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On a stepladder model walking (with and without a decorator)

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1. Introduction

This work is related to our previous studies on underactuated biped robot models and has been motivated by the need to implement the previously developed sensor and control algorithms for the real-time movement of the laboratory walking robot, designed and built at the Department of Control Theory of the Institute of Information Theory and Automation of the Czech Academy of Sciences [1, 6, 7].

Underactuated biped robots with an upper body form a subclass of legged robots, see, e.g., [4] for a review on the control of underactuated mechanical systems and [2] for a study of an asymptotically stable walking for biped robots. It is obvious that in general, the walking control of underactuated walking robots is a more challenging problem than walking control of fully actuated walking robots.

As follows, we examine the well-known mechanical system of the stepladder model with and without a decorator, whose role is substituted by an external inertial force according to the D'Alembert principle. It is well known, that stepladder walking is possible due to the periodic movement (pendulating) of an operator – decorator[1] The rigorous dynamical analysis of stable cyclic walking of a class of stepladder models is presented in the next section.

2. Stepladder model formulation

The stepladder model with, respectively without a decorator is schematically depicted in Fig. 1. It is similar to the so-called Compass gait biped walker (with neither ankles nor knees), which is the simplest underactuated mechanical system hypothetically able to walk, see, e.g., [3] and references therein. This three or two-link planar mechanism with two rigid legs, each one with a lumped mass (no inertia), connected at the revolute (hip) joint, is alternatively called the Acrobot (with or without an upper body). It has, in general, four or five degrees of freedom in 2-dimensional space. More precisely, using the notation introduced in Fig. 1: There are 2 degrees for the angles of two legs of equal length ($\theta_1,\ \theta_2$), 2 degrees for the position of hips (position of the center of mass O_H, and one degree of freedom for the angle θ_3 describing the angular position of a decorator (an upper body), respectively.

There is an important difference between the general Compass gait biped and stepladder walking: While the Compass gait usually switches the legs, stepladder walking resides in preserving the leg order.

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[1] One curious case of a 'self-induced' stepladder walking on a sloped plane is reported in YouTube (https://www.youtube.com/watch?v=v5yRvop08t0).

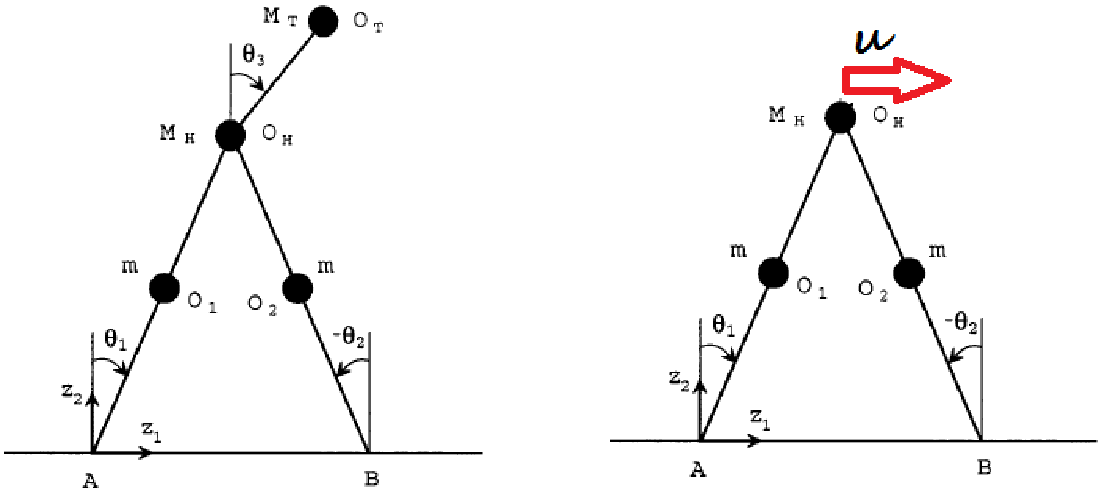

Fig. 1. (left) The stepladder model (with a decorator), (right) Compass gait biped walker (Acrobot) with external force applied on hip joint: parameters and coordinates

Next, we set up the system model for both the swing phase of the motion and the impact model, which has to be applied when both legs touch the ground.

First, the swing phase of the motion where either the forward or backward leg touches the ground: a dynamic equation, in the well-known form for mechanical systems obtained from the usual Lagrangian approach, follows

$$\mathbf{D}(\theta)\ddot{\theta} + \mathbf{C}(\theta, \dot{\theta})\dot{\theta} + \mathbf{G}(\theta) = \mathbf{u}, \tag{1}$$

where $\mathbf{D}(\theta)$ is the inertia matrix, $\mathbf{C}(\theta, \dot{\theta})$ contains Coriolis and centrifugal terms, $\mathbf{G}(\theta)$ contains gravity terms, $\mathbf{u}$ stands for the vector of external (and inertial) forces, θ, and $\dot{\theta}$ are 3-dimensional configuration vectors of angular positions and velocities, $\ddot{\theta}$ is the vector of accelerations.

Second, the very short, phase of the motion for the velocities just before (x^-) and just after (x^+) the impact, is governed by the impact model. The result of solving the corresponding equations yields an expression

$$x^+ = \Delta x^-, \tag{2}$$

which should then be used to re-initialize the model (1) with appropriate use of coordinates since the former swing leg is now in contact with the ground. The function Δ and other details are given, e.g., in [2].

Finally, the overall model can be expressed as a system with impulse effects. Striving for a (stable) cyclic walking, four consecutive time intervals can be distinguished:

1. Swing phase A: Point A is moving forward, and the governing equations are (1) with suitable initial conditions (of type A).
2. Impact A: Point A is touching ground, the governing equations are (2) with a suitable form of the compact operator (of type A).
3. Swing phase B: Point B is moving forward, and the governing equations are (1) with suitable initial conditions (of type B).
4. Impact B: Point B is touching the ground, the governing equations are (2) with a suitable form of the compact operator (of type B).

Two variants of the stepladder model are derived. The more complex one with a decorator (an upper body) with an autonomous movement $\theta_3 = f(t)$, and the simpler one, for the special form of the external force $u = g(t)$. The implementation is made in the computer algebra system *Mathematica.*

3. Conclusion

In this work, we formulated a model for stable cyclic walking for the stepladder with and without an operator. We used the idea of substituting the dynamic effects of the operator with the inertial forces applied to the revolving joint connecting both legs (center of mass O_H).

Now, encouraged by the successful implementation of the model, we are open to the possibility of running a study on the stability of cyclic walking for different values of model parameters and the operator movement. Here, the expected result is to find in some sense optimal (e.g., minimizing the energy input) stepladder walking regime.

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