

Influence of static unbalance on rotors with various journal bearing types

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A mathematical model of the rotor operating at angular velocity ω and supported on the journal bearing is written in the following form:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{y}_J \\ \ddot{z}_J \end{bmatrix} = \begin{bmatrix} -mg \\ 0 \end{bmatrix} + \begin{bmatrix} F_{hd,y} \\ F_{hd,z} \end{bmatrix} + \begin{bmatrix} \Delta m E \omega^2 \sin(\omega t) \\ \Delta m E \omega^2 \cos(\omega t) \end{bmatrix}, \quad (1)$$

where g is the gravitational acceleration of applied gravity load, $F_{hd,y}$, $F_{hd,z}$ are the components of hydrodynamic force and $\Delta m E$ is the rotor static unbalance. Employing substitutions

$$\tau = \omega t, \quad \bar{y}_J = \frac{y_J}{c}, \quad \text{and} \quad \bar{z}_J = \frac{z_J}{c}, \quad (2)$$

where τ stands for non-dimensional time and \bar{z}_J and \bar{y}_J are non-dimensional coordinates of the journal centre related to radial bearing clearance c , allows to transform the equations of motion into the dimensionless form which is more suitable for deeper numerical analyses. The equations of motion can be further rewritten as

$$\begin{bmatrix} \bar{y}_J'' \\ \bar{z}_J'' \end{bmatrix} = -\frac{1}{\bar{\omega}^2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{m c \omega^2} \begin{bmatrix} F_{hd,y} \\ F_{hd,z} \end{bmatrix} + \begin{bmatrix} A_u \sin \tau \\ A_u \cos \tau \end{bmatrix}, \quad (3)$$

where the temporal derivatives, i.e., $\dot{\square} = \square' \omega$ etc., where $\dot{\square} = \frac{d}{dt} \square$ and $\square' = \frac{d}{d\tau} \square$, are held and the following non-dimensional parameters are defined:

$$\bar{\omega} = \omega \sqrt{\frac{c}{g}}, \quad A_u = \frac{\Delta m E}{m c}. \quad (4)$$

The design parameters of the rotor-bearing system are summarised in the non-dimensional parameter λ based on [3]

$$\lambda = \frac{\mu R L^3}{m c^{2.5} g^{0.5}}, \quad (5)$$

where R is the inner shell bearing radius and L is the bearing length.

The presented model (3) of the rotor-bearing model was analysed by the numerical continuation method employed in the open-source software MATCONT [1]. Using this method allows for tracing solution branches [2] of limit cycle oscillation (LCO) solution based on chosen bifurcation parameters $\bar{\omega}$, A_u , checking the stability criteria and revealing bifurcation points such as period-doubling (PD) and Neimark-Sacker (NS) bifurcation points. The obtained results (adopted from [3]) for two systems with various λ parameters of cylindrical journal bearing and

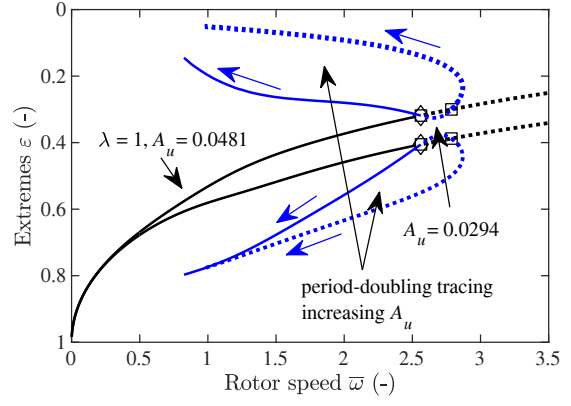
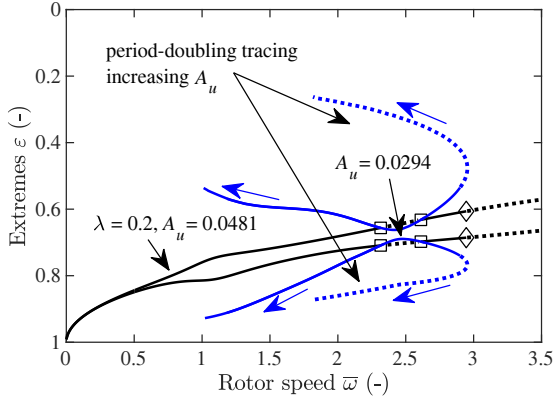


Fig. 1. Bifurcation diagram of the system response: synchronous LCO branches (stable '—', unstable '...'); codim-2 branch of PD bifurcation points in $(\bar{\omega}, A_u)$ space (stable '—', unstable '...'); NS bifurcation point '◇', PD bifurcation point '□'

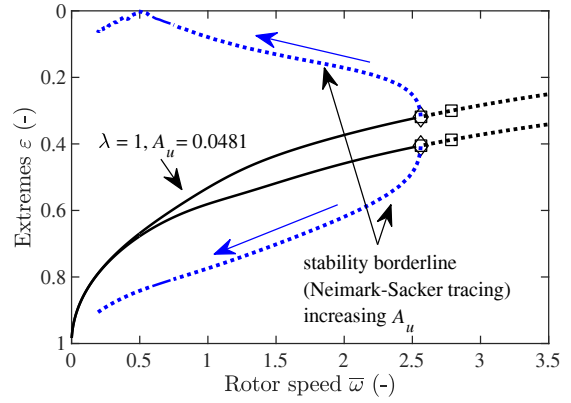
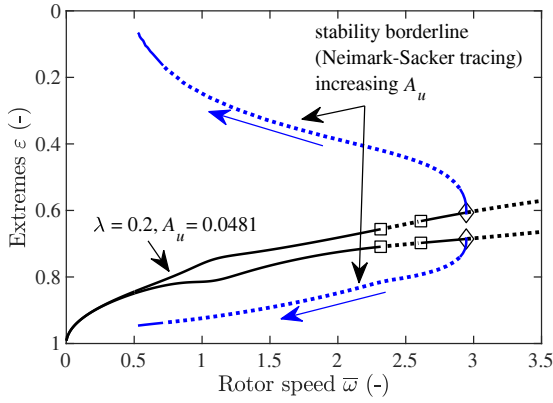


Fig. 2. Bifurcation diagram of the system response: synchronous LCO branches (stable '—', unstable '...'); codim-2 branch of NS bifurcation points in $(\bar{\omega}, A_u)$ space (stable '—', unstable '...'); NS bifurcation point '◇', PD bifurcation point '□'

tracing the bifurcation points for varying static unbalance A_u are depicted in Figs. 1–2. The hydrodynamic force is calculated based on the infinitely short bearing theory. The system stability is globally lost after crossing the NS bifurcation point. Moreover, there is an area between PD points when the 1-periodic stable solution is locally lost and new 2-periodic solution is born.

Acknowledgement

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References

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