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Rodrigo L. S. Silva*, Clemens Verhoosel, Erik Quaeghebeur

Eindhoven University of Technology
5612 AZ Eindhoven
r.lima.de.souza.e.silva@tue.nl, c.v.verhoosel@tue.nl, e.quaeghebeur@tue.nl

ABSTRACT

We present an inverse analysis for the estimation and uncertainty quantification of a temperature dependent thermal conductivity. For that, we consider the one-dimensional problem of a slab made of steel with a temperature-dependent thermal conductivity, to which a constant heat flux is applied at both edges. After defining the mathematical formulation, we can numerically solve the direct problem and derive transient temperature values across the slab. A third-degree polynomial is used to model the temperature-dependence of the conductivity [1], so the estimation of this physical property is obtained through the estimation of the polynomial's coefficients. We use a Bayesian framework [2-4] to model the uncertainties involved, consisting of a likelihood and a prior. The likelihood models the temperature measurements and their uncertainties, and is chosen to be a normal distribution with mean at the solution of the direct problem and a standard deviation characterizing the errors. The prior models our pre-existing knowledge about the coefficients that describe the thermal conductivity. To gain insights about the impact of using different priors, we investigate both uniform and normal distributions. The prior and likelihood are combined into a posterior, which describes the coefficients for a given set of temperature measurements. This posterior cannot be computed analytically, and therefore we solve the inverse problem with the Metropolis–Hastings algorithm [3, 4], a Markov chain Monte Carlo method. Results showed that our approach could deal with all uncertainties involved, and not only provided an estimation of the thermal conductivity curve but also delivered uncertainty quantification using credible intervals. It can furthermore be modified to estimate thermal conductivity values instead of coefficients, allowing for a more physical formulation of the prior.

References

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