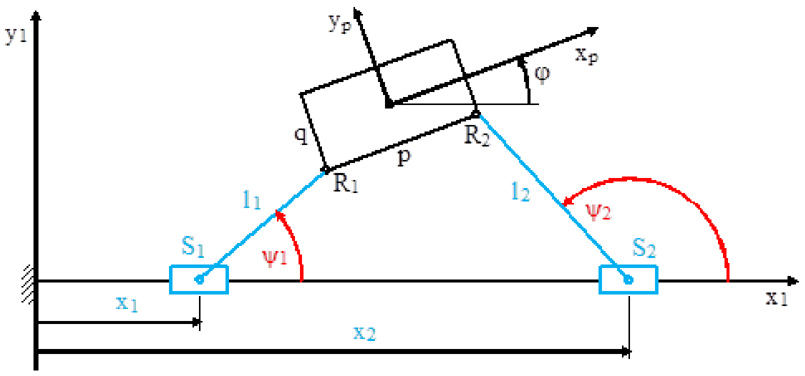

Fig. 2. Planar variant of calibration device

Ten different sets of equations and particular unknowns (e.g. $\sin(\varphi), cos(\varphi)$ instead of φ) were used for determination of platform coordinates x_{1Op}, y_{1Op}, φ.

Then the concurrent measurement of angles ψ_1 and ψ_2 was considered. It leads to the constraints

$$x_{\mathrm{R}1} = x_1 + l_1 \cos\psi_1, \tag{5}$$

$$y_{\mathrm{R}1} = l_1 \sin\psi_1, \tag{6}$$

$$x_{\mathrm{R}2} = x_2 + l_2 \cos\psi_2, \tag{7}$$

$$y_{\mathrm{R}2} = l_2 \sin\psi_2. \tag{8}$$

And again ten different sets of equations and particular unknowns were used for determination of platform coordinates x_{1Op}, y_{1Op}, φ.

The solution of the constraint equations was done not only with precise measured values but also with varied values due to the influence of errors within the measurement. The ideal leg lengths were gradually shortened by $1 \cdot 10^{-5}$ m and corresponding errors in angles. The best results were obtained for unknowns x_{1Op}, y_{1Op} , sin φ and cos φ.

The resulting deviations were for the case without measurements of angles Δx_{1Op} = 18,38 $\cdot\, 10^{-6}$ m, Δy_{1Op} = 55,0 $\cdot\, 10^{-6}$ m, $\Delta\varphi$ = 366 $\cdot\, 10^{-6}$ rad and for the case with measurements of angles Δx_{1Op} = 4,66 $\cdot\, 10^{-6}$ m, Δy_{1Op} = 2,22 $\cdot\, 10^{-6}$ m, $\Delta\varphi$ = 30,4 $\cdot\, 10^{-6}$ rad. This means 10 times improved accuracy.

4. Spatial analyses

The spatial variant of the calibration device is in Fig. 3. Similar description as in the case of planar variant is used. And again ten different sets of equations and particular unknowns were used for determination of platform coordinates x_{1Op}, y_{1Op}, z_{1Op}, Cardan angles φ_x, φ_y and φ_z.

The solution of the constraint equations was done not only with precise measured values but also with varied values due to the influence of errors within the measurement. The ideal leg lengths were gradually shortened by $1 \cdot 10^{-5}$ m and corresponding errors in angles. It was considered the variant of measurement of both azimuth and elevation for all legs, the variant of angular measurement of only azimuth or only elevation. The results of accuracy for variant of measurement of both angles and just measurement of elevation angles are similar. The case of only measurement of azimuths is worse. Therefore the variant with measurement of elevation angles is favourite one. The dimensions were d = 0,8 m, p = 0,24 m, q = 0,18 m and leg lengths $\langle 0{,}8\,;\ 1{,}3\,\rangle$ m.

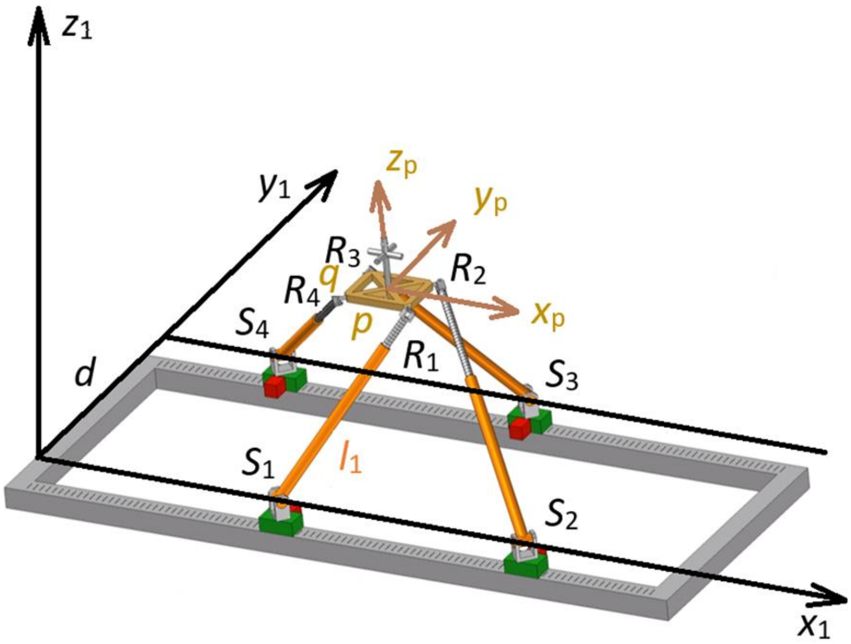

Fig. 3. Spatial variant of calibration device

The resulting deviations were for the case without measurements of angles Δx_{1Op} = 123,8 $\cdot\, 10^{-6}$ m, Δy_{1Op} = 4740,0$\cdot\, 10^{-6}$ m, Δz_{1Op} = 333,0 $\cdot\, 10^{-6}$ m, $\Delta\varphi_x$ = 9380 $\cdot\, 10^{-6}$ rad, $\Delta\varphi_y$ = 31600 $\cdot\, 10^{-6}$ rad, $\Delta\varphi_z$ = 1858 $\cdot\, 10^{-6}$ rad, for the case with measurements of both azimuth and elevation angles Δx_{1Op} = 8,54 $\cdot\, 10^{-6}$ m, Δy_{1Op} = 7,86 $\cdot\, 10^{-6}$ m, Δz_{1Op} = 5,73 $\cdot\, 10^{-6}$ m, $\Delta\varphi_x$ = 68,2$\cdot\, 10^{-6}$ rad, $\Delta\varphi_y$ = 82,0 $\cdot\, 10^{-6}$ rad, $\Delta\varphi_z$ = 61,0 $\cdot\, 10^{-6}$ rad and for the case with measurements of only elevation angles Δx_{1Op} = 11,1 $\cdot\, 10^{-6}$ m, Δy_{1Op} = 19,42 $\cdot\, 10^{-6}$ m, Δz_{1Op} = 5,93 $\cdot\, 10^{-6}$ m, $\Delta\varphi_x$ = 75,4 $\cdot\, 10^{-6}$ rad, $\Delta\varphi_y$ = 43,2 $\cdot\, 10^{-6}$ rad, $\Delta\varphi_z$ = 68,2 $\cdot\, 10^{-6}$ rad. This means 10 to 100 times improved accuracy.

5. Conclusion

The measurement by the described calibration device requires the measurement of some angles besides the displacements. The particular suitable variants are described above.

This can be generalized. The determination of positions of points is possible with good precision just from measurement of lengths, i.e. in our case the lengths of legs from different displacements. But the determination of positions of bodies requires the measurement of some angles. The measurement of only displacements is not sufficient for reasonable accuracy.

Acknowledgement

The authors thank for initial ideas and concept to Frantisek Petru.

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