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Abstract: In the literature, new types of neutrosophic sets have been introduced in the meantime by the growing neutrosophic community. We present a few.

Keywords: Pythagorean Neutrosophic Set, Fermatean Neutrosophic Set, Generalized Fermatean Neutrosophic Set, n-power Neutrosophic Set, Cubic Spherical Neutrosophic Set, Spherical Neutrosophic Set, n-HyperSpherical Neutrosophic Set, Refined n-HyperSpherical Neutrosophic Set.

0. Introduction

We present several of the new types of neutrosophic sets have been introduced in the meantime by the growing neutrosophic community.

Let all $T, I, F \in [0,1]$, that represent the degree of Truth, degree of Indeterminacy, and degree of Falsehood respectively. Then:

1. The **Pythagorean Neutrosophic Set**:

$$0 \leq T^2 + I^2 + F^2 \leq 2$$

In a more general way, we may take any number strictly less than 3 as superior limit of the above sum:

$$0 \leq T^2 + I^2 + F^2 \leq b < 3$$

For example,

$$0 \leq T^2 + I^2 + F^2 \leq 1.7$$

or

$$0 \leq T^2 + I^2 + F^2 \leq 2.6$$

Etc.

2. The **Fermatean Neutrosophic Set** [2]

$$0 \leq T^3 + I^3 + F^3 \leq 2$$

where it is considered that $0 \leq T^3 + F^3 \leq 1$ and of course $0 \leq I^3 \leq 1$

3. The **Generalized Fermatean Neutrosophic Set**

In a more general way, we may take any number strictly less than 3 as superior limit of the above sum:

$0 \le T^3 + I^3 + F^3 \le b < 3$

For examples:

$$0 \le T^3 + I^3 + F^3 \le 2.5$$

or

$$0 \le T^3 + I^3 + F^3 \le 1.8$$

etc.

4. The ***n*-Power Neutrosophic Set**, $n > 1$, more general than the previous ones:

$$0 \le T^n + I^n + F^n \le b < 3$$

The varying parameters *n > 1* and *0 < b < 3* depend on each specific application.

For example:

$$0 \le T^7 + I^7 + F^7 \le 2.2$$

5. The **Cubic Spherical Neutrosophic Set** from the paper [1] is totally different.

Let $S = \{< a_1(T_1, I_1, F_1) >, < a_2(T_2, I_1, F_2) >, ..., < a_n(T_n, I_n, F_n) >\}$ be a single-valued neutrosophic set, where $a_1, a_2, ..., a_n \in U$ that is a universe of discourse.

One makes the averages of T's, then of I's, and of F's to get the center of the sphere.

$$T_a = \frac{1}{n}\sum_{i=1}^{n} T_i, I_a = \frac{1}{n}\sum_{i=1}^{n} I_i, F_a = \frac{1}{n}\sum_{i=1}^{n} F_i$$

The center (C) of the sphere of this neutrosophic set is:

$$C = (T_a, I_a, F_a)$$

Then the sphere's radius (*r*) is computing as:

$$r = \min\{\max\sqrt{[(T_1 - T_a)^2 + (I_1 - I_a)^2 + (F_1 - F_a)^2] + [(T_2 - T_a)^2 + (I_2 - I_a)^2 + (F_2 - F_a)^2] + ... + [(T_n - T_a)^2 + (I_n - I_a)^2 + (F_n - F_a)^2]}, 1\}$$

meaning the biggest distance, that does not exceed *1*, from each element (T_i, I_i, F_i) to the sphere's center (T_a, I_a, F_a).

6. **Single-Valued Spherical Neutrosophic Set**

Spherical Neutrosophic Set (SNS) was introduced by Smarandache [3] in 2017.

A Single-Valued Spherical Neutrosophic Set (SNS), of the universe of discourse U, is defined as follows:

$$A_{SNS} = \{<x, T_A(x), I_A(x), F_A(x)> \mid x \in U\},$$

where, for all x ∈ U, the functions $T_A(x)$, $I_A(x)$, $F_A(x) : U \to [0, \sqrt{3}]$, represent the degree of membership (truth), the degree of indeterminacy, and degree on nonmembership (falsity) respectively, that satisfy the conditions:

$$0 \le T_A^2(x) + I_A^2(x) + F_A^2(x) \le 3.$$

The Spherical Neutrosophic Set is a generalization of Spherical Fuzzy Set, because we may restrain the SNS's components to the unit interval $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$, and the sum of the squared components to 1, i.e. $0 \le T_A^2(x) + I_A^2(x) + F_A^2(x) \le 1$.

Further on, if replacing $I_A(x) = 0$ into the Spherical Fuzzy Set, we obtain as particular case the Pythagorean Fuzzy Set.

7. Single-Valued n-HyperSpherical Neutrosophic Set (n-HSNS)

The Single-Valued n-HyperSpherical Neutrosophic Set (n-HSNS) [3], which is a generalization of the Spherical Neutrosophic Set and of n-HyperSpherical Fuzzy Set, of the universe of discourse U, for $n \geq 1$, is defined as follows:

$$A_{n\text{-}HNS} = \{<x, T_A(x), I_A(x), F_A(x)> \mid x \in U\},$$

where, for all $x \in U$, the functions $T_A(x), I_A(x), F_A(x) : U \to [0, \sqrt[n]{3}]$, represent the degree of membership (truth), the degree of indeterminacy, and degree on nonmembership (falsity) respectively, that satisfy the conditions:

$$0 \leq T_A^n(x) + I_A^n(x) + F_A^n(x) \leq 3.$$

8. Single-Valued Refined n-HyperSpherical Neutrosophic Set (R-n-HSNS)

The Single-Valued Refined n-HyperSpherical Neutrosophic Set (R-n-HSNS) [3], which is a generalization of the n-HyperSpherical Neutrosophic Set and of Refined n-HyperSpherical Fuzzy Set.

On the universe of discourse U, for $n \geq 1$, we define it as:

$$A_{R-n-HSNS} = \{x(T_A^1(x), T_A^2(x), ..., T_A^p(x); I_A^1(x), I_A^2(x), ..., I_A^r(x);$$
$$F_A^1(x), F_A^2(x), ..., F_A^s(x)), p + r + s \geq 4, x \in U\},$$

where *p, r, s* are nonzero positive integers, and for all $x \in U$, the functions $T_A^1(x), T_A^2(x), ..., T_A^p(x), I_A^1(x), I_A^2(x), ..., I_A^r(x), F_A^1(x), F_A^2(x), ..., F_A^s(x) : U \to [0, m^{1/n}]$, represent the degrees of sub-membership (sub-truth) of types *1, 2, …, p,* the degrees of sub-indeterminacy of types *1, 2, …, r,* and degrees on sub-nonmembership (sub-falsity) of types *1, 2, …, s* respectively, that satisfy the condition:

$$0 \leq \sum_1^p (T_A^j)^n + \sum_1^r (I_A^k)^n + \sum_1^s (F_A^l)^n \leq m, \text{ where } p + r + s = m.$$

9. Conclusion

Many types of extended or hybrid neutrosophic sets have been introduced by the neutrosophic community. We revealed only a few of them. Certainly, more in the future will follow.

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