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Efficient methods for calculating equivalent stiffness of circular composite tubes: Insights for structural design optimisation

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This paper presents the comparative analysis of various analytical methods used to estimate the equivalent stiffness of circular cross-section composite tubes manufactured using filament wound technology with long fibre laminates. Four computation methods, namely ABD matrices theory (ABD), minimum total potential energy theory (E), minimum complementary energy theory (C), and an averaged approach (avg), are evaluated. The comparison involves analysing a cantilever beam subjected to bending, torsion, and combined loading with varying force-to-moment ratios (as illustrated in Fig. 1).

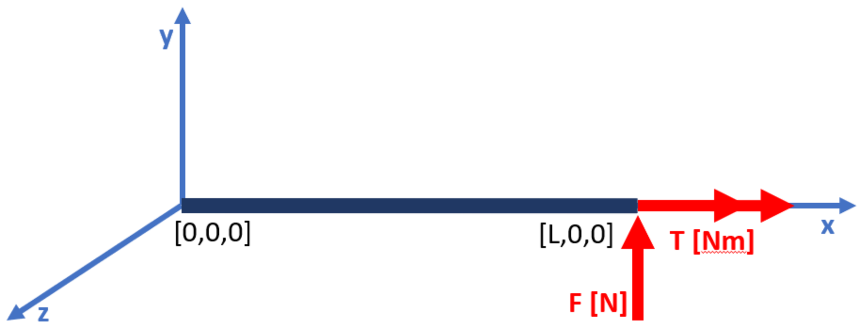

Fig. 1. Cantilever beam loaded with bending (bending force F) and torsion (torque moment T)

These theories are derived from the extended Hooke´s law for orthotropic materials, defining the stiffness matrix of a ply in the laminate coordinates E_{ti} akin to its inversion (compliance matrix C_{ti}), as found in references like [1]. Both matrices are determined by material mechanical parameters (Young´s module in the fibre direction E_L and perpendicular to the fibre direction E_T, shear module G_{LT}, and Poisson's ratio ν_{LT}), as well as the ply orientation angle (α_i).

Important geometric parameters used in calculations include tube outer diameter (d_{out}), inner diameter (d_{in}), outer radius (r_{out}), mean radius (r), inner radius (r_{in}), the number of plies in the laminate (n), ply thickness (t), and the thickness of the tube (T_T). The index 'i' refers to '$i-th$' ply of the laminate.

The equations for determining equivalent stiffnesses, such as tensile $(EA)_{eq}$, bending $(EJ)_{eq}$, torsional $(GJ_p)_{eq}$ and shear $(GA)_{eq}$), are provided in Table 1. The cross-sectional area $\tilde{A} = \pi(r_{out}^2 - r_{in}^2)$, the moment of inertia $J = \frac{\pi(d_{out}^4 - d_{in}^4)}{64}$, and the polar moment of inertia $J_p = \frac{\pi(d_{out}^4 - d_{in}^4)}{32}$. ABD matrices theory employs matrix A for calculation, representing laminate extensional stiffness. It is defined by the equation (1) as the sum of the product of individual ply stiffnesses and the ply thicknesses. The laminate tension module E_{eqABD} and shear module

G_{eqABD} are determined by members of the A matrix in equations (2) and (3), incorporating the laminate thickness (T_T) and auxiliary variable M (4).

$$A=\sum_{i=1}^{n} E_{ti}t_i\ , \tag{1}$$

$$E_{eqABD}=\frac{M}{T_T(A(3,3)A(2,2)-A(2,3)A(2,3))}\ , \tag{2}$$

$$G_{eqABD}=\frac{M}{T_T(A(1,1)A(2,2)-A(1,2)A(1,2))}\ , \tag{3}$$

$$M=A(1,1)A(2,2)A(3,3)+2A(1,2)A(1,3)A(2,3)-A(1,1)A(2,3)^2-A(2,2)A(1,3)^2-A(3,3)A(1,2)^2\ , \tag{4}$$

Table 1. Equations for calculating equivalent stiffnesses according to individual calculation methods

	Stiffness			
Theory	$(EA)_{eq}$	$(EJ)_{eq}$	$(GJ_p)_{eq}$	$(GA)_{eq}$
ABD	$E_{eqABD}\tilde{A}$	$E_{eqABD}J$	$G_{eqABD}J_p$	$G_{eqABD}\tilde{A}$
E	$\sum_{i=1}^{n}\pi(r_{outi}^2-r_{ini}^2)E_{ti}(1,1)$	$\sum_{i=1}^{n}\pi r_i^3 t_i E_{ti}(1,1)$	$\sum_{i=1}^{n}\frac{4A_{ci}^2 t_i E_{ti}(3,3)}{c_i}$	$\sum_{i=1}^{n}\pi(r_{outi}^2-r_{ini}^2)E_{ti}(3,3)$
C	$\sum_{i=1}^{n}\frac{\pi(r_{outi}^2-r_{ini}^2)}{C_{ti}(1,1)}$	$\sum_{i=1}^{n}\frac{\pi r_i^3 t_i}{C_{ti}(1,1)}$	$\sum_{i=1}^{n}\frac{4A_{ci}^2 t_i}{c_i C_{ti}(3,3)}$	$\sum_{i=1}^{n}\frac{\pi(r_{outi}^2-r_{ini}^2)}{C_{ti}(3,3)}$
avg	the average of the stiffness values obtained from the previous three methods			

The set of tubes were loaded by bending force $F = 1000$ N, torsion moment $T = 1000$ Nm, or by its combination, where the ratios $T/F = 1,\ 10,\ 100$ or 1000 with constant value of bending force $F = 1000$ N. The value of displacement for pure bending, rotation of pure torsion, and minimum equivalent energy of combined loading was shown in a dependence graph of the given value on the tube winding angle. These dependencies were plotted for tubes of different thicknesses ($t = 0.2, 0.5, 1, 2, 5, 10$ and 20 mm), which winding is composed of four layers with thicknesses $t/4$ and winding angles $\pm\alpha$.

Inner diameters of analysed tubes were $d = 5, 10, 20, 40, 80$ mm, and the tube length-to-inner diameter ratios $L/d = 1, 5, 10, 20$ and 50. Material parameters of analysed tubes were set as T700/epoxy laminate properties: $E_L = 128000$ MPa, $E_T = 5060$ MPa, $G_{LT} = 3400$ MPa and $\nu_{LT} = 0.345$.

A typical angle-to-displacement dependence for the bended beam, with a length-to-diameter ratio greater than 5, is displayed in Fig. 2. Similarly, the angle-to-rotation dependence is presented in Fig. 3. The angle-to-equivalent energy dependence for beams subjected to combined loading highly depends on the torsion moment-to-bending force ratio, which affects the position of the extremes of the given dependencies, as shown in Fig. 4. The top graph shows an example of a load with a bending force of 1000 N and a torque of 1000 Nmm, while the bottom graph shows a load with a bending force of 1000 N and a torque of 1000 Nm.

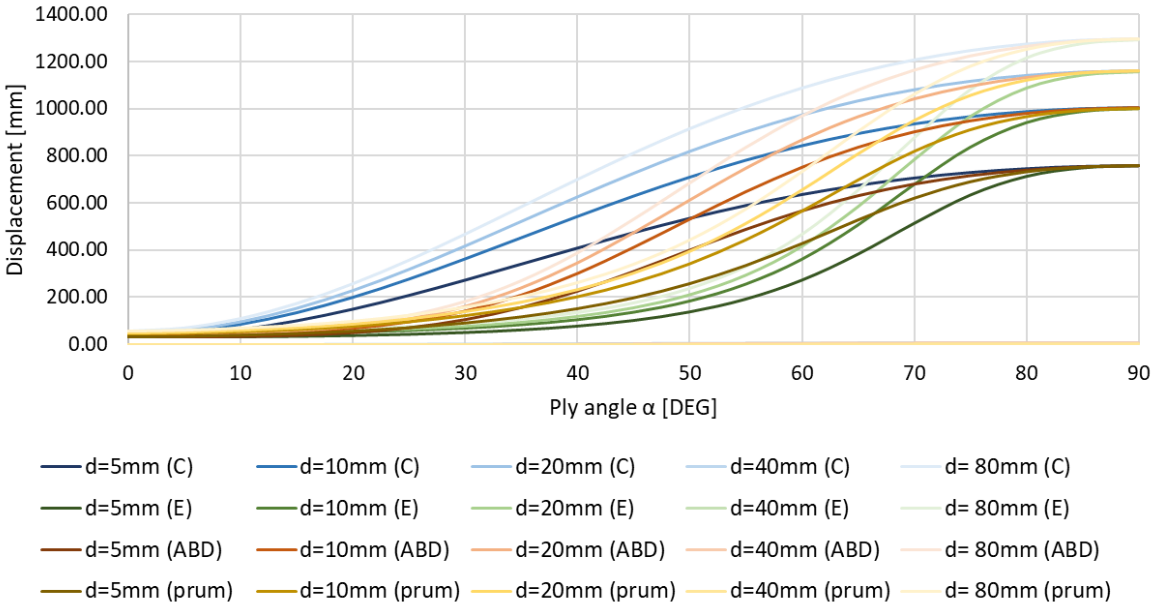

Fig. 2. A typical angle-to-displacement dependence for bended beam

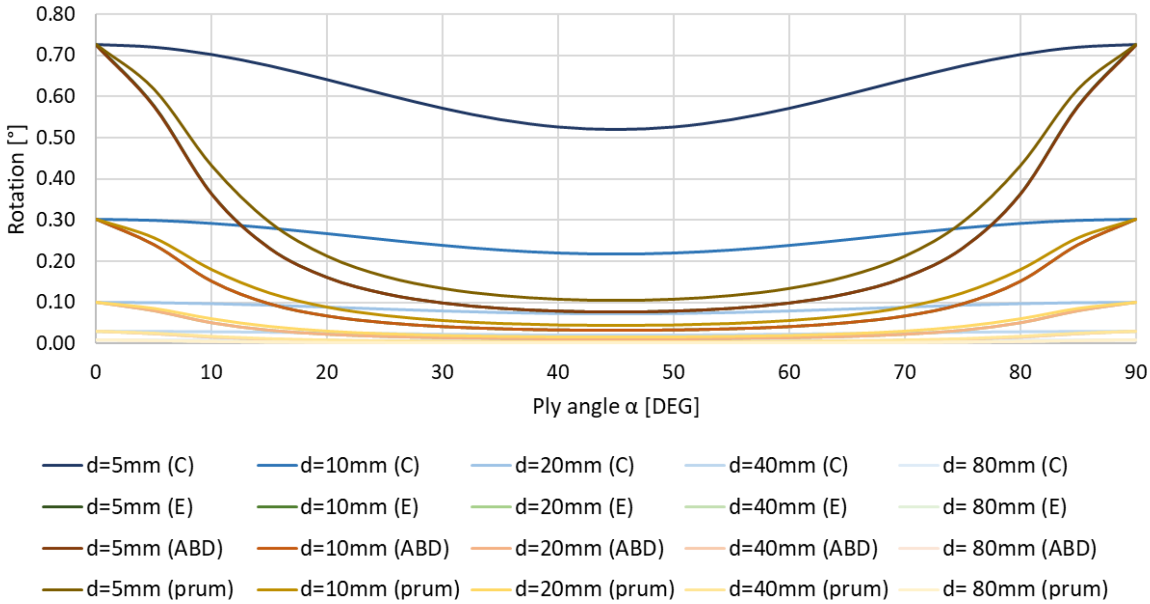

Fig. 3. A typical angle-to-rotation dependence for torsion

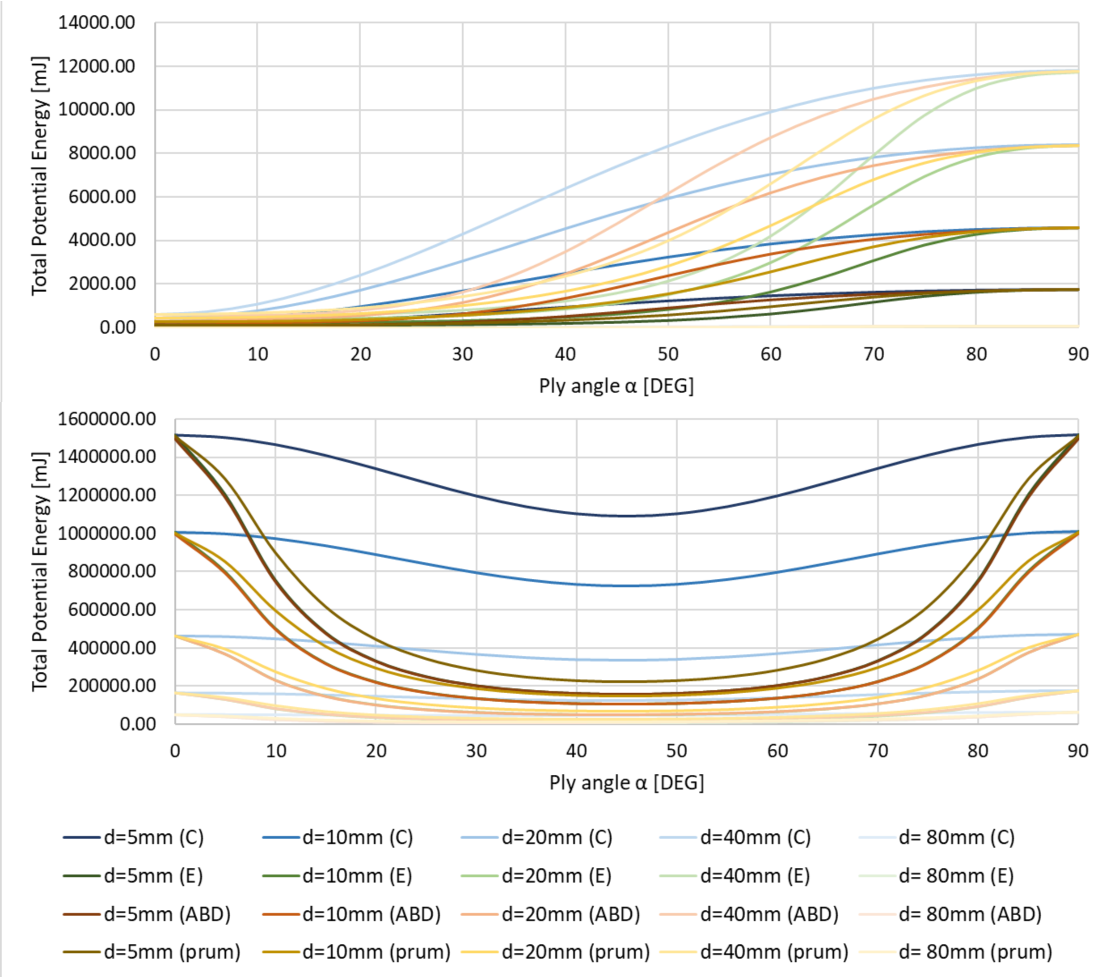

Fig. 5. Angle-to-total potential energy dependence

As evident from the aforementioned dependencies, the minimum complementary energy theory exhibits the highest absolute values, while the trends remain similar across the selected theories. This underscores the preference for utilizing the minimum complementary energy approach in optimizing the winding parameters for composite tubes. The theory's higher values correspond to lower stiffness, which enhances the safety margin in the design process. It is worth noting that extreme cases involving beams with a length-to-diameter ratio of one have been excluded from consideration for the given structures, as such geometries are no longer deemed as beams in this context.

Acknowledgement

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References

[1] Jones, R. M, Mechanics of Composite Materials, second ed., Taylor & Francis Ltd., London, 1999.