Three Essays in Commodity Futures and Options Price Performance

by

Marin Božić

A dissertation submitted in partial fulfillment of

the requirements for the degree of

Doctor of Philosophy

(Agricultural and Applied Economics)

at the

UNIVERSITY OF WISCONSIN-MADISON

2011

Acknowledgements

A single name may stand on the front page of this thesis, but more than a few mentors, teachers and friends were the influence, support and inspiration without which this endeavor could hardly be completed, and perhaps not even be embarked upon. First, I want to express my gratitude to my advisor, Dr. T. Randall Fortenbery, who has supervised this work over the past three years, and has provided me with essential financial and intellectual support. Randy's understanding of commodity markets is without par, and through many debates he has challenged me to think more deeply, write more clearly and has corrected more omitted definite articles in my drafts than I would like to admit.

I have also benefited immensely from discussions with Professors Jean-Paul Chavas, Brian W. Gould, Bruce E. Hansen and James E. Hodder. I am deeply thankful for their willingness to serve on my PhD thesis committee and read through early drafts of these essays. Their substantial scholarly contributions, sharp intellectual instincts and, above all, kind nature and spirit illuminate all students fortunate to be able to work with them.

A special thanks goes to my colleagues and friends at The Institute of Economics, Zagreb where I started my post-college career before coming to US for graduate studies. Without the encouragement I got from Dr. Nenad Starc, the head of my division and a dear mentor and friend, and Dr. Sandra Švaljek, the Director of the Institute, America would perhaps remain only a dream. Financial support of the Institute allowed me, among other things, to visit Croatia each summer and present my work at a workshop in Zagreb.

A unique influence to my thinking, and an unfading inspiration in integrity, Dr. Ivo Bićanić of the Faculty of Economics at the University of Zagreb, more than any other single person, set me on the path I thread today. I keenly remember one lecture on macroeconomics of Croatia where he scolded me in front of the entire class for not paying attention – and even the fact that I was at that moment reading his own newly published article did not help!

My parents, my brother, and my late grandma were with me from the beginning of my graduate studies, through all my efforts and many Skype conversations that were often more successful at conveying their love than clear audio of their voice. By ourselves, we are a bunch of headstrong individuals, but together we share deep trust and unyielding silent support that gives space when space is asked for, and support when support is needed.

This acknowledgement would be unforgivingly incomplete if I do not mention Josip Glaurdić, my dear friend since high school days, who set the bar as the first kid I knew that went to the USA to study; Jeremy Weber, a genius of balanced life and an unmatched dedicated spirit; and, Dan Prager, with whom I shared many nice moments discussing with much passion issues both deep and shallow, U.S. politics being often the latter.

Finally, I dedicate this thesis to my partner and my best friend, Debanjana Chatterjee. Others may benefit from this work, but she has borne most of the cost. Through all-nighters spent over Gauss code and many precious moments of peace I chose to forfeit in pursuit of my degree, she has given me her full heart and mind, and I am forever grateful for her support, patience and wisdom.

List of Figures

List of Tables

THREE ESSAYS IN COMMODITY FUTURES AND OPTIONS PRICE PERFORMANCE

Marin Božić

Under the supervision of RENK Professor of Agribusiness T. Randall Fortenbery

At the University of Wisconsin-Madison

Abstract

In the first essay I propose a novel pricing model for options on commodity futures motivated from the economic theory of optimal storage, and consistent with implications of plant physiology on the importance of weather stress. The model is based on a Generalized Lambda Distribution (GLD) that allows greater flexibility in higher moments of the expected terminal distribution of futures price. I find a statistically significant negative relationship between ending stocks-to-use and implied skewness, as predicted by the theory of storage. Intra-year dynamics of implied skewness reflect the fact that resolution of uncertainty in corn supply is resolved during the corn growth phase from corn silking through maturity. Impacts of storage and weather on the distribution of terminal futures prices jointly explain upward sloping implied volatility curves.

In the second paper, a partially overlapping time series (POTS) model is estimated to examine price behavior in simultaneously traded Class III milk futures contracts. POTS is a latent factor model that measures price changes in futures as a linear combination of a common factor, i.e. information affecting all traded contracts, and an idiosyncratic term specific to each contract. The importance of a common factor in price volatility determination for dairy is related to capital production factors, i.e. the dairy herd. It is shown that Class III volatility decreases as contracts approach maturity. The importance of the common factor declines as one approaches maturity, implying that individual contract months are poor substitutes in hedging a specific month's cash

price risk. Thus, despite relatively low liquidity in the market, it is useful to have 12 contract delivery months per year.

The third essay examines price discovery, volatility spillovers and the impacts of speculation in the dairy sector. I find that the flow of information in the mean prices is predominantly from futures to cash, while volatility spillovers are bidirectional. I propose an extension of the BEKK variance model that I refer to as GARCH-MEX. Utilizing the model to evaluate the impact of speculation I find strong evidence against the hypothesis that excessive speculation is increasing the conditional variance of futures prices.

Table of Contents

1. Pricing Options on Commodity Futures: The Role of Weather and Storage

Abstract: Options on agricultural futures are popular financial instruments used for agricultural price risk management and to speculate on future price movements. Poor performance of Black's classical option pricing model has stimulated many researchers to introduce pricing models that are more consistent with observed option premiums. However, most models are motivated solely from the standpoint of the time series properties of futures prices and need for improvements in forecasting and hedging performance. In this paper I propose a novel arbitrage pricing model motivated from the economic theory of optimal storage, and consistent with implications of plant physiology on the importance of weather stress. I introduce a pricing model for options on futures based on a Generalized Lambda Distribution (GLD) that allows greater flexibility in higher moments of the expected terminal distribution of futures price. I use times and sales data for corn futures and options for the period 1995-2009 to estimate the implied skewness parameter separately for each trading day. An economic explanation is then presented for inter-year variations in implied skewness consistent with the theory of storage. After controlling for changes in planned acreage, I find a statistically significant negative relationship between ending stocks-to-use and implied skewness, as predicted by the theory of storage. Furthermore, intra-year dynamics of implied skewness reflect the fact that resolution of uncertainty in corn supply is resolved between late June and middle of October, i.e. during corn growth phases that encompass corn silking through grain maturity. Impacts of storage and weather on the distribution of terminal futures price jointly explain upward sloping implied volatility curves.

JEL Codes: G13, Q11, Q14

Keywords: arbitrage pricing model, options on futures, generalized lambda distribution, theory of storage, skewness

1.1. Introduction

Options written on commodity futures have been investigated from several aspects in the commodity economics literature. For example, Lence (1994), Vercammen (1995), Lien and Wong (2002), and Adam-Müller and Panaretou (2009) considered the role of options in optimal hedging. Use of options in agricultural policy was examined by Gardner (1977), Glauber and Miranda (1989), and Buschena (2008). The effects of news on options prices has been investigated by Fortenbery and Sumner (1993), Isengildina-Massa, Irwin, Good, and Gomez (2008) and Thomsen (2009). The informational content of options prices has been looked into by Fackler and King (1990), Sherrick, Garcia and Tirupattur (1996), and Egelkraut, Garcia, and Sherrick (2007). Some of the most interesting work done in this area considers modifications to the standard Black-Scholes formula that accounts for non-normality (skewness, leptokurtosis) of price innovations, heteroskedasticity, and specifics of commodity spot prices (e.g. mean-reversion). Examples include Kang and Brorsen (1995), and Ji and Brorsen (2009).

In this article I revisit the well-known fact that the classical Black's (1976) model is inconsistent with observed option premiums. Previous studies like Fackler and King (1990) and Sherrick et al. (1996) address this puzzle by identifying properties of futures prices that deviate from assumptions of Black's model, i.e. leptokurtic and skewed distributions of the logarithm of terminal futures prices and stochastic volatility. A common feature of past studies is the grounding of their arguments in the time-series properties of stochastic processes for futures prices and the distributional properties of terminal futures prices. In other words, their arguments are primarily statistical. In contrast to previous studies, I offer an economic explanation for the observed statistical characteristics. In this paper I analyze in detail options on corn futures. The focus is on presenting an alternative pricing model that is not motivated by improving the forecasts of options premiums compared to Black's or other models, but by linking option pricing models with the economics of supply for annually harvested storable agricultural commodities. In particular, I demonstrate the effect of storability and crop physiology (i.e. susceptibility to weather stress) on higher moments of the futures price distribution. Only by understanding these fundamental economic forces can I truly explain why classical option pricing models work so poorly for commodity futures.

The article is organized as follows. In the next section I examine in detail the implications of Black's classical option pricing model on the shape and dynamics of the futures price distribution. I follow by summarizing the rational expectations competitive equilibrium model with storage (Williams and Wright,1991; Deaton and Laroque, 1992), and a testable hypothesis on conditional new crop price distributions that follows from those models. In addition to storage, I present the agronomical research on the impact of weather on corn yields. I then develop a novel arbitrage pricing model for options on commodity futures based on the Generalized Lambda Distribution (GLD) which I propose to use in calibrating skewness of new crop futures price to match observed option premiums. The third section describes the econometric model. In the fourth section I summarize the data used in econometric analysis. Finally, I describe the estimation procedure and present results of statistical inference, followed by a set of conclusions and directions for further research.

1.2. Theory

1.2.1. Foundations of Arbitrage Pricing Theory for Options on Futures

Black (1976) was the first to offer an arbitrage pricing model for options on futures contracts. Despite numerous extensions and modifications proposed in the literature, and the inability of the model to explain observed option premiums, traders still use this model in practice. This is likely due to its simplicity and ability to forecast option premiums after appropriate "tweaks" are put in place. Black proposes that futures prices follow a stochastic process as described below:

$$dF = \sigma F dz \tag{1.1}$$

where F stands for futures price, σ for volatility, and dz is an increment of Brownian motion.

The implication is that futures prices are unbiased expectations of terminal futures prices (ideally equal to the spot price at expiration), and the stochastic process followed by futures prices is a geometric Brownian motion.

Under this scenario the option premium V is equal to the present value of the expected option payoff under a risk-neutral distribution for terminal prices. For example, for a call option with strike K, volatility σ, risk-free interest rate r and time left to maturity T:

$$V\left(K, F_0, T, \sigma, r\right) = e^{-rT} \int_0^{\infty} Max\left(F_T - K, 0\right) f\left(F_T; F_0, \sigma, r, T\right) dF_T \tag{1.2}$$

Because delta hedging with futures does not require a hedger to pay the full value of the futures contract due to margin trading, a risk-neutral terminal distribution for futures prices is equivalent to a risk-neutral terminal distribution for a stock that pays a dividend yield equal to the risk-free interest rate:

$$\ln F_T \sim N\left(\ln F_0 - \frac{1}{2}\sigma^2, \sigma^2 T\right) \tag{1.3}$$

Thus, Black's model postulates that the distribution of terminal futures prices, conditional on information known at time zero, is lognormal with the first four moments fully determined by the current futures price and volatility parameter σ. In particular, the first four moments of the risk-neutral terminal distribution are equal to:

$$\tilde{\mu} = F_0 \quad \tilde{\sigma}^2 = F_0^{\ 2}\left(e^{\sigma^2 t} - 1\right) \quad SKEW = (e^{\sigma^2 t} + 2)\sqrt{e^{\sigma^2 t} - 1} \quad KURT = e^{4\sigma^2 t} + 2e^{3\sigma^2 t} + 3e^{2\sigma^2 t} \tag{1.4}$$

For example, if a futures price is $2.50, volatility is 30%, and there are 160 days left to maturity, the standard deviation of the terminal distribution would be $0.50, skewness would be 0.60 and kurtosis would be 3.64. Therefore, the standard Black's model implies that the expected distribution of terminal prices would be positively skewed, and leptokurtic. When complaints are raised that Black's model imposes normality restrictions, it is the logarithm of the terminal price that the critique refers to.

The standard way to check if Black's model is an appropriate pricing strategy is to exploit the fact that for a given futures price, strike price, risk-free interest rate, and time to maturity, the model postulates a one-to-one relationship between the volatility coefficient and the option premium. Thus, the pricing function can be inverted to infer the volatility coefficient from an observed option premium. Such coefficients are referred to as *implied volatility* and the principal testable implication of Black's model is that implied volatility does not depend on how deep in-the-money or out-of-money an option is. If the logarithm of terminal price is not normally distributed, then Black's model is not appropriate, and implied volatility (IV) will vary with option moneyness – a flagrant violation of the model's assumptions. Black's model gives us a pricing formula for European options on futures, i.e. options that can only be exercised at contract maturity. Prices of American options on futures that are assumed to follow the same stochastic process as in Black's model must also account for the possibility of early exercise. For that reason, their prices cannot be obtained through a closed-form formula, but must be estimated through numerical methods such as the Cox, Ross and Rubinstein (CRR) (1979) binomial trees.

Implied volatility curves for storable commodity products are almost always upward sloping. As an example consider the December 2006 corn contract. The futures price on June 26, 2006 was $2.49/bu. As seen in Figure 1.1, the implied volatility curve associated with calculating IV using various December option strikes is strongly upward sloping, with the implied volatility coefficients for the highest strike options close to 15 percentage points higher than the implied volatility for options with lower strikes.

Geman (2005) calls this phenomenon an "inverse leverage effect," after the "leverage effect" proposed to explain downward sloping implied volatility curves for individual company stocks. However, this is a complete misnomer. As Black (1976b) explains, the leverage effect arises from the fact that as stock price declines, the ratio of a company's debt to equity value, its leverage, increases. If the volatility of company assets is constant, then as the equity share of assets declines, volatility in equity will increase. While the leverage effect has a coherent causal model to justify the term, nothing explains "inverse leverage effect."

We can gain further insight as to how Black's model performs if we plot the implied volatility curve for a single contract at different time-to-maturity horizons. As an example, consider December corn contracts in the years 2004 and 2006. As Figure 1.2 shows, three distinct patterns are noticeable. First, except when options are very near maturity, we always see an upward sloping implied volatility curve. Second, implied volatility of at-the-money options, i.e. options that have the strike price equal to the current futures price, rises almost linearly until the end of June, declines throughout the summer months, and then starts rising again. Finally, near maturity, volatility skews give way to symmetric volatility smiles. The implied volatility coefficient measures volatility on an annual basis, and the variance of the terminal price, conditional on time remaining to maturity, is $\sigma^2 \left(T-t\right)$. So if uncertainty about the terminal price is uniformly resolved as time passes, implied volatility will not decrease, but will stay the same. Likewise, when the same amount of uncertainty needs to be resolved in a shorter time interval implied volatility will increase. Therefore, linear increases in implied volatility from distant horizons up until June is best interpreted not as increases in day to day volatility of futures price changes, but a market consensus that the conditional variance of terminal prices is not much reduced before June.

While CRR binomial trees preserve the basic restrictions of Black's model, i.e. the normality of the log-prices terminal distribution, Rubinstein (1994, 1998) shows how that can be relaxed to allow for non-normal skewness and kurtosis. To illustrate the effect of skewness and kurtosis on Black's implied volatility I used Edgeworth binomial trees (Rubinstein, 1998). This allows for pricing options that exhibit skewed and leptokurtic distributions of terminal log-prices. As can be seen in panel 1 in Figure 1.3, zero skewness and no excess kurtosis (S=0, K=3) corresponds to a flat IV curve, i.e. CRR implied volatility estimated from options premiums is the same no matter what strike is used to infer it, just like Black's model would have it. A leptokurtic distribution will cause so called "smiles", i.e. options with strikes further away from the current futures price will produce higher implied volatility coefficients. Positive skewness creates an upward sloping curve, and negative skewness a downward sloping IV curve.

Faced with the inability of Black's model to explain observed option premiums, researchers and traders have pursued three different approaches to address this issue:

1) Start from the end: relax the assumptions concerning risk-neutral terminal distributions of underlying futures prices, i.e. allow for non-lognormal skewness and kurtosis. As long as delta hedging is possible at all times (i.e. markets are complete), it is still possible to calculate option premiums as the present value of expected option payoffs. Examples of this approach include Jarrow and Rudd (1982), Sherrick et al. (1996), and Rubinstein (1998). While the formulas that derive option premiums as discounted expected payoffs

assume that options are European, one can still price American options using implied binomial trees calibrated to the terminal distribution of choice (Rubinstein, 1994).

2) Start from the beginning: start by asking what kind of stochastic process is consistent with a non-normal terminal distribution? By introducing appropriate stochastic volatility and/or jumps, one might be able to fit the data just as well as by the approach above. Examples of this approach are Kang and Brorsen (1995), Hilliard and Reis (1998) and Ji and Brorsen (2009).
3) "Tweak it so it works good enough" approach: if one is willing to sacrifice mathematical elegance, the coherence of the second approach, and insights that might emerge from the first approach, and if the only objective is the ability to forecast day-ahead option premiums one can simply tweak Black's model. An example of such an approach would be to model the implied volatility coefficient as a quadratic function of the strike. Even though it makes no theoretical sense (this is like saying that options with different strikes live in different universes), this approach will work good enough for many traders. Just as in that famous saying by Yogi Berra (2010): "In theory, there is no difference between theory and practice. In practice, there is." A seminal article that evaluates the hedging effectiveness of such an approach is Dumas, Fleming and Whaley (1998). The authors find that for hedging purposes such an ad-hoc approach seems to work equally well compared to the more sophisticate and theoretically coherent models they evaluate.

In this article I take the first approach, and modify the Black's model by modifying the terminal distribution of futures price. Instead of a lognormal, I propose a generalized lambda distribution (GLD) developed by Ramberg and Schmeiser (1974) and introduced to options pricing by Corrado (2001). An alternative would be to use Edgeworth binomial trees, but preliminary analysis showed that such an approach may not be adequate for situations where skewness and kurtosis are rather high. In addition, Edgeworth trees work with the skewness of terminal log-prices, while I prefer to have implied parameters for the skewness of terminal futures prices directly, not their logarithms. In addition, the GLD pricing model allows for a higher degree of flexibility in terms of skewness and kurtosis, i.e. its' parameters are rather easy to calibrate from observed options prices and it is straightforward to develop a closed-form solution for pricing options. While these are all favorable characteristics, it is in fact the ability to gain additional economic insight that truly justifies yet another option pricing model. GLD allows us to get an explicit estimate of skewness and kurtosis of the terminal distributions, that can be used to make a strong connection between the economics of supply for storable agricultural commodities and financial models for pricing options on commodity futures. As is usually the case with option pricing, we are estimating risk-neutral, rather than physical (i.e. true) moments of the price distribution. In subsequent analysis we assume that all risk-adjustements are contained in the first moment of the distribution, i.e. the level of a futures price.

1.2.2. Storage and Time-series Properties of Commodity Spot and Futures Prices

Deaton and Laroque (1992) used a rational expectations competitive storage model to explain nonlinearities in the time series of commodity prices: skewness, rare but dramatic substantial increases in prices, and a high degree of autocorrelation in prices from one harvest season to the next. The basic conclusion of their work was that the inability to carry negative inventories introduces a non-linearity in prices that manifests itself in the above characteristics.

This is an example of theory being employed in an attempt to replicate patterns of observed price data. In a similar fashion, but subtly different, Williams and Wright (1991) postulate that the moments of expected price distributions at harvest time vary with the current (pre-harvest) price and available carryout stocks, as shown in Figure 1.4. According to them, when observed at annual or quarterly frequency, spot prices exhibit positive autocorrelation that emerges because storage allows unusually high or low excess demand to be spread out over several years. Furthermore, the variance of price changes depends on the level of inventory. When stocks are high, and the spot price is low, the abundance of stored stocks serves as a buffer to price changes, and variance is low. When stocks are low, and thus the spot price is high, stocks are not sufficient to buffer price changes. Finally, the third moment of the price change distribution also varies with inventories. Since storage can always reduce the downward price pressure of a windfall harvest, but cannot do as much for a really bad harvest, large price increases are more common than large decreases. The magnitude of this cushioning effect of storage depends on the size of the stocks. In conclusion, one should expect commodity prices to be mean-stationary, heteroskedastic and with conditional skewness, where both the second and third moments depend on the size of the inventories.

Testing the theory proceeds with this argument: if we can replicate the price pattern using a particular set of rationality assumptions, then we cannot refute the claim that markets indeed behave as described above. That is the road taken by Deaton and Laroque (1992) and Miranda and Rui (1995). However, since in the spot price series we only see the realizations of prices, not the conditional expectations of them, we cannot use spot price data to directly test what the market *expected* to happen. As such, predictions from storage theory focused on the scale and shape of expected distributions of new harvest spot prices have remained untested. In this paper I use options data to infer the conditional expectations of terminal futures prices, and therefore test the following prediction of the theory of storage:

- The lower inventories are relative to consumption, the more positive will be the skewness of the conditional harvest futures price distribution

Without building a complete model of production with storage it is not feasible to ascertain the sensitivity of predictions emerging from Figure 1.4 to values of particular parameters. For example, a more elastic supply response could perhaps weaken the link between expected ending stocks-to-use and skewness of expected new-crop harvest price. Likewise, trade with countries whose growth cycle does not coincide with the U.S. could allow for quicker adjustments to scarce domestic stocks. While these extensions are needed for a complete account of the impact of storability on harvest price distributions, in this paper I focus on developing methods that would allow me to test the predictions on price behavior postulated by classic works of Williams and Wright, as well as Deaton and Laroque. In pursuing this analysis I are thus assuming that extensions of the cited papers that would incorporate richer supply structure would still preserve the viability of the central hypothesis of this paper, i.e. inverse relationship between skewness of terminal prices and relative abundance of stocks that can serve as buffer in face of supply or demand shocks.

In addition, it is worth emphasizing that it is not claimed here that storage affects only skewness, as the impact will likely be on all moments of the distribution. However, as the primary task of the paper is to explain upward sloping implied volatility curves, based on preliminary analysis of implied volatility curves in section 1.2.1 it seems reasonable to put primary focus on skewness.

My plan is to use an options pricing formula based on the generalized lambda distribution to calibrate the skewness and kurtosis of expected (conditional) harvest futures price distributions. Implied parameters from the model are then used to test the hypothesis above.

1.2.3. The Role of Weather in Intra-year Resolution of Price Uncertainty

As illustrated in section 1.2.1, a very small share of uncertainty concerning the terminal price of a new crop futures contract is resolved before June. A large part of the uncertainty is resolved between late June and early October. The reason lies in corn physiology and the way weather stress impacts corn throughout the growing season. In the major corn producing areas of the U.S., corn is planted starting the last week of April. It takes about 80 days after planting for a plant to reach its reproduction stage, also known as corn silking. At this juncture the need for nutrients is highest, and moisture stress has a large impact on final yield. Weather continues to play an important role through the rest of the growing cycle, as summarized by Figure 1.5, taken from Shaw et al. (1988).

Beginning in July, the United States Department of Agriculture (USDA) publishes updated forecasts of corn yield per acre. At the beginning of the growing season, before corn starts silking, production forecasts ae generally based on estimated acres and historical trend yields. As can be seen in Figure 1.6, June forecasts of final yield deviated from the historical trend value essentially the same in both what was at the time the record-setting yield year 2004/2005 when final yield was 15 bushels above the trend, and the major draught year of 1988/89 when final yields were 32 bushels below the trend. However, uncertainty is quickly resolved in July and August. As shown in Figure 1.7, whereas June forecasts deviated from final estimates from the low of -11% in 1994/95 to high of 45% in 1988/89, the September estimate deviations ranged only from -7% to 12%. Besides weather, more precise methods used by USDA from August onwards estimate final yields also contribute to decrease in uncertainty. Starting in late July, and first reported in August edition of the *Crop Production* report, final yields are estimated not only based on statistical models that control for trend and crop condition, but also include information obtained through grower-reported yield survey and objective measurement survey.

A testable hypothesis that emerges from these stylized facts concerns the fundamental role of seasonality in uncertainty resolution, as well as pronounced negative skewness in deviations of final yields from trend values. In other words, do seasonal yield deviations contribute to a positive skewness of the terminal price distribution and the dynamics of skewness throughout the marketing year? In particular, we might expect implied skewness to decrease throughout the growing season.

1.2.4. Option Pricing Formula Using Generalized Lambda Distribution

The generalized lambda distribution (GLD) was developed by Ramberg and Schmeiser (1974), with Ramberg et al. (1979) further describing its properties. It was introduced to options pricing by Corrado (2001) who derived a formula for pricing options on non-dividend paying stocks. Here I review the properties of GLD and adopt Corrado's formula to options on futures.

GLD is most easily described by a percentile function[1] (i.e. inverse cumulative density function):

$$F\left(p\right)=\lambda_1+\frac{p^{\lambda_3}-\left(1-p\right)^{\lambda_4}}{\lambda_2} \tag{1.5}$$

For example, to say that for $p=0.90, F\left(p\right)=4.5$ means that the market expects with a 90% probability that the terminal futures price will be lower than or equal to \$4.50/bu.

GLD has four parameters: λ_1 controls location, λ_2 determines variance, and λ_3 and λ_4 jointly determine skewness and kurtosis. In particular the mean and variance are calculated as follows:

---

[1] F here stands for futures price, not for cumulative density function.

$$\mu = \lambda_1 + A/\lambda_2$$
$$\sigma^2 = \left(B - A^2\right)/\lambda_2^2 \tag{1.6}$$

with $A = \frac{1}{1+\lambda_3} - \frac{1}{1+\lambda_4}$ and $B = \frac{1}{1+2\lambda_3} + \frac{1}{1+2\lambda_4} - 2\beta\left(1+\lambda_3, 1+2\lambda_4\right)$,where $\beta(\ )$ stands for the complete beta function. Ramberg et al. give expressions for the third and fourth central moments of the distribution:

$$\mu_3 = E\left(x-\mu\right)^3 = \frac{C - 3AB + 2A^3}{\lambda_2^3}$$
$$\mu_4 = E\left(x-\mu\right)^4 = \frac{D - 4AC + 6A^2B - 3A^4}{\lambda_2^4}$$

The skewness and kurtosis formulas are:

$$\alpha_3 = \frac{\mu_3}{\sigma^3} = \frac{C - 3AB + 2A^3}{\lambda_2^3\sigma^3} = \frac{C - 3AB + 2A^3}{\left(B - A^2\right)^{3/2}}$$
$$\alpha_4 = \frac{\mu_4}{\sigma^4} = \frac{D - 4AC + 6A^2B - 3A^4}{\sigma^4\lambda_2^4} = \frac{D - 4AC + 6A^2B - 3A^4}{\left(B - A^2\right)^2} \tag{1.7}$$

where expressions for *C* and *D* are:

$$C = \frac{1}{1+3\lambda_3} - \frac{1}{1+3\lambda_3} - 3\beta\left(1+2\lambda_3, 1+\lambda_4\right) + 3\beta\left(1+\lambda_3, 1+2\lambda_4\right)$$

$$D = \frac{1}{1+4\lambda_3} + \frac{1}{1+4\lambda_3} - 4\beta\left(1+3\lambda_3, 1+\lambda_4\right) - 4\beta\left(1+\lambda_3, 1+3\lambda_4\right) + 6\beta\left(1+2\lambda_3, 1+2\lambda_4\right)$$

We see that the λ_3 and λ_4 parameters influence both location and variance, however λ_1 influences only the first moment, and λ_2 influences only the first two moments. Thus, skewness and kurtosis do not depend on λ_1 and λ_2.

A standardized GLD has a zero mean and unit variance, and has a percentile function of the form:

$$F(p)=\frac{1}{h(\lambda_3,\lambda_4)}\left(p^{\lambda_3}-(1-p)^{\lambda_4}+\frac{1}{\lambda_4+1}-\frac{1}{\lambda_3+1}\right) \tag{1.8}$$

with $\lambda_2=h(\lambda_3,\lambda_4)=sign(\lambda_3)\times\sqrt{B-A^2}$ and $\lambda_1=\left(\frac{1}{\lambda_4+1}-\frac{1}{\lambda_3+1}\right)/h(\lambda_3,\lambda_4)$.

From here, we can move more easily to an options pricing environment. We wish to make GLD an approximate generalization of the log-normal distribution so I keep the mean and the variance the same as in (1.4), while allowing skewness and kurtosis to be separately determined by the λ_3 and λ_4 parameters. Therefore, the percentile function relevant for option pricing will be

$$F(p)=F_0\left(1+\frac{\sqrt{e^{\sigma^2 t}-1}}{h(\lambda_3,\lambda_4)}\left(p^{\lambda_3}-(1-p)^{\lambda_4}+\frac{1}{\lambda_4+1}-\frac{1}{\lambda_3+1}\right)\right) \tag{1.9}$$

Note that this is equivalent to (1.5) with $\lambda_1=F_0+\frac{\sqrt{e^{\sigma^2 t}-1}}{h(\lambda_3,\lambda_4)}\left(\frac{1}{\lambda_4+1}-\frac{1}{\lambda_3+1}\right)$

and $\lambda_2=\frac{h(\lambda_3,\lambda_4)}{\sqrt{e^{\sigma^2 t}-1}}$. This will guarantee that the first two moments of the terminal distribution will be $\tilde{\mu}=F_0 \quad \tilde{\sigma}^2=F_0{}^2\left(e^{\sigma^2 t}-1\right)$, just as in Black's model.

The pricing formula for European calls is

$$V(K,F_0,T,\sigma,r,\lambda_3,\lambda_4)=e^{-rT}\int_0^\infty Max(F_T-K,0)\,dp(F) \tag{1.10}$$

As shown by Corrado (2001), I can simplify this through a change-of-variable approach where $F(p)=F_T$:

$$\int_0^\infty Max(F_T-K,0)\,dp(F)=\int_K^\infty (F_T-K)\,dp(F)=\int_{p(K)}^1 (F(p)-K)\,dp \tag{1.11}$$

Here $p(K)$ stands for the cumulative density function, evaluated at K. While there is no closed form formula for the function, values can be easily found with numerical approaches by using the percentile function.

Integrating $F(p)$ I get

$$\int_{p(K)}^{1} F(p)\,dp = F_0\left(p + \frac{\sqrt{e^{\sigma^2 t}-1}}{h(\lambda_3,\lambda_4)}\left(\frac{1}{\lambda_3+1} p^{\lambda_3+1} + \frac{(1-p)^{\lambda_4+1}}{\lambda_4+1} + \frac{1}{\lambda_4+1} p - \frac{1}{\lambda_3+1} p \right)\right)\Bigg|_{p(K)}^{1}$$

$$= F_0\left(1 - p(K) + \frac{\sqrt{e^{\sigma^2 t}-1}}{h(\lambda_3,\lambda_4)}\left(\frac{p(K) - p(K)^{\lambda_3+1}}{\lambda_3+1} + \frac{1 - p(K) - (1-p(K))^{\lambda_4+1}}{\lambda_4+1} \right)\right)$$

For clarity, denote

$$G_1 = 1 - p(K) + \frac{\sqrt{e^{\sigma^2 t}-1}}{h(\lambda_3,\lambda_4)}\left(\frac{p(K) - p(K)^{\lambda_3+1}}{\lambda_3+1} + \frac{1 - p(K) - (1-p(K))^{\lambda_4+1}}{\lambda_4+1} \right)$$

with the final European call pricing formula being:

$$V(K, F_0, T, \sigma, r, \lambda_3, \lambda_4) = F_0 e^{-rt} G_1 - e^{-rt} K G_2 \tag{1.12}$$

where G_1 is defined above and $G_2 = 1 - p(K)$

In a similar way it can be shown that the price for a put is

$$V_P(K, F_0, T, \sigma, r, \lambda_3, \lambda_4) = e^{-rt} K (1 - G_2) - F_0 e^{-rt} (1 - G_1) \tag{1.13}$$

1.3. Econometric Model

1.3.1. Estimating Implied Skewness

Implied skewness is used as a dependent variable in subsequent econometric models, thus the first task at hand is to estimate implied higher moments of the terminal futures price distribution for a particular underlying futures contract. The GLD option pricing model can be used to price only European options, that is, options that can only be exercised at contract maturity. As mentioned before, options on corn futures are American options, i.e. they can be also exercised at any time before contract maturity. Therefore, for each option trade I use in fitting implied GLD higher moments, I first need to calculate the price at which such an option would trade if it indeed were of the European type. To do this, for each data point, I separately estimate implied volatility using CRR binomial trees with 500 steps. Then, for each observation separately, I use Black's model to calculate the price of a European option with same futures price, strike, interest rate and time to maturity as that record for actually traded American option.

Using calibrated premiums for European options on corn futures, I then fit the following option pricing model to options of a particular contract month:

$$O_i^E = V\left(K_i, F_{0i}, \tau, r, \sigma, \lambda_3, \lambda_4\right) + \varepsilon_i \qquad (1.14)$$

where function used is as in (1.12) for calls or (1.13) for puts, O_i^E would be the previously calibrated option premium for trade *i* for an option with strike K_i and with F_{0i} being the last observed traded futures price prior to this trade. Observed parameters common to all options of the same contract month traded on the same day include the interest rate r and the time to maturity measured in calendar days, denoted as τ .

The unobserved generalized lambda distribution parameters $\sigma, \lambda_3, \lambda_4$ jointly determine variance, skewness and kurtosis of the implied terminal distribution of futures prices, and are assumed to be the same for all trades occurring on a single trading day. Implied parameters are fitted by a nonlinear least squares model, minimizing squared differences between calibrated option premiums for European options, and option premiums that arise from the GLD option pricing model. Models are estimated separately for each trading day and each contract month traded at that day.

1.3.2. Modeling Intra-year Dynamics of Implied Skewness

As I postulated in section 1.2.3, corn physiology in conjunction with weather patterns should play a major role in governing the intra-year dynamics of implied skewness. The panels in Figure 1.8 present scatter diagrams of estimated implied skewness over the life of particular contract months. Each dot represents the estimated implied skewness on a particular trading day, with bolded diamonds being averages for a particular time-to-maturity horizon over the 15 marketing years used in estimation (1995-2009). Visual inspection does not contradict patterns I expected to see. In particular, new-crop contracts (September and December), exhibit near flat average implied skewness until late June, followed by a concave decrease for the September contract, and linear downward trend for December. Patterns for carry contracts (March, May and July) share strong and concave decreases in implied skewness over the last four months of contract life, with the effects on implied skewness during corn growth period not as distinct as for new-crop contracts. All five patterns stand in stark contrast to Black's model where variance of the terminal futures price distribution is assumed to be decreasing linearly in time. Given that Black's model stipulates the terminal distribution to be lognormal, a linear decrease in variance would correspond to a slightly convex and smooth decline in implied skewness.

If skewness in options on corn futures arises due to asymmetry in the ability of old-crop stocks to mitigate price effects of unexpected weather events during the growing season then skewness should exhibit different dynamics before corn silking, during the growing season, and post-harvest. To test this hypothesis, I fit implied skewness as a function of time using several models. Let implied skewness be denoted with IS_t. If options expire at time T, then the remaining time to maturity $T-t$ is denoted as τ. The models I test can then be written as

Linear model:

$$IS_t = \alpha + \beta_1\tau + \varepsilon_t \tag{1.15}$$

Quadratic model:

$$IS_t = \alpha + \beta_1\tau + \beta_2\tau^2 + \varepsilon_t \tag{1.16}$$

Linear model with one change in regime (timing is estimated endogenously):

$$\begin{aligned} &IS_t = \left[\alpha_1 + \beta_1\tau\right]\left[\tau > \tilde{\tau}_1\right] + \left[\alpha_2 + \beta_2\tau\right]\left[\tau \le \tilde{\tau}_1\right] + \varepsilon_t \\ &s.t.\ \ \alpha_1 + \beta_1\tilde{\tau}_1 = \alpha_2 + \beta_2\tilde{\tau}_1 \end{aligned} \tag{1.17}$$

Quadratic model with one change in regime (timing is estimated endogenously):

$$\begin{aligned} &IS_t = \left[\alpha_1 + \beta_1\tau + \gamma_1\tau^2\right]\left[\tau > \tilde{\tau}_1\right] + \left[\alpha_2 + \beta_2\tau + \gamma_2\tau^2\right]\left[\tau \le \tilde{\tau}_1\right] + \varepsilon_t \\ &s.t.\ \ \alpha_1 + \beta_1\tilde{\tau}_1 + \gamma_1\tilde{\tau}_1^2 = \alpha_2 + \beta_2\tilde{\tau}_1 + \gamma\tilde{\tau}_1^2 \end{aligned} \tag{1.18}$$

Quadratic model with two changes in regime (timing is estimated endogenously):

$$\begin{aligned} &IS_t = \left[\alpha_1 + \beta_1\tau + \gamma_1\tau^2\right]\left[\tau > \tilde{\tau}_1\right] + \left[\alpha_2 + \beta_2\tau + \gamma_2\tau^2\right]\left[\tilde{\tau}_2 \le \tau \le \tilde{\tau}_1\right] + \left[\alpha_3 + \beta_3\tau + \gamma_3\tau^2\right]\left[\tau \le \tilde{\tau}_2\right] + \varepsilon_t \\ &s.t.\ \ \alpha_1 + \beta_1\tilde{\tau}_1 + \gamma_1\tilde{\tau}_1^2 = \alpha_2 + \beta_2\tilde{\tau}_1 + \gamma_2\tilde{\tau}_1^2 \\ &\qquad \alpha_2 + \beta_2\tilde{\tau}_2 + \gamma_2\tilde{\tau}_2^2 = \alpha_3 + \beta_3\tilde{\tau}_2 + \gamma_3\tilde{\tau}_2^2 \end{aligned} \tag{1.19}$$

Simple linear (1.15) and quadratic models (1.16) are used as benchmarks. In particular, it is interesting to compare the performance of model (1.16) to more complicated models as model (1.16) together with a restriction that β_2 be positive (i.e. IS exhibiting a convex pattern over time) follows as an implication of Black's option pricing model. Different skewness dynamics through a marketing year would be captured either by estimating higher polynomial or multiple-regime models. In the multiple-regime models fit here, the restrictions listed above result in continuity of predicted implied skewness at points of regime change, but smoothness at those points is not imposed.

The points at which regimes changes, i.e. $\tilde{\tau}_1$ in models (1.17) and (1.18) and $\tilde{\tau}_1, \tilde{\tau}_2$ in model (1.19) are also treated as parameters that need to be estimated, rather than being pre-determined. Conditional on a particular choice of these parameters, the rest of the model can be estimated using restricted least squares. For one-switch models, similar to Hansen (1999), denote the sum of square errors for restricted least squares estimates conditional on a particular value of τ_1 as $SSE(\tau_1)$. The optimal point for the regime switching time is found as the minimizer of the conditional restricted sum of square errors:

$$\tilde{\tau}_1 = \arg\min_{\tau \in \Gamma} SSE(\tau) \tag{1.20}$$

For models with two switches, I can find the optimal switching points through a three-step minimization. First, conditional on particular values of τ_1, τ_2 I can find the optimal slope coefficients by restricted least squares estimation. Then, like above

$$\begin{aligned} \tilde{\tau}_2 \mid \tau_1 &= \arg\min_{\tau_2 \in \Gamma(\tau_1)} SSE(\tau_1, \tau_2) \\ \tilde{\tau}_1 &= \arg\min_{\tau_1 \in \Gamma_1} SSE(\tau_1, \tilde{\tau}_2 \mid \tau_1) \end{aligned} \tag{1.21}$$

where $\Gamma_1 = \{\tau_1 : \tau_{MAX} - 20 \le \tau_1 \le 50\}$ and $\Gamma_2(\tau_1) = \{\tau_2 : \tau_1 - 30 \le \tau_2 \le 20\}$.

To implement this when estimating optimal points for regime switching, conditional on stipulating the number of regime switch points, simple grid search is used, and then the sum of squared errors (SSE) from the estimated restricted least squares are ranked. I stipulate that regime switching cannot be less than 20 days to expiry or closer than 20 days to the maximum time to maturity. For models with two switch dates, I also stipulate that the two switch dates cannot be less than 30 days apart. The model with the lowest SSE is chosen as best in its class.

Models are estimated separately for each contract month (March, May, July, September and December), using daily values of implied skewness for the period 1995-2009. To repeat, implied skewness is itself estimated using high-frequency data as described in the previous section. As such, implied skewness estimates become very unstable on a day-to-day basis for very high time to maturity horizons. One reason could be a lack of liquidity in options markets for options far from expiry, and another the low number of years for which options with such long horizons have even been traded. To eliminate the effect of noise in the estimation of implied skewness for long time to maturity horizons, I truncate the maximum allowable time to maturity for each contract separately at the point where simple visual inspection indicates noise starts to dominate.

In selecting the optimal model specification among the five models listed, I have used the theory developed by Hansen (1996, 1999, 2000) and used in Cox, Hansen and Jimenez (2004). As Hansen (1996) explains, the problems of inference in the presence of nuisance parameters (i.e. regime switching times) is that they are not identified under the null hypothesis of no-regime change. If I fixed the regime switching-time to a particular value, I could perform a standard Wald test to see if parameters for intercept and slopes are equal for observations occurring before and after $\tilde{\tau}$ days to maturity. However, since I cannot restrict the possible threshold time a priori, as Hansen (1996) explains, the asymptotic distribution of standard tests are nonstandard and nonsimilar, which means that tabulation of critical values is impossible. The finite sample distribution of the Wald statistic under the null hypothesis is calculated by simulation and the null hypothesis is rejected if the test statistic is higher than the desired percentile of the simulated Wald statistic distribution under the null. Details of the bootstrapping method used in testing for the optimal model class are presented in section 1.3.2.

1.3.3. **Inter-year Variation of Implied Skewness**

Finally, I turn to explaining the inter-year variations in implied skewness. As argued in the previous section, skewness will likely be impacted by weather once corn silking starts. Therefore, if we are to infer an impact of storage on skewness across many years, each with its own weather peculiarities, we should choose the time before the reproductive growth phase starts, i.e. no later than third week of June. If we were to choose skewness observed much earlier than that, we would risk falling in the endogeneity trap. Before a marketing year is close to the end, consumption can react to changes in futures price, possibly even to changes in options premiums, thus increasing or decreasing carryout stocks. It would make little sense then to use expected ending stocks-to-use as a predetermined explanatory variable and implied skewness as a dependent variable. To avoid this problem, the expected ending stocks-to-use ratio of the previous marketing year, as reported in June edition of World Agricultural Supply and Demand Estimates (WASDE) report[2] is employed for explanatory variable for storage adequacy.

If the price elasticity of supply for corn is not zero, we would expect producers to react to tighter expected stocks and higher new crop prices with an increase in planted acreage, so acreage response is the second variable I need to include in the model. Specifically, I use the measure of change between intended plantings for a given year as reported in the USDA Prospective

[2] WASDE is produced by World Agricultural Outlook Board, inter-agency body at United States Department of Agriculture. Historical WASDE reports can be accessed at http://usda.mannlib.cornell.edu/MannUsda/viewDocumentInfo.do?documentID=1194

Plantings[3] report published at the end of March, and the actual acreage planted in the previous marketing year.

In addition to supply side covariates, I need to address possible asymmetries in uncertainty of demand. Domestically, corn is used as a livestock feed, an industrial sweetener and as an input in ethanol production. All three of these derived demand categories are likely impacted by macroeconomic shocks. Therefore, as a measure of demand uncertainty I use the June-to-June change in the national unemployment rate as published by the Bureau of Labor Statistics.

The final econometric model has the following form:

$$IS_t = \alpha + \beta_1 E_t \Delta A_T + \beta_2 E_t \left[S_T / D_t \right] + \beta_3 \Delta U_t \tag{1.22}$$

Where IS_t stands for implied skewness for a December contract of year *t* estimated as the average of implied skewness for the 10 trading days following the June WASDE report. The change in acreage planted is ΔA_T. Since in June I only observe intended plantings, this is written as the expected change in acreage. Expected ending stocks-to-use is $E_t \left[S_T / D_t \right]$ and ΔU_t is the June-to-June change in the U.S. unemployment rate. Theory predicts that all coefficients except the constant should be negative. A stronger acreage response and higher carryout stocks relative to demand imply more ability to buffer adverse weather shocks, and will thus reduce skewness. Likewise, a more unstable macroeconomic environment will decrease demand for fuel and possibly even for meat, thus reducing upward pressure on corn prices.

---

[3] Prospective Plantings is a government report produced annually by the National Agricultural Statistics Service, an agency of the United States Department of Agriculture. Historical Prospective Plantings reports can be accessed at http://usda.mannlib.cornell.edu/MannUsda/viewDocumentInfo.do?documentID=1136

1.4. Data

Commodity futures for corn as well as options on futures are traded on the Chicago Mercantile Exchange (formerly the Chicago Board of Trade). A dataset comprising all recorded transactions, i.e. times and sales data (also known as "tick data") for both futures and options on futures, for the period 1995 through 2009, was obtained. It includes data for both the regular and electronic trading sessions. The total number of transactions exceeds 30 million, including 22 million observations on futures contract trades, and about 10 million trades in options contracts. Options data were matched with the last preceding futures transaction. LIBOR interest rates were obtained from British Bankers' Association, and represent the risk-free rate of return. Overnight, 1 and 2 weeks, and 1 through 12 months of maturity LIBOR rates for period the 1995 through 2009 were used to obtain the arbitrage-free option pricing formulas. In particular, each options transaction was assigned the weighted average of interest rates with maturities closest to the contract traded. To avoid serial correlation in residuals from estimating implied coefficients, the data frequency was reduced to not less than 15 minutes between transactions for the same options contract. This resulted in data sets of between 200 to 800 recorded transactions for a particular trading day for a total of around 1.1 million observations used in estimation. For each data point I separately estimate implied volatility using CRR binomial trees with 500 steps. Then, for each data point, the price of a European option using Black's formula is calculated using the same parameters (futures price, interest rate, time to maturity) as that recorded for the American option. In addition, volatility is set equal to the one implied for American options. These 'artificial' European options are then used in fitting parameters of GLD option pricing model for each trading day separately.

As stated in the previous section, the implied skewness used in the econometric analysis is calculated as a simple average over 10 business days following the June WASDE report. Due to the high incidence of limit-move days and days with high intraday price changes the year 2008 is excluded from the sample. Including 2008 would render the calculation of higher moments unreliable. Descriptive statistics of the variables used in econometric analysis are given in Table 1.1, and corn supply/demand balance sheets are in Table 1.2.

Figure 1.9 presents a scatter diagram of expected ending stocks-to-use vs. implied skewness. Note the inverse relationship between these variables and the beneficial impact of the acreage response. For example, in the summer of 1996, carryout stocks-to-use were only 4.03%, two standard deviations below the average for 1995-2009. However, skewness was below the mean, due to a 12.2% increase in expected acreage, which is 2.2 standard deviations above the average increase of 1.4%. Similarly, in 2007 carryout stocks were only 8.56% of demand, but a massive acreage increase of 15.5%, by far the largest in this sample, reduced the skewness below the mean. It is instructive to look at 2006 as well. Although ending stocks were bountiful at 19.67% of demand, a reduction in acreage of 4.6% made for the third largest skewness in the sample.

1.5. Estimation Procedure and Results

1.5.1. Estimating Parameters of GLD Distribution and Implied Higher Moments

As stated in section 1.3.1, for each contract, for each trading day, I separately estimate the parameters $\sigma, \lambda_3,$ and λ_4 in the GLD option pricing formula. In particular, I minimize the squared difference in option premiums calculated with the GLD formula, and prices of European options as implied by Black's model. To do so, I first need a starting value for the implied volatility of an option with a strike price closest to the underlying futures price. The starting values for the λ_3

and λ_4 parameters were chosen to correspond to the skewness and kurtosis of the terminal futures price as they would be under the restriction that the logarithm of the terminal price is normally distributed with variance equal to $\sigma^2 t$, where σ^2 is the square of the starting value for the implied sigma parameter. Excel Solver is used to run the minimization problem, utilizing a FORTRAN compiled library (.dll file) created by Corrado (2001) that estimates GLD European Call prices. A formula for the GLD European put option was then programmed in Visual Basic for Applications.

Estimated lambda parameters are employed to calculate implied skewness and kurtosis. GLD option prices seem to work rather well, with an average absolute pricing error about 3/8 of a cent per bushel, and a maximum pricing error usually reaching not more than 2 cents (this occurs for the least liquid and most away from the money options). While there may be issues regarding the robustness of implied parameters with respect to starting values, the implied parameters seem to be rather stable from one day to the next. For December 2007 corn, for example, the skewness estimated between June 11 and June 25, 2007 varies between 1.15 and 1.26. For that year, the average absolute pricing error was 7/8 of a cent per bushel, with a maximum pricing error of 7.9 cents.

For all years in the sample, the implied skewness is 1.2 to 3 times higher than it would be if the logarithm of the terminal futures price was really expected to be normal. Implied kurtosis is 1.2 to 1.6 times higher than that predicted by Black's model. I thus see that deviations from Black's model are particularly pronounced in implied skewness.

1.5.2. Dynamics of Intra-year Implied Skewness: Results

The results of intra-year models for dynamics of implied skewness are presented in Table 1.3, and predicted implied skewness for each contract month is plotted in Figure 1.10. For all five contracts, the quadratic model improves fit dramatically over the linear model. To perform a formal test whether a model with one switching time and quadratic segments fits the data better than the quadratic model, I have used the bootstrapping procedure described by Cox, Hansen and Jimenez (2004). In doing this test, I shall refer to the quadratic model (1.16) as the restricted model and model (1.18) as the unrestricted model. These models are nested, i.e. model (1.16) is obtained by imposing restrictions $\alpha_1 = \alpha_2, \beta_1 = \beta_2, \gamma_1 = \gamma_2$. Under the null hypothesis that these restrictions hold the switching time τ_1 is not identified. To test the null hypothesis, I first make 2000 bootstrap samples using the fixed-regressors residual bootstrapping method. In particular, for each simulation values of implied skewness are calculated by adding a draw from the empirical distribution of residuals to predicted value of the dependent variable. Fitting is done using the estimated coefficients from the restricted model, in this case model (1.16). Then, for each bootstrapped sample, parameters of the unrestricted model, including switching time, are calculated by the same method as before, i.e. combining a grid search and concentrated restricted least squares. A Wald statistic $W_n = n\frac{\left(SSE_0 - SSE_1\right)}{SSE_1}$ is then calculated for that particular replication, where n is the number of observations in the sample, SSE_0 is the sum of square errors in the restricted model (zero switching points) using bootstrapped data and SSE_1 is the sum of square errors of model (1.18) using bootstrapped data. The entire process is repeated 2000 times to obtain a finite sample distribution of the Wald statistic. The null hypothesis is rejected if the Wald statistic obtained using the original data is higher than the 95th percentile of the simulated distribution. We see from Table 1.3 that model (1.16) is strongly rejected in favor of model (1.18) for all five contract months.

I also estimate a model with two regimes changes. For the May contract, the optimal first switching time solves to a corner solution, i.e. 20 days less than the maximum time-to-maturity used in estimation. I interpret this as evidence that for the May contract, a model with two regime switching times does not explain the data any better than models with one change in regime, and is in fact a misspecification, i.e. number of break points is stipulated to be higher than actually exist. For other contract months, the optimal switching time solves out to the interior of the allowable set of times, and I need to perform a formal test to investigate if models with two switching times are indeed better representations of the data. Bootstrapping is again employed. In particular, the null hypothesis now is that true model is model (1.18), and the unrestricted model is model (1.19). Model (1.18) can be obtained from model (1.19) by restricting it, such that coefficients satisfy: $\alpha_1 = \alpha_2, \beta_1 = \beta_2, \gamma_1 = \gamma_2$.

I again use fixed-regressors residual-bootstrap technique and add draws from the empirical distribution of residuals obtained from model (1.18) to the implied skewness measures predicted using the estimated coefficients of model (1.18). For each replication, a Wald statistic $W_n = n\dfrac{\left(SSE_1 - SSE_2\right)}{SSE_2}$ is calculated, where SSE_1 is the sum of square errors obtained by estimating model (1.18) on bootstrapped data, and SSE_2 is calculated by estimating the model with two switching times on bootstrapped data. As before, the entire process is repeated 2000 times to obtain a finite sample distribution of the Wald statistic. The null hypothesis is rejected if the Wald statistic obtained using original data is higher than the 95th percentile of the simulated distribution. I find that the Wald statistics obtained using the original data are low enough that the null hypothesis cannot be rejected for any contract month, and p-values are exceptionally large. In conclusion, statistical tests show that a model with 1 regime change is superior. To test if model (1.18) explains the data any better than model (1.17) with two linear segments I can use standard critical values in Wald test, as both models have the same number of regimes. I find that the null is rejected for all contract months.

The next issue to investigate and explain concerns evaluated knot times and their confidence intervals. Point estimates are found using the already explained estimation procedure. Residual-based bootstrap is then used to obtain confidence intervals. For a particular contract month, simulated data is created by adding draws from the empirical distribution of residuals to the predicted implied skewness using the same model for which I evaluate confidence intervals of the knot. The model is then re-estimated on simulated data, and a new optimal knot value is noted. The procedure is repeated 2000 times, with the confidence interval obtained using the 2.5^{th} and 97.5^{th} quantile as the lower and upper bounds, respectively. For model (1.18) with quadratic segments, I find that the confidence intervals for switching times are substantial for May and July contracts. A possible reason is that the first segment in the model is convex, and the second concave, creating a rather smooth transition. In such a setting, changing the knot value can be very easily compensated for by changes in the slopes parameters. For the September, December and March contracts, both segments are estimated with concave curves, and exhibit much tighter confidence intervals of the switching times. Results are presented in Table 1.3. As a robustness check, I also calculate asymptotic confidence intervals using a method developed by Hansen (2000) that involves inverting a likelihood ratio statistic. I find that our bootstrapping method matches closely the results obtained using asymptotics for all contract months except July. For that contract month, the curve for the likelihood ratio statistic is rather flat and close to the asymptotic critical value for time-to-maturity values included in the bootstrapped confidence interval. In that sense, we perhaps could say that the bootstrap produces more conservative estimates for the confidence intervals. Another likely reason for observed differences could be that I estimate our model with the additional restriction of continuity in predicted variable, whereas asymptotic distribution is developed for unrestricted least squares estimation.

In the model with 1 regime change and quadratic segments, optimal switching time for September contract is 69 days to maturity, and for December it is 160 days. It will help us to be able to map time-to-maturity measures to a particular date in a year. Option contract specifications state that last trading day is *"The last Friday preceding the first notice day of the corresponding corn futures contract month by at least two business days."* The first notice day is the first day of the delivery month. For simplicity, I approximate the last option trading day to be 25th of the month preceding the delivery month. Under such an approximation, regime switching times for new-crop contracts correspond to June 18th for the September contract and June 19th for the December contract. To test if regime switching times for these two contracts really fall on the same calendar date, I perform a Wald test. This is a non-standard test and I use residual-based bootstrapping to generate data under the null hypothesis that calendar switching dates are the same, which is equivalent to restriction that $\tilde{\tau}^{DEC} = \tilde{\tau}^{SEP} + 90.$ The null hypothesis is not rejected, and p-value is 0.9995, with the original Wald statistic is higher than only one out of 2000 Wald statistics simulated under the null hypothesis. The *Crop progress* report[4] published in

---

[4] Crop Progress report is a government report produced weekly from April through November of each year by the National Agricultural Statistics Service, an agency of the United States Department of Agriculture. Historical Prospective Plantings reports can be accessed at
http://usda.mannlib.cornell.edu/MannUsda/viewDocumentInfo.do?documentID=1048

last week of June is normally the first such report to list corn silking progress. These reports suggest that on average about 5% of the U.S. corn crop is silking by June 26th. Thus, dynamics of implied skewness for new-crop contracts appear to change right at the start of the corn silking period.

For carry contracts I find quite a different dynamic. Regime changes for the March, May and July contracts occur respectively at 130, 125 and 112 days to maturity. Suspecting that these days account for similar patterns across contracts, I tested whether regime changes occur at statistically significantly different time-to-maturity horizons. Similar to a previous hypothesis test, I express that hypothesis as restrictions on switching times: $\tilde{\tau}^{MAR} = \tilde{\tau}^{MAY} = \tilde{\tau}^{JUN}$. The optimal switching time under the null hypothesis (that the restriction holds) is 128 days to maturity, and the null is not rejected. This common switch is manifested in Figure 1.8 as concave and substantial decrease in implied skewness close to contract expiry. This likely reflects the decline in overall uncertainty concerning terminal prices as maturity approaches. It is more interesting to note that for carry contracts, the first segment (i.e. before the switch) is convex for the May and July contracts, but concave for March. This could reflect that fact that for March contract, corn growth-sensitive period falls in the middle of the March contract lifetime, while for May and July this growth period is at the beginning of the contract life.

Table 1.4 showcases the relative contribution of the corn growth period (silking through harvest, approximated by the dates June 20 to October 20) to skewness reduction during a contract's lifetime. For carry contracts the days spent in second regime at the end of the contract life are excluded. For example, for the July contract, the maximum time to maturity was 350 calendar days. The contract traded for 230 calendar days prior to entering the "finish-line" period, i.e. the last 120 days in which I find a strong reduction of skewness. Out of those 230 days, 113 days, or

49.1% of time, falls in the growth sensitive period. At the maximum time-to-maturity horizon predicted implied skewness is 1.419, and at switch time it is 1.218. Although the growth sensitive period constitutes only one half of that time, it accounts for 76.5% of the difference between the maximum time-to-maturity horizon and switch time implied skewness. For the March contract, I see a situation that is even more extreme – the sensitive growth period constitutes 59.5% of the pre-switch life, but accounts for 94.3% of the difference between skewness at maximum time-to-maturity and at the switch-time.

1.5.3. Intra-year Variation in Implied Skewness: Results

Results of the previous section further justify using implied skewness for December contract over 10 days after June WASDE report in investigating effect of expected stocks-to-use at the end of a marketing year (Aug. 31) on implied skewness. To test this, a simple linear regression is estimated for the period 1995-2009 using implied skewness as the dependent variable. The independent variables include a constant, the expected ending stocks-to-use, the expected planned change in planted acreage and changes in the unemployment rate. Regression statistics are reported in Table 1.5. Due to very low degrees of freedom (10), I have to rely on t-table for critical values, and use a one-tail test for the stocks-to-use coefficient.

An 1 % increase in stocks-to-use reduces skewness by 0.015. This coefficient is statistically significant at the 95% confidence level. To put this number in perspective, the difference between the lowest and the highest ending stocks-to-use recorded in the sample reduces skewness from 1.47 to 1.24, which is 47% of the difference between the highest and the lowest recorded skewness in the sample. Coefficients for demand uncertainty and acreage response are also statistically significant and have the expected sign.

1.6. Conclusions and Further Research

An option pricing model based on a generalized lambda distribution provides a useful heuristic in thinking about determinants of the shape of terminal futures price conditional distributions. Results indicate that crop inventories and plant physiology play a significant role in determining the expected asymmetry of the terminal distribution. In particular, results reveal that implied skewness is much more persistent than implied by Black's model. In years with low implied volatility implied skewness remains much higher than would be the case under the lognormality restriction, and dynamics are dominated not by time to maturity, but by temporal patterns in the resolution of uncertainty regarding crop yields.

Further research will focus on extending this analysis to soybeans and wheat. The U.S. is a major world player in corn, with 55.6% of world exports. That is higher than 45.3% of world exports of soybeans, and much higher than 17.7% percent in wheat. Extending the analysis to other crops will identify the effect of trade and non-overlapping growing seasons in different countries on the magnitude, inter-year differences and intra-year dynamics on implied higher moments of the terminal price distribution.

Thus far the literature has focused on evaluating the impacts of government reports on implied volatility coefficients. The model presented here allows us to extend this to higher moments and examine how reports (i.e., information) influence the entire distribution of prices, not just the second moment. For example, I could use weekly crop progress reports to explain inter-year differences in the evolution of skewness through the summer months.

In the absence of high frequency data, many researchers use end of day reported prices for futures and options to evaluate implied higher moments. By re-estimating this model using only end of day data it is possible to examine the amount of noise and possible direction of bias such an approach brings to estimates of implied higher moments.

What happens when storage is not available to partially absorb the shocks to supply? It would be interesting to use the GLD option pricing model to examine the evolution and determinants of higher moments of non-storable commodities. Further research is needed to examine the impact of durability of production factors for commodities that are themselves not storable.

Finally, impacts of market liquidity and trader composition on the levels and stability of implied higher moments is a promising new area for research. With careful design of the analysis, we may be able to find a way to separate the part of the option price that is due to implied terminal price distributions from additional premium influences incurred due to hedging pressure or lack of market liquidity.

1.7. References

Adam-Müller, A.F.A., and A. Panaretou. 2009. "Risk management with options and futures under liquidity risk." Journal of Futures Markets 29(4):297-318.

Berra, Y. 2010. *"The Yogi Book: I didn't really say everything I said"*, New York: Workman Publishing.

Black, F. 1976. "The pricing of commodity contracts." *Journal of Financial Economics* 3(1-2):167-179.

Black, F. 1976b. "Studies of Stock Price Volatility Changes." Proceedings of the 1976 Meetings of the American Statistical Association, pg. 177-181.

Buschena, D., and K. McNew. 2008. "Subsidized Options in a Thin Market: A Case Study of the Dairy Options Pilot Program Introduction." *Review of Agricultural Economics* 30(1):103-119.

Corrado, C.J. 2001. "Option pricing based on the generalized lambda distribution." *Journal of Futures Markets* 21(3):213-236.

Cox, D., Hansen, B.E. and Jimenez, E. 2004. "How responsive are private transfers to income?" *Journal of the Public Economics*, 88:2193-2219.

Cox, J.C., S.A. Ross, and M. Rubinstein. 1979. "Option pricing: A simplified approach." *Journal of Financial Economics* 7(3):229-263.

Deaton, A., & Laroque, G. 1992. "On the Behaviour of Commodity Prices." *The Review of Economic Studies*, 59(1):1-23.

Dumas, B., J. Fleming, and R.E. Whaley. 1998. "Implied Volatility Functions: Empirical Tests." *The Journal of Finance* 53(6):2059-2106.

Egelkraut, T. M., P. Garcia, and B.J. Sherrick. 2007. "The Term Structure of Implied Forward Volatility: Recovery and Informational Content in the Corn Options Market." *American Journal of Agricultural Economics*, 89(1):1-11.

Fackler, P.L., and R.P. King. 1990. "Calibration of Option-Based Probability Assessments in Agricultural Commodity Markets." *American Journal of Agricultural Economics* 72(1):73-83.

Fortenbery, T.R., and D.A. Sumner. 1993. "The effects of USDA reports in futures and options markets." *Journal of Futures Markets* 13(2):157-173.

Gardner, B. L. (1977). "Commodity Options for Agriculture." *American Journal of Agricultural Economics*, 59(5):986-992.

Geman, H. 2005. Commodities and commodity derivatives. West Sussex: John Wiley & Sons Ltd.

Glauber, J. W., and M. J. Miranda. 1989. "Subsidized Put Options as Alternatives to Price Supports." Proceedings of the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management. Chicago, IL. [http://www.farmdoc.uiuc.edu/nccc134].

Hansen, B.E. 1996. "Inference when a nuisance parameter is not identified under the null hypothesis" *Econometrica* 64:413-430.

Hansen, B.E. 1999. "Testing for Linearity" *Journal of Economic Surveys* 13, 551-576.

Hansen, B.E. 2000. "Sample splitting and threshold estimation," *Econometrica*, 68:575-603.

Hilliard, J.E., and J. Reis. 1998. "Valuation of Commodity Futures and Options Under Stochastic Convenience Yields, Interest Rates, and Jump Diffusions in the Spot." *The Journal of Financial and Quantitative Analysis* 33(1):61-86.

Isengildina-Massa, O., S.H. Irwin, D.L. Good, and J.K. Gomez. 2008. "Impact of WASDE reports on implied volatility in corn and soybean markets." *Agribusiness*, 24(4):473-490.

Jarrow, R., and A. Rudd. 1982. "Approximate option valuation for arbitrary stochastic processes." *Journal of Financial Economics* 10(3):347-369.

Ji, D., and W. Brorsen. 2009. "A relaxed lattice option pricing model: implied skewness and kurtosis." *Agricultural Finance Review* 69(3):268-283.

Kang, T., and B.W. Brorsen. 1995. "Conditional heteroskedasticity, asymmetry, and option pricing." *Journal of Futures Markets* 15(8):901-928.

Lence, S.H., Y. Sakong, and D.J. Hayes. 1994. "Multiperiod Production with Forward and Option Markets." *American Journal of Agricultural Economics* 76(2):286-295.

Lien, D., and K.P. Wong. 2002. "Delivery risk and the hedging role of options." *Journal of Futures Markets* 22(4):339-354.

Ramberg, J.S., and B.W. Schmeiser. 1974. "An approximate method for generating asymmetric random variables." *Commun. ACM* 17(2):78-82.

Ramberg, J.S., P.R. Tadikamalla, E.J. Dudewicz, and E.F. Mykytka. 1979. "A Probability Distribution and Its Uses in Fitting Data." *Technometrics* 21(2):201-214.

Rubinstein, M. 1994. "Implied Binomial Trees." *The Journal of Finance* 49(3):771-818.

Rubinstein, M. 1998. "Edgeworth Binomial Trees." *The Journal of Derivatives* 5(3):20-27.

Rui, X., and M.J. Miranda. 1995. "Commodity Storage and Interseasonal Price Dynamics." Paper presented at the Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management., Chicago, IL.

Shaw, R. H., J. E. Newman, D. G. Baker, R. V. Crow, R. F. Dale, Rich Feltes et al. 1988. "Weather Stress in the Corn Crop." OSU current report - Oklahoma State University, Cooperative Extension Service, Vol. 2099.

Sherrick, B.J., P. Garcia, and V. Tirupattur. 1996. "Recovering probabilistic information from option markets: Tests of distributional assumptions." *Journal of Futures Markets* 16(5):545-560.

Thomsen, M.R., A. McKenzie, and G.J. Power. 2009. "Volatility Surface and Skewness in Live Cattle Futures Price Distributions with Application to North American BSE Announcements." Paper presented at 2009 AAEA & ACCI Joint Annual Meeting. Milwaukee, WI.

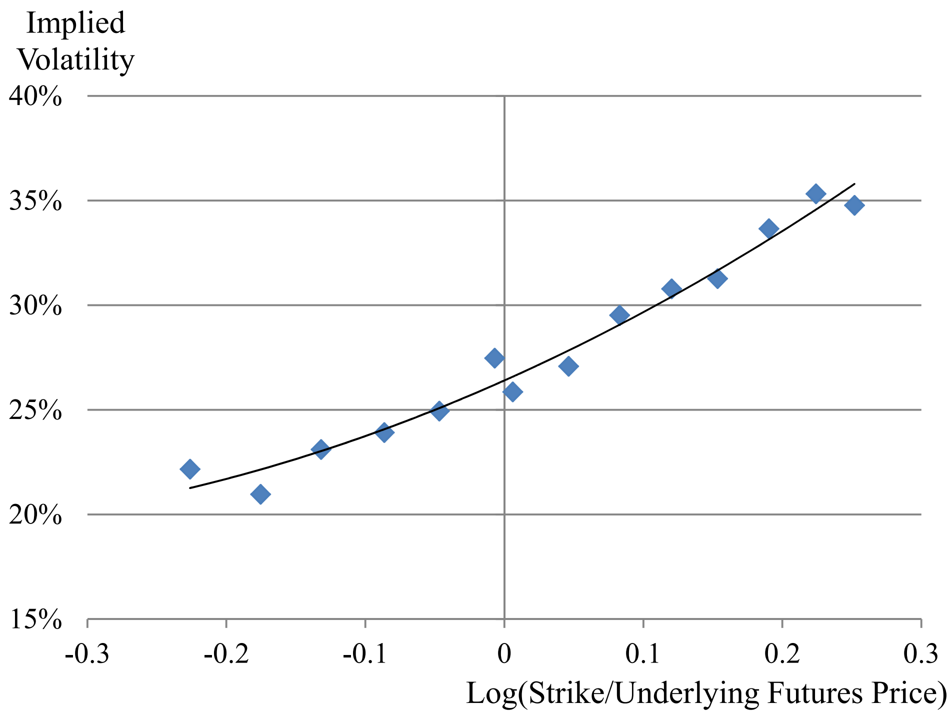
World Agricultural Outlook Board. 2010. "World Agricultural Supply and Demand Estimates." United States Department of Agriculture. Retrieved October 4, 2010 from http://usda.mannlib.cornell.edu/MannUsda/viewDocumentInfo.do?documentID=1194

National Agricultural Statistics Service. 2010. "Prospective Plantings." United States Department of Agriculture. Retrieved October 4, 2010 from http://usda.mannlib.cornell.edu/MannUsda/viewDocumentInfo.do?documentID=1194

Vercammen, J. 1995. "Hedging with Commodity Options When Price Distributions Are Skewed." *American Journal of Agricultural Economics* 77(4):935-945.

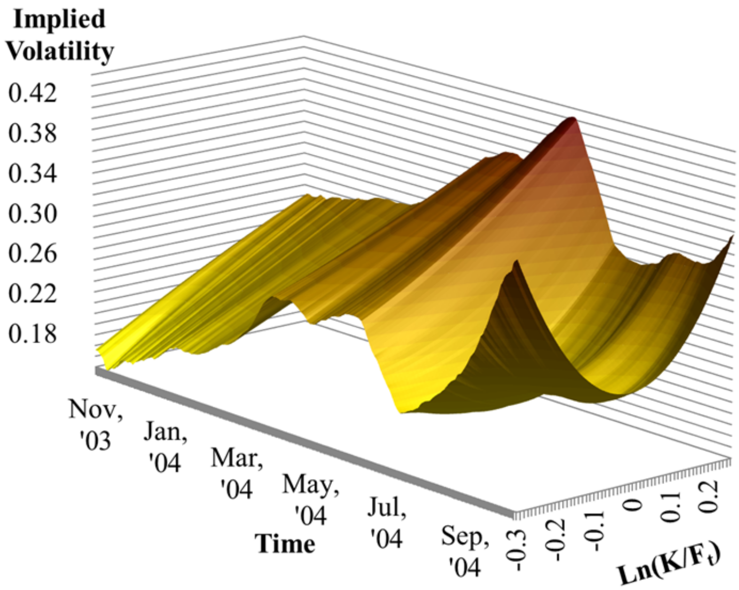
Williams, J.C., and B. Wright. 1991. *Storage and commodity markets*. Cambridge ; New York: Cambridge University Press.

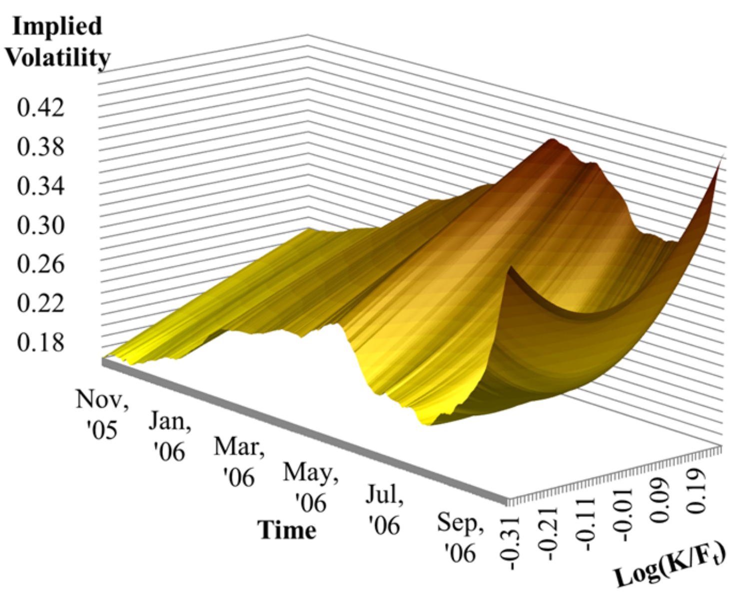
Figure 1.1. Typical Pattern for Implied Volatility Coefficients for Options on Agricultural Futures



Notes: Implied volatility coefficients are estimated for options on the December 2006 corn futures contract, on 6/26/2006 using Cox, Ross and Rubinstein's binomial tree with 500 steps. Using high-frequency data, out-of-money puts and calls were matched with the last previously observed futures price. The average underlying futures price was $2.49/bu. Dots represent implied volatility coefficients for each strike, and the smooth line is a fitted quadratic trend.

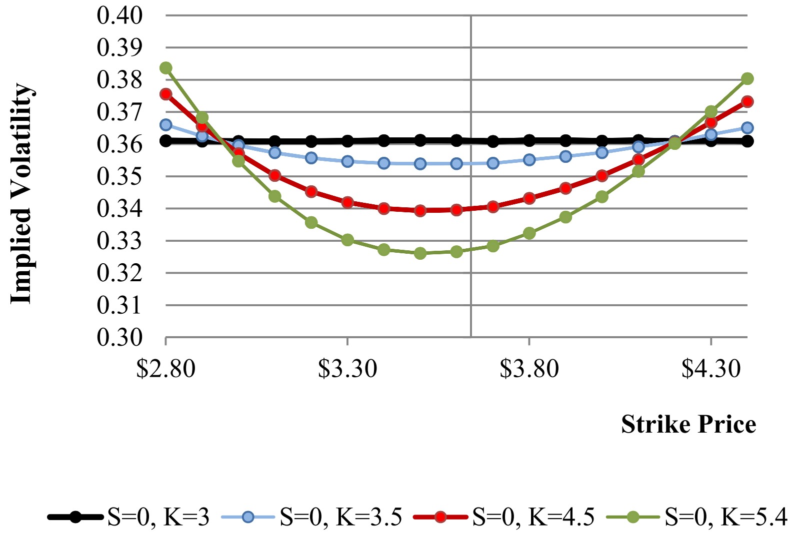
Figure 1.2. Evolution of Implied Volatility Curve for Options on Dec' 04 and Dec '06 Corn Futures.

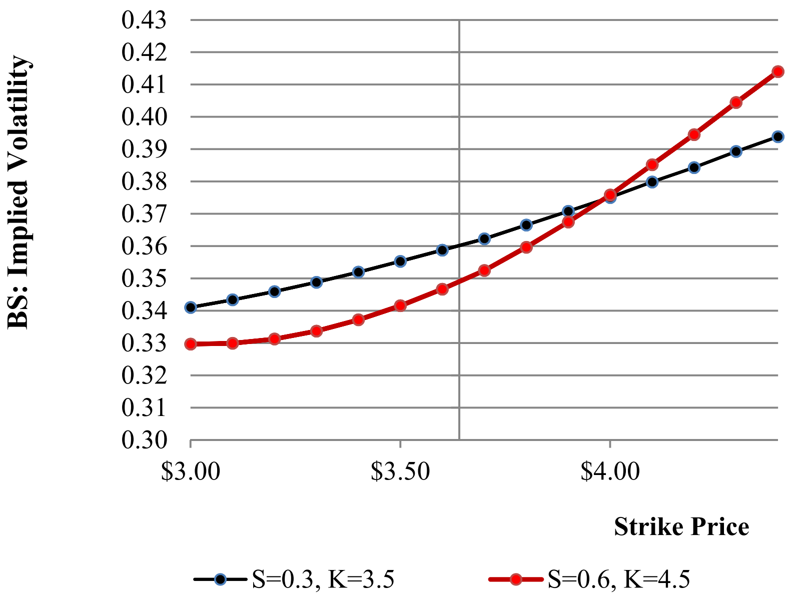




Notes: For each day, implied volatility is estimated for each traded option using 15 minute interval data. A quadratic trend curve is fitted to produce the implied volatility curve. A 30 day moving average is calculated to increase smoothness of the volatility surface and make it easier to see principal characteristic of the IV curve evolution. The Z-axis shows option moneyness calculated as the logarithm of the ratio between option strike (K) and underlying futures price F_t. When the option strike price is equal to the current futures price moneyness is zero.

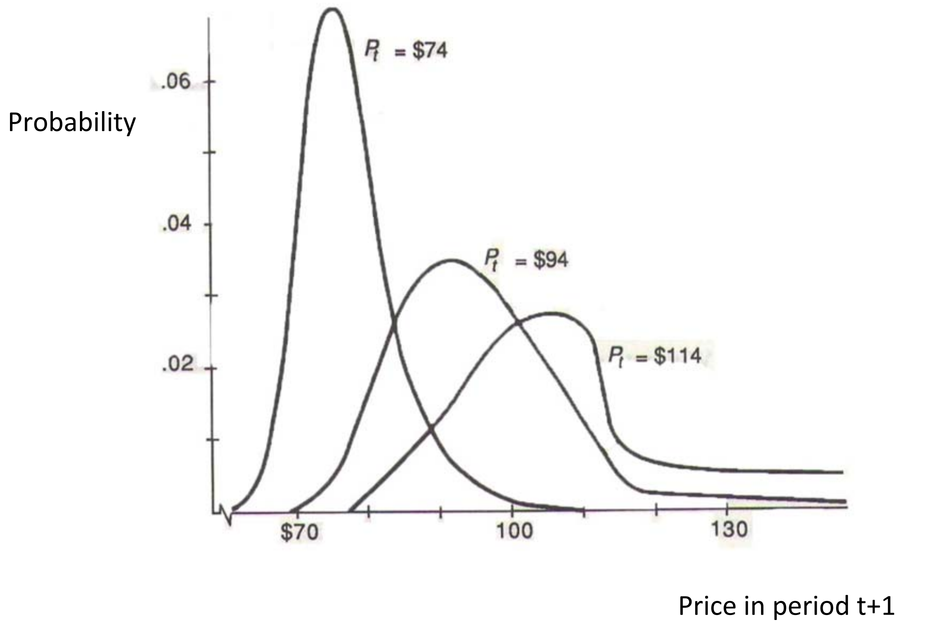
Figure 1.3. Effects of Excess Kurtosis and Positive Skewness on Implied Volatility





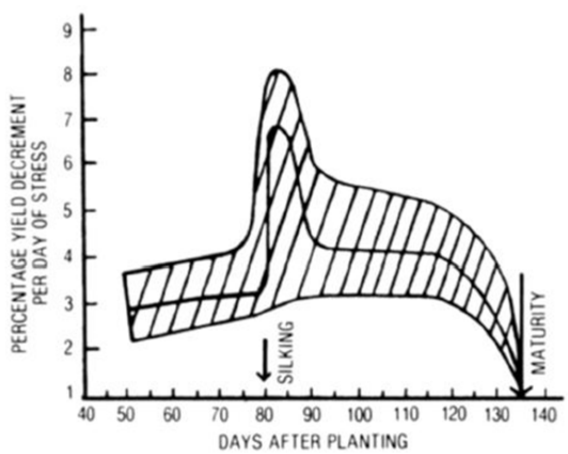
Notes: S stands for skewness, and K for kurtosis of terminal futures log-prices. Option premiums are calculated via Rubinstein's Edgeworth binomial trees that allow for non-normal skewness and kurtosis, and implied volatility is inferred using Cox, Ross and Rubinstein's binomial tree which assumes normality in terminal futures prices. The black line in the above diagram with S=0 and K=3 corresponds to assumptions of Black's model, where implied volatility curve is flat across all strikes. Excess kurtosis (K>3) creates convex and nearly symmetric "smiles", and positive skewness produces an upward sloping implied volatility curve.

Figure 1.4. New Crop Price Distributions Conditional on Storage Adequacy



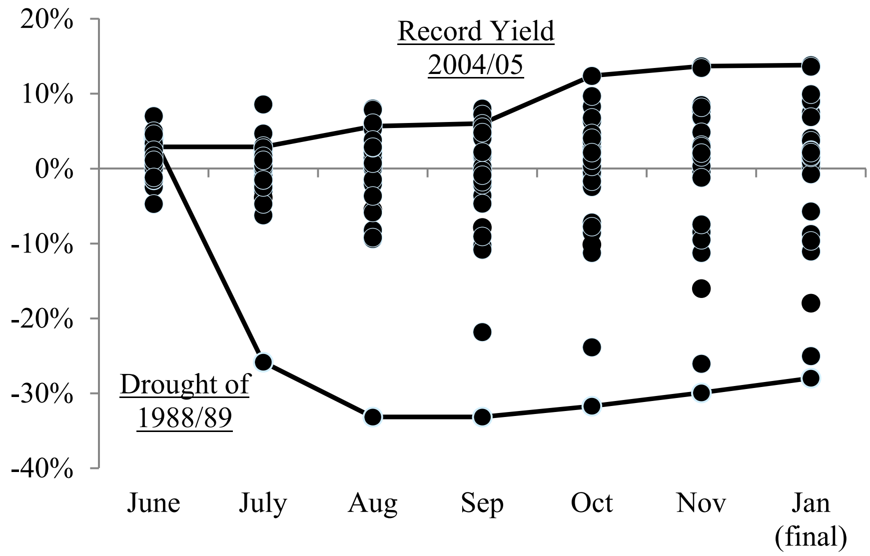
Note: Reproduced from Williams and Wright (1991). Conditional price distributions obtained from a rational expectations competitive equilibrium model with storage. The time frequency is one year, i.e. t+1 represents the next harvest. New crop price distributions are conditional on information known after old crop carryout stocks have been determined, but before weather shocks are revealed. Prices and quantity are standardized to make non-stochastic equilibrium at $100 and 100 units. Higher prices at time *t* reflect lower carryout stocks, and correspond to higher skewness of new crop price distribution.

Figure 1.5. Weather Stress in the Corn Crop



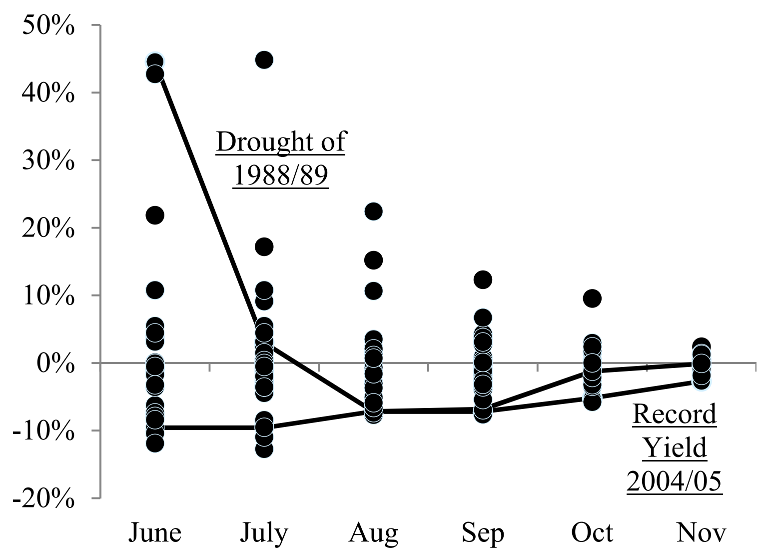
Note: Reproduced from Shaw et al. (1988). This figure shows the relationship between the age of the corn crop and the percentage yield reduction due to one day of moisture stress. Outer lines show boundaries of experimental results, while the middle line shows the average.

Figure 1.6. Monthly Projected Corn Yield 1980-2008 - Deviation from Trend



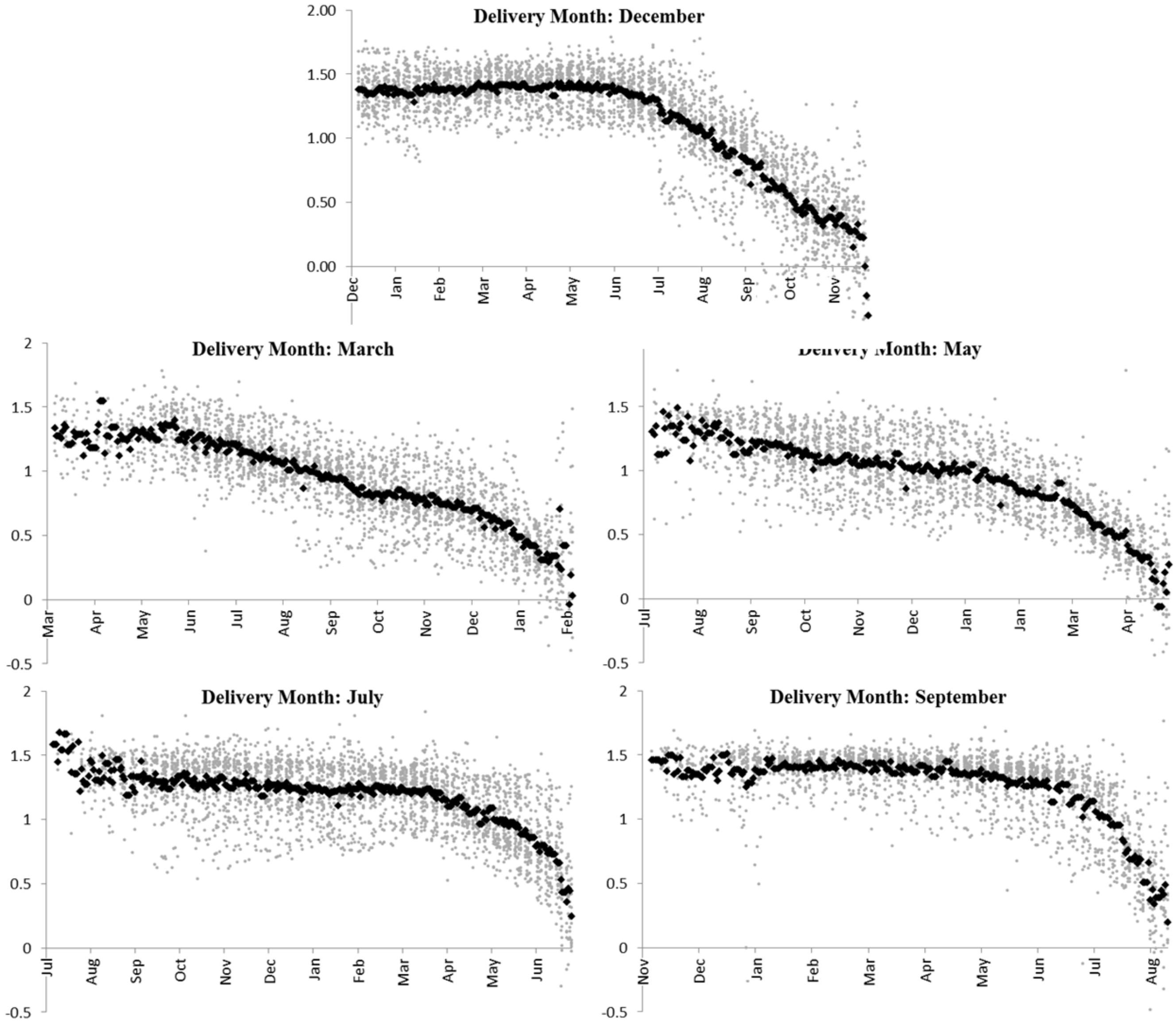
Note: For each year, trend yield was calculated as a simple linear regression over previous years, starting in 1960. Monthly projected yields were obtained from the WASDE report either directly or by calculations based on projected planted area and expected production size.

Figure 1.7. Monthly Projected Corn Yield 1980-2008 - Deviation from the Final Estimate (January)



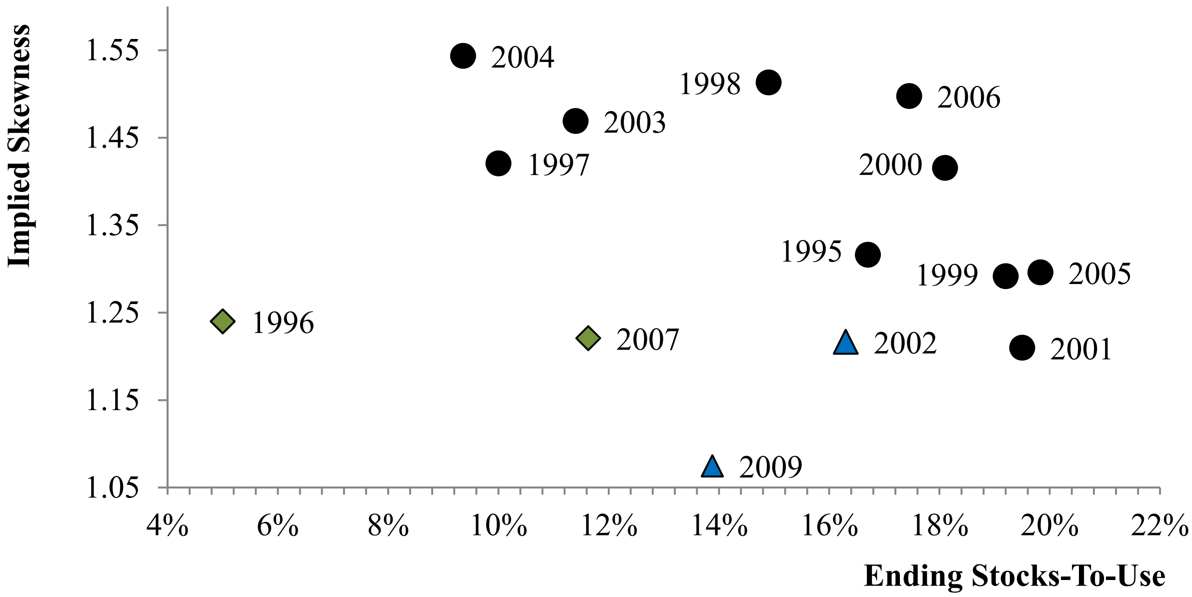
Note: For each month, projected yield was obtained from WASDE reports. Final estimates are taken from WASDE reports published in January.

Figure 1.8. Options on Corn Futures: Dynamics of Implied Skewness



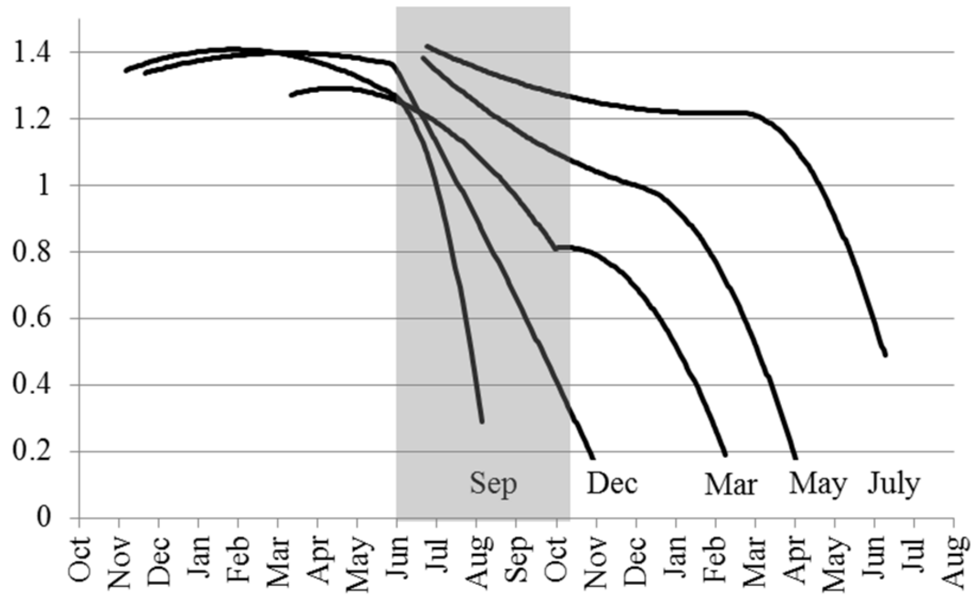
Note: Implied skewness is estimated for each contract and for each trading day separately using the generalized lambda distribution pricing model for options on commodity futures. Graphs show individual estimates over 1995-2009 for each contract and each time to maturity as gray scatter diagrams with the average for a particular time-to-maturity bolded.

Figure 1.9. Relationship between Implied Skewness and Expected Ending Stocks-to-Use



Note: Years with increase in intended cultivated acreage of 5 or more percent are drawn using green rhombs. Years with the June-to-June increases in the unemployment rate of 1 percent or more are drawn using blue triangles.

Figure 1.10. Predicted Intra-Year Dynamics of Implied Skewness for Options on Corn Futures Contracts



Note: Implied skewness is modeled with quadratic segments with one regime switch. The grayed area covers the corn growth sensitive period, i.e. silking through harvest time (approximately Jun 20 – Oct 20).

Table 1.1. Determinants of Implied Skewness: Descriptive Statistics

Variable	Mean	Standard Deviation	Min	Max
Implied Skewness	1.33	0.14	1.07	1.54
Ending Stocks-to-Use (%) WASDE June Projection	14.4	5.36	4.03	21.23
Intended Acreage Planted – Percentage Change	1.37	5.89	-4.84	15.48
Unemployment Rate Change	0.17	0.23	-0.7	4.00

Note: Implied skewness was calculated for December corn contracts as the average for implied parameters over 10 trading days following the June WASDE report. On average, 100-150 data points were used in estimating implied parameters for each trading day in the stated periods.

Table 1.2. Corn Supply/Demand Balance Sheet 1995-2009

		Marketing year	1995	1996	1997	1998	1999	2000	2001	2002
SUPPLY	Exp.	Exp. acres planted	73.3	79.0	81.4	80.8	78.2	77.9	76.7	78.0
		Exp. acreage chg.	-7.4%	11.0%	2.4%	0.7%	-2.5%	0.6%	-3.5%	2.9%
		Exp. yield	119.7	126.0	131.0	129.6	131.8	137.0	137.0	135.8
	Realized	Acres planted	71.2	79.5	80.2	80.2	77.4	79.5	75.8	79.1
		Acres harvested	65.0	73.1	72.7	72.6	70.5	72.4	68.8	69.3
		%Harvested	91.3%	91.9%	90.6%	90.5%	91.1%	91.1%	90.8%	87.6%
		Yield	113.5	127.1	127.0	134.4	133.8	137.1	138.2	130.2
		Production	7,374	9,293	9,366	9,761	9,437	9,968	9,507	9,008
		Beginning stocks	1,558	426	883	1,308	1,787	1,718	1,899	1,596
		Imports	16	13	9	19	15	7	10	14
		Total supply	8,948	9,732	10,258	11,088	11,239	11,693	11,416	10,618
DEMAND	Exp.	Exp. total demand	8,600	8,820	9,000	9,360	9,480	9,645	9,725	9,535
		Exp.ending stocks	347	909	1,259	1,727	1,759	2,048	1,621	1,084
		Exp. stocks-to-use	4.03%	10.3%	13.9%	18.4%	18.5%	21.2%	16.7%	11.4%
	Realized	Feed & residual	4,696	5,360	5,505	5,472	5,664	5,838	5,877	5,558
		Food/Seed/Ind.	1,598	1,692	1,782	1,846	1,913	1,967	2,054	2,340
		Ethanol	N/A	N/A	N/A	N/A	N/A	N/A	N/A	996
		Exports	2,228	1,797	1,504	1,981	1,937	1,935	1,889	1,592
		Total demand	8,522	8,849	8,791	9,299	9,514	9,740	9,820	9,490
		Ending stocks	426	883	1,467	1,789	1,725	1,953	1,596	1,128
		Stocks-to-use	5.0%	10.0%	16.7%	19.2%	18.1%	20.1%	16.3%	11.9%
		Avg. farm price	3.24	2.71	2.43	1.94	1.82	1.85	1.97	2.32

Table 1.2. Corn Supply/Demand Balance Sheet 1995-2009 (continued)

		Marketing year	2003	2004	2005	2006	2007	2008	2009
SUPPLY	Exp.	Exp. acres planted	79.0	79.0	81.4	78.0	90.5	86.0	85.0
		Exp. acreage chg.	-0.1%	0.4%	0.6%	-4.6%	15.6%	-8.1%	-1.2%
		Exp. yield	139.7	145.0	148.0	149.0	150.3	148.9	153.4
	Realized	Acres planted	78.7	80.9	81.8	78.3	93.6	86.0	86.5
		Acres harvested	70.9	73.6	75.1	70.6	86.5	78.6	79.6
		%Harvested	90.1%	91.0%	91.8%	90.2%	92.4%	91.4%	92.0%
		Yield	142.2	160.4	147.9	149.1	151.1	153.9	164.7
		Production	10,114	11,807	11,112	10,535	13,074	12,101	13,110
		Beginning stocks	1,087	958	2,114	1,967	1,304	1,624	1,673
		Imports	14	11	9	12	20	15	8
		Total supply	11,215	12,776	13,235	12,514	14,398	13,740	14,792
DEMAND	Exp.	Exp. total demand	10,405	10,560	11,060	11,525	12,960	12,140	13,190
		Exp.ending stocks	806	2,215	2,176	987	1,433	1,600	1,603
		Exp. stocks-to-use	7.7%	20.9%	19.6%	8.5%	11.1%	13.2%	12.2%
	Realized	Feed & residual	5,798	6,162	6,141	5,598	5,938	5,205	5,159
		Food/Seed/Ind.	2,537	2,686	2,981	3,488	4,363	4,993	5.938
		Ethanol	1,168	1,323	1,603	2,117	3,026	3,677	4,568
		Exports	1,897	1,814	2,147	2,125	2,436	1,858	1,987
		Total demand	10,232	10,662	11,269	11,211	12,737	12,056	13,084
		Ending stocks	983	2,114	1,966	1,303	1,661	1,684	1,708
		Stocks-to-use	9.61%	19.8%	17.5%	11.6%	13.0%	14.0%	13.1%
		Avg. farm price	2.42	2.06	2.00	3.04	4.20	4.06	3.55

Note: Acres planted and harvested are measured in million acres, yield in bushels per acre, beginning and ending stocks, imports, exports and other demand categories are measured in million bushels. Average farm price measured in U.S. dollars per bushel. Corn marketing year starts on September 1 of the current calendar year, and ends on August 31 the following calendar year. Expected acres planted based on “Prospective Plantings” report published at the end of March preceding the marketing year. Expected total demand, ending stocks, and stocks-to-use are taken from June WASDE report. For example, marketing year 2001/02 (denoted in the table simply as 2001) started on 09/01/2001, and ended on 08/31/2002. For that year, expected acres planted was published on 03/31/2001 and expected total demand, ending stocks and stocks-to-use were taken from WASDE report published in 06/12/2002. Variables used in econometric analysis are bolded.

Table 1.3. Models of Intra-Year Skewness Dynamics: Regression Results

	March	May	July	September	December
Maximum time to maturity	330	290	350	275	350
Number of observations	2736	2558	3058	2321	3448
1. Linear model with no regime change					
SSE	178.49	156.39	180.62	166.28	295.67
R^2	0.55	0.56	0.30	0.34	0.58
2. Quadratic model with no regime change					
SSE	171.32	135.61	159.60	114.36	163.98
R^2	0.57	0.62	0.38	0.55	0.77
3. Linear model with one change in regime (timing is estimated endogenously)					
SSE	170.94	127.50	143.83	98.90	158.80
R^2	0.57	0.65	0.45	0.61	0.77
Switch time	63	64	62	56	162
4. Quadratic model with one change in regime (timing is estimated endogenously)					
SSE	167.36	126.36	141.17	96.10	158.30
R^2	0.58	0.65	0.46	0.62	0.77
Switch time	130	125	112	69	160
Switch date	Oct, 19	Dec, 22	Mar, 6	Jun, 18	Jun, 19
Switch time 95% CI (boot.)	(119-140)	(86-167)	(80-149)	(49-94)	(151-169)
Switch time 95% CI (asy.)§	(113-144)	(86-151)	(70-95)	(51-95)	(142-183)
Wald test: (2) vs. (4)					
Critical val. (simul., 95%)	10.89	10.94	10.57	10.73	10.77
Wald-statistic	64.72	187.14	399.30	441.11	123.76
(p-value)	0.0000	0.0000	0.0000	0.0000	0.0000
Wald test: (3) vs. (4)					
Critical val: $\chi^2(0.95, 2)$	5.99	5.99	5.99	5.99	5.99
Wald-statistic	58.56	23.06	57.65	67.64	10.98
(p-value)	<0.001	<0.001	<0.001	<0.001	0.004
Wald tests for knots:					
Null: $\tilde{\tau}^{DEC} = \tilde{\tau}^{SEP} + 90$			Wald: 4.16, Crit. val.: 24.74 (p-value: 0.001)		
Null: $\tilde{\tau}^{MAR} = \tilde{\tau}^{MAY} = \tilde{\tau}^{JUL}$	Wald: 1.35, Crit. val.: 6.83 (p-value: 0.662)				
5. Quadratic model with two changes in regime (timing is estimated endogenously)					
SSE	167.28	126.19	140.96	95.59	158.16
R^2	0.58	0.65	0.46	0.62	0.77
Switch time 1	120	268†	270	220	270
Switch time 2	75	137	110	77	159
Wald test: (4) vs. (5)					
Critcal val. (simul., 95%)	13.82		13.79	11.57	12.15
Wald-statistic	1.30	N/A	4.55	1.47	3.05
p-value	0.9265		0.9094	0.9690	0.9575

† Model estimates to a corner solution, and is therefore treated as misspecified.

§ Estimated using asymptotic likelihood ratio test developed by Hansen (2000), which does not impose restriction of continuity in predicted variable.

Table 1.4. Relative Contribution of Corn Growth-Sensitive Period to Reduction in Implied Skewness

Contract		
March	% of contract life	% skewness reduction
non-growth period	40.5%	5.7%
in growth period	59.5%	94.3%
May	% of contract life	% skewness reduction
non-growth period	39.1%	19.6%
in growth period	69.1%	80.4%
July	% of contract life	% skewness reduction
non-growth period	50.9%	23.5%
in growth period	49.1%	76.5%
September	% of contract life	% skewness reduction
non-growth period	76.0%	7.5%
in growth period	24.0%	92.5%
December	% of contract life	% skewness reduction
non-growth period	64.6%	33.47%
in growth period	35.4%	76.53%

Note: For carry contracts (March, May, and July), percentages reported refer to contract life before a regime switch, i.e. excluding the last four months of contracts life. For new-crop contracts (September and December), percentage reported are over the entire contract life.

Table 1.5. Determinants of Implied Skewness: Regression Results

Explanatory variables	Dependent Variable: GLD Implied Skewness
Constant	1.55
	(0.10)
Ending Stocks-to-Use (%)	-1.28
	(0.64)
Intended Acreage Planted –	-1.52
Percentage Change	(0.58)
Unemployment	-0.07
Percentage Change	0.02
Degrees of Freedom	10
Mean Root Square Error	0.075
R^2	0.66

Note: The critical t-statistic for 10 d.f. at 95% confidence is 1.81 for one-tail tests and 2.22 for two-tail tests. All coefficients are statistically significant at the 95% confidence level (Ending stocks-to-use coefficient is significant at the 95% using one-tailed test, or 90% using two-tailed test).

2. Volatility Dynamics in Non-storable Commodities: A Case of Class III Milk Futures

ABSTRACT. This paper begins with an account of the evolution of dairy futures markets in the U.S. A partially overlapping time series (POTS) model is then estimated to examine price behavior in simultaneously traded Class III milk futures contracts. POTS is a latent factor model that measures price changes in futures as a linear combination of a common factor, i.e. information affecting all traded contracts, and an idiosyncratic term specific to each contract. This paper contributes to the literature by showing that the importance of a common factor in price volatility determination for dairy is related to capital production factors, i.e. the dairy herd. Finally, it is shown that Class III milk futures volatility decreases as contracts approach maturity. This "Inverse Samuelson Effect" comes from the fact that Class III milk futures are cash-settled contracts that settle against a formula-based price. The results suggest that the importance of the common factor declines as one approaches maturity, implying that individual contract months are poor substitutes in hedging a specific month's cash price risk. Thus, despite low liquidity in the market, it is useful to have 12 contract delivery months per year.

JEL Codes: G13, Q14

Keywords: Partially overlapping time series model, EM algorithm, dairy futures, volatility

2.1. Introduction

The U.S. Dairy industry has undergone significant change over the last two decades. Some of the more salient features include continued increases in yield, regional shifts in production accompanied by an increasing role of large farms, and major changes in dairy policy and milk pricing (Blayney (2002), Blayney et al. (2006), Bozic and Gould (2009)). Over the last two decades federal price supports have been low relative to market prices, and the volatility of milk price volatility has increased dramatically across all categories of milk utilization. This increasingly uncertain economic environment resulted in the development of new instruments for managing price risk in the early 1990's. Beginning in 1993, futures contracts were introduced to meet the hedging needs of the dairy sector. However, in response to regular changes in both industry practices and public policy, dairy futures contracts have gone through continuous re-design.

Several papers have been written on the performance of specific dairy futures contracts (Fortenbery, Cropp, and Zapata (1997), Fortenbery and Zapata (1997), and Maynard, Wolf, and Gearhardt (2005)) but a detailed exposition of their history and performance is still lacking. This represents the first contribution of this paper.

To evaluate the performance of a futures contract, it is essential to understand the price volatility dynamics including seasonality, time-to-maturity effects, and cash-futures volatility spillovers, as well as the effects of speculation and liquidity. This paper addresses the first two topics using the Class III milk futures contract. This contract exhibits the highest trade volume among the family of dairy futures contracts. Cash-futures volatility and speculative impacts on dairy prices are the subject of a parallel study.

There are two characteristics that differentiate the Class III milk contract from many other agricultural futures contracts. First, milk is essentially a non-storable product, and the only way to partially preserve or enhance the economic value of milk over time is to undertake an irreversible conversion to one of the derived dairy products. Another differentiating characteristic is that Class III milk contracts are not settled by physical delivery. They are cash settled against the USDA announced Class III milk price based on a publicly known formula. This raises two research questions. First, for storable commodities, temporal arbitrage opportunities guarantee that information affecting a futures price for a certain maturity will also have an impact on all other traded delivery periods. Does the perishable nature of milk result in information pertinent to one contract having little or no influence on the price of any other contract? Second, in storable commodities, volatility normally increases as time to maturity declines (Smith (2005)). This is called the 'Samuelson Effect,' after Paul Samuelson (1965) who first proposed the conditions under which such a relationship exists. But if milk futures are cash settled against a formula-based price, and the USDA formula is both transparent and based to a large extent on information known prior to contract expiration, should we still expect to see a Samuelson Effect? Addressing these issues is the second and most important contribution of this paper.

The paper proceeds as follows: in the next section we provide an outline of the economic environment faced by the US milk industry. This is followed by a description of the history of dairy futures contracts including a review of previous work analyzing dairy futures. Section 2.4 introduces the POTS model and the data used to analyze dairy futures price performance. In section 2.6 we present the main econometric results. The final section discusses consequences of the results, and draws conclusions related to the structure of the Class III milk futures market.

2.2. Overview of Dairy Futures Contracts

2.2.1. Evolution of Dairy Futures

U.S. milk prices were relatively stable through the late 1980's as government price supports were generally binding. In the early1990's, however, milk price volatility increased significantly as both support prices and government stocks of dairy products were reduced. This has been widely reported and can be seen in Figure 2.1.

In response to increased price volatility tools were developed to help the dairy sector manage its new market price risk. The first dairy futures contracts were introduced by the Coffee, Sugar, and Cocoa Exchange (CSCE - now part of the New York Board of Trade) in New York in 1993. The initial impetus for the development of dairy futures came from the cheese and confectionary industries. Cheese makers were concerned about price volatility in cheddar cheese and milk, as cheddar is used as the reference price for many cheeses produced domestically. At the same time the confectionary industry was concerned with increased price volatility in non-fat dry milk, an important ingredient in candy making (Fortenbery (2009)). Many of the confectionary firms were hedging cocoa and sugar at the CSCE, and once they began to experience price volatility in non-fat dry milk they wanted a way to manage that risk as well. Since the confectionary firms were already using contracts traded at the CSCE, this seemed the logical place to launch the non-fat dry milk contract. As a result, the CSCE rolled out a cheddar cheese and a non-fat dry milk futures contract in 1993. Since prices for cheddar cheese and milk are highly correlated, it was assumed that other dairy firms, including dairy farmers, would cross hedge their milk price risk with the cheddar cheese futures contract. However, despite a significant promotional effort on the part of the exchange, initial trade volume was very thin.

One reason for thin volume was the lack of cross hedging of milk in cheese futures. In response, the CSCE developed and launched a futures contract for fluid milk in 1995, but this contract also suffered low trade volume. Despite the low participation in dairy contracts at the CSCE, however, the Chicago Mercantile Exchange (CME) launched a nearly identical fluid milk contract in 1996. This marked the first time since wheat in the 1950's that two domestic futures exchanges competed head to head for trade in nearly identical products. The competition intensified when both exchanges launched a butter futures contract in 1996. Like the earlier dairy contracts, however, trade in butter futures was also nearly non-existent.

The next change in dairy contract design was in response to changes in federal milk policy. In 1997, both exchanges re-designed their fluid milk contracts, and converted them to a Basic Formula Price contract (BFP). The BFP was introduced in 1995 and was the USDA's monthly announced price from which prices paid to individual dairy farmers were derived. The BFP was based on USDA survey data adjusted by a product price formula. Since the BFP contracts were explicitly cash settled against the USDA BFP announced price that determined individual farmers' prices they provided a direct hedge opportunity for dairy producers.

In addition to its BFP contract, the CME launched a cheddar cheese contract in 1997. Once this happened the only unique contract still in existence in New York was the non-fat dry milk contract. However, this did not last long. In 1998, the Chicago Exchange rolled out both a dry milk and a dry whey futures contract. In 2000, the New York market eliminated its dairy contracts, and Chicago became the sole market for the trade of dairy futures.

Following requirements set forth in the 1996 Farm Bill, new milk pricing classes were introduced in January 2000. These replaced the BFP formula price. The Class III milk price was developed to reflect the minimum price for milk used to make cheese, while Class IV was introduced for milk used in products such as butter and non-fat dry milk. The fundamental change was that the BFP price had been based on surveyed purchase prices of manufacturing grade milk, while the new formulas were based on wholesale prices of products that used manufacturing milk as an input. Consequently, the BFP and cheddar cheese futures contracts were replaced with a Class III milk contract. In addition, the nonfat dry milk and dry whey contracts were discontinued, and a new Class IV milk contract was introduced.

Responding to requests from industry, CME continued to make changes to their dairy futures products over the next several years. A cash-settled butter contract was introduced in 2005 with a contract size equal to half the old physically delivered butter contract. Cash settled dry whey was introduced in March 2007. A nonfat dry milk cash-settled contract, discontinued in 2000, was redesigned and introduced in 2008. This was followed by a physically delivered nonfat dry milk contract in 2009, and deliverable cheese and international skim milk powder in 2010. Table 2.1 summarizes the development and abandonment of the various dairy contracts from 1993 through 2010.

2.2.2. Price Relationships among Current Contracts

Correlations between selected dairy prices over the last 10 years are given in Table 2.2. Note that the Class III price is strongly correlated with both cheddar cheese prices (National Agricultural Statistics Service (NASS) survey prices for the last week of the month) and fluid milk prices. The Class IV announced price is strongly correlated with fluid milk and nonfat dry milk prices. Butter and dry whey show weak correlation with Class III and Class IV prices. This suggests a need for separate futures contracts for these products if futures are to be used as risk management vehicles. However, neither nonfat dry milk nor butter futures contracts reveal much trade volume. Also, as of February 2011 open interest in the recently introduced deliverable cheese contract was extremely low, and may be due to the high correlation with Class III futures.

2.3. Literature Review

Early papers (Fortenbery and Zapata (1997), Thraen (1999)) examined pricing issues in dairy markets by measuring cointegration relationships between cheddar cheese cash and futures prices. Fortenbery and Zapata (1997) failed to find a significant cointegrating vector between prices, but argued that the market might not have been mature enough to establish a stable long-run equilibrium at the time of their analysis. Using a longer time series, Thraen does find a stable relationship between cash and futures prices for cheddar cheese.

Later, Fortenbery, Cropp and Zapata (1997), Zylstra, Kilmer and Uryasev (2004), and Maynard, Wolf, and Gearhardt (2005) examined the hedging opportunities for fluid milk with milk futures contracts. Fortenbery et al. evaluated the performance of the BFP milk contract, while the later papers focused on the Class III contract. It was found that the BFP contract was successful in reducing price risk for milk producers in the Upper Midwest, but less useful for producers on the West Coast. Zylstra et al. examined basis risk for Class I milk producers hedging with the Class III contract, and determined that basis risk prevented producers from locking in a minimum Class I milk price using only the Class III futures. Hedging performance could be improved by using a lagged futures position, and offsetting the hedge the day the Class I price is announced. Maynard et al. found that hedging generally resulted in a 50 to 60 percent reduction in price variance, but the markets favored large, sophisticated producers in major cheese manufacturing regions.

Market efficiency issues were studied by Tondel and Maynard (2004) and Sanders and Manfredo (2005). Tondel and Maynard focused on the butter market, and failed to reject the null hypothesis of market efficiency in the deliverable butter contract. Sanders and Manfredo studied the milk contract, and argued that milk futures do not encompass all the information contained in USDA forecasts at a two-quarter horizon. However, they also found that the revenue obtained by exploiting the difference in forecast performance is likely below transaction costs.

Buschena and McNew (2008) investigated the market impacts of the 1999 USDA Dairy Options Pilot Program. The program sought to teach dairy producers how to use put options to hedge downside price risk, and subsidized the purchase of the options. They found that options purchased under the Pilot Program generally resulted in higher options premiums compared to options not purchased through the program.

The low liquidity of dairy futures was a common theme in earlier work. This suggests there might be a trade-off between liquidity and the number of different futures products offered, with the introduction of new products leading to reductions of trading volume in existing products. In addition, most successful agricultural futures contract do not trade for delivery every month. Would decreasing the number of traded contract months be beneficial for milk futures in the sense of higher liquidity and lower hedging costs? Our analysis of volatility dynamics in Class III futures provides a conceptual framework for answering that question.

There has not been an extensive analysis of potential cyclical behavior in milk pricing, nor an examination of possible structural breaks in milk price dynamics induced by changes in federal dairy policy. It seems that U.S. dairy policy changes may have opened the door for a classic boom-bust cycle in milk production, where initial periods of very low milk prices result in a contraction of milk production that then leads to much higher prices. Higher prices then induce new entrants or expansions of existing farms.[5] This is the focus of Appendix 1.

2.4. Econometric Analysis

2.4.1. The POTS Model - Introduction

A common approach to studying volatility in futures markets is to construct a so called "nearby" series, where only data from contracts closest to expiry are used. In such an approach, when the nearby contract expires, or time-to-maturity falls below a certain predetermined number of trading days, data used for the "nearby" series are drawn from the next-to-nearby contract. This strategy creates a single time series which can then be used in econometric estimation. However, this approach is not without its problems. By deciding to use information from only a single contract on any given trading day, information from all other contracts is discarded. Furthermore, the "rollover" procedure for patching consecutive contracts may introduce complex non-linear dynamics .

In this paper we employ a partially overlapping time series (POTS) model similar to Smith (2005) and Suenaga, Smith and Williams (2008). The POTS model utilizes information from all contracts trading concurrently. The difference between a "nearby" and POTS approach is illustrated in Figure 1.2 for Class III milk futures. In the POTS approach, each line represents a new variable, while in the "nearby" approach only the bold segments of each contract trading period are used and patched together consecutively to create a continuous time series. In this context, the POTS model can be interpreted as an unbalanced panel method, as each contract

---

[5] In Appendix 1 we a present spectral analysis of milk prices and find that the primary difference in sample spectrums for the periods 1970-1991 vs. 1988-2009 is the emergence of 3 year cycle in the later period.

constitutes a separate time series that originates on the first day the contract is traded, and terminates at its' last trading day.

POTS is a latent factor model where price changes on contracts are assumed to be linear combinations of a common factor, i.e. information affecting all traded contracts, and an idiosyncratic term specific to each contract. This ability to differentiate the impact and relative importance of common factor and idiosyncratic effect is lost when considering only a nearby price series. Smith finds that for corn and natural gas, the information sets pertinent to contracts with different maturities show almost perfect overlap, i.e. the common factor explains close to 100 percent of futures price variance. However, it is the storable nature of these two commodities that renders this an expected result.

In explaining the POTS model, we closely follow the terminology and notation of Smith (2005). Let F denote the price of a futures contract, d the number of trading days until maturity for a particular contract, and t the date of the observation. Then subscripting $F_{d,t}$ suffices to uniquely identify any point in a partially overlapping panel data set. Smith models sources of volatility in futures prices as originating from a latent common factor ε_t, influencing all currently trading contracts, and idiosyncratic errors $u_{d,t}$, specific to each contract.

Modeling jointly all concurrently traded contracts, we can write:

$$\mathbf{\Delta F}_t = \boldsymbol{\theta}_t \varepsilon_t + \boldsymbol{\lambda}_t \mathbf{u}_t \tag{2.1}$$

where $\mathbf{\Delta F}_t$ is the $n_t \times 1$ vector of price changes for contracts traded at time t. The common factor is modeled as the scalar ε_t with $E\left(\varepsilon_t\right)=0, E\left(\varepsilon_t^2\right)=1$ and $\boldsymbol{\theta}_t$ is a $n_t \times 1$ vector of factor loadings for each of the n_t contracts that traded on date t. The vector $\mathbf{u}_t$ represents idiosyncratic shocks specific to each contract. For identification purposes, we assume that $\mathbf{u}_t \sim N\left(0, I_{n_t}\right)$, $E\left(\varepsilon_\tau \mathbf{u}_t\right) = 0, \forall \tau, t$ and $E\left(\mathbf{u}_t \mathbf{u}'_\tau\right) = 0$ for $t \neq \tau$. Finally, $\boldsymbol{\lambda}_t$ is the diagonal $n_t \times n_t$ matrix of the innovations' standard deviations. We assume futures prices are unbiased, i.e. $E\left(\boldsymbol{\Delta}\mathbf{F}_t \mid \mathfrak{I}^{t-1}\right) = 0$ where $\mathfrak{I}^{t-1}$ denotes the information set at $t-1$.

The factor loadings and innovation standard deviations are modeled using cubic splines with two knots. As Smith explains, spline functions capture deterministic seasonal and time-to-delivery effects. Separate splines are estimated for each delivery month. For a specific delivery month, factor loadings and innovation standard deviation splines have the following functional form:

$$
\begin{aligned}
\theta_{d,t} &= \sum_{j=1}^{3}\left(\phi_{0j} + \phi_{1j}\left(d_t - k_{j-1}\right) + \phi_{2j}\left(d_t + k_{j-1}\right)^2 + \phi_{3j}\left(d_t - k_{j-1}\right)^3\right) I_{jt} \\
\lambda_{d,t} &= \sum_{j=1}^{3}\left(\gamma_{0j} + \gamma_{1j}\left(d_t - k_{j-1}\right) + \gamma_{2j}\left(d_t + k_{j-1}\right)^2 + \gamma_{3j}\left(d_t - k_{j-1}\right)^3\right) I_{jt} \qquad (2.2) \\
I_{jt} &= 1\left(k_{j-1} \leq d_t \leq k_j\right)
\end{aligned}
$$

where I_{jt} is an indicator function, ϕ_{ij} and γ_{ij} are parameters estimated by the model, d_t is the number of trading days to maturity on date t for a particular contract, and knots connecting consecutive cubic functions, k_1, k_2 are chosen a priori to be 30 and125 trading days to maturity. Additional nodes are $k_0 = 0$ at contract expiry and $k_K = \max\left(d_t\right)$, i.e. maximal time-to-horizon trade observed for this specific contract month. We impose several constraints on the splines. First, consecutive cubic functions they must equal in value at the nodes. Letting $d_t = k_j$, at knot k_j we have

$$\phi_{0\,j+1} = \phi_{0j} + \phi_{1j}\left(k_j - k_{j-1}\right) + \phi_{2j}\left(k_j - k_{j-1}\right)^2 + \phi_{3j}\left(k_j - k_{j-1}\right)^3$$
$$\lambda_{0\,j+1} = \lambda_{0j} + \lambda_{1j}\left(k_j - k_{j-1}\right) + \lambda_{2j}\left(k_j - k_{j-1}\right)^2 + \lambda_{3j}\left(k_j - k_{j-1}\right)^3 \quad (2.3)$$

Next, to stipulate smoothness, slopes of the adjoining cubic functions must be equal at knots. Differentiating cubic functions with respect to d_t and setting the first derivatives equal to each other, we obtain

$$\phi_{1\,j+1} = \phi_{1j} + 2\phi_{2j}\left(k_j - k_{j-1}\right) + 3\phi_{3j}\left(k_j - k_{j-1}\right)^2$$
$$\lambda_{1\,j+1} = \lambda_{1j} + 2\lambda_{2j}\left(k_j - k_{j-1}\right) + 3\lambda_{3j}\left(k_j - k_{j-1}\right)^2 \quad (2.4)$$

Finally, we force the slope of both splines to equal zero at end points, i.e. at contract expiry and at maximum time-to-maturity horizon observed in the data. Denoting maximal time-to-maturity with k_K (maximum time-to-maturity is the last node), we get

$$\phi_{11} = 0, \phi_{1K} = 2\phi_{2K}\left(k_K - k_{K-1}\right) + 3\phi_{3K}\left(k_K - k_{K-1}\right)^2 = 0$$
$$\lambda_{11} = 0, \lambda_{1K} = 2\lambda_{2K}\left(k_K - k_{K-1}\right) + 3\lambda_{3K}\left(k_K - k_{K-1}\right)^2 = 0 \quad (2.5)$$

Each spline is thus fully determined by 12 parameters, but given the restrictions imposed only 6 free parameters need to be estimated per spline. Class III milk futures trade for 12 contract months per year, which brings the number of spline parameters that we need to estimate to 144; 72 coefficients for common factor splines, and as much for contract-specific innovation splines.

The time-varying conditional volatility in the common factor is modeled using a GARCH(1,1) process. Denoting the conditional variance of the common factor as $h_t^2 = E\left(\varepsilon_t^2 \mid \Im^{t-1}\right)$, we have

$$h_t^2 = \omega + \beta h_{t-1}^2 + \alpha E\left[\varepsilon_{t-1}^2 \mid \Im^{t-1}\right] \quad (2.6)$$

To restrict the unconditional variance to be equal to 1, free parameter is constrained to be $\omega = 1 - \alpha - \beta$.

Given the assumptions that idiosyncratic shocks are uncorrelated across contracts and over time, the conditional covariance matrix, just like in Smith (2005), is given by

$$\Sigma_t = E\left(\Delta\mathbf{F}_t\Delta\mathbf{F}_t' \mid \Im^{t-1}\right) = \boldsymbol{\theta}_t\boldsymbol{\theta}_t' h_t^2 + \boldsymbol{\lambda}_t^2 \tag{2.7}$$

2.4.2. Using Kalman Filter to Obtain Conditional Variance of the Common Factor

The only data directly observable are futures price changes, index classifying each observation to one of 12 contract months and time to maturity. In particular, we do not observe the common factor, and we need to use Kalman filter to obtain linear least squares forecast of the common factor at time *t*, given the information set $\Im_t$. To do so, let us first summarize fundamentals of state-space representation of dynamic systems. We follow closely exposition found in Hamilton (1994, ch. 13). We first write down the most general state-space system using Hamilton's notation, and Kalman filter results pertinent to our analysis. We then map our model to state-space representation and after considerable cancelations develop simple expressions for conditional expectation $\varepsilon_{t|t} = E\left(\varepsilon_t \mid \Im_t\right)$, and conditional variance of common factor

$P_{t|t} = E\left(\left(\varepsilon_t - \varepsilon_{t|t}\right)\left(\varepsilon_t - \varepsilon_{t|t}\right) \mid \Im_t\right)$.

Let us start by writing down the general state-space system equations. Following Hamilton, let $\mathbf{y}_t$ denote an $(n\times 1)$ vector observed at time t. Possibly unobserved $(r\times 1)$ vector $\boldsymbol{\xi}_t$ is called the *state vector*. The *state-space representation* of the dynamics of $\mathbf{y}_t$ is given by the *state equation*

$$\boldsymbol{\xi}_{t+1} = \mathbf{G}\boldsymbol{\xi}_t + \mathbf{v}_{t+1} \tag{2.8}$$

where $\mathbf{G}$ is the matrix of parameters of dimensions $(r\times r)$ and *observation equation*

$$\mathbf{y}_t = \mathbf{A}'\mathbf{x}_t + \mathbf{J}'\boldsymbol{\xi}_t + \mathbf{w}_t \tag{2.9}$$

where $\mathbf{x}_t$ is a $(k \times 1)$ vector of predetermined variables and $\mathbf{A}'$ and $\mathbf{J}'$ are matrices of parameters of dimensions $(n \times k)$ and $(n \times r)$.

The disturbances $\mathbf{v}_t$ and $\mathbf{w}_t$ are distributed normally with zero mean and covariances

$$E\left(v_t v_\tau\right) = \begin{cases} \mathbf{Q}_t & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases} \tag{2.10}$$

$$E\left(\mathbf{w}_t \mathbf{w}_\tau^{'}\right) = \begin{cases} \mathbf{R}_t & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases} \tag{2.11}$$

Finally, disturbances are uncorrelated at all lags:

$$E\left(v_t \mathbf{w}_\tau\right) = 0, \forall t, \tau \tag{2.12}$$

Hamilton motivates Kalman filter as an algorithm for calculating linear least squares forecasts of the state vector on the basis of data we have observed through date t.

$$\hat{\xi}_{t+1|t} = \hat{E}\left(\xi_{t+1} \mid \Im_t\right) \tag{2.13}$$

Kalman filter calculates these forecasts using a recursion, starting with $\hat{\xi}_{1|0}$ then using that forecast in obtaining $\hat{\xi}_{2|1}$ and so on. Recursion is started with a forecast of $\hat{\xi}_1$ based on no previously observed data, i.e. using unconditional mean of ξ_1

$$\hat{\xi}_{1|0} = E\left(\xi_1\right) \tag{2.14}$$

with mean square error (MSE) of the forecast

$$P_{1|0} = E\left\{\left[\xi_t - E\left(\xi_1\right)\right]\left[\xi_t - E\left(\xi_1\right)\right]'\right\} \tag{2.15}$$

Next, formula for forecasting $\mathbf{y}_t$ is obtained

$$\hat{\mathbf{y}}_{t|t-1} = \mathbf{A}'\mathbf{x}_t + \mathbf{J}'\xi_{t|t-1} \tag{2.16}$$

Inference about the current value of $\boldsymbol{\xi}_t$ is updated on the basis of the observation of $\mathbf{y}_t$ using formula for updating a linear projection

$$\begin{aligned}\hat{\boldsymbol{\xi}}_{t|t} = \hat{\boldsymbol{\xi}}_{t|t-1} + \left\{ E_t\left[\left(\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{t|t-1}\right)\left(\mathbf{y}_t - \hat{\mathbf{y}}_{t|t-1}\right)'\right]\right\} \times \\ \left\{ E_t\left[\left(\mathbf{y}_t - \hat{\mathbf{y}}_{t|t-1}\right)\left(\mathbf{y}_t - \hat{\mathbf{y}}_{t|t-1}\right)'\right]\right\}^{-1} \times \left(\mathbf{y}_t - \hat{\mathbf{y}}_{t|t-1}\right)\end{aligned} \tag{2.17}$$

Finally, MSE of the updated projection, $E\left[\left(\boldsymbol{\xi}_t - \boldsymbol{\xi}_{t|t}\right)\left(\boldsymbol{\xi}_t - \boldsymbol{\xi}_{t|t}\right)' \mid \Im_t\right]$, denoted $\mathbf{P}_{t|t}$ is

$$\mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{J}\left(\mathbf{J}'\mathbf{P}_{t|t-1}\mathbf{J} + \mathbf{R}\right)^{-1}\mathbf{J}'\mathbf{P}_{t|t-1} \tag{2.18}$$

where $\mathbf{P}_{t|t-1}$ is MSE of one-step ahead forecast of the state vector

$$\mathbf{P}_{t|t-1} = E_{t-1}\left\{\left[\boldsymbol{\xi}_t - E_{t-1}\left(\boldsymbol{\xi}_t\right)\right]\left[\boldsymbol{\xi}_t - E_{t-1}\left(\boldsymbol{\xi}_t\right)\right]'\right\} \tag{2.19}$$

Rewriting the POTS model in state-space format, we see that there are no pre-determined variables $\mathbf{x}_t$, i.e. $\mathbf{A}' = 0$, state equation is simplified by having $\mathbf{G} = 0$. Common factor ε_t is $\left(1\times1\right)$ state vector, with state equation being

$$\varepsilon_{t+1} = 0 \cdot \varepsilon_t + v_{t+1} \tag{2.20}$$

Vector of observed variables is $\Delta\mathbf{F}_t$ and coefficient next to state vector is time dependent, $\mathbf{J}_t = \boldsymbol{\theta}_t$ so observation equation is

$$\Delta\mathbf{F}_t = \boldsymbol{\theta}_t\varepsilon_t + \mathbf{w}_t \tag{2.21}$$

Common factor is uncorrelated through time, thus

$$\hat{\varepsilon}_{t|t-1} = 0 \tag{2.22}$$

Together with equation (2.16) this implies that

$$\Delta\hat{F}_{t|t-1} = 0 \tag{2.23}$$

To simplify expression (2.17) for updating the inference on the common factor at time *t* based on data observed at time *t*, notice that

$$E_{t-1}\left(\varepsilon_t \Delta \mathbf{F}_t\right) = E_{t-1}\left(\varepsilon_t\left(\boldsymbol{\theta}_t \varepsilon_t + \mathbf{w}_t\right)\right) = \boldsymbol{\theta}_t E_{t-1}\left(\varepsilon_t^2\right) = \boldsymbol{\theta}_t h_t^2 \tag{2.24}$$

Based on (2.22), (2.23), (2.24) and (2.7), we find that updated inference on the common factor can be written as

$$\varepsilon_{t|t} = h_t^2 \boldsymbol{\theta}_t' \Sigma_t^{-1} \Delta \mathbf{F}_t \tag{2.25}$$

Notice that equation (2.18) can be rewritten using the fact that $\mathbf{P}_{t|t-1} = h^2$, $\mathbf{R}_t = \boldsymbol{\lambda}_t^2$ and

$\mathbf{J}'\mathbf{P}_{t|t-1}\mathbf{J} + \mathbf{R} = \boldsymbol{\theta}_t \boldsymbol{\theta}_t' h_t + \boldsymbol{\lambda}_t^2$:

$$P_{t|t} = h_t^2 - h_t^4 \boldsymbol{\theta}_t' \Sigma_t^{-1} \boldsymbol{\theta}_t \tag{2.26}$$

In equation (2.6) expression $E\left(\varepsilon_{t-1}^2 \mid \Im_{t-1}\right)$ can now be expressed with the help of (2.25) and (2.26)

:

$$E\left(\varepsilon_{t-1}^2 \mid \Im_{t-1}\right) = \varepsilon_{t-1|t-1}\varepsilon_{t-1|t-1}' + P_{t-1|t-1} \tag{2.27}$$

Now we have everything in place to recursively calculate conditional variances of the common factor. Unconditional variance of the common factor is one, so recursion is started by setting $h_1^2 = 1$. To calculate h_2^2 we need to calculate $E\left(\varepsilon_1^2 \mid \Im_1\right) = \varepsilon_{1|1}^2 + P_{1|1}$. From (2.25) we calculate $\varepsilon_{1|1} = h_1^2 \boldsymbol{\theta}_1 \Sigma_1^{-1} \Delta \mathbf{F}_1$ and $P_{1|1} = h_1^2 - h_1^4 \boldsymbol{\theta}'_t \Sigma_1^{-1} \boldsymbol{\theta}_t$. After obtaining h_2^2 we repeat the same procedure to calculate h_3^2, and so on. Of course, everything that is written in this section assumes that parameters of the model are known, while in fact they need to be estimated. We explain the estimation procedure in detail in the next section.

2.4.3. Estimating POTS Model

Conditional on information available at $t-1$, futures price change is assumed to be normally distributed with zero mean and conditional variance Σ_t developed in (2.7). Unbiasedness of futures prices is evident from Figure 2.4, thus zero mean assumption does not seem to be violated. Assumption of normality is not as binding due to classical results on consistency and asymptotic normality of coefficient estimates obtained using quasi-maximum likelihood. Therefore, to estimate the model, we need to maximize the Gaussian log-likelihood function:

$$\begin{aligned} L &= \sum_{t=1}^{T} \log f\left(\Delta \mathbf{F}_t \mid \Im_{t-1}\right) \\ &= -\frac{\overline{n}T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log\left|\Sigma_t\right| - \frac{1}{2}\sum_{t=1}^{T}\Delta \mathbf{F}_t' \Sigma_t^{-1} \Delta \mathbf{F}_t \end{aligned} \tag{2.28}$$

where $\overline{n} = T^{-1}\Sigma_{t=1}^{T} n_t$ is the average number of contracts traded each day, and n_t is the number of contracts traded on day t. Therefore, $\overline{n}T$ is simply the total number of observations. All parameters that need to be estimated enter the likelihood function through the conditional covariance matrix Σ_t. Combining (2.6) and (2.7) we can write

$$\Sigma_t = \boldsymbol{\theta}_t \boldsymbol{\theta}_t' \left[\omega + \beta h_{t-1}^2 + \alpha E\left[\varepsilon_{t-1}^2 \mid \Im^{t-1}\right]\right] + \boldsymbol{\lambda}_t^2 \tag{2.29}$$

Now we immediately see that unobserved common factor will make estimation of the likelihood function complicated. To facilitate estimation we rewrite the likelihood function for a single trading day as follows:

$$f\left(\Delta \mathbf{F}_t \mid \Im_{t-1}\right) = f\left(\Delta \mathbf{F}_t \mid \varepsilon_t, \Im_{t-1}\right) \times f\left(\varepsilon_t \mid \Im_{t-1}\right) \tag{2.30}$$

Taking logs, we get

$$\log f\left(\Delta \mathbf{F}_t \mid \Im_{t-1}\right) = \log f\left(\Delta \mathbf{F}_t \mid \varepsilon_t, \Im_{t-1}\right) + \log f\left(\varepsilon_t \mid \Im_{t-1}\right) \tag{2.31}$$

The first term in the sum in (2.31) is now much simplified due to conditioning on information about the common factor. If we could observe the common factor ε_t, we could be directly maximizing the 'complete data' likelihood written below.

$$\begin{aligned} L_C &= -\frac{\overline{n}T}{2}\log(2\pi) - \sum_{t=1}^{T}\log\left|\boldsymbol{\lambda}_t\right| - \frac{1}{2}\sum_{t=1}^{T}\left(\Delta \mathbf{F}_t - \boldsymbol{\theta}_t \varepsilon_t\right)' \boldsymbol{\lambda}_t^{-2}\left(\Delta \mathbf{F}_t - \boldsymbol{\theta}_t \varepsilon_t\right) \\ &\quad - \frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log\left|h_t^2\right| - \frac{1}{2}\sum_{t=1}^{T} h_t^{-2}\varepsilon_t^2 \end{aligned} \tag{2.32}$$

However, we do not observe ε_t so the way forward is to iterate between using Kalman filter to compute expected complete data likelihood, given parameters, and maximizing expected likelihood with respect to the parameters. Using (2.27), we write the expected complete data likelihood as follows:

$$\begin{aligned} E\left(L_C \mid \Im_T\right) = & -\frac{\overline{n}T}{2}\log\left(2\pi\right) - \sum_{t=1}^{T}\log\left|\boldsymbol{\lambda}_t\right| - \frac{1}{2}\sum_{t=1}^{T}\left(\Delta\mathbf{F}_t - \boldsymbol{\theta}_t\varepsilon_{t|t}\right)'\boldsymbol{\lambda}_t^{-2}\left(\Delta\mathbf{F}_t - \boldsymbol{\theta}_t\varepsilon_{t|t}\right) \\ & -\frac{1}{2}\sum\boldsymbol{\theta}_t'\boldsymbol{\lambda}_t^{-2}\boldsymbol{\theta}_t P_{t|t} - \frac{T}{2}\log\left(2\pi\right) - \frac{1}{2}\sum_{t=1}^{T}\log\left|h_t^2\right| - \frac{1}{2}\sum_{t=1}^{T}h_t^{-2}\varepsilon_{t|t}^2 - \sum_{t=1}^{T}\frac{1}{2}h_t^{-2}P_{t|t} \end{aligned} \quad (2.33)$$

In maximizing (2.33) we use EM algorithm developed by Dempster et al. (1977), modified as in Smith (2005). Computing the expectation of the complete data likelihood with respect to the latent variable is called the E-step, and maximizing the expected likelihood with respect to the parameters is referred to as the M-step. Algorithm is the following:

1) Make an initial guess about the parameters. For conditional variance of the common factor, initial guess for α coefficient in equation (2.6) is 0.08, and for coefficient β it is 0.90. To obtain initial guess for spline parameters notice first that from **Error! Reference source not found.** it follows that each spline is linear in $\mathbf{x}$, where $\mathbf{x}$ consists of products of indicator functions and exponents of time-to-next-knot counters. For example, $x_0 = 1; x_1 = \left(d_t - k_0\right)\times I\left\{k_0 \le d_t \le k_1\right\}; x_2 = \left(d_t - k_0\right)^2 \times I\left\{k_0 \le d_t \le k_1\right\}, etc.$

   Initial guesses for theta-splines (splines for common factor, separate spline for each contract month) are obtained by projecting absolute values of dependent variable (futures price changes) on $\mathbf{x}$, then multiplying projection coefficients by 0.7. For lambda-splines (contract-specific shocks) vectorized mean absolute value of dependent variable is projected on $\mathbf{x}$. Both projections are appropriately modified to account for restrictions (2.3), (2.4) and (2.5).

2) Using current guess for spline and GARCH parameters, use the Kalman filter to obtain $\varepsilon_{t|t}$ and $P_{t|t}$.
3) Keeping both free GARCH parameters fixed, Newton-Raphsen is used in M-step to maximize (2.33)
4) Steps 2 (E-step) and 3 (M-step) are iterated until convergence, not changing GARCH parameters.
5) Now keeping spline parameters fixed, maximize likelihood with respect to GARCH parameters.
6) Step 2-5 are repeated until convergence.
7) Parameter estimates obtained upon convergence in step 6 are used as starting values in final numerical optimization over both spline and GARCH parameters using BHHH algorithm.

2.5. Data

The POTS model for dairy futures is estimated using the Class III futures contracts because they have the highest daily trading volume of all dairy futures, and an open interest that is generally four times larger than the open interest of all other dairy futures combined. Class III futures are cash settled against the announced USDA Class III milk price. Although this contract is used for hedging manufacturing and fluid grade milk, both of which are non-storable commodities, the settlement price is determined by the price of three dairy products, two of which are storable: cheddar cheese (not storable), butter (storable) and dry whey (storable). [6]

[6] One could argue that cheddar cheese is storable, but through storage it is aged and that changes its value. The USDA price surveys used to calculate the Class III milk price exclude cheese sold that is over 30 days old.

Every week, NASS surveys all cheese, butter, dry whey and nonfat dry milk plants that commercially produce more than 1 million pounds of these products, and computes national average prices for each product. The Class III milk price is then calculated once a month using a USDA formula that is based on the monthly weighted average prices of the products. The milk pricing formula is rather complex. The three product prices are first used to calculate imputed prices of milk components, i.e. protein, butterfat and other solids. In the second step, milk component prices enter the formula to get a skim milk price. Finally, the Class III milk price is calculated using the base skim milk price and assuming 3.5% butterfat content. Conflating this multi-step USDA formula into a simple linear function of cheese, butter and dry whey prices suggests that most of the manufacturing grade milk value comes from cheese. For example, Class III milk price for October 2008 was announced the Agricultural Marketing Service on October 31, 2008 at $17.06 per cwt. It was based on wholesale prices cheddar cheese, dry whey and butter observed in weeks ending on 10/4, 10/11, 10/18, and 10/25. Weighted average monthly prices for these commodities were calculated using weekly sales data. In the end, cheese accounted for 86.08% of the milk price, while butter and dry whey contributed 2.55% and 11.37%, respectively. Full calculation is presented in Table 2.3.

Figure 2.3 presents the open interest (aggregated over all traded contract months) and the nearby Class III futures price for the period 2000-2009. Three patterns emerge from this figure. First, open interest shows an upward trend from 2000 through 2007, then flattens to an average level of 25,000 contracts. Second, open interest exhibits strong seasonality that corresponds to the production patterns in milk, with a peak in May or June. Finally, the milk price and open interest are strongly positively correlated. This is not un-expected - as milk prices increase, users of manufacturing milk will likely hedge more of their input costs, and farmers may feel more inclined to lock in a good price for their product.

The Class III price announcement refers to the Class III price for the previous month. For example, the October 2010 Class III milk price was announced on Nov, 5 2010. This implies that in contrast to other agricultural contracts, Class III futures trade through their delivery month. Trade ends the business day immediately preceding the day on which USDA announces the Class III price.

Data used for model estimation covers the period January 2000 through October 2009. Initially, trading in Class III futures was allowed for up to 12 months prior to maturity. This was later expanded to 18 months, and then to 24 months. However, there is very little volume for contracts more than 1 year out so we restrict our analysis to the daily closing prices for the 12 most nearby contracts. This results in 28,145 data points over 118 contracts, classified in 12 categories based on the contract delivery month. Over the entire data period, average futures price change is 0.125 cents with a standard deviation is 13.21 cents. The largest day-to-day drop in price is $1.00 per cwt., and the largest increase is 95 cents. The distribution of price changes is leptokurtic, with an excess kurtosis of 7.82. It is also slightly left-skewed with a skewness of -0.14.

2.6. Results

To understand price dynamics in dairy markets, we first investigate whether futures prices appear unbiased. As noted earlier, this is an explicit assumption in the POTS model. If prices are unbiased, then we would expect the prediction errors associated with using futures price as an expectation of later cash price to average zero. In other words, futures would not systematically over or under predict cash prices. This is a necessary condition for futures price efficiency and thus for the market to effectively provide for price discovery. To examine this we calculate, for each futures contract at time t with d days to maturity, the percentage difference between the current futures price $F_{d,t}$ and the terminal settlement price P_T. Note that P_T is *unknown* before expiration time T, and only discovered ex post. Realized percentage prediction errors, $E_{d,t}$ are thus calculated as

$$E_{d,t} = \frac{F_{d,t} - P_T}{F_{d,t}} \times 100 \tag{2}$$

Using scatter diagrams, plots of prediction errors are developed, with time to maturity d on the x-axis and percentage prediction errors $E_{d,t}$ on the y-axis. If the mean of $E_{d,t}$ is below zero, evidence exists that d days to maturity futures prices are systematically downward biased. This result would be consistent with the Keynesian concept of a positive marginal risk premium. If, on the other hand, mean of $E_{d,t}$ is significantly above zero, that implies that futures prices exhibit a systematic upward bias.

Figure 2.4 presents realized prediction errors for the period 2000-2009. Gray dots represent individual prediction errors, while the bold black line plots root mean square prediction error for each time-to-maturity d. Market efficiency postulates that prediction errors would be centered around zero, so the appropriate measure of prediction accuracy is the root mean square error (RMS). The RMS at 30 days to maturity is 2.77%, but rises to 14.20% 90 days and 27.76% one year to maturity. We find that futures price prediction errors are centered around zero up to 180 days to maturity, but exhibit weak evidence of under prediction bias for time-to-maturity horizons higher than 60 days, reaching a maximum 4.5% negative bias 350 days to maturity. We do not explicitly examine the statistical properties of this bias, but given the high degree of correlation of prediction errors at such distant horizons, and a root mean square error of 26.65% at 350 days to maturity, we do not expect the bias to be statistically significant. We therefore do not reject the assumption of un-biased futures prices. As a result, the POTS model was estimated, and the empirical results are given in Appendix II. A description of the estimation implications follows.

Given un-biasedness, the first issue to be addressed focuses on seasonal and time-to-maturity effects on the volatility of futures prices. Second, the degree to which a common information set is relevant to pricing futures with different maturities is examined. From equation (2.1) it follows that the price change for an *i*th-nearby contract at time t is given by $\Delta F_{i,t} = \theta_{d,t}\varepsilon_t + \lambda_{d,t}u_{i,t}$ where subscript i on both the price change and idiosyncratic shock identifies a specific contract. The subscript $d = T_i - t$ is the number of trading days left to maturity for this contract. The factor loading $\theta_{d,t}$ and standard deviation of the idiosyncratic shock $\lambda_{d,t}$ are both deterministic functions of time to maturity and contract month. By imposing unit variances on both the common factor and idiosyncratic shock, we can obtain the unconditional variance of the price change for a specific contract as

$$Var\left(\Delta F_{i,t}\right) = \theta_{d,t}^2 + \lambda_{d,t}^2 \tag{3}$$

We can then easily calculate the fraction of variance due to information innovations pertinent to all contracts, i.e. the common factor as

$$CFI\left(\Delta F_{i,t}\right) = \frac{\theta_{d,t}^2}{\theta_{d,t}^2 + \lambda_{d,t}^2} \tag{4}$$

where we denote this as the *common factor importance* to the i-th nearby contract at time t.

Figure 12.5 presents the square root of the unconditional variance, i.e. the unconditional standard deviation as a function of time to maturity for each of the 12 contract months considered. To illustrate the interpretation while avoiding clutter, only the January and August contracts are emphasized in the figure. Three systematic patterns can be seen. First, there is evidence of seasonal variation in milk price volatility, with the highest volatility occurring in May and June. This corresponds to the seasonal pattern in milk production associated with what is called the "spring flush" (in spring, cows are brought out to graze on fresh pasture, and milk production increases). The second thing to notice is that up to two months from maturity, there is evidence of increasing volatility as time-to-maturity decreases. This phenomenon is the well known Samuelson effect, which postulates that shocks to production influence nearby contracts more strongly than contracts with more distant maturities. However, in each contract analyzed, there is also a strong *decline* in volatility over the last eight trading weeks. This pattern stands in sharp contrast to grain futures contracts (i.e. corn, as analyzed by Smith) where the Samuelson effect extends all the way through the last trading day. The reason behind this decline, which we call the "Inverse Samuelson Effect", may lie in the design of milk futures contracts. As explained in the previous section, the Class III contract is cash settled against a known formula calculated using a set of prices that is partially revealed up to 3 weeks before contract maturity. Hence, much of the uncertainty concerning the announced cash price is resolved in the last several weeks of trading.

The next objective is to understand what percentage of price variance across contracts is explained by a common factor. When a commodity is storable, intertemporal arbitrage between cash and futures prices implies that a common pricing factor should explain nearly all of the observed price variation. This has been confirmed by results in Smith (2005) and Suenaga, Smith, and Williams (2008). While manufacturing milk is itself not storable, two of its price components are. This leads to the question of whether a common factor explains milk price changes across contract months, or whether unique contract month characteristics are more important in driving price changes.

In addition to component storability the capital nature of milk production may link prices across contract maturity dates. Dairy cows produce one calf every 14 months, with the gestation period being 9.5 months. In recent years the average length of time a cow spends in the dairy herd is 3 lactations, or around 4 years. Because the dairy herd is indeed a capital factor of milk production it will help to conceptualize total annual milk production as a product of dairy herd size and yield per cow, as in Bozic and Gould (2009). The annual U.S. average yield per cow exhibits a strong and stable upward trend that is almost completely determined by technological progress, i.e. genetic improvements. Milk supply reactions to price changes come almost entirely through adjustments in the size and age composition of the dairy herd. Furthermore, the short-run own-price elasticity of milk is almost zero, while the 10-year price elasticity is close to unity (Bozic and Gould (2009)). In other words, an annual price increase from $15.00 to $19.00 per hundredweight, not an uncommon magnitude in recent periods, would, if prices were to remain fixed at that higher level, lead to an increase in the dairy herd of 2.5 million cows over ten years. An inelastic demand for milk in conjunction with high differentials between long-run and short-run supply elasticities leads inevitably to highly volatile milk prices. This suggests that the importance of a common factor in milk futures price may arise from capital factors of milk production, i.e. the dairy herd, in addition to component storability.

Table 2.4 lists the average proportion of unconditional variance explained by the common factor over the life of each Class III contract over the sample period. These averages hide the fact that for each contract the common factor at some point in time explains over 80% of variance. More insight can be gained from Figure 2.6, where we plot the proportion of variance explained by a common factor as a function of time to maturity, for each contract separately.

We find two principal regularities. First, as the overall price variance collapses near maturity, the share of variance explained by the common factor declines even faster. Second, the common factor explains the highest share of variance close to six months prior to expiration. The fact that the importance of the common factor peaks so early could be due to the fact that, if there were no imports of dairy cattle to U.S., the upper bound of the U.S. national dairy herd at contact maturity would be determined about 285 days prior to contract expiration. This is the average length of the cow gestation period.[7]

In addition to examining peak importance in the common factor, we identify the percentage of a contract's life for which the common factor explains more than 70% of the price variance. This is shown in Figure 2.7. Note that for summer contracts the common factor's importance stays above this threshold until 50 days from maturity. For other contract months the common factor fades in importance as much as 100 days from maturity. While it is not clear what drives this difference, it does coincide with generally higher cheese stocks in summer months.

---

[7] In addition to domestic herd growth dairy cattle are imported from Canada. Average annual imports are about 50,000 cows. This may seem small compared to the U.S. dairy herd of 9.2 million cows, but it is significant compared to the average annual changes in dairy herd size of 61,000 head from 1998-2008. However, exports of dairy cows from Canada were banned from May 2003 through November 2008 due to incidence of bovine spongiform encephalopathy, i.e. mad cow disease (DiGiuseppe, 2010). Therefore, for the better part of the estimation period, the upper bound of dairy herd size at contract expiry was indeed determined close to nine months out.

Finally, we can ask how much of the *market variance* does the common factor explain. For each trading day, we would like to know how much of the price variability across delivery months is due to a common factor. Once we have obtained this measure we can determine if the common factor's influence exhibits seasonal variation. A central issue in calculating market-level common factor importance is how to aggregate the 12 contract-level CFIs to one number. For example, one could use a simple weighted average with either contract open interest or daily volume as weights, or one could treat the entire market for Class III futures as a balanced portfolio (a simple non-weighted average). It turns out that in either case the common factor explains most of the market price variance in spring and fall months. In addition, weighted measures reveal a double-hump seasonality, where the common factor increases in importance in both spring and fall, and is less important in summer months. Figure 2.8 shows the proportion of market variance explained by the common factor, using a weighted average method, with relative daily volume as the weight. The expression for CFI_t is :

$$CFI_t = \frac{1}{\overline{V}_t} \sum_{i=1}^{12} \overline{V}_{i,t} \frac{\theta_{i,t}^2}{\theta_{i,t}^2 + \lambda_{i,t}^2} \qquad (5)$$

where $\overline{V}_{i,t}$ is the average volume of *i*-th nearby contract on day *t* of a year, with the average taken over the period 2001-2009. Market average volume $\overline{V}_t$ is the sum of average volumes of all 12 contracts. To reduce the serrated shape induced by a declining CFI in nearby contracts, we calculate a 30-day smoothed series where the smoothed CFI at time *t* is an average of the market CFI over period *t-15* to *t+15*. A partial explanation of the seasonality observed is offered in Figure 2.9. As argued before, adjustments to milk production come primarily through dairy herd culling and replacement decisions. In the second part of a year, decisions on dairy cow slaughter exhibit seasonality similar to CFI. In particular, slaughter is lowest in May through July, and reaches a peak in October and November. Seasonality of CFI influences may be partially attributable to seasonality in the information flow regarding the future size of the dairy herd.

2.7. Conclusions

This paper describes the evolution of dairy futures markets, and explicitly examines the volatility dynamics of Class III milk futures contracts. The design of this contract allows it to be used as a hedging instrument for cheese products as well as fluid and manufacturing grade milk. Even though Class III milk is itself a nonstorable commodity, the analysis reveals a rather important overlap of information pertinent to futures for different maturities. This is attributed to the capital structure of the dairy herd and the storability of some dairy products underpinning the USDA formula for minimum Class III milk prices. Seasonal variation reveals that contracts expiring in summer months much more closely resemble contracts on storable commodities like corn, while contracts expiring in winter months are more influenced by idiosyncratic shocks.

There are two principal implications of this study for contract design and dairy policy. First, the modest importance of a common factor for most contract months implies that contracts of different maturities are poor substitutes in a risk management program, and product cross-hedging may not be particularly effective either. Corn, a much deeper agricultural futures contract in terms of volume traded, has only 5 contract months per year. However, even though Class III futures often suffer from illiquidity, it is likely optimal to have separate contracts for all 12 months in the year.

An important question for policy makers concerns the ability of futures markets to substitute for binding milk price supports as an instrument for enhancing cash price stability. As shown in the spectral analysis in Appendix 1, a major contributor to cash milk price variance comes from a cycle whose period is about 3 years. While Class III futures are listed for up to 24 months ahead, contracts with more than 260 trading days to maturity do not generate any significant trading interest. The ability of futures markets to dampen the low-frequency cash price oscillations relies on two factors. The first is the degree to which farmers use futures as forecasts of milk prices in later years in making dairy herd replacement decisions. Decisions regarding the dairy herd are investment decisions with a relevant planning horizon of 3-5 years, and forecasts of next year's prices will guide their investment decisions only if they believe price expectations for longer horizons are accurate, and that their individual production decisions will not impact future prices. If the first condition is satisfied, we would still need farmers to have confidence that futures prices carry economically meaningful information over the horizon used in investment decisions. Futures prices are likely unbiased predictors of future cash milk prices, but the average absolute prediction errors for horizons longer than 60 days is rather high. For example, at 9.5 months to maturity – the duration of a cow's gestation period - futures price prediction errors are on average 28%. That means that if the futures price is $16.00 per hundredweight, the final price would be expected to be between $20.50 per cwt. (which would render milk production rather profitable), and $12.00, which is low enough to initiate countercyclical price supports (i.e. MILC payments). In face of such high prediction uncertainty, it is questionable whether futures serve a major role in coordinating expectations to a degree that meaningfully reduces the amplitude of the boom-bust cycle.

2.8. Appendix 1. Spectral Analysis of Manufacturing Milk Prices

We use a frequency domain approach to construct the sample spectrums of milk prices. We do this separately for the periods January 1970 – April 1991 (the period of price stability) and September 1988 – December 2009 (the period of increased price volatility) to identify if there was any change in the relative contribution to variance across frequencies. Results are presented in Figure 2.110 and Figure 2.11.

To construct the sample spectrums we have followed Klingenberg (2005). The cutoff points for the sub-samples were chosen in order to satisfy constraints imposed by the Fourier analysis function in Excel. This constraint requires that the number of observations in each sample must equal 2^n for some n . We chose periods corresponding to 256 months, using monthly data. A useful reference for the interpretation of periodograms is Hamilton (1994, chpt. 7). The results reveal peaks in both periods at frequencies corresponding to a 1 year and 1 month cycle. The principal difference between the periods, however, is that in the later period there is a very high peak at a frequency of 0.33, which corresponds to a 3 year cycle. This corroborates a simple reading of Figure 2.1, where we find dips in milk prices occurring in 1997, 2000, 2003, 2006 and 2009. However, some of these major price movements were brought about by export shocks and macroeconomic downturns, so we judge this evidence as preliminary.

2.9. Appendix 2. POTS Estimation Results

Log Likelihood value = -56905246.64
Overall % Explained by Common Factor = 64.21%
GARCH parameters

	Value	Robust s.e.	t-ratio
Alpha	0.1149	0.003487	32.95
Beta	0.9847	0.002203	447.0

Spline #	Time-to-maturity
1	0-30
2	31-125
3	126+

Factor spline parameters (theta)

Month	Spline	Free	Linear	Quadratic	Cubic
January	1	4.50^{-3}	4.50^{-3}	3.30^{-4}	-6.97^{-6}
		(4.68^{-4})	(4.68^{-4})	(1.18^{-5})	(2.65^{-7})
	2	1.12^{-1}	1.55^{-3}	-4.23^{-5}	2.53^{-7}
		(3.84^{-3})	(1.03^{-4})	(8.66^{-7})	(8.50^{-9})
	3	9.57^{-2}	4.45^{-4}	-1.85^{-5}	9.03^{-8}
		(6.48^{-3})	(1.05^{-4})	(5.42^{-7})	(2.60^{-9})
February	1	-1.88^{-3}	-1.88^{-3}	2.75^{-4}	-5.41^{-6}
		(3.93^{-4})	(3.93^{-4})	(6.89^{-6})	(1.15^{-7})
	2	9.79^{-2}	2.33^{-3}	-5.27^{-5}	2.84^{-7}
		(3.27^{-3})	(1.71^{-4})	(1.97^{-6})	(1.83^{-8})
	3	8.78^{-2}	7.71^{-5}	-9.79^{-6}	5.18^{-8}
		(9.83^{-3})	(1.59^{-4})	(3.12^{-7})	(4.71^{-9})
March	1	-5.85^{-3}	-5.85^{-3}	2.56^{-4}	-5.04^{-6}
		(2.63^{-4})	(2.63^{-4})	(7.13^{-6})	(1.63^{-7})
	2	8.67^{-2}	2.14^{-3}	-3.61^{-5}	1.51^{-7}
		(2.64^{-3})	(1.50^{-4})	(1.07^{-6})	(3.74^{-9})
	3	9.33^{-2}	-6.07^{-4}	1.50^{-6}	4.35^{-9}
		(6.46^{-3})	(1.17^{-4})	(3.93^{-7})	(3.42^{-9})
April	1	2.19^{-3}	2.19^{-3}	1.16^{-4}	-1.45^{-6}
		(1.04^{-3})	(1.04^{-3})	(1.21^{-6})	(8.73^{-8})
	2	6.41^{-2}	3.05^{-3}	-4.90^{-5}	2.19^{-7}
		(1.18^{-3})	(1.80^{-4})	(8.38^{-7})	(1.10^{-8})
	3	9.81^{-2}	-3.20^{-4}	-5.59^{-6}	3.61^{-8}
		(6.62^{-3})	(1.70^{-4})	(2.72^{-7})	(4.53^{-9})

Factor-spline parameters (theta) – cont'd.

Month	Spline	Free	Linear	Quadratic	Cubic
May	1	2.71^{-3}	2.71^{-3}	3.32^{-4}	-7.39^{-6}
		(2.81^{-4})	(2.81^{-4})	(7.86^{-6})	(1.87^{-7})
	2	1.01^{-1}	5.84^{-4}	-5.63^{-6}	-4.03^{-9}
		(2.17^{-3})	(5.49^{-5})	(2.14^{-7})	$(4.64-^{10})$
	3	1.02^{-1}	-6.09^{-4}	-8.83^{-7}	1.65^{-8}
		(4.32^{-3})	(4.19^{-5})	(7.78^{-7})	(3.63^{-9})
June	1	-6.23^{-3}	-6.23^{-3}	5.42^{-4}	-1.22^{-5}
		(3.16^{-4})	(3.16^{-4})	(2.03^{-5})	(5.10^{-7})
	2	1.53^{-1}	7.08^{-4}	-3.37^{-5}	2.14^{-7}
		(4.71^{-3})	(1.69^{-4})	(8.91^{-7})	(1.40^{-8})
	3	9.94^{-2}	1.58^{-4}	-1.20^{-5}	5.83^{-8}
		(5.92^{-3})	(1.58^{-4})	(7.06^{-7})	(6.03^{-9})
July	1	1.15^{-3}	1.15^{-3}	4.95^{-4}	-1.02^{-5}
		(3.13^{-4})	(3.13^{-4})	(1.39^{-5})	(4.15^{-7})
	2	1.68^{-1}	2.92^{-3}	-8.07^{-5}	4.50^{-7}
		(2.44^{-3})	(2.99^{-4})	(2.07^{-6})	(2.25^{-8})
	3	1.03^{-1}	-1.30^{-4}	-7.11^{-6}	4.03^{-8}
		(1.09^{-2})	(2.40^{-4})	(3.54^{-7})	(3.46^{-9})
August	1	-1.33^{-2}	-1.33^{-2}	2.98^{-4}	-4.32^{-6}
		(6.76^{-4})	(6.76^{-4})	(6.11^{-6})	(2.39^{-7})
	2	1.32^{-1}	6.41^{-3}	-1.32^{-4}	6.64^{-7}
		(2.49^{-3})	(3.00^{-4})	(3.72^{-6})	(1.26^{-8})
	3	1.16^{-1}	-6.36^{-4}	-1.86^{-7}	1.37^{-8}
		(7.35^{-3})	(1.40^{-4})	(6.47^{-7})	(1.57^{-9})
September	1	-1.79^{-3}	-1.79^{-3}	1.35^{-4}	-5.84^{-7}
		(9.25^{-4})	(9.25^{-4})	(3.62^{-6})	(4.13^{-8})
	2	9.72^{-2}	6.33^{-3}	-1.04^{-4}	4.54^{-7}
		(1.87^{-3})	(1.40^{-4})	(3.54^{-6})	(2.40^{-8})
	3	1.44^{-1}	-1.17^{-3}	3.65^{-6}	4.41^{-9}
		(7.20^{-3})	(1.05^{-4})	(3.14^{-7})	(2.64^{-9})
October	1	-2.61^{-3}	-2.61^{-3}	2.75^{-4}	-4.98^{-6}
		(3.60^{-4})	(3.60^{-4})	(8.19^{-6})	(8.86^{-8})
	2	1.07^{-1}	3.36^{-3}	-4.45^{-5}	1.45^{-7}
		(7.47^{-3})	(5.76^{-4})	(4.48^{-6})	(3.32^{-9})
	3	1.47^{-1}	-1.19^{-3}	3.17^{-6}	7.68^{-9}
		(2.00^{-2})	(3.37^{-4})	(2.70^{-7})	(6.81^{-9})
November	1	1.67^{-4}	1.67^{-4}	2.49^{-4}	-4.89^{-6}
		(4.91^{-4})	(4.91^{-4})	(2.86^{-6})	(5.41^{-8})
	2	9.04^{-2}	2.12^{-3}	-2.39^{-5}	7.01^{-8}
		(1.58^{-3})	(6.66^{-5})	(6.19^{-7})	(2.28^{-9})
	3	1.35^{-1}	-5.45^{-4}	-7.03^{-6}	4.75^{-8}
		(4.41^{-3})	(3.66^{-5})	(3.81^{-7})	(2.21^{-9})
December	1	-5.29^{-3}	-5.29^{-3}	5.17^{-4}	-1.20^{-5}
		(3.81^{-4})	(3.81^{-4})	(1.16^{-5})	(2.77^{-7})
	2	1.36^{-1}	-3.37^{-4}	-2.86^{-7}	2.80^{-8}
		(2.64^{-3})	(4.04^{-5})	(2.47^{-8})	$(4.40-^{10})$
	3	1.26^{-1}	3.81^{-4}	-2.06^{-5}	1.03^{-7}
		(2.79^{-3})	(4.03^{-5})	(5.01^{-7})	(2.89^{-9})

Innovation-spline parameters (lambda)

Month	Spline	Free	Linear	Quadratic	Cubic
January	1	1.96^{-2}	1.96^{-2}	4.74^{-4}	-1.15^{-5}
		(1.25^{-3})	(1.25^{-3})	(6.90^{-6})	(1.67^{-7})
	2	1.37^{-1}	-1.55^{-3}	-5.10^{-6}	1.25^{-7}
		(2.62^{-3})	(5.62^{-5})	(2.61^{-7})	(3.02^{-9})
	3	5.16^{-2}	9.12^{-4}	-1.35^{-5}	5.39^{-8}
		(2.87^{-3})	(5.45^{-5})	(1.28^{-6})	(6.43^{-9})
February	1	2.78^{-2}	2.78^{-2}	3.45^{-4}	-8.16^{-6}
		(4.10^{-4})	(4.10^{-4})	(5.56^{-6})	(1.01^{-7})
	2	1.19^{-1}	-5.65^{-4}	-2.32^{-5}	2.11^{-7}
		(2.26^{-3})	(1.02^{-4})	(7.23^{-7})	(5.31^{-9})
	3	3.78^{-2}	8.16^{-4}	-6.63^{-6}	1.90^{-8}
		(3.91^{-3})	(1.07^{-4})	(4.31^{-7})	(3.85^{-9})
March	1	2.57^{-2}	2.57^{-2}	3.95^{-4}	-9.43^{-6}
		(4.97^{-4})	(4.97^{-4})	(1.63^{-5})	(3.86^{-7})
	2	1.28^{-1}	-8.83^{-4}	-1.49^{-5}	1.41^{-7}
		(4.21^{-3})	(4.72^{-5})	(1.26^{-6})	(1.03^{-8})
	3	3.13^{-2}	1.74^{-4}	7.02^{-6}	-4.01^{-8}
		(2.98^{-3})	(9.72^{-5})	(7.60^{-7})	(2.39^{-9})
April	1	1.45^{-2}	1.45^{-2}	3.08^{-4}	-6.64^{-6}
		(3.64^{-4})	(3.64^{-4})	(4.89^{-6})	(1.67^{-7})
	2	1.12^{-1}	1.11^{-3}	-5.05^{-5}	3.11^{-7}
		(2.08^{-3})	(2.10^{-4})	(2.63^{-6})	(8.52^{-9})
	3	2.80^{-2}	1.45^{-5}	7.61^{-6}	-4.05^{-8}
		(4.28^{-3})	(1.02^{-4})	(3.34^{-7})	(2.80^{-9})
May	1	1.40^{-2}	1.40^{-2}	3.15^{-4}	-7.07^{-6}
		(4.10^{-4})	(4.10^{-4})	(7.51^{-6})	(1.92^{-7})
	2	1.06^{-1}	4.31^{-4}	-2.75^{-5}	1.57^{-7}
		(2.39^{-3})	(1.11^{-4})	(8.28^{-7})	(5.24^{-9})
	3	3.32^{-2}	-5.11^{-4}	1.49^{-5}	-6.63^{-8}
		(3.73^{-3})	(8.39^{-5})	(8.45^{-7})	(4.62^{-9})
June	1	1.77^{-2}	1.77^{-2}	3.84^{-4}	-8.93^{-6}
		(5.34^{-4})	(5.34^{-4})	(4.54^{-6})	(1.41^{-7})
	2	1.23^{-1}	-2.57^{-4}	-1.78^{-5}	1.17^{-7}
		(1.95^{-3})	(1.61^{-4})	(7.48^{-7})	(4.12^{-9})
	3	3.78^{-2}	-4.35^{-4}	1.01^{-5}	-4.33^{-8}
		(7.02^{-3})	(8.81^{-5})	(1.27^{-6})	(8.09^{-9})

Innovation-spline parameters (lambda) – cont'd.

Month	Spline	Free	Linear	Quadratic	Cubic
July	1	1.67^{-2}	1.67^{-2}	5.90^{-4}	-1.48-5
		(1.28^{-3})	(1.28^{-3})	(5.89^{-6})	(1.88-7)
	2	1.53^{-1}	-3.04^{-3}	3.28^{-5}	-1.36^{-7}
		(3.59^{-3})	(2.47^{-4})	(2.01^{-6})	(3.96^{-9})
	3	4.27^{-2}	-5.00^{-4}	9.32^{-6}	-3.91^{-8}
		(7.91^{-3})	(1.64^{-4})	(3.80^{-7})	(4.99^{-9})
August	1	2.62^{-2}	2.62^{-2}	3.08^{-4}	-6.84^{-6}
		(1.36^{-3})	(1.36^{-3})	(4.33^{-6})	(1.44^{-7})
	2	1.19^{-1}	6.13^{-4}	-3.85^{-5}	2.48^{-7}
		(2.05^{-3})	(1.51^{-4})	(1.07^{-6})	(1.30^{-8})
	3	4.16^{-2}	6.87^{-5}	-1.38^{-6}	5.81^{-9}
		(7.52^{-3})	(1.37^{-4})	(1.99^{-7})	(2.57^{-9})
September	1	2.64^{-2}	2.64^{-2}	2.59^{-4}	-5.11^{-6}
		(9.81^{-4})	(9.81^{-4})	(2.40^{-6})	(9.04^{-8})
	2	1.20^{-1}	2.13^{-3}	-7.02^{-5}	4.14^{-7}
		(2.04^{-3})	(1.37^{-4})	(1.24^{-6})	(1.02^{-8})
	3	4.36^{-2}	1.03^{-4}	-2.53^{-6}	1.09^{-8}
		(6.50^{-3})	(1.04^{-4})	(4.27^{-7})	(3.39^{-9})
October	1	2.07^{-2}	2.07^{-2}	4.20^{-4}	-9.37^{-6}
		(1.05^{-3})	(1.05^{-3})	(8.32^{-6})	(1.24^{-7})
	2	1.45^{-1}	7.01^{-4}	-4.30^{-5}	2.72^{-7}
		(6.46^{-3})	(3.76^{-4})	(1.96^{-6})	(1.06^{-8})
	3	5.70^{-2}	-2.71^{-5}	-7.14^{-8}	9.59-^{10}
		(1.57^{-2})	(2.73^{-4})	(2.91^{-7})	(6.92^{-9})
November	1	1.09^{-2}	1.09^{-2}	4.30^{-4}	-9.65^{-6}
		(2.41^{-4})	(2.41^{-4})	(9.58^{-6})	(2.88^{-7})
	2	1.37^{-1}	5.91^{-4}	-4.38^{-5}	3.04^{-7}
		(1.69^{-3})	(2.12^{-4})	(1.95^{-6})	(7.05^{-9})
	3	5.94^{-2}	5.88^{-4}	-1.33^{-5}	5.73^{-8}
		(5.53^{-3})	(1.19^{-4})	(7.41^{-7})	(2.70^{-9})
December	1	1.49^{-1}	1.49^{-1}	-1.42^{-4}	3.83^{-6}
		(4.56^{-3})	(4.56^{-3})	(5.43^{-6})	(1.72^{-7})
	2	1.23^{-1}	1.41^{-3}	-5.93^{-5}	3.95^{-7}
		(4.41^{-3})	(1.46^{-4})	(3.52^{-6})	(1.97^{-8})
	3	6.02^{-2}	9.32^{-4}	-1.78^{-5}	7.66^{-8}
		(3.10^{-3})	(6.74^{-5})	(9.24^{-7})	(4.18^{-9})

Note: To simplify notation, all numbers for spline parameters are expressed in abbreviated scientific notation. For example, $3.10^{-3}=3.10*10^{-3}=0.0031$.

2.10. References

Bamba, I. and L. Maynard (2004) Hedging Effectiveness of Milk Futures using Value-at-risk Procedures, Paper presented at the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management, St. Louis, MO.

Blayney, D. P. (2002). The Changing Landscape of U.S. Milk Production, USDA-ERS Statistical Bulletin No. 978. Retrieved October 4, 2010, from http://www.ers.usda.gov/publications/err28/err28.pdf

Blayney, D. P., M. Gehlhar, C. H. Bolling, K. Jones, S. Langley, and M. A. Normile (2006). U.S. Dairy at a Global Crossroads, Economic Research Report No. 28. Washington, D.C.: U.S. Department of Agriculture, Economic Research Service.

Bozic, M. and B. W. Gould (2009). The Dynamics of the U.S. Milk Supply: Implications for Changes in the U.S. Dairy Policy, Agricultural & Applied Economics Staff Paper No. 540, University of Wisconsin-Madison, Department of Agricultural & Applied Economics, Retrieved October 4, 2010, from http://www.aae.wisc.edu/pubs/sps/pdf/stpap540.pdf

Buschena, D. and K. McNew (2008). Subsidized Options in a Thin Market: A Case Study of the Dairy Options Pilot Program Introduction. *Review of Agricultural Economics*, 30, 103-119.

Chavas, J-P. and R. M. Klemme (1986). Aggregate Milk Supply Response and Investment Behavior on U.S. Dairy Farms. *American Journal of Agricultural Economics*, 68, 55-66.

Dempster, A.D., N. M. Laird, and D. B. Rubin (1977). Maximum Likelihood from Incomplete Data via the EM Algorithm. *Journal of the Royal Statistical Society. Series B (Methodological),* 39, 1-38.

DiGiuseppe, G. (2010). Replacement Heifers. Retreived January 6, 2011, from http://www.americandairymen.com/articles/replacement-heifers.

Fortenbery, T. R. (2009). The Development of Dairy Futures Contracts: Experiences and Implications for New Contract Development. Presented at the USDA-CFTC Conference on Contract Design in Futures Markets, Washington, D.C.

Fortenbery, T. R., R. A. Cropp and H. O. Zapata (1997). Analysis of Expected Price Dynamics Between Fluid Milk Futures Contracts and Cash Prices for Fluid Milk. *Journal of Agribusiness*, 15, 215-234.

Fortenbery, T. R. and H. O. Zapata (1997). An Evaluation of Price Linkages Between Futures and Cash Markets for Cheddar Cheese. *Journal of Futures Markets*, 17, 279-301.

Hamilton, J.D. (1994). Time Series Analysis. Princeton, NY: Princeton University Press.

Jarvis, L.S. (1974). Cattle as a Capital Good and Ranchers as Portfolio Managers: An Application to the Argentine Cattle Sector. *Journal of Political Economy*, 82, 480-520.

Klingenberg, L. (2005). Frequency Domain using Excel. San Francisco State University. Retrieved online on January 6, 2011 from http://userwww.sfsu.edu/~larryk/Common%20Files/Excel.FFT.pdf

Maynard, L. J., C. Wolf, and M. Gearhardt (2005). Can Futures and Options Markets Hold the Milk Price Safety Net? Policy Conflicts and Market Failures in Dairy Hedging. *Review of Agricultural Economics*, 27, 273-286.

Samuelson, P.A. (1965) Proof That Properly Anticipated Prices Fluctuate Randomly. Industrial Management Review, 6, 41-49.

Sanders, D. R. and M. R. Manfredo (2005). Forecast Encompassing as the Necessary Condition to Reject Futures Market Efficiency: Fluid Milk Futures. *American Journal of Agricultural Economics*, 87, 610-620.

Smith, A. (2005). Partially Overlapping Time Series: A New Model for Volatility Dynamics in Commodity Futures. *Journal of Applied Econometrics*, 20, 405-422.

Suenaga, H., A. Smith, A. and J. Williams (2008). Volatility Dynamics of NYMEX Natural Gas Futures Prices. *Journal of Futures Markets*, 28, 438–463.

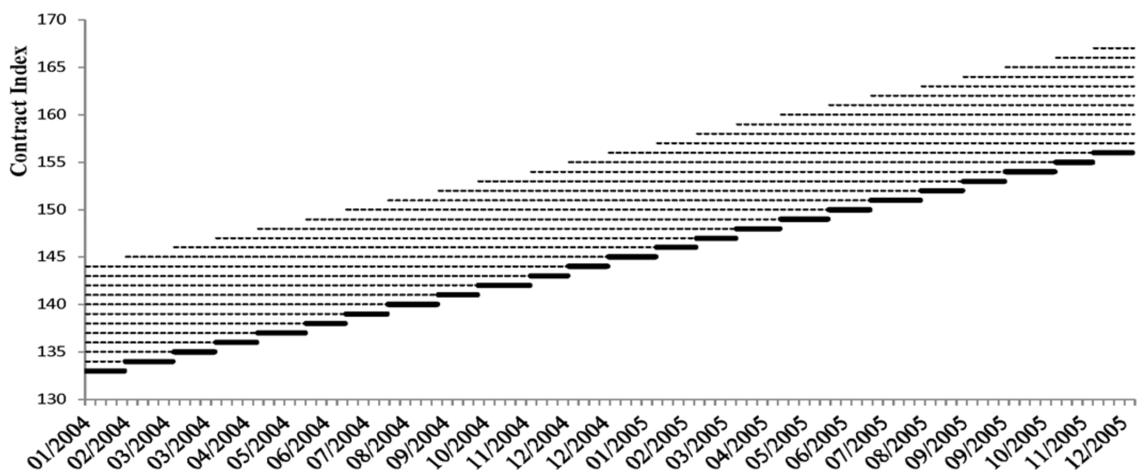
Thraen, C. S. (1999). A Note: The CSCE Cheddar Cheese Cash and Futures Price Long-term Equilibrium Relationship Revisited. *Journal of Futures Markets*, 19, 233-244.

Tondel, F., Maynard, L. (2004) Is the Thinly-Traded Butter Futures Contract Priced Efficiently? Paper presented at the Southern Agricultural Economics Association Annual Meeting, Tulsa, OK.

Zylstra, M. J., R. L. Kilmer, S. Uryasev. (2004). Hedging Class I Milk: The "Acceleration" and "Mover" Effect. Paper presented at the American Agricultural Economics Association Meetings. Denver, CO.

Figure 2.1. Manufacturing Milk Price: 1970-2009

Figure 1.2. Class III Milk futures - Partially Overlapping Time Series vs. "Nearby" Series



Note: This figure illustrates the difference between the standard analysis of futures prices and POTS model, as applied to the Class III milk futures. Contracts are denoted with indexes (e.g. 2004-Jan: 133, 2004-Feb: 134, …). Dashed line represents the period when a particular Class III contract traded. Bold line represents data used to create continuous 1st nearby series. Unlike the standard analysis of futures prices that use only data segments represented with bolded line, POTS model utilizes all traded data for all contracts (both dashed and bold line segments).

Figure 2.3. Class III Milk Futures: Open Interest and Nearby Price 2000-2009.

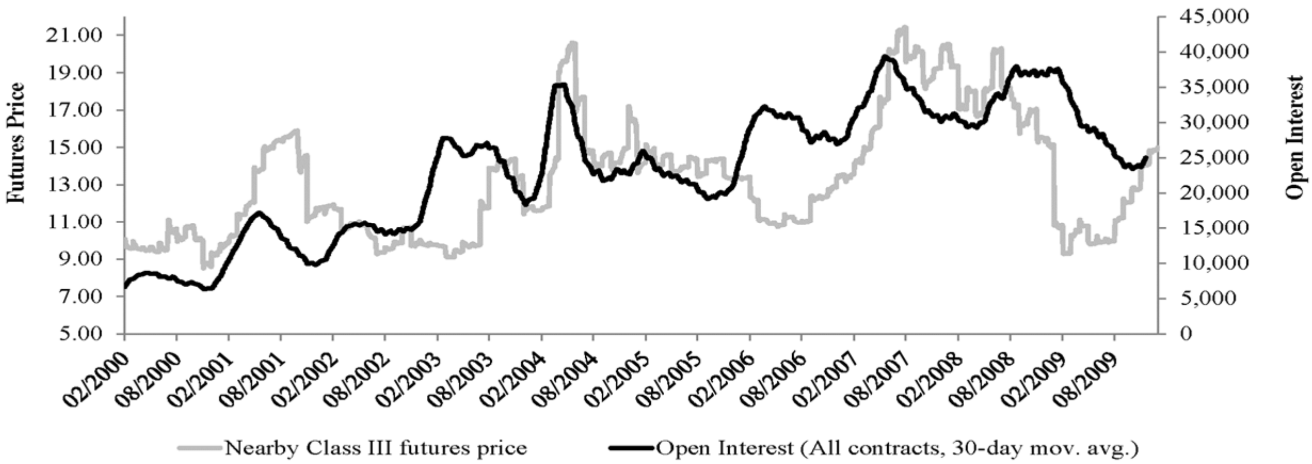


Figure 2.4. Realized Prediction Errors of Class III Milk Futures Prices.

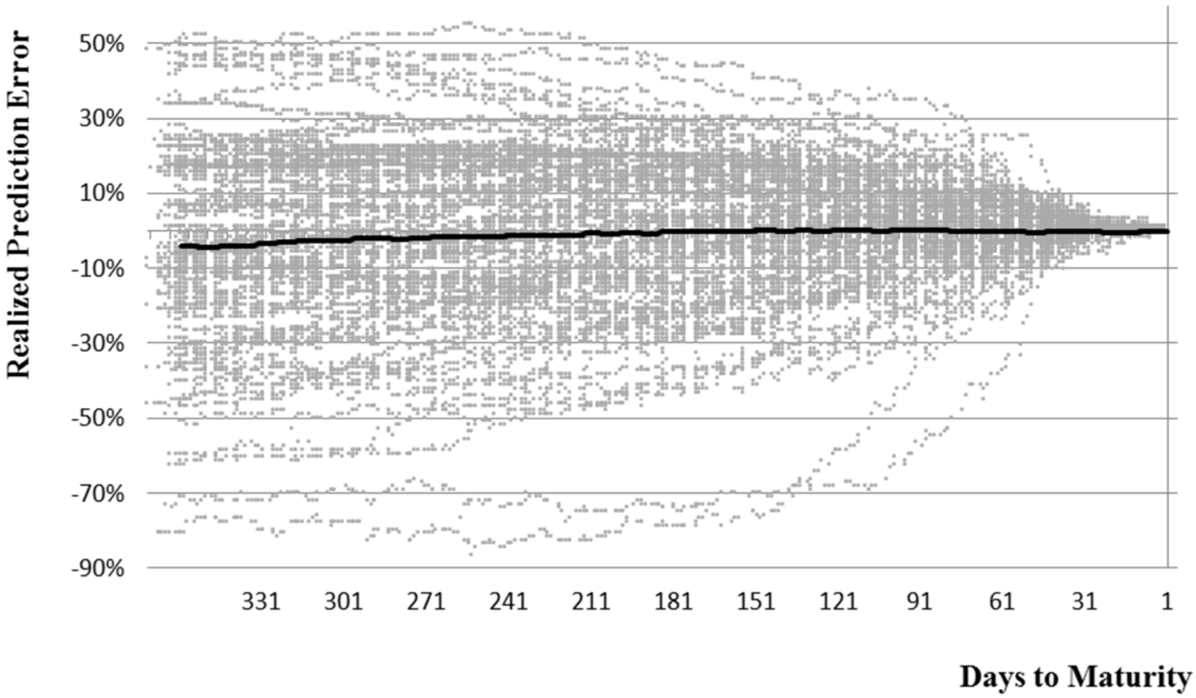


Figure 12.5. Class III Milk Futures: Unconditional Standard Deviation of Price Change as a Function of Time to Maturity.

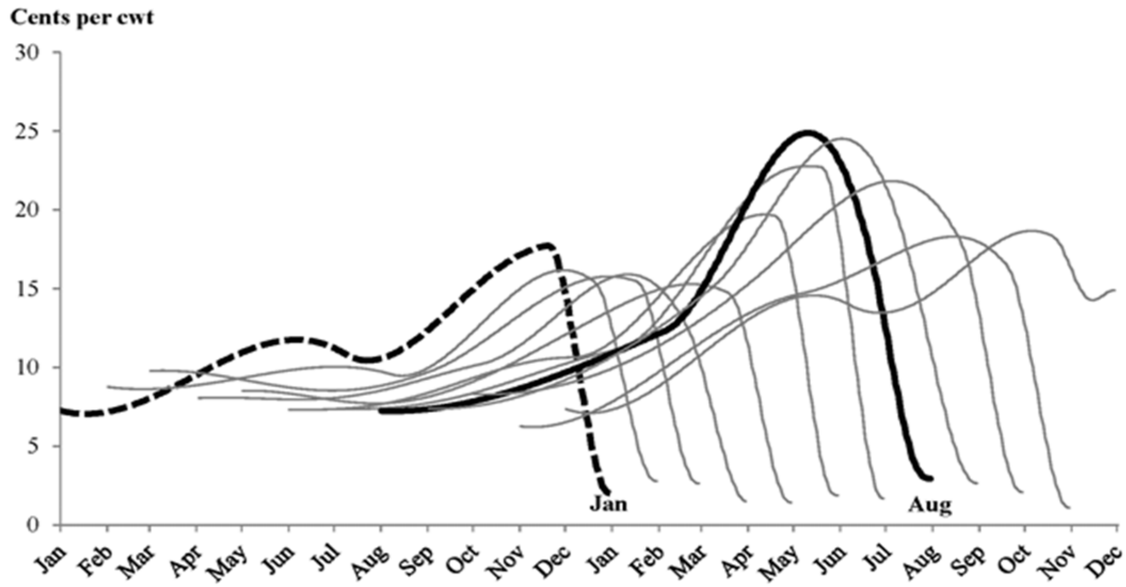


Figure 2.6. Proportion of Variance Explained by the Common Factor.

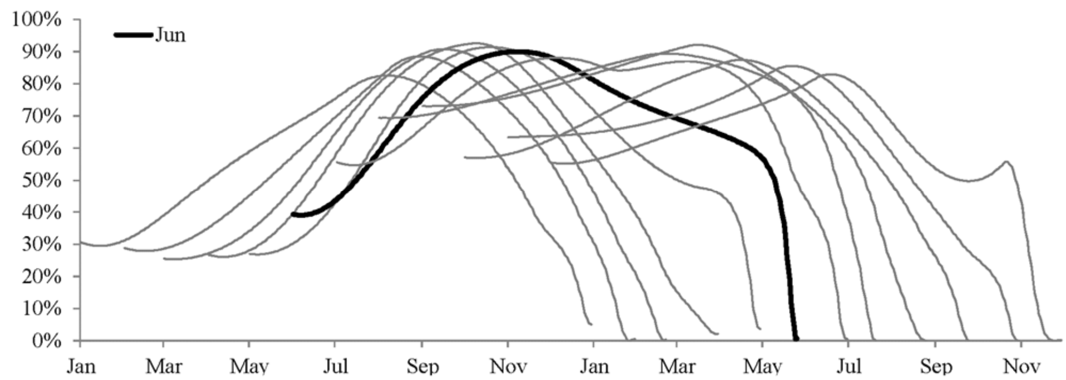


Figure 2.7. Common Factor Importance for Class III Milk Futures.

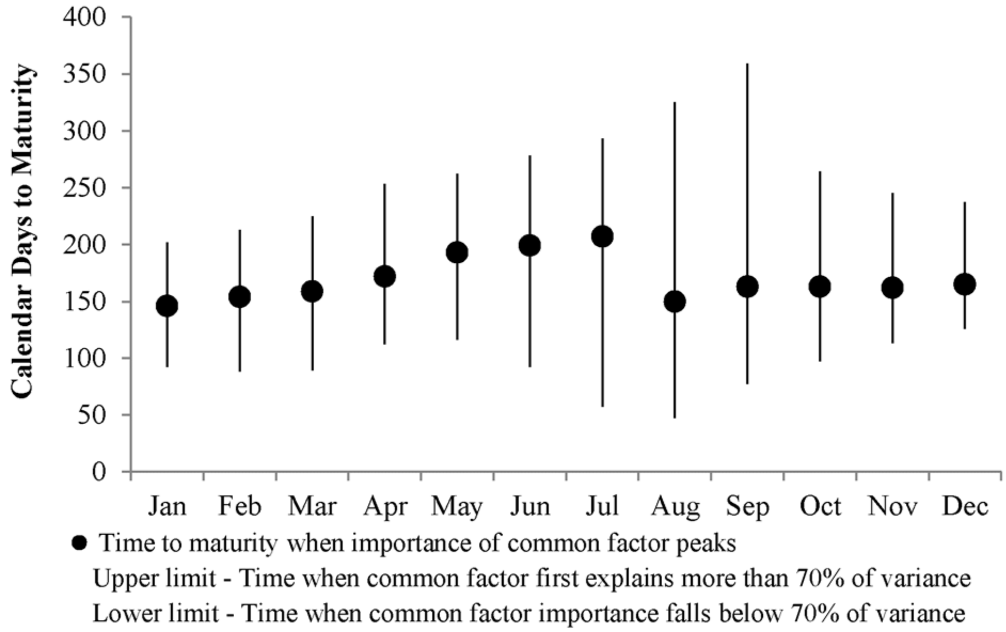


Figure 2.8. Proportion of Market Variance Explained by the Common Factor.

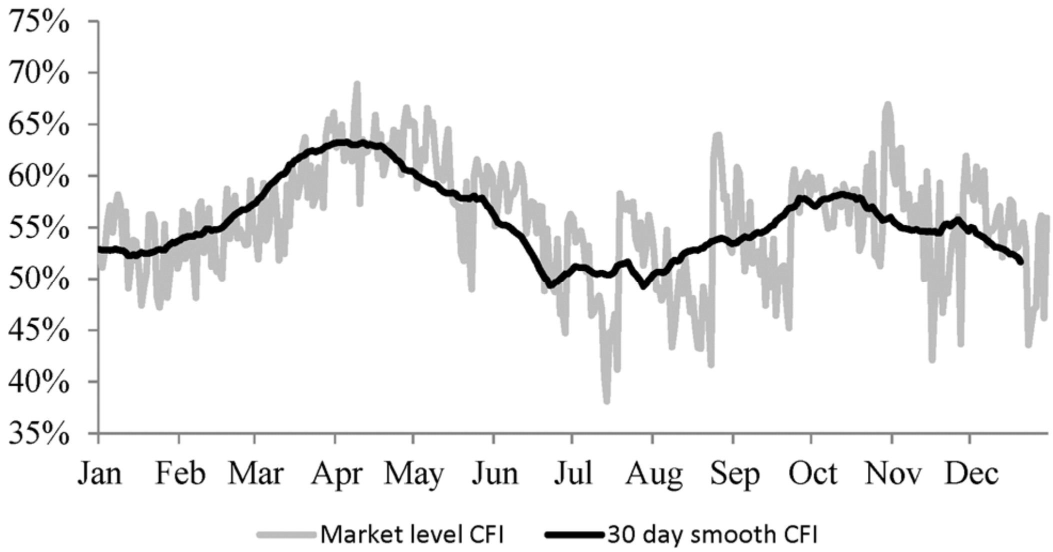


Figure 2.9. U.S. Commercial Dairy Cow Slaughter, Percentage of Annual Cull.

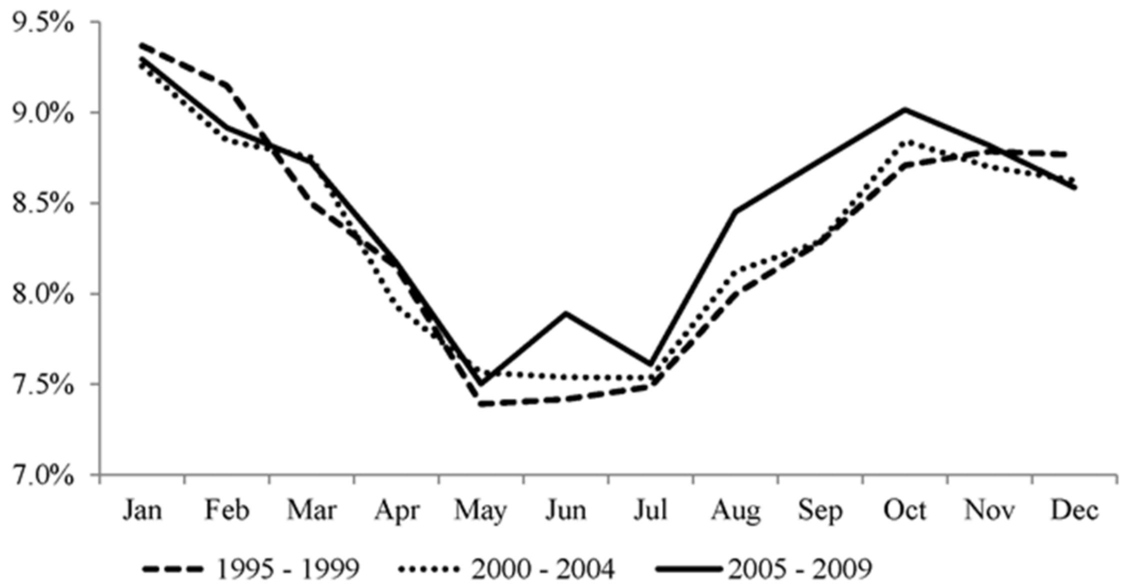


Figure 2.110. Manufacturing Milk Price: Sample Spectrum (Jan 1970-Apr 1991).

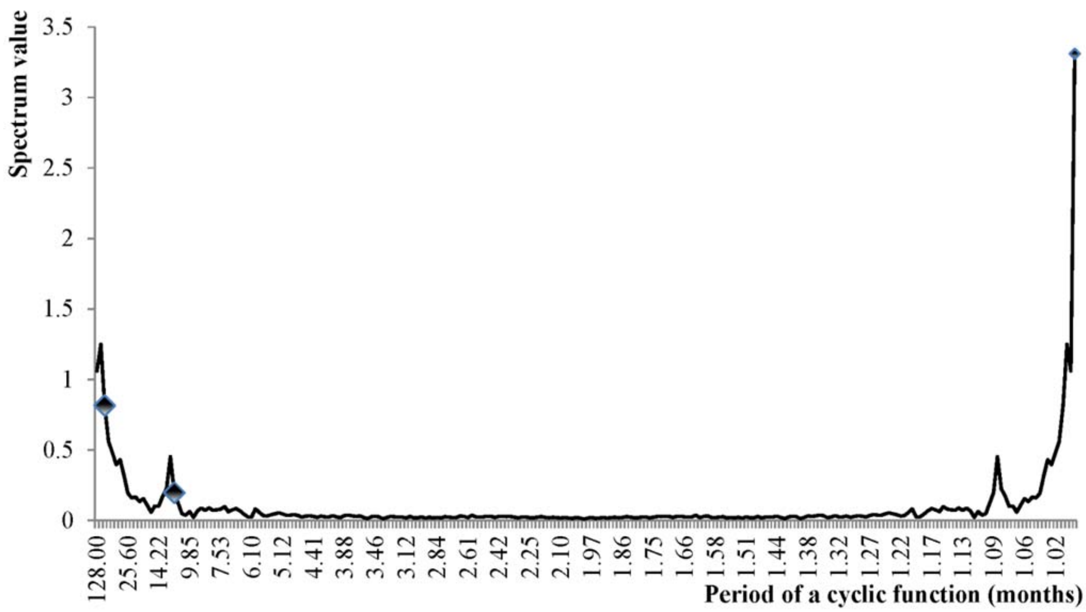


Figure 2.11. Manufacturing Milk Price: Sample Spectrum (Sep 1988-Dec 2009).

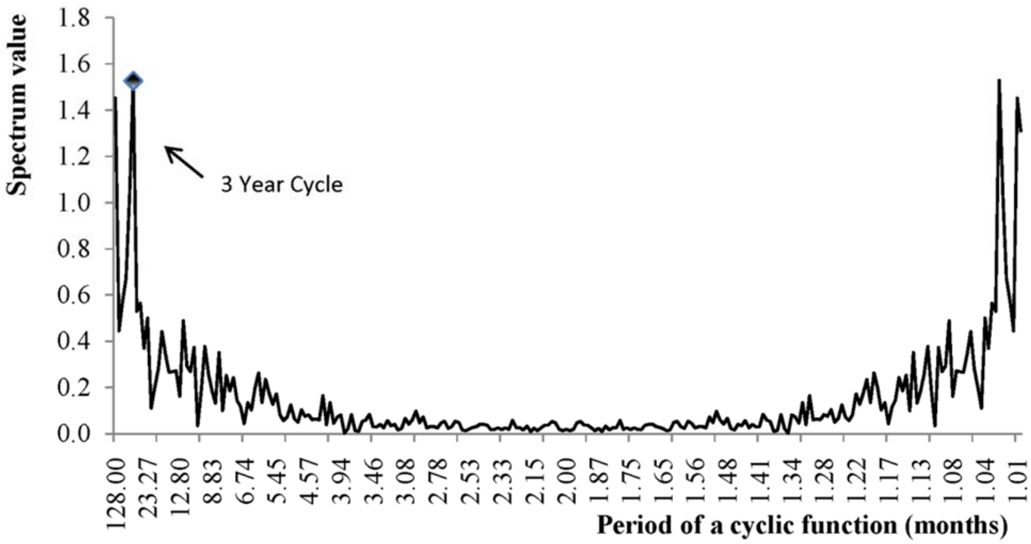


Table 2.1. Specifications of Dairy Futures Contracts.

Contract	Contract Size	Terminal Price/Settlement	Date First Traded
Cheddar Cheese (NYCSCE)	10,500 lbs of Cheddar cheese, in 40-lbs blocks	Physical Delivery	June 1993 (discontinued)
Nonfat Dry Milk (NYCSCE)	11,000 lbs in 25-kilo bags	Physical Delivery	June 1993 (discontinued)
Fluid-milk Contract (BFP) (NYCSCE)		Cash Settled	April 8, 1997 (discontinued)
BFP Milk contract (CME)	200,000 lbs (50,000lbs and 100,000lbs available)	BFP price, Cash settled	1997 (changed to Class III Milk contract in 2000)
Butter	40,000 lbs	Physical Delivery	March 20, 1997 (discontinued)
Class III Milk	200,000 lbs of Class III Milk	USDA Announced Class III Price for contract month, Cash Settled	February 1, 2000 (Replaced BFP)
Class IV Milk	200,000 lbs of Class IV Milk	USDA Announced Class IV Price for contract month, Cash Settled	July 10, 2000
Cash-Settled Butter	20,000 lbs	USDA Announced Butter price for contract month, Cash Settled	September 19, 2005
Dry Whey	44,000 lbs	USDA Announced Dry Whey price for contract month, Cash Settled	March 19, 2007
Nonfat Dry Milk	44,000 lbs of Nonfat Dry Milk	USDA Announced Nonfat Dry Milk price, Cash Settled	October 10, 2008 (discontinued)
Deliverable Nonfat Dry Milk	44,000 lbs	Physical Delivery	April 20, 2009
International Skim Milk Powder	20 Metric Tonnes	Physical Delivery	May 10, 2010
Cheese	20,000 lbs	Physical Delivery	June 21, 2010

Table 2.2. Dairy Prices: Correlations 2000-2009.

	Class III	Class IV	Nonfat Dry Milk	Butter
Class IV	0.83	1		
Nonfat Dry Milk	0.65	0.89	1	
Butter	0.54	0.46	0.04	1
Dry Whey	0.54	0.64	0.71	0.14
Cheddar Cheese	0.95	0.78		
Fluid Milk	0.97	0.90		

Table 2.3. Determination of Manufacturing Grade Milk Price in Federal Milk Marketing Orders

	Weekly average price and total sales volume				Monthly weighted average price and cumulative sales
Commodity	Oct 4, 2008	Oct 11, 2008	Oct 18, 2008	Oct 25, 2008	October 2008
40lbs cheddar	$1.9194	$1.9176	$1.8781	$1.8557	1.8927
cheese blocks	10,938,573 lbs	11,069,262 lbs	11,338,644 lbs	10,822,753 lbs	44,169,232 lbs
500lbs cheddar cheese	$1.9253	$1.9124	$1.8612	$1.8752	$1.8928
barrels†	9,180,226 lbs	9,027,224 lbs	9,758,321 lbs	9,548,341 lbs	37,514,112 lbs
Announced average cheddar cheese price††					$1.9065
Grade AA butter	$1.6833	$1.7007	$1.7002	$1.7202	
	4,288,319	2,609,238	3,642,963	3,186,005	$1.6997
Dry Whey	$0.2021	$0.1929	$0.1916	$0.1923	$0.1945
	9,630,436	11,752,139	10,804,469	9,751,256	
value of milk in cheddar cheese (9.64 X Cheese Price)					$18.37
+ value of milk in butter (0.42 X Butter Price)					$0.72
+ value if milk in dry whey (5.86 X Dry Whey Price)					$1.14
- value added by manufacturer through processing milk to dairy products (– 3.18)					-$3.18
= Class III milk price					$17.06

Note: † Adjusted to 38% moisture. †† 500lbs cheddar cheese barrels price increased by 3 cents prior to averaging.

Table 2.4. Class III Futures: Proportion of Variance Explained by the Common Factor.

January	53.5%
February	56.4%
March	57.7%
April	58.7%
May	63.2%
Jun	65.2%
July	71.8%
August	74.7%
September	72.3%
October	65.6%
November	60.2%
December	56.0%

3. Price Discovery, Volatility Spillovers and Adequacy of Speculation in Cheese Spot and Futures Markets

ABSTRACT. The task of this essay is to examine price discovery, volatility spillovers and impacts of speculation in the dairy sector. I have developed a method that allows for the calculation of implied cheese futures prices for a period before cheese futures actually started trading. Evidence for periods when cheese futures did trade suggests that utilized approximation methods perform very well. Examining the time series properties of cheese cash and implied futures price I find that the unit root hypothesis is strongly rejected for cash prices, while unit roots cannot be rejected for nearby futures prices in the framework that carefully controls for rollovers. To explain this result, I built a model that illustrates the time series properties of the nearby futures price series for a futures contract written on a second-order stationary cash series and identified the mean-reverting nonlinear dynamics that will occur at rollovers. Given the time series properties of the cash and futures series I propose an error-correction model using spreads between cash and the second nearby futures instead of the cointegration vector. To account for volatility dynamics I employ the GARCH-BEKK structure. I find that the flow of information in the mean model is predominantly from futures to cash, while volatility spillovers are bidirectional. It is possible that cash prices that include unfilled bid/offers react differently to increases in volatility in futures prices than sales cash prices, though this result may not be robust and further research is needed to identify if liquidity in the cash market is reduced with increase in conditional volatility of the futures price. I propose an extension of the BEKK variance model that I refer to as GARCH-MEX. That model does not restrict the sign of the additional regressors on the conditional variances, and can easily insure positive-definiteness of the conditional covariance matrix. Utilizing that model to evaluate the impact of speculation I find strong evidence against the hypothesis that excessive speculation is increasing the conditional variance of futures prices. If anything, speculation may in fact be inadequate, and further research with daily speculative positions and high-frequency futures prices is needed to identify the effect of increased speculation on realized volatility of futures prices, bid-ask spread and magnitude of slippage.

JEL Codes: G13, Q13, C22

Keywords: implied cheese futures, unit root tests, volatility spillovers, speculation, GARCH-MEX

3.1. Introduction

There has been considerable interest in recent years concerning the overall performance of commodity futures markets, and the extent to which futures activity has led to price instability in cash markets. Much of the recent work in futures/cash price relationships has focused on the first moment of the price distribution and deep (large volume) markets (e.g. Irwin, Sanders and Merrin, 2009; Sanders, Irwin and Merrin, 2010; Hamilton, 2009; Gilbert, 2010). However, equally important are the relationships between the second and higher moments of futures/cash price distributions. Specifically, does price action in the futures market result in increased instability (volatility) in cash markets? As noted by Witherspoon (1993), market composition may impact market stability, and, as noted by Fortenbery and Zapata (2004), this may be more apparent in thin markets.

Dairy markets are unique for several reasons, not the least of which is the relative age of the futures markets for dairy. Dairy futures markets have existed since 1993, but underwent continual re-design through the early 2000's. The re-designs were in response to both changes in dairy market structure, and changes in dairy policy. Early work on dairy suggested that there were problems with the relationships between dairy futures and cash markets (Fortenbery and Zapata, 1997). In later work, it appeared that the issues had resolved themselves (Fortenbery, Cropp and Zapata, 1997; Thraen 1999). However, recent price action has again called into question the relationship between futures and cash markets for dairy, the impacts of technical innovation in the dairy sector on price performance, and the role of public policy in promoting price stability. Past work on price performance is dated given recent changes in both production and price policy.

This paper investigates price action and performance in dairy markets in several ways. First, the actual relationship between cash and futures price is studied. Both futures and cash markets have undergone significant changes over the last ten years, and their relationship to each other has not been examined recently. Futures changes include changes in futures contract design, delivery specifications, and the actual dairy commodities traded. On the cash side, the closing of the Green Bay Cheese Exchange, its replacement by a cash market in Chicago, and changes in both cash market structure and technology adoption in production may have impacted the cash/futures relationships.

I open the essay with a brief review of literature examining the information flow between cash and futures markets and the impact of speculators activity on both. In the second section, I describe the daily wholesale cash market for cheese as well as futures contracts for cheese and other dairy futures. A technique for calculating 'implied' cheese futures price for the period before actual cheese futures contract started trading is then discussed. Time series properties of cash and nearby futures cheese price are evaluated next. The fourth section discusses in further detail the concepts of causality as they are commonly used in the applied econometrics literature. An error-correction model with a GARCH-BEKK variance structure is then proposed as an appropriate analytical framework given the results of unit root tests, followed by the discussion of empirical results. To examine the influence of speculators on price volatility new GARCH model is proposed that nests BEKK variance structure while allowing flexibility in the direction of impact of additional regressors included in the variance model. I apply this model to evaluate the adequacy of speculation in the Class III milk futures. Paper concludes with policy implications and suggestions for further research.

3.2. Literature Review

Do futures markets, by facilitating speculation, increase cash price volatility? Early work by Working (1960, Feb) in onion futures demonstrated that speculative support at harvest time reduced both seasonal price range and price adjustments at the end of the marketing year, as needed adjustments were better anticipated and incorporated in prices earlier. Gray (1963) extended Working's analysis to include seasonal patterns in onion cash prices after the trade in onion futures was prohibited. He found that pronounced seasonality in cash prices, reduced during the years of intense futures trading, had returned after onion futures were discontinued.

In addition to reductions in seasonality and earlier anticipation of adjustments needed at the end of a marketing year, futures may reduce year-to-year price fluctuations. Whereas the first two effects are mediated through the impacts of futures prices on storage decisions, the last effect will be present if the futures prices are perceived as reliable guides to production planning, and farmers engage in what Working (1962) calls anticipatory hedging.

Powers (1970) investigated live beef and pork belly markets around the time futures contracts were first introduced for these commodities. He modeled weekly cash prices as the sum of two components assumed to be uncorrelated: the systematic component associated with fundamental economic conditions and the error or random component which represents noise and disturbance in the price system. He employed a variate difference method to separate systematic and random components, and then calculated the variance of the random part separately for four year periods before and after the introduction of futures markets. For both commodities he analyzes, he found that the variance of the noise decreased after futures contracts were introduced, but offered no statistical evidence that reduction can be attributed to information services performed by the futures market. Taylor and Leuthold (1974) examined the impact of new futures market on the variance of average annual, monthly and weekly of livestock cash prices, and found that while weekly and monthly variances decreased, the effect on annual variability was not statistically significant. They conjecture that the differential effect of futures market introduction on different frequencies may be due to a contract design, as livestock contracts at that time did not trade for horizons sufficiently long to influence the behavior of livestock breeders, given the long reproductive cycle of cattle. Brorsen, Oellerman and Farris (1989) extended the analysis to daily live cattle cash price variability. In their theoretical model, faster information assimilation was enabled by the introduction of a futures market, and the transmission of that information to the cash market resulted in increased daily variability in cash prices, and a reduction in cash price autocorrelation. Empirical analysis encompassing periods before and after live cattle futures were introduced confirmed their hypotheses. They concluded that live cattle futures improved the cash market efficiency, but increased short-term price risk.

Cox (1976) proposed that introducing of a futures market may attract a new set of traders who acquire and process information in order to predict future cash prices, but do not handle the physical commodity. Speculators participating in the futures market may be more informed about the future supply and demand conditions than commercial parties. If that is the case, the addition of a futures market will enable the cash market to more quickly absorb the most recent information. The testable hypotheses emerging from Cox's work are twofold. First, the addition of a futures market will change the time series properties of the cash prices, with autoregressive components in cash price process fading in importance. In addition, expected prices will be more reliable predictors of future cash prices, i.e. the variance of the price-forecast error will decrease.

Turnovsky (1983) adds to the literature by showing that in the case where producers are risk averse, introduction of a futures market will affect not only the information set based on which the expectation of future spot prices are made, but also the slopes of the supply and inventory demand functions as they depend on the degree of price stability. He found that under a wide range of behavioral assumptions the futures market reduces both cash price volatility and long run average spot price.

Newbery (1987) extended Turnovsky's basic idea that futures markets can insure against risk, and thus increase the supply of otherwise risky activities. He built a model in which farmers must choose among two competing plant breeds, one that produces less output but with no uncertainty and another that is risky, but produces higher average yield. Once hedging with futures becomes available farmers may trade some price risk for increased production risk. If a sufficient number of producers exhibited such behavior, then the volatility of cash price may in fact increase. In his study the futures market did not destabilize cash price through speculative activities but through the impact on producer decision-making.

The price discovery function of the futures market is the ability of the futures prices to quickly absorb new price-relevant information and transmit it through to the cash prices. Price discovery has been the subject of a vast empirical literature, and some examples include Garbade and Silber (1983), Oellerman and Farris (1985), Schroeder and Goodwin (1991), Fortenbery and Zapata (1993, 1997), Zapata and Fortenbery (1996), and Yang and Bessler (2001). In early work, dynamic models in either price levels or differenced prices were utilized. With development of time series methods that can appropriately address nonstationarity of prices, researchers have started using co-integration models to analyze price discovery. Of particular interest for the present study are articles analyzing the price discovery in thin markets. In such settings, Brockman and Tse (1995), Fortenbery and Zapata (1997), Mattos and

Garcia (2006) and Ivanov and Cho (2011) find that price discovery can be hampered by the lack of liquidity or institutional constraints. This is manifested as either lack of cointegrating relationship between cash and futures market or lower information share of futures market in the price discovery process.

Impact of speculation on price levels and volatility dynamics has been recently investigated in several papers. For example, in his testimony before the U.S. Senate, Masters (2008) argued that institutional investors are among the major factors affecting commodities prices, and Gilbert (2010) argues that index futures investment was the principal channel through which monetary and financial activity have affected food prices in the second half of 2000s. However, Irwin, Sanders and Merrin (2009) argue that bubbles in futures prices are not likely, and find that speculative positions do not Granger cause futures price changes. Earlier work by Streeter and Tomek (1992) finds that volatility of soybeans futures decreases as Working's T index of speculation increases. Du, Yu and Hayes (2011) find a positive impact of speculation on crude oil price volatility. Using more detailed data, Brunetti and Büyükşahin (2009) and Brunetti, Büyükşahin and Harris (2011) find that increased speculative activity does not destabilize financial markets, and in fact predicts lower realized volatility in crude oil and other markets they analyze. However, Tang and Xiong (2010) find that futures prices of different commodities in the US became increasingly correlated with each other and this trend was significantly more pronounced for commodities in the two popular GSCI and DJ-UBS commodity indices. Büyükşahin and Robe (2011) note that correlations between the returns on commodity and on equity indices increase significantly amid greater activity by speculators in general and one type of traders in particular – hedge funds. These results may indicate that while speculators overall help increase market liquidity, they increase the sensitivity of commodity prices to macroeconomic shocks.

3.3. Data

In this paper, we are interested in evaluating information flows between spot and futures markets in the dairy sector. While we are ultimately interested in price discovery for milk price, there is in fact no national spot market for fluid milk. Given that the fluid milk prices are linked to the prices of milk used in cheese production, the second best approach to investigating cash-futures relations seems to be to look at the cash and futures markets for cheese. This section describes the aspects of the milk pricing environment in the U.S. that are relevant to my research question.

Pricing of milk in the U.S. is highly regulated under Federal Milk Marketing Orders (FMMO). Three main objectives of the FMMOs are: 1) insuring market price stability, 2) preventing processors from exercising market power over milk producers and 3) insuring adequate supply and orderly marketing of fluid milk.

The primary instrument FMMOs use to achieve these objectives is to set minimum prices that handlers of Grade A milk must pay to farmers. The fundamental principle currently used to determine minimum milk price is measure the value of milk as a function of milk ingredients that have desirable nutritional qualities: milk protein, butterfat, and milk solids (lactose, whey proteins, minerals, lactic acid).

Values of the principal milk components are inferred from derived dairy products like cheese, butter, dry whey and non-fat dry milk. Finally, in order to calculate a minimum price of milk, a standard composition of milk in terms of percentages of each ingredient is assumed. In particular, the standard used by the USDA assumes that milk used for the production of cheese consists of 3.5% butterfat, 2.99% protein and 5.69% other solids. The USDA differentiates between milk used for cheese production and milk used in production of dry products. The former is referred to as Class III milk, and the latter Class IV milk. Similarly, milk used for fluid consumption is termed Class I milk, and its minimum price exceeds the price of manufacturing milk. For milk produced for consumption, the 'ingredient' that carries the additional value is the location of marketing.

The flowchart in Figure 3.1 presents the procedure the USDA uses to arrive at the Class III and Class IV manufacturing milk prices. First, major producers of butter, dry whey, nonfat dry milk and cheese are surveyed weekly. Monthly averages of these prices, with weeks weighted by volume, are used to infer the average monthly price of the ingredients. In particular, let P_t^B be the average surveyed price of butter in month t. Then the value of butterfat is calculated as

$$bf_t = \left(P_t^B - C_t^B\right) \times Y^B \qquad (3.1)$$

where C_t^B is the USDA's estimate of the national average cost of manufacturing a pound of butter, termed *make allowance* in industry jargon, and Y^B is the *yield*, i.e. the pounds of butter that can be manufactured from one pound of butterfat. This is assumed equal to 1.20. Make allowances for dairy products change very infrequently, and only after a lengthy administrative process that involves public hearings where manufacturers present arguments on what should be deemed a fair assessment of production costs. Currently, the butter make allowance stands at $0.1715. This value changed only 4 times since the beginning of 2000. Similar to butterfat, the value of *other milk solids* is inferred from a surveyed value of dry whey:

$$s_t = \left(P_t^W - C_t^W\right) \times Y^W \qquad (3.2)$$

where P_t^W is the price of dry whey and C_t^W is the dry whey make allowance, currently set at \$0.1991 per pound. Although the name of the final product may indicate that it contains nothing but solids, it is the fact that dry whey does contains some moisture. USDA formula assumes that yield Y^W equals 1.03, i.e. a pound of other milk solids will make 1.03 pounds of dry whey. Calculating the value if nonfat milk solids, nfs_t , proceeds in exactly the same fashion. The nonfat dry milk make allowance, currently at \$0.1678/lbs, is deducted from the surveyed price for nonfat dry milk, and the difference is then multiplied by 0.99.

The dairy product that serves as the base for the calculation of the protein price is cheddar cheese that is 4 to 30 days old, sold in 40 pound blocks or 500 pound barrels. Cheese yield depends nonlinearly on the amount of protein and butterfat in milk, as the interaction of these components is recognized as an important contributor to yield. The following formula accounts for that effect

$$pr_t = \left(P_t^C - C_t^C\right) \times Y^{CP} + \left\{\left[\left(P_t^C - C_t^C\right) \times Y^{CPB}\right] - 0.9 \times bf_t\right\} \times 1.17 \qquad (3.3)$$

where P_t^C is the surveyed price of cheese, C_t^C is the cheese make allowance, currently at \$0.2003/lbs, Y^{CP} is the cheese yield from protein, and Y^{CPB} is the multiplier accounting for interaction effects between protein and butterfat. The assumed ratio of protein to butterfat in cheese is 1.17 which explains the last multiplier.

After the prices of all ingredients have been calculated, arriving at the final Class III and Class IV prices is a simple two-step process. First, the price of skim milk is calculated for both classes. For Class III, the skim milk price is calculated as the weighted average of protein and other solids.

$$C3skim_t = 3.1 \times pr_t + 5.9 \times os_t \tag{3.4}$$

Similarly, the Class IV skim milk price is

$$C4skim_t = 9.0 \times nfs_t \tag{3.5}$$

Finally, Class III and Class IV milk prices are obtained by adding a butterfat price to skim milk prices.

$$\begin{aligned} C3_t &= 0.965 \times C3skim_t + 3.5 \times bf_t \\ C4_t &= 0.965 \times C4skim_t + 3.5 \times bf_t \end{aligned} \tag{3.6}$$

Unlike prices for particular ingredients, the prices for skim milk and final class prices are expressed as U.S. dollars per hundredweight (100lbs). At each step of the process, derived prices are rounded to four decimal points.

The Chicago Mercantile Exchange (CME) operates a spot market for cheese that trades each business day from 10.45-10.55 a.m. Cheese trades in carloads weighing between 40,000 and 44,000 pounds, packed as either 40lbs blocks or 500lbs barrels. Cheese may not be less than four days or more than 30 days of age on the date of sale. This market is often regarded as thin, given that only a handful of trades occur each day, and on 40% of the trading days no sale occurs at all. Nevertheless, it is precisely this market that serves as a the price discovery center for many cash dairy products, and the weekly NASS national survey price for cheese exhibits a 0.99 correlation with the previous week average spot cheese price.

Perhaps due to the thinness of the CME cash cheese market, it is a custom to report the last bid or last offer as the daily closing price if they remained uncovered or unfilled. This renders public data on spot cheese prices problematic for econometric analysis, as the prices are often not transaction prices and are not indicative of the current market equilibrium price.

To address this issue, I have obtained the intraday cash market data that specifies each price quote as either sales, bid or offer, and have used only the last sales price of the day in my analysis. If no trade has occurred on a particular day, I use the last observed transaction price from an earlier date.

Next, I need to obtain a cheese futures price. Although cheese futures were among the first dairy futures contracts created in the early 1990s, the contracts were discontinued after the federal milk marketing order reform of 2000. Since 2000 on there were no cheese futures available, until new cash-settled cheese futures contract started trading in July 2010. As presented in the second essay of this thesis, the correlation between announced Class III price and the monthly average NASS survey cheese price is 0.95, which means we could use Class III futures, appropriately scaled, as a proxy for cheese futures prices. However, if we seek to pin down current market expectation of the price of cheese in the future, then such an approach is still imperfect and subject to substantial measurement error, as it disregards the changes in expectations regarding prices of other milk components that enter the Class III milk price formula: namely, butterfat and other milk solids.

From 2000 until September 2005, the only dairy futures contracts publicly available where Class III and Class IV milk and a deliverable butter contract. We can use the butter contract to infer the implied futures price of butterfat. To calculate the implied futures price of cheese, we would need to know the implied futures price of protein, in addition to butterfat. Using Class III and Class IV futures and the implied price for butterfat, we calculate the implied futures price for Class III and Class IV skim milk. In order to obtain an implied protein price, we need implied futures price for other milk solids. Class III and IV skim milk prices are functions of butterfat and other milk solids, and butterfat and nonfat milk solids respectively. Using only the implied skim milk prices we cannot uniquely identify the implied futures price of the three ingredients that enter the formulas for skim milk prices. However, if we make an assumption about the future price ratio of nonfat milk solids to other milk solids then we can use implied Class III and Class IV skim milk price to estimate the conditional expectation of implied futures price for other milk solids as well as protein.

In particular, I assume that the ratio of the monthly announced average price for nonfat dry milk and dry whey is an AR(1) process. When calculating conditional expectations of the implied cheese futures at time t, I only use the observations of dry whey and nonfat dry milk prices available on that date in fitting the coefficients of the stated AR(1) regression. In this way the conditional expectation of the implied futures cheese price is obtained using only information available to traders.

Measurement errors using this approximation method arises from two sources. First is uncertainty regarding the ratio of nonfat dry milk to dry whey prices. The other potential source of error comes from the fact that the butter contract is not cash-settled against the NASS survey price. Instead, physical delivery is required at certified warehouses. This may be cause for additional differences at settlement between the CME cash cheese price and the implied cheese futures price calculated using the butter futures contract.

In September 2005 cash-settled butter contract was introduced. From that point on I use the cash-settled butter contract in calculating the implied cheese futures price. In May 2007, a dry whey contract was introduced. This allows us to identify the implied futures price of other milk solids, and consequently the last source of measurement error is removed.

Figure 3.3 compares the implied cheese futures obtained using all three approximation methods and, for the period after July 2010, actually observed cheese futures prices. While there is some difference between the implied cheese futures price in the early months of 2007 as obtained using various approximation methods, all methods give very similar results for the predominant part of the past four years. In particular, I find that the absolute difference between implied and observed cheese futures is never higher than 2 cents per cwt., which probably reflects the transaction costs of riskless arbitrage (i.e. bid-ask spread, assuming that exchange members can trade without paying any transaction fees) combined with the effects of rounding to four decimal points at each step in deriving the various class milk prices.

This results in a total sample period that spans July 11, 2000 (the first day Class IV contract traded) through April 4, 2011, the last day for which cash price data was available to us. The total number of observations is 2670.

3.4. Time Series Properties of Cheese Cash and Futures Prices.

In order to appropriately model the information flow between cash and futures markets, it is important to understand the time series properties of both cash and futures price series. In particular, when prices are non-stationary, estimating models in price levels may result in spurious regressions. On the other hand, if prices are co-integrated, estimating models with differenced series will result in a misspecified model, and the cointegration framework should be utilized instead.

In this section I evaluate the time series properties of cheese cash and futures prices. I find that the null hypothesis of unit roots presence is strongly rejected for cash prices. Results are mixed for nearby futures prices, and vary with data frequency, horizon to maturity and the method of constructing lagged prices in regressions used for estimating Augmented Dickey-Fuller tests.

Further, the simple difference between concurrent cash and nearby futures price is strongly mean-reverting. In what follows I review the theoretical predictions for time series properties of cash and futures prices and build a simple model to illustrate the kind of nonlinearities that a nearby futures price series may exhibit when a cash price series is second-order stationary. The patterns observed in unit-root results closely match predictions of the illustrative model.

3.4.1. Unit Root Tests

I employ three types of unit root tests: the Dickey-Fuller (DF), Augmented Dickey-Fuller (ADF) with automatic lag selection based on AIC criteria and the Phillips-Perron test (PP). For the Dickey-Fuller (1979) test for unit roots in the absence of serial correlation I estimate a regression with an included constant term but no time trend. The null hypothesis assumes that true process is a random walk.

The estimated regression is:

$$y_t = \alpha + \rho y_{t-1} + u_t \tag{3.7}$$

with the assumed true process under the null hypothesis of

$$y_t = y_{t-1} + u_t, u_t \sim N\left(0, \sigma^2\right) \tag{3.8}$$

Under the null hypothesis, $\left(\hat{\rho} - 1\right) / \hat{\sigma}_{\hat{\rho}}$ has a non-standard distribution, and for large samples critical values for rejecting the null at 10%, 5% and 1% confidence level are -2.57, -2.86 and -3.43 respectively.

Augmented Dickey-Fuller tests (Said and Dickey, 1984) correct for serial correlation in residuals by including higher-order autoregressive terms in the regression. Similar to equation (3.7) above, I estimate an autoregression that includes a constant term. The null hypothesis is that the data are generated by a unit root autoregression with no drift.

The estimated regression is

$$\Delta y_t = \zeta_1 \Delta y_{t-1} + \zeta_2 \Delta y_{t-2} + \ldots + \zeta_{p-1} \Delta y_{t-p+1} + \alpha + \rho y_{t-1} + \varepsilon_t \tag{3.9}$$

where $\Delta y_t = y_t - y_{t-1}$. The true process is assumed to be the same specification as in (3.9) with $\alpha = 0$ and $\rho = 0$. The OLS t test for $\rho = 0$ has a non-standard distribution and critical values are the same as in Dickey-Fuller test listed above. To select the appropriate lag structure, I estimate the model with 0 through 20 lags, and choose the specification with the lowest AIC criteria.

Finally, I also estimate the Phillips-Perron (1988) tests for unit roots in presence of serial correlation. The estimated regression is

$$y_t = \alpha + \rho y_{t-1} + u_t \tag{3.10}$$

with the true process assumed to be:

$$y_t = y_{t-1} + u_t \tag{3.11}$$

The test statistic used in this test is

$$Z_t = \left(\hat{\gamma}_{0,T} / \hat{\lambda}_T^2\right)^{1/2} \frac{\left(\hat{\rho}_T - 1\right)}{\hat{\sigma}_{\hat{\rho}_T}} - \frac{1}{2}\left(\hat{\lambda}_T^2 - \hat{\gamma}_{0,T}\right)\frac{T\hat{\sigma}_{\hat{\rho}_T}}{s_T} \tag{3.12}$$

where $\hat{\gamma}_{j,T} = T^{-1} \sum_{t=j+1}^{T} \hat{u}_t \hat{u}_{t-j}$, $\hat{u}_t$ are OLS sample residuals from the estimated regression, $s_T = \left(T-k\right)^{-1} \sum_{t=1}^{T} \hat{u}_t^2$, k is the number of parameters in estimated regression, $\hat{\sigma}_{\hat{\rho}_T}$ is the OLS standard error for $\hat{\rho}$, and $\hat{\lambda}_T^2 = \hat{\gamma}_{0,T} + 2\sum_{j=1}^{q}\left[1 - \frac{j}{q+1}\right]\hat{\gamma}_{j,T}$ is the Newey-West estimator of error variance with q lags chosen based on minimial AIC criteria in the ADF lag selection process. Critical values for Z_t are the same as in Dickey-Fuller and Augmented Dickey-Fuller tests.

RATS 8.01 was used for these tests as it provides the user with easy-to-use commands for the tests as well as selecting the optimal lag structure to be used in the ADF and Phillips-Perron tests.

It has been noted that results of unit-root test may vary with data frequency (Tomek and Wang, 2007), so I estimate these tests for both daily and weekly data frequency. In addition, since CME cash market for cheese is not very liquid, and sales transactions do not occur on about 40 percent of trading days in the sample, I construct an irregular frequency data keeping only those days when the cash market did in fact record a sales transaction either in 40lbs cheese blocks or 500lbs cheese barrels.

Perhaps due to low cash market liquidity, it is a custom in the cheese industry to report the last uncovered offer or last unfilled bid as the closing cash price for the day. While our primary cash price series only contains sales records, it may be of interest to examine if any of the results regarding cash/futures information flow are sensitive to the choice of cash price series (i.e. closing sales price vs. closing prices that may be a bid or offer). For that reason, I perform unit root tests on these publicly reported closing cash prices as well, and denote them with a (B/O) suffix to differentiate from the regular sales price series. In addition, I account for potential seasonality in cash prices by testing for the presence of unit roots in the residuals from regressions of cash prices on quarterly indicator variables rather than testing for unit roots in cash prices directly.

I examine the sensitivity of ADF test results to the specification of lagged prices in the estimated regression. Recall that the futures price series is always an n-th nearby series, i.e. concatenation of segments taken from different futures contract months at the time when those contracts were the n-th contract to maturity. Denote by f_t^i a futures price on time t for a contract expiring at T_i.

Then jth-nearby is a sequence of futures prices that can be represented as

$$F_j = \left(\ldots f_t^i, f_{t+1}^i, \ldots, f_{T_{i-j+1}}^i, f_{T_{i-j+1}+1}^{i+1}, f_{T_{i-j+1}+2}^{i+1}, \ldots, f_{T_{(i+1)-j+1}}^{i+1}, f_{T_{(i+1)-j+1}+1}^{i+2} \ldots \right) \tag{3.13}$$

For example, the 2nd nearby futures price series is constructed by taking the prices of the February contract in the month of January, the March contract in the month of February, etc. Notice that in such a nearby series from time to time two consecutive prices will correspond to different contract months. For example, when the January contract settles, the 2nd nearby futures price will be the one from March, while one day earlier, i.e. on the last day the January contract traded, the 2nd nearby futures price will refer to a February contract futures price. As a consequence, using inbuilt software commands that do not account for this information will produce differenced prices for equation (3.9) that are occasionally corresponding to different contract months. To check if this is influencing the test results, I estimate a regression that insures that differenced prices always come from the same contract month, but is other otherwise equivalent to regression estimated by the inbuilt RATS command, i.e. the number of lags are determined via AIC criteria. In particular, I estimate the following regression

$$\Delta f_t = \zeta_1 \Delta f_{t-1} + \zeta_2 \Delta f_{t-2} + \ldots + \zeta_{p-1} \Delta f_{t-p+1} + \alpha + \rho f_{t-1} + \varepsilon_t \tag{3.14}$$

where $\Delta f_t = f_t^i - f_{t-1}^i$ and right hand side variable $f_{t-1} = f_{t-1}^i$.

Finally, to account for possible sensitivity to the choice of rollover date, I perform unit root tests of nearby series that are constructed by rolling over included contracts on the days when the 1st nearby contract expires, as well as on the day when the 1st nearby has 3 trading days to maturity left.

Results of the unit root tests are presented in Table 3.1. The null hypothesis of unit roots is strongly rejected for cash cheese prices. This holds true irrespective of the data frequency used, whether 40lbs blocks or barrels are examined, and whether the series is based on sales prices or publicly reported daily closing prices that may include unfilled bids and uncovered offers.

The situation is more complex in unit root test results for nearby futures prices. The ADF test for unit roots rejects the null for the second nearby futures series at a 5% confidence level irrespective of the data frequency used, while data frequency seems to matter for the 3rd, 4th and 5th nearby series. Irrespective of the rollover procedure used, tests done with weekly data are more likely to reject a unit root. The Phillips-Perron statistics are generally higher for weekly data frequency than those obtained using daily and sales only frequency. For daily and sales-only data, with the exception of the 1st nearby contract, the ADF test statistic falls as time-to-maturity increases, i.e. the ADF t-statistic is higher for the 2nd nearby than for the 3rd nearby, etc. The most striking result is the difference in t-statistics from the regular ADF regression that does not account for contract rollovers in constructing differenced prices, and the equivalent regressions that do. When regressions like (3.14) are estimated, the t-statistics next to lagged price of the same contract are very small, and the null hypothesis of unit roots is never rejected. This happens regardless of data frequency, time to maturity horizon and rollover method used. Regressions (3.14) evaluate the time series properties of within-contract segments, ignoring dynamics at contract rollovers. Therefore, these results suggest that unit root results based on ADF regressions may be driven by the nature of the price changes at the rollover time. In particular, this indicates that nearby futures price series are nonlinear - martingales within each contract segment, and mean-reverting at contract rollovers. As rollovers occur more frequently for weekly data the explanation offered above is consistent with unit roots tests more strongly rejecting the null for weekly data series.

Of special interest is evaluating the time series properties of the cash-futures price spread, denoted $d_{n,t}$ and defined as the difference between contemporaneous average cash prices and n-th nearby futures price $d_{n,t} = c_t - f_{n,t}$. In this context the cash price is a simple average of blocks and barrels cash sales prices for a particular day. Results are reported for spreads calculated using second and fifth nearby futures, and spreads are always found to be strongly stationary. If the fifth nearby futures were indeed nonstationary, and if the cash price was truly stationary, then no linear combination of the two series could be stationary. The fact that the null is strongly rejected for spreads using the fifth nearby futures is additional evidence suggesting there are nonlinearities in the nearby futures prices.

There are three questions that naturally arise as a reaction to these results. First, what does economic theory suggests about the time series properties of cash and futures commodity prices? Second, what kind of nonlinearities should we expect in nearby futures prices, when cash prices are stationary? And finally, what is the appropriate way to model information flows between cash and futures markets in the face of observed stationarity in cash prices and nonlinearities in nearby futures prices. The first two questions are answered in the next subsection, while the model specification issues are left for the next part of the paper.

3.4.2. Economic Theory and Time Series Properties of Agricultural Cash and Futures Prices

Early studies of cash/futures linkages used regression in price levels or differenced series (e.g. Oellerman and Farris, 1985). However, in many commodities, and especially when using daily data, researchers have found that both cash and futures contain unit root (e.g. Schneider and Goodwin, 1991; Ivanov and Cho, 2011). These findings have been disputed both on theoretical and statistical grounds. As far as theory is concerned, it has been claimed that agricultural price theory does not support the hypothesis that all shocks to prices are persistent (Tomek and Wang, 2007). Furthermore, misspecified models that do not account for structural breaks may bias the results towards accepting the null hypothesis of unit root presence, and unit roots test that have low power will not be effective in differentiating between integrated and stationary, but highly persistent time series (Geweke and Porter-Hudok, 1983).

Theoretical priors regarding time series properties of cash and futures prices are remarkably different. As far as cash price is concerned, the fundamental property of prices emerging in perfectly competitive markets is the necessity of zero long-run economic profit for the marginal producer. That condition implies that profit margind will be a mean-reverting time series. Consequently, if the long-run industry average cost curve is flat (a case of constant returns to scale), any permanent shift in the demand function will produce only temporary shock to cash prices, while permanent changes in input costs will shift the long-run average cost curve and thus induce a structural change in cash price series. Even with constant long run average costs, if production cannot adjust quickly to demand shocks in the short run, cash prices may exhibit high a degree of persistency and rather slow reversion to long-run averages. Finally, if returns to scale are either decreasing or increasing, shifts in demand will manifest as permanent shocks to cash price series.

While the time series properties of cash prices are argued based on production theory, the time series characteristics of futures price series emerge from finance theory. If the futures market is efficient (i.e. if futures prices fully account for all available information) then prices within a single contract will be martingales if the marginal risk premium is zero, submartingales if marginal risk premium is positive (i.e. futures are downward biased and traders having long position are rewarded), and supermartingales if the marginal risk premium is negative (i.e. futures are upward biased and traders having short position are rewarded). In each case, by deducting the marginal risk premium we can arrive at a martingale series whose direction of change cannot be predicted based on concurrently available information. From this it follows that whether the risk premium is present or not, efficient futures prices will be nonstationary, i.e. all shocks to futures prices are permanent.

Suppose now that there exists a second-order stationary cash price series for some commodity, and that a futures contract is written on that commodity. Assume further that there is no basis at futures contract expiry, i.e. the terminal futures price equals the cash price prevailing at contract expiry. Finally, assume that futures prices are efficient and embody no risk premium. What will be the time series properties of an n-th nearby futures price series?

Let μ be the unconditional mean of the cash price, and σ_c^2 be the unconditional variance. By the Wold decomposition theorem (Wold, 1954) we know that there exists the unique fundamental moving average representation of the cash price stochastic process:

$$c_t = \mu + \sum_{i=0}^{\infty} \alpha_i \varepsilon_{t-i} \tag{3.15}$$

where $\alpha_0 = 1$. Denote futures price at time t for a contract that expires at time T by f_t^T. Efficient futures prices that do not incorporate risk premiums will be unbiased predictors of cash prices at contract expiry:

$$f_t^T = E_t\left[c_T\right] \tag{3.16}$$

Using Wiener-Kolmogorov prediction formula (Hansen and Sargent, 1980) we can express futures prices at time t as

$$f_t^T = \mu + \left(\frac{\alpha(L)}{L^{T-t}}\right)_+ \varepsilon_t = \mu + \sum_{i=0}^{\infty} \alpha_{T-t+i}\varepsilon_{t-i} \tag{3.17}$$

where the annihilation operator $(\ )_+$ replaces all negative lag values by zero. An alternative, and equivalent expression for (3.17) is

$$f_t^T = \mu + \sum_{i=T-t}^{\infty} \alpha_i \varepsilon_{T-i} \tag{3.18}$$

We first exploit notation in (3.18) to show that under the assumptions of this model, any change to futures prices of a single contract must come from unanticipated information shocks ε_{t+1}.

First, express f_{t+1}^T using Wiener-Kolmogorov formula as

$$f_{t+1}^T = \mu + \sum_{i=T-(t+1)}^{\infty} \alpha_i \varepsilon_{T-i} = \mu + \sum_{i=T-t}^{\infty} \alpha_i \varepsilon_{T-i} + \alpha_{T-(t+1)}\varepsilon_{t+1} = f_t^T + \alpha_{T-t-1}\varepsilon_{t+1} \tag{3.19}$$

Since $E_t(\varepsilon_{t+1}) = 0$ it follows that $E_t\left[f_{t+1}^T\right] = f_t^T$ and we have established the martingale property of futures prices of the same contract. If in addition fundamental moving average coefficients increase in absolute value as their index decreases (this would be the case for AR(1) models for example) then the conditional variance of futures prices

$$Var_t\left[f_{t+1}^T\right] = \alpha_{T-t-1}^2 \sigma_c^2 \tag{3.20}$$

will be increasing as time to maturity decreases. This is the well-known "Samuelson Effect". From the analysis undetaken above, it would be wrong to conclude that because prices within a single futures contract are martingales that such must also be true for an n-th nearby futures price series. Let the first nearby futures price series be constructed by rolling contracts over one day before the delivery date:

$$F_1 = \left(f_1^{T_1}, \ldots, f_{T_1-1}^{T_1}, f_{T_1}^{T_2}, \ldots f_{T_2-1}^{T_2}, f_{T_2}^{T_3}, \ldots \right) \tag{3.21}$$

Let us take a closer look at the MA representation of futures prices around the rollover date:

$$\begin{aligned} f_{T_k-1}^{T_k} &= E_{T_k-1}\left[c_{T_k} \right] = \mu + \sum_{i=1}^{\infty} \alpha_i \varepsilon_{T_k-i} \\ f_{T_k}^{T_{k+1}} &= E_{T_k}\left[c_{T_{k+1}} \right] = \mu + \sum_{i=T_{k+1}-T_k}^{\infty} \alpha_i \varepsilon_{T_{k+1}-i} \end{aligned} \tag{3.22}$$

The difference in consecutive futures prices of this nearby series at rollover time is

$$f_{T_k}^{T_{k+1}} - f_{T_k-1}^{T_k} = \alpha_{T_{k+1}-T_k} \varepsilon_{T_k} + \sum_{i=1}^{\infty} \left(\alpha_{T_{k+1}-T_k-i} - \alpha_i \right) \varepsilon_{T_k-i} \tag{3.23}$$

Only the first part of the difference, $\alpha_{T_{k+1}-T_k} \varepsilon_{T_k}$, is not known at time $T_k - 1$, while the second part, the infinite sum, is fully known at that time. It follows that

$$E_{T_k-1}\left[f_{T_k}^{T_{k+1}} \right] = f_{T_k-1}^{T_k} + \sum_{i=1}^{\infty} \left(\alpha_{T_{k+1}-T_k-i} - \alpha_i \right) \varepsilon_{T_k-i} \neq f_{T_k-1}^{T_k} \tag{3.24}$$

The first nearby futures price series will *not* have the martingale properties, and changes in the nearby price sequence at rollover time are partially predictable. To give a simple example, suppose that current first nearby contract is the March contract, and tomorrow the first nearby contract will be the futures price for delivery in April, i.e. rollover is to occur tomorrow. Then the expected change in the first nearby price series is the simple difference between today's price for April delivery and today's price for March delivery.

We can extract some further insight from (3.24). If it so happens that the cash price at time $T_k - 1$, $c_{T_{k-1}} = \mu + \sum_{i=1}^{\infty} \alpha_i \varepsilon_{T_k-i}$ is above the long run mean μ, then the sum $\sum_{i=1}^{\infty} \alpha_i \varepsilon_{T_k-i}$ will be positive. When fundamental moving average coefficients are monotonically declining in absolute value, i.e. $i < j \Rightarrow |a_i| > |\alpha_j|, \forall i, j \geq 0$ then the infinite sums from expression (3.22) can be ordered in absolute value:

$$\left|\sum_{i=T_{k+1}-T_k}^{\infty} \alpha_i \varepsilon_{T_{k+1}-i}\right| < \left|\sum_{i=1}^{\infty} \alpha_i \varepsilon_{T_k-i}\right| \tag{3.25}$$

If condition (3.25) holds, and $c_{T_{k-1}} > \mu$ then $\sum_{i=1}^{\infty}\left(\alpha_{T_{k+1}-T_k-i} - \alpha_i\right)\varepsilon_{T_k-i} < 0$. In other words, the predictable component in the first nearby price change at contract rollover will be mean-reverting. In addition, because cash price is assumed to be second-order stationary, moving average coefficients are square summable, i.e. $\sum_{i=0}^{\infty} \alpha_i^2 < \infty$. This implies that

$$\lim_{i\to\infty} \alpha_i = 0 \tag{3.26}$$

Since for a fixed t $k \to \infty \Rightarrow T_k - t \to \infty$ it follows that

$$\begin{aligned}\lim_{k\to\infty} E_t\left[f_{T_{k-1}}^{T_k}\right] &= \lim_{k\to\infty} E_t\left[E_{T_k-1}\left[c_{T_k}\right]\right] = \lim_{k\to\infty} E_t\left[c_{T_k}\right] \\ &= \lim_{k\to\infty} E_t\left[\mu + \sum_{j=0}^{\infty} \alpha_j \varepsilon_{T_k-j}\right] = \lim_{k\to\infty}\left[\mu + \sum_{i=o}^{\infty} \alpha_{T_k-t+j}\varepsilon_{t-j}\right] = \mu\end{aligned} \tag{3.27}$$

In words, the long-run expected value of the first nearby futures price series is the unconditional mean of the cash price. This characteristic is shared with any second-order stationary series: if a variable is second-order stationary then forecasts of the variables value far into the future will eventually converge to an uninformed prior which is the unconditional mean of the variable. That must be so since any shocks that explain current deviations of that variable from its unconditional mean will eventually die out. The result that the long-run forecast of the first nearby futures price series is the unconditional mean of the cash price stands in sharp contrast to characteristics of series that exhibit martingale properties. For such a series, $\lim_{k\to\infty}\left[x_{t+k}\right] = x_t$, i.e. all shocks are permanent, and the long-run forecast is equal to the last observed value of the variable.

The argument that the first nearby price series will be mean-reverting at contract rollover carries forward to the n-th nearby series. Let us compare the first and an n-th nearby price series at rollover. For the first nearby series, price change at rollover time is given in (3.23). Since futures prices are assumed unbiased predictors of future cash prices, we can rewrite (3.23) as

$$f_{T_k}^{T_{k+1}} - f_{T_k-1}^{T_k} = E_{T_k}\left[c_{T_{k+1}}\right] - E_{T_k-1}\left[c_{T_k}\right] \tag{3.28}$$

Similarly, for the n-th nearby price series, $f_{T_k}^{T_{k+n}} - f_{T_k-1}^{T_{k+n-1}} = E_{T_k}\left[c_{T_{k+n}}\right] - E_{T_k-1}\left[c_{T_{k+n-1}}\right]$. From (3.26) it follows that

$$\lim_{n\to\infty} f_{T_k}^{T_{k+n}} - f_{T_k-1}^{T_{k+n-1}} = 0 \tag{3.29}$$

In other words, the predictabile part of price change for n-th nearby contract at rollover time will be smaller the higher the *n* is. Mean-reverting changes at contract rollover will be most pronounced in the first nearby, less so in the second nearby, and even less in the third nearby price series, etc.

In conclusion, I have illustrated using a simple example that when the cash price series is second-order stationary, futures prices for a specific contract will be a martingale, but not a random walk, as random walk assumes constant variance of shocks, and in we expect to see the Samuelson effect, i.e. increases in futures price volatility as time to maturity declines. Furthermore, the n-th nearby futures series will be nonlinear, having martingale properties within each contract segment, and mean-reverting changes at contract rollover. When fundamental MA coefficients are monotonically declining in absolute value, mean-reverting change at contract rollover will be less prononuced for further horizon series.

Suppose that we apply unit root tests that assume linearity in the variable being tested and posit as a null hypothesis that the process contains a unit root, e.g. a Dickey-Fuller type test or Phillips-Perron test. Based on these insights we would expect to see the following:

1) The null hypothesis will likely be rejected for cash prices
2) The null hypothesis will likely not be rejected for a single contract futures price series
3) The null hypothesis will be more likely to be rejected for n-th nearby than for $n+1$-th nearby.
4) The more observations there are between rollover periods, the less likely the null hypothesis will be rejected. Consequently, reducing data frequency increases the likelihood of rejecting the null hypothesis.

The results we obtained for unit root tests applied to cheese cash and nearby futures prices are consistent with these predictions. In particular,

1. The null is always rejected for cash prices
2. Regressions like (3.14) that test for unit root presence in contract segments, and ignore dynamics at contract rollovers never reject the null hypothesis.
3. For daily and sales-only data frequencies, test statistics for ADF and Phillips-Perron tests mostly decline from 2nd to 5th nearby series, although I not test if they are statistically significantly different.
4. Unit roots are rejected for all tested nearby series when weekly frequency is used, but tests fails to reject unit roots for a majority of the nearby series when higher data frequency is employed.

We should also notice that not all predictions of the model above hold for cash markets. In particular, for daily and sales-only data the test statistic used in the ADF test is higher for second than for the first nearby series, indicating a stronger mean-reversion at rollover time for the second series. This should not be surprising, however, as I have shown in another part of the thesis that volatility of futures prices declines dramatically in the last 4 weeks of contract life. This may be due to formula-based contract settlement procedure. In conclusion, my simple forecasting model demonstrates a high ability to explain observed patterns in unit-root tests results.

3.5. Information flows between cheese cash and futures markets

When examining the information flow between cash and futures markets, it is standard practice to use the cointegration framework developed by Johansen and Juselius (1990). Bessler and Covey (1991) were among the first to introduce this method to commodity price analysis. Examples relevant for this chapter include Fortenbery and Zapata (1997) and Thraen (1999), papers that applied co-integration to analyses of dairy futures and cash markets. In this section, I first define the concepts of causality as they are commonly understood and used in modern applied econometrics. Next I discuss causality in mean, better known as ‘Granger causality’, causality in variance and second-order causality as well as testable restrictions on model parameters that correspond to these concepts. I then propose an error-correction model incorporating GARCH structure on errors as a framework to examine information flows between cheese cash and futures markets. Given the apparent stationarity of cheese cash prices, coupled with nonlinearities in nearby futures prices, I build a model similar to an error-correction model, with the role of cointegrating vector taken by the spread between cash and the 2nd nearby futures price. The addition of a GARCH error structure allows us to examine volatility spillovers between the cash and the futures markets. In addition, the GARCH structure results in a framework that allows us to perform a preliminary examination of the role of speculators on market volatility when available data on trader positions has low frequency.

3.5.1. Concepts of Causality

Operational definitions of causality are summarized in Granger (1980). Consider a universe in which all variables are measured at prespecified time points at constant intervals $t = 1, 2, ...$ We are interested in the possibility that a series y_t causes another series x_t. Let I_n be an information set available at time n, consisting of terms of the series x_t

$$I_n : x_{n-j},\ \ j \geq 0 \tag{3.30}$$

Denote an information set that includes information on both series x_t and y_t with J_n

$$J_n : x_{n-j}, y_{n-j}\ \ j \geq 0 \tag{3.31}$$

Let $F\left(x_{n+1} \mid I_n\right)$ be the conditional distribution of x_{t+1} given I_n with conditional mean $E\left[x_{n+1} \mid I_n\right]$.

Non-causality in distribution occurs if y_n does not cause x_{n+1} with respect to J_n, or

$$F\left(x_{n+1} \mid I_n\right) = F\left(x_{n+1} \mid J_n\right) \tag{3.32}$$

In other words, knowing the history of variable y does not improve probabilistic forecasts of the variable x. For the sake of completeness, we should state that Granger defined *non-causality* and *causality* differently. Let Ω_n be the universal information set, i.e. a set containing all the information in universe available at time n. Then y_n is said to *cause* x_{n+1} if

$$F\left(x_{n+1} \mid \Omega_n\right) \neq F\left(x_{n+1} \mid \Omega_n - y_n\right) \tag{3.33}$$

The difference between the concepts of non-causality and causality is large, if we adhere strictly to definitions (3.32) and (3.33). The former equation requires us to observe only histories of the series x_t and y_t while the latter expression stipulates omniscience. Given the definitions above, rejecting non-causality is necessary, but not sufficient to demonstrate causality. In defining the non-causality in mean and variance, we follow the exposition by Comte and Lieberman (2000).

Non-causality in mean occurs if y_n does not Granger cause x_{n+1} in mean, denoted as $y_n \overset{G}{\not\to} x_{n+1}$, or

$$E\left[x_{n+1} \mid I_n\right] = E\left[x_{n+1} \mid J_n\right] \tag{3.34}$$

Given some information set $\Im_n$, the conditional variance of the one-step ahead forecast will be $E\left(\left[x_{t+1} - E\left(x_{t+1} \mid \Im_n\right)\right]^2 \middle| \Im_n\right)$. Non-causality in variance was first introduced in Granger, Robins and Engle (1986). We follow Comte and Lieberman (2000) in differentiating between *non-causality in variance* and *second-order non-causality*.

Non-causality in variance happens when variable y_n does not cause x_{n+1} in variance, denoted $y_n \overset{V}{\not\to} x_{n+1}$, or

$$E\left(\left[x_{n+1} - E\left(x_{n+1} \mid I_n\right)\right]^2 \middle| I_n\right) = E\left(\left[x_{n+1} - E\left(x_{n+1} \mid J_n\right)\right]^2 \middle| J_n\right) \tag{3.35}$$

Second-order non-causality, denoted $y_n \overset{GRE}{\not\to} x_{n+1}$, is obtained if

$$E\left(\left[x_{n+1} - E\left(x_{n+1} \mid J_n\right)\right]^2 \middle| I_n\right) = E\left(\left[x_{n+1} - E\left(x_{n+1} \mid J_n\right)\right]^2 \middle| J_n\right) \tag{3.36}$$

Unlike non-causality in variance which fully restricts the information set used in forecasting, in second-order non-causality information from y_n is allowed to be utilized in calculating the conditional mean of x_{t+1}, but not the expected square deviation from the conditional mean. In this terminology, the concept of causality in second moments, as introduced by Granger, Robins and Engle (1986), corresponds to second-order non-causality. The relationship between non-causality in mean, non-causality in variance and second-order non-causality is as follows:

$$y_n \overset{V}{\not\to} x_{n+1} \Leftrightarrow \left(y_n \overset{G}{\not\to} x_{n+1} \text{ and } y_n \overset{GRE}{\not\to} x_{n+1} \right) \tag{3.37}$$

It suffices to show that y_n Granger causes x_{n+1} in the mean to establish variance causality. This renders second-order non-causality a more interesting concept, for it is neither necessary nor sufficient for causality in means to exist for second-order causality to be present.

3.5.2. Testing for Second-Order Non-Causality

Cheung and Ng (1996) propose a two-step approach where the first step consists of estimating univariate time series models that allow for a time-varying conditional mean and variance, i.e. a univariate GARCH(1,1) model. In the second stage, residuals of the univariate models are squared and standardized by dividing with the conditional variance. The square-standardized residuals are then utilized in a cross-correlation function (CCF) to test for the presence of causality in variance. The authors argue that this method may be superior to multivariate GARCH modeling because it is simpler to implement, and does not rely on correctly specifying the functional form of inter-series dynamics.

Comte and Lieberman (2000) show that in GARCH models with a BEKK conditional covariance structure, second-order non-causality is equivalent to specific restrictions on ARCH and GARCH parameters. Consider a BEKK representation of a bivariate GARCH(1,1) model as suggested by Engle and Kroner (1995). The conditional covariance, H_t is given by the formula

$$H_t = C_0{}'C_0 + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + G'H_{t-1}G \tag{3.38}$$

with

$$H_t = \begin{pmatrix} h_{11} & h_{21} \\ h_{21} & h_{22} \end{pmatrix} \quad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad G = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \quad \varepsilon_t = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}$$

In the covariance matrix H_t, h_{11} is the conditional variance of the first series, h_{22} is the conditional variance of the second series and $h_{12} = h_{21}$ is the conditional covariance between the series. Given that expressions for ARCH and GARCH in the conditional covariance are written in (3.38) as a quadratic form, it will help our intuition to rewrite it in a simpler way.

$$\begin{aligned} h_{11} &= c_1 + a_{11}^2\varepsilon_{1,t-1}^2 + 2a_{11}a_{21}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}^2\varepsilon_{2,t-1}^2 + g_{11}^2h_{11,t-1}^2 + 2g_{11}g_{21}h_{12,t-1} + g_{21}^2h_{22,t-1}^2 \\ h_{12} &= c_2 + a_{11}a_{12}\varepsilon_{1,t-1}^2 + \left(a_{21}a_{12} + a_{11}a_{22}\right)\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}a_{22}\varepsilon_{2,t-1}^2 + \\ &\quad g_{11}g_{12}h_{11,t-1} + \left(g_{21}g_{12} + g_{11}g_{22}\right)h_{12,t-1} + g_{21}g_{22}h_{22,t-1} \\ h_{22} &= c_3 + a_{22}^2\varepsilon_{1,t-1}^2 + 2a_{22}a_{12}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{12}^2\varepsilon_{2,t-1}^2 + g_{22}^2h_{22,t-1}^2 + 2g_{22}g_{12}h_{12,t-1} + g_{12}^2h_{22,t-1}^2 \end{aligned} \tag{3.39}$$

Let x_t be indexed as the first and y_t as the second series in the model above. Comte and Lieberman show that the second-order non-causality and coefficients of the BEKK GARCH(1,1) correspond as given below.

$$y_n \overset{GRE}{\not\to} x_{n+1} \Leftrightarrow a_{21} = 0, g_{21} = 0 \tag{3.40}$$

One reason Cheung and Ng opted to pursue a cross-correlation function approach was that at the time, asymptotic theory was not worked out for multivariate GARCH. In fact, Comte and Lieberman warn us in 2000 that while it was a common practice to employ likelihood-based tests, such as Wald, Lagrange multiplier test or likelihood ratio test to evaluate statistical properties of restrictions on GARCH parameters, the asymptotic distributions of these tests in the general multivariate GARCH(p,q) were still unknown. Fortunately, Comte and Lieberman (2003) and Ling and McAleer (2003) prove both consistency and asymptotic normality of quasi-maximum likelihood estimator. That enables us to use the standard approach in testing second-order non-causality. In particular, to test the restrictions of (3.40) we need to estimate both restricted and unrestricted model. Under the null hypothesis of second-order non-causality, critical value of the test statistic will be given by $\chi^2\left(2, 0.95\right) = 5.9915$

3.5.3. Model for Evaluating Information Flows Between Cash and Futures Cheese Prices.

In previous section the time-series properties of cash and nearby futures price were evaluated. Given the results, it makes little sense to pursue a standard cointegration approach, for cash price is clearly not an integrated process, and nearby futures price seems to be a nonlinear concatenation of unit-roots within-contract segments and mean-reverting changes at contract rollovers. The basic idea of cointegration is that if two integrated variables get too far apart, at least one of them will adjust to bring the variables closer together. A framework that allows the difference between cash and futures cheese price to carry valuable forecasting information seems like a rather reasonable approach. We have seen that such spreads exhibits strong mean-reverting characteristics, so the model that naturally presents itself is the following:

$$\begin{aligned} \Delta c_t &= \mu_1 + \delta_{11}\Delta c_{t-1} + \ldots + \delta_{1p}\Delta c_{t-p} + \gamma_{11}\Delta f^i_{t-1} + \ldots \gamma_{1p}\Delta f^i_{t-p} + \theta_1 d_{t-1} + \varepsilon_{1t} \\ \Delta f^i_t &= \mu_2 + \delta_{21}\Delta c_{t-1} + \ldots + \delta_{2p}\Delta c_{t-p} + \gamma_{21}\Delta f^i_{t-1} + \ldots \gamma_{2p}\Delta f^i_{t-p} + \theta_2 d_{t-1} + \varepsilon_{2t} \end{aligned} \qquad (3.41)$$

where $d_{t-1} = c_{t-1} - f^i_{t-1}$, and all futures prices are for the second nearby contract, with i being the contract month/year index of the second-nearby contract at time t. In the above model, information flow between the two markets can arise from two effects. First, short-term dynamics may be important. For example, if the futures market closes the day higher than yesterday, that may help predict the direction of cash cheese prices the following morning. The predictive power in this example may originate from several different causal mechanisms. First, it may be that futures prices have discovered new information, and the cash price will incorporate it in the next trading session. Alternatively, it may be that traders in both markets get the information at the same time, but if futures market is open for trading, and cash market already closed for the day then the first lag of the changes in futures prices could have predictive power simply due to differences in market trading hours. This could happen even if no additional price discovery in futures markets took place. The other source of information flow may come from the spread between the cash and the second nearby futures price. Second nearby contracts have time to maturity between 22 and 44 trading days. The cheese futures contract is financially settled against the USDA announced national average monthly cheese price that will closely match the average of CME cash prices observed from 30 to 10 trading days to maturity. If the spread between the cash and futures price is large, one of these two variables will have to eventually adjust and reduce the spread. If futures prices accurately anticipate the average cash price near the contract expiry, then the spread between cash and the futures price will have forecasting power in predicting changes in cash prices.

Given the definition of Granger non-causality in mean, we can state that futures prices do not Granger cause cash prices if the following restrictions on model coefficients hold:

$$\begin{aligned} &\gamma_{1i} = 0, \forall i = 1,.., p \\ &\theta_1 = 0 \end{aligned} \tag{3.42}$$

For non-causality from cash to futures prices the same restrictions are to be applied to the corresponding coefficients in the second equation of the model (3.41).

In addition to forecasting price changes, we are interested in volatility dynamics and spillovers between the two markets. One approach would be to add the BEKK structure to errors as in (3.38). In addition to spillovers, we are interested in evaluating the impact of speculative activity on volatility levels and dynamics. A standard method to include additional regressors to BEKK variance model is to expand the structure to what is called BEKK-X:

$$\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim N\left(0, H_t\right) \qquad H_t = C_0{}'C_0 + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + G'H_{t-1}G + D'Dx_{t-1} \tag{3.43}$$

Given that the coefficients next to the additional regressor enter variance equation (3.43) as a quadratic form, the sign of the coefficients for the impact on variance of both cash and futures prices is always restricted to be positive. To see that, expand $D'DS_{t-1}$ as follows

$$D'Dx_{t-1} = \begin{bmatrix} d_{11} & 0 \\ d_{21} & d_{22} \end{bmatrix}' \begin{bmatrix} d_{11} & 0 \\ d_{21} & d_{22} \end{bmatrix} x_{t-1} = \begin{bmatrix} d_{11}^2 + d_{21}^2 & \\ d_{21}d_{22} & d_{22}^2 \end{bmatrix} x_{t-1} \tag{3.44}$$

It turns out that the BEKK specification can only test if the increase in an additional regressor in the variance equation is associated with an increase in the conditional variance of cash and futures prices. As such, a hypothesis that speculation may reduce the conditional variance of futures price cannot be easily tested with this model specification, and an alternative model structure needs to be developed.

One possibility would be a bivariate EGARCH model introduced first by Koutmos and Tucker (1996) to evaluate dynamic interactions between spot and futures in the stock markets. Following Nelson (1991), it is the logarithm of the conditional variance in their EGARCH model that is modeled as a linear function of past conditional log-variances and magnitude of realized shocks in the previous period. Exponential form allows this modeling approach to admit additional regressors in the variance equation while preserving the positive definiteness of the conditional covariance matrix. Similar models were used by Tse (1999), Bhar (2001), Zhong, Darrat and Ottero (2005) and Bhar and Nikolova (2009).

While variations of the bivariate EGARCH model used by these authors allows for high degree of flexibility, the exponential form may present estimation problems. In addition, it may be of help to have a model that nests a BEKK model with means as in (3.41) and variance model as in (3.38) as a special case where additional regressors in the variance equation are found to have no influence on the variance dynamics.

In what follows, we expand the BEKK model without imposing a particular sign on the coefficient of the additional regressor in the variance equation, while preserving the positive-definiteness of the conditional covariance matrix. Let the mean model be as in (3.41).

We introduce additional regressors in the variance equation through an exponential function that multiplies the BEKK structure. In a standard BEKK model, we have

$$\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim N\left(0, \tilde{H}_t\right) \qquad \tilde{H}_t = C_0{}'C_0 + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + G'\tilde{H}_{t-1}G \tag{3.45}$$

We can expand the variance model in the following way

$$\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim N\left(0, H_t\right) \qquad H_t = X_t \odot \tilde{H}_t \tag{3.46}$$

where $\tilde{H}_t$ is given by the expression in (3.45), the symbol $\odot$ stands for Hadamard product, i.e. element-by-element multiplication, and the matrix X_t is defined as

$$X_t = \begin{bmatrix} e^{\xi_1 x_{t-1}} & \\ e^{\xi_{12} x_{t-1}} & e^{\xi_2 x_{t-1}} \end{bmatrix} \tag{3.47}$$

Due to the BEKK form, $\tilde{H}_t$ will be positive definite, and to insure positive definiteness of H_t it will suffice to impose the following restriction on the parameter ξ_{12}:

$$\xi_{12} = \frac{1}{2}\left(\xi_1 + \xi_2\right) \tag{3.48}$$

As I shall demonstrate, restriction (3.48) is equivalent to restricting the impact of additional regressors to have influence only on conditional variances of the two series, but not on their conditional correlation. Denote the elements of $\tilde{H}_t$ as

$$\tilde{H}_t = \begin{bmatrix} \tilde{\sigma}_{11,t} & \\ \tilde{\sigma}_{12,t} & \tilde{\sigma}_{22,t} \end{bmatrix} \tag{3.49}$$

Since the exponential form is used for all elements of X_t, the diagonal elements of H_t will be positive. To insure positive definiteness of H_t and it will be sufficient that the determinant of the H_t matrix is positive.

$$\left|H_t\right| = e^{\xi_1 x_{t-1}} \tilde{\sigma}_{11,t} e^{\xi_2 x_{t-1}} \tilde{\sigma}_{22,t} - \left(e^{\xi_{12} x_{t-1}} \tilde{\sigma}_{22,t}\right)^2 > 0 \tag{3.50}$$

With restriction (3.48) this is reduced to

$$\left|H_t\right| = e^{\xi_1 x_{t-1} + \xi_2 x_{t-1}} \left[\tilde{\sigma}_{11,t} \tilde{\sigma}_{22,t} - \left(\tilde{\sigma}_{22,t}\right)^2\right] \tag{3.51}$$

The positive range of the exponential function together with positive-definiteness of $\tilde{H}_t$ imposed by the BEKK structure jointly guarantees that H_t will be positive-definite. In practice, I recommend starting with the unrestricted version given in (3.47), and after the model is estimated checking for positive-definiteness of the conditional covariance matrix for each observation. If that is violated for any data point, the stated restriction will resolve the problem.

I propose that this new GARCH model be called GARCH-MEX, or BEKK-MEX, where MEX stands for multiplicative exponential heteroskedasticity induced by additional regressors. GARCH-MEX may be estimated in two variations, depending on whether a researcher believes that the impact of an additional regressor is exhausted in the present period or propagated forward through the GARCH structure. In the first case, we would model the variance with

$$\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim N(0, H_t)$$
$$H_t = X_t \odot \tilde{H}_t \tag{3.52}$$
$$\tilde{H}_t = C_0{}' C_0 + A' \varepsilon_{t-1} \varepsilon'_{t-1} A + G' \tilde{H}_{t-1} G$$

and in the second case, the BEKK structure for $\tilde{H}_t$ would be modified to

$$\tilde{H}_t = C_0{}' C_0 + A' \varepsilon_{t-1} \varepsilon'_{t-1} A + G' H_{t-1} G \tag{3.53}$$

In words, specification (3.53) allows the impact of additional regressors to influence future variances through the GARCH structure.

GARCH-MEX has four important characteristics:

1) If additional regressors do not explain volatility, i.e. $\xi_1 = \xi_{12} = \xi_2 = 0$, the model collapses to a standard BEKK model.
2) Since the exponential function is always positive, signs of coefficients ξ_1, ξ_2 do not need to be restricted as in (3.44).
3) The covariance matrix is always positive definite, as demonstrated in (3.51).
4) With restriction (3.48) additional regressors impact only conditional variances of individual series, but not conditional correlation directly.

$$\rho = \frac{\sigma_{12,t}}{\sigma_{11,t}\sigma_{12,t}} = \frac{\tilde{\sigma}_{12,t} e^{\frac{1}{2}(\xi_1+\xi_2)S_{t-1}}}{\tilde{\sigma}_{11,t} e^{\frac{1}{2}\xi_1 S_{t-1}} \tilde{\sigma}_{22,t} e^{\frac{1}{2}\xi_2 S_{t-1}}} = \frac{\tilde{\sigma}_{12,t}}{\tilde{\sigma}_{11,t}\tilde{\sigma}_{22,t}} \tag{3.54}$$

However, conditional correlation is time-varying, and in specification (3.53) influenced by additional regressors indirectly, through impacts on lagged conditional variances that enter the BEKK structure.

The complete model for evaluating information flows between cash and futures prices, and the influence of speculators on volatility dynamics is as follows:

$$\begin{aligned}\Delta c_t &= \mu_1 + \delta_{11}\Delta c_{t-1} + \ldots + \delta_{14}\Delta c_{t-4} + \gamma_{11}\Delta f^i_{t-1} + \ldots \gamma_{14}\Delta f^i_{t-4} + \theta_1 d_{t-1} + \varepsilon_{1t} \\ \Delta f^i_t &= \mu_2 + \delta_{21}\Delta c_{t-1} + \ldots + \delta_{24}\Delta c_{t-4} + \gamma_{21}\Delta f^i_{t-1} + \ldots \gamma_{24}\Delta f^i_{t-4} + \theta_2 d_{t-1} + \varepsilon_{2t}\end{aligned} \quad (3.55)$$

$$\begin{pmatrix}\varepsilon_{1t} \\ \varepsilon_{2t}\end{pmatrix} \sim N\left(0, H_t\right)$$

$$H_t = X_t \odot \tilde{H}_t \quad (3.56)$$

$$\tilde{H}_t = C_0{}'C_0 + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + G'H_{t-1}G$$

$$X_t = \begin{bmatrix} e^{\xi_1 S_{t-1} + \phi_1 c_{t-1}} & \\ e^{\xi_{12} S_{t-1} + \phi_{12} c_{t-1}} & e^{\xi_2 S_{t-1} + \phi_2 c_{t-1}} \end{bmatrix} \quad (3.57)$$

In the MEX matrix, I have used lagged cash prices in addition to a measure of speculative adequacy, to control for possible confounding if speculative activity coincides with cycles in milk prices.

The model can be estimated in several variations. First, prices can be expressed as either levels or logarithms. Second, data frequency can be daily, ‘sales-only’ or weekly. However, for daily and ‘sales-only’ frequencies we cannot estimate the impact of speculators as that data is only available on a weekly basis. I test 14 different measures of speculative adequacy which are described in the next section. We may test both variations of GARCH-MEX model, i.e. allowing additional regressors to have an effect that propagates dynamically through the GARCH structure as in (3.56) or restrict the BEKK structure to

$$\tilde{H}_t = C_0{}'C_0 + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + G'H_{t-1}G \quad (3.58)$$

Finally, we may impose the restriction

$$\begin{aligned}\xi_{12} &= \frac{1}{2}\left(\xi_1 + \xi_2\right) \\ \phi_{12} &= \frac{1}{2}\left(\phi_1 + \phi_2\right)\end{aligned} \quad (3.59)$$

to test if conditional correlation is impacted by speculative activity and cash price levels.

3.5.4. Measures of Speculative Adequacy

To evaluate the impact of speculators on cheese markets, we use a measure of speculative activity in Class III milk futures prices. We chose Class III milk futures as it is the most liquid dairy futures market, and tied to cheese futures through a no-arbitrage condition that connects futures prices for butter, cheese, whey and Class III contracts. In the absence of trading costs, cheese futures price can be represented as simple linear combinations of butter, whey and Class III milk prices. Thus, whichever variable influences the conditional variance of Class III milk futures, it will influence the conditional variance of cheese futures price as well. Data for speculators' position are obtained from Commitments of Traders (COT) report published weekly by the Commodity Futures Trading Commission (CFTC). COT only presents data for selected markets that the CFTC considers to be important to monitor closely, and cheese futures are not currently included in the COT report.

A classical measure of speculative adequacy is called Working's T, and was introduced by Working (1960, May). Denote commercial (hedging) long positions with H_L, H_S is the commercial short position, S_L is the noncommercial (speculative) long, and S_S speculative short positions, all measured by the amount of contracts held. When short hedging exceeds long hedging, Working's T is calculated as

$$T = \frac{S_L + 2H_L}{H_S + H_L} = 1 + \frac{S_S}{H_S + H_L} \tag{3.60}$$

If the hedging position is net long, i.e. $H_L > H_S$ then the formula becomes

$$T = 1 + \frac{S_L}{H_S + H_L} \tag{3.61}$$

To calculate T, all open interest (excluding noncommercial spreading) has to be allocated to these four categories (H_L, H_S, S_S, S_L). That means that nonreportable positions, for which we have no information relative to their speculative or hedging nature, have to be allocated to the above categories. Working's T measures the 'adequacy' of speculation. The minimum value is equal to 1. Markets with T-index less than 1.15 are considered to have insufficient liquidity (Irwin and Sanders, 2010).

There are at least nine ways to allocate nonreportable positions in the calculation of Working's T-index. First, we may allocate them in such way as to obtain the upper or lower bounds of the index as in Peck (1980). We denote these calculations as "*upper bound*" and "*lower bound*". Another approach would be to base the allocation rules on informal feedback based on personal conversations from brokers managing accounts of traders in class III milk. It was suggested to me by Mr. Phil Plourd, the manager of a company that was among the first to offer risk management services to dairy sector, that it might be reasonable to treat all nonreportable short positions as commercial, and split nonreportable long positions equally between the speculative and hedging categories. Working's T calculated in such way is reffered to as *"industry"*. Irwin and Sanders (2010) follow Rutledge (1977) and allocate nonreportable positions to the commercial, noncommercial and index trader categories in the same proportion as that which is observed for reporting traders. Denote index obtained using their method as "*proportional"*.

The problem with this approach is that it ignores the similarity between position profiles of nonreportables and the stated three categories. To illustrate this point with a simple example we shall ignore the role of index traders for a moment and assume all reportable positions are classified to the noncommercial and commercial categories only. If commercials represent the predominant percentage of open interest, the above approach would classify most nonreportables as commercials. However, it may be the case that commercial traders have a very strong net-long positions, while both non-commercials and nonreportables have strong net-short position. In that sense, nonreportables seem to have a position profile that is more like non-commercials than like commercials.

For example, consider the situation in Class III milk futures market on September 12, 2000. Ignoring noncommercial spreading positions, the relative noncommercial short position measured as $S_S / (H_S + S_S)$ was only 13.24%, and the relative noncommercial long position, measured as $S_L / (S_L + H_L)$ was even lower, only 5.90%. According to the Irwin and Sanders approach, only 13.24% of nonreportable shorts and 5.90% of nonreportable long positions would be allocated to noncommercials short and long positions respectively. Now consider the percentage net-long positions for all trader groups. Define relative net-long position for a particular group as

$$NL\% = \frac{L - S}{L + S + Spr} \times 100 \qquad (3.62)$$

where L stands for long, S for short, and Spr for spreading.

On 9/12/2000, the percentage net-long position was -29.68% for noncommercials, 13.7% for commercials and -29.77% for nonreportable positions. According to this metric, nonreportables were very similar in their position profile to noncommercials, and it may make sense to allocate more than just a marginal fraction of those contracts to the noncommercial group.

Consider a following allocation rule. Let $H_{NL}\%, S_{NL}\%$ and $NR_{NL}\%$ be percentage net-long positions for commercial, noncommercial and nonreportable positions in that order. The percentage of nonreportable contracts allocated to the noncommercial category is

$$\lambda = \frac{\left(NR_{NL}\% - H_{NL}\%\right)^2}{\left(NR_{NL}\% - H_{NL}\%\right)^2 + \left(NR_{NL}\% - S_{NL}\%\right)^2} \tag{3.63}$$

In the example above, $\lambda = 0.999$ and practically all nonreportable positions would be allocated to noncommercials.

However, the relative net-long position does not capture the full picture. For example, on 6/26/2007 nonreportables where -8.51% net-long, much like commercials traders which where -9.51% net-long, while noncommercials traders where 28.36% net-long. The distance measure λ would indicate we should assign almost all contracts to the commercial category, for the position profiles match closely. However, a closer look at the Figure 3.5 reveals that nonreportable positions where strongly negatively correlated with commercial traders position both before and after this date. If nonreportables were really small commercial traders, wouldn't they be moving similarl to that category? Consider a measure

$$\phi_t = \frac{\rho_{N,t} - \rho_{C,t}}{2} \tag{3.64}$$

Where $\rho_{N,t}$ is the correlation between noncommercial and nonreportable net-long positions over the previous three months, and $\rho_{C,t}$ measures the correlation over the same period between commercial and nonreportable net-long positions. ϕ_t is bounded between -1 and 1. A value close to 1 would indicate that $\rho_{N,t}$ is high and positive, and $\rho_{C,t}$ is strong and negative, and therefore would suggest that nonreportables behave like noncommercials. If ϕ_t is close to -1 situation is reverse and nonreportables take positions similar to commercials. When ϕ_t is close to zero either correlations are of the same sign, or are both rather small. In that case, past comovements offer us little clue as to the composition of nonreportables, and the distance metric λ_t seems like a measure of choice. Finally, a composite assignment rule that takes into account both static (position distance) and dynamic (comovement) factors is presented in the equation (3.65). The percentage of nonreportable contracts assigned to the noncommercial category is denoted as α_t and calculated as follows

$$\alpha_t = |\phi_t|\left(\frac{\phi_t + 1}{2}\right) - \left(1 - |\phi_t|\right)\lambda_t \tag{3.65}$$

In words, the primary assignment criteria is based on dynamics. When comovements are not very informative, the distance metric is given higher weight. Finally, it should be stated that the applicability of this method rests on the implicit assumption that risks faced by reportables and nonreportables is not negatively correlated. Given that the available data does allow us to test this directly, this approach can at best be used as just one of several robustness checks, rather than a definitive solution in itself. Denote the Working's T calculated based on this rule as "*dynamic*". While Working originally excluded the noncommercial spreading contracts from the calculation of the T-index, it may make sense to test if results are robust to the inclusion of spreading contracts in noncommercial long and short positions. For all except "*industry*" based T-index calculations we develop a variation of the method that includes spreading contracts as indicated above. We denote such measures with the letter 'S' in parenthesis following the original notation.

Finally, besides Working's T, we may measure the speculative impact by a simple percentage of open interest held by the speculators. We may measure that with either speculative long or short positions divided by the total open interest, as well as the sum of speculative short and long positions divided by twice the open interest. As above, spreading may be included or excluded, which is indicated by '(S)' where appropriate.

3.6. Model Results

To start, I first estimate the mean model (3.55) without explicitly modeling the volatility dynamics in model residuals and ignoring the speculative influence on price dynamics. Results using all three data frequencies and both price levels and log-prices are presented in Table 3.2 and Table 3.3. Results are consistent and suggest that short-term dynamics in cash prices do not influence the direction of futures prices. Futures prices seem to have one-day momentum dynamics as the coefficient of differenced futures prices lagged once is significant. In both model specifications cash prices adjust to the cash-futures spread. However, weak evidence of futures prices adjusting to the spread is found only when using publicly reported closing cash prices that may be bids or offers (not transactions), and that seems to be the only observable difference between the model specifications. It seems that the data frequency only influences the magnitude of the spread coefficient in the cash equation. The coefficient is -0.068 for daily frequency and -0.33 for weekly frequency. The relative magnitude closely resembles the change in frequency – one week contains 5 trading days, and the coefficient in the weekly equation is about 5 times as big as the one in the model estimated using daily frequency.

The next topic of interest includes the volatility spillovers between cash and futures markets. I estimate the standard BEKK model using weekly frequency and price data in levels. Results are reported in

Table 3.3. Several results are interesting. First, when the mean model was estimated without a specified variance structure, we found very little evidence that futures prices adjust to the spread between cash and futures. That adjustment coefficient is now found to be significant. Next, we find that volatility spillovers are bidirectional. Coefficients α_{21} and γ_{21} capture the spillovers from futures to cash markets. We see that both coefficients are negative, although only α_{21} is significant. To measure the sign of an average impact of these spillovers, I calculate the share of the cash conditional variance attributable to futures for each observation, and take an average. I find that on average, volatility spillovers are negative. Estimating the model for cash data that include unfilled bids and uncovered offers, I find that this result is reversed and that volatility spillovers are positive. While this analysis is preliminary, and robustness of these results needs to be further checked, it is interesting to consider what this would really mean in practice. Cash markets for cheese are very thin, and not more than a handful of sales occur per one trading session, and often it is the case that no sales happen. This result could indicate that in turbulent times price discovery in cash markets is done via bids/offers, while sales are few. That would make sense, since this is a surplus market. In thin markets, an agent that is one of the few sellers with a surplus of cheese to sell and believes that price will continue to increase is likely to wait and try to capture a better price in one of the following trading sessions. A recent Government Accountability Office study (2007) of the cheese spot market brings further examples that fit well with this story:

> "*Between January 1, 1999, and February 2, 2007, the closing price for block cheese fluctuated based on unfilled bids and uncovered offers on at least 17 percent of the trading days. During the same time period, the barrel market closing price fluctuated based on unfilled bids and uncovered offers 28 percent of trading days.*
>
> *• Between March 1, 2004, and April 16, 2004, block cheese prices increased from $1.49 to $2.20 per pound, or 48 percent, on the CME spot cheese market based primarily on unfilled bids to buy cheese, with only four carloads of block cheese bought or sold during this period.*
>
> *• Between October 26, 2004, and November 19, 2004, block cheese prices rose from $1.57 to $1.80 per pound, or 14 percent, with completed transactions for only three carloads of cheese completed during this period."*

In conclusion, my reading of these results is that higher volatility in the futures market translates to higher volatility in cash market if we look at closing prices that may be bids or offer quotes. But when it comes to sales prices, they are less likely to follow as fast, and sales cash forecast variance declines in times of high volatility in the futures.

Finally, we estimate the full model (3.55)-(3.57). I have estimated four variations of the model. Model 1 imposes restriction (3.58), i.e. effects of additional regressors does not enter the GARCH structure in the subsequent period, while Model 2 allows such effects to be present as in equation (3.56). For both model versions, I first allow the conditional correlation to be directly impacted by speculators, and check if the conditional covariance matrix was positive definite at each data point after the estimation is completed. Results from estimation that insures positive definiteness of the covariance matrix ex ante by restricting correlation via (3.59) are also presented. To further check robustness of the results, I estimate the model for 14 alternative measures of speculative influence described in the previous section. It needs to be noted that these robustness checks cannot completely compensate for the potential measurement errors that can arise from the fact that imputed, rather than observed cheese futures prices were used prior to July 2010.

Results, presented in Table 3.4, reveal that Working's T index in the Class III milk futures are on average much lower than any of the twelve agricultural futures markets analyzed recently by Irwin and Sanders (2010). For example, they find that average T-index for corn was 1.15, 1.17 in soybeans, 1.44 in CBOT wheat, 1.86 in feeder cattle, and 1.14 in cocoa. Using their method, I find that T-index is only 1.03 in the class III milk futures. My alternative calculations based on dynamics of nonreportable positions bring the T-index up to 1.10, and in fact the theoretical upper bound obtained by allocating all of the nonreportables to noncommercial traders is 1.16.

Irwin and Sanders state that value of 1.15 would be historically considered an inadequate amount of speculation to efficiently meet hedging demands and facilitate the transfer of risk.

We find that Model 1 with unrestricted correlation almost always fails to converge. Overall, the highest likelihood is obtained using variation 2 of the model with unrestricted conditional correlations. In each case the conditional covariance is found to be positive definite for all observations. In most specifications, likelihood ratio test rejects restricted correlation version of model 2 in favor of unrestricted correlationFor conditional variance of the futures price, coefficients of speculative adequacy are always either negative or not statistically significantly different from zero. This constitutes strong evidence that there is no excessive speculation in the Class III milk prices. In fact, according to standards suggested by Irwin and Sanders (2010), it could be that speculation in Class III milk futures market is inadequate. My current results indicate that on a weekly frequency, more noncommercial presence would predict lower conditional variance of the futures prices.

3.7. Conclusions and Directions for Future Research

The task of this essay was to examine price discovery, volatility spillovers and impacts of speculation in the dairy sector. I have developed a method that allows for the calculation of implied cheese futures prices for a period before cheese futures actually started trading. Evidence for periods when cheese futures did trade suggests that utilized approximation methods perform very well. Examining the time series properties of cheese cash and implied futures price I find that the unit root hypothesis is strongly rejected for cash prices, while unit roots cannot be rejected for nearby futures prices in the framework that carefully controls for rollovers. To explain this result, I built a model that illustrates the time series properties of the nearby futures price series for a futures contract written on a second-order stationary cash series and identified the mean-reverting nonlinear dynamics that will occur at rollovers. Given the time series properties of the cash and futures series I propose an error-correction model using spreads between cash and the second nearby futures instead of the cointegration vector. To account for volatility dynamics I employ the GARCH-BEKK structure. I find that the flow of information in the mean model is predominantly from futures to cash, while volatility spillovers are bidirectional. It is possible that cash prices that include unfilled bid/offers react differently to increases in volatility in futures prices than sales cash prices, though this result may not be robust and further research is needed to identify if liquidity in the cash market is reduced with increase in conditional volatility of the futures price. I propose an extension of the BEKK variance model that I refer to as GARCH-MEX. That model does not restrict the sign of the additional regressors on the conditional variances, and can easily insure positive-definiteness of the conditional covariance matrix. Utilizing that model to evaluate the impact of speculation I find strong evidence against the hypothesis that excessive speculation is increasing the conditional variance of futures prices. If anything, speculation may in fact be inadequate, and further research with daily speculative positions and high-frequency futures prices is needed to identify the effect of increased speculation on realized volatility of futures prices, bid-ask spread and magnitude of slippage.

Strong desirability of higher liquidity is clearly seen in the dairy futures products. CME offers seven dairy futures products, but Class III milk has by far the highest open interest, volume and is probably the only dairy futures market to attract speculative interest. While processors hedge their input prices with class III milk, most dairy farmers are interested in protecting their milk mailbox prices which depend on both Class III and Class IV futures prices. In fact, in the second quarter of 2011, the minimum price of fluid milk was calculated using Advanced Class IV, not Class III milk prices. However, Class IV milk futures did not attract significant additional volume, and Class III milk futures remain the center for price risk transfer. Several dairy groups have recently proposed legislative measures that would simplify milk pricing. From a price risk management point of view, simplifying the milk pricing process could enhance the price discovery role of the futures prices, bring new hedging interest to the futures contract and consequently attract more speculative capital. My results indicate more speculative presence could lead to a more stable futures market.

The finding of bidirectional volatility spillovers could indicate that cash prices do also help discover the price levels in the futures market. However, since the cash market trades for only ten minutes in the morning, if the new information is incorporated in the futures market by the end of the day, then a model that uses only daily closing data will fail to uncover the information transmission arising from lagged cash price, as the information flow occurs at a higher frequency. However, this market setting can also be turned to a researcher's advantage. Given that the cash market is open for only short periods of time each day makes it rather easy to uncover information flows from cash to futures if high-frequency futures prices are used. For example, one dairy broker (Schalla, 2011) summarized in his blog the events of one particularly turbulent day in the cheese markets, following several weeks of increases in cheese blocks prices on the spot market, mostly on unfilled bids and with little or no sales transactions:

- *"9:05 a.m. – July Milk opens the day trading session at $20.13/cwt., off 4 cents from the day before.*
- *10:45 a.m. – Heading into the daily cash cheese session, July drops a minimal 4 cents from the open to $20.09.*
- *10:48 a.m. – Cash cheese session closes with no action seen in either the Block or Barrel market. This spooks the milk market, and the July contract free-falls to $19.87, 22 cents lower than just three minutes earlier."*

By focusing on activity of the futures markets immediately after the close of the cash market we can identify the information contribution of the cash market. Should we find it to be substantial, that will be an additional cause for worry. The CME cash market is very thin, and agents trading on this market may exhibit market power that is inconsistent with perfect competition that agricultural cash markets are usually taken to be. A previous study of the National Cheese Exchange (NCE) undertaken by Mueller et al. (1996) found the presence of cash price manipulation. Shortly afterwards NCE was closed and cheese spot trading moved to the Chicago Board of Trade. While a recent GAO (2007) study indicates that safeguards against market manipulation are stronger at CBOT than they were at the NCE, further analytical study of cash market behavior is needed.

In conclusion, while the specifics of milk pricing, the nature of milk production, and the thinness of cash and futures cheese markets presented me with interesting challenges in evaluating the market characteristics, the overall conclusion is actually quite simple – effective price discovery needs deep and liquid futures as well as cash markets. Any proposed policy change should be evaluated by its potential to simplify and deepen market based price discovery and risk management. If there is any excessive volatility in milk prices, it is more likely to be a consequence of overregulation which leads to fragmented and thin markets, rather than animal spirits of profit-seeking speculators.

3.8. References

Bessler, D. A. and T. Covey (1991). Cointegration: Some results on U.S. cattle prices. *The Journal of Futures Markets 11* (4), 461-474.

Bhar, R. (2001). Return and volatility dynamics in the spot and futures markets in Australia: An intervention analysis in a bivariate EGARCH-x framework. *The Journal of Futures Markets 21* (9), 833-850.

Bhar, R. and B. Nikolova (2009). Return, volatility spillovers and dynamic correlation in the BRIC equity markets: An analysis using a bivariate EGARCH framework. *Global Finance Journal 19* (3), 203-218.

Brockman, P. and Y. Tse (1995). Information shares in canadian agricultural cash and futures markets. *Applied Economics Letters 2*, 335-338.

Brorsen, B. W., C. M. Oellermann, and P. L. Farris (1989). The live cattle futures market and daily cash price movements. *The Journal of Futures Markets 9* (4), 273-282.

Brunetti, C. and B. Büyükşahin (2009). Is speculation destabilizing? Working Paper, Available from URL: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1393524 [accessed 25 June, 2011]

Brunetti, C., B. Büyükşahin, and J. H. Harris (2011). Speculators, prices and market volatility. Working Paper, Available from URL: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1736737 [accessed 25 June, 2011]

Büyükşahin, B. and M. A. Robe (2011). Speculators, commodities and Cross-Market linkages. Working Paper, Available from URL: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1707103 [accessed 25 June, 2011]

Cheung, Y. and L. K. Ng (1996). A causality-in-variance test and its application to financial market prices. *Journal of Econometrics 72* (1-2), 33-48.

Comte, F. and O. Lieberman (2000). Second-Order noncausality in multivariate GARCH processes. *Journal of Time Series Analysis 21* (5), 535-557.

Comte, F. and O. Lieberman (2003). Asymptotic theory for multivariate GARCH processes. *Journal of Multivariate Analysis 84* (1), 61-84.

Cox, C. C. (1976). Futures trading and market information. *The Journal of Political Economy 84* (6), 1215-1237.

Dickey, D. A. and W. A. Fuller (1979). Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association 74* (366), 427-431.

Du, X., C. L. Yu, and D. J. Hayes (2011). Speculation and volatility spillover in the crude oil and agricultural commodity markets: A Bayesian analysis. *Energy Economics 33* (3), 497-503.

Fortenbery, T. R. and H. O. Zapata (1993). An examination of cointegration relations between futures and local grain markets. *The Journal of Futures Markets 13* (8), 921-932.

Fortenbery, T. R. and H. O. Zapata (1997). An evaluation of price linkages between futures and cash markets for cheddar cheese. *The Journal of Futures Markets 17* (3), 279-301.

Fortenbery, T. R., R. A. Cropp, and H. Zapata (1997). Analysis of expected price dynamics between fluid milk futures contracts and cash prices for fluid milk. *Journal of Agribusiness 15* (2), 215-234.

Fortenbery, T. R. and H. O. Zapata (2004). Developed speculation and underdeveloped markets – the role of futures trading on export prices in less developed countries. *European Review of Agricultural Economics 31* (4), 451-471.

GAO (2007). Spot Cheese Market - Market oversight has increased, but concerns remain about potential manipulation. Technical report. U.S. Government Accountability Ofice.

Garbade, K. D. and W. L. Silber (1983). Price movements and price discovery in futures and cash markets. *The Review of Economics and Statistics 65* (2), 289-297.

Geweke, J. and S. Porter-Hudak (1983). The estimation and application of long memory time series models. *Journal of Time Series Analysis 4* (4), 221-238.

Gilbert, C. L. (2010). How to understand high food prices. *Journal of Agricultural Economics 61* (2), 398-425.

Granger, C. W. J. (1980). Testing for causality: A personal viewpoint. *Journal of Economic Dynamics and Control 2* (2-4), 329-352.

Granger, C. W. J., R. P. Robins, and R. F. Engle (1986). *Wholesale and Retail Prices: Bivariate Time-Series Modeling with Forecastable Error Variances* in *Model Reliability* (eds. David Belsley and Edwin Kuh), pp. 1-16. Cambridge, MA: The MIT Press.

Gray, R. W. (1963). Onions revisited. *Journal of Farm Economics 45* (2), 273-276.

Hamilton, J. D. (2009). Causes and consequences of the oil shock of 2007–08. *Brookings Papers on Economic Activity*, (Spring), 215-261.

Hansen, L. P. and T. J. Sargent (1980). Formulating and estimating dynamic linear rational expectations models. *Journal of Economic Dynamics and Control 2*, 7-46.

Irwin, S. H., D. R. Sanders, and R. P. Merrin (2009). Devil or angel? the role of speculation in the recent commodity price boom (and bust). *Journal of Agricultural and Applied Economics 41* (2), 377-391.

Irwin, S. H. and D. R. Sanders (2010). The impact of index and swap funds on commodity futures Markets: Preliminary results. *OECD Food, Agriculture and Fisheries Working Papers* 27, OECD Publishing.

Ivanov, S. and J. H. Cho (2011). A study of the factors affecting price discovery in commodity markets. Proceedings of the *2011 Southwestern Finance Association Conference*, Available at URL: http://www.southwesternfinance.org/conf-2011/swfa2011_submission_24.pdf [accessed 25 June, 2011]

Johansen, S. and K. Juselius (1990). Maximum likelihood estimation and inference on cointegration - with applications to the demand for money. *Oxford Bulletin of Economics and Statistics 52* (2), 169-210.

Koutmos, G. and M. Tucker (1996). Temporal relationships and dynamic interactions between spot and futures stock markets. *The Journal of Futures Markets 16* (1), 55-69.

Ling, S. and M. McAleer (2003). Asymptotic theory for a vector ARMA-GARCH model. *Econometric Theory 19* (02), 280-310.

Masters, M. W. (2008). Testimony before the commodity futures trading commission. May 20, 2008. Accessed from URL: http://hsgac.senate.gov/public/_files/052008Masters.pdf. [accessed June 25, 2011]

Mattos, F. and P. Garcia (2006). Identifying price discovery and risk transfer in thinly traded markets: Evidence from Brazilian agricultural futures markets. *The Review of Futures Markets 14* (4), 469-480.

Mueller, W. F., B. W. Marion, M. H. Sial, and F. E. Geithman (1996, March). Cheese pricing: A study of the National Cheese Exchange. Technical report, Deparment of Agricultural Economics, University of Wisconsin-Madison, Madison, WI.

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica 59* (2), 347-370.

Newbery, D. M. (1987). When do futures destabilize spot prices? *International Economic Review 28* (2), 291-297.

Oellermann, C. M. and P. L. Farris (1985). Futures or cash: Which market leads live beef cattle prices? *The Journal of Futures Markets 5* (4), 529-538.

Peck, A. E. (1980). The role of economic analysis in futures market regulation. *American Journal of Agricultural Economics 62* (5), 1037-1043.

Phillips, P. C. B. and P. Perron (1988). Testing for a unit root in time series regression. *Biometrika 75* (2).

Powers, M. J. (1970). Does futures trading reduce price fluctuations in the cash markets? *The American Economic Review 60* (3), 460-464.

Rutledge, D. J. S. (1977). Estimation of hedging and speculative positions in futures markets - an alternative approach. *Food Research Institute Studies 16* (3), 205-211.

Said, S. E. and D. A. Dickey (1984). Testing for unit roots in autoregressive-moving average models of unknown order. *Biometrika 71* (3), 599-607.

Sanders, D. R., S. H. Irwin, and R. P. Merrin (2010). The adequacy of speculation in agricultural futures markets: Too much of a good thing? *Applied Economic Perspectives and Policy 32* (1), 77-94.

Schalla, S. (2011). Control the emotions and impulsive decisions that milk market volatility can produce. Available at URL: http://www.agweb.com/livestock/dairy/blog/Know_Your_Market_281/?Year=2011&Month=6. [accessed 25 June, 2011]

Schroeder, T. C. and B. K. Goodwin (1991). Price discovery and cointegration for live hogs. *The Journal of Futures Markets 11* (6), 685-696.

Streeter, D. H. and W. G. Tomek (1992). Variability in soybean futures prices: An integrated framework. *The Journal of Futures Markets 12* (6), 705-728.

Tang, K. and W. Xiong (2010). Index investment and financialization of commodities. *NBER Working Paper* No. 16835. Available from URL: http://www.nber.org/papers/w16385 [accessed 25 June, 2011]

Taylor, G. S. and R. M. Leuthold (1974). The influence of futures trading on cash cattle price variations. *Food Research Institute Studies 13* (1), 29-35.

Thraen, C. S. (1999). A note: The CSCE cheddar cheese cash and futures price long-term equilibrium relationship revisited. *The Journal of Futures Markets 19*(2), 233-244.

Wang, D. and W. G. Tomek (2007). Commodity prices and unit root tests. *American Journal of Agricultural Economics 89* (4), 873-889.

Wold, H. (1954). *A Study in the Analysis of Stationary Time Series*. Almqvist & Wiksell.

Turnovsky, S. J. (1983). The determination of spot and futures prices with storable commodities. *Econometrica 51* (5), 1363-1387.

Tse, Y. (1999). Price discovery and volatility spillovers in the DJIA index and futures markets. *The Journal of Futures Markets 19* (8), 911-930.

Yang, J., D. A. Bessler, and D. J. Leatham (2001). Asset storability and price discovery in commodity futures markets: A new look. *The Journal of Futures Markets 21* (3), 279-300.

Witherspoon, J. T. (1993). How price discovery by futures impacts the cash market. *The Journal of Futures Markets 13* (5), 469-496.

Working, H. (1960, Feb). Price effects of futures trading. *Food Research Institute Studies 1* (1), 3-31.

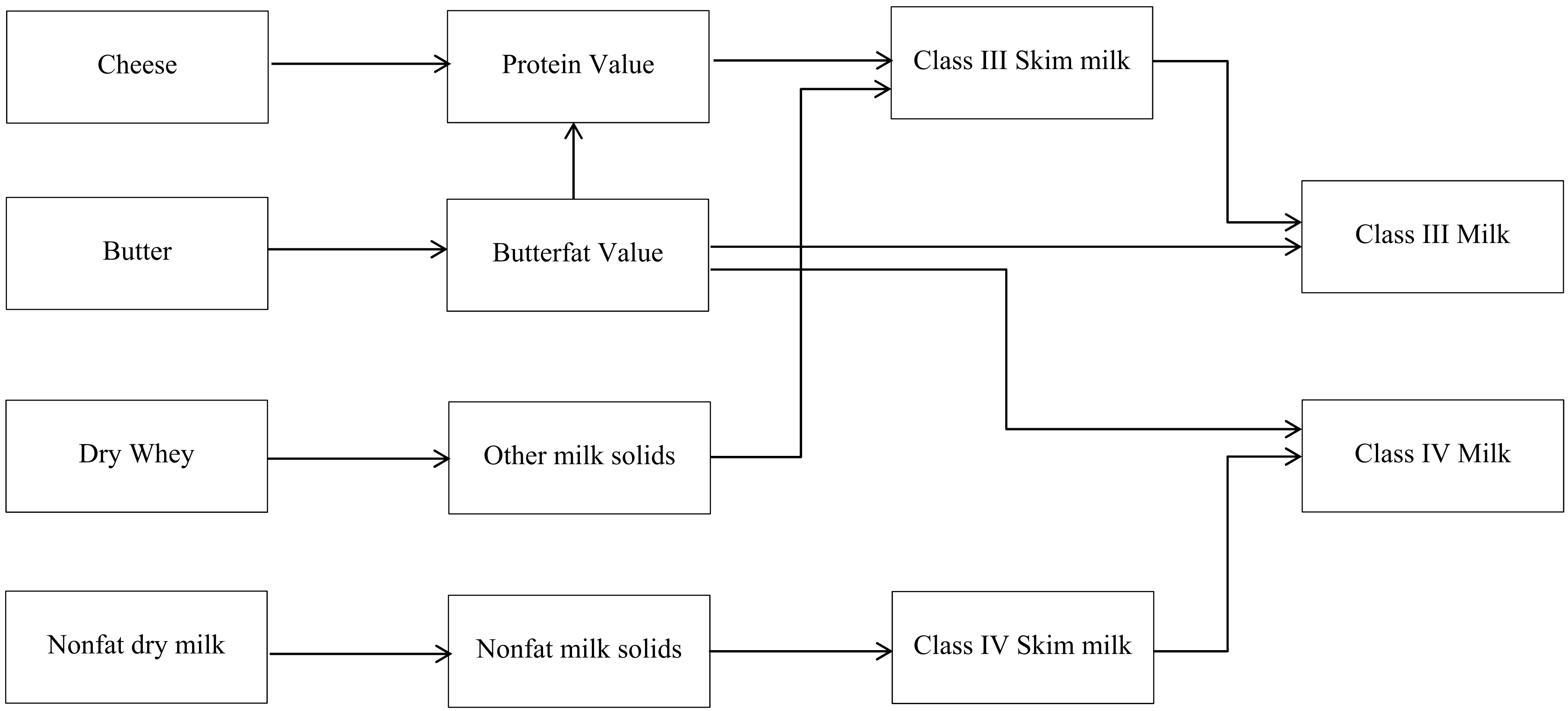
Working, H. (1960, May). Speculation on hedging markets. *Food Research Institute Studies 1* (2), 185-220.

Working, H. (1962). New concepts concerning futures markets and prices. *The American Economic Review 52* (3), 431-459.

Zapata, H. O. and T. R. Fortenbery (1996). Stochastic interest rates and price discovery in selected commodity markets. *Review of Agricultural Economics 18*, 643-654.

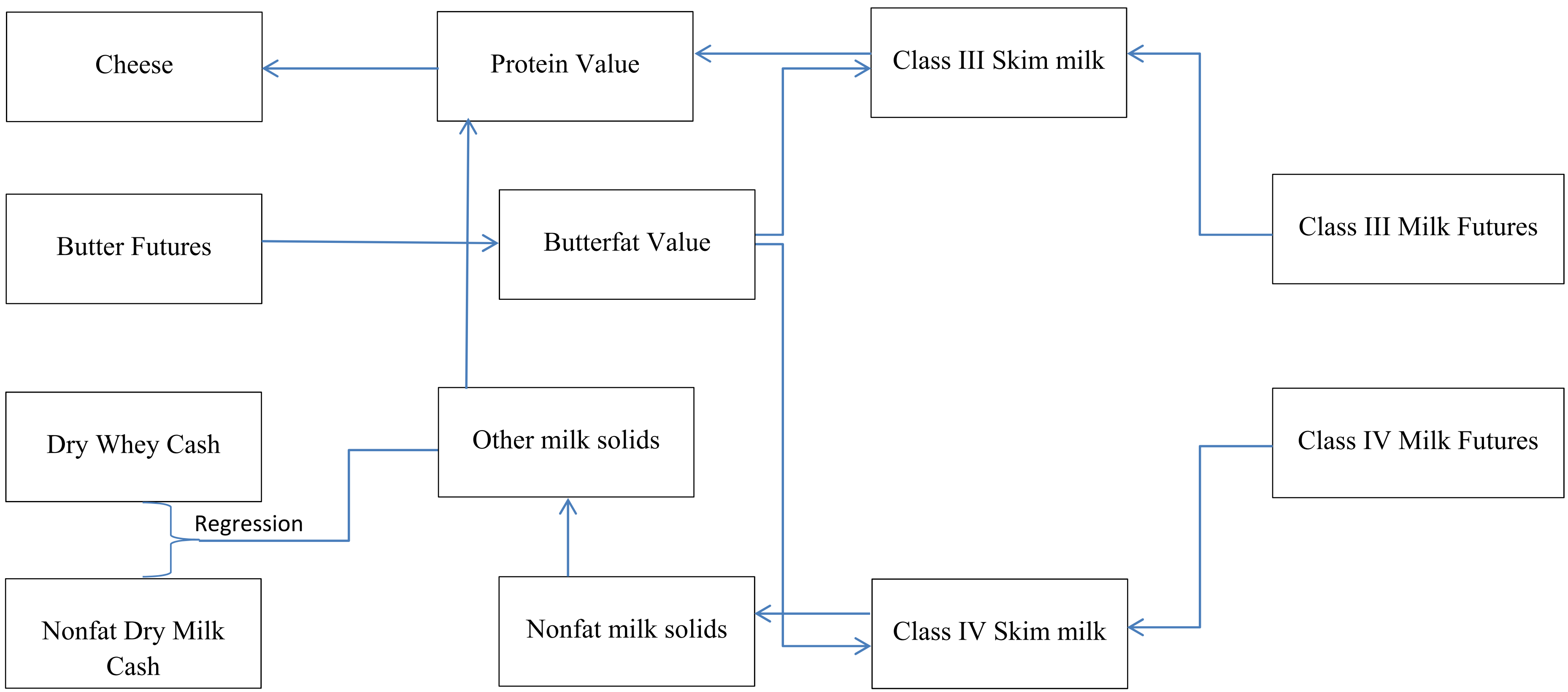
Zhong, M., A. Darrat, and R. Otero (2004). Price discovery and volatility spillovers in index futures markets: Some evidence from Mexico. *Journal of Banking & Finance 28* (12), 3037-3054.

Figure 3.1. Flowchart Diagram of Classified Milk Pricing in Federal Milk Marketing Orders



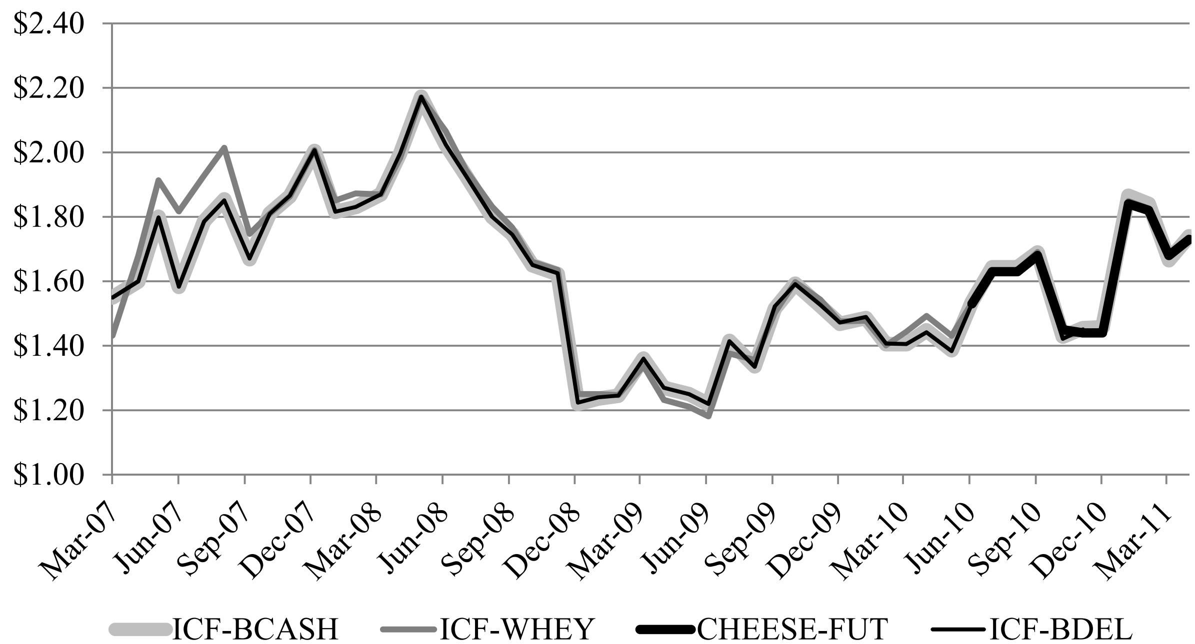
Note: Surveyed national average cash price of butter is used to infer value of butterfat. Protein value is calculated using survey price of cheese and imputed value of butterfat. Other milk solids are imputed from surveyed cash price for dry whey, and nonfat milk solids are imputed from surveyed cash nonfat dry milk price. Class IV skim milk price is obtained from imputed nonfat milk solids. Class IV milk price is obtained from imputed butterfat value and class IV skim milk value. Imputed other milk solids and imputed protein value are used to calculate class III skim milk price. Finally, class III skim milk price together with imputed butterfat value give us Class III milk price.

Figure 3.2. Calculating Implied Cheese Futures Price



Note: Butter futures are used to infer butterfat value. Class IV milk futures and implied butterfat values are used to infer implied values of nonfat milk solids. Class III milk futures and implied butterfat value are used to obtain the implied Class III skim milk price. Implied nonfat milk solids price and projection coefficients obtained using past nonfat dry milk to dry whey cash price ratios are used to obtain fitted other milk solids. Correlations between nonfat milk solids and other milk solids is 0.77. Combining Class III skim milk and other milk solids give us protein price. Finally, butterfat value and protein value give a best guess of implied futures cheese prices.

Figure 3.3. Implied vs. Observed Cheese Futures



Note: Three different methods were used to calculate cheese futures price implied from other dairy futures prices.

1. ICF-BDEL (7/11/2000-9/19/2005) uses deliverable butter, and Class III and IV milk futures prices as well as forecasted ratio of nonfat dry milk to dry whey.
2. ICF-BCASH (9/20/2005-3/20/2007) is similar to ICF-BDEL, but cash-settled butter is used instead of deliverable butter.
3. ICF-WHEY (3/21/2007-8/3/2010) uses Class III, cash-settled butter and dry whey futures prices.
4. CHEESE-FUT (8/4/2010-) are observed cash-settled cheese futures prices.

Figure 3.4. Average Absolute Value of Cheese Cash-Futures Spread, as a Function of Time to Maturity.

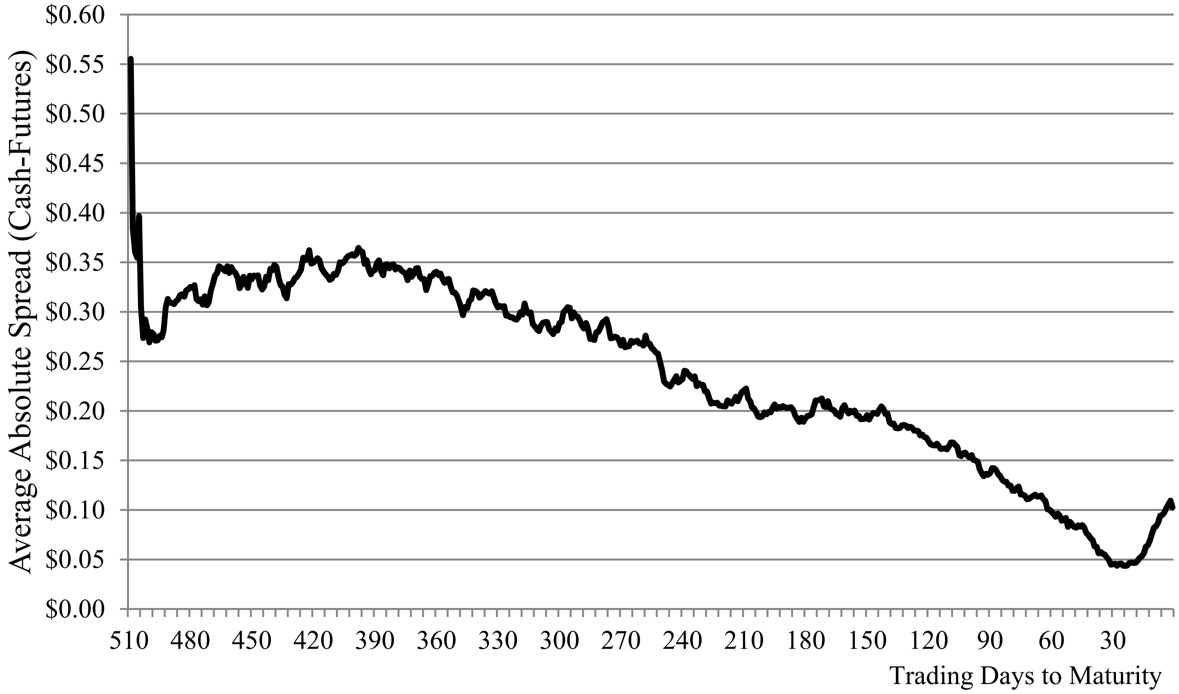


Figure 3.5. Percentage Net Long Positions for all Trader Groups

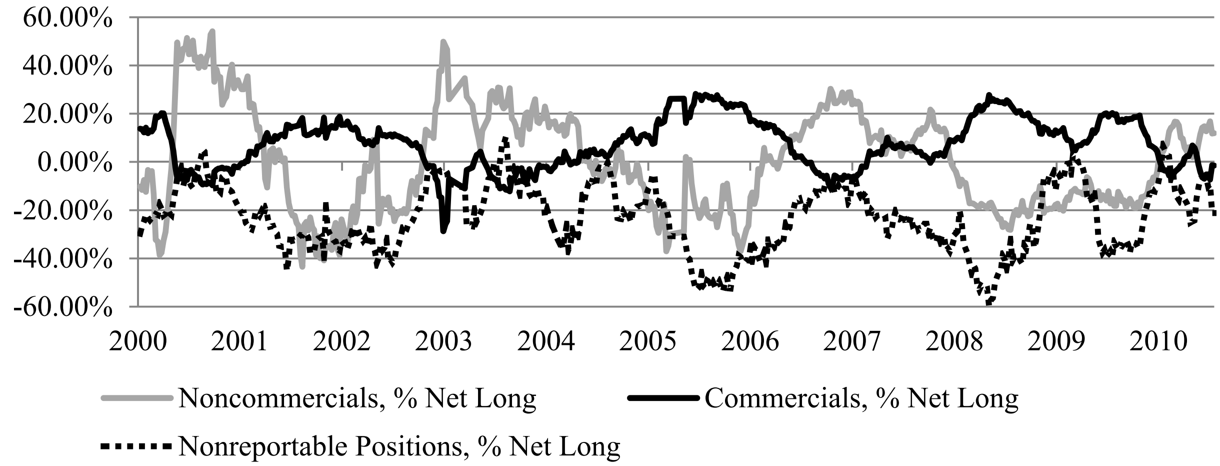


Table 3.1. Unit Root Tests

		Rollover: at expiry				Rollover: 3 days to expiry	
	Dickey-Fuller	ADF (lags)		Phillips Perron	Reg. t-stat.	Phillips-Perron	Reg. t-stat.
Daily							
Cash – 40# Blocks		-3.67***	(4)	-3.35**	-3.67***	-3.35***	-3.67***
Cash - Barrels		-3.16**	(19)	-3.58***	-3.16**	-3.58***	-3.16***
Cash – 40# Blocks (B/O)		-3.81***	(8)	-3.34**	-3.81***	-3.34**	-3.81***
Cash – Barrels (B/O)		-3.63***	(8)	-3.12**	-3.63***	-3.11**	-3.63**
Futures: 1st nearby	-2.87**	-2.87**	(0)	-2.87**	0.15	-2.88**	0.15
Futures: 2nd nearby	-2.73*	-2.92**	(1)	-2.83*	0.65	-2.83*	0.65
Futures: 3rd nearby	-2.67*	-2.55	(1)	-2.58*	-0.05	-2.58*	-0.05
Futures: 4th nearby	-2.58*	-2.23	(8)	-2.41	-0.10	-2.40*	-0.10
Futures: 5th nearby	-2.35	-2.32	(19)	-2.22	0.03	-2.22	0.04
Diff: Cash – Futures 2nd n.	-11.36***	-11.36***	(0)	-11.36***		-11.36***	
Diff: Cash – Futures 5nd n.	-4.87***	-5.50***	(2)	-5.19***		-5.19***	
Sales-only							
Cash – 40# Blocks		-3.42**	(28)	-3.31**	-3.42**	-3.44***	-3.42**
Cash – Barrels		-3.14**	(0)	-3.15**	-3.14**	-3.15**	-3.14**
Cash – 40# Blocks (B/O)		-3.77***	(6)	-3.42**	-3.77***	-3.42**	-3.77***
Cash – Barrels (B/O)		-3.62***	(7)	-3.28**	-3.62***	-3.28**	-3.62***
Futures: 1st nearby	-2.41	-2.53	(2)	-2.45	0.91	-2.88**	0.19
Futures: 2nd nearby	-2.75*	-3.10**	(15)	-3.02**	0.75	-3.05**	0.57
Futures: 3rd nearby	-2.68*	-2.63*	(5)	-2.72*	-0.04	-2.81*	-0.14
Futures: 4th nearby	-2.54	-2.54	(2)	-2.54	-0.28	-2.49	-0.12
Futures: 5th nearby	-2.26	-2.26	(7)	-2.28	-0.38	-2.31	-0.10
Diff: Cash – Futures 2nd n.	-13.36***	-7.35***	(15)	-13.79***		-11.65***	
Diff: Cash – Futures 5nd n.	-4.29***	-3.70***	(23)	-4.54***		-4.67***	
Weekly							
Cash – 40# Blocks		-3.69***	(1)	-3.49***	-3.69***	-3.49***	-3.69***
Cash - Barrels		-2.67*	(10)	-3.23**	-2.67*	-3.23**	-2.67*
Cash – 40# Blocks (B/O)		-3.69***	(1)	-3.32**	-3.69***	-3.32**	-3.69***
Cash – Barrels (B/O)		-3.49***	(4)	-3.28**	-3.49***	-3.28**	-3.49***
Futures: 1st nearby	-2.31	-3.24**	(4)	-2.53	0.86	-2.93**	1.11
Futures: 2nd nearby	-2.68*	-2.95**	(1)	-2.83*	0.59	-3.02**	0.35
Futures: 3rd nearby	-2.70*	-2.58*	(1)	-2.66*	0.06	-2.73*	0.01
Futures: 4th nearby	-2.60*	-3.39**	(8)	-2.81*	0.13	-2.78*	-0.20
Futures: 5th nearby	-2.32	-3.24**	(8)	-2.64*	-0.17	-2.38	-0.38
Diff: Cash – Futures 2nd n.	-11.36***	-9.69***	(4)	-11.53***		-11.39***	
Diff: Cash – Futures 5nd n.	-5.35***	-5.66***	(1)	-5.54***		-5.37***	
COT: Noncom. % OI long	-3.91***	-3.15**	(3)	-3.69***	-3.15**	-3.69***	-3.69***
COT: Noncom. % OI short	-3.56***	-3.79***	(4)	-3.63***	-3.79***	-3.63***	-3.63***
COT: Noncom. % OI net l.	-3.06**	-3.05**	(4)	-3.02**	-3.05**	-3.02**	-3.02**

Note: Reported numbers are t-statistics used for unit-root tests. In all tests, intercept is included in the estimated regression. Unit-root test are run using inbuilt commands of RATS 8.01 software. Null hypothesis in all tests is that unit root is present. Significance at 10%, 5% and 1% is indicated with one, two, and three stars respectively. For augmented Dickey-Fuller (ADF) and Phillips-Perron tests optimal lag length is determined using AIC criteria. For cash prices, deseasonalized series is used in tests. “Reg t-stat” is the t-statistic from regression equation that should be used in ADF test.

Table 3.2. Simple Error-Correction Model: Comparing Results Across Data Frequency and Cash Series Type

	Daily		Sales-only		Weekly	
	Trans.	Bid/Offer	Trans.	Bid/Offer	Trans.	Bid/Offer
Dep. Variable: Δc_t						
Δc_{t-1}	0.023	0.152***	-0.005	0.094*	0.054	0.095
Δc_{t-2}	0.108**	0.041	0.044	0.066*	0.09*	-0.053
Δc_{t-3}	0.037**	0.046	0.034	0.011	-0.001	-0.066
Δc_{t-4}	0.056**	0.059**	0.057**	0.085**	-0.016	0.031
Δf_{t-1}	0.202***	0.178***	0.334***	0.164***	0.223***	0.223***
Δf_{t-2}	0.109***	0.120***	0.173***	0.089	0.054	0.176**
Δf_{t-3}	0.061*	0.032	0.092***	0.049	-0.066	0.048
Δf_{t-4}	0.022	0.031	0.035	-0.014	-0.028	0.044
$c_{t-1} - f_{t-1}^{2nd}$	-0.082***	-0.079***	-0.132***	-0.118***	-0.365***	-0.369***
Dep. Variable: Δf_t						
Δc_{t-1}	0.016	0.053	0.018	0.022	-0.013	0.080
Δc_{t-2}	0.016	0.035	0.035	0.059**	0.017	-0.044
Δc_{t-3}	0.028	0.013	0.018	0.025	-0.032	-0.009
Δc_{t-4}	-0.001	0.029	0.023	0.027	-0.046	-0.030
Δf_{t-1}	0.103***	0.077**	0.041	0.028	0.068	-0.003
Δf_{t-2}	-0.010	-0.036	-0.014	-0.046	0.117**	0.124*
Δf_{t-3}	-0.011	-0.029	0.029	0.015	0.045	0.050
Δf_{t-4}	0.029	0.004	-0.009	-0.021	0.018	0.014
$c_{t-1} - f_{t-1}^{2nd}$	0.002	0.025***	0.009	0.019	0.007	0.040

Table 3.3. Simple Error-Correction Model: Comparing Results Across Data Frequency and Cash Series Type – Log-Prices

	Daily		Sales-only		Weekly	
	Trans.	Bid/Offer	Trans.	Bid/Offer	Trans.	Bid/Offer
Dep. Variable: Δc_t						
Δc_{t-1}	0.036	0.160***	0.008	0.114***	0.036	0.098
Δc_{t-2}	0.086**	0.043	0.053**	0.080***	0.075*	-0.039
Δc_{t-3}	0.035**	0.045	0.044	0.016	0.003	-0.045
Δc_{t-4}	0.058***	0.055**	0.044**	0.074**	-0.011	0.025
Δf_{t-1}	0.206***	0.173***	0.309***	0.157***	0.259***	0.226***
Δf_{t-2}	0.110***	0.119***	0.164***	0.068	0.084	0.164**
Δf_{t-3}	0.059**	0.038	0.086**	0.046	-0.059	0.029
Δf_{t-4}	0.053	0.043	0.045	0.000	-0.004	0.063
$c_{t-1} - f_{t-1}^{2nd}$	-0.081***	-0.075***	-0.129***	-0.111***	-0.351***	-0.360***
Dep. Variable: Δf_t						
Δc_{t-1}	0.013	0.054*	0.014	0.032	0.000	0.092*
Δc_{t-2}	0.021	0.024	0.051**	0.062***	0.008	-0.038
Δc_{t-3}	0.029*	0.015	0.021	0.027	-0.022	-0.003
Δc_{t-4}	-0.006	0.026	0.013	0.021	-0.039	-0.024
Δf_{t-1}	0.098***	0.070**	0.048	0.025	0.059	-0.017
Δf_{t-2}	0.000	-0.021	-0.024	-0.055	0.113**	0.108*
Δf_{t-3}	-0.019	-0.034	0.029	0.015	0.052	0.050
Δf_{t-4}	0.041*	0.017	-0.004	-0.014	0.033	0.029
$c_{t-1} - f_{t-1}^{2nd}$	0.004	0.025***	0.010	0.023*	0.014	0.050

Table 3.4. Volatility Spillovers

	Price levels	
	Weekly	
	Trans.	Bid/Offer
Mean Model		
Dep. Variable: Δc_t		
$c_{t-1} - f_{t-1}^{2nd}$	-0.349***	-0.336***
Dep. Variable: Δf_t		
$c_{t-1} - f_{t-1}^{2nd}$	0.049**	0.057**
Variance Model:		
ARCH		
α_{11}	0.104*	-0.202*
α_{12}	-0.464***	-0.495***
α_{21}	-0.396***	0.142
α_{22}	0.572***	0.639***
GARCH		
γ_{11}	0.977***	0.851***
γ_{12}	0.300***	-0.010
γ_{21}	-0.059	0.182***
γ_{22}	0.640***	0.929***
Likelihood		
Unrestricted	1877.1	1948.7
$f_t \overset{GRE}{\not\to} c_{t+1}$	1821.5***	1907.1***
$c_t \overset{GRE}{\not\to} f_{t+1}$	1849.5***	1909.9***
Average volatility spillover (% of c. var.)		
$f_t \to c_{t+1}$	-8.27%	15.72%
$c_t \to f_{t+1}$	27.07%	10.45%

Table 3.5. Evaluating Adequacy of Speculation in Cheese Futures Market: Selected Results from GARCH-MEX Model

				Model 1				Model 2				
Variations on Working's T				Unrestricted corr.		Restricted corr.		Unrestricted corr.		Restricted corr.		LR test§
	Avg.	Min	Max	Log-like.	Coef.	Log-like.	Coeff.	Log-like.	Coeff.	Log-like.	Coeff.	
Proportional	1.03	1.00	1.08	2293.0†		2309.6	-7.76***	2306.0	-2.72***	2302.2	0.26**	7.60***
							(0.44)		(0.24)		(0.10)	
Upper bound	1.16	1.06	1.29	2294.0†		2304.7	-2.33***	2306.7	-0.61***	2301.5	0.28**	10.40***
							(0.12)		(0.20)		(0.13)	
Lower bound	1.02	1.00	1.09	2295.0†		2306.5†		2302.5	-1.05***	2301.6	0.30***	1.80
									(0.06)		(0.11)	
Industry	1.03	1.00	1.11	2294.0†		2306.5	-8.66***	2308.9	-2.66***	2297.0†		
							(0.67)		(0.07)			
Dynamic	1.10	1.00	1.26	2301.0†		2299.0	-3.16***	2307.6	-1.04***	2298.9†		
							(1.06)		(0.18)			
Proportional (S)	1.12	1.03	1.22	2295.2†		2295.5	-1.19***	2314.6	-2.89***	2305.7	0.24**	17.80***
							(0.38)		(0.75)		(0.10)	
Dynamic (S)	1.21	1.03	1.41	2297.8†		2297.4	-1.48***	2299.6†		2295.2†		
							(0.63)					
Upper bound (S)	1.28	1.12	1.45	2307.4†	-1.13**	2295.7	-1.01***	2303.9	-0.40***	2298.7†		
					(0.56)		(0.19)		(0.06)			
Lower bound (S)	1.12	1.03	1.23	2296.8	-0.52	2303.2	-0.43	2313.9	-1.95*	2305.6	0.28***	16.60***
					(0.37)		(0.69)		(1.05)		(0.05)	

Table 3.5. Evaluating Adequacy of Speculation in Cheese Futures Market: Selected Results from GARCH-MEX Model (continued)

				Model 1				Model 2				
Percentage of OI				Unrestricted corr.		Restricted corr.		Unrestricted corr.		Restricted corr.		LR test§
	Avg.	Min	Max	Log-like.	Coef.	Log-like.	Coeff.	Log-like.	Coeff.	Log-like.	Coeff.	
Total	0.06	0.01	0.18	2306.0†		2298.5	-4.68*	2303.2	0.43	2300.54	1.72***	5.32**
							(2.75)		(0.84)		(0.56)	
Total (S)	0.21	0.09	0.32	2298.0†		2297.8	0.12	2314.0	-1.83	2298.6	0.67**	30.80***
							(0.95)		(1.21)		(0.34)	
Long	0.07	0.00	0.33	2300.7	-0.29	2296.5	2.94*	2308.3†		2302.9	1.39***	
					(1.71)		(1.60)				(0.41)	
Long (S)	0.20	0.07	0.46	2300.6†		2297.2	1.62*	2305.9	0.45*	2302.2	0.72**	7.40***
							(0.93)		(0.27)		(0.29)	
Short	0.07	0.00	0.17	2297.0†		2299.2	-3.83***	2304.6	-2.58***	2298.9	-0.01	11.40***
							(1.32)		(0.85)		(0.57)	
Short (S)	0.20	0.05	0.38	2308.2†		2305.7	-0.35	2305.5	-0.12	2299	-0.75*	13.00***
							(0.71)		(0.25)		(0.44)	

Notes: † Model does not converge. § Likelihood ratio test of Model 2 uncrestricted vs. restricted correlation. Critical values of Chi-square distribution with 1 d.f. are 2.706 (10%, *), 3.841 (5%, **), and 6.635 (1%, ***).