THE INCORPORATION OF *GEOGEBRA* AS A VISUALISATION TOOL TO TEACH CALCULUS IN TEACHER EDUCATION INSTITUTIONS: THE ZAMBIAN CASE

A thesis submitted in fulfilment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

of

RHODES UNIVERSITY, MAKHANDA, SOUTH AFRICA

(Education Department)

by

LEMMY KANGWA

January 2022

ACKNOWLEDGEMENTS

I extend my heartfelt gratitude to the following people without whose contribution I could not have completed this study:

My supervisor Professor Marc Schafer for his unwavering professional support, guidance and dedication to work. Prof, your advice and words of encouragement, support, both academic and materially, played a significant role in helping me to persevere with this journey. I had reached a point where I felt I could not continue, but your words gave me courage and confidence to continue, I am grateful that I had the rare opportunity to study under your guidance.

Dr. Clemence Chikiwa for his encouragement and guidance.

Ms. Zipho Mqingwana for her diligence in administrative issues in the mathematics education department.

The SARChi Chair in Mathematics Education for the generous financial support.

Mr Robert Kraft for hosting me during my stay in Makhanda.

Members of staff of Rhodes Education Department for their professional support and my course mates for sharing valuable information and company.

All individuals who sacrificed their valuable time to enable me collect data for the study.

Management of Chalimbana University who granted me leave from work to enable me concentrate on my study.

Ms. Jean Schafer for proofreading my work.

Mr. Edgar Zulu for guidance on type-setting.

My wife Francisca, and children, Lukonde and Malama, for bearing my absence when I was away on study leave.

ABSTRACT

This qualitative case study investigated teacher educators' (lecturers) use of the dynamic mathematics software, *GeoGebra*, to teach calculus in three teacher education institutions (TEIs) in Zambia. Visualisation, a key characteristic of *GeoGebra*, is increasingly gaining recognition of playing a critical role in mathematics teaching and learning, especially in problem solving tasks. It is considered a powerful didactical tool for students to construct mental and physical representations that can enhance conceptual understanding of mathematics. *GeoGebra* is a visualisation tool that can be used for problem-oriented teaching and foster mathematical experiments and discoveries. *GeoGebra*'s inherent visualisation characteristics align well with the teaching of calculus, the mathematical domain of this study. The study (whose research methodology was underpinned by the interpretive paradigm) was undertaken with a broader goal of designing and implementing *GeoGebra* applets and instructional materials on various calculus topics. The study is located within the "Teaching and Learning Mathematics with *GeoGebra* (TLMG) project" – a project that involves mathematics teachers and lecturers in Zambia. The case in this study is the six mathematics lecturers who co-designed and used *GeoGebra* applets to teach derivatives and integrals to pre-service mathematics teachers in TEIs. The unit of analysis therefore is the six lecturers' use of *GeoGebra* as a visualisation tool to teach calculus to enhance conceptual understanding, their perceptions and experiences of using *GeoGebra* and the enabling and constraining factors of using *GeoGebra* to teach and learn mathematics. The data for the study were video recordings of observations and interviews of lecturers. The data was analysed thematically and was guided and informed by an analytical framework adopted from the theory of constructivism – the umbrella theoretical framework of this study – and the models of Technological Pedagogical Content Knowledge (TPACK), and the Technology Acceptance Model (TAM). A detailed analysis of the lecturers' interactions with the applets enabled me to gain insights into the participants' experiences and perceptions of *GeoGebra* applets in the teaching and learning process. The findings of the study revealed that the visualisation characteristics of *GeoGebra* generally enhanced the conceptual understanding of calculus. It also revealed that adequate training, coupled with sufficient knowledge of the subject matter in calculus, were necessary for lecturers to use *GeoGebra* effectively, and that the lack of resources and expertise were major hindrances in the use of *GeoGebra* to teach mathematics in TEIs. It also revealed that there is a need to equip *GeoGebra* with other features that would make it more versatile, and suggested a teaching approach that would complement the use of conventional methods and *GeoGebra* to provide a link between abstract and concrete concepts of calculus.

DEDICATION

This thesis is dedicated to my late parents, Mr. Vitaliano Socks Kangwa and Mrs. Joanna Mulenga Mwansa Kangwa, and to our late son, Shoka Kangwa

DECLARATION OF ORIGINALITY

I, Lemmy Kangwa (Student number 613K6284), declare that this doctoral thesis entitled: "The Incorporation of *GeoGebra* as a visualisation tool to teach calculus in teacher education institutions: The Zambian case", is my own work written in my own words. Where I have drawn on the words or ideas of others, these have been fully acknowledged and referenced in the manner required by the Rhodes University Department of Education Guide to referencing.

Lemmy Kangwa (Signature) January (2022)

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LIST OF ABBREVIATIONS

AIDs	Acquired Immune Deficiency Syndrome
BA Ed	Bachelor of Arts with Education
BERA	British Educational Research Association
BYOD	Bring your own Device
BYOT	Bring your own Technology
CAS	Computer Algebra System
CDC	Curriculum Development Centre
CK	Content Knowledge ase - 2019
CPD	Continuing Professional Development
DGM	Dynamic Geometric Software
DMS	Dynamic Mathematics Software
ECZ	Examination Council of Zambia
ECZ	Examination Council of Zambia
EFA	Education for All
FGD	Focus Group Discussion
GLIP	The GeoGebra Literacy Initiative Programme
HDEC	Higher Degrees Ethics Committee
HEIs	Higher Education Institutions
HIV	Human Immunodeficiency Virus
ICT	Information and Communication Technology
JICA	Japanese International Cooperation Agency

MCT	Ministry of Communications and Transport
MDGs	Millennium Development Goals
MoE	Ministry of Education
NCTM	National Council of Teachers of Mathematics
NIF	National Implementation Framework
OLPC	One Laptop Per Child
PCK	Pedagogical Content Knowledge
PK	Pedagogical Knowledge
PTMT	Preparing to Teach Mathematics with Technology
RUHDEC	Rhodes University Higher Degrees Ethics Committee
SMASTE	Strengthening of Mathematics, Science and Technology Education
TAM	Technology Acceptance Model
TCK	Technological Content Knowledge
TEIs	Teacher Education Institutions
TK	Technology Knowledge
TLMG	Teaching and Learning Mathematics with *GeoGebra*
TLMG	Teaching and Learning Mathematics with GeoGebra
TPACK	Technological Pedagogical Content Knowledge
TPK	Technological Pedagogical Knowledge
TRA	Theory of Reasoned Action
UNESCO	United Nations Educational, Scientific and Cultural Organisation
UNZA	University of Zambia
ZAME	Zambian Association of Mathematics Education

ZAMSTEP Zambia Mathematics and Science Teacher Education Project

ZECF Zambia Education Curriculum Framework

CHAPTER 1: INTRODUCTION

The purpose of this introductory chapter is to present the contextual background and goals of the study, whose focus is the incorporation of *GeoGebra* as a visualisation tool to teach calculus in Teacher Education Institutions (TEIs) in Zambia. The chapter briefly introduces the theoretical underpinnings, the significance of the study, and the methodology. In conclusion, an outline of each chapter is presented.

1.1 INTRODUCTION TO THE RESEARCH

The rapid increase in technological advancement has led to profound changes in almost all aspects of life globally and has attracted much debate on education practice. This has resulted in important implications in the teaching and learning process, particularly in mathematics – the focus of my study. Consequently, several education reforms (Tondeur et al., 2017a) have advocated for the use of modern technology in classrooms, including the mathematics classroom, under the premise that it has the potential to modernise teaching approaches, make them more relevant and yield better mathematics results. In mathematics education, the specific focus is on the shift from teaching that promotes rote, algorithmic drilling and of facts, to problem solving, reflective thinking and reasoning; from pencil and paper computation to model building and conceptual learning; and from static and verbal mathematics to evolving mathematics that is highly dynamic, visual, and interactive (Hegedus & Kaput, 2004). Reputable mathematics education organisations have acknowledged the critical role of technology in mathematics with the National Council of mathematics Teachers (NCTM) in the United States of America indicating its "technology principle" as one of the six principles for high-quality mathematics instruction: "[t]echnology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000, p. 11). These developments have resulted in a notably increased use of technological tools in learning institutions. Despite the steady increase in the use of technology, Inan and Lowther (2010) and Berrett (2012a) maintain that achieving technology integration to desirable levels is still a complex process in educational change. This is evidenced by the use of technology in schools which is often uncoordinated, and in most instances, inadequate (Kim & Md-Ali, 2017). Wanjala (2016) and Kozma and Andersen (2002) concur with this view and elaborate that despite many countries having revised their education curricula to include Information and Communication Technology (ICT), its integration, and in particular, the teaching and learning of mathematics remains a major challenge and is yet to achieve the anticipated goals. Žilinskiene and Demirbilek (2015) observe that "even though teachers have access to computers and appropriate software is available both in schools and at home, technology is rarely integrated substantially into everyday teaching" (p. 139). This study specifically investigates the incorporation and use of a dynamic mathematic software, *GeoGebra*, as a visualisation tool, to teach Calculus in TEIs in Zambia to enhance conceptual understanding. It investigates lecturers' experiences and perceptions of incorporating *GeoGebra* as a visualisation tool to teach Calculus to undergraduates in TEIs. It also investigates the factors that enable and constrain lecturers' adoption of technology in teaching.

1.2 THE CONTEXT OF THE STUDY

The Zambian Ministry of Education (Zambia, [MoE], 1996) policy recognises the importance of mathematics and reiterates that "one of the purposes of teaching mathematics is to equip the learner with knowledge and skills to live effectively in this modern age of science and technology, and to enable the learner to contribute to the social and economic development of the nation, (Zambia, [MoE], 1996) . The emphasis on mathematics has been necessitated by students' low performance in this subject at almost all levels of education. For instance, in the Zambian school certificate mathematics examinations, the mean performance in the 2020 mathematics examinations at Grade 12 was 24.28 percent compared to 22.04 percent in 2019, showing a slight increase in performance, but still very much below the desired performance (Examination Council of Zambia, 2020) At Teacher Education level diploma examination results, the national assessment findings of 2020 conducted by the Examination Council of Zambian indicated that "across all subjects, the lowest mean performance in 2020 was recorded in mathematics content at 26.85 percent as compared to 39.50 percent in the 2019 survey", (Examination Council of Zambia, 2020) These national trends of mathematics performance at teacher education and secondary school levels correlate with the performances of students in other Higher Education Institutions (HEIs), (Atchoarena, 2016). Some stakeholders have attributed the poor performance in mathematics at HEIs to the students' weak mathematical foundation at primary and secondary school levels coupled with the teaching methods in HEIs which generally, in my experience, focus on drilling and memorising algorithmic procedures at the expense of student exploration and investigation. This has prompted an advocacy for a shift in approaches used in teaching and learning at all levels of education to include teaching strategies that are innovative, including methods that make optimal and strategic use of technology.

The Zambian government policy recognises and encourages the use of ICT in education. This is evidenced by the country's development of an ICT policy on education and the subsequent creation of the National Implementation Framework for ICT (Ministry of Communications and Transport, 2006). Major recommendations in the ICT policy on education include strategies to equip schools and TEIs with modern ICT tools to support teaching and learning, and for TEIs and teacher continuing professional development programmes (CPD) to include programmes to produce technology-skilled personnel to support schools in enhancing their use of technology in the teaching and learning process (Atchoarena, 2016).

1.2.1 Motivation to undertake the study

I was motivated to undertake a study on *GeoGebra* (www.geogebra.org) following my realisation of the lack of use and awareness of the software by teachers, during a joint presentation with my students on *GeoGebra* at the 2018 Zambian Association of Mathematics Education (ZAME) national conference. The teachers showed overwhelming enthusiasm to learn how to use *GeoGebra*. This prompted me to carry out an informal survey among lecturers in three TEIs in Zambia to get baseline data, and the findings confirmed a lack of awareness and use of *GeoGebra* in the institutions (see Table 1 below and Appendix C). Eleven out of thirteen lecturers (85 per cent) indicated that their institutions did not use *GeoGebra* in their teaching and ten lecturers (77 per cent) stated that they had no personal experience with the software.

Table 1.1: Use and awareness of GeoGebra among lecturers in TEIs

	Current use of *GeoGebra* by lectuerers in TEIs		Personal experience of lecturers in using *GeoGebra* for teaching	
	Number of lecturers	Percentage	Number of Lecturers	Percentage
Yes	2	15	3	23
No	11	85	10	77
Total	13	100	13	100

The question I asked myself was: "Why is there this lack of awareness and use of *GeoGebra* among teachers and lecturers when the software is a free open resource with a lot of potential to enhance the teaching and learning of mathematics?".

1.3 TECHNOLOGY USE IN MATHEMATICS EDUCATION

Technology, particularly Dynamic Mathematics Software (DMS), serves a myriad of significant roles in mathematics education. Its dynamic interactive environment has the potential to facilitate manipulations of virtual objects, graphic visualisations, numeric and symbolic representations to promote engagement and deep understanding of mathematics concepts (Konold & Lehrer, 2008). Its flexible human-computer-interaction interface fosters mathematical investigations, exploration, and communication. Several affordances of technology support innovative and novel ways of doing and learning mathematics. Hegedus and Kaput (2004) single out the main affordance of technology in mathematics as the unique potential for representations to promote "dynamic, interactive, animate, linked, and multiple representation capabilities of technological displays" (p. 12).

Technology offers teachers opportunities for flexible incorporation and interpretation into the teaching and learning of mathematics concepts that are represented in various ways (Zbiek et al. 2007). For instance, different representations of the same concept may provide different aspects of the concept that invariably necessitate different approaches such as graphical, algebraic, or numeric approaches to understand the concept. Consequently, such representational affordances of technology help students to explore various approaches that provide meaning to the same mathematical idea, encouraging them to connect different representations of the concept, hence supporting them to achieve thorough understanding (Zbiek et al., 2007).

The dynamic and interactive environment that technology often provides, offers a means to engage students in conceptual conversations about mathematical relationships, solutions and concepts, thereby promoting deeper understandings of mathematics. In this way, technology makes mathematical concepts visible to the class and encourages teachers and students to share their mathematical experiences (Bos, 2008; Kaput et al., 2007; Zbiek et al., 2007).

Despite the many affordances associated with technology integration in mathematics education, research has revealed some constraints to teachers' adoption of technology. Tondeur et al., (2017a) classified the types of constraining factors to teachers' technology integration into external and internal factors. The external barriers they pointed out were a lack of in-service training, access to technology, and restricted curriculum. On the other hand, internal factors cited were teacher confidence, beliefs about student learning, and value of technology in the classroom. From my experience with teachers, in a number of cases where an attempt to incorporate technology was made, it was mostly generic and not subject or topic specific, making it difficult for teachers to appropriately design learning activities for students in a technology environment. Other constraining factors mentioned were teachers' attitudes, lack of time and examination demands. Lack of supportive institutional policies, technophobia, rapid changes in ICT tools, fear of loss of instructional authority and resistance to change were other reasons identified (Agyei & Voogt, 2015). These factors (Zulnaidi & Zamri, 2017) have been exacerbated by a dearth of research examining teachers' perception of technology use in mathematics teaching.

1.4 CONCEPTUAL LANDSCAPE – VISUALISATION

Visualisation is increasingly gaining recognition of having a critical role in the learning of mathematics especially when learners are solving mathematical problems. It is considered a powerful tool for students to construct mental and physical representations that correctly mirror mathematical concepts and relationships (Zimmermann & Cunningham, 1991). Rogness (2011) acknowledges the importance of visualisation in the mathematics classroom and proposes "initial engagement of students' interest; improving student understanding of a concept, particularly for visual learners; development of visual reasoning skills; and a tool for mathematical exploration and research" (p. 6).

Despite a lack of consensus on the definition of visualisation, several authors have placed emphasis on gaining insights into a concept to deepen understanding as an important aspect of visualisation. A more generic definition from Arcavi (2003) states that:

> "Visualisation is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understanding." (p. 217)

Zimmermann & Cunningham (1991) described visualisation as "the process of producing or using geometrical or graphical representations of mathematical concepts, principles or problems, whether hand drawn or computer generated (p. 1). They add that visualisation does not equate to just forming a mental image, but that it is also about visualising a concept or a problem rather than just an idea.

The Zambian Ministry of Education (Zambia, ECZ, 2015) in its revised mathematics curriculum has recognised visualisation as an important skill that is indispensable in mathematics and a vital cognitive tool in problem solving. Visualisation (Zambia, ECZ, 2015) encompasses mental manipulation of various alternatives for solving a problem related to a situation or an object without the benefit of concrete manipulatives.

Hohenwarter and Fuchs (2004) argue that to achieve deep understanding, visualisation cannot be isolated from mathematics since learners need to learn how to represent ideas symbolically, numerically and graphically, and to navigate between these modes. In calculus, the mathematical domain of my study, students' understanding could be enhanced by dynamic symbolic representations of a function and its corresponding graphical representation; and the subsequent conversions between the function and its graph when a function is differentiated or integrated. For instance, Figure 1.1 below enables learners to observe how the derivative is defined as the limit of a sequence of secant slopes in the *GeoGebra* interface. The secant is dynamically illustrated to converge to a unique tangent line by dragging a point. Alternative cases, such as where the derivative does not exist, can also be dynamically illustrated.

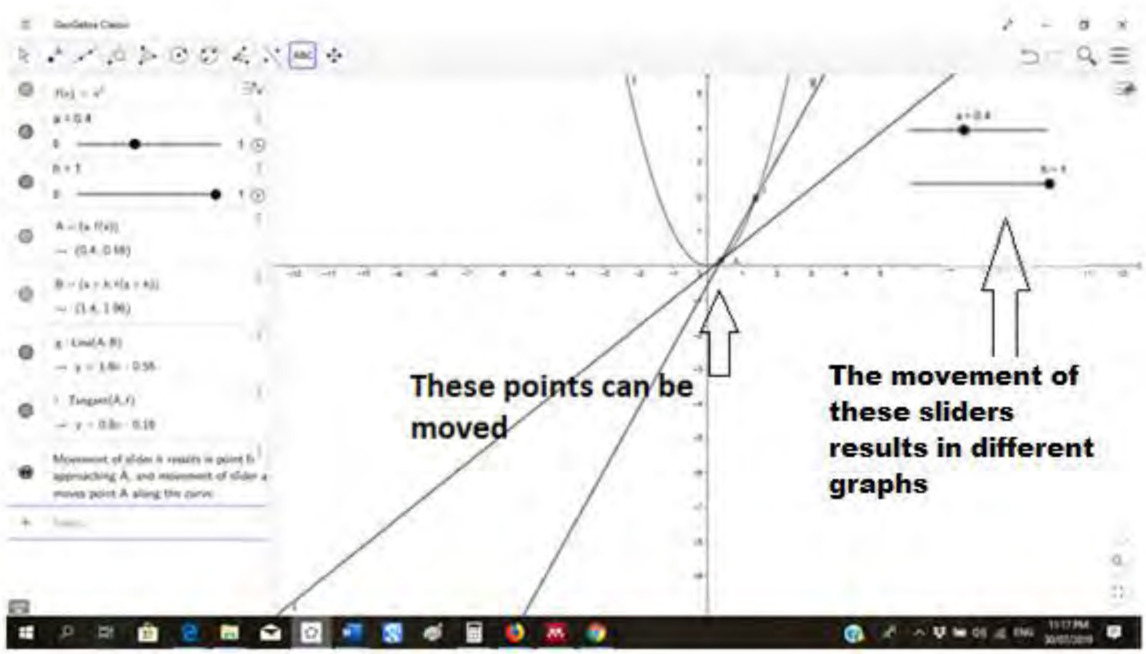

Figure 1.1: Visualisation of the limit process in *GeoGebra*

While some visualisation tasks may involve routine observations, the focus in this study is on visualisation tasks that are analytical and involve various thought processes. These are tasks that draw on experimentation and explorations. Such activities will not only arouse learners' interest in learning, but will increase their curiosity to explore and investigate. In affirmation, Preiner (2008) posits that "using dynamic visualisation to explain a concept, makes students grasp the mathematical ideas easier than with traditional teaching methods" (p. 49).

1.5 GEOGEBRA

Several studies acknowledge that the technological tools that use computer algebra systems (CAS) and dynamic geometry software (DGS) have capacities to help teachers and students think mathematically and present concepts in multiple ways to foster understanding Artigue (2019); Khalil et al. (2018); Kastberg and Leatham (2005). Among the mathematical technological software tools equipped with both CAS and DGS is *GeoGebra*.

GeoGebra is a free dynamic open-source mathematics software designed to enhance teaching and learning of various mathematics topics. Initially designed for students' use for algebra, geometry and calculus, it has been developed to include statistics, probability, vectors, complex numbers and spreadsheets in a single integrated package and has the potential to revolutionise mathematics teaching and learning (Hohenwarter & Lavicza, 2013). *GeoGebra* provides a virtual environment where the functionality of DGS and CAS enable users to simultaneously view both a numeric algebraic component (e.g. equations or coordinates) and the geometric corresponding features of an object (Hohenwarter & Jones, 2007); Preiner (2008). A major strength of *GeoGebra* is facilitating visualisation of mathematics concepts. "One of the most powerful and widely recognised didactical components of dynamic mathematics software is visualisation" (Kadunz, 2002, p. 198).

1.5.1 Teaching calculus using *GeoGebra*

Several topics in mathematics are conducive to technology aided learning environments which will promote student understanding. One such area is calculus. In calculus, the visualisation features and versatility offered by *GeoGebra* suits exploration.

Calculus is a vital course in many HEIs. This is evidenced by many academic departments such as teacher education, natural sciences, business and engineering that include it in the first year of their undergraduate mathematics courses. However, studies have indicated that many students enrolled in HEI calculus classes tend to acquire superficial and incomplete understanding of basic concepts of calculus (Sabella & Redish, 2007). They add that the failure to develop a conceptual understanding of calculus topics can in part be attributed to teaching practices that emphasise rote, algorithmic drilling and manipulative learning.

Calculus is a topic which emphasises rates of change and the relationship of one quantity to another quantity. Teaching strategies for fundamental concepts of calculus should take advantage of *GeoGebra*'s dynamic and visualisation characteristics and the concepts of change, movement and relationships in calculus. The software's versatile capabilities can help presentations of calculus content in a manner that promotes 'meaning making', for instance, the illustration of the limit process illustrated in Figure 1.1 above. In their study which they termed "*GeoGebra* and Calculus: An interesting partnership", Caligaris et al. (2015) concluded that the incorporation of *GeoGebra* applets and the teaching situations arising, is a much more effective teaching methodology than the traditional one to facilitate the learning of the fundamental concepts of calculus. This partnership can be viewed from different perspectives. For instance, calculus requires a coordination of algebraic and geometric concepts (Little, 2011), and on the other hand, the software (*GeoGebra's* main characteristics) are in essence anchored on geometry and algebra. The synergy between geometry and algebra windows coupled with logical clarity of the *GeoGebra* screen helps to reinforce the understanding of the algebraic and geometric formulations of calculus concepts. This would motivate teachers to use *GeoGebra* in order to employ a diversity of underlying visualisation themes that may result in emphasis on visual thinking in a manner that promotes teachers' and students' awareness of multiple representations. The use of multiple representations is envisioned to provide the missing link between mathematics education and technology (Özmantar et al., 2010).

1.6 MATHEMATICAL PROFICIENCY

Proficiency in teaching mathematics, according to Kilpatrick et al. (2002) relates to effectiveness and consistency in helping students learn mathematical content that is meaningful. Kilpatrick et al. (2002) used the term mathematical proficiency to describe what they considered was necessary for a person to learn mathematics successfully. They reiterated that teaching for mathematics proficiency encompasses "focusing on the interactions between teachers and students around educational materials and how teachers develop proficiency in teaching mathematics (p. 48). They identified five strands essential for developing mathematics proficiency, namely: conceptual understanding; procedural fluency; strategic competence; adaptive reasoning and productive disposition (p. 330). An important aspect about the five strands is that though itemised separately, they are interwoven and interdependent, and therefore represent different aspects of a complex whole of developing proficiency in mathematics. Notwithstanding the importance of all the five strands of mathematics proficiency to the learners' understanding of mathematics, this study will only focus on the first two strands: conceptual understanding and procedural fluency, based on their relevance to the study and their important implications to teaching. Kilpatrick et al. (2002) suggest that mathematical proficiency requires similarly interrelated components that includes conceptual understanding of the core knowledge of mathematics and procedural fluency in carrying out instructional procedures.

1.7 RESEARCH PROBLEM AND SETTING

1.7.1 Significance of the research

Despite the potential of *GeoGebra* in enhancing students' learning of mathematics that includes visualisation, manipulation, and exploration of geometrical figures and mathematical concepts, lecturers in TEIs in Zambia rarely use *GeoGebra* for teaching. Several studies on technology integration in the teaching and learning process have in the recent past focused on secondary schools, with very little research being undertaken in TEIs. There has been a dearth of research in Zambia examining lecturers' perception of technology use in mathematics teaching. This study locates itself in TEIs and investigates lecturers' experiences and perceptions of using *GeoGebra* as a visualisation tool to teach calculus to undergraduates in TEIs. It also investigates the factors that enable and constrain lecturers' adoption of technology in teaching. The findings of this study may contribute to literature on good practices in teaching calculus. The findings may also guide and provide a basis to design appropriate instructional materials for the consolidation of the Teaching and Learning Mathematics with *GeoGebra* (TLMG) Project for quality teacher professional development in the use of *GeoGebra* for mathematics teachers, with a view to incorporating *GeoGebra* in the Zambian teacher education curriculum.

1.7.2 Research goals and research questions

The main goal of my study is to investigate the use of *GeoGebra* as a visualisation tool by lecturers in Zambia, to teach calculus in TEIs to pre–service teachers, to enhance conceptual understanding.

The specific objectives are to:

- investigate how *GeoGebra* can be used as a visualisation tool to teach calculus to pre-service student teachers in TEIs to enhance conceptual understanding;
- find out the perceptions and experiences of lecturers in using *GeoGebra* as a visualisation tool to teach calculus in TEIs in Zambia; and
- analyse the enabling and constraining factors of using *GeoGebra* in the teaching and learning of mathematics.

Research Questions

Arising from these objectives, the research questions are:

- How can *GeoGebra* be used as a visualisation tool to teach calculus to pre-service student teachers in TEIs to enhance conceptual understanding?
- What are the perceptions and experiences of lecturers in using *GeoGebra* as a visualisation tool to teach calculus in TEIs in Zambia?

- What are the enabling and constraining factors of using *GeoGebra* in the teaching and learning of mathematics?

1.8 THEORETICAL FRAMEWORK

This section establishes the theoretical framework for the study.

> "A theoretical framework is an idea or a group of ideas that provide structure to a theory in a research study. Researchers may use theoretical frameworks to guide their studies, discover or analyse new perspectives, or find connections between seemingly unrelated concepts. Researchers may choose one or more theoretical frameworks that are appropriate to the study." (Borgatti, 1999, p. 1)

The umbrella theoretical framework for this study is constructivism (Piaget, 1967; Cobb, 2016). The study draws on the Technology Acceptance Model, (TAM) (Davis, 1989) and the Technological Pedagogical Content Knowledge (TPACK) model (Mishra & Koehler, 2006) as enabling theoretical frameworks. The inter-relationship between the three frameworks as used in this study is shown in Figure 1.2 below:

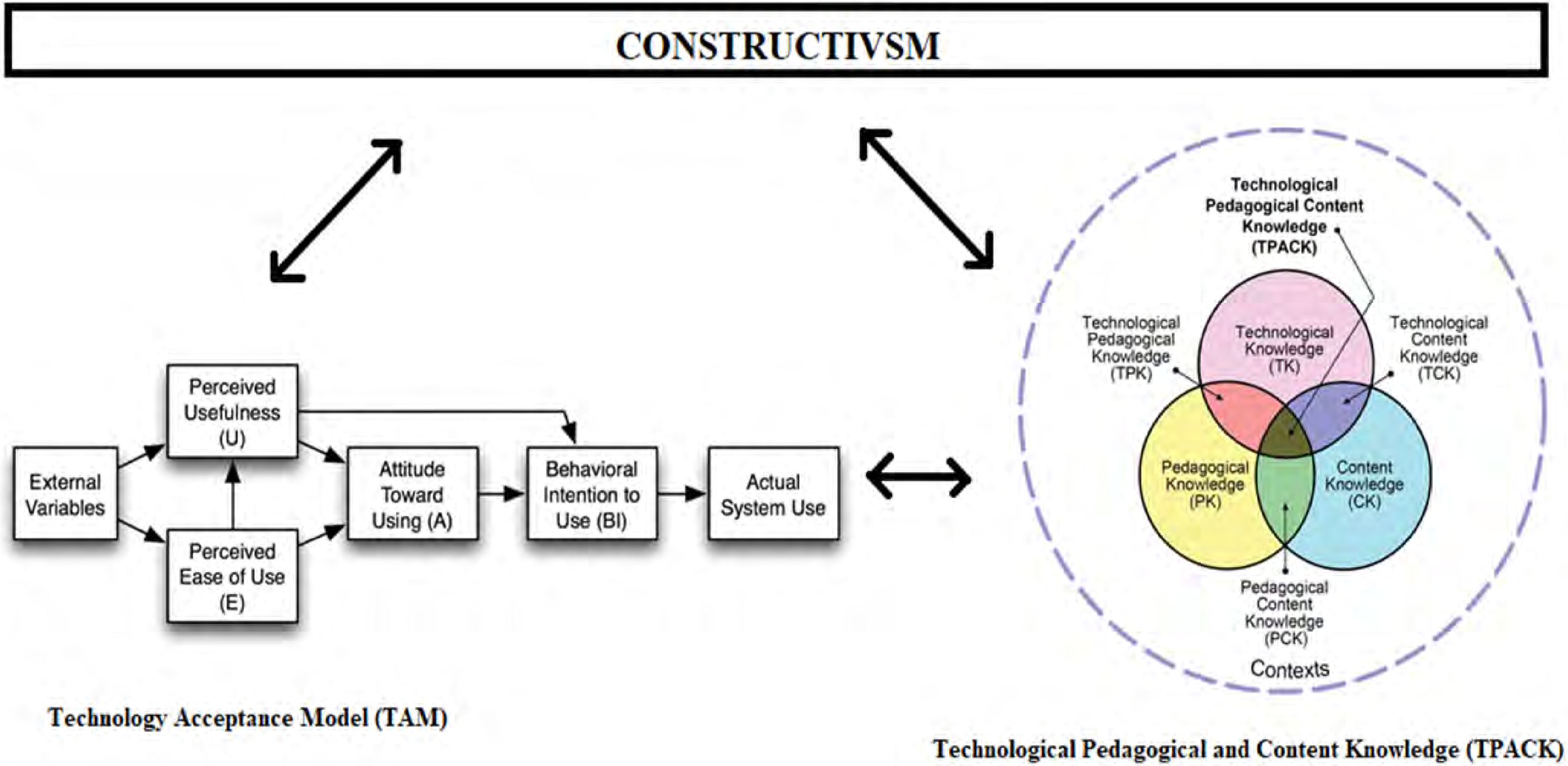

Figure 1.2: Theoretical Frameworks of the study

Constructivism

Constructivism focuses on learning of an individual through interactions with the external world, physical and social aspects and derives from the writings of Piaget and Vygotsky (Confrey, 2000; Von Glasersfeld, 1987). Piaget is credited with conceiving of radical constructivism, one of the branches of constructivism and highly associated with mathematics education. It is framed by the theoretical constructs of assimilation, accommodation and reflective abstraction (Liu & Chen, 2016). On the other hand, social constructivism draws on the works of Vygotsky (Popkewitz, 2019) . Social constructivism operates on the premise that individuals' construction of knowledge is strongly related to social interaction, discourse and the patterns in language (Crawford, 2016; Popkewitz, 2019). It addresses the learning of the individual, in relation to the external world. In a social constructivist context, learning is perceived as a process of constructing knowledge by individuals as opposed to being passive receivers from the teacher (Kolb & Kolb, 2019; Von Glasersfeld, 1987), where the learners' new knowledge is linked to their previous knowledge.

Mathematics teaching in HEIs in general, and in Zambia in particular, has for a long time embraced non-interactive ways of teaching in which the student receives knowledge from the lecturer, with very minimal participation (Abaté & Cantone, 2005)**.** This type of method, generally referred to as "traditional" is usually dominated by drills, manipulation of facts and algorithms, and falls short of addressing the needs of most students. Recently, there have been calls to reform mathematics instruction in HEIs by giving consideration to more innovative pedagogical approaches, underpinned in constructivist theory with a view to promoting students' conceptual understanding (Sang et al., 2011; Jaworski, 2006; Mokhtar et al., 2013; Orton & Roper, 2008).

An important aspect in constructivist pedagogy is contextualising learning in authentic environments and using real-world examples. Many students have difficulties in connecting mathematics to real world applications and this has been cited as one of the reasons for poor performance in mathematics (Palincsar, 1998; Kolb & Kolb, 2019). Research further suggests that students who adopt constructivist approaches to learning mathematics, tend to follow a conceptual approach when solving problems, while those who follow traditional teaching approaches tend to incline more towards procedural approaches (Wilson & Thornton, 2002).

GeoGebra is an activity-based learning tool which has a lot of potential for investigation and exploration and therefore aligns well with the constructs of the constructivism theory in various ways. Firstly, the dynamic applets created using the software tools are designed to be used in an exploratory and investigative manner. Engaging in exploration and investigation of mathematics concepts offers students opportunities to construct knowledge and be responsible for their learning, which are major attributes of constructivism. Using technology (Appelbaum et al., 2009) in mathematics teaching can encourage students to be more responsible for their learning, increase their confidence, and motivate them by providing them with novel experiences (p. 15). Technological environments provide many opportunities for a teacher to be a guide rather than lead the learning process – which is an important attribute of constructivism.

Teaching mathematics from a constructivist perspective involves the provision of activities. The constructivist approach to the use of technology in teaching mathematics provides opportunities to alter the nature of the material to be taught and learnt from routine-based to discovery and inquiry-based activities. This can be achieved by employing mathematical software embedded with visualisation characteristics and equipped with various features, which facilitate a constructive approach to learning mathematics. With its multiple representation capabilities, *GeoGebra* allows the user to drag and move points, lines and graphs on the screen, whilst observing changes in parameters and the effects of such changes on geometric shapes. It also enables the user to switch easily between numeric, symbolic and visual representations of information. These characteristics of the software can enhance constructive learning and encourage 'what if' situations for students to explore.

Technology Acceptance Model (TAM)

Research findings state that teachers' attitudes towards ICTs have a strong influence on the acceptance of the usefulness of ICTs in their lessons, and a bearing on whether teachers integrate ICTs into their classrooms (Teo & Milutinovic, 2015; Huang & Liaw, 2005). This is echoed by Cuban et al. (2001) who add that other than beliefs, effective implementation of education reforms is also dependent on teachers' knowledge, attitudes and skills. The significance of teachers' contribution to this process is underscored by the NCTM (NCTM, 2000) in their declaration of the teacher being one of the six major factors in the effective use of new technology in mathematics education

TAM is a model related to technology adoption and an empirically tested theory (Davis, 1989). It originated from the Theory of Reasoned Action (TRA) which claims that the intention to use a computer-driven technology is influenced by its users' beliefs and perceptions (Ajzen & Fishbein, 1980). Venkatesh et al. (2007) claim that "TAM currently enjoys the status of being the prime tool for testing user acceptance of new technologies" (p. 139). It is underpinned by a social psychological approach to explain the adoption of technology and the factors that influence individuals' decisions to adopt technology in their work. TAM theorises that an individual's behavioural intention to use technology is essentially determined by two beliefs: perceived usefulness and perceived ease of use (Joo et al., 2018). Perceived usefulness is the extent to which a person believes that using the system will enhance work performance, whereas perceived ease of use is the extent to which a person believes that using the system will be effortless (Venkatesh & Davis, 2016) The full constructs of TAM are External variables, Perceived usefulness, Perceived ease of use, Attitudes towards ICT, Intention to use and Actual use (Davis, 1989).

Ajzen (1991) explains that external factors in the TAM framework include institutional policies, beliefs about the environment such as support staff, infrastructure, and access to ICTs. Internal factors encompass skills, abilities and attitudes. In the context of ICT in education, TAM has been perceived useful by several researchers as a strong determinant of user intentions (Venkatesh & Davis, 2016).

As Kriek and Stols (2018) observe, *GeoGebra* combines its ease of use aspect with the construction features of a DGS and the functionality of a CAS. It lends itself to a wide range of possible applications for teaching mathematics This aligns well with the constructs of TAM of 'Perceived usefulness' and 'Perceived ease of use'. TAM contends that the attitude of the user towards use of technology for teaching and learning is very critical. Hew and Brush (2007) concur with this view and elaborate that changing attitudes and beliefs about technologies is an important factor and should take precedence in the teachers' ability to integrate technology into teaching. These observations resonate with Ertmer et al. (2012) view who elaborate that "If we truly hope to increase teachers' uses of technology, especially uses that increase student learning, we must consider how teachers' current classroom practices are rooted in, and mediated by existing pedagogical beliefs" (p. 19).

Tondeur et al., 2017b) point out that teachers who embrace constructivist beliefs in their pedagogies actively are more likely to adopt ICTs compared to those with low constructivist beliefs. Echoing this view, Ananiadou and Claro (2009) contend that teachers with constructivist beliefs use technology as a means to assist students develop higher order problem-solving and thinking skills and to support students' capacity to ''apply knowledge and skills in key subject areas and to analyse, reason, and communicate effectively as they raise, solve, and interpret problems in a variety of situations'' (p. 7).

Technological Pedagogical Content Knowledge (TPACK)

TPACK theorises the intersection among three domains of teacher knowledge: content. pedagogy, and technology. In order to describe a framework for the teacher's knowledge necessary to integrate technology in the classroom, Mishra and Koehler (2006) introduced the model TPACK. TPACK was a build-up on the earlier work of Shulman (1986) in which he articulated the intersection between pedagogy, content and knowledge (PCK). Ruthven (2014) pointed out that the idea of TPACK draws attention to how the new technological resources reshape pedagogical knowledge, content knowledge and pedagogical content knowledge.

TPACK describes the complexities and challenges of technology integration, informs strategies required to better prepare future teachers for learning and teaching in the 21st Century, and articulates the importance of teacher training (Koehler et al., 2017). One of the benefits of using TPACK is that it allows teachers to make thoughtful decisions about what technology best suits their teaching and students (Oberdick, 2015). The interaction of these bodies of knowledge both theoretically and in practice, produces the types of flexible knowledge needed to successfully integrate technology use into teaching. The resulting knowledge components of TPACK are: Technology Knowledge (TK), Content Knowledge (CK), Pedagogical Knowledge (PK), Pedagogical Content Knowledge (PCK), Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK); and, Technological Pedagogical Content Knowledge (TPCK) (Baran et al., 2011).

The TPACK model is essential to this study as quality teaching with the help of technology requires a thorough understanding of the complex relationships between technology, content, and pedagogy. Teachers require TPACK to understand how to effectively use technology to present concepts in constructivist approaches. Since TPACK focuses on the knowledge of the teacher (Koehler et al., 2017), it would help lecturers to make informed decisions about which topics can effectively be taught in a *GeoGebra* environment and in understanding the teaching and learning theories that are appropriate. As teachers create dynamic worksheets for their lessons using

GeoGebra, they also synthesise their TPACK. Mishra and Koehler (2009) argue that there is no perfect approach to integrating technology into the curriculum, claiming that the process is complex. It is therefore incumbent upon teachers to develop and understand this complexity in their quest for successful integration of technology into mathematics teaching.

1.9 METHODOLOGY

This is a qualitative case study underpinned by an interpretive research paradigm. The interpretive paradigm facilitated a deeper understanding of situations of how lecturers interacted with the *GeoGebra* applets from planning to implementation and reflection. It also provided insights on how they used their technological knowledge to make decisions during classroom practice in their lesson presentations. It sought to investigate the subjective understanding and interpretations that are the experiences of the participants concerning the use of *GeoGebra* applets as a visualisation tool in the teaching of calculus.

The data collection and analysis occurred in four calculus cycles. During the data collection process, a variety of techniques were utilised: stimulated recall interviews, audio-video recordings, observations and field notes. The data collected from the video recordings of observations of the lecturers' lessons consisted of two data sets: observation data and interview data. These were analysed separately. Data was analysed thematically (O'Neill et al., 2018) The observation data was analysed based on the analytical tools generated from the constructs of the enabling theoretical frameworks of TAM and TPACK. These analytical frameworks also took into consideration the constructs of constructivism and Kilpatrick et al.'s (2002) framework, analysed in three stages and based on the three research questions that guided the study. The analysis process is presented sequentially to illustrate multiple perspectives of this study. A more detailed presentation of the research methodology is provided in Chapter 3.

1.10 STRUCTURE OF THE THESIS

In addition to this chapter, there are four more chapters in this thesis.

Chapter 2: Literature Review

This chapter reviews the literature relevant to my research study. Firstly, the situational analysis of teacher education in Zambia from a technological perspective is presented. The chapter then discusses visualisation, the conceptual landscape of the study, and how it relates to *GeoGebra* in the teaching and learning process of mathematics, with particular emphasis to calculus. The chapter concludes with the discussion of the theoretical framework and models that underpin the study.

Chapter 3: Methodology

This chapter presents the qualitative case study approach adopted in this study. It discusses the various methods of data collection, viz. interviews, focus group discussions, document analysis, audio recordings and video recordings. The chapter then discussed the data analysis methods and how the methods enhance the reliability and validity of the study. The chapter concludes with a discussion of ethical considerations and a synopsis of the challenges encountered during the data collection process.

Chapter 4: Data Analysis

In this chapter, I present the analysis of the data. The analysis of the participating lecturers' interaction with the applets is guided by the analytical framework adopted from the literature. The findings are presented in relation to each of the three research questions for each mathematical topic taught.

Chapter 5 Conclusion and Recommendations

In this final chapter, I present a summary of the findings, the limitations and a proposal of recommendations based on the research. A discussion of the contributions to the field and suggestions for future research are presented.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter, I discuss the issue of teacher education in Zambia, with a specific focus on the teaching of mathematics in Teacher Education Institutions (TEIs) in Zambia. The use of Information and Communication Technology (ICT) in Zambia, and visualisation – the conceptual landscape of this study will also be discussed – followed by a discussion on the dynamic mathematics software, *GeoGebra*. Finally, I will discuss mathematical proficiency, the theoretical frameworks that guided the study and how they align to each other.

2.2 TEACHER EDUCATION IN ZAMBIA

The philosophical rationale for educational provision the world over is to promote the social and economic welfare of society and to nurture the holistic development of all individuals. This implies that educational policies of every nation should value and promote a multifaceted development of all learners, taking into account their individual differences so that they can participate in the cultural, economic, and social affairs of the nation.

Teaching is a key area of focus in current education reform in Zambia due to a number of factors. These include low capacity of teacher education institutions, high rates of attrition and the impact of HIV/AIDS (UNESCO, 2015). Initial training and professional development underline what a teacher should accomplish in the classroom. Mulenga and Luangala (2015) stress that the quality of teachers is largely determined by the teacher education experience that a prospective teacher goes through. The quality of a teacher, says Zientek (2007) relies heavily on teacher education. Teacher educators therefore need to be highly knowledgeable in the content and pedagogical knowledge areas of their subjects. This view has been acknowledged by several renowned international organisations. UNESCO (2015) acknowledged that teacher educators are significant players for maintaining and improving the teaching workforce as they have a huge impact on the quality of teaching and learning in schools. The commission, however, noted that despite the importance of teacher educators, they are rarely consulted in policy making. Atchoarena, (2016) reaffirmed this, but stressed the importance of having sufficient numbers of lecturers in teacher education institutions and equipping future teachers with necessary and adequate learning materials.

The Zambian Ministry of Education, developed the National Policy on Education, 'Educating Our Future' (MoE, 1996), to respond to the developmental needs of individual learners as well as the nation. This document superintends all issues concerning education in Zambia, including teacher education. According to the Zambian education policy document, Educating our Future (MoE, 1996), the aim of school education is "to promote the full and well-rounded development of the physical, intellectual, social, affective and spiritual qualities" (p. 29). The Zambian MoE, taking into account the complexity of the modern world as skills, knowledge and technology change rapidly, endeavours to provide a very clear articulation of the direction of the Zambian curriculum. The Zambian Ministry of Education [MoE] (2010) recognises the importance of teacher education and stresses that teacher education should assist teachers to develop their planning and instructional skills through the use of a variety of teaching methods and techniques. The Zambia Education Curriculum Framework (ZECF) is a guide that spells out regulations for all levels of learning institutions involved in the provision of formal education in Zambia. It functions as a tool to assist teachers and teacher-educators in the implementation of the national policy on education, which stipulates the provision of education in Zambia (MoE, 2013).

The Ministry of Education considers teachers as key human resources in the educational system in the country (MoE, 2010). It has often been acclaimed that the quality of an educational system cannot be greater than the quality of its teachers, but as noted by Bunyi (2013), little attention is paid to understanding how mechanisms that produce teachers can be made more effective, to impact positively on learning outcomes. The effectiveness and quality of an education system depends to a great extent on the quality of its teachers. Teachers' competence, commitment, and resourcefulness (Manchishi & Mwanza, 2016) play a major role in students' success. This inevitably implies that the quality of training teachers undergo is an important factor.

The quality of Zambia's schools reflects the quality of the teachers manning these schools, while the quality of the teachers reflects the effectiveness of the institutions that train them. The focus of concern in an effective teacher education institution is on transforming its students into competent and committed teachers. The programme for teacher education, therefore, must be kept under constant review to ensure that it responds to the real needs of Zambian schools (ibid., p. 97).

Teacher education in Zambia faces a number of challenges. These include a lack of facilities and resources, weak capacity and qualifications of staff at TEIs, weak utilisation of ICT. Zambia accounts for low levels of technology integration in the education sector (Sintema, 2018). The country is also faced with a challenge of disconnect between the curriculum offered in teacher education institutions and that offered in schools (Atchoarena, 2016).

In response to these challenges, coupled with the necessity and urgency to achieve education quality targets, the MoE launched a number of policy actions intended to produce a qualitative and quantitative improvement in the management of teacher education and supply (MoE, 2013). These included the MoE (MoE, 2013) "improving the quality and relevance of teacher education; increasing the output of pre-service teacher education in order to achieve the Millennium Development Goals (MDGs) and EFA goals; improving efficiency and effectiveness in college education; and improving equity in teacher training" (p. 33).

The importance of mathematics in any curriculum cannot be over emphasised. Mathematics is one of the most important subjects in any curriculum. In the Zambian education system, mathematics is a compulsory subject at both primary and secondary school levels. To underscore the importance of mathematics as a subject, the Examinations Council of Zambia (ECZ, 2014) stated that "one of the objectives of teaching the mathematics curriculum is to build up understanding and appreciation of mathematical concepts and computational skills in the learners in order for them to apply them in other subject areas and everyday life" (p. 31). There are a number of competencies and skills that mathematics can offer. These include analytical skills, abstract or spatial thinking, creativity, critical thinking, problem solving and effective communication skills (Fátima et. al., 2007).

The development of mathematics student teachers is highly dependent on the quality of mathematics education. In Zambian TEIs, the training of student teachers is made up of a large component of high-level mathematics content. However, over the last three decades, some researchers have observed that teachers who have studied advanced mathematics may not necessarily understand secondary school mathematics (Akkoç, 2015; Ball & Bass, 2000; Simsek & Clark-Wilson, 2018; Mueller et al., 2008). While the study of advanced mathematics with future engineers and mathematicians (as is the case in most TEIs in Zambia) can form a foundation for understanding school mathematics – this in itself is not a guarantee that the students will understand the school mathematics they have to teach (Berrett, 2012b; Wilson et al., 2005; Hiebert, 1999). It is argued that secondary school mathematics has its own aspects, different from mathematics in higher institutions (Rösken, et al., 2007). It was noted that in a number of Southern African TEIs, the training of pre-service mathematics teachers does not often include training to teach the fundamentals (Malambo, 2020).

It is assumed that since the pre-service mathematics teachers did well in mathematics at school, they possess what it takes to teach effectively in schools. This view was exemplified at the University of Zambia (UNZA), (Malambo, 2020), where mathematics content courses are offered by the School of Natural Sciences (whose focus is not on teacher preparation), while methodology courses are offered in the School of Education. This results in a situation where mathematics courses studied by student teachers are the same as those being taken by future mathematicians, with the expectation that teachers will reacquaint themselves with the school mathematics when they start teaching.

Mathematics textbooks used in schools provide both structure and content. In theory this enables newly trained mathematics teachers to refresh their understanding and teach the basic principles that did not form part of their teacher education. The study by Malambo and Putten (2019) to investigate mathematics student teachers' understanding of key function concepts at school, revealed that studying advanced mathematics during teacher training did not entail developing an understanding of school mathematics sufficient enough to explain concepts and justify reasoning – as is the case in most TEIs in Zambia. The study also revealed that a number of final year mathematics student teachers may possess procedural knowledge but lacked the conceptual knowledge of the mathematics required to teach. The shortage of qualified teachers in Zambia is extremely acute in mathematics and the sciences, and the country is also faced with the challenges of over enrolment in schools and HEIs.

To address the challenges of the shortage of teachers in mathematics, the MoE launched what it termed the 'Fast Track Training' initiative for teachers of mathematics and science by upgrading the qualifications of in-service teachers of mathematics and science through a fast-track three-year distance learning programme through a partnership by government and the private sector. Further, the (MoE, 2013) embarked on transforming selected colleges of education into universities. While this action solved some problems, it created others. I am located in one of the institutions that was recently upgraded into a university which lacks adequate infrastructure and qualified personnel.

In an effort to improve the qualification profile of teachers, the MoE upgraded all teacher qualifications offered at colleges of education to three-year diploma programmes from the traditional two-year certificate programmes. Despite these changes, the MoE noted that qualified human resources, infrastructure and teaching and learning materials still remained the main challenges in all colleges of education (MoE, 2010, p. 36). According to the MoE (2013)), teachers can obtain one of three qualifications: (i) the teaching diploma, which is required to teach at primary and junior secondary level; and (ii) a teaching degree – either at a bachelor's or master's level – which is a requirement to teach in senior secondary education (Grades 10–12) (p. 73). To teach in a college of education or a university, the minimum qualification is a Master of Education.

There have been other teacher professional growth initiatives that have been government-driven and donor-funded to improve teacher competence and qualifications in mathematics. For instance, the Japanese International Cooperation Agency (JICA) and the MoE implemented a project termed 'Strengthening of Mathematics, Science and Technology Education' (SMASTE) which began in 2005 (MoE, 2010). Similarly, the British government supported a one-year advanced diploma programme called the Zambia Mathematics and Science Teacher Education Project (ZAMSTEP), which upgraded mathematics diploma holders to an advanced diploma.

However, it has been noted that despite the number of teachers holding university degrees increasing at a fast rate in Zambian learning institutions, the performance of learners in mathematics is below the desired standard (ECZ, 2015). In School Certificate mathematics results, the overall poor performance in mathematics and science has largely been attributed to the bad performances in mathematics and science. In presenting the 2015 School Certificate results, the (MoE, 2015) report in the MoE document reiterated that "the overall unsatisfactory performance in School Certificates is attributed in large measure to poor performance in mathematics and science" (p. 53).

2.3 THE USE OF INFORMATION AND COMMUNICATION TECHNOLOGY IN EDUCATION

2.3.1 The role of technology in teaching and learning mathematics

Over the last three decades, it has been recognised that technologies such as computers and graphing calculators have a significant impact on the teaching and learning of mathematics in schools (Lawrence & Tarr, 2018; Ruggiero & Mong, 2015; Gardner et al., 2018). The predictions were mostly concerned with opportunities for enhancing student learning; for instance, by enabling connections to be made between graphical, algebraic and numeric representations of mathematical concepts. It was also highly anticipated that technology would bring about changes in teachers' and students' roles, based on the premise that teachers would act as facilitators of student collaborative exploration and discussion with peers (Petko et al., 2018; McCulloch et al., 2018; Ebrahim, 2018).

In response to emerging trends, several education institutions ventured into research on how to integrate new technologies in the curriculum, in a quest to reshape their curriculum with technology (Ottenbreit-Leftwich et al., 2018; Tømte et al., 2015). However, Trust (2018) observes that although recent frameworks of professional standards have stressed the importance of promoting student teachers' competences in teaching with technology, she noted that is rarely reflected in teacher education classrooms. Regarding teacher education, it was noted that TEIs were struggling to provide student teachers with adequate inspiring role models because of the insufficient number of teacher educators that used technology effectively (Tondeur et al., 2018; Prestridge, 2017). In view of this, teacher educators were encouraged to support student teachers' ability and their knowledge to choose optimal technologies to reach specific pedagogical goals with specific groups of learners (Koehler et al., 2017; Chien et al., 2012).

Tondeur et al. (2019) argue that the innovative use of technology in education lags far behind expectations. They observed that several teachers are in the early stages of integrating technology into their classes, and that the level of use varies widely between and within schools. They further claim that technology is predominantly used to support existing 'traditional' practices and not as a way to transform pedagogical practices. The use of technology for changing pedagogical practices is limited among student teachers, new teachers and in-service teachers. Student teachers and in-service teachers feel that they are not sufficiently equipped for teaching and learning with technology in their classrooms (Ottenbreit-Leftwich et al., 2018; Tondeur et al., 2012; Becuwe et al., 2017).

The question that arises is "how can teacher education institutions meet these demands, and especially what is required of teacher educators within this context? Teacher educators are known as second-order teachers (Murray & Male, 2005), as opposed to first-order teachers who work directly with pupils, because they educate student teachers who will be working with pupils in the future. Therefore, in addition to being teachers, teacher educators also serve as role models for their students in teaching with technology as well as in fostering students' technological knowledge. As role models in teaching, this implies that teacher educators' pedagogical behaviour should be in congruence with the pedagogical behaviour they want to promote in their students (Wright & Wilson, 2011; Lunenberg et al., 2007).

Teacher educators not only deliver the course content to their students, but they also teach and model technology use, instructional strategies and pedagogical beliefs ((Ertmer et al., 2012; Uerz et al., 2018). Modelling is one of the effective techniques to help student teachers learn to use technology (Teo & Milutinovic, 2015, Akinde, 2016). However, to effectively prepare student teachers to integrate technology as a tool in their future practice, teacher educators need to do more than just model technology use. They need to substantiate the underlying pedagogical and educational choices, and connect aspects of technology, pedagogy and content and their underlying relationships explicitly, to justify their modelled behaviour (Koehler et al., 2014; Lunenberg et al., 2007).

It can therefore be argued that teacher educators are faced with an even more complex task than first-order teachers on the use of technology in education. The complexity has been exacerbated by the fact that while research on teacher educators' teaching with technology in pre-service teacher education is expanding, it is still far less than research on learning and teaching with technology by teachers in secondary and primary schools.

Several TEIs are conducting research into how to integrate new technologies in the curriculum, in a quest to reshape their curriculum with technology (Ottenbreit-Leftwich et al., 2018; (Tømte et al., 2015). However, Trust (2018), observes that although recent frameworks of professional standards have stressed the importance of promoting student teachers' competences in teaching with technology, this is rarely reflected in the teacher education classrooms. Other studies claim that TEIs are struggling to provide student teachers with adequate inspiring role models because of the insufficient number of teacher educators that use technology effectively (Gronseth et al., 2010, Tondeur et al., 2012). In view of this, teacher educators are encouraged to support student teachers' ability and their knowledge to choose optimal technologies to reach specific pedagogical goals with specific groups of learners (Chien et al., 2012; Koehler et al., 2017). I argue in this study that *GeoGebra* could be one of those technologies.

Mathematics curriculum and policy documents currently place increased emphasis on processes of problem solving, communication and reasoning, and promote student discussion of mathematical ideas as a means of reflecting and developing their understanding (NCTM, 2000). These actions for curriculum reform are embraced by current research in mathematics education that promotes sociocultural theories of learning (Vygotsky, 1994; Wertsch & Rupert, 1993).

This theoretical perspective holds that all human development involves learning from others and from the culture that precedes us, and that thinking and reasoning are mediated by cultural tools such as material artefacts, signs, language, symbol systems and diagrams (Trouche, 2016). Mathematics teaching and learning therefore calls for the formation of a classroom community of learners where epistemological values and discourse conventions of the mathematics community are progressively enacted and appropriated (Goos, 2014; Schoenfeld, 1989). In such learning environments, collaboration and discussion are valued in building an atmosphere of intellectual challenge. As opposed to relying on the teacher as an unquestioned authority, students are encouraged to be analytical and defend mathematical conjectures and ideas, and to thoughtfully respond to mathematical arguments of their peers.

Despite technology's recognised potential for teaching and learning, its integration into mathematics education lags behind the high expectations that many researchers and educators have promised (Lagrange, 2005). The role of the teacher has been acknowledged as both a problematic and critical factor in this integrative process (Artigue, 2019; Trgalová et al., 2018; Hennessy et al., 2005), Ruthven, 2014). It is critical in the sense that the way in which teachers approach technology use has major consequences for the effects of its use in the mathematics classroom (Kendal & Stacey, 2003). It is problematic, because some teachers do not conceive the use of technology in their teaching as valuable for their educational goals, and in most cases avoid it, unless they are required to do so by curriculum or institutional constraints. Additionally, teachers quite often experience difficulties in adapting their teaching techniques to situations in which technology plays a role (Drijvers et al., 2015).

Therefore, in order to help teachers to benefit from technology in mathematics teaching, it is necessary to have sufficient knowledge about the new teaching techniques that emerge in the technology-oriented classroom and how they relate to teachers' views on mathematics education and the role of technology. The technological tools recommended for use in the teaching and learning processes of mathematics include statistical software packages, graphical calculators, data analysis routines, educational software classroom response systems, applets, spreadsheets, web-based statistics related resources, online texts and. data repositories (Devi, 2017).

Computers and internet technologies support novel ways of teaching and learning mathematics by not simply allowing students and teachers to do what they have done before in another way (Noor-Ul-Amin, 2013). It has been observed that for teaching and learning to improve, technologies must not simply be used as an alternative delivery platform, but rather, as cognitive tools for teaching and learning (Herrington & Herrington, 2008).

Integrating technology in a mathematics classroom has the potential to promote the development of computational skills as well as develop higher order mathematical skills. Bansilal (2015) is of the view that using technological tools can significantly improve the teaching and learning of mathematics by allowing learners to focus more on underlying properties and relationships instead of paying attention to tedious complicated calculations that may detract from the intended outcomes. Technology provides opportunities for learning by helping learners to spread, access, and share ideas and information, which is transmitted in integrated communication designs. In addition, technological tools can open up access to a wider variety of problem-solving strategies compared to those limited to pencil and paper strategies (Bansilal, 2015). Umugiraneza et al. (2018) elaborate that tools such as online videos enable students to vary the pace at of learning new material in mathematics.

By providing access to different representations that help in the visualisation of mathematical objects, certain mathematics software can contribute to a conceptual understanding of concepts. Technology also opens up possibilities for developing mathematics concepts by enabling the visualisation of the concepts; it can demonstrate complex abstract ideas clearly, while providing multiple examples (Griqua, 2019).

2.3.2 ICT and Teacher Education

There is an emerging consensus that to a great extent, ICT has impacted significantly on the way teaching and learning is conducted globally (Lim & Wang, 2016). The growth of ICT has opened up many opportunities for the improvement of education, particularly in teaching, where teaching and learning can take place anywhere and anytime (Adarkwah, 2021). The potential for teachers and students to harness the power of ICT to improve and enhance the quality of teaching and learning in the classroom is tremendous. The skills and knowledge students need in the digital age have resulted in a growing demand on educational institutions to adopt ICT in their daily activities. In a globalised digital age, the adoption and integration of ICT into teaching and learning environments provide more opportunities for teachers and students to work interactively. Technological innovations are increasingly being used to drive changes intended to deliver significant improvement in education (Lawrence & Tarr, 2018).

Information and communication technology (ICT) includes all technologies specific to education, and used in the handling and communication of information. This includes desktops, laptops, recording equipment, mobile telephony, projection technology, digital, intranet, internet, software applications, multimedia resources, information systems, tablets, PCs and e-readers (Lawrence & Tarr, 2018). These devices provide great opportunities and challenges for education in general, particularly in the teaching and learning process. ICT offers a new paradigm shift globally in how education is delivered, and it is changing the face of education. The adoption and integration of ICT in education has continued to gain momentum in educational literature (Wanjala, 2016). Studies have shown that the appropriate use of ICT has the potential to connect learning to real-life situations and raise educational quality (Lowther et al., 2008; Alt, 2018).

The adoption and integration of ICT is extremely important both in the access of knowledge and keeping pace with modern developments (Plomp et al., 2007). There is a remarkable increase in the availability of global resources such as digital libraries, where teacher educators, teachers and students can access and share research and course materials anytime and anywhere. UNESCO (2015) elaborates that new educational approaches are possible in the teaching and learning process through the adoption and integration of ICT, which leads to the provision of higher order skills such as collaborating across time and place and solving complex real-world problems, thus improving and enhancing the perception and understanding of the learning process.

Technology has various effects on education, particularly in enhancing students' learning (Tarmizi, 2010). When technology and appropriate teaching methods are integrated in the teaching and learning process, a positive impact may be observed in the affective, cognitive and psychomotor domains. The various opportunities that technology provides for improving classroom instruction have been demonstrated in mathematics education. The NCTM urges teachers of mathematics to use technology in teaching and learning mathematics (NCTM, 2000). However, the NCTM cautions that technology should not replace the mathematics teacher. The teacher plays many important roles in a technology-rich classroom and makes decisions that affect students' learning in a number of ways (NCTM, 2000). The use of technology as a tool to communicate with others enables learners to play an active role, compared to the passive role they play when they receive information transmitted by a textbook or broadcast. Several educators are of the view that technology has the power to provide enrichment and illustrate concepts that go beyond what teachers can provide. Technology also encourages students to think actively about information, and to make choices and execute skills that are typical in teacher-led lessons (Englund et al., 2017). On the other hand, students need guidance in applying the latest technology to solve various mathematical problems. The computer is now widely used as a teaching aid in mathematics in order to enhance students' self-motivation and self-confidence (Oldknow, 2009).

Studies, such as those done by Dockendorff and Solar (2018) and Suárez-Rodríguez et al. (2018)) have indicated that the use of ICT in the classroom is essential for providing opportunities for students to learn to operate effectively in an information age. The conventional educational environments are not sufficient for preparing learners to function or be productive in the workplaces of today's society. They emphasise that educational organisations that do not incorporate the use of ICT in schools cannot genuinely claim to prepare their students for life in the 21st century). They acknowledge that ICT can play various roles in learning and teaching. They reiterate several studies that have reviewed the literature on ICT and learning and conclude that it has great potential to enhance the learning and teaching process.

Many researchers affirm that the use of technology can help students to become knowledgeable and reduce the amount of direct instruction given to them. It can also give teachers an opportunity to assist those students with particular needs (Bransford et al., 2000). While ICT can assist students in their learning, it can also help teachers enhance their pedagogical practice. Lowther et al., (2008) posit that ICT can play a role in developing student skills, knowledge and motivation. They further claim that ICT can be used as a tool to present information to students and help them complete learning tasks. Reid (2002) indicates that the success of the integration of new technology into education varies from curriculum to curriculum and depends on the ways in which it is applied. He adds that in mathematics education, there are some areas where ICT has shown to have a positive impact.

Inan and Lowther (2010) elaborate that ICT presents a unique opportunity for teaching and learning to enhance and improve learning activities by providing course materials online where they can be accessed anytime and anywhere. This implies that that learning is not restricted to a geographical location but can occur anywhere. Reid (2002) posits that ICT offers students time to explore beyond the mechanics of course content – which provides opportunities to understand concepts more efficiently. Technology use does not only change the conventional methods of teaching, but also accords teachers opportunities to be more creative in customising and adapting their own teaching materials and strategies (Reid, 2002).

Juan et al. (2011) concur with Reid and add that ICT assists in transforming the teaching environment into a learner-centred one, since learners are involved actively in the learning processes. While appreciating that it is difficult to measure the impact of ICT on learning, researchers Juan et al. (2011) and Lowther et al. (2008) have highlighted some of the potential opportunities that can be gained from the effective use of ICT in education generally, and in enhancing teaching and learning activities:

- to provide opportunities for students to learn from local and/or international experts;
- to provide opportunities for students to develop global understanding and cultural sensitivity;
- to collaborate and cooperate with students from other countries;
- to students in accessing digital information efficiently and effectively;
- to support student-centred and self-directed learning;
- to produce a creative learning environment;
- to improve and enhance teaching and learning quality;
- to support teaching by facilitating access to course content;
- to promote problem solving and develop critical high order thinking skills;
- to improve communication skills;
- to motivate and engage learners; and
- to tailor learning to the learner.

(Lawrence & Tarr, 2018, p. 83).

Teachers are therefore encouraged to embrace the current changes and strive to realise the use of the latest technology in the classroom. Teacher educators of mathematics, through the use of ICT, should endeavour to make mathematics an interesting subject in order to attract students' interest and at the same time to help them to consciously focus on important mathematical concepts. It is the teachers' responsibility to prepare students to focus on the future world which undoubtedly depends on mathematics, science and technology (Furner & Marinas, 2007).

However, despite the huge investment in ICT infrastructure, equipment and professional development, to improve the quality and delivery of education in many countries, the expected results have not been realised. Research (for example Cuban et al., 2001; Eteokleous, 2008; Hayes, 2007) has shown that although many governments have invested substantially in the integration of technology in institutions of learning, the results have fallen short of the expectations and the intended educational outcomes have not been achieved (Houghton & Keynes, 2013). Gülbahar (2007) concurs with these findings and adds that huge educational investments have produced little evidence of ICT adoption and use in teaching and learning. There are, however, some projects where the integration of ICT in education has been relatively successful.

2.3.3 Successful projects integrating technology in education

Motivated by the prospect of greater economic, social, educational and technological gains, both developing and developed countries are bringing about education reform with a clear focus on ICT integration in education (Farjon et al., 2019; Anthony et al., 2019). To this effect, countries have invested substantially in terms of money, resources, expertise and research to integrate technology in education in a quest to make the classroom environment more conducive for enhanced teaching and learning. To compete favourably in the global information and knowledge-based economy, there is a need for a workforce that is skilled in the use of technology. Therefore, it is no longer a question of whether technology should be integrated in education, but rather when and how to integrate technology so that it benefits all stakeholders ranging from teachers, students, administrators, the community and parents to compete favourably in the global economy (Farjon et al., 2019; Anthony et al., 2019). This section focuses on some successful projects on ICT integration in education.

The GeoGebra Literacy Initiative Programme (GLIP)

The *GeoGebra* Literacy Initiative Programme (GLIP) is a teacher development project in Mthatha in the Eastern Cape, in South Africa (Mavani, 2019). It aimed to equip teachers and students with the skills to use the technological tool, *GeoGebra,* a dynamic mathematics software (DMS), for teaching and learning mathematics. The GLIP project was primarily designed to promote collaborative reflection and engagement among teachers adopting technological tools for effective use in mathematics classrooms. Launched in 2015, the project had twelve participants who were all drawn from secondary schools, spearheaded by two researchers under the guidance of Rhodes University. The GLIP intervention programme had two phases, with the first phase comprising training of teachers in the basic use of *GeoGebra*.

In the second phase, which was the intervention stage, the participants used the *GeoGebra* applets they had collaboratively developed, to implement in mathematics classrooms. The focus of the researchers was twofold. One researcher focused on how the learners engaged with the applets while the other focused on how the teachers implemented the applets. Both studies harnessed the visualisation opportunities of the applets to develop 'meaning-making' and enhance conceptual understanding.

The findings of the GLIP study revealed that "collaborative teamwork among the GLIP teachers is a feasible way to bridge the gap between having access to technology and adapting it for effective use in the classrooms" (Mavani et al., 2018). The study also supported that view that teaching with the aid of DGS offered teachers other teaching strategies that would not have been possible to implement in conventional classrooms.

It was also observed that elsewhere in South Africa, the integration of dynamic mathematics technology into the South African classroom was very tentative (Ndlovu et al., 2013). Ndlovu et al. further argued that despite *GeoGebra* being implemented in South Africa, it is done in isolated cases, and that the implementation is still a challenge since most teachers do not have adequate prior experience with computers. It was recommended that there was a need to develop teacher competencies so as to speed up the rate of integration of ICT into mathematics classrooms.

Project determining the effects of Google Sketchup based geometry activities on the spatial visualisation ability of student mathematics teachers

Spatial visualisation ability refers to the ability to manipulate, rotate and change the position of an object in the mind depicted as a picture (Kurtuluş & Uygan, 2010). Spatial ability plays a key role in the teaching of mathematics and geometry. Spatial thinking enables an individual to draw shapes during mathematical problem-solving to visualise verbal problems in the mind and to categorise the given data in tables. Spatial thinking has many important applications in many disciplines, including geometry education. Hence the purpose of this project conducted in Turkey, was to determine the effects of *SketchUp* based geometry activities and projects on spatial visualisation ability of pre-service mathematics teachers (Kurtuluş & Uygan, 2010).

In this project, both experimental and control groups each had twenty-four mathematics student teachers. In order to obtain data, the Santa Barbara Solid Test (SBST) to identify cross sections that are two-dimensional (2D) slices and three-dimensional (3D) objects, was used (Ferla et al., 2009).

A post-test and pre-test experimental design was used. For the experimental group, problem-based activities related to how solid objects were designed and solved in the *SketchUp* environment were used. The experimental group solved problem-based activities by using dynamic tools and a project study carried out on the *SketchUp* environment. On the other hand, traditional geometry activities requiring the use of only paper and pencil were given to the control group.

The researchers had earlier introduced the basic tools of the software to the experimental group. The students then solved problems on solid objects while manipulating and analysing 3D simulations of objects in a dynamic environment. The *SketchUp* software gave students opportunities such as sketching, rotating and cutting solid objects. In contrast, the control group solved the problems on paper without using any technological tools. The experimental group designed objects having different complex geometric shapes on *SketchUp*; then they measured their surface area and volume using the measurement tools of the software and illustrated their surface developments on paper. Finally, they made virtual presentations of their products.

The findings revealed that *SketchUp*-based activities illustrated spatial visualisation abilities. However, the conventional applications of static tools with examples of 2D objects did not show any significant effect on spatial visualisation. The result, in the context of *SketchUp,* could be used beneficially for improving spatial visualisation. The results of the project indicated that 3D dynamic software was more effective on spatial skills than conventional instructions and that *Google SketchUp* could be used more effectively in geometry education as an alternative to other dynamic geometry software to improve spatial visualisation ability.

The Bring your own Technology Project

Among the most popular educational projects in the United States of America (USA) is the "Bring your own technology' (BYOT) also referred to as "Bring your own device' (BYOD). Many schools in districts in the USA are increasingly promoting BYOT initiatives as a way to increase access to technological gadgets such as laptops and tablets within schools. While the BYOT project has been well embraced, it has nonetheless raised issues such as costs, equity, digital safety and maintenance (Ballagas et al., 2004).

The Preparing to Teach Mathematics with Technology Project

The 'Preparing to Teach Mathematics with Technology' (PTMT) is another successful project based in the USA. Funded by grants from the National Science Foundation since 2005, the aim of this longitudinal project was to create instructional materials. It also focused on the implementation and sustenance of the use of new instructional strategies in teacher education programmes, which included developing faculty expertise, evaluating and conducting research on the effectiveness of the PTMT approach (Lee & Hollebrands, 2008). The PTMT project impacted practices of about eighty faculties in teacher education programmes across Canada and the USA and its materials have been used with over one thousand five hundred prospective and practicing mathematics teachers (Lee & Hollebrands, 2008).

The One Laptop Per Child project

In Peru, the one laptop per child (OLPC) XO project was initiated in 2008. The main goal of the OLPC XO laptops project was to distribute a laptop to each student to help integrate technology in the mathematics classroom. With an initial focus of distributing laptops to learners in rural and remote schools, one million OLPC XO laptops (Cristia et al., 2020), were given to students.

A randomised evaluation of the impact of the OLPC project was undertaken by the American Development Bank (ADB). While the OCPL project served the educational institutions substantially positively with increased engagement of students in learning practices, the findings, however, brought out some important issues for educational reformers and technology proponents. These findings raised debate on whether large scale introduction of new technologies will, in and of themselves bring about the promised positive changes in educational systems (Cristia et al., 2020).

2.3.4 Opportunities and constraints of using technology in mathematics education

Research on the use of technological tools for teaching and learning mathematics has been widely conducted and there is agreement that incorporating technology into the teaching and learning of mathematics is very desirable and recommendable (Scherer et al., 2019). However, there is relatively little research focused on factors that influence technology integration in mathematics instruction; the way that technology is positioned in the teaching and learning of mathematics; and a synthesis of studies on preparing teachers to teach mathematics using technology. Afshari et al. (2009), Hew & Brush (2007), Cuban et al. (2001) and Goos & Bennison (2008) concur and elaborate that while a number of studies have been conducted to describe technology use in the mathematics classroom, not much research has been done on factors that influence teachers' decisions to integrate technology into their teaching (Bray & Tangney, 2017). An insight into these factors may be significantly beneficial to mathematics teacher educators, who consequently equip pre-service teachers with the tools needed to teach mathematics with technology successfully and prepare them to be better teachers.

Prudent use of technology can inevitably foster the migration from the concrete and externalisation of mathematics, to the abstract and internalisation of mathematical concepts respectively. It may also support the idea of iteration between processes as postulated by (Mudaly & Rampersad, 2010). Technology's influence on mathematical teaching and learning is either amplified or limited through the types of mathematical tasks and activities teachers provide in a learning environment,

Opportunities

Students need guidance in the use of the latest technology to solve various mathematical problems (Oldknow, 2009). The computer is widely used as a teaching aid in mathematics in order to enhance students' self-confidence and self-motivation (İsleyen & Sivin-kachala, 2019). Over the last two decades, the variety and number of technological tools that teachers and students have access to in schools has risen greatly, and the mathematics classroom is no exception (Gray et al., 2010; Snyder et al., 2016). The teacher plays a critical role in determining effective ways of how this technology is used.

Teachers are encouraged to embrace the current changes and strive to realise the use of the latest technology in the classroom. Educators should endeavour to make mathematics a very interesting subject in order to attract students' interest and at the same time help them to consciously focus on important mathematical concepts. It is the teachers' responsibility to prepare students to focus on the future world which will undoubtedly depend on mathematics, science and technology (Furner & Marinas, 2007). Technology-based learning provides symbols, formulas, tables, graphs, numbers, equations and manipulative materials to link them with various real-life situations. Technology application in teaching and learning mathematics helps students to better understand basic mathematical concepts and to experience intuition in solving certain mathematical problems (Prestridge, 2017). Hollebrands et al. (2017) are of the view that learners can benefit in various ways from the integration of technology into everyday teaching and learning and state that:

> Technological environments have the potential to offer students opportunities to engage with various mathematical skills and levels of understanding through a variety of mathematical tasks and activities.

Van Voorst (1999) emphasised the notion that technology was "useful in helping learners view mathematics less passively, as a set of procedures, and more actively as reasoning, exploring, solving problems, generating new information, and asking new questions… he asserted that technology helps learners to "visualises certain mathematics concepts better and adds a new dimension to the teaching of mathematics" (p. 2).

With an emphasis on mathematical processes, technology in the mathematics teaching and learning process becomes increasingly necessary. With the help of technology, laborious computations are easily performed and multiple representations of concepts are produced with minimal effort. With dynamic, vivid visuals, technology can provide strategies that foster mathematical thinking. This ultimately allows teachers and learners more time to concentrate on mathematical processes in the classroom. The illustrative properties of technological software allow learners to visualise and make reference to graphs, images, charts, and diagrams. This has the potential to facilitate the conceptualisation of the mathematical ideas and concepts and in reasoning and conjecturing during their engagement with tasks.

Findings from a study conducted by Ottenbreit-Leftwich et al. (2018) indicated that one of the most important factors when making decisions about use of technology in mathematics was how well it aligned with the goals of a lesson. The findings further revealed that when considering how to infuse technology into teacher education programmes, it is necessary for teachers to focus more broadly on ease of use for both their students and themselves, and on how particular activities align with specific mathematics learning objectives and outcomes.

The teachers' role regarding technology use in mathematics teaching and learning is underscored by the NCTM, whose position on the use of technology is that "effective teachers optimises the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics" (NCTM, 2000). Thus, teachers should not only integrate technology in their mathematics teaching, but should integrate it competently. However, anecdotal evidence suggests that very few teachers use technology in ways that align with NCTM's position. Many mathematics teachers primarily use technology for data storage, as a simple calculation tool, for the display of static materials or in ways that are unlikely to stimulate their students' interest, develop student understanding, or increase their proficiency in mathematics (Cuban et al., 2001; Ertmer, 2005).

Most developed countries have good and reliable access to technology, thus alleviating the severity of external barriers; however, some internal barriers still persist (Vongkulluksn et al., 2018; Kaleli-Yilmaz, 2015; Wachira & Keengwe, 2011). These include beliefs regarding the role of the teacher across different levels and beliefs about the nature of mathematics teaching and learning.

While an increase in the use of technology in a classroom is a positive step towards enhancing students' learning, its mere presence does not however guarantee successful outcomes. The success of technology implementation depends, to some extent, on the type of software (Khan, 2017) and the type of tasks to be implemented (Sherman, 2014). The software used mostly for drill-and-practice, for example, has very little impact on student achievement (Hennessy et al., 2005). What is of significance is the type of software used, and the role played by the teacher in its implementation. The decisions the teacher makes when integrating technology is key to its successful implementation (Escuder, 2013; Li & Ma, 2010); Drijvers et al., 2015; NCTM, 2000).

In their review of the study on teachers' uses of technology, Zbiek et al. (2007) described the various concerns teachers had about integrating technology, which included personal, managerial and technological concerns. They posited that teachers' decisions about whether to include technology in their instruction was based on "their comfort with the tool, their perceptions of whether students would be able to use it, and their confidence in the technology working" (p. 23). These roles are to a large extent influenced by teachers' beliefs on instructional practices.

Research on the effects of computers on teachers' instructional practices has been limited. Ertmer (2005) reported that "although many teachers are using technology for numerous low-level tasks (word processing, Internet research), higher level uses are still very much in the minority" (p. 25). Cuban et al. (2001) examined two high-tech high schools in California and came to a similar conclusion.

Constraints

Notwithstanding the fact that ICT has impacted teaching and learning positively in several ways, literature however also indicates that there are some impediments in integrating and adopting ICT in the teaching and learning process. Although research has been limited regarding factors that inhibit significant changes to instructional style, a number of possibilities have been put forward. Tearle (2003) categorises the barriers for the successful implementation of ICT into three levels: teacher-level, school-level and system-level. At the teacher level or individual level, among the major constraining factors identified are teachers' beliefs (Ottenbreit-Leftwich et al., 2018). The constraining factors associated with beliefs include teachers' beliefs about the following: technological skills; the nature of teaching and learning; the nature of mathematical knowledge; the role of computers in the classroom and their possible effects on student outcomes; and beliefs about their students' capabilities (Kendal & Stacey, 2003).

Charles (2012) identified school level barriers as lack of effective training to solve the technical problems and lack of access to resources. Teacher-level barriers, on the other hand, include resistance to change and lack of confidence. In Pelgrum's (2001) view, the obstacles to ICT adoption could be material or non-material. The material condition comprises insufficient software and hardware while the non-material condition includes lack of teachers' ICT skills and knowledge, challenges of integrating ICT-based instruction and insufficient time for teachers to prepare ICT integrated lessons.

Ertmer (2005) categorised barriers to technology integration into two parts: external barriers and internal barriers. External barriers include the availability of computers, professional development opportunities involving technology and level of administrative support. On the other hand, internal barriers principally involve teachers' attitudes and beliefs. They include teacher concerns about whether students should use technology to learn mathematics and whether the students will become over-reliant on technological tools.

Other constraining factors at system level are: the amount of curricular freedom afforded to the teacher (Becker, 2000), prior teaching experiences with technology (Escuder, 2013), adequate preparation and training of teachers (Afshari et al., 2009; Becker, 2000), adequate time for planning (Hadjerrouit, 2019), preferred teaching style (Brenner & Brill, 2016)), lack of appropriate software (Carver, 2016) and the time when the software was adopted (Rosdi et al., 2020).

2.4 VISUALISATION

2.4.1 Visualisation processes in mathematics education

Many researchers have emphasised the importance of visual reasoning and visualisation for learning mathematics, as visualisation is an important facet of students' understanding of the construction of mathematical concepts (Haciomeroglu & Haciomeroglu, 2020). Some proponents of visualisation have suggested that visual thinking can be both an alternative and powerful resource for students learning mathematics (Pettigrew & Shearman, 2014). They added that it is a resource that can pave the way to various ways of thinking about mathematics other than the logico- propositional thinking and linguistic way of traditional proofs and manipulation of symbols of traditional algebra.

Visualisation is a critical aspect of mathematical thinking, reasoning and understanding. A growing body of knowledge is in support of the assertion that understanding of mathematics is strongly linked to the ability to use analytic and visual thinking. Researchers further contend that for students to construct a sound understanding of mathematical concepts, visual and analytic reasoning must both be present and integrated (Aspinwall & Miller, 2001; Pfeiffer, 2017).

Visualisation in the teaching of mathematics has become a research focus for many researchers (Arcavi, 2003; Presmeg, 2014; Rösken & Katrin, 2006). But despite being studied by many researchers, there is no consensus on the definition of the term 'visualisation'.

Different authors have defined and explained visualisation differently. The term "visualisation" (Zimmermann & Cunningham, 1991) is concerned with a concept or problem involving visualising. Nobre et al., (2016) defined visualisation as a tool that penetrates or travels back and forth between learners' mental perceptions and external representations. Goldin (2002) put emphasis on the relationships between conceptual understanding, mathematical visualisation, and representation. Dreyfus and Eisenberg (1990) contended that what learners 'see' in a representation is linked to their conceptual structure. They suggested that visualisation should be regarded as a learning tool. This resonates with Presmeg (2014), who views visualisation as "processes involved in constructing and transforming both visual images and all of the representations of a spatial nature that may be used in drawing figures or constructing or manipulating them with pencil and paper" (p. 73). The emphasis of Presmeg's definition is that in mathematical thinking and problem solving, graphs that can be used as problem-solving tools, may be drawn to represent mathematical concepts.

As for Dubinsky et al.,(1996) "visualisation is the act in which an individual establishes a strong connection between an internal construct and something to which access is gained through the senses" (p. 46). On the other hand, Konyalioglu (2008) defined it as "the bridge between the experimental world, thinking and reasoning" (p. 123). A more generic definition given by (Arcavi, 2003) is that:

> Visualisation is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understanding. (p. 217)

From these numerous definitions, it can be deduced that visualisation takes different meanings in different contexts. Based on Arcavi's definition of visualisation, it can be interpreted that there is a need for 'objects' to interact with people for visualisation to take place Arcavi (2003). Therefore, for the purposes of this study, Arcavi's (2003) definition is key as it synchronises the major issues raised in the other definitions.

Research has indicated different ways to visualise learning. Examples of the means of visualisation include two-dimensional (2D) or three dimensional (3D) physical manipulatives; graphs, diagrams and pictures; technological tools such as in dynamic geometry environments (Cabri, *GeoGebra*));

graphing calculators and other computer software (Tall et al., 2008; Toptaş et al., 2012; Konyalioglu, 2008; Hollebrands et al., 2017; Kastberg & Leatham, 2005).

Visualisation tools are among the most innovative technologies to emerge in the last few years in educational settings. They offer new potentialities for mathematical learning by means of dynamic representations and animations, interactive simulations, and live streaming of lessons. Additionally, visualisation tools have the potential to promote dynamic, visual, distributed and embodied mathematics, rather than individual achievements and static representations (Zimmermann & Cunningham, 1991)

Sedig and Summer (2015) outline in more detail the benefits of interactive visualisation to include:

> making latent properties visible, hence amplifying their epistemic utility and extending their communicative power"; provide opportunities for "experimentation and exploration of hypothetical 'what if?' situations"; "guide and transform the path of reasoning and understanding"; and coordinate users' "internal mental models with external visual models of objects and processes. (p. 345)

Presmeg (2014) interpreted visualisation processes as learners' individual ways of problem solving that tend to be centred on either visual or analytical approaches. She adds that the nature of mathematics frequently requires the use of both these approaches. An awareness of these different preferred approaches, she emphasises, is important for effective teaching. In her study on the visualisation process and learner difficulties, Presmeg (2006) concluded that students performed better in managing analytic processes but struggled in managing visual processes. This is echoed by Lowrie and Kay (2001) who reiterated that analytic learners were more successful at problem solving than visual learners, but they were quick to point out that this could be related to other aspects of the teaching environments such as students' prior experiences.

There is a growing body of research study focused on the significance accorded to visualisation as an essential component in the teaching and learning of mathematics. Current trends in mathematical inquiry include an exploration of the role of visual representations which are illustrated through data representations, graphs, pictures, diagrams, words, symbols and patterns in the development of mathematical concepts. Presmeg (2006) states that "mathematics is a subject that has diagrams, tables, spatial, arrangements of signifiers such as symbols, and other inscriptions as essential components" (p. 206). This implies that mathematical language is rich in visuals. Secondly, Arcavi (2003) is of the view that visualisation plays more than a perceptual role and is directly linked to conceptual understanding. Arcavi (2003) remarks further that visual representations have unfortunately been considered by some as 'second-class citizens'. This can be attributed to the prominence of conventional teaching methodologies that stress the application of rules, procedures and logical sequential steps in problem solving. However, Presmeg, (2014) emphasises the relationship between external representations and visual processes in school mathematics – which she claims are essential foundations of visualisation.

2.4.2 The role of visualisation in teaching and learning mathematics

Due to the important role visualisation plays in mathematical activities, it is imperative that teachers and students clearly understand the visualisation roles in a given mathematical situation. Haciomeroglu et al., (2010) allege that visual presentation is very helpful, especially for mathematics concepts that are of an abstract nature and require students to consider situations which do not physically exist.

Mathematics is built on abstract concepts (Bhagat & Chang, 2015), and mathematics students are often required to comprehend the abstract processes and concepts. During problem solving, students usually deal with symbolic representation of questions and then attempt to give meaning to algebraic and symbolic equations. Even in non-challenging questions, students normally base their thoughts on abstract concepts of axioms and theorems (Caligaris et al., 2015). In such features of mathematics, students have a great need to navigate from the abstraction level of mathematical solutions and formulas to the concrete level (Goldin, 2002). This can be performed with mental activities or physical constructions. In such circumstances, visualisation or visual representations of the same mathematical concept play a vital role.

Corter and Zahner (2007) categorised visual representations into two categories: internal (e.g. mental imagery) and external (e.g. pictures, graphs, diagrams). They elaborated that external visual representations used by problem solvers include those concepts with visuospatial relationships such as mathematical symbols, tree diagrams, graphs, Venn diagrams, contingency tables, formulas and pictures. External visual representations can help problem solvers (Veřmiřovský, 2018) build models (internal visuals), of the described problem situation. These can further augment cognitive capabilities by aiding memory and attracting attention (Crompton et al., 2018) and facilitate discovery and inference.

Konyalioglu (2008), separated visualisation into two forms, which he referred to as: non-mathematical representations and mathematical representations. Mathematical representations, he elaborated, are tangible or visible features, such as graphs, arrangements of manipulatives, concrete objects, diagrams, written words, mathematical expressions, formulas or depictions on the screen of a computer or calculator. These encode mathematical relationships or ideas. On the other hand, non-mathematical representations encompass non-tangible concepts such as spoken mathematical language, movements, gestures, interjections, facial expressions and postures.

From the students' perspectives, the abstract nature of mathematics makes it challenging for them to learn and comprehensively understand concepts. Many researchers indicate that visualisation plays an important role to overcome this challenge, (Arcavi, 2003; Corter & Zahner, 2007; Konyaloglu et al., 2011; and Presmeg, 2014). Corter and Zahner (2007) stated that, by placing emphasis on the external visual representations such as diagrams and graphs, visual tools support students' memory, provide models for students and attract their attention, thereby facilitating the finding of correct solutions of the problems.

There are numerous advantages of using visualisation in mathematics teaching and learning. For instance, in classroom applications, it is apparent that while students can see different representation types at the same time, they can easily make connections and can see the effect of any change in one representation type on another (Hur et al., 2016). Another advantage of using visualisation, according to Özkaya et al. (2016) is related to the fact that the major source of gathering information is the sense of seeing. He claimed that depending on what we see in an object, we can deduce information about it.

It is therefore logical to argue that visualisation may provide students with enhanced understanding of mathematical concepts that have an abstract and complex nature. The importance of figures, pictures and observation samples, emphasized by Mainali & Key (2008), may provoke mental operations such as constructing relations among abstract mathematics concepts. (Konyaloglu et al., 2011) add that in the process of mathematics education, visualisation has positive effects on both cognitive and emotional response.

Another advantage of using visualisation is that it deters students from learning by anchoring onto memorisation. As a matter of fact, some scholars do not consider memorisation to be learning (Soylu et al., 2009). Soylu et al. (2009) advocated for the enhancement of meaningful learning compared to memorisation-based learning. When students solve questions without comprehensive understanding, they are likely to encounter difficulties when they face new questions based on the same principles (Englund et al., 2017; Lawless & Pellegrino, 2007). According to Bansilal (2015) several studies have indicated that students can comprehend mathematical subjects and concepts by means of visual aids.

Evidence suggests that graphical representation of mathematical problems inspires students to gain more insight into a question (Dikovic, 2009). He elaborates that students benefit from the visual tools used in teaching and learning mathematics concepts by constructing multiple representations of the concepts and linking the mental images with such representations. Another positive attribute of visualisation is that students can, without much effort, accommodate the procedures needed in successful problem-solving processes, transfer the knowledge obtained from prior experiences to new situations and subsequently become successful in the new tasks (Hsieh et al., 2015). Additionally, students can deepen their problem-solving abilities and their understanding of the mathematical concepts through visualisation (Hitt et al., 2017). He further asserted that visualising mathematics is a way to transform the abstract nature of mathematics into concrete, which assists students to comprehend concepts easily. In a nutshell, using visual aids to learn and teach mathematics subjects is necessary, and the use of visualisation methods are gaining more prominence. However, if not used well, just like other approaches, using visualisation may not produce the desired results.

While most studies assert the positive aspects of visual tools and their use in teaching and learning mathematics, some researchers caution people about the possible misapplication of visual manipulatives. Shaw (2018) emphasises a positive correlation between the structure of the problem given to students and the appropriateness of the visual tool, otherwise problems in students' understanding and knowledge construction may arise. There is also a risk of students focusing only on the given visual material or model (Topuz & Birgin, 2020) at the expense of the knowledge to be learnt. It is therefore necessary for students to understand the relationship between the symbolic form of the mathematical notion and its visual representation (Ruthven, 2014). A lack of understanding of this relationship may result in a negative effect on students' understanding of the new knowledge. Zazkis (2016) explains that "this situation as perhaps the most harmful, yet quite a common difficulty with visualisation is that students have shown a lack of ability to connect a diagram with its symbolic representation, a process some authors consider to be an essential companion to visualisation" (p. 91). Kimmons and Hall (2018) add to this assertion. They postulate that there is a belief that visual representations and models cannot be a part of a proof, because these models and representations are difficult to comprehend and construct from students' perspectives.

Furthermore, Pantziara and Philippou (2007) point out other possible misleading features of the use of visualisation. They assert that the possible challenges of using diagrams can be the particularity of the diagrams, such as standard diagrams that neither support flexible thinking nor promote students' recognition and understanding of a concept, and that the same diagram can be perceived in different ways.

Research has also revealed that there is a prejudgment of visualisation among mathematicians (Zimmermann & Cunningham, 1991). Owing to the fact that most of the successes in formal mathematics were achieved through symbolic studies in the 19^{th} and 20^{th} centuries, visual approaches in mathematics are looked down upon. Therefore, there is a tendency among students, teachers and other mathematicians to think that mathematics requires formal and symbolic illustrations, not visual representations. Therefore, even in cases where visualisation is encouraged, teachers and students prefer to use formal mathematics when they are asked to show a proof or solve questions (Caglayan, 2014).

From another perspective, as teachers use visual tools to enhance students' learning, there is a likelihood of students getting engaged more in the figure or diagram being used than the analogical meaning of what it presents. This results in situations where students perceive the visual tool as the ultimate goal instead of simply a means to enhance their learning (Zimmermann & Cunningham, 1991). Another aspect is that the use of visualisation may only represent a restricted part of the given condition, while a lot of reliance on visualisation may prevent students from mathematical thinking (Presmeg, 2014).

Thus, visualisation in mathematics education facilitates a broader coverage of mathematical topics, and gives students access to new ways to approach mathematics (Elliott et al., 2000). Students are able to conceptualise ideas with visual arguments. Zimmermann and Cunningham, 1991) assert that:

> visualisation supplies depth and meaning to understanding, serving as a reliable guide to problem solving, and inspiring creative discoveries. In this sense, visualisation cannot be isolated from the rest of mathematics; in other words, symbolic, visual and numerical representations must be connected. (p. 35)

The use of applications presenting various semiotic registers in the process of learning mathematical concepts enables students to interact dynamically with different semiotic representations of the object studied, thus promoting conceptual learning (Caligaris et al., 2015). Among the education technology tools that are equipped with visualisation features and have potential to enhance the teaching and learning of mathematics is *GeoGebra* (www.geogebra.org).

2.5 GEOGEBRA

2.5.1 Use of *GeoGebra* as a visualisation tool to teach mathematics

The developments of technology applications such as drill-practice software, Computer algebra System (CAS) and Dynamic Geometry Software (DGS), have been comprehensive. In the last two decades DGS has been developed for both algebra and geometry. Among the software developed is *GeoGebra* (www.geogebra.org).

Created by Markus Hohenwarter in 2001 for his master's thesis study at the University of Salzburg, Austria and initially designed for students' use for algebra, geometry and calculus, *GeoGebra* has been developed to include statistics, probability, vectors, complex numbers and spreadsheets, and it is increasingly being used both as a learning and teaching tool. The functionality of DGS enables users to work with points, vectors, segments and lines, while the capabilities of CAS enables dynamic manipulation of coordinates and equations. These two characteristics of *GeoGebra* provide its default screen with two windows in which each object in the geometry window corresponds to an object in the algebra window, and vice versa, (Hohenwarter & Jones, 2007; Hohenwarter & Fuchs, 2004) It allows diverse representations of mathematical objects where points, vectors, lines, conic sections and functions can be graphed and modified dynamically.

GeoGebra is a DMS designed for teaching and learning mathematics from kindergarten to tertiary education. The software combines the ease of use of a DGS with features of a CAS, and therefore, allows for bridging the gap between the mathematical disciplines that include geometry, algebra, vectors, statistics, and calculus (Hohenwarter et al., 2007). *GeoGebra* can be used to visualise mathematical concepts as well as to create instructional materials. It also has the potential to foster active and student-centered learning by allowing for mathematical experiments, interactive explorations and discovery learning. Additionally, it is user friendly for beginners and has an easy-to-use interface for both students and teachers (Dikovic, 2009).

Unlike other technological tools which are equipped with only one of the features of DGS or CAS, *GeoGebra* has the added advantage of being equipped with both DGS and CAS, therefore making it easy to show synchronous multiple representations algebraically and graphically.

Its unique characteristic of establishing a relationship between geometry and algebra in one interface makes it more relevant and practical to the mathematics curriculum (Hohenwarter &

Jones, 2007). These attributes contributed largely to *GeoGebra* being adopted for this study, as calculus – the mathematical domain of this study – requires a coordination of algebraic and geometric concepts. By utilising DGS and CAS synchronously, DGS enables users to work with points, vectors, lines segments, and conic sections, while CAS is used to enter functions and equations algebraically. The data entered can be modified dynamically where necessary in the *GeoGebra* interface, a task which would not be easily undertaken using the conventional method of 'pencil and paper'.

GeoGebra can be used by both teachers and by students. Teachers can use it for instructing their students while students can use it for solving tasks. Findings from research indicate that students benefit from visualisations during demonstrations (Drijvers al., 2010). This is one major reason for teachers to use *GeoGebra*. Systematic use of the visualisation features of *GeoGebra* can help students to explore, solve problems, receive prompt feedback and to engage in reasoning (Jones et al., 2009; Hohenwarter & Jones, 2007; Tatar & Zengin, 2016). Kadunz concurs with this and states that "[o]ne of the most powerful and widely recognised didactical components of dynamic mathematics software is visualisation" (Kadunz, 1998, p. 198)).

Many studies have shown that DMS supports reasoning, problem solving and the development of conceptual understanding (Bu et al., 2010). During problem solving, students need clear reference to mathematical concrete objects, such as visual representations like geometric figures, graphs and algebraic expressions, (Sedig, 2015). *GeoGebra* enables students to create mathematical objects, which can be displayed in multiple representations. The representations are dynamically linked, so that if anything is altered, for instance in an algebraic representation, the graphical representation will accordingly adjust. Unlike in the use of pen and paper, *GeoGebra* accords students opportunities to construct supporting mathematical objects from direct instructions and to investigate relations between different representations and properties. Ultimately, through frequent interaction with the software during problem solving, students may construct a method to solve the problem (Villarreal & Borba, 2010).

The software enables visualisations of mathematical concepts and multiple representations. This attribute enables the users to create activities with multiple representations of mathematical concepts that are linked dynamically (Zengin et al., 2012). To effectively engage in problem solving and reasoning, students require visual mathematical representations for reference, for example algebraic expressions, graphs and geometric figures, (Tatar & Zengin, 2016). *GeoGebra* may be used to explore, construct and manipulate representations, for example, the calculus functions and their derivatives and integrals. When a mathematical expression is entered in *GeoGebra*, and its representation is automatically displayed, *GeoGebra* can adjust it in either the algebraic or the corresponding graphical representation (Preiner, 2008). Thus, as students engage in problem solving, they can use *GeoGebra* to make their ideas visual, confirm or falsify their assumptions, and to obtain immediate feedback. Additionally, *GeoGebra* is equipped with tools that students may use to examine the visual representations and to explore mathematical processes, relationships and properties. Furthermore, *GeoGebra* takes care of time-consuming constructions such as graphs. By using tools like sliders and drag-and-drop, students can easily construct variations of a graphical representation that can be used to generalise and explore concepts (Arcavi, 2000; Marrades & Gutiérrez, 2000). While it is appreciated that representations like graphs or geometric figures could also be constructed using pen and paper, however, the accuracy and speed with which the dynamic software enables students to construct multiple representations, to explore the dynamic representations, and to receive immediate feedback on their actions is not easy to accomplish with pen and paper. Research has furthermore revealed that the exploration, construction and the use of feedback is very beneficial to students. For instance, since *GeoGebra* takes care of time-consuming procedures like the drawing of graphs, students have more time to concentrate on other more cognitive aspects like reasoning and problem solving (Heid and Edwards., 2001; Zazkis, 2016). The way students use *GeoGebra* to explore various mathematical prosperities, processes, and relationships has shown to support their reasoning and their problem solving.

As students interact with *GeoGebra* and plan for the next step in their problem-solving process they need to engage in reasoning. Similarly, to construct a mathematical activity to submit to *GeoGebra*, students need to consider relations between concepts and mathematical properties (Drijvers et al., 2010; Olsson, 2017). Such interaction with *GeoGebra* may encourage students to predict the outcome of a computer activity. Research has revealed that students who engage in predictive reasoning are more efficient in reasoning generally (Lee et al., 2015). This potential of *GeoGebra* to support students' reasoning and problem solving could be used as a justification to include *GeoGebra* in a didactic design that expects students to take an explorative approach to solving tasks.

In summary, positive impacts of *GeoGebra* in mathematics include: enhancing mathematics teaching; conceptual development; enriching visualisation of concepts; laying a foundation for analysis and deductive proof; creating opportunities for creative thinking; and providing easy access to multiple representations of mathematics content (Hohenwarter & Fuchs, 2004; Hohenwarter & Jones, 2007; Hohenwarter & Lavicza, 2013; Ocal, 2017).

2.5.2 Teaching calculus using *GeoGebra*

With the pervasiveness of technology in almost all aspects of life globally, there is a notable increased use of computer software in the teaching and learning process. The integration of technology into the curriculum has availed the classroom to accommodate more flexible teaching methods that result in more engagement of students' learning processes, and in the changing roles of the teacher and the student. Consequently, the technology-backed learning environment has enhanced the learning process, and enabled students to access information, communicate with one another in real time and develop applications that make learning an active process.

In mathematics education, research has revealed that the use of technology has the potential to contribute to mathematical problem solving and develop creative thinking skills that can significantly contribute to mathematical reasoning and thinking (Tatar & Zengin, 2016). According to Samuels (2010) and Jaworski (2010), calculus is the starting point of higher mathematical thinking, and the limit function is the main concept of calculus. He argues that students often face difficulty in understanding the concepts of calculus and limit, but he emphasises that these are the foundation for all standards of modern analysis, and also form the basic conventional pedagogy in the introduction of calculus.

Researchers Zazkis (2016) and Aspinwall & Miller (2001) contend that in order for students to construct deep understanding of mathematical concepts, both analytic and visual reasoning must be present and integrated. Despite the significance given to visual thinking in understanding mathematics, Dreyfus & Eisenberg (1990) concluded that calculus students have a strong tendency to think analytically rather than visually. The use of dynamic software in calculus teaching helps make the learning of abstract concepts easier (Tatar & Zengin, 2016). Research has shown that technology use in teaching some mathematics content is more effective than traditional teaching methods (Zengin et al., 2012; Ross and Bruce, 2009; Reis, 2010).

Calculus is conducive to technology-aided learning environments in that its visualisation features and versatility offered by *GeoGebra* suits exploration. Therefore, teaching strategies for fundamental concepts of calculus should take advantage of *GeoGebra*'s dynamic and visualisation characteristics and the concepts of change, movement and relationships in calculus. The software's versatile capabilities can help presentations of calculus content in a manner that promotes 'meaning making'. One of the uses of *GeoGebra* in calculus is to enable the user to investigate the parameters of the equation of a curve using a mouse to drag the curve and observe the equation change, and conversely to change the equation of the curve directly and observe the change of the objects in the geometry window (Hohenwarter & Jones, 2007).

Sabella and Redish (2007) recount that the use of the technological tools for performing the procedures of calculus and algebra can free students from routine tasks, to concentrate more on exploring the underlying concepts. They elaborate that learning in a technology-supported environment may be more effective in promoting students' understanding of concepts of calculus. They added that:

> Topics in mathematics which lend themselves to computer implementation, have visual aspects which can be well represented on a computer screen; have transformational aspects which necessitate a dynamic implementation; have technical computational aspects which are not very relevant to the essence of the topic and are thus better being taken care of by the computer; and are intimately connected to the relationship between two different representations of the same concept; which can be dealt with in parallel by the computer program. (p. 7)

Bu et al. (2010) reaffirmed that:

> without having to spend a significant amount of classroom time on drawing figures, objects, or functions, students can explore mathematical concepts and dynamically connect algebraic, graphic and numeric representations of these concepts. (p. 25)

Tall (2009) reiterates that "the use of the computer as a tool for performing the procedures of calculus and algebra can free students to explore applications. The course can then de-emphasise skills and concentrate on the underlying concepts" (p. 14). In the study of definite integrals, Kendal and Stacey (2003) revealed that visual-spatial abilities play a crucial role in finding the volume formed by the curves of the objects and the area between curves. They added that spatial visualisation ability has an influence on the use of multiple representations in problem solving involving definite integrals.

A study was conducted by Engelbrecht et al. (2005) to elicit the opinions of teachers on using *GeoGebra* to teach definite integrals with a focus on the Riemann's sum. The findings showed that *GeoGebra* facilitates visualisation of lessons as the visual representation of the Riemann sums helped students' internalisation of concepts. They added that it facilitated a grasp of concepts, increased retention, enabled manipulation, fostered conceptual learning, and minimised memorisation. The study further revealed that the use of the software fostered the recognition of conceptual learning in the relationship between lower sum, upper sum and Riemann sums. The teachers indicated that the method was motivating, interesting, fun and facilitated concretisation of concepts. In affirmation, a participating teacher acknowledged "in particular, dynamic software keeps mathematics away from abstraction and helps to visualise the concept, which positively contributes to achievement in understanding the concept of definite integrals" (p. 31). Furthermore, Zengin et al., (2012) noted that the software enhances visuality and assists learners to discern the relationships between mathematical concepts. The examples below illustrate how *GeoGebra* can be used to enhance understanding of the concepts of slope in derivatives, and area under a curve in definite integrals.

Example 1: In differential calculus, the graph of the general polynomial function,

$y = ax^3 + bx^2 + cx + d$ can be created in the *GeoGebra* interface and setting sliders, $a, b, c \ and \ d$ from -10 to +10 for the minimum and maximum values respectively. *GeoGebra* can be used to make visual the underlying mathematical meaning attached to the constants of a polynomial, as well as explore the concept of the slope of the tangent by simply changing the values of the parameters, $a, b, c \ and \ d$. Students will be able to observe synchronously the changes made to the graph and to the equation in the geometric and algebraic views respectively, as shown in Figure 2.1. Such illustrations can motivate students to explore further.

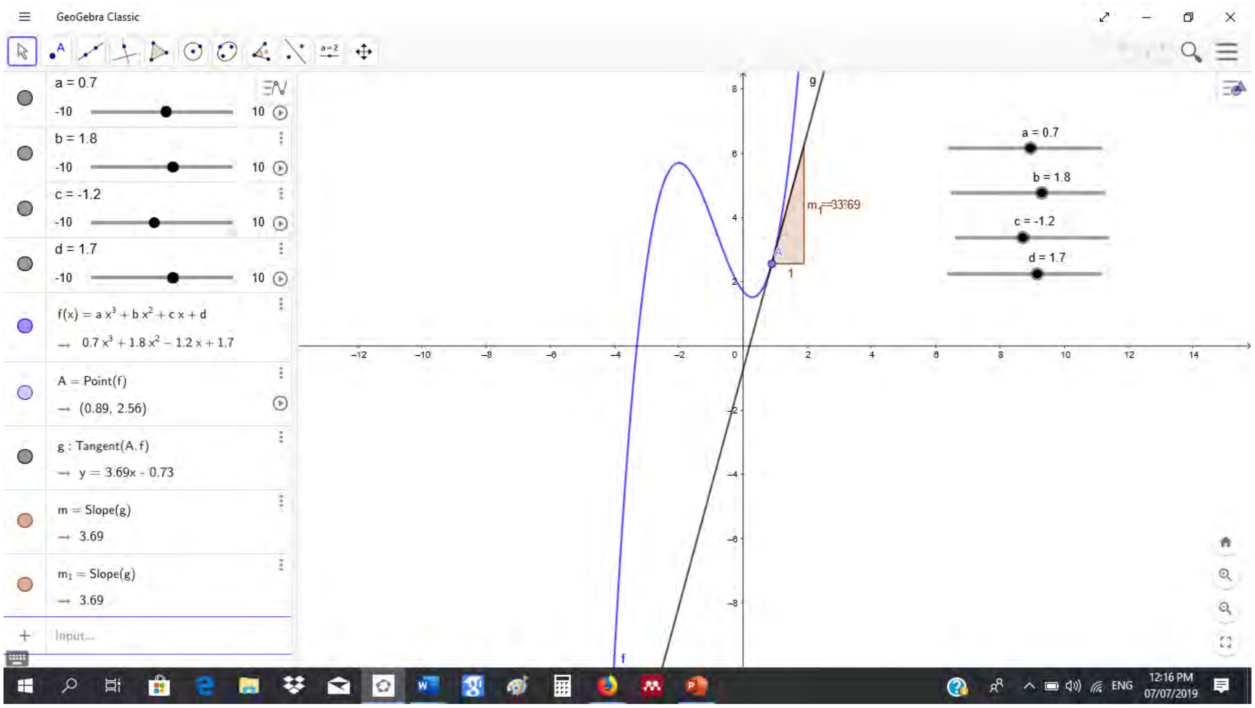

Figure 2.1: The cubic function drawn with the aid of GeoGebra

Example 2: (Adapted from Sanella, 2008)

GeoGebra can be used in the example below, (see Figure 2.2) to investigate some concepts on the definite integral using the Riemann sum. By writing the general equation $f(x)= ax^3 + bx^2 + cx + d$ and using sliders from $a, b, c \text{ and } d$ from -5 to +5 in the input bar of the *GeoGebra* window, and using the lower sum command of (f, a, b, n) a dynamic worksheet is obtained.

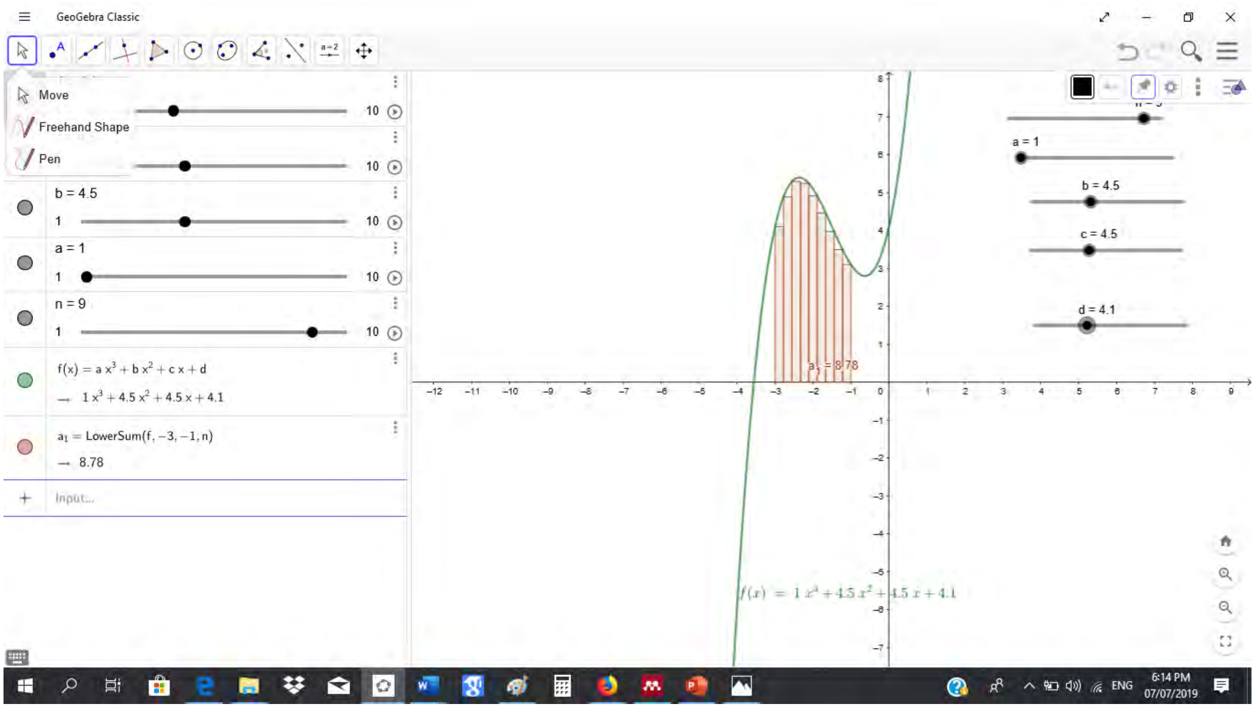

Figure 2.2: The Riemann sum visualisation using GeoGebra

The Riemann sum visualisation using *GeoGebra*

By dragging the endpoints a and b of the intervals with the mouse, learners can observe the characteristics of the rectangles of the lower and upper sums and discuss their observations with colleagues. Additionally, by moving the slider with the mouse, they can change the number of rectangles n, observe the effect of the difference between the lower and upper sum, write down their conjectures and discuss the results. Finally, by changing both the interval endpoints a and b and the number of rectangles n, they can investigate further and gain deeper insights into underlying concepts of the definite integral.

However, teachers are cautioned that technology should not just be used for its own sake, but should be used judiciously. From my experience, in most learning institutions, typical uses of technology tend to simply complement conventional teaching instead of making underlying changes to the dominant teacher-centred instructional paradigm.

Despite the potential of *GeoGebra* to enhance students' learning of mathematics including visualisation, manipulation and exploration of geometrical figures and mathematical concepts, lecturers in TEIs in Zambia hardly use *GeoGebra* for teaching. This is despite *GeoGebra* being a free open source software, with a lot of potential to enhance teaching and learning of mathematics. Unlike other software, *GeoGebra* does not require any licence fees, and is therefore well suited to low-income countries such as Zambia. Against this background, during a joint presentation with my students on *GeoGebra* at the 2018 Zambia Association of Mathematics Education (ZAME) national conference, teachers, lecturers and administrators showed overwhelming enthusiasm to learn how to use *GeoGebra.* Furthermore, having been a teacher and teacher educator for over three decades in Zambia, I have come to realise that one of the challenging topics both in secondary schools and TEIs is calculus. Calculus is a topic which emphasises rates of change and the relationship of one quantity with another quantity. Teaching strategies for fundamental concepts of calculus should take advantage of *GeoGebra*'s dynamic and visualisation characteristics and the concepts of change, movement and relationships in calculus. The software's versatile capabilities can help presentations of calculus content in a manner that promotes 'meaning making' (Caligaris et al., 2015). I was therefore motivated to undertake this study due to the overwhelming desire shown by teachers and lecturers to learn how to use *GeoGebra* and the students' poor performance in calculus at both secondary school and in TEIs. Additionally, calculus lends itself well to exploration using the features of *GeoGebra*.

This study locates itself in TEIs and investigates lecturers' experiences and perceptions of incorporating *GeoGebra* as a visualisation tool to teach calculus to undergraduates in TEIs. It also investigates the factors that enable and constrain lecturers' adoption of technology in teaching. The findings of this study may contribute to literature on good practices of teaching calculus. The findings may also guide and provide a basis to design appropriate instructional materials for the consolidation of the Teaching and Learning Mathematics with *GeoGebra* (TLMG) project for quality teacher professional development in the use of *GeoGebra* for mathematics teachers, with a view to incorporating *GeoGebra* in the Zambian teacher education curriculum.

2.6 MATHEMATICAL PROFICIENCY

2.6.1 Conceptual understanding and procedural knowledge

Proficiency in teaching mathematics, according to Kilpatrick et al. (2002), relates to effectiveness and consistency in helping students learn mathematical content that is meaningful. Kilpatrick et al. (2002) used the term 'mathematical proficiency' to describe what they considered was necessary for a person to learn mathematics successfully. They reiterated that teaching for mathematics proficiency encompasses "focusing on the interactions between teachers and students around educational materials and how teachers develop proficiency in teaching mathematics" (p. 48). They identified five strands essential for developing mathematics proficiency, namely: conceptual understanding; procedural fluency; strategic competence; adaptive reasoning and productive disposition (p. 330). An important aspect about the five strands is that though itemised separately, they are interwoven and interdependent, and therefore represent different aspects of a complex whole of developing proficiency in mathematics. Notwithstanding the importance of all the five strands of mathematics proficiency to the learners' understanding of mathematics, this study will only focus on the first two strands: conceptual understanding and procedural fluency, based on their relevance to the study and their important implications to teaching. Kilpatrick et al. (2002) suggest that mathematical proficiency requires similarly interrelated components that include conceptual understanding of the core knowledge of mathematics and procedural fluency in carrying out instructional procedures.

Several studies in mathematics teacher education are in agreement that the hallmark of the reform processes in mathematics education is teaching mathematics for understanding (Cockroft, 1982; (NCTM, 2000). Ball and Bass (2000) appreciate this view, but cautions that teaching mathematics for understanding is a complex process that calls for considerable review of content and pedagogical knowledge. Kilpatrick et al. (2002) refer to conceptual understanding as "an integrated and functional grasp of mathematical ideas" (p. 42). They explain that students with conceptual understanding know more than isolated facts and methods. They understand the kinds of contexts in which a mathematical idea is useful and why the idea is important. Such students, they add, organise their knowledge coherently, which enables them to learn new ideas by connecting them to their prior knowledge.

The connection between facts and methods learned with understanding plays a critical role in the acquisition of conceptual knowledge. This connection helps in retention and construction of knowledge (Ball & Bass, 2000). An important indicator of conceptual understanding, suggest Kilpatrick et al. (2002), is the ability to interpret mathematical concepts in multiple representations and knowing how the multiple representations can be useful in different situations. These multi-representations could be in visualisation features such as diagrams, symbols or computer graphics.

In mathematics education, understanding connections between similarities and differences between various connections is an important aspect. “The degree of students’ conceptual understanding is related to the richness and extent of the connections they have made” (Kilpatrick et al., 2002, p. 122). Knowledge anchored on conceptual understanding provides a strong basis for solving new and unfamiliar problems and generating new knowledge. In a study by Mhlolo and Schafer on learners’ responses in geometric tasks in South Africa, it was observed that when solving mathematics tasks, learners erroneously made connections based on subjective impressions. The study revealed that geometrical problems required both visual and conceptual understanding (Mhlolo & Schafer, 2013).

Kilpatrick et al. (2002) are of the view that “when learners have acquired conceptual understanding in an area of mathematics, they see the connections between concepts and procedures and can give arguments to explain why some facts are consequences of others” (p. 119).

On the other hand, “[p]rocedural fluency refers to knowledge of procedures, of when and how to use them appropriately, and skills in performing them flexibly, accurately, and efficiently” (Kilpatrick et al., 2002). Procedural knowledge is demonstrated Engelbrecht et al. (2005) by learners’ ability to solve a problem through the manipulation of mathematical rules, skills, formulas, algorithms, procedures and symbols used in mathematics.

Engelbrecht et al. (2005) observed that mathematics teaching at almost all levels of education tends to be generally procedural. The common trend, he adds, is that most students enter university with well-developed skills for manipulation but with little exposure to deeper conceptual understanding. This often leads to students being proficient in procedural ways of thinking but lacking sound basic concepts of mathematics. He observes that from a teachers’ perspective, communication systems for calculus and other mathematics topics in schools tend to follow the procedural approach to teaching mathematics and not laying much emphasis on the conceptual base. Affirming this view, Aspinwall and Miller (2001) recount that “students regard computation as the essential outcome of calculus and thus end their study of calculus with little conceptual understanding” (p. 17). Figure 2.3 below illustrates the difficulties students encounter in solving problems on derivatives and integrals.

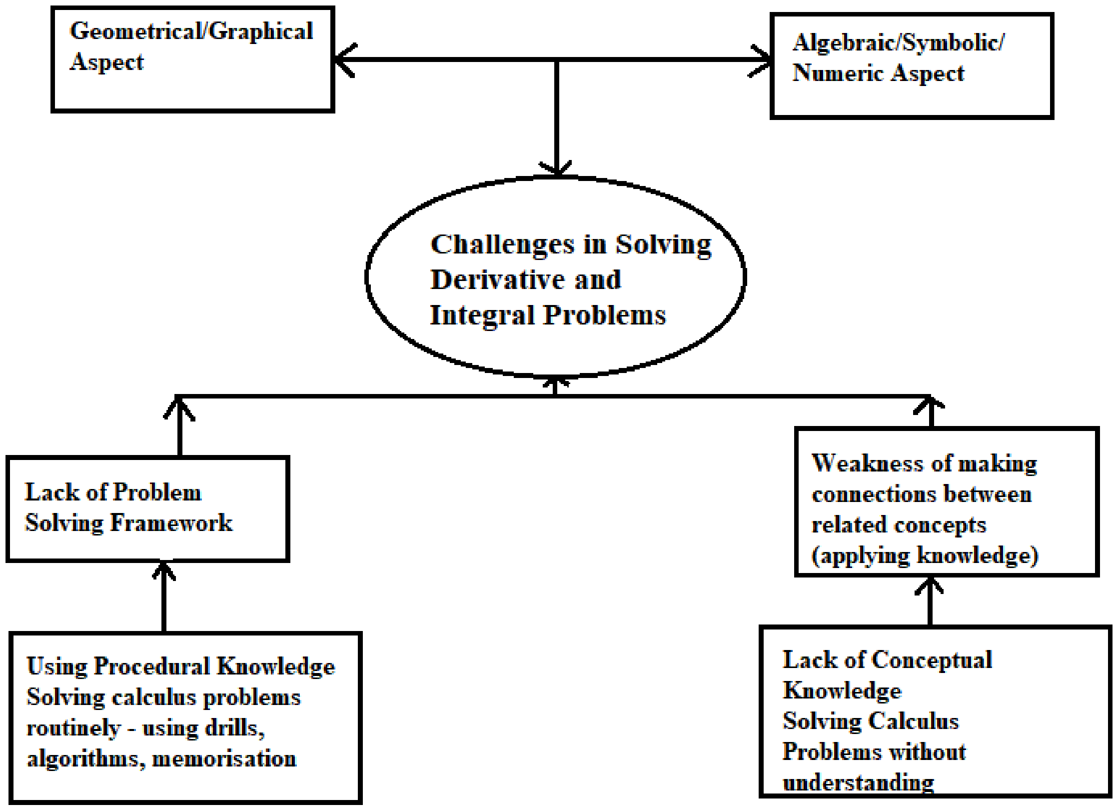

Figure 2.3: Difficulties in Solving Derivative and Integral Problems – adapted from Hashemi et al. (2015)

From a technology perspective, the NCTM (2000) emphasises that the use of appropriate technology is key to developing students' conceptual understanding of mathematics sense-making (p. 25). Research indicates that learning environments that foster conceptual understanding through multiple representations could be created through the use of technology. They argue that technology provides students with many opportunities to learn abstract concepts based on individual learning styles and interests (Alacaci & McDonald, 2012; Özgün-Koca & Meagher, 2012; Meagher & Edwards, 2011).

Kilpatrick et al. (2002) emphasise that to acquire mathematics proficiency, a close link between conceptual understanding and procedural fluency needs to be taken into consideration. In their study, Miller and Upton (2008) discovered that graphical representations link more appropriately with conceptual understanding. By contrast, symbolic expressions, they assert, lend themselves to procedural operations. Such representations, explain Kay and Knaack (2008), can concretise abstract concepts if undertaken with a sound pedagogical approach. *GeoGebra* is a tool that possesses features to represent mathematical concepts symbolically and graphically and is therefore well suited for mathematics pedagogies that foster conceptual understanding and procedural fluency. It can therefore be argued that successful teaching of mathematics requires a combination of both conceptual understanding and procedural fluency.

Among the reasons for teachers' use of *GeoGebra* in mathematics classrooms is to make students learn mathematics conceptually and meaningfully (Ocal, 2017). Research in mathematics education indicates that effective use of *GeoGebra* supports and has a positive impact on students' conceptual understanding and performances in a variety of mathematics topics including calculus (Aydos, 2015; Tatar & Zengin, 2016; Tekin et al., 2021).

Aspinwall and Miller (2001), Mahir (2009) and Tatar and Zengin (2016) have reaffirmed that one of the major reasons for students experiencing challenges in learning calculus emanates from inadequate conceptual knowledge. This is exacerbated by the students' perception that computations done in solving calculus questions is the ultimate goal in calculus courses, thus their focus is mainly on procedural knowledge (Nedaei et al., 2021).

In a study conducted by Tatar and Zengin (2016) to determine the impact of *GeoGebra* software on the achievement of pre-service secondary mathematics teachers in their conceptual understanding of definite integrals, it was discovered that *GeoGebra* software:

> … facilitates visualisation in lessons, enables a better grasp of concepts, increases retention, provides an environment of conceptual learning instead of memorisation, and enables practicing; in particular, it was recognised as enabling conceptual learning of the relationship between lower sum, upper sum, and Riemann sums. (p. 122)

Additionally, Tatar and Zengin (2016), in their study on first grade university students' discovered that conceptual understanding on the definite integral was significantly higher in the experimental group where the mode of instruction was done with *GeoGebra*. Similarly, Aydos (2015), conducted an investigation into the influence of teaching mathematics with *GeoGebra* on secondary school students' conceptual understanding of continuity and limits. The findings revealed that the students in the experimental group that received intervention with *GeoGebra* performed significantly better in the test that measured their conceptual knowledge than those who received conventional instructions. From these findings, it can be deduced that teaching the application of derivatives and integrals with the help of *GeoGebra* has the potential to impact positively on students' achievement regarding their conceptual knowledge. This development can be attributed to the nature of the topic – calculus – coupled with the characteristics of *GeoGebra*. Rasslan and Tall (2002) attest to this notion and affirm that the concepts in calculus are abstract – which may require students to deal with formal proofs and definitions.

Though the above studies were done using the pre- and post- experiment methods, this study takes a qualitative approach and focuses on the perceptions and experiences of teacher educators as they incorporate *GeoGebra* as a visualisation tool to teach calculus in TEIs to pre-service teachers.

2.7 THEORETICAL FRAMEWORK

2.7.1 Constructivism

Different researchers have acknowledged the complexities of integrating technology into learning and teaching (Angeli and Valanides, 2009; Artigue, 2019; Drijvers et al., 2015; Niess, 2011 and Gueudet & Trouche, 2011). The complexities of integrating technology in teaching and learning have led researchers to advocate for various models of integrating technology in mathematics education. Some researchers like Wang, (2008) proposed a generic model, consisting of social interaction, pedagogy and technology, while others, eg Margaret and Happiness (2019) proposed a framework that guides and describes the process of teachers' learning as they develop their model. Christou et al. (2006) affirm that sound theoretical frameworks with reliable innovative reference models are indispensable in informing the design of technology-oriented learning environments.

The umbrella theoretical framework for this study is constructivism (Piaget, 1967; Cobb, 2016). This study also draws on two key theoretical models on the integration and adoption of ICT in teaching and learning: technological pedagogical content knowledge (TPACK), Mishra & Koehler, 2006), and the technology acceptance model (TAM), (Davis, 1989). These theoretical perspectives offer insight into the factors that influence teachers to adopt and integrate technologies in general and ICT in particular. These perspectives touch on crucial insights that could not have been achieved by reliance on a single perspective. These models will widen the scope of inquiry and pool the issues learned from research that spans methodologies and disciplines, and thus provide insights that may help to extend the depth of understanding of the integration and adoption of ICT in teaching and learning.

The constructivist theory has been very prominent in past research on mathematics learning and teaching and has provided a basis for many recent reforms in mathematics teaching (Cobb, 2016). In the recent past, the constructivist approach to teaching was central to empirical and theoretical research in mathematics education (Cobb, 2016; von Glasersfeld, 1987; Confrey & Kazak, 2006). Constructivism is an epistemological and a philosophical approach which describes learning as a change in meaning that is constructed from experience (Ottenbreit-Leftwich et al., 2018). Constructivism places learners at the centre of the learning process. It holds that knowing is a process of actively constructing and interpreting individual knowledge representations. A concern for a lived experience as experienced and understood by the learner is the core of constructivism.

Roblyer and Doering (2015) contend that in a constructivist environment, knowledge is constructed by learners through experience-based activities as opposed to direct instruction which is based on behaviourist and information-processing models of learning. It is concerned with how learners construct knowledge. One of its main tenets is that human beings construct knowledge of the world from their experiences and perceptions, which are mediated through previous knowledge and acknowledge the existence of the external reality (Confrey et al., 2010).

The constructivist environment accords learners an opportunity to give meaning based on their experiences, socially or individually (Fox, 2010; Narayan, 2016). Barak (2017) explains that constructivism postulates that knowledge is not transferred to the student, but constructed by a student. The students are therefore not blank slates, as they bring prior knowledge and experiences to the learning situations. The experiences and prior knowledge can be integrated into the new knowledge that students construct and make an impact (Merriam et al., 2007). On the contrary, in a more traditional, teacher-centered classroom, information transfer is prevalent, and students are less likely to be engaged as the meaning is in most cases interpreted on their behalf by the teacher. Olofson et al. (2016) reminisces that there should be interaction among learners and knowledge of prior social experiences for knowledge to be constructed. Recent efforts to integrate technology into the process of teaching and learning in the classroom have been within a constructivist framework.

The two constructivist perspectives that have dominated research on technology in mathematics education are radical constructivism and social constructivism. Cobb (2016) argues that social and radical constructivism cannot be separated since they complement each other. Both perspectives share the common epistemological assumption that knowledge or meaning is not discovered, but rather constructed by the human mind. Knowledge is invented or actively created by the learner and not passively received from the environment (Cobb, 2016). Students, therefore, need to construct their own understanding of mathematical concepts. There are, however, some differences between radical and social constructivism. The major difference between the two is centred on knowledge construction. Despite both perspectives having the same general view of how individuals learn or construct knowledge, they differ with respect to the mechanisms they see at work.

Radical constructivism is associated with von Glasersfeld, whose thinking was greatly influenced by the theories of Piaget. Von Glasersfeld (1987) defines radical constructivism based on the conceptions of knowledge. He assumed that external reality cannot be known and that the learner constructs knowledge ranging from observations to scientific knowledge. He focuses on individual learners and pays little attention to the social processes in knowledge construction. Radical constructivists take a cognitive perspective and argue that students must discover knowledge by themselves without explicit instruction. Its focus is on the individual's construction of knowledge. Olofson et al., (2016) are of the view that even though social interaction is seen as an important aspect in radical constructivism, the focus is on the reorganisation of individual cognition.

However, this research is anchored in the social constructivist theoretical perspective, with particular emphasis on the social construction of knowledge through the mediation of technology. The social constructivist perspective influenced the selection of a constructivist epistemology which assumes that intellectual development is preceded by learning through mediated transactions and that social interaction plays a vital role in the children's acquisition of knowledge (Kalina & Powell, 2009; Wertsch & Rupert, 1993; Ernest, 1994; Narayan, 2016, and Gurung, 2019).

The social or realist constructivist theory is often said to derive from the works of Vygotsky. Ernest's (1994) opinion was that the social constructivist theory acknowledges that both individual sense and social processes have an essential part to play in the learning of mathematics. Notwithstanding the varied views among social constructivist theorists, they all subscribe to the idea that the social environment plays a central role in learning. They believe that learners are to be enculturated into their learning community and appropriate knowledge through their interaction with the environment. Students are therefore encouraged to construct their own understandings through social negotiation and validation of new perspectives (Ertmer & Newby, 2013).

Social constructivist pedagogy emphasises teaching approaches that focus on concepts and contextualisation as opposed to instructing isolated facts (Brooks & Brooks, 1999). The theory also gives credence to students' social interaction with peers and the teacher and suggests that student's preferred learning styles should be given consideration (Palincsar, 1998; Kolb & Kolb, 2019). Research further suggests that students who adopt constructivist approaches to learning mathematics tend to follow a conceptual approach in solving problems, while those who follow traditional teaching approaches tend to incline more to procedural approaches (Summit & Rickards, 2013). Constructivist approaches are considered to be cardinal to deeper understanding and internalising concepts (Narayan, 2016). Fosnot and Perry (2005) emphasise that the constructivist approach focuses on a holistic view of learning mathematics, and on deep understanding and strategies, rather than rote memorisation.

Mvududu and Thiel-Burgess (2012) claim that constructivism is touted as an approach where learners' levels of understanding are probed, with a view to increase their understanding to higher level thinking. Constructivism attempts to explain how students can make sense of concepts and also how the concepts can be taught effectively. Goodwin and Webb (2014) affirm that constructivism changes the learners' roles to that of active participants in the learning process from that of passive recipients of knowledge. This has the potential to enable learners to become involved in applying their existing knowledge and real-life experiences, learn to test their conjectures and in due course assess their findings.

Social constructivism considers learning as a collective process through which people learn by interacting with signs, artefacts, peers and adults (Goos, 2014; Galbraith et al., 2001). It focuses on how individuals construct knowledge and make sense of their world. From a technological perspective, social constructivists argue that the formation of learning requires the use of technology and symbolic tools to act as mediators of knowledge acquisition (Artigue, 2019).

Vygotsky, a key proponent of social constructivism, focused on the social factors that influence learning. Digital technology, which has significantly changed the learning environment from what Vygotsky originally conceived it to be, did not exist in Vygotsky's days. However, the introduction of technology compelled some researchers (Ridgway, 2016; Verillon & Rabardel, 1995) to revisit

Vygotsky's socio-constructivist theories with a view to reconceptualise them in line with the emerging research field of technology, by starting to infuse technology where appropriate into mathematics education (Simsek & Clark-Wilson, 2018).

Jonassen (2019) contends that in a constructivist-learning environment, students use technology to explore relationships and manipulate data. He further suggests that, in their quest to respond to tasks and challenges that come from actively engaging in mathematics problems and environments, learners form models that construct mathematical knowledge. This implies that learners do not simply take in information. According to Lerman (1989), "constructivism is defined by a widely accepted hypothesis which states that knowledge is actively constructed by the cognising subject, not passively received from the environment" (p. 211).

The fast-increasing developments in technology have led to new dimensions to necessitate technology inclusion within the domain of cultural artefacts accessible in mathematics learning environments. Crawford (2016) considers technology as cultural artefacts that teachers and students can use to mediate and internalise mathematics learning.

The social constructivist perspective adopted for this study entails understanding how interaction with *GeoGebra* as a visualisation tool can enhance the teaching of calculus. Kaptelinin (2005) argues that by understanding the ways in which people use an artefact and the needs it serves, the nature of an artefact can be understood within the framework of human activity. They further state "… technology is just another artefact that mediates the interaction between learners and their learning environment, but it has the potential to empower the individual learner to develop cognitive structures" (Kaptelinin, 2005, p. 56).

The need for social interactions (Cobb et al., 2015), is underscored, owing to the fact that mathematical meanings are socially constructed and culturally situated. For instance, some major developments of mathematics, such as the history of the derivative, reveals that they were in response to the needs of the specific generations and the ideas were influenced by society. On the basis of this, mathematics teaching must contextualise, as much as possible, the mathematical concepts being taught. Ernest (2006)) surmises that mathematics cannot be understood outside its history.

Constructivism provides mathematics education with insights on how students learn mathematics. It also guides various stakeholders on how to use instructional strategies that begin with children rather than adults or teachers (Fosnot & Perry, 2005). In their description of constructivist compatible instruction, Zbiek et al. (2007)) noted that it stems from the theory of learning that suggests that understanding arises through prolonged engagement of the learner in relating new ideas to the learner's own prior beliefs.

Constructivist approaches generally utilise student-centred methods in mathematics instruction and encourage students to search for information that stimulates thinking (Mokhtar et al., 2013). Lecturers in TEIs should therefore strive to use technology to foster student-centred methods as they teach pre-service students. Utilisation of student-centred methods in mathematics instruction has been reported to have positive effects on student learning, which includes an increase in students' interest in the subject and their success rate, an increase in students' appreciation of the role of mathematics in life and the motivation to learn mathematics and realise its applicability (Ruggiero & Mong, 2015; Edwards & Ward, 2008). Research has also revealed that in general, student-centred approaches enhance students' motivation in learning mathematics.

Constructivist teaching beliefs are more likely to adopt learner-centered teaching methods and are inclined to innovation in instruction approaches compared to the teachers that hold conventional teaching beliefs. Furthermore, Tondeur et al. (2008) point out that teachers who embrace high constructivist beliefs in their pedagogies actively use ICTs more compared to those with low constructivist beliefs.

The constructivist approach to teaching accords a teacher an opportunity to examine learning from the child's perspective (Olivier, 1989). Misconceptions therefore form an important entry point for teaching. For instance, students learning about definite integrals in a context that involves area under the $x-$ axis would develop a misconception that the numerical value for area can be negative. Such misconceptions (Bell, 1993), should be challenged by the use of counterexamples. Appropriate use of *GeoGebra* can help mitigate such misconceptions. Almeida (2010) further points out that teaching is more effective if it focuses on challenging, identifying and ameliorating the misconceptions. From a constructivist perspective, students should be given authentic tasks that allow them to observe, experiment, explore, make conjectures and construct generalisations.

In specific reference to mathematics teaching and learning, Cobb (1988), a constructivist proponent, states that:

> A fundamental goal of mathematics instruction should be to help students build structures that are more complex, powerful, and abstract than those that they possess when instruction commences. The teacher's role is not merely to convey to students' information about mathematics. One of the teacher's primary responsibilities is to facilitate profound cognitive restructuring and conceptual reorganisaisation. (p. 89)

This resonates with what students, teachers and lecturers need in the 'Teaching and Learning Mathematics with *GeoGebra* (TLMG) project' in Zambia – to have a deeper, conceptual understanding of mathematical concepts in calculus and other mathematics topics. Learners can be supported in this by providing logical explanations to justify a generalisation. The software *GeoGebra* has the potential to undertake such tasks. Since *GeoGebra* provides learners with an opportunity to actively construct, experiment and explore mathematical concepts, it can be argued that tools within a *GeoGebra* interface can be utilised to facilitate such cognitive restructuring and conceptual reorganisation as espoused in a social constructivist perspective.

2.7.2 Technological Pedagogical Content Knowledge (TPACK)

A number of researchers in the field of technology integration in teaching and learning adopt the TPACK framework to explore the development of teacher knowledge about technology integration (Lee & Hollebrands, 2008; Mishra & Koehler, 2006; Harrington et al., 2019). Building on Shulman's (1986) pedagogical content knowledge (PCK) framework, Mishra and Koehler (2006), developed the TPACK framework. This framework has emerged as a particularly important way of conceptualising research and practice in technology-driven classrooms (Koehler et al., 2014). They argued that teacher knowledge for technology integration is built on interaction among three domains of knowledge: content knowledge, pedagogy, and technology. TPACK describes the complexities and challenges of technology integration, informs strategies required to better prepare future teachers for learning and teaching in the 21st century and articulates the importance of teacher training (Koehler et al., 2017). Ruthven (2014) pointed out that the idea of TPACK draws attention to how the new technological resources reshape pedagogical knowledge, content knowledge and pedagogical content knowledge.

Confrey et al. (2010) emphasised the importance of TPACK in understanding mathematical concepts and their inter-relationships, in order to effectively determine how these can be represented within the mathematics software. They further stressed that teachers' content knowledge is transformed in problem-solving contexts as well as in multiple representations of concepts. The focus of TPACK is on the knowledge teachers need to meaningfully teach with technology.

One of the benefits of using TPACK is that it allows teachers to make thoughtful decisions about what technology best suits their teaching and students (Oberdick, 2015). Kopcha et al. (2014) elaborate that TPACK is a body of professional knowledge that teachers need to have in order to significantly incorporate pedagogy and technology into the content that they teach. Mishra and Koehler (2009) emphasise that:

> this professional knowledge is about effective teaching with technology, requiring an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge to develop new epistemologies or strengthen old ones. (p. 9)

The interaction of these bodies of knowledge, as shown in Figure 2.4, both theoretically and in practice, produces the types of flexible knowledge needed to successfully integrate technology use into teaching.

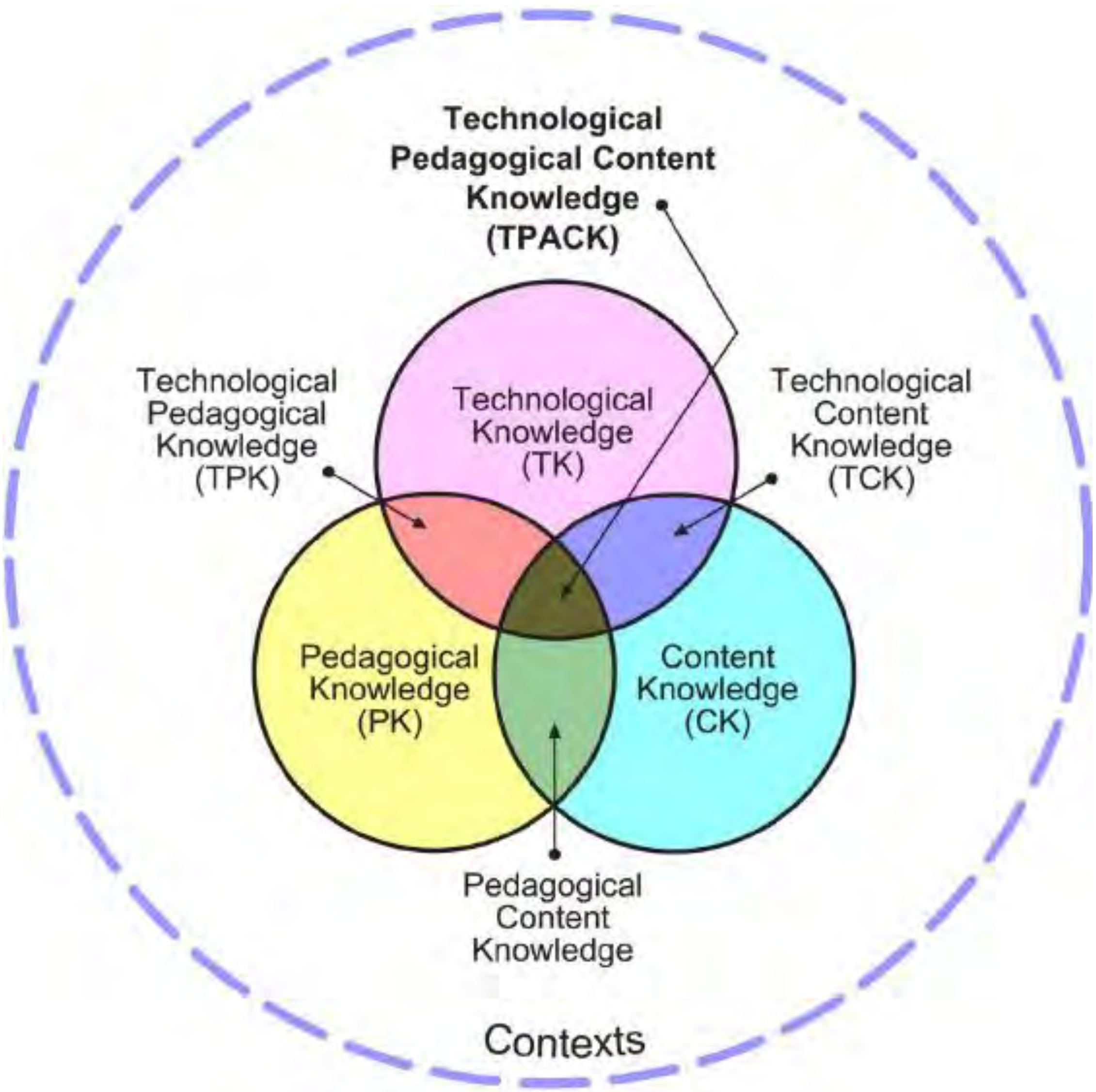

Figure 2.4: TPACK framework and its knowledge components

The resulting knowledge components of TPACK are: Technology Knowledge (TK), Content Knowledge (CK), Pedagogical Knowledge (PK), Pedagogical Content Knowledge (PCK), Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK); and Technological Pedagogical Content Knowledge (TPCK) (Baran et al., 2011).

The TPACK model is essential to this study as quality teaching with the help of technology requires thorough understanding of the complex relationships between technology, content and pedagogy. Teachers require TPACK to understand how to effectively use technology to present concepts in constructivist approaches. TPACK is considered an appropriate lens for the study of mathematics teacher educators' knowledge development as they work on *GeoGebra* tasks where the tasks are designed to advance both mathematics and technology knowledge. Since TPACK focuses on the knowledge of the teacher (Mishra & Koehler, 2009), it would help lecturers to make informed decisions about which topics can effectively be taught in a *GeoGebra* environment and in understanding the teaching and learning theories that are appropriate. As teachers create dynamic worksheets for their lessons using *GeoGebra*, they also synthesise their TPACK. Mishra & Koehler, (2009) argue that there is no perfect approach to integrate technology into the curriculum, claiming that the process is complex. It is therefore incumbent upon teachers to develop and understand this complexity in their quest for successful integration of technology into mathematics teaching.

The TPACK framework is a criterion for effective integration of technology in education (Mishra & Koehler, 2006). These authors clarify that TPACK is the interaction of these bodies of knowledge, theoretically and in practice, to produce the types of knowledge needed to successfully integrate technology use into teaching. (Mishra & Koehler, 2006, p. 63) describe the seven knowledge constructs as follows:

> Content Knowledge (CK) is teachers' knowledge about the subject matter that incorporates knowledge of theories, concepts, and organisational frameworks. It also includes knowledge of proof and evidence and established approaches and practices toward developing such knowledge.
>
> Pedagogical Knowledge (PK) is teachers' knowledge about the processes, practices and methods of teaching and learning. It includes knowledge about techniques used in the classroom; the nature of the audience targeted, and strategies to evaluate students' understanding.
>
> Technological Knowledge (TK) is the knowledge about different technologies and requires a mastery and thorough understanding of thinking about and working with technology.
>
> Pedagogical Content Knowledge (PCK) is knowledge of pedagogy that is appropriate to the teaching of specific content. It covers the fundamental aspects of teaching, learning and assessment. It includes issues that foster links among curriculum, pedagogy, and assessment.
>
> Technological Content Knowledge (TCK) is the knowledge about pedagogy needed to understand the specific technologies that are best suited to address the subject-matter when learning various domains. It also incorporates how content may dictate or alter the technology and vice-versa.

> Technological Pedagogical Knowledge (TPK) is the knowledge needed for deep understanding of the affordances and constraints of technologies and the contexts within which their functions are applied.

To sum up, TPACK is the intersection of the three domains of knowledge that teachers need to implement the curriculum while they support students' learning with technologies for specific content. It is the desirable knowledge required for effective teaching with technology, demanding an understanding of the interactions among the three knowledge domains: content, pedagogy, and technology, based on the interrelation of these domains and their contextual parameters.

In the context of my study, the TPACK framework constructs are defined as: CK is the teacher educators' knowledge about calculus including knowledge of concepts, theorems, symbols and graphs; PK is the teacher educator's knowledge about the processes and methods of teaching and learning calculus; TK is the knowledge about *GeoGebra* that requires a deep understanding and mastery of effective ways of working with *GeoGebra*; PCK is knowledge of pedagogy that is relevant to the teaching of calculus; TCK is knowledge required to understand how *GeoGebra* is best suited to address the learning of calculus; TPK is knowledge needed for a thorough understanding of the affordances and constraints of using *GeoGebra* to teach calculus; and finally, TPACK is the knowledge required for teaching calculus with *GeoGebra* effectively.

In view of the above, to understand TPACK well, there is a need to view the three knowledge domains as interrelated and not in isolation.

2.7.3 Technology Acceptance Model (TAM)

One of the most challenging issues in educational research is understanding why organisations accept or reject technological innovation (Luhamya et al., 2017). TAM (Davis, 1989) is an adaptation of the theory of reasoned action (TRA) (Ajzen & Fishbein, 1980), which explains factors that influence users' acceptance of information technology. Technology acceptance, according to Masrom et al. (2009) is "an individual's psychological state about his or her voluntary or intended use of a particular technology" (p. 139). It is a model related to technology adoption and an empirically tested theory (Venkatesh & Davis, 2016). It is underpinned by a social psychological approach to explain the adoption of technology and the factors that influence individuals' decisions to adopt technology in their work. It is a model that adapts the belief-attitude-intention-behaviour relationship of the TRA to clarify users' acceptance of technology.

The TRA operates on the premise that the intention to use a computer-driven technology is influenced by its users' beliefs and perceptions (Ajzen & Fishbein, 1980). In the context of ICT in education, TAM has been perceived as useful by several researchers and as a strong determinant of user intentions (Venkatesh & Davis, 2016)). TAM provides one of the perspectives for understanding the integration and adoption of ICT in the teaching and learning process. It attempts to provide an explanation of the determinants of computer acceptance in general and user behaviour across a wide range of end-user technologies in computing.

Research findings state that teachers' attitudes towards ICTs have a strong influence on the acceptance of the usefulness of ICTs in their lessons and a bearing on whether teachers integrate ICTs into their classrooms (Teo, 2011; Huang & Liaw, 2005)). This is echoed by Cuban et al. (2001) who add that, other than beliefs, effective implementation of education reforms is also dependent on teachers' knowledge, attitudes and skills. The significance of teachers' contribution to this process is underscored by the NCTM (NCTM, 2000) in their declaration of the teacher being one of the six major factors in the effective use of new technology in mathematics education.

Venkatesh et al., (2007) claim that "TAM currently enjoys the status of being the prime tool for testing user acceptance of new technologies" (p. 139). It is underpinned by a social psychological approach to explain the adoption of technology and the factors that influence individuals' decisions to adopt technology in their work. TAM theorises that an individual's behavioural intention to use technology is essentially determined by two beliefs: perceived usefulness and perceived ease of use (Davis et al., 1989). Perceived usefulness is the extent to which a person believes that using the system will enhance work performance, whereas perceived ease of use is the extent to which a person believes that using the system will be effortless (Davis et al., 1989). The full constructs of TAM are External variables, Perceived usefulness, Perceived ease of use, Attitudes towards ICT, Intention to use and Actual use ((Davis, 1989)), as shown in Figure 2.5.

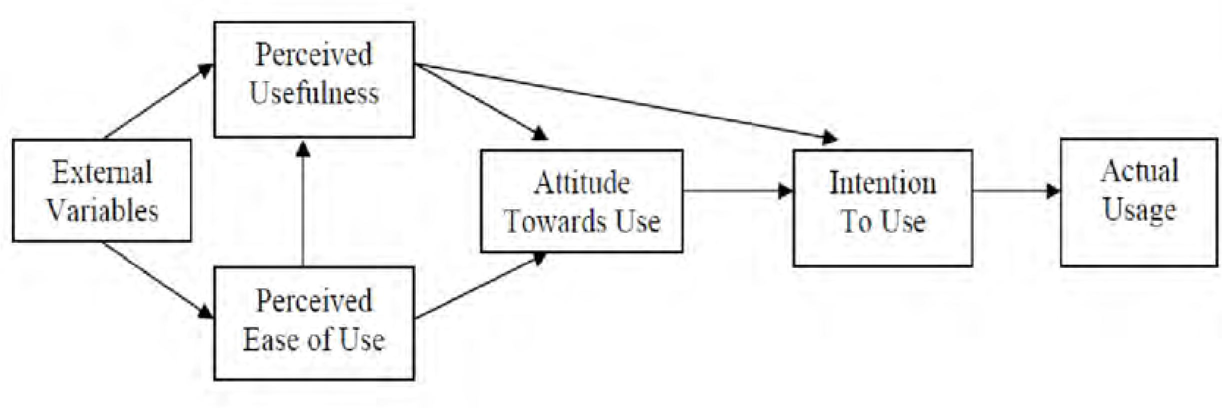

Figure 2.5: Technology Adoption Model

Ajzen (1991) explains that external factors in the TAM framework include institutional policies, beliefs about the environment such as support staff, infrastructure and access to ICTs. Internal factors, he adds, encompass skills, abilities and attitudes. In the context of ICT in education, TAM has been perceived useful by several researchers as a strong determinant of user intentions (Venkatesh et al., 2007).

TAM contends that the attitude of the user towards use of technology for teaching and learning is very vital Hew and Brush (2007) concur with this view and elaborate that changing attitudes and beliefs about technologies are an important factor and should take precedence in teachers' ability to integrate technology into teaching. These observations resonate with Ertmer (2005) who elaborates that "[I]f we truly hope to increase teachers' uses of technology, especially uses that increase student learning, we must consider how teachers' current classroom practices are rooted in, and mediated by, existing pedagogical beliefs" (p. 19).

Tondeur et al. (2017a) point out that teachers who actively embrace constructivist beliefs in their pedagogies are more likely to adopt ICTs compared to those with low constructivist beliefs. Echoing this view, Ananiadou and Claro (2009) contend that teachers with constructivist beliefs use technology as a means to assist students to develop higher order problem-solving and thinking skills and to support students' capacity to "apply knowledge and skills in key subject areas and to analyse, reason, and communicate effectively as they raise, solve, and interpret problems in a variety of situations" (p. 7).

Systems which are considered highly in perceived usefulness, are those in which users believe in the existence of a positive use-performance relationship. In contrast, perceived ease of use (Davis, 1989) is "the degree to which a prospective user believes that using a particular system would be free of effort" (p. 8). Anecdotal evidence suggests that an application perceived to be easier to use than another is more likely to be accepted by users. The TAM model posits that both perceived usefulness and perceived ease of use correlate with system use, a relationship that explains why teacher educators may accept or reject technological innovations (Maslin, 2007).

As Stols and Kriek (2011) observe, *GeoGebra* combines its ease of use aspect with the construction features of a DGS and the functionality of a CAS. It lends itself to a wide range of possible applications for teaching mathematics This aligns well to the main constructs of TAM of 'Perceived usefulness' and 'Perceived ease of use'.

In this study, the three frameworks discussed, namely the TAM, TPACK and constructivism, explain the connections between technology, pedagogy, content, beliefs, attitudes and practice. The frameworks are aligned with each other as TPACK can help to identify the nature of knowledge required by lecturers for technology integration using *GeoGebra* (as an artefact in a learning environment that mediates in the learning process), while TAM can provide meaningful information on the link between intention and motivation to integrate technology in the teaching and learning process from a constructivist perspective. As alluded to earlier, social constructivism lays emphasis on the importance of prior knowledge and the construction of knowledge. This resonates with Salomon (2006) who argues that the balance between accessing prior knowledge and constructing new knowledge with technology tools has changed, with the scale inclining towards construction of new knowledge. In a constructivist classroom, learning is constructed actively, reflectively and collaboratively, and it is enquiry based and evolving. Constructivism also holds that problem solving is at the heart of learning, thinking and development.

Furthermore, this study is located in interpretivism, where knowledge is believed to be acquired through involvement with content as opposed to repetition or imitation. The constructivist approach to learning contends that learning is personally constructed and is attained through interaction with tools or artefacts (Maloney et al., 2004). Among such tools is *GeoGebra,* which is equipped with the attributes of a constructivist environment as outlined in Chapter 1.

Constructivist learning environments also provide multiple representations of reality (Gilakjani et al., 2013) and also provide opportunities for hands-on activities.

GeoGebra is a technological tool equipped with these constructivist features and therefore, in this study, as lecturers interact with *GeoGebra* when teaching calculus concepts to pre-service students, their focus is aligned to these constructivist goals.

This is in resonance with what the calculus reform movement emphasised on the use of multiple representations in the presentation of concepts: that they should be represented graphically, numerically, algebraically – tasks that can be done accurately and effectively with *GeoGebra* (Haciomeroglu & Haciomeroglu, 2020). Integrating technology into discourses of learning and teaching in using technology within the existing curriculum, implies that curricula should be flexible to incorporate technology tools so that teachers can create new learning environments that engage students in constructivist approaches to learning.

However, to mediate teaching and learning in a technology environment, the (NCTM, 2000) states that "teachers select or create mathematical tasks that take advantage of what technology can do efficiently and well-graphing, visualising, and computing" (p. 97). This is not something that teacher educators can achieve by a simple combination of software and hardware. Studies indicate that teachers encounter challenges in the integration of ICT into the classroom. Some of these barriers can be attributed to a lack of TPACK, and teacher resistance, as espoused in the TAM constructs, to integrate technology in a constructive teaching and learning discourse.

2.8 CONCLUSION

In this chapter, I discussed the use of ICT in education. Visualisation processes in mathematics education and the role of visualisation in teaching and learning mathematics were also discussed. I then discussed the use of the DMS, *GeoGebra,* as a visualisation tool to teach mathematics in general and calculus in particular. I finally looked at mathematics proficiency and the theoretical frameworks that guide the study.

CHAPTER 3: METHODOLOGY

3.1 INTRODUCTION

The aim of this research project was to specifically explore how *GeoGebra* could be incorporated as a visualisation tool to teach mathematics, with a particular focus on teaching calculus to enhance conceptual understanding. Consequently, this study sought an in-depth investigation of how *GeoGebra* could be used as a visualisation tool to teach calculus to pre-service students in TEIs in Zambia to enhance conceptual understanding. Furthermore, the interviews and observations I conducted, sought an in-depth understanding of the perceptions and experiences of lecturers on using *GeoGebra* as a visualisation tool to teach calculus in TEIs in Zambia; and finally the study sought to gain insights into the enabling and constraining factors of using *GeoGebra* to teach mathematics. All these are discussed in view of how they relate to answer the research questions of the study as presented in Chapter one, which are:

- How can *GeoGebra* be used as a visualisation tool to teach calculus to pre-service student teachers in TEIs to enhance conceptual understanding?
- What are the perceptions and experiences of lecturers in using *GeoGebra* as a visualisation tool to teach calculus in TEIs in Zambia?
- What are the enabling and constraining factors of using *GeoGebra* in the teaching and learning of mathematics?

3.2 THE RESEARCH ORIENTATION

This study is oriented in the qualitative research framework. Qualitative research designs focus on data gathering that occur in natural phenomena (Williams & Morrow, 2009). Qualitative data is usually in the form of words and is generated through the use of various methods in order to gain a deep understanding of events. In line with the key research questions and the need to deeply understand the perceptions and experiences of teacher educators on the incorporation of *GeoGebra* as a visualisation tool in the teaching and learning process, the qualitative approach underpinned by the interpretive research paradigm was chosen to answer the research questions. It is affirmed that "to understand the subjective world of human experience is fundamental in the context of an interpretative paradigm', (Cohen et al. 2018, p. 21).

The qualitative approach allows the researcher to make sense of the participants' engagement towards a phenomenon under consideration in a natural setting, such as a mathematics classroom. This is done through the use of interviews, focus group discussions, observations, audio and video recordings, field notes and photographs, among others, to interpret or make sense of the participants' engagement or response to a phenomenon under consideration (Denzin & Giardina, 2017). The active involvement of participants is therefore critical to constructing meaningful understanding.

Patton (2002) postulates that a paradigm is a worldview of breaking down the complexities of the world and that paradigms are deeply ingrained in the socialisation of practitioners and adherents. The researchers' worldview determines what they choose to place emphasis on during the data gathering and analysis processes. Interpretivists interpret the world through the conceptual lens formed by their previous experiences, existing knowledge, beliefs and assumptions about the world, and their theories about knowledge and how it is accumulated (Carroll & Swatman, 2017).

In an interpretive paradigm, researchers rely heavily on participants' perspectives when reaching a conclusion. The interpretive perspective is anchored on the premise that a person's way of making sense of the world should be respected (Patton, 2002). In view of this, in this study I engaged the participating lecturers in interviews and observations in an attempt to figure out how they arrived at their generalisations and conjectures. Additionally, the tasks the lecturers undertook as they taught pre-service students using *GeoGebra* applets on derivatives and integrals provided feedback vital to arriving at the findings. According to Cohen et al. (2018), "What characterises the interpretive paradigm is that the researcher tries to make sense of the world from the participants' point of view. The intention is to get 'inside' the participant and to understand his/her world from within" (p. 152). The actual words of the participants are vital in conveying the meaning of the participants – which ultimately guide the findings of the study. The researcher's role is crucial as he or she interprets the participants' interactions and responses based on the interventions. In this study, I drew inferences on how teacher educators incorporated *GeoGebra*, a visualisation tool, to teach calculus to pre-service teachers in TEIs to enhance conceptual understanding.

3.3 RESEARCH METHODOLOGY – CASE STUDY

In the field of education, research is situated in diverse epistemological settings that create a diverse methodological and intellectual landscape. Research methodology, according to Cresswell (2014), is a systematic and theoretical analysis of the methods used in a field of study. Daymon and Holloway (2001), affirmed this view and accentuated that "a methodology is a systematic framework which guides the researcher in the selection of participants, collection and analysis of data, and presentation of the findings" (p. 129). In educational research, there are generally three major methodological approaches: quantitative and qualitative research, or a combination of the two – mixed methods.

This is a qualitative case study oriented in the interpretive paradigm. Qualitative research aligns itself with the interpretivist paradigm, and aims to understand the social world (Newby et al., 2012). Qualitative research is described in various ways by different authors. Daymon and Holloway (2001) refer to qualitative research as a type of social enquiry with a focus on the way people make sense and interpret their experiences. Quick and Hall (2015) contend that a qualitative approach is systematic and is used to describe life situations and experiences to provide meaning. On the other hand, Creswell and Cresswell (2017) envision that qualitative researchers retain a focus on the meanings that are held by participants about an issue. It is a holistic approach that explores experiences, behaviour, views, and emotions of the participants (Cohen et al., 2018; Williams, 2007). Qualitative research seeks to understand and describe human experiences (Myers, 2000). Approaches that are used in qualitative research include narrative enquiry, grounded theory, ethnography, phenomenology and case study (Creswell & Cresswell, 2017).

As is the case in this study, in qualitative research, a researcher often approaches reality from a constructivist perspective, which allows for various meanings of individual experiences (Denzin et al., 2011). Qualitative data can provide rich information about human behaviour based on participants' own meanings (Creswell et al., 2011).

The rationale for adopting a qualitative case study approach was because the aim of this study was to explore the understanding and interpretation of lecturers' perceptions and experiences of incorporating *GeoGebra* as a visualisation tool to teach derivative and integral concepts to pre-service students. Case studies, according to Ertmer and Newby (2013), can be excellent at giving researchers a rich understanding of a situation. Cohen et al. (2018) claim that there is a resonance between interpretive methodologies and case studies. A case study is a systematic and in depth study in its context, where the case may be a person (such as a teacher, a learner, a principal or a parent), a group of people (such as a family or a group of learners, a school, a community or an organisation (Rule & John, 2011, p. 4). Yin (2014), adds that case study designs allow for a study to examine phenomena empirically within real-life contexts when the boundary between context and phenomena are not apparent. I explored selected lecturers' perceptions and experiences (phenomena) of teaching calculus within a teacher education mathematics programme (context), and as such, the nature of the inquiry appropriately suited the case study design. Case studies are styles of research that are often used by researchers in the interpretivist paradigm where the researcher aims to capture the reality of the participants' lived experiences of, and thoughts about a particular situation (Cohen et al., 2018, p. 182). This study involved interactions with lecturers from three TEIs.

The data collection occurred over a period of six months between June 2020 and March 2021. The case study approach enabled me to gather holistic and rich information from the participants' perspectives in a natural classroom setting where technological software was used. Newby (2014) states that a case study is a "detailed analysis of an individual circumstance or event that is chosen because something new is in operation" (p. 51). In this regard, the use of *GeoGebra* in TEIs in Zambia was a novel phenomenon that required empirical reflection in order to understand the complexities of teaching with technological devices. In context, as *GeoGebra* was used to teach calculus to students, the study aimed to understand the processes that teacher educators used in the process. I studied several cases in a single project in order to gain a comprehensive understanding of the phenomenon. In this single project, the case comprised six lecturers from three different TEIs.

Though case studies may be used in some cases in quantitative analysis, Farquhar et al. (2020), observe that in a qualitative study, case studies opt for analytic as opposed to statistical generalisation, as they can develop a theory which can assist researchers to understand other similar cases, situations or phenomena. Cohen et al. (2018) suggest that the case study approach is notably valuable when the researcher has little or no control over events. They further suggest that a case study has several characteristics, that include:

- concern with a rich and vivid description of events relevant to the case;

- providing a chronological narrative of events relevant to the case;
- blending a description of events with analysis;
- focusing on individual actors or groups of actors, and seeking to understand their perceptions of events;
- highlighting specific events that are relevant to the case;
- integral involvement of the researcher in the case; and
- portraying the richness of the case in writing up the report (p. 253).

The six mathematics lecturers from three TEIs who interacted with *GeoGebra* applets to teach derivatives and integrals to pre-service mathematics teachers constitute the case in this study. Therefore, the unit of analysis is the six lecturers' use of *GeoGebra* as a visualisation tool to teach calculus to enhance conceptual understanding, their experiences and perceptions of using *GeoGebra,* and the enabling and constraining factors of using *GeoGebra* to teach and learn mathematics.

The research sites were the three TEIs in Zambia. I selected two lecturers from each TEI. All three TEIs are government owned and they train students in a three-year secondary teaching diploma programme in mathematics. The calculus activities on derivatives and integrals that were used to create applets for the intervention in the study were based on activities that were common in the curricula of all the three TEIs.

3.1.1 Participants and selection criteria

In qualitative research, the selection of research sites and participants is purposeful and intentional to best understand the central phenomenon under study (Creswell & Cresswell, 2017). There are various purposeful sampling strategies, and the onus is on the qualitative researcher to select one based on their intent regarding the sampling. The researcher selects individuals because they are convenient, available and represent some characteristics the researcher seeks to study.

The six participants were purposively selected. In qualitative research, purposeful sampling is widely used for the selection and identification of information-rich cases linked to the phenomenon of interest. Six mathematics lecturers, two from each of the three TEI research sites in Zambia comprised the sample. In purposive sampling (Cohen et al., 2018), researchers "handpick the cases to be included in the sample on the basis of the judgement of their typicality or possession of the particular characteristics being sought. They assemble the sample to meet their specific needs" (p.

218). This sampling technique is useful for selecting participants from a group of people who hold information relevant to the study based on their experiences (Palinkas et al., 2015). The six participants were all lecturers in TEIs who are computer savvy and have experience of teaching calculus to undergraduate students. I present a brief background of each of the six participant lecturers who took part in this study.

Lecturer 1, from TEI 1, had been teaching for over fifteen years. Having initially trained as a primary school teacher, where she obtained a certificate in primary school teaching, she furthered her studies and obtained a secondary teachers' diploma with specialisation in mathematics and geography. She once again furthered her studies at a local public university and obtained a bachelor's degree with education (BA Ed), in mathematics and geography, and was subsequently transferred to a TEI. Despite having specialised in both mathematics and geography, she only taught mathematics at college at the TE1. TEI 1 offered mathematics education using both full time and distance education modes: in early childhood education, (ECE), primary and secondary education and Lecturer A was involved in all the programmes.

Prior to her involvement in this project, Lecturer 1had not used any technological software in her teaching. She had, however, used Microsoft Paint.net, to draw diagrams mostly for preparing her assessment papers. She was one of the two participants who had the *GeoGebra* software installed on her laptop, but had never used it. During the training, she was very committed and eager to learn, and showed the same commitment during her lesson presentations to students. She offered both mathematics methodology and content courses at TEI 1.

Lecturer 2, also from TEI 1, had been teaching for between eleven and fifteen years. Of all the six participants, he was the only one who had gone directly to university after leaving secondary school, as the others had first gone to college before proceeding to university as in-service students. Upon completing his BA Ed degree programme in mathematics, he taught mathematics in various secondary schools and later read for a master's degree programme in mathematics content at a public university. He was promoted as lecturer at TEI 1 even before he graduated with his master's degree. He was only offering mathematics content at TEI 1. During the *GeoGebra* training course, he was quick to learn and exhibited sound knowledge of content in calculus. He presented his lessons with confidence and did not have major challenges in learning how to use the software.

On the use of technology in his teaching, Lecturer 2 revealed that he used PowerPoint to present his lessons occasionally.

Lecturer 3 of TEI 2 had been teaching for over fifteen years. He initially trained as a junior secondary school teacher at a public college and obtained a secondary teachers' diploma. Despite having trained to teach at junior secondary school, (Grades 8 and 9), due to the shortage of teachers in schools he was assigned to teach senior classes (Grades 10, 11 and 12), in ordinary level mathematics. When the teacher who was qualified to teach additional mathematics was not available, he stood in for him and he subsequently developed an interest in teaching additional mathematics. After teaching for close to twenty years, he furthered his studies and obtained a BA Ed qualification in mathematics. Upon completion of the degree programme he was promoted as a lecturer at TEI 2.

While at secondary school, he hardly used technology in his teaching, as he always used the 'chalk and talk' method. When he moved to teacher education, he was introduced to Microsoft Excel, which he mostly used to compile students' results. On his experience of teaching calculus, he said students did not generally have challenges with differentiation, but encountered difficulties with integration, especially with questions involving graphs, area and volume. At TEI 2, lecturer 3 taught both content and methodology in mathematics education.

Lecturer 4 of TEI 2 had been teaching for over 15 years. Like a couple of the other participants, he initially trained as secondary school teacher, and obtained a diploma in mathematics and geography. After teaching for a couple of years, he went for further studies and obtained a BA Ed degree in mathematics and geography. He was transferred to TEI 2, and later obtained a master's degree in mathematics content. Prior to taking part in this study, lecturer 4's interaction with technology was limited to PowerPoint presentations during lessons, and using Microsoft Excel to manage students' results and records.

He found the teaching of calculus in TEIs interesting, but pointed out that students generally had challenges in certain components. These components included differentiation and integration of trigonometric, exponential and logarithmic functions. He attributed the challenges to poor foundation in from secondary school and lack of practice by students. He further explained that students had fewer challenges with questions that only required the use of the power rule.

Lecturer 5 of TEI 3 had been teaching for over fifteen years. Likewise, he initially trained as a primary school teacher, where he obtained a certificate and taught at primary school. Before he had even finished a year in service, he enrolled as a distance education student for a diploma programme, and obtained a secondary teachers' diploma in mathematics. He was consequently transferred to a secondary school. After teaching for some years in secondary school, he again enrolled through the distance mode for a BA Ed programme with Mathematics as major and geography as minor subjects. At the time the study was being conducted, he been a lecturer at TEI 3 for seven years and he was offering mathematics only, both content and methodology. His interaction with technology in his teaching process was confined to PowerPoint presentations of some of his lessons, and he asserted that he had never used any packaged software to present a lesson. On his experience of teaching calculus, he said it was good, as the students were able understand certain concepts, and made an effort to research on their own when they faced challenges.

Lecturer 6 of TEI 3 had also been teaching for over fifteen years, and just like his colleagues, he had started from primary school and worked his way up. For his professional qualification, he held a primary teachers' certificate, a secondary teachers' diploma and a BA Ed, in mathematics. At the time of the study, he was pursuing a masters' degree in mathematics education. He used technology from time to time in class was to present lectures on Power Point. He affirmed that his students encountered challenges learning calculus and attested that there were some concepts in calculus that even he was not very comfortable to teach as well.

3.4 RESEARCH DESIGN

A research design, essentially, is a plan of how the researcher systematically collects and analyses the data that is required to answer the research question (Bertram & Iben, 2014). Kothari concurs and accentuates that "a research design is the conceptual structure for collection and analysis of data in a manner that aims to combine relevance to the research purpose with economy in procedure" (Kothari, 2004, p. 48). Cohen et al., (2018) echo this view and expound that:

> Producing knowledge includes, inter alia: a rigorous and coherent research design that demonstrates fitness for purpose; appropriate sampling, methodology and instrumentation; transparency, usefulness and validity; scholarly and scientific merit; significance and advancement of the field (e.g. substantively, conceptually, methodologically); scientific value; risk assessment and minimisation; and transparency." (p. 29)

This study sought to investigate the subjective understanding and interpretations that are the experiences of the participants concerning the use of *GeoGebra* applets as a visualisation tool in the teaching of calculus. Key to the study involved drawing on the judgment and inferences of the participants' engagement with the *GeoGebra* software and applets and their individual and their joint understanding of information as I interacted with them for six months.

This research project involved an in-depth study of six lecturers as they interacted with the *GeoGebra* software in their preparation and presentation of lessons on four calculus cycles in three TEIs. The training sessions I had with the participants on the use of *GeoGebra* in their respective TEIs accorded me an opportunity to interact with them and thereby establish a mutual and professional relationship. The intervention was done in the classrooms when the lecturers implemented their lessons using *GeoGebra* applets on derivative and integral calculus topics.

I adopted a cyclic approach for the research design. The research design unfolded in four cycles, where each cycle was determined by a topic of the calculus curriculum. The cycles were 'slope of a tangent on a curve',l imits from differential calculus, Riemann sum and area between curves from integral calculus i.e.

Cycle 1: Slope of a Tangent on a Curve
Cycle 2 Limits:
Cycle 3: Riemann Sum
Cycle 4: Area between Curves

Each cycle consisted of three stages: planning, implementation and reflection, as shown in Figure 3.1 below

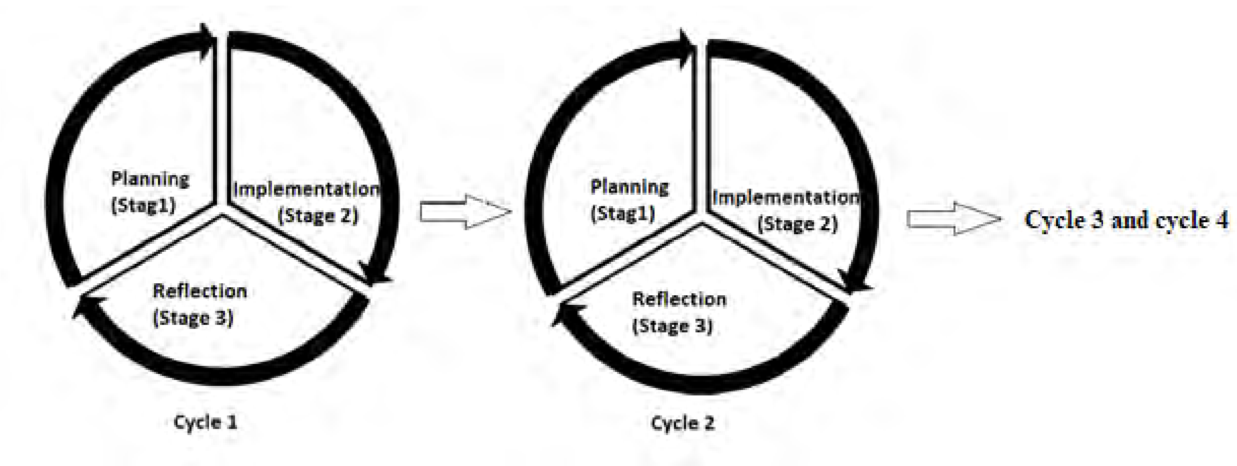

Figure 3.1: Stages of each calculus cycle

Stage 1: Planning

In this stage, the two lecturers and I in each TEI planned four lessons based on each cycle. We then manufactured a *GeoGebra* applet for each cycle.

Stage 2: Implementation

Each of the two lecturers from each institution conducted two lessons with the aid of the applets, one on differentiation and the other on integration to the same group of students in their respective TEIs. The total number of lessons in all the three institutions was therefore twelve. I observed and video recorded each of the twelve lessons. Video recordings of the lessons generated the data for the study.

Stage 3: Reflection

I split the reflection stage into two parts:

(a) Stimulated Recall Interviews

Using the stimulated recall interview, each lecturer and I analysed their lesson according to my analytical framework. Video recordings of the lessons generated data for the study.

(b). Enabling and constraining factors

In this part, each lecturer and I reflected on the enabling and constraining factors of using *GeoGebra* as a visualisation tool to enhance the teaching of calculus, based on the cycle that was taught. Refinement of each cycle informed the planning of the next cycle.

Each of these three stages were repeated for the three groups of lecturers where Group 1 was: Lecturer 1 and Lecturer 2 in in TEI 1; Group 2: Lecturer 3 and Lecturer 4 in TEI 2; and Group 3: Lecturer 5 and Lecturer 6 in TEI 3.

3.4.1 The Teaching and Learning of Mathematics with *GeoGebra* (TLMG) project

This qualitative case study underpinned in the interpretive paradigm, was undertaken with a broader goal of designing *GeoGebra* applets and instructional materials on various mathematical topics in the context of a project called 'Teaching and Learning Mathematics with *GeoGebra* (TLMG) project', among teachers and lecturers in Zambia.

The findings of this study may inform policy, guide and provide a basis to design appropriate instructional materials for the consolidation of the TLMG project for quality teacher professional development in the use of *GeoGebra* for mathematics teachers, with a view to incorporating *GeoGebra* in the Zambian teacher education curriculum. It is envisioned that the TLMG project can also be used as a springboard to establish the Zambian mathematics education community in the International *GeoGebra* Institute, (IGI), which incorporates a global network of students and teachers, that fosters support for learning, improvements and innovations in mathematics education.

3.4.2 The Pilot Study

Pilot studies can serve as prior research, but with more focus on the researcher's concerns and theories. They can be designed specifically to test ideas and methods and explore their implications. Kothari (2004) argues that many features of the research design cannot be rigorously determined without exploratory research. It is therefore advisable to conduct a pilot study before a study is undertaken as it helps, among other things, to reveal the weaknesses of the data collection instruments. A pilot study highlights the weaknesses of the instruments and also of the methods and techniques.

For my pilot study, I conducted a training workshop for three mathematics lecturers who were not part of my study sample, at the institution where I am based, starting with the basic use of *GeoGebra.* My experience of using *GeoGebra* provided a guide to identify the actions and responses of lecturers when they used the technological tools. Initially, some lecturers experienced challenges in using the technological tools and lacked confidence, but after a number of trials, their confidence and ability improved. I then worked with them following the stages as outlined in the research design in Section 3.4 above and administered the pilot test by asking lecturers to present lessons based on calculus cycles

The insights gained from the pilot project helped me to refine the analytical framework and sharpen my data gathering and analysis instruments. Cresswell (2014) put forth that "there is often a fine line between questions being too detailed or too general., therefore a pilot test of the questions on a few participants can usually help one to decide which ones to use" (p. 226). Suffice therefore to say that before using the data collection instruments, it is always advisable to conduct a pilot study for testing, reviewing, editing and adapting the instruments.

3.4.3 The Research Instruments

As soon as I was granted the ethical clearance approval from Rhodes University and the six participants gave consent to take part in my research project, my field work immediately commenced. I engaged with the lecturers during the *GeoGebra* training sessions to familiarise them with the *GeoGebra* tools and how to use the software. My data collection schedule was determined by completion of the training session in each of the three TEIs. I used multiple techniques to gather data from the participants. These were: interviews, focus group discussions, observations, and audio and video recordings.

Cohen et al. (2018) affirm that case studies accept and recognise that there are different variables that operate in a single case. Therefore, there is a need to use more than one tool for data collection in order to capture the implications of the variables. In view of this, and to add to the validity of this study, I triangulated the data. Data was collected from six lecturers in three TEIs in Zambia through interviews, focus group discussions and observations over a period of six months at three TEIs. I used interviews and observation protocols (see Appendix C) during the data collection process. The study drew on the constructs of constructivism – the umbrella theoretical frameworkand those of TPACK and TAM – that underpin this study (see Appendix C).

3.5 DATA ANALYSIS

According to Newby (2014), data analysis is an activity to enable "data to release the information we need to answer our research question" (p. 395). It is through data analysis that people make sense of the data and also communicate the essence of its revelations. The process involves explaining and organising the data from the participants' perspective, accounting for categories, noting themes, regularities and patterns (Cohen et al., 2018). Newby (2014) suggested four stages of generic protocol of qualitative analysis: preparing the data, identifying basic units of data, organising data and interpreting data.

For an analysis to be considered an accurate representation of what transpired during the course of the study, the different methods with which data was collected must agree with each other and this emerges through a process of data triangulation. An analytical framework provides a guide to organisation of data to be collected and how to analyse it (Cohen et al., 2018). The analytical framework is used to identify information that is useful for analysis, from that which may be discarded. The analytical framework tools in Appendices C guided the data analysis.

Data, collected from video recordings of observations of the lecturers' lessons, consisted of two data sets: observation data and interview data, which I analysed separately. I initially analysed the observation data using the video recordings of each of the lecturers' lessons, based on the analytical tools generated from the constructs of the enabling theoretical frameworks of TAM and TPACK. Teachers require TPACK to understand how to effectively use technology to present concepts in constructivist approaches. Additionally, as espoused in the TAM constructs, teachers' attitudes and beliefs about technology are important factors in making decisions to integrate technology into teaching (Hew & Brush, 2007).

The analysis of the interview data was done in three stages based on the three research questions that guided my study. Each lecturer and I analysed the interview data using the respective video recordings of the lessons in a stimulated recall environment. Stimulated recall (Gass & Mackey, 2009) is an introspective research technique where participants' cognitive processes can be investigated. Participants report on their thoughts on task interactions while viewing the video and by listening to their audio recordings in order to elicit data. The analysis of the interview data was done thematically.

The focus of the analysis was on gaining the lecturers' insights on each of the following research goals:

- lecturers' use of *GeoGebra* as a visualisation tool to teach calculus to pre-service students to enhance conceptual understanding;
- the perceptions and experiences of lecturers in using *GeoGebra* as a visualisation tool to teach calculus in TEIs; and
- the enabling and constraining factors of using *GeoGebra* in the teaching and learning of mathematics.

3.5.1 Analytical Framework

The umbrella theoretical framework for this study is constructivism (Fosnot & Perry, 2005; Piaget, 1967). The study also drew on two key theoretical models on the integration and adoption of ICT in teaching and learning: TPACK, (Mishra & Koehler, 2006) and technology acceptance model (TAM) (Davis, 1989). These theoretical perspectives, coupled with Kilpatrick's two strands of mathematical proficiency, conceptual understanding and procedural knowledge, (Kilpatrick et al., 2002), offered insights into the factors that constituted the analytical framework (see Appendices C). An analytical framework provides a guide to organisation of data to be collected and how to analyse it (Cohen et al., 2018). The analytical framework is used to identify information useful for analysis from that which may be discarded.

I fully transcribed and reviewed all the verbatim data from various sources. The data was then organised by assigning codes using my analytical framework. While qualitative research orientations may differ operationally and theoretically relative to managing the data collected, each employs a unique method for organising and coding the data. Coding in qualitative research comprises processes that enable the data collected to be categorised and thematically organised, thus providing an organised basis for the construction of meaning. (Williams & Moser, 2019). Coding methods employ procedures that uncover themes that embed the data, thereby suggesting thematic direction toward categorising data through which meaning can be conveyed, codified and presented. Coding, therefore, is an important structural operation in qualitative research, that enables data analysis and subsequent steps to achieve the purpose of the study.

Qualitative research embraces a description of its analytic framework. This provides a rationale that justifies coding decisions with well-defined steps. According to Stenfors et al. (2020), the researcher's ability to perform analysis that develops a novel conceptual framing is evident of rigorous qualitative research.

A description of the coding of the analytical rubrics developed from the TPACK, TAM and constructivist constructs is presented. It focused on the tasks utilised in the study that were designed to investigate the lecturers' actions and responses to the tasks, as they interacted with the students.

I identified the following codes for this the research: visualisation processes, TPACK, TAM, mathematics proficiency, prior knowledge and multiple representation. This implies that the lecturers' responses and observed actions were classified according to the codes itemised above.

The coding structure was developed in line with the literature which facilitated the reorganisation of my data and extraction of the information to answer the research questions. I therefore came up with the following broad indicators for each codes: for visualisation of concepts, the code is yellow (VC), and the observable indicators under this code are: visualisation processes (VP 1)and use of sliders and dragging, USD 1.

For the TPACK constructs, the code is TPACK and the colour is green. The observable indicators are: CK1 - content knowledge; PK1 – pedagogical knowledge; TK1 – technical knowledge; PCK 1 – pedagogical content knowledge; TPK 1 – technical pedagogical knowledge; TCK 1 – technical content knowledge; and TPACK 1 – technical pedagogical content knowledge.

For the TAM constructs, the code is TAM, and the colour is pink. The observable indicators are: EU 1 – easy to use; AU 1 – actual use; and ITUI 1 - intention to use ICT.

For the constructs of mathematics proficiency, the code is MP and the colour is red. The observable indicators are CU1 – Conceptual Understanding and PF 1 – Procedural Fluency. The analytic framework (see Appendix C) displays how the deconstruction of the tasks by a description of the rubric was used to qualify the responses to each task. The analytic rubrics were designed to capture analytic framework evidence.

3.6 VALIDITY AND RELIABILITY

While it is appreciated that it may not be possible to completely eliminate bias in social science research, a good design generally endeavours to minimise bias and maximise the validity and reliability of the data collected and analysed. I triangulated my data by employing different methods of data collection to ensure validity. The methods included workshops with students and orientation lessons with participants, interviews, observations, video and voice recordings, worksheets, field notes and reflective interviews. Employing this wide variety of data collection sources enabled me to gain a thorough, objective and in-depth understanding of the dimensions of the study. According to Rule and John (2011) triangulation is "the process of using multiple sources and methods to support propositions or findings generated in a case study" (p. 109).

Triangulation is considered as a means to achieve rigorous, quality research, from a diversity of sources, to strengthen the validity of the assertion or findings, to eliminate the bias or inaccuracy that may possibly be introduced by relying on a single source (Rule & John, 2011). Cohen et al. (2018) echo this and state that "triangulation is the researcher's attempt to understand the richness and complexity of human behaviour within a phenomenon, from different standpoints" (p. 256). Triangulation, therefore, reduces bias, verifies the integrity of participants' responses and enhances various dimensions of a phenomenon.

Case studies employ multiple techniques of data collection. Multiplicity of data sources in a case study is encouraged as it provides rigorous (theoretically, empirically and grounded) evidence that supports the triangulation of results (Cobb et al. (2015). In their advocacy for the use of multiple sources to ensure validity, Yin (2014) and Onwuegbuzie and Leech (2007) argued that the sources of evidence of data should measure what they are supposed to measure. In my quest to collect robust and rich data in this study, I therefore triangulated my data collection process to enhance both reliability and validity of the data and the results.

Validity, in education research, is the process of collecting data accurately that reflects the aspects that they are meant to measure (Newby, 2014, p. 129). Reliability, on the other hand implies that the outcomes of measurement are stable over time, on the assumption that other factors remain the same. These factors are key to objectivity in a study, in the sense that that a researcher's judgement is dispassionate – implying that if the same study were to be conducted by another disinterested researcher using the same evidence, they would reach the same conclusion.

Validity of research therefore requires careful consideration. Cohen et al. (2018) affirm that validity and reliability of data are essential components to rigorous research and have an important effect on the on the authenticity of research and the interpretation of the data. The quality or trustworthiness of a research study is a strong indicator of its reliability (Merriam et al., 2007). For the validity and reliability of my study, I adopted different strategies as outlined below.

Merriam et al. (2007) outlined four major principles that validity and reliability in qualitative research is derived from:

- the nature of interaction between the researcher and the participants;
- triangulation of data;
- the interpretation of perceptions; and
- descriptions to make the researcher's conclusions sensible.

To ensure validity and reliability in my study and make my study trustworthy, I adhered to the above principles, by employing various strategies.

As alluded to in Section 3.4 above, the training sessions I had with the participating lecturers in their respective TEIs on the use of *GeoGebra,* accorded me an opportunity to interact with them, and henceforth establish a mutual and professional relationship. I encouraged them to relax and feel free as they interacted with the software and I reassured them that the challenges they encountered as they began to use the software were normal, and that in due course they would gain confidence. The training workshops were structured and timed in a manner that before the lecturers could use the software in a class with students, they had to reach an accepted level of expertise on the use of the software. The workshops focussed on basic use of GeoGebra, with a focus on calculus topics. Their knowledge of calculus content, and basic knowledge of computer use provided extra confidence. I also encouraged them to interact among themselves and work collaboratively. Ndlovu et al. (2013) acknowledge that working collaboratively in professional communities with constant engagement in reflective practice can help develop common identities and overcome lack of confidence. As is the case with most qualitative case studies, I triangulated my data collection process.

Triangulation, according to Ramsook (2018), is a method of ensuring that qualitative research is less subjective and more objective, thus making it more scientific. A researcher can gain confidence when they employ more than one method of data collection, and generate results that correspond (Cohen et al., 2000). Credibility of research can therefore be enhanced through triangulation. In line with the rationale of triangulation, I collected my data through document analysis, interviews, focus group discussions, observations, videos and recordings. These various methods of data collection enabled me to get detailed insights into the lecturers' engagement with differential and derivative calculus applets in *GeoGebra*. Use of multiple cases, observes Farquhar et al. (2020), is considered an appropriate strategy for enhancing the validity of a research study.

Some of the participants in the study my colleagues as we interacted during professional meetings. Despite this, my rapport with the participants was purely professional and I encouraged them to give their honest views. I ensured that this relationship did not mitigate against any issues of positionality and I ensured that our relationship did not in any way influence their opinions. Being fully aware that the integrity of the research is determined to a large extent by the authenticity of data, I informed the participants that giving an honest and correct account of the events regarding the situation was cardinal to research. Ethical issues related to research relations, according to Denzin and Giardina (2017), could be overcome through collaboration in participatory approaches to research, where the roles of both research participants and the researcher are respected. Throughout the course of my research project, I adhered to the rules and standards of Rhodes University Education Department and the Rhodes University Ethical Standards Committee, and upheld the institution's professional, academic and integrity standards. The authentic data that I collected provided a basis for my findings, and my analysis was not based on my assumptions and opinions, but on my empirical work.

3.7 ETHICAL CONSIDERATIONS

Ensuring validity and reliability in qualitative research involves conducting the investigation in an ethical manner (Creswell & Cresswell, 2017). By coincidence, this study was conducted during the period of transition when Rhodes University was introducing the new online ethical clearance system. The university's Higher Degrees Ethics Committee's (RUHDEC) gatekeeping requirements required every researcher to apply online for ethical clearance, through the Rhodes University Ethical Review Application system, which I did. In addition to obtaining the ethical clearance, (see Appendix A), I observed all the other Rhodes University ethical protocols for conducting research. I also obtained clearance letters from other gatekeepers, viz: the respective principals of each of the three TEIs (see Appendix A); and the consent letters from each of the six lecturers who participated in the study (see Appendix B).

I communicated to the participants all the information that could influence their choice to take part in the research project and informed them what the research was all about. According to the British Educational Research Association (BERA), the initial step in obtaining consent for researchers is to ensure that participants understand the process that they are engaging in, why their participation is needed, how the findings will be reported and who will use the research findings (BERA, 2011). I made it explicitly clear to the participants that they had the right to withdraw their participation at any time without any consequences. Their identity and institutions would remain anonymous as pseudonyms were used in the write up. The data collected was used for research purposes only and will at all times be kept confidential. It will only be shared with my supervisor and with the Rhodes University Ethics Department. upon request.

The audio and video recordings and all other information that came into my custody by virtue of this research project has been kept strictly secure at all times and will not become a part of participants' records. All identifying information was deleted as soon as the data was collected. The hard copies of the data have been securely kept under lock and key and the soft copies have been stored with a strong password. The recordings were coded and transcribed with no means of tracing them to the participants. The data will be destroyed after five years in accordance with the Rhodes University Ethics department regulations

Connelly (2013) surmises that a researcher should "obtain informed consent from potential research participants; minimise the risk of harm to participants; protect their anonymity and confidentiality; avoid using deceptive practices; and give participants the right to withdraw the research" (p. 13).

I explained to the principals and the lecturers the potential benefits that may accrue from taking part in the research. There was potential for participants to adopt innovative instructional practices in their teaching of calculus. They could also enrich their knowledge and skills in integrating technology in teaching and learning mathematics. The research could also guide and provide a basis for participants to contribute to the design of appropriate instructional materials for the consolidation of the Teaching and Learning Mathematics with *GeoGebra* (TLMG) project. This could contribute to quality teacher professional development in the use of *GeoGebra* for mathematics lecturers and teachers, with a view to incorporate *GeoGebra* in the Zambian teacher education Curriculum.

3.8 CHALLENGES ENCOUNTERED IN DATA COLLECTION

As alluded to in Section 3.7, this study was undertaken during the period of transition when Rhodes University was introducing the new online ethical clearance system. This entails that every student is obliged to apply online for ethical clearance, prior to commencement through the university's Higher Degrees Ethics Committee (HDEC). The gatekeeping conditions require every researcher to apply online for ethical clearance.

However, the HDEC's gatekeeping requirements subjected me to a bureaucratic and rigorous process and took a considerable period. The application process was intricate and had some technological hitches. Despite interventions from my supervisor to expedite the process, the whole process took around six months. This had drastic negative implications on my data collection process, as appointments with my participants had to be adjusted., By the time the ethical clearance letter was ready, lecturer 4 at TEI B informed me that his students had gone on student teaching practice (STP), and would only return to campus the following term.

The advent of the Coronavirus (COVID-9), further compounded the problem. In Zambia, in an effort to control the spread of COVID -19, all learning institutions were ordered by the government to close on 20 March 2020 and all citizens advised to observe self-isolation (Sintema, 2020).

> Around mid-March 2020, the Zambian government through the Minister of Health announced at a press briefing that all schools, colleges and universities would close indefinitely on Friday, 20 March 2020 amid fears of the Coronavirus (COVID-19) outbreak that had reportedly ravaged most parts of China, United States of America, Italy, Spain and other parts of Europe and Africa. (Sintema, 2020, p. 1)

This inevitably implied that I had to put my whole data collection process on hold.

During the same period, I was supposed to travel from Zambia to Namibia on five different occasions, and on multiple occasions to Rhodes University, in South Africa, to meet my supervisor for contact sessions, but this could not materialise as travel restrictions had been introduced as a result of COVID-19. This robbed me opportunities to interact physically with my supervisor and engage with him in areas where I needed guidance. As a recourse to this, we engaged virtually from time to time. When the COVID-!9 situation improved, I was able to go out into the field and collect data, but I had to make a lot of adjustments from the initial plan.

A challenge that I generally encountered among the participants was locating the right construction tool on the *GeoGebra* interface. Furthermore, once the construction tool had been identified, the other challenge was how to use that tool. This was apparent in components of cycles that required a relatively high number of steps. This resulted in taking more time than anticipated to complete a session.

3.9 CONCLUSION

This chapter presented a discussion of the methodology used to execute this qualitative case study. The methods used to collect data and the analytical frameworks for the case study data were presented. I also discussed the measures that I undertook to ensure the validity and reliability of the research study, and the ethical measures that were undertaken to ensure the protection of the rights of research participants.

CHAPTER FOUR: ANALYSIS AND DISCUSSIONS OF LECTURERS' ENGAGEMENT WITH GEOGEBRA APPLETS

4.1 INTRODUCTION

The data being analysed in this chapter was collected through interviews, focus group discussions, lesson observations, and audio and video recordings. The data was analysed based on the analytical tools generated from the constructs of the enabling theoretical framework models of TAM, TPACK and mathematics proficiency. This helped me to describe and explore individual cases and obtain insightful information about my participants' interactions with *GeoGebra*.

The analysis consists of two parts: the vertical and the horizontal analysis. For each lesson presented by the lecturers, I present a vertical analysis, followed by a horizontal analysis. In the vertical analysis I analyse how each lecturer interacted with their applet during their lesson presentation. In the horizontal analysis I analyse across all six participating lecturers, basing my analysis on each of the four cycles, in consideration of their views, for the purposes of answering the three research questions. Similarities and differences in the participants' views are presented and discussed. Codes that I used to identify the participants were as follows: Lecturer 1's views in the interview on line 3 for example, is coded as IL1L3 (I for interview, L1 for Lecturer 1, L3 for line 3), and similarly Lecturer 5's views in the interview on line 29 is coded as IL5L29. Similarly, lecturer 6's view in the focus group discussion on line 8 is coded as FGD L6L8.

I transcribed the recorded interviews verbatim, coded the responses of the participants according to the common themes that emerged, and then cross-checked them for consistency and commonalities. The themes were reviewed and categorised again for emerging sub-themes. Data from the interviews are presented in a narrative form, and the interpretation is presented in the discussion. This data accurately recorded the general views of the participants. Field note excerpts from the researcher's perspective supplemented the data.

The goal of this study was to investigate lecturers' experiences and perceptions of incorporating *GeoGebra* as a visualisation tool to teach calculus to undergraduates in TEIs in Zambia. It also investigated the factors that enabled and constrained participating lecturers' adoption of this technology into their teaching.

My cyclical approach adopted for the research design unfolded in four cycles, where each cycle was determined and characterised by a topic of the calculus curriculum. I used the same order as in the first paragraph of Section 4.2 below. There were two lecturers from each of the three TEIs. Each lecturer conducted two lessons from the four cycles with the aid of *GeoGebra* applets, to the same group of students in their respective TEIs. The two lecturers in each of the three TEIs and I planned the lessons and then created a *GeoGebra* applet for each cycle.

I observed and video recorded each of the six lessons. These video recordings of the lessons generated the data for my study. As outlined in the methodology chapter, each cycle consisted of three stages: planning, implementation and reflection. I analysed each cycle of the calculus topics by analysing my classroom observations of the teacher educators. The data collected from video recordings of observations of the lecturers' lessons consisted of two data sets: observation data and interview data.

My research design is underpinned by a case study methodology. The following narrative analysis documents how each of the participating lecturers interpreted and used the calculus cycles as they interacted with *GeoGebra* applets. Where necessary, the narrations are interspersed with excerpts of interviews and screen shots of the applets used.

In Chapter 3, I outlined the observable indicators of the lecturers' teaching activities based on TPACK and TAM, which formed the core of my analytical framework, I also used Kilpatrick et al.'s (2002) first two strands of mathematics proficiency – conceptual understanding and procedural knowledge. The details of my analytical framework tools that guided the data analysis are in Appendix C. The analytical framework is used to identify information useful for analysis from that which may be discarded (Cohen et al., 2018).

The codings and observable indicators have been explained in detail in the methodology chapter (Chapter 3). These observable indicators enabled me to obtain more insights into the lecturers' engagement and interpretation with the applets in order to present a detailed account. The reflective interview data was used to affirm or contradict the lecturers' actions and communication during the lesson presentations.

In addition, the visualisation processes, outlined in Chapter 2 in the literature review, supplement the analytical framework tool to facilitate a deep analysis of the lecturers' interaction with the applets. The relevant applets and diagrams generated by the lecturers to provide more clarity are referred to as figures, whereas screen capture video clips are referred to as screenshots.

4.2 A BRIEF OUTLINE OF EACH OF THE CYCLES TAUCHT

The lessons taught by each of the six lecturers were based on four cycles of the calculus curriculum at TEIs: The slope of the tangent, limits, the area above and below the x -axis, and the Riemann sum. Each lecturer and I collaboratively designed all the applets that were used in the lesson presentations in each of the four cycles.

4.3 VERTICAL ANALYSIS

4.3.1 Lecturer 1

4.3.1.1 The slope of the tangent - brief description of the lesson and how the applets were used

Lecturer 1 presented the cycle – the slope of the tangent (Cycle 1). In this presentation, the lecturer discussed how the secant could be used to promote conceptual understanding, (CU 1), of finding the slope of a tangent line on a curve, with a view to exploring the limit process and the definition of the derivative. As shown in Applet 1.1 in Figure 4.1, the lecturer introduced the lesson with the function $f(x) = x^2$ and a tangent at P on the curve, using the prior knowledge of the slope of the straight line, from coordinate geometry.

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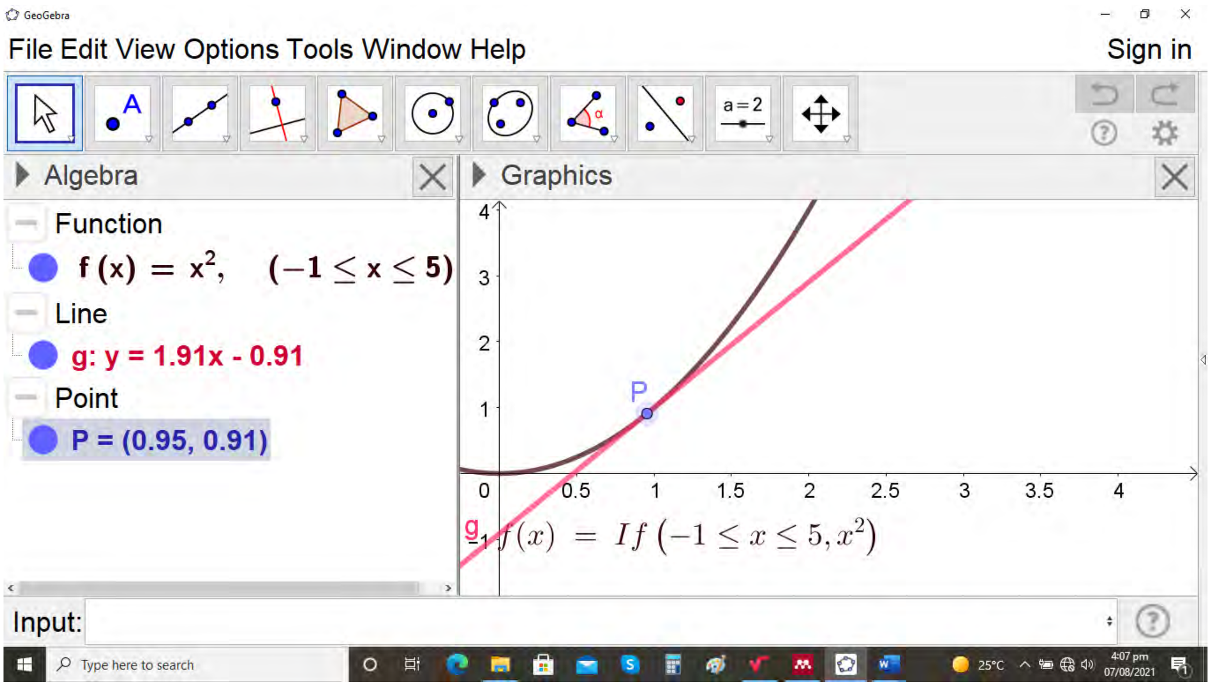

Figure 4.1: Applet 1.1, Tangent line through point P

The point P was marked on the function $f(x)=x^2$, and a tangent to the curve drawn with P at the point of tangency. The lecturer asked the students about their views on finding the gradient of a straight line (CK 1), when only one point was known. This concept was discussed with the assumption that the students were familiar with the idea of finding the tangent of a straight line using the formula $m=\dfrac{y_2-y_1}{x_2-x_1}$. The challenge in the applet above was that only one point, P, was known, so that the formula for the slope of the straight line could not be applied directly, as it required two points (x_1, y_1) and (x_2, y_2).

Applet 1.1 was modified by introducing another point Q on the curve, such that the line PQ is a secant with the point Q as a point of estimate as shown in Figure 4.2, Applet 1.2. Using a slider, (USD 1), the point Q could be moved closer and closer to P. As the lecturer moved the point Q close to P, he asked the students what they observed about the secant line in relation to the tangent line.

Since there were now two points – P and Q on the curve, it was possible to find the slope of the secant line PQ. The slope of the secant line is approximately the slope of the tangent, though the values will vary significantly due to the difference in the alignment of the tangent and the secant.

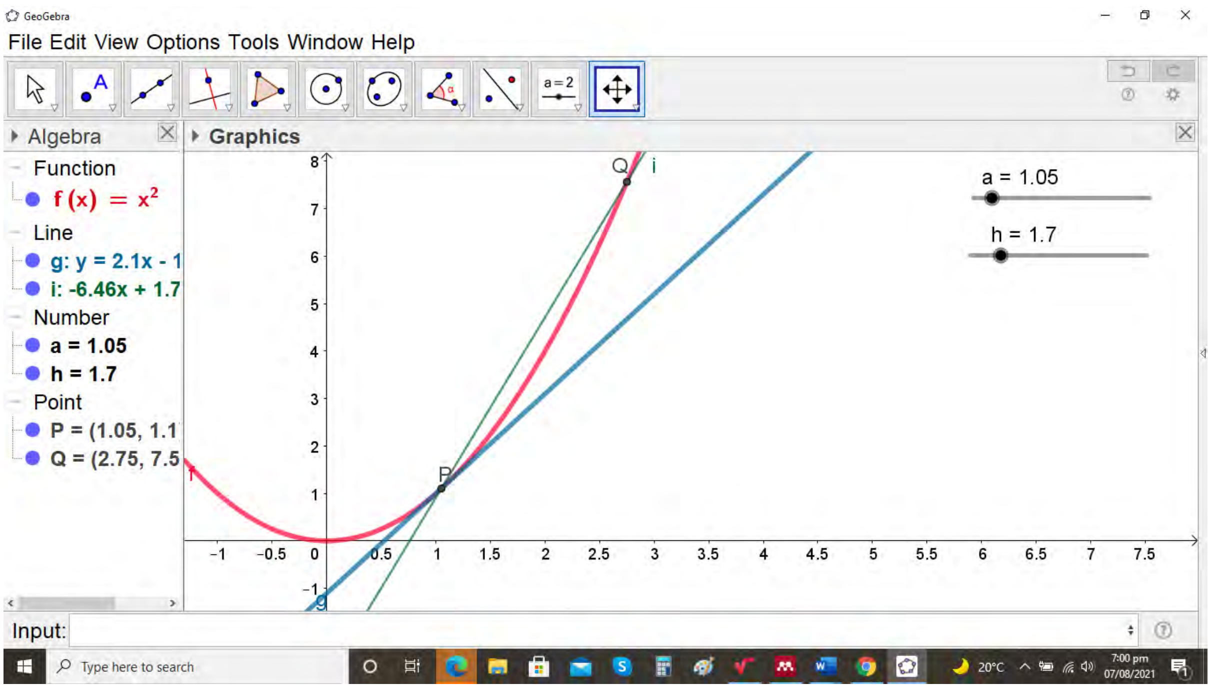

Figure 4.2: Applet 1.2 – Secant line PQ and the tangent line at point P

For the estimate to be better, the estimation point Q has to move closer to the point of tangency, P, as shown in Figure 4.3, applet 1.3, below.

Using the slider, the lecturer moved the estimation point closer to P as shown in Figure 4.3, Applet 1.3. The Applet 1.3 shows an estimate point R getting closer to P. With the new point R as the point of estimate, the lecturer illustrated how the secant line PR aligned more to the tangent line (TPK 1). As the lecturer dragged the point R to P, she asked the students to observe (VP1) what was happening to the values of the slopes of the secant line PR and that of the tangent line, as shown in the algebra view of the *GeoGebra* interface. Of interest to note, was that as the lecturer was dragging the point R to P, the students could see the secant line aligning to the tangent line, and the value of the slope simultaneously changing (MR1), and getting closer to that of the tangent line.

It was observed that the secant line PR was a better estimate of the tangent line, since the point R was closer to the point P. It was also noticed that as the estimation point moved (VP1) closer to the point of tangency, the secant line was aligning more to the tangent line, implying that the slope of the secant line was getting closer to that of the tangent line. This became more evident when the estimation point got closer and closer to the point of tangency as shown Figure 4.1 in Applet 1.4.

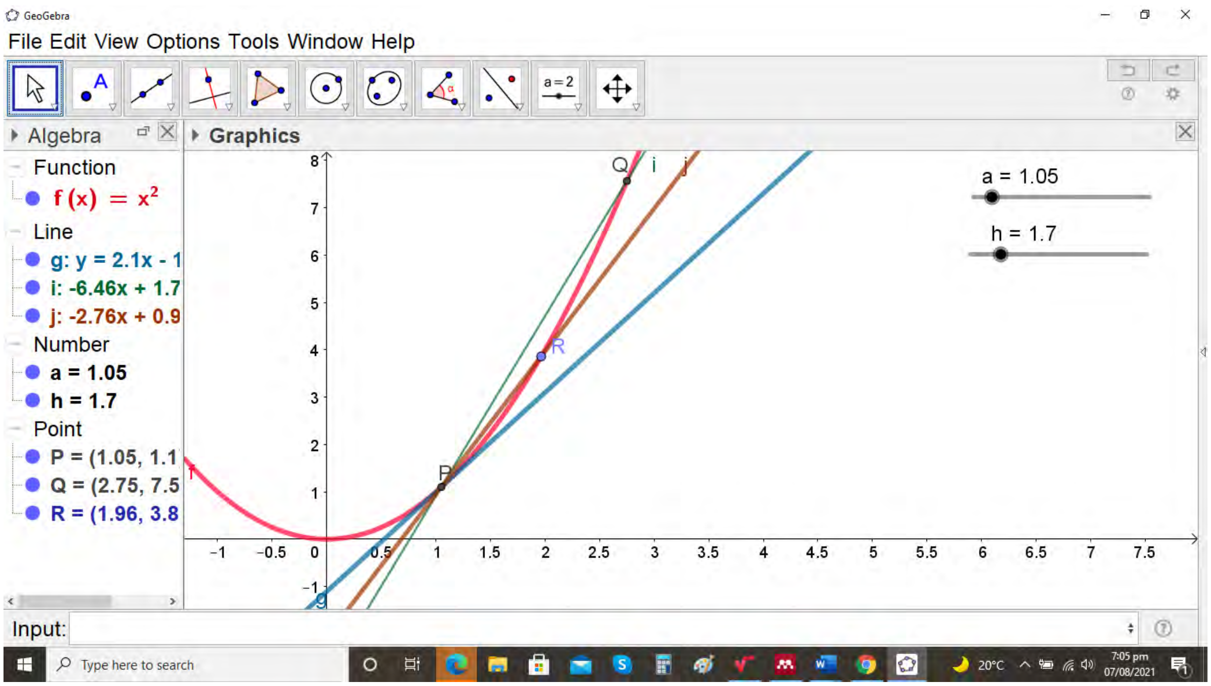

Figure 4.3: Applet 1.3 – Secant line PR and the tangent line at point P

In Applet 1.4, Figure 4.4, the new point of estimate, S gets closer and closer to P, with the secant line, PS, almost overlapping the tangent line. The lecturer showed that the values of the slopes of the secant line PS and that of the tangent became equal.

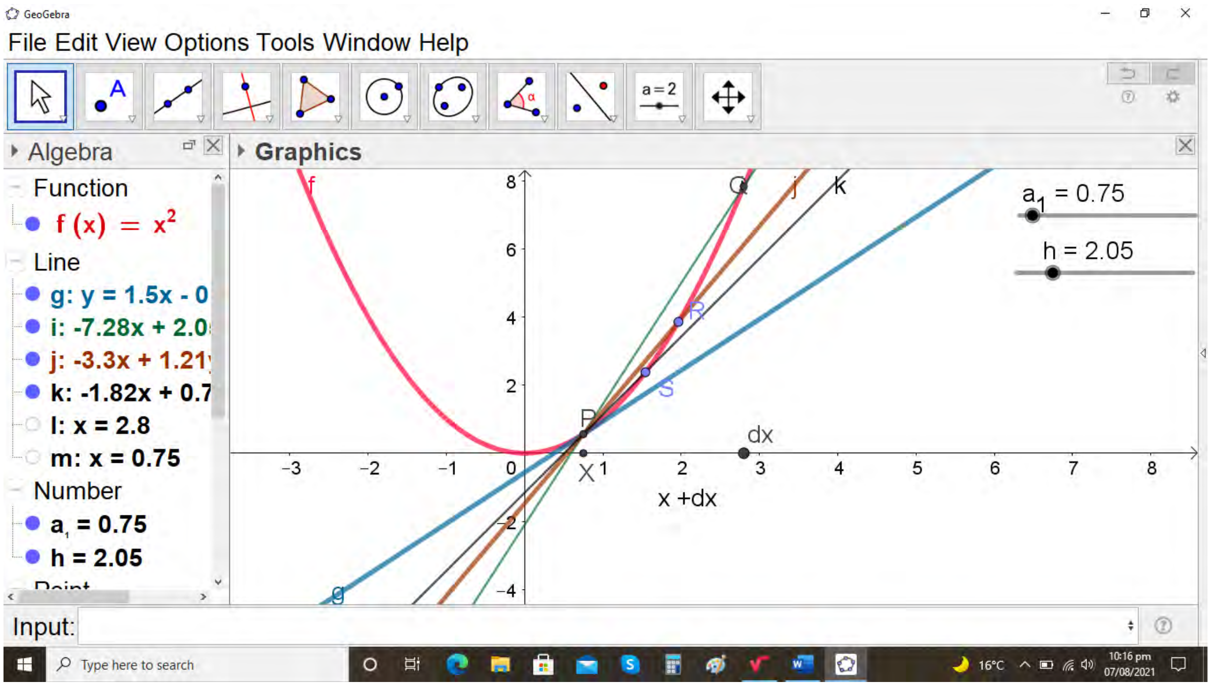

Figure 4.4: Applet 1.4 – secant line and the tangent line at point

With reference to Figure 4.4, the lecturer then proceeded as follows:

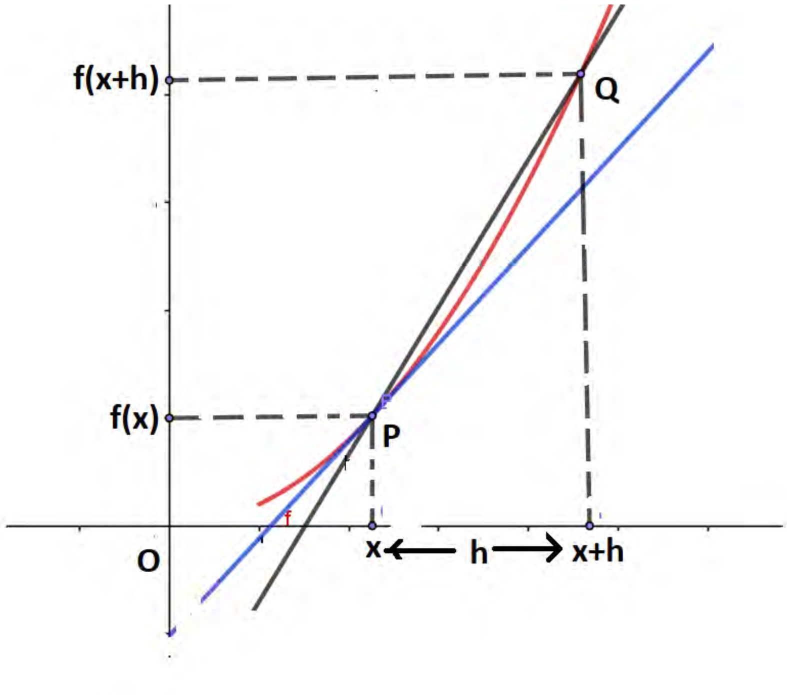

Figure 4.5: Deriving the slope of tangent line at point from that of the secant line

If we let the distance from the origin to the $x$coordinate be x, and the distance from x- coordinate to the estimation point be h, then the x coordinate of the original estimation point Q is $x+h$. This implies that the corresponding y - coordinate of $x+h$ is $f(x+h)$.

The lecturer went on further to discuss that since there were now two points on the curve whose coordinates were (x, $f(x)$) and ($x+h$, $f(x+h)$, the slope of the secant line could be found, ((PF)), and its value would approximately be equal to that of the slope of the tangent. Thus, from the concept that the slope of the straight line is given by the change in y divided by the change in x, this would be given by:

$\frac{f(x+h)-f(x)}{x+h-x}$, which simplifies to $\frac{f(x+h)-f(x)}{h}$

The slope of this secant line is approximately equal to the slope of the tangent. But as we drag the estimation point closer to the point of tangency (TCK 1), what happens to h? (Refer to Figure 4.5, Applet 1.5,).

As the estimation point moves closer and closer to the point of tangency, h gets smaller and smaller (infinitely small). Therefore, taking the limit as h approaches zero, gives

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, and the approximated slope of the tangent becomes the exact slope.

Thus, taking the limit as h approaches zero, implies that h will be arbitrarily small. This makes the two points (the point of tangency and the estimation point) to be arbitrarily close. As a matter of fact, they become infinitely close, implying that the estimate becomes exact because the two points are infinitely close, and we therefore get the exact slope of the tangent line. This limit is essentially the definition of the tangent line and the definition of the derivative, implying that the derivative is the slope of the tangent line.

As I observed the lesson, I noted down the following:

> *An interesting point is how close can you move P to Q, and when do you stop? What happens when P and Q coincide, then PQ ceases to be a secant, instead, it becomes a point (We cannot make Q equal to P, because we need two points to make a line). How close can one point get to another without becoming one point? Between any two points, there is a gap, and within that gap, you can fill it with another point. So, the idea behind calculus is that we can get Q so close to P, that there is literally no difference between a secant and a tangent. They can get so close such that the secant line is the same as the tangent line. If we let Q get closer and closer to P, the secant will be identical to the tangent (approximately equal to the tangent). The idea of being very close (closer and closer to is the limit –the idea of Q being closer to P, but not touching P is the idea of the limit. As Q gets closer to P, the secant line more closely approximates to the tangent line.* **(FN 1**)

To further visualise the concepts that have been discussed above, the lecturer asked the students to solve the following problem first without the aid of the software, and then with the aid of *GeoGebra.*

Find the slope on the curve $f(x) = \frac{7}{20}x^3 + \frac{11}{20}x^2 - \frac{9}{10}x$, at the point where $x = 1$

Solution

$$f'(x) = \frac{21}{20}x^2 + \frac{22}{20}x - \frac{9}{10}$$

$f'(1) = 1.25$

The lecturer then worked out the same question with the aid of *GeoGebra* as shown in Applet 1.5, Figure 4.6).

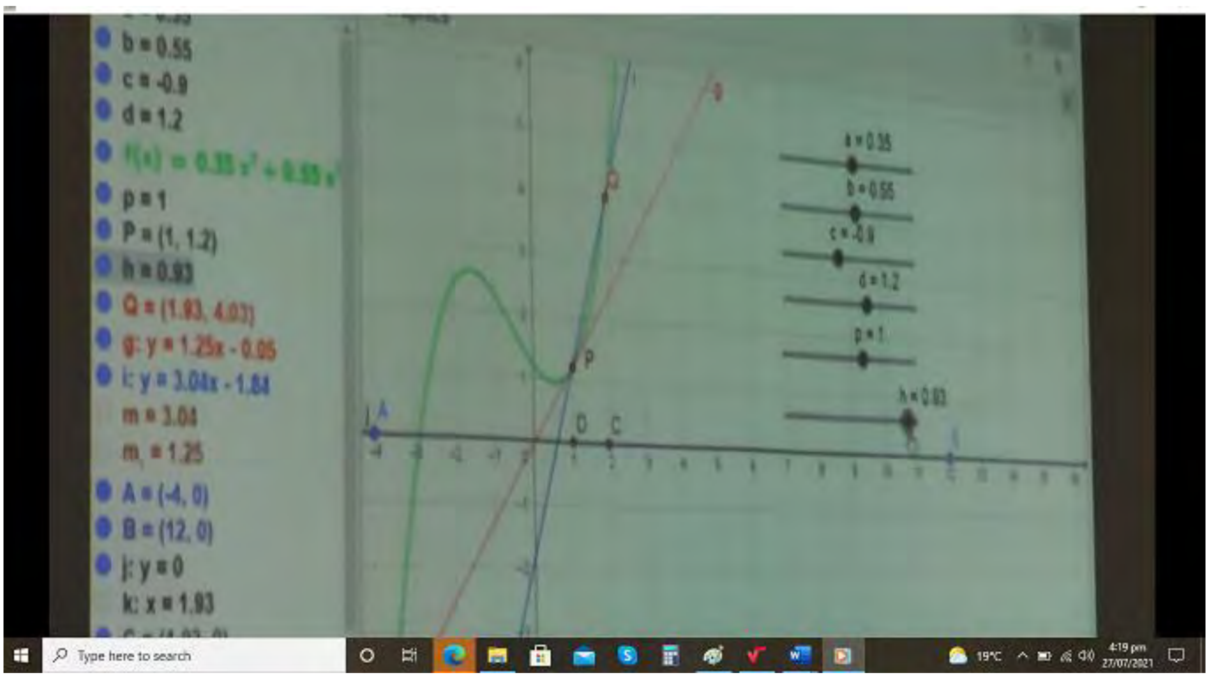

Figure 4.6: Applet 1.5: Finding the slope of the tangent using GeoGebra

4.3.1.2 Interview with Lecturer 1

Lecturer 1 had not interacted with *GeoGebra* prior to the commencement of this study. She had, however, taught calculus in TEIs for over ten years. After interacting with the software, she felt that *GeoGebra* was helpful to both students and lecturers, especially from the visual perspective. She observed that:

> *Okay when we come to the use of GeoGebra to explore the concept of limit it was quite interesting because when we are finding the gradient, we mostly just take the change in y, the change in x without looking typically at the limiting factor towards that gradient point* (PCK 1). *But when we used GeoGebra, it was giving us a clear indication of how when we*

get to a particular point, how the gradient of that particular curve or that line, giving us that figure so the visualisation was quite clear when using the concept of *GeoGebra*. *The fact that the visual aspect* (VP 1) *is brought into the lesson, students are able to understand better). The aspect of limit, the limiting process, so, when you are saying this value, or those small changes, along the x – axis and the y – axis, the aspect of approaching zero, so that can be seen* (VP 1*). Because from the conventional way, the traditional method, when you just say, it is approaching zero, it is abstract, it is not seen, so, where is it approaching zero, so if you go to GeoGebra, those lines, red and blue colours of the tangent line and the secant sine in the GeoGebra applet there is that movement* (VP 1*), either way, it's going towards zero, until it reaches zero, so that is the difference, the seeing aspect.* (VP1) (**IL9L7)**

She elaborated that in her lesson presentation, *GeoGebra* was very useful (AU1) in reinforcing the concept of first principle of differentiation, as the learners were able to understand the concept of the derivate and how the formula for first principles was arrived at (TPK 1). She explained that as h was approaching zero (VP1), the concept of the limit is not very easy to explain without the visualisation features of *GeoGebra,* because as h was reducing, this gradient for PQ was getting closer to the gradient at the tangent line. So, without a good visualisation, even when you illustrate it on the board, it is not very clear, but that with *GeoGebra*, she was able to demonstrate the gradient very precisely to the students (EU 1) and they were able to see the line getting closer and closer to the tangent, until it coincided with the tangent line (VP).

She added that:

I think there, GeoGebra, it being a software that I can say is practical in such a way that, if we are talking about, maybe, moving the secant line, from its original position, it can move like, it can visually move, as we are moving sliders (USD 1), *so there I found it to be very easy,* (EU 1), *when explaining the slope of tangent using the secant, to the students, it was something that we could see,* (VP 1), *and it was very easy* (EU 1)**.** *I would say the concepts when using this tool, GeoGebra, provide a clear link on how to develop the gradient at a point because the moment this line is, the imaginary point is coming closer,* (VC 1), *to the point where we want to draw the gradient from, it gives that clear indication as you get into*

> *action so the secant providing that extra view, (MR 1), of how to draw the tangent at a certain point of the, tangent, yes.* **(IL1L14)**

She added that it was very easy (EU 1) to grasp the concept of how P was moving to Q, because *GeoGebra* clearly demonstrated the effect when P was moving towards Q (VC 1) and how the secant was transforming into the tangent, so it was easy to visualise and see what was happening. It was all about seeing it move and observing the effect it was causing – in other words, it was easy to see.

She elaborated that *GeoGebra,* if used properly (TPACK 1) as a visualisation tool (VC 1) on the concept of the slope, can clarify the concept accurately and clearly. She explained that *GeoGebra* enables the students to understand (CU1) how the gradient occurs, especially when they can see the way the two points (P and Q) move closer (VC 1) to each other. This enables learners to understand why the gradient of the tangent is equal to the gradient of the curve at a particular point (PCK 1). Unlike when conventional and manual methods are used (such as chalk-and-board methods which are often difficult to use to produce accurate diagrams), *GeoGebra* produces very accurate and precise representations to visualise concepts effectively. She explained:

> *…so the focus was on how the secant was moving (VP 1) towards the tangent at a particular point. Because when you have two points on the secant, as one point is drawing closer to the other one, the secant inclines to the tangent (TPK 1), so they were able to see how as the gap between P and Q is being closed up, or should I say the gap between two points is being closed up. The students were able to see how the secant, that passes through those two points gets reduced to the tangent (CK1). So since they were able to see that, and to understand how the secant is being used to find the slope of the tangent. The fact that the visual aspect (VP 1) is brought into the lesson, and students are able to understand better (CU1), then they will be motivated to do more (ITUI 1). If anything, they will even come up with their own areas of concern. Like if you tackle specific areas of concern and they have interaction with the software (AU 1), they can explore and get to those points that you didn't touch as a teacher.* **(IL1L6)**

During the FGD, lecturer 2 asserted:

Okay, the...okay GeoGebra brought out the concept so clear, (CU1), *because when you want to introduce the problem of the tangent to the learners and when you are just using the board sometimes it's not very clear for them to see that point* Q *is approaching point* P *and that* h *is reducing but with the GeoGebra application, I think it clearly showed us* (VP 1) *how* h *decreases as it approaches zero* (TPK 1), *so that the gradient of the secant is closer to the gradient of the tangent, and therefore, solving the tangent problem.* (PF1) **(FGD L1L7)**

Lecturers in the FGD had this to say:

We know quite alright that once we integrate the curve at a particular point, what we are getting is the gradient. (CK1). *Now with this one we are able to locate the actual point using this software GeoGebra. We are able to locate the point at the same time* (MR1) *we are able to draw the tangent at that particular point without any problem* (EU1). *Then once the tangent is drawn, then we can even come up with another point within the curve then from there we can do some movement of a point,* (VP 1), *towards if it is Q towards P, then even there we are able to visualise. Unlike a situation whereby we just solve without seeing. Just like I indicated earlier on. Once you attach the two calculations plus the aspect of vision, then the whole process is complete. This part will help us, one do the calculations* (PF1), *and at the same time learners are able to see. If we say this line will move closer to this, we are able to see.* (VP 1) **(FGD L4L93)**

When asked to elaborate in the FGD, lecturer 4 recounted that the software enhanced the concept of the slope of the tangent where the point Q moved to point P so visually that one was able to see what was happening to the secant as it was almost becoming a tangent. He added that when it was almost a tangent then you knew (CK) that the gradient of the secant was almost the same as that of a tangent, as opposed to simply talking in abstract terms. Seeing the movement really enabled the students to understand the concept (CU 1) and it would be retained in their minds since they saw it actually happening (VP1).

4.3.2 Lecturer 2

4.3.2.1 Limits - brief description of the lesson and how the applets were used

Lecturer 2 taught Cycle 2 on limits. He started his lesson by emphasising that students generally find the concept of limits very challenging, but stressed that limits form the fundamental basis of calculus and it was therefore very important to have an intuitive understanding (CU1) of the concept of the limit function in the study of calculus. In this cycle, the lecturer explored the limit of a function using *GeoGebra.* In mathematics, a limit is the sequence or value that a function approaches as the input gets closer to some value (Muhtadi et al., 2018). Limits are absolutely essential to calculus and mathematical analysis, and are used to define continuity, integrals, and derivatives.

In this cycle, the applet that was used to explore the concept of the limit of the function was to evaluate the limit of the function $\frac{x^2 - x - 6}{x - 3}$ as x approached 3. In his presentation, the lecturer asked the students about the methods that they could use to evaluate the above limit.

The lecturer, working collaboratively with the students, proceeded as follows:

Evaluate: $f(x) = \lim_{x \to 3} \left(\frac{x^2 - x - 6}{x - 3} \right)$

By direct substitution of x for 3 in $f(x)$, the function is undefined, as division by zero has no solution. The lecturer asked the students about any alternative methods of evaluating the limit in such a case.

The lecturer discussed with the students that in such a scenario, it is possible to evaluate the limit by plugging into the function a value close to the number x is approaching (in such a case, values close to 3). In this case, numbers close to 3 but not exactly 3, for instance 2.999 or 3.001, were considered, and the value obtained was close to 5. Notice that as you get closer and closer to 3, the limit approaches 5. We can therefore say that the limit of the function $f(x)$ as x approaches 3, is equal to 5. This technique works for any limit, as long as you plug in a number that is very close to the given number, but not exactly that number, if the limit exists, it is going to converge to a certain value. However, at times, you have to use other techniques to find the solution.

One such method is the factor method as worked out below:

$$\lim_{x\to 3}\left(\frac{x^2-x-6}{x-3}\right)=\lim_{x\to 3}\left(\frac{x+2)(x-3)}{(x-3)}\right.$$
$$=\lim_{x\to 3}(x+2)$$
$$=3+2$$
$$=5$$

Therefore $\lim_{x\to 3}\left(\frac{x^2-x-6}{x-3}\right)=5$. This implies that when x approaches 3, the function value of $f(x)$ approaches 5.

Using the *GeoGebra* applets, the lecturer presented the lesson to the students as follows:

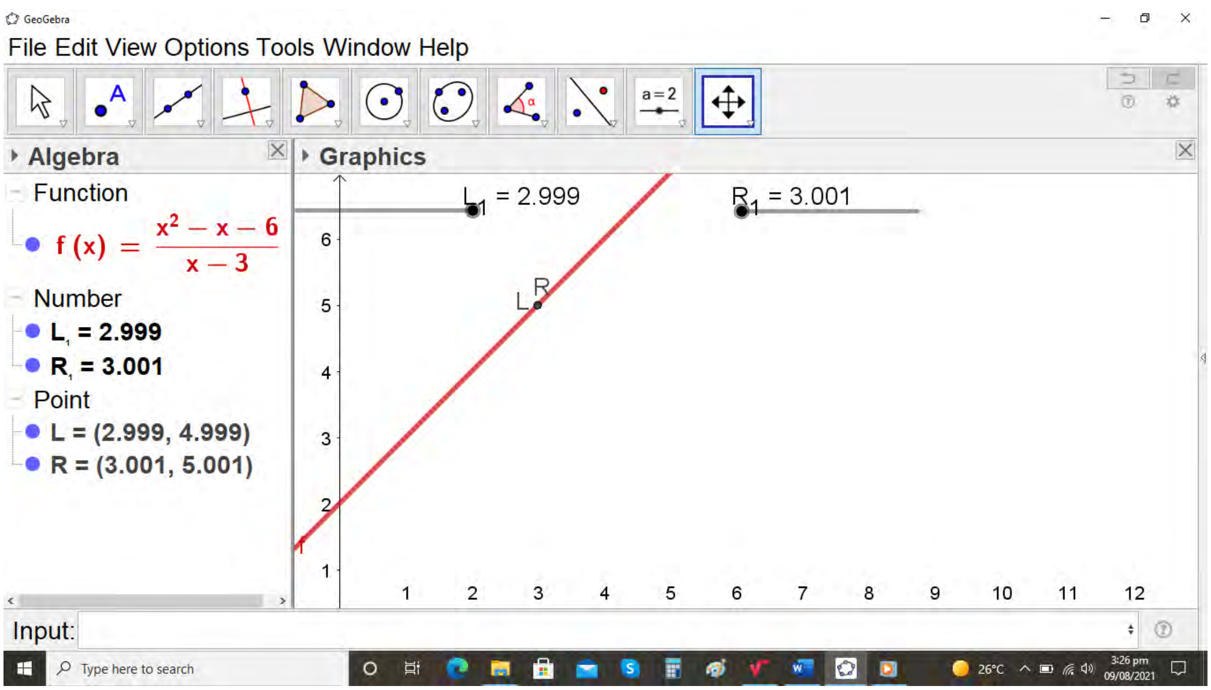

Figure 4.7: : Applet 2.1 – Limit function in algebra view and graphic view

On the graph of $f(x)$, as shown in Applet 2.1, Figure 4.6, in the *GeoGebra* interface, two points were selected: L, on the left hand side of $x=3$, (3), and R, on the on the right hand side of $x=3$, (3^+) of $f(x)$ and their coordinates simultaneously appeare (MR1), in the algebraic view. As the points L and R were dragged towards 3 using the sliders (USD 1), it was observed that at the point where $x = 3$, the function was not defined, but as the point approached 3 from either side, the function values approached 5.

On the same applet, the same concept was further investigated using the spreadsheet (MR1). The spreadsheet was used to explore values of the function as x approached 3 from either side. The lecturer took values of x from 2.9 to 3,04 to obtain the values close to 3 from either side. The corresponding values of $f(x)$ were obtained by the spreadsheet function as shown in Figure 4.7 in the Applet 2.2.

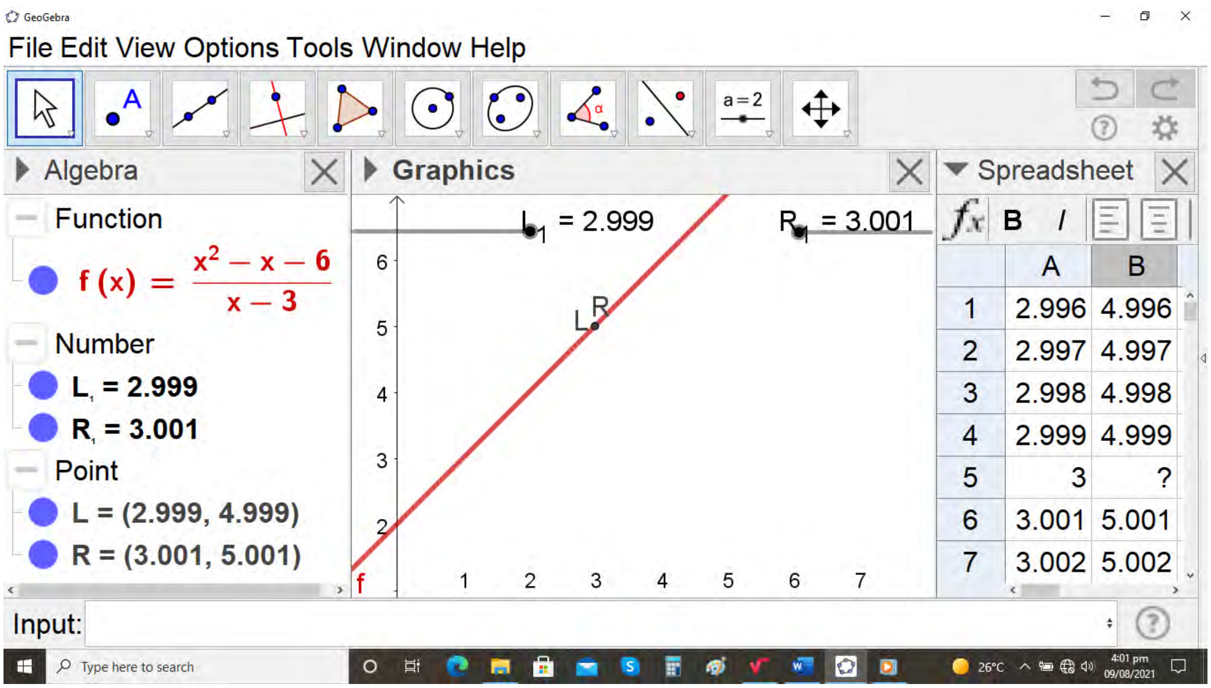

Figure 4.8: Applet 2.2 – Limit function in algebra view, graphic view and in spreadsheet view

In the spreadsheet view the function is not defined when $x = 3$, but the function values were approaching 5, as x approached 3 from either side.

On the same applet, the limit of the same function was evaluated using the CAS option under the 'view' tab as shown in Applet 1.3, Figure 4.3.

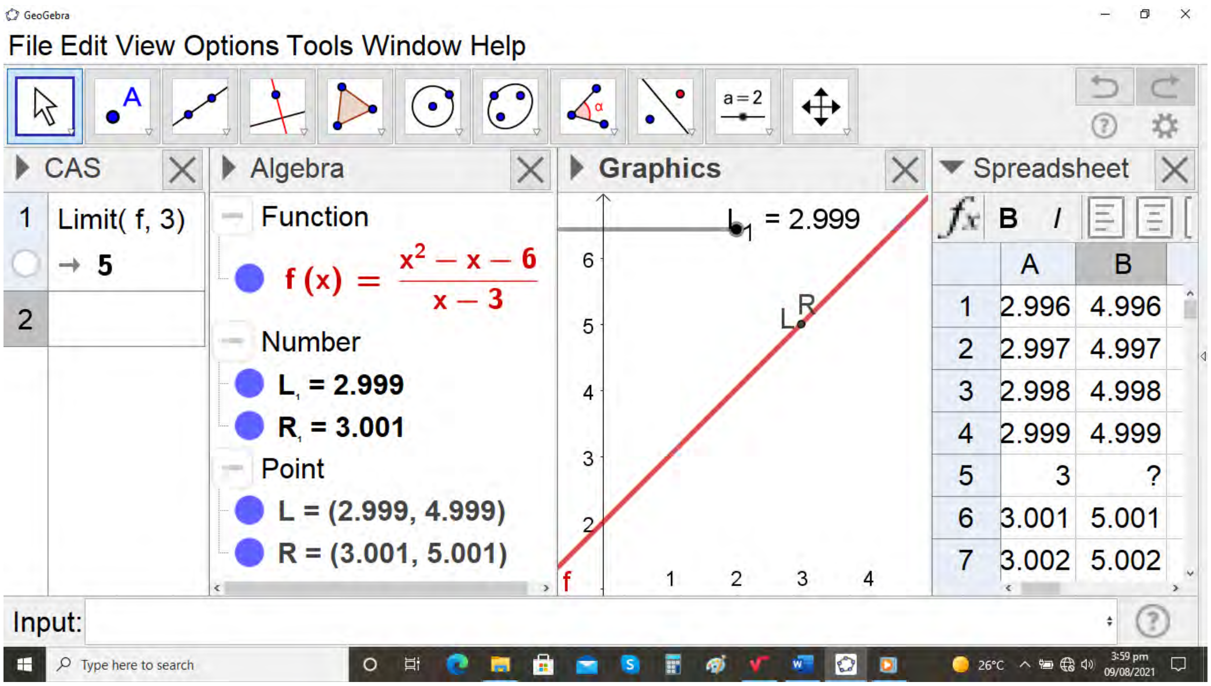

Figure 4.9: Applet 2.3 – Limit function in algebra view, graphic view, spreadsheet view and in CAS

The above applets indicate that the lecturer explored the concept of the limit in multiple ways (MR1): graphically, algebraically, by using spreadsheets and using CAS.

4.3.2.2 Interview with Lecturer 2

The lecturer acknowledged that from his interaction with *GeoGebra*, he discovered that the concept of limits could be explored in multiple ways (MR1), viz: algebraic, graphic, and with spreadsheets.

He emphasised that it was very clear (CU1) when using the applets that when x was equal to 3, the function $f(x)$ was not defined. However, when x was very close but less than 3 (for instance when x was 2.97. 2.98 and 2.99), the values of $f(x)$ were 4.97. 4.98 and 4.99 respectively – all very close to, but less than 5. On the other hand, when the values very close to but greater than 3 were considered (for instance 3.03, 3.02 and 3.01), the values of $f(x)$ were 5.04, 5.03 and 5.01 – all close to, but greater than 5.

Lecturer 2, reflected:

Teaching of calculus, I think it has not been easy because when you look at calculus, mainly we teach that topic in abstract. Most of the issues we explain when for example, the issue of limits, you say okay this is approaching a certain figure but meanwhile we end up substituting the actual figure (PF1). *So to explain certain concepts, we explain those concepts based on issues that are abstract. There I would say GeoGebra helps. There, we never used to know what was transpiring or happening when x is approaching any number. We were just taking it by faith that it just happens like that but GeoGebra was able to iron out all those doubts,* (CU1), *yes. Also, I was able to know that, oh!, this is the way things are, not the way we were taught, yes, it became so easier* (EU1), *in the sense that visualisation, you are able to see* (VP1), *how it is changing, not by calculating, to say, no, we now approaching 3, using various methods* (MR1) **(IL2L43)**

During the FGD, Lecturer 6, elaborated:

This one, (pointing to the image on the laptop) equally enhanced (CU1) *in a way that instead of just suggesting numbers and calculating in abstract* (PF1), *one is able to see* (VP1), *that as one point is moving towards the other you are seeing what is happening to the coordinates* (MR1), *so that you see the values, you are able to see, the number that they are approaching right from the left and from the right, so that in the end it becomes easy even to tell that oh, the limit is this one because from both ends the number that has been approached is this, one is seeing the values as they are changing.* (VP1) **(FGDL6L119)**

A common view among the interviewees was that a number of calculus concepts could be better illustrated using the *GeoGebra* software (TPACK 1), than by the conventional method (PF1).. This is underscored by one participant during the FGD, who explained:

I think I can start, personally I have been teaching calculus for quite a long period of time when it comes to teaching of calculus, there are areas which we teach in abstract (PF1). *I can give an example of limits. You may teach learners that for example you take a certain figure for example as numbers are approaching 3 they will not reach 3, but when it comes to the actual solving you are solving a problem under limit, you substitute the same 3 which you said numbers cannot reach 3. So when it comes to explanation, you cannot give the correct explanation as to why numbers cannot reach 3 but you are substituting 3. So in short what I am saying is, the way we teach calculus, there are some concepts which we* *teach in abstract we just explain, you cannot really show, showing becomes difficult but there are some concepts which we when it comes to oh differentiate and people will take it that way of course without diagrams. There are some which we draw those simple ones but there are some again which we cannot demonstrate like the issue of approaching a certain figure we can't demonstrate you can demonstrate but not really reaching the point as we saw on the software* (TPACK 1). *As you are using the app we will be able to move the points* (USD1*), and there it will help us even explain* (CU1) *how the limit in the formula of the first principle comes about as it approaches. So there is that visualisation* (VP 1*), One is able to see how all the coordinates that are involved are changing and how actually the limiting value is arrived at.* (MR1) **(FGD 1L2 L11)**

He went on to clarify that when you just compute limits (PF1) using conventional methods, you will obtain the solution, but it does really show the conceptual understanding (CU1) of limits. However, when now you use *GeoGebra* (TPACK 1), he added, *you* are able to create sliders (USD1), move points and really see (VP1) where the limits are approaching the required value, thus relating the abstract concept to the concrete one.

In the FGD, Lecturer 5 pointed out that:

When you are using the concept in abstract (PF1), *the learners wouldn't really see the approach you make, but now when you are using GeoGebra* (TPACK1), *even when you are saying we are approaching from the left or we are approaching from the right they will be seeing the values* (VP1), *getting closer to what the correct value, especially with the use of the sliders* (USD 1), *that was quite good providing mathematical reasoning towards that particular limiting value to the left or to the right.* **(FGD 1L20)**

Lecturer 2 further asserted that the use of the software worked in such a manner that instead of just suggesting numbers and calculating in abstract (PF1), one was able to see the points moving, the values of the $x$and y coordinates changing and simultaneously (MR1), see the number that is being approached, from both the left and the right (VP1).

Underscoring his view, he clarified:

That one for me it will greatly help learners. One, when it comes to the points(for example when you talk of limits), I will cite an example of first of all the first principle. Where you *have a point, of course you can name it and you have another point, then as this point is moving closer to the other point of course not reaching the actual point. When it comes to GeoGebra, you are able to see that physically* (VP 1) *that this point is moving but it is not reaching this point. Now when it comes to the real teaching of limits, we don't show that, we just explain that this is approaching this either from the left or the right but it will not reach this* (PF1). *Now graphically* (VP1*), under this software we were able to see and I will ensure that I will be able to teach learners using this and they will not forget. Usually when we are teaching limits, we just teach it in a… I can say a robotic way* (PF1*), even if we can come up with that table where you enter the value of x to get the f(x) as a number, as taking the number approaching that value and those a bit up. You will discover that it takes time actually for learners to calculate. Using the app, GeoGebra, it will be time saving actually* (EU1*) and it will also give them a skill, not just the knowledge but also the skill to work with the app, a computer skill yeah.* (TK 1) **(IL2L10**)

He further explained that as he was presenting the lesson on limits, *GeoGebra* enabled him to focus on the movement of points (VP1), while the corresponding changes were being observed on the interface by students. He emphasised that what was so interesting, was that he could demonstrate synchronously the limit approaching the required value from both the left and the right, thereby bringing in the visualisation aspect (MR1).

He stressed:

For the limits I would say that in most cases if you are talking of limits without this software (TPACK 1), *it is actually in abstract*(PF1). *But for this one the way the table showed us* (VP1) *in spreadsheets, when the sliders were being dragged, we were able to see as h was approaching 3, from both the left and the right, the limit was approaching 5. I don't think this would be easy to illustrate using chalk and talk.* **(IL2L15)**

4.3.3 Lecturer 3

4.3.3.1 Area above the $x-$ axis and below the $x-$ axis $-$ brief description of the lesson and how the applets were used

Lecturer 3 taught the area above and below the x axis in Cycle 3. In this cycle, the focus was on how to use *GeoGebra* applets to explore the concept of area bounded by the curve, with one part above and the other below the x axis.

Before using the *GeoGebra* applets, the lecturer presented his lesson using the conventional method. He emphasised that given a function y and asked to find the area, in integration, that function y is considered to be $f(x)$, thus:

$y = \frac{dA}{dx}$, which implies that

$$A = \int_a^b y dx = A(b) - A(a)$$

$$A = \int_a^b f(x) dx$$

He explained that finding the integral of a function entails finding its area.

He then gave an example as follows:

Find the area under the curve $y = x(x-1)(x-2)$, between $x = 0$ and $x = 2$.

The lecturer asked the students what method they thought could be appropriate to find the solution. The students responded that the function needed to be integrated first, and then substitute the lower and upper limits, and then find the difference. They foumd zero as the solotion and the lecturer asked them about their views on area having a value of zero. With help of the lecturer, they realised that they needed to first find the area from $x = 0$, to $x = 1$, and then from $x = 1$ to $x = 2$. With participation from the students, he proceeded as follows:

$$A = \int_a^b f(x) dx$$

$$A = \int_0^2 x(x-1)(x-2) dx$$

$$A=\int_0^2 (x^3-3x^2-2x)dx = \left[\int_0^1 (x^3-3x^2-2x)\right] -- \left[\int_1^2 (x^3-3x^2-2x)\right] dx$$

$A=\left[\frac{x^4}{4}-x^3+x^2\right]_0^1$ The lecturer worked out the first part as shown below and in figure 4.10, and asked students to work out the rest.

$$A=\left[\frac{x^4}{4}-x^3+x^2\right]_0^1 = \left(\frac{1}{4}-1+1\right)-(0) = \frac{1}{4} \text{ square units}$$

The lecturer then illustrated the work above using *GeoGebra* as follows:

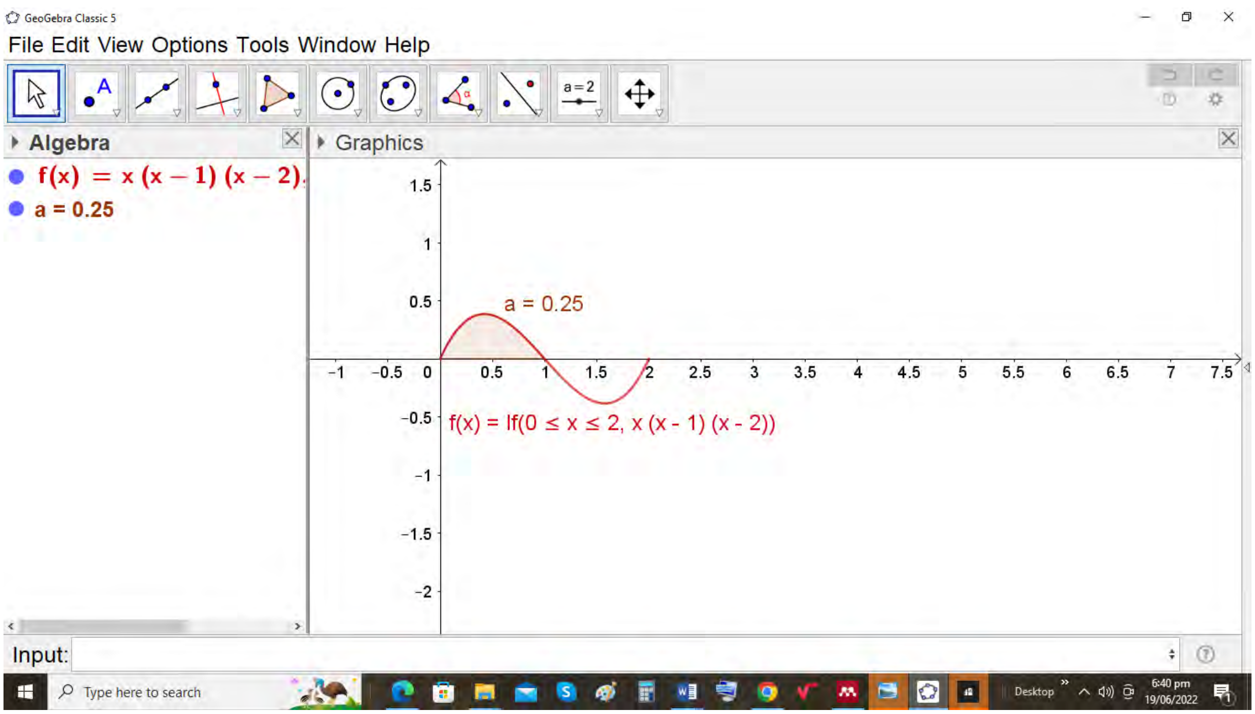

Figure4.10: Applet 3.1: Estimating area bounded by a curve between 0 and 1, for a cubic function

The lecturer gave the second example based on the applet prepared:

Find the area between the curve. $y = s$ from $x = 0$ to $x = 2\pi$

The lecturer asked the students for their views about finding the solution. One student suggested that the first thing was to find the integral of the given function (PF1). The lecturer responded that if he had to go by what one of the students had suggested, he would proceed as follows:

$$A = \int_0^{2\pi} \sin x dx$$

$$A = [-\cos x]_0^{2\pi}$$

$$A = (-\cos 2\pi) - (-\cos 0)$$

$$A = (-1) - (-1)$$

$$A = 0$$

The lecturer asked the students about their views on the value of area being 0. He then suggested that the same question be explored using the *GeoGebra* applet as shown in Applet 3.2, Figure 4.11.

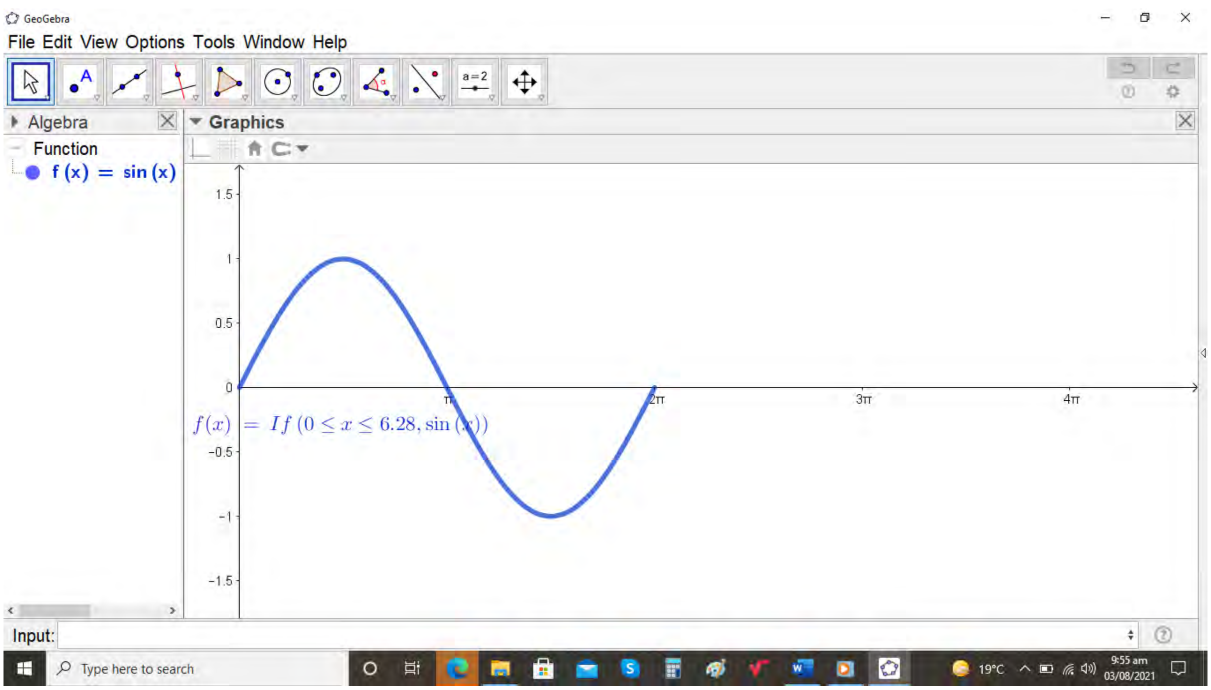

Figure4.11: Applet 3.2. Estimating area bounded by a curve between 0 and Pi, below and above the x - axis

He emphasised that the area to be calculated in the Applet 3.2, was the one bounded by the curve and the x axis, from $x = 0$ to $x = 2\pi$. Referring to the earlier calculation, he asked the students how the area could be 0, when the graph was covering space above and below the graph. He pointed out that this was where the visual aspect (VP 1), played an important role, and emphasised that in problems involving area under the curve, a sketch was very necessary, especially in cases where it involved parts above and below the x axis. He then proceeded to find the areas using the software and picked the integral function in the input bar 'Function, start value, end value'. So, in the illustration in Figure 4.12, Applet 3.3, he used the start value 0, and the end value 2π and he obtained 0:

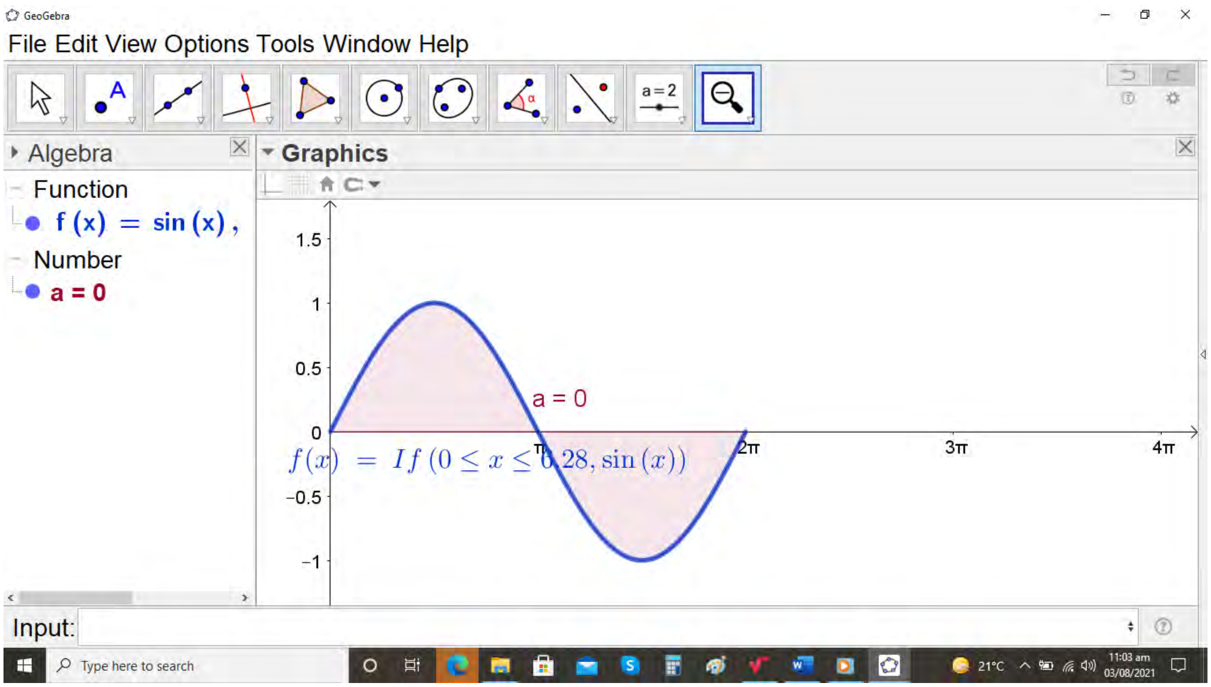

Figure 4.12: Applet 3.3. Estimating area bounded by a curve between 0 and 2 pi showing area zero square units

He asked the students why this was the case, and they responded that the intervals needed to be separated, from 0 to π, and then from π to 2π (PCK 1). He then used the *GeoGebra* tool 'start value' 0 and the 'end value' π, then in the next part used the start value of π and end value 2π, to obtain the area 4 square units as shown in Figure 4.12, Applet 3.3.

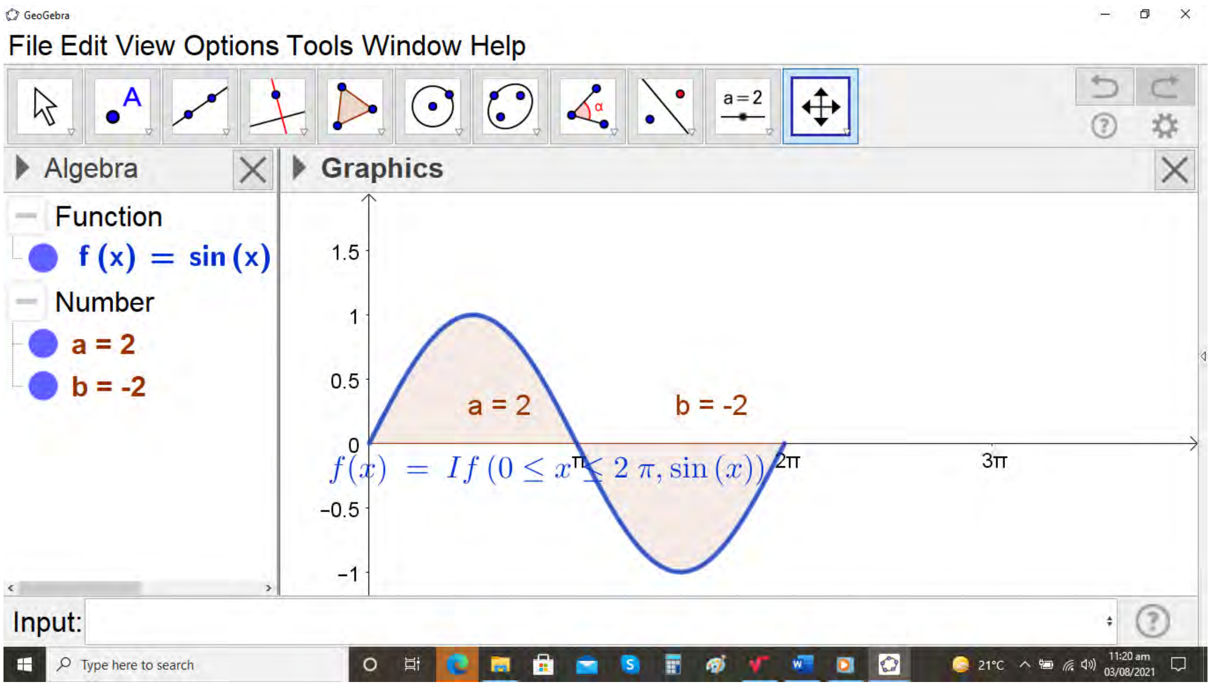

Figure 4.13: Applet 3.3. Estimating area bounded by a curve between 0 to π and $\pi$to 2 π, showing an area of four square units

For the area under the x- axis, the value indicates negative 2. Lecturer 3 clarified that this was because the area was below the x - axis and therefore its absolute value should be considered, which was 2 in this case, making the total area four square units. He emphasised that in such cases, it was necessary to separate the intervals, as failure to do so will result in erroneously obtaining 0 as the area.

4.3.3.2 Interview with Lecturer 3

Lecturer 3 presented the calculus cycle on area under the curve, with a specific focus on a curve that had one part above and the other part below the x-axis. He pointed out that from his experience, students faced some challenges in conceptualising (CU1) the area under the curve, especially when it involves one part above and the other below the $x - axis$. He said the challenges mostly arose from having to deal with 'negative' or 'zero' area. The lecturer explained that after his interaction with *GeoGebra*, (TPACK 1) he realized that the software could enhance the students' understanding CU1) of the area under the curve. He elaborated that it was for this reason that he suggested questions that would give values of zero and a negative value when the upper and lower limits were directly substituted in the integral function.

He commented:

> *I think there the enhancement was that when we looked at the area under the curve, and the software GeoGebra which was used (TCK 1). Initially, when we are finding the area during the traditional method, we just calculate, for example, we just integrate, and we will fuse in our limits, and we get the exact area* (PF1). *But in that case, I found it to be very easy* (EU 1), *in such a way that after the area has been identified with the limits that were given, maybe it is just a matter of clicking, and it clearly shows you the portion which you – actually there is accurate measurement of the area there. It can be a bit difficult if you have plotted using your free hand and using a ruler, but there, the actual area was clearly visible.* (VP 1) **(IL3L8)**

4.3.4 Lecturer 4

4.3.4.1 The Riemann sum - brief description of the lesson and how the applets were used

Lecturer 4 taught cycle 4 on the Riemann sum. The focus in this cycle was to estimate the area under the curve using the lower and upper sums. The question used to develop the concept was as follows: Find the area under the curve $f(x)=x^2$, between x= 0, and x =3. Of course, this can easily be calculated algebraically (PF 1), by definite integral using the algebraic formula as follows:

$\int_0^3 x^2 dx$, to obtain $\left[\frac{x^3}{3}\right]$ and get 9 square units.

The solutions worked out by some of the students on paper are shown below:

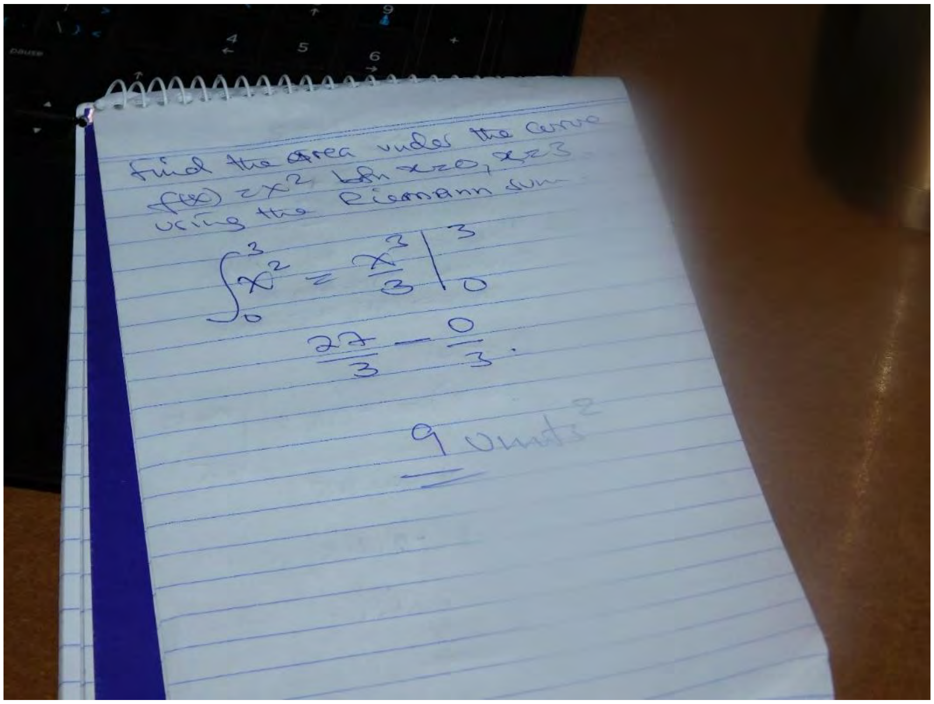

Figure 4.14: Screenshot 4.1: Solution worked out by a student on Riemann sum cycle

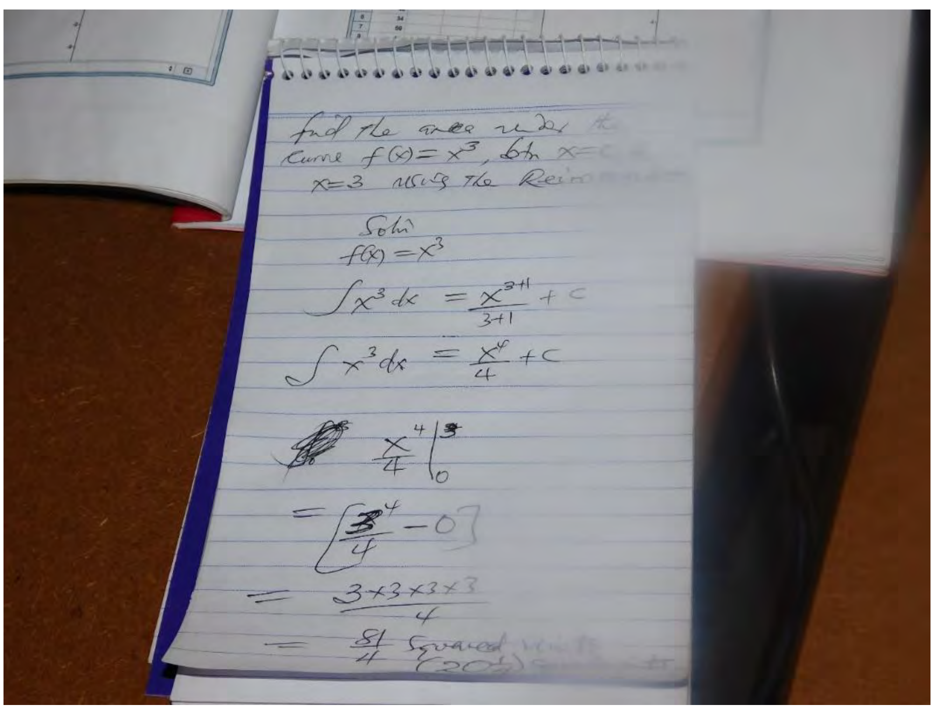

Figure 4.15: Screenshot 4.2: Solution worked out by a student on Riemann Sum cycle

The lecturer asked the students for their views on alternative methods of how they could calculate the area under the curve (PCK 1). The responses included making squares, trapeziums or rectangles under the curve, then calculating the area and finding the sum of all the shapes under the curve. Then using *GeoGebra*, he used the construction tools for upper and lower sums.

4.3.4.2 Estimating area using the lower sum

Using the lower sum and the slider tools (USD1), rectangles were created under the curve, with the number of rectangles set to n, where n ranged from 1 to 50, as shown in Figure 4.15, Applet 4.1. The slider tool is one feature of *GeoGebra* that can be used to decrease or increase the number of objects by dragging it back and forth. It is more effective than having a fixed quantity, as it can easily be used to make the necessary adjustments to a diagram without necessarily starting from scratch.

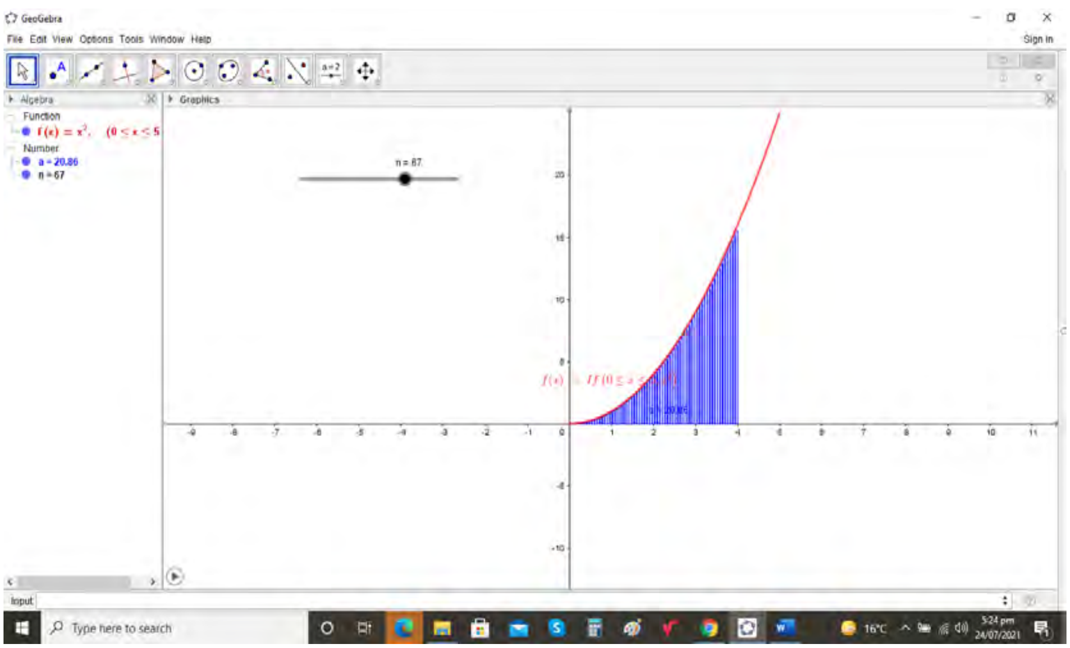

Figure 4.16: Applet 4.1. Finding the area under a curve using Riemann sum

Setting n to 6 on the slider, the applet with six rectangles was created as shown in Figure 4.16, Applet 4.2 below:

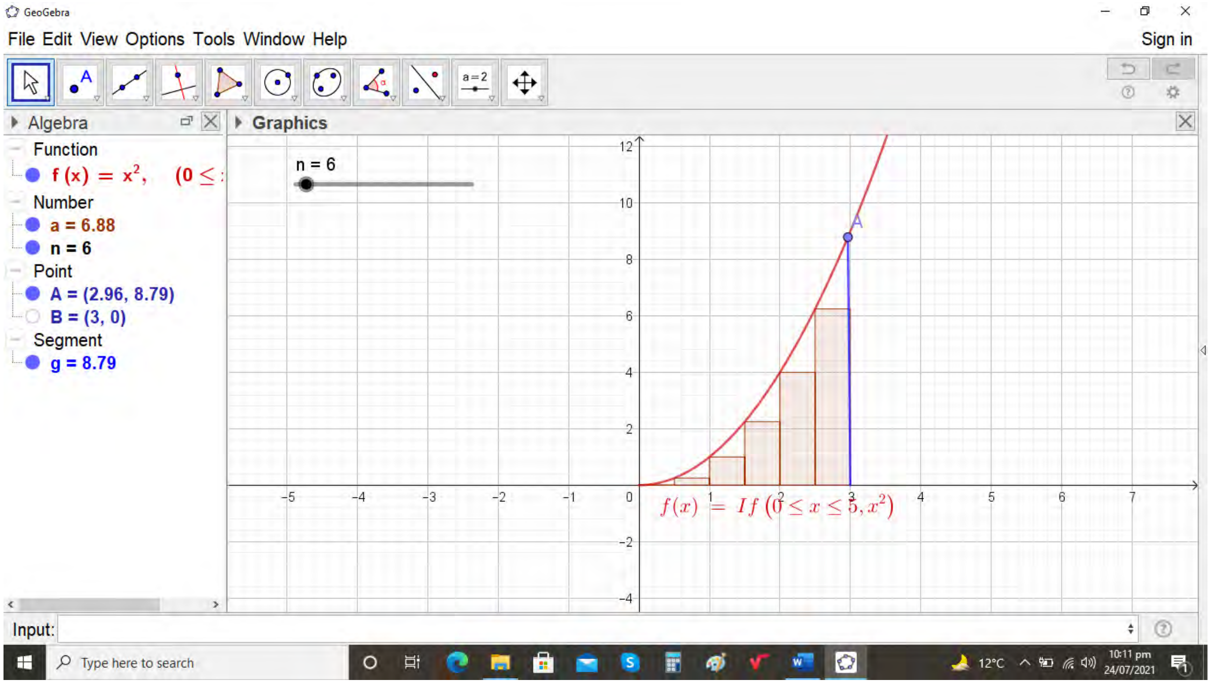

Figure 4.18: Applet 4.2: Estimating area under the curve using the lower sum with six rectangles

The lecturer then asked the students to compare the area found by the lower sum method to that of the actual area. The students were asked to give reasons for their answers. The difference in the value of the approximated area and the actual area was attributed to the gaps in the rectangles in the lower part of the curve (TPK).

Using the slider tool (USD 1), the lecturer increased the number of rectangles to fifteen, as shown in the Applet 4.3 in Figure 4.18.

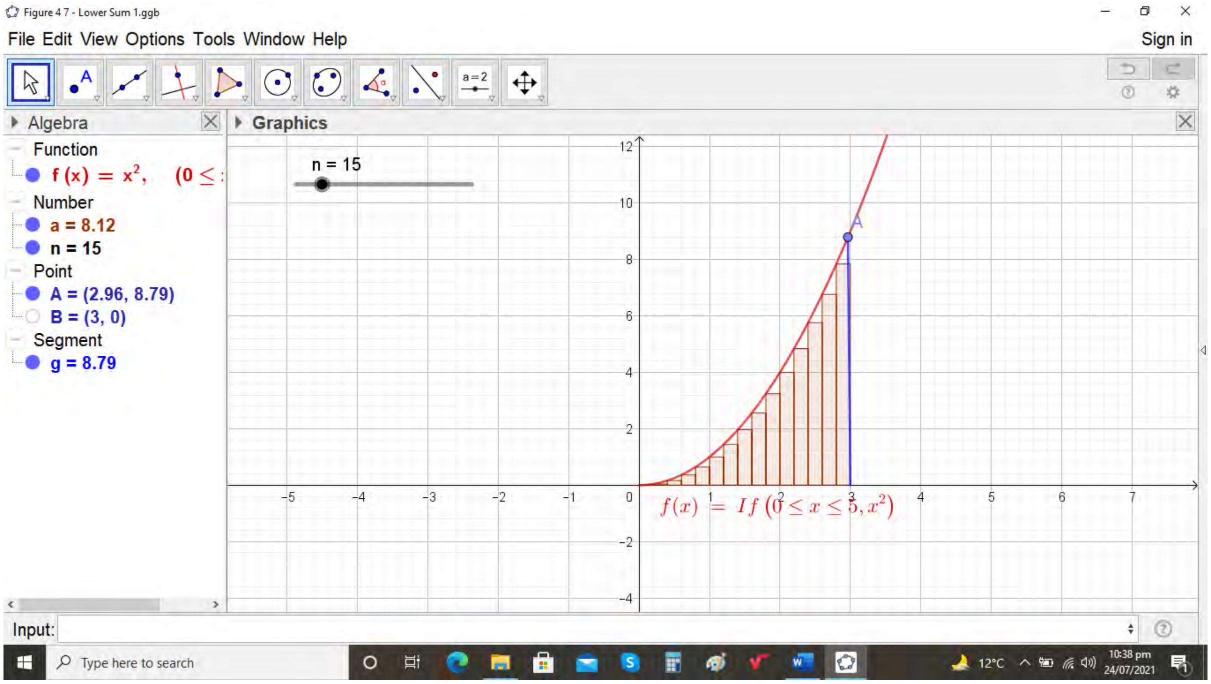

Figure 4.19: Applet 4.3: Estimating area under the curve using the lower sum with fifteen rectangles

With the increased number of rectangles, it was discovered that the approximated area was closer to the actual area as the sizes of the gaps on the top of rectangles was significantly reduced. The value of the approximated area was very close to the actual area, but still less than the actual area at 8.12 square units. As the term lower sum implies, the gaps in the rectangles are on the lower part of the curve on top of the rectangles (CK). It was noted that as the number of rectangles below the curve increased, the more the value of the estimated area became close to, but still less than, the actual area for the lower sum.

4.3.4.3 Estimating area using the upper sum

The lecturer also investigated the area under the curve using the upper sum as shown in Figure 4.18, Applet 4.4.

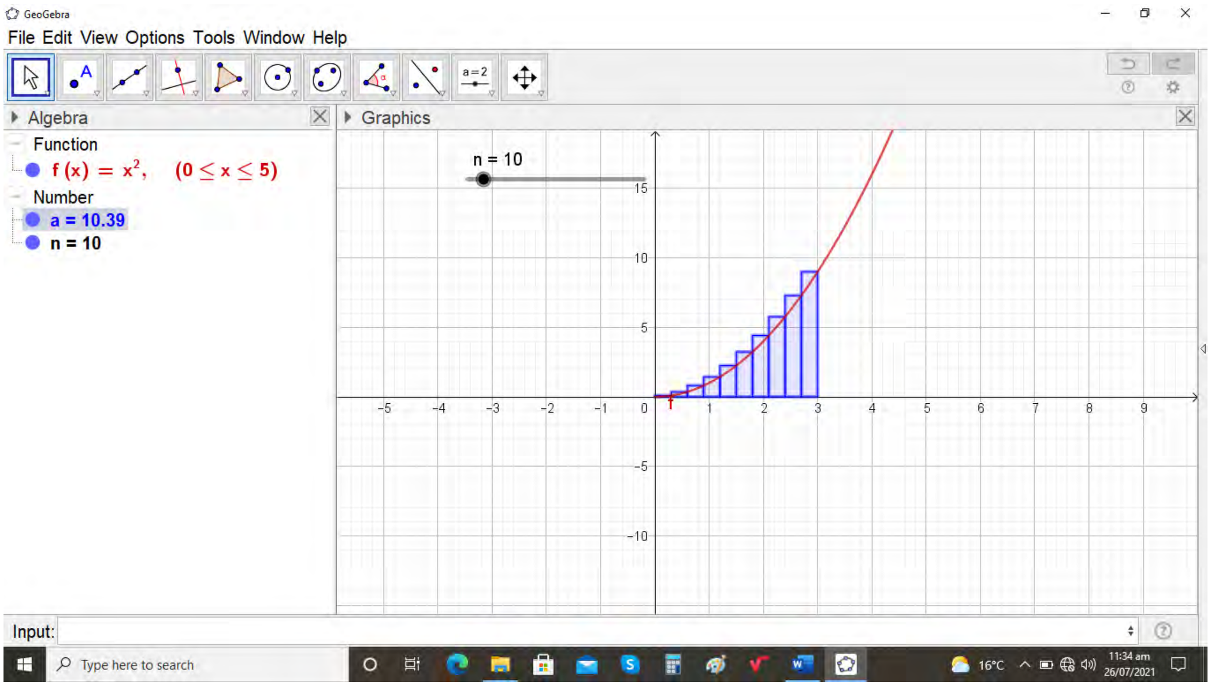

Figure 4.20: Applet 4.4: Estimating area under the curve using the upper sum method

The lecturer illustrated Applet 4.4 showing ten rectangles below the curve (VP1). It was discovered that unlike in the lower sum method (where there were some gaps on top of the rectangles), in the upper sum method, some parts of the rectangles overlapped (protruded) on the upper part of the curve. The lecturer asked the students what they observed about the estimated area compared to the actual area. The observation was that the estimated area was close but higher than the actual area. This was attributed to the parts that overlapped the curve on top of the rectangles (CK1).

By adjusting the value of n on the slider to 31, the number of rectangles increased (VP1) to 31 as shown in Figure 4.18, Applet 4.5.

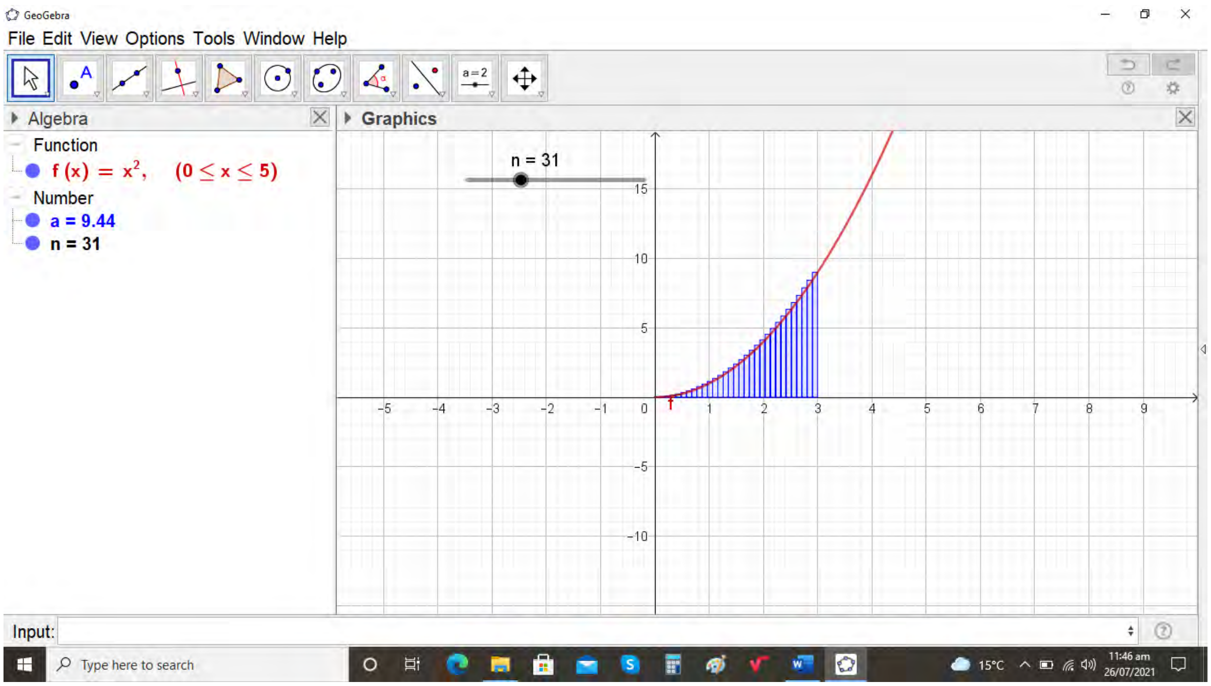

Figure 4.21: Applet 4.5: Estimating area under the curve using the upper sum method with an increase in the number of rectangles

With the increased number of rectangles, it was observed that the estimated area (9.44 square units) was very close to the actual area (9 square units).

4.3.4.4 Interview with Lecturer 4

The lecturer reflected that it was interesting and easy (EU 1) to explore the concept of the area under the curve, using the Riemann sum with the *GeoGebra* software, compared to the conventional method. He reflected that using the conventional method involves a lot of unwieldy work as one is required to draw a number of trapeziums or rectangles, and when changing the quantity of shapes drawn, one has to erase the whole diagram and draw it afresh. In addition to accuracy and prompt feedback, the slider tool of *GeoGebra* makes it easier to visualise (VP1), adjustments in a more concrete manner. Lecturer 4 :

> *It also made the students understand that the more rectangles you have, the more accurate the result* (PK1). *When we had fewer rectangles below the curve, for example two or three, the value obtained for the area was not close to the actual area, so we were able to see that in order to get an accurate result, you need to have more rectangles, so that there are fewer cut offs.* **(IL4L59)**

GeoGebra made the concept of area very clear. The use of sliders to increase or reduce the number of rectangles when calculating the area under the curve made it easier and simpler (TPK 1). During the lecturer's lesson presentation on the cycle of the Riemann sum, it was observed that when he increased the number of rectangles in the applet, the area under the curve was closer to the actual area and when he decreased the number of rectangles under the curve, the estimated area became less and less accurate.

He said:

> *Very good. There we say, when those rectangles were created, we just draw the bigger ones, and we say as we increase the number of rectangles, the area becomes as closer as possible, it is not easy to see because you can't draw so many using chalk and board, but with what we saw with GeoGebra* (TPACK 1), *although it didn't eventually come to the same, but we could see that the area was, the lower was 6.2 and the upper one was 6.7. So it is very easy* (EU 1) *to visualise) that* (VCP1. *They would actually see that the results are bounded – he upper sums, and the lower sums – so it will be clear to understand that these are the lower Riemann integrals, these are the upper Riemann integrals.* **(IL3L9)**

Asked further what he meant by not eventually coming to the same, he explained that there were some differences in the value of area in the decimal part. This was the result because the lecturer did not use the tool under the options tab, 'Rounding off", to increase the number of decimal places.

In the Focus Group Discussion, Lecturer 1, pointed out:

> *This one, is equally… was quite very helpful, because by simply considering the limits or the boundaries you find maybe people have different answers and sometimes you may not even be able to justify the answer* (PF1). *But this software* (TPACK) *was able to show* (VP1) *how close one will be by say increasing the number of rectangles or so, so that the answer actually…, you are able to see, that the more you increase the closer you are to the answer than just saying okay, let me just take between three or four rectangles or so, you find that that way you may not be certain but with this one. one is able to be to tell with certainty that definitely that the answer should be very close to what is shown on the graph.* **(FGDL1L151)**

The lecturer summed it up by waying that when using the Riemann's sum method in the *GeoGebra* interface, the focus was on the number of rectangles that would be formed under a particular curve. For both the lower and upper sum, the goal was to have as many rectangles as possible so as to get as close as possible to the actual answer, and reduce the amount of space that remained uncovered under the graph (CK1).

4.3.5 Lecturer 5

4.3.5.1 Area above the the $x-$ axis and below the $x-$ axis - brief description of the lesson and how the applet was used

The cycle that ecturer 5 presented to the pre-service student teachers was on the slope of the tangent (Cycle 1) – the same one that was presented by Lecturer 1. Unlike Lecturer 1 whose focus was on investigating the slope of the tangent using a secant, the focus in Lecturer 5's presentation was to explore the relationship between the graph of the original function $f(x)$ and its derivative $f'(x)$. His focus was that while *GeoGebra*'s construction tools could simultaneously generate (MR1) the graph of the derivative and the corresponding function, the students may not grasp the underlying concept (CU1) that connects the graph of $f(x)$ to that of $f'(x)$. To explore this concept, the lecturer gave the following activity to his students:

Given the function $f(x) = x^3 + 2x^2 - 1$, find the slope of $f(x)$, at the point where $x = -2, x = -1, x = 0, x = 1$ and $x = 2$. Use your results to draw the graph of $f'(x)$.

The students worked out the questions on paper as follows:

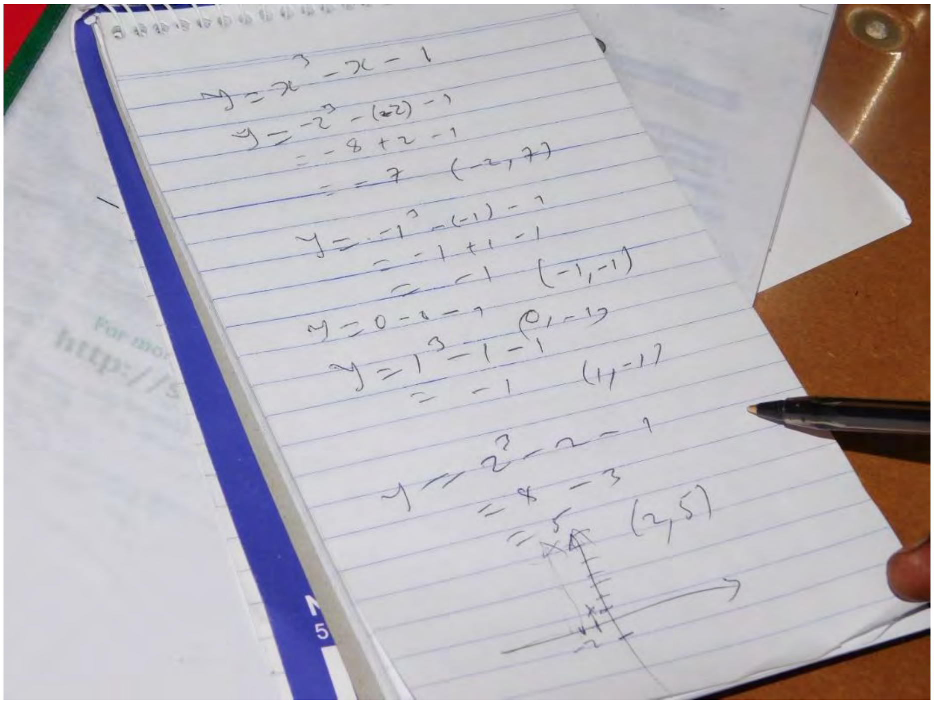

Figure 4.22: Investigating the relationship between graphs of $f(x)$ and of $f'(x)$

Some students, for instance the group that worked out the question as depicted in Screenshot 5.1, faced a challenge in working out the question. They erroneously substituted the values of x x directly in $f(x)$ before finding the first derivative and then substitute in $f'(x)$. However, other groups, such as the one depicted in Screenshot 5.2 below, worked it out correctly with the help of a sketch (VC 1), thus incorporating the notion of visualisation.

The lecturer then worked out the question as follows:

$$f(x) = x^3 - x - 1$$

$$f'(x) = 3x^2 - 1$$

Thus: $f'(-2) = 3(-2)^2 - 1 = 11$ $f'(-1) = 3(-1)^2 - 1 = 2$

$$f'(0) = 3(0)^2 - 1 = -1$$

$$f'(1) = 3(1)^2 - 1 = 2$$

$f'(2) = 3(2)^2 - 1 = 11$ (PF1)

After substituting the integer values of x from -2 to 2, he obtained the following coordinate points: (-2, 11), (-1, 2), (0, -1), (1, 2), and (2, 11). A group of students made a sketch of the graphs of $f'(x)$ and $f(x)$ as shown in Screenshot 5.2 below.

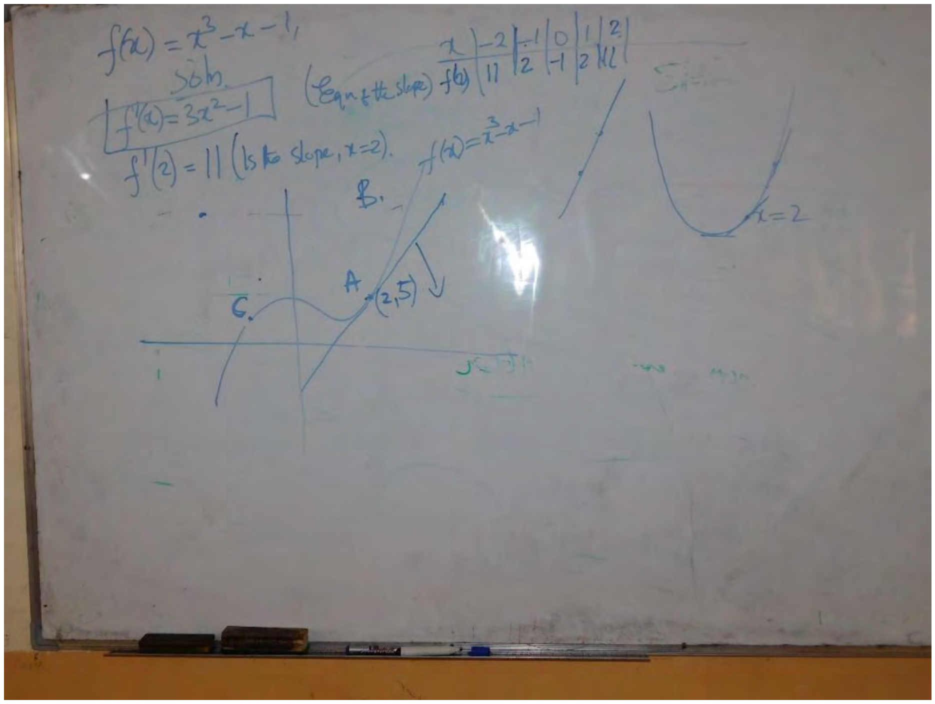

Figure 4.23: Screenshot 5.2 : Investigating the relationship between graphs of $f'(x)$ and of $f(x)$

The lecturer further probed the students to comment on the connection that they observed between the two graphs $f(x)$ and $f'(x)$. The students had challenges and the lecturer proceeded by first drawing the graph of $f(x)$ using *GeoGebra* as follows:

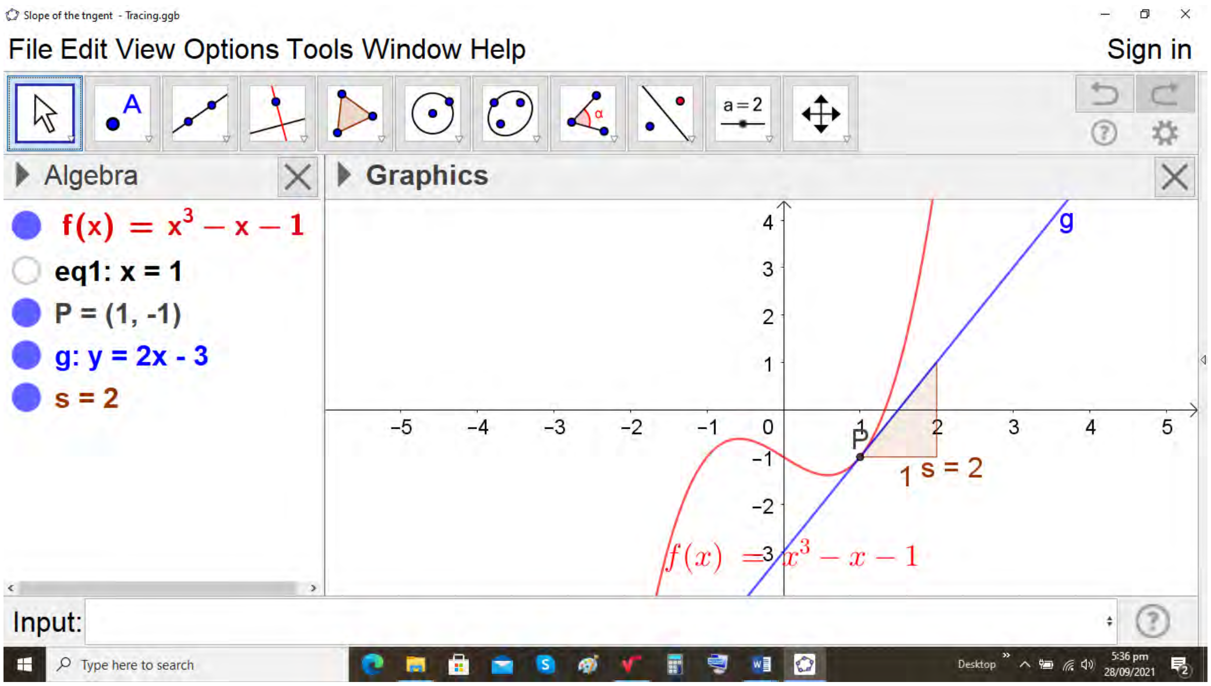

Figure 4.24: Applet 5.1: Investigating the relationship between graphs of $f(x)$ and of $f'(x)$

Having drawn the graph of $f(x)$, the lecturer constructed a tangent on the curve and found the slope of the tangent at point P, as shown in Applet 4.24 He went on further to explain that the derivative is the slope of the tangent at a point on the curve, and the slope at P, where x is 1, $f'(1) = 2$. This implies that the slope on the curve where x is 1 (the abscissa), is 2, and coincidentally, 2 is the y coordinate (the ordinate) where$x$ is 1, which gives the point (1,2). He then asked the students to test the rest of the points and it was established that the value of $f'(x)$ corresponded to the value of the slope (TCK 1) and PCK 1), which was also the same value as the y coordinate. He elaborated that this concept could be used to plot the graph of the derivative using the conventional method, and by tracing using *GeoGebra* as shown in Figure 4.22 below, in Applet 5.2.

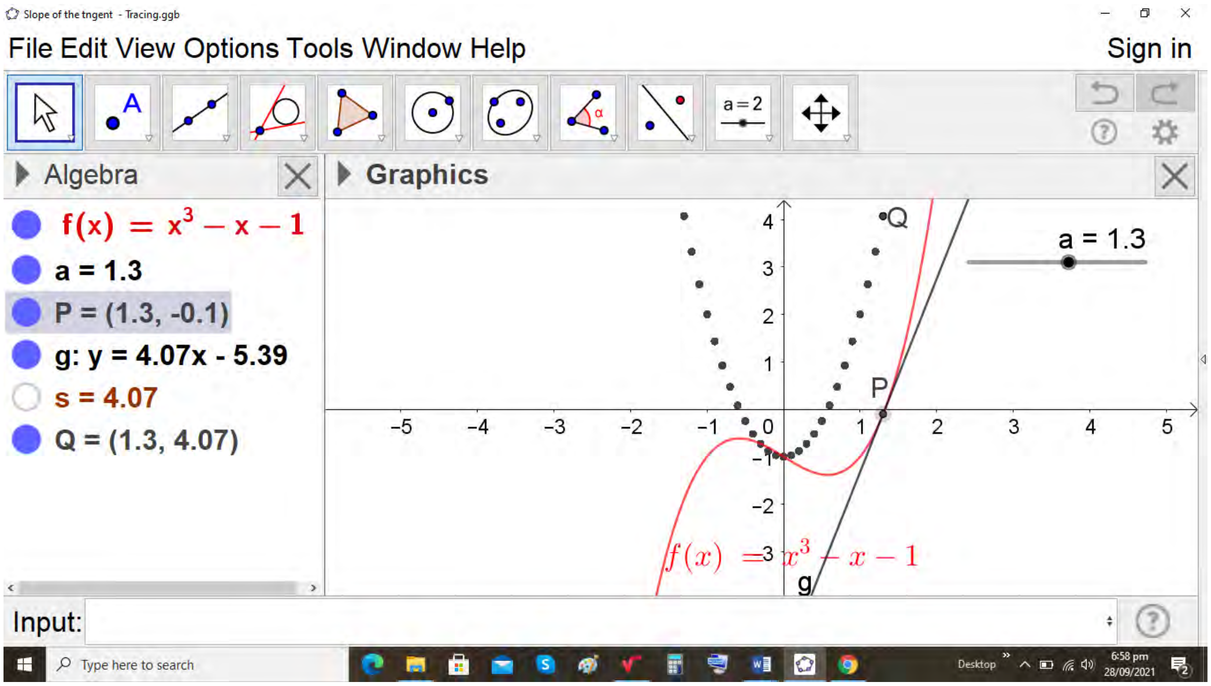

Figure 4.25: Applet 5.2: Investigating the relationship between graphs of f(x) and of f'(x) - 1

After creating a slider called *a*, for $f(x)$, to obtain a general point P on the curve, the lecturer let $P=(a, f(a))$, constructed a tangent at P, and then found the slope at point P. To create a general point on the derivate, he set Q =(x(P), s)), having set *s* as the slope of the tangent at P. Then, moving the slider between its minimum and positive values, the graph of the derivative was traced, using the trace construction tool (TPACK 1).

It was observed that for $f(x)$ and $f'(x)$, the x coordinates for P were the same (as can be seen in Applet 5.2), where Q is directly above P, but the y coordinates for $f(x)$ and $f'(x)$, were different (CK 1). He encouraged students as they worked on activities on differentiation, to draw graphs (VP 1) of functions of their corresponding derivatives, as this may enhance their understanding of concepts (CU1).

To further illustrate the concept –being aware that students were very familiar with the idea that the derivative of sine x is cosine x – the lecturer asked the students to use *GeoGebra* to trace the result. The illustration included the following, shown in Figure 4.23 and 4.24, Applet 5.3a and 5.3b.:

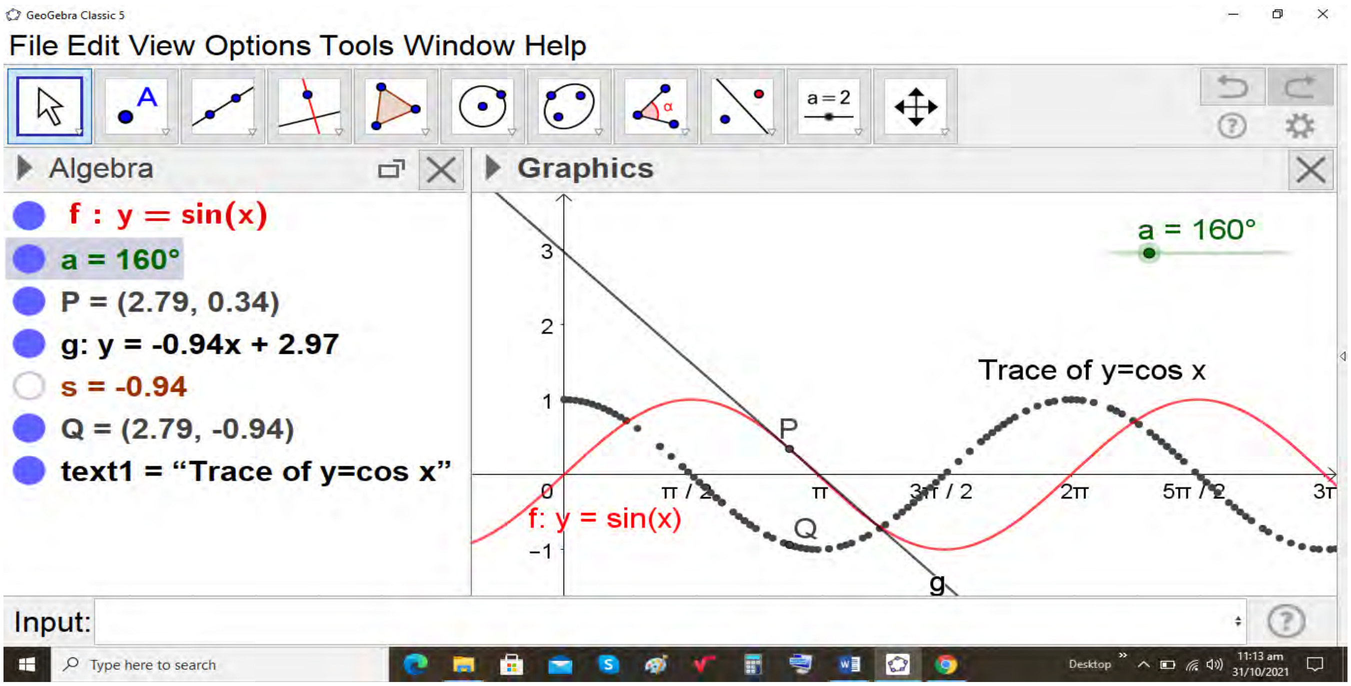

Figure 4.26: Applet 5.3a - Investigating the relationship between graphs of f(x) and of f’(x) - 2

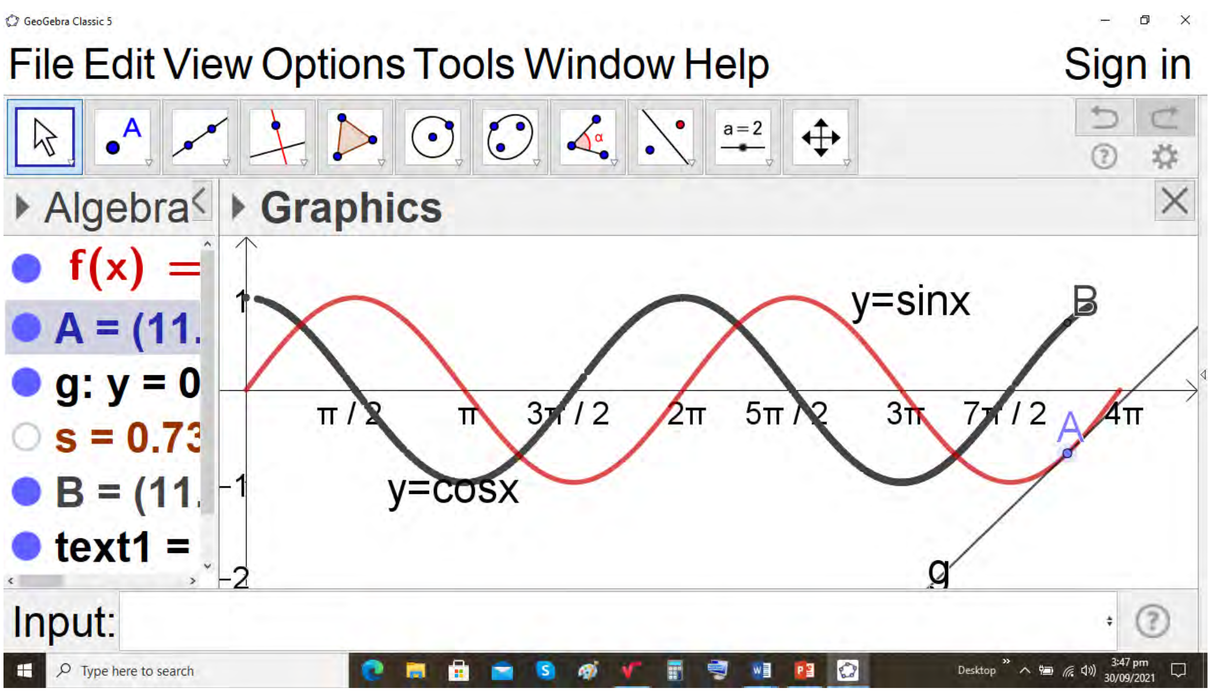

Figure 4.27: Applet 5.3b: Investigating the relationship between graphs of f(x) and of f’(x) - 3

4.3.5.2 Interview with Lecturer 5

The interviewees expressed a variety of perspectives on the concept of the derivative. Lecturer 5 narrated:

> *Usually, as you are introducing the derivative function, we usually draw a kind of a diagram like that which we just drew, in abstract* (PF1). *Learners don't even appreciate what we are doing. But if we can draw using GeoGebra, they are able to see exactly* (VP 1) *how a tangent line is drawn, where it is coming, how the graph of the function relates to the graph of the derivative. All those* (MR1) *and then it will give them a picture*, (VP1) *on how the derivative will come out or is done***,** *because in our teaching, we normally just differentiate, and it ends there, without really knowing how that particular concept was arrived at.* **(IL5L7)**

The lecturer reflected that unlike the *GeoGebra* construction tool of differentiating (Derivative <Function>) which gave the derivative of a function directly (VP1), the aspect of tracing provided more insight into the concept of the derivative. He further acknowledged that prior to his interaction with the software, he had hardly investigated the relationship between a function and its derivative (PF1), and he rarely brought in the visual aspect of graphs. Most of the work was merely done procedurally and in an abstract manner (VP1). He explained that from the applet of $f(x)$ and $f'(x)$, it was apparent to the students that when a cubic function whose degree is 3, is differentiated, its degree decreased by one, to that of a quadratic function whose graph is a parabola (CK). The students were able to see all these on one *GeoGebra* interface (MR1). In the FGD, Lecturer 6 noted:

> *First of all, I will acknowledge that this GeoGebra is probably the best in the teaching of calculus. It has come at a point when I think I introduced the topic of calculus I think a week ago and some of the concepts which I had difficulty teaching were made simple* (EU 1). *So since the topic is not yet complete I will ensure that I familiarise with this software even some of the parts which I taught I will ensure that I will be able to show the learners. Of course, there are some concepts which I explained but I was not satisfied but I will be able to of course to invite my learners and show them what I meant by saying the derivative is the slope of the tangent at a point on the curve* (PCK). *The issue of the derivative, how*

it is done (TPK), so to me, this one is a very good software which all of us who are teaching mathematics must embrace. **(FGDL6L209)**

Echoing this view, Lecturer 5, acknowledged that:

> *That experience in the lesson, where, instead of going straight away to finding the derivative* (PF1), *we had to differentiate through the method of tracing* (TPK), *really helped me and the students conceptualise derivatives better* (CU1). *For me, I think, it's the best tool for me I can say because it will give the know-how* (PCK 1) *for both the lecturer and the learner on how things are done and how they came up because as we are using it we will be able to see how things are done and I can also recommend that going forward this tool should be introduced in the teaching of mathematics especially calculus and other topics such as functions and many others.* **(FGD 3L5)**

In the FGD, Lecturer 5 summed up that a number of calculus concepts could be better illustrated using the *GeoGebra* software than by the conventional method.

4.3.6 Lecturer 6

4.3.6.1 Area above the x −axis and below the x −axis – brief description of the lesson and how the applet was used

Just as Lecturer 3, Lecturer 6 presented Cycle 3 on the area above and below the x −axis. However, Lecturer 6's approach was different. After asking the students the question: 'Find the area between the curve $y = s\text{ in } x$ from $x = 0\ to\ x = 2\pi'$, in the conventional method, he asked each student to explain what each answer represented (PCK 1). Some of the working done by students is presented in Screenshots 6.1 and 6.2 below:

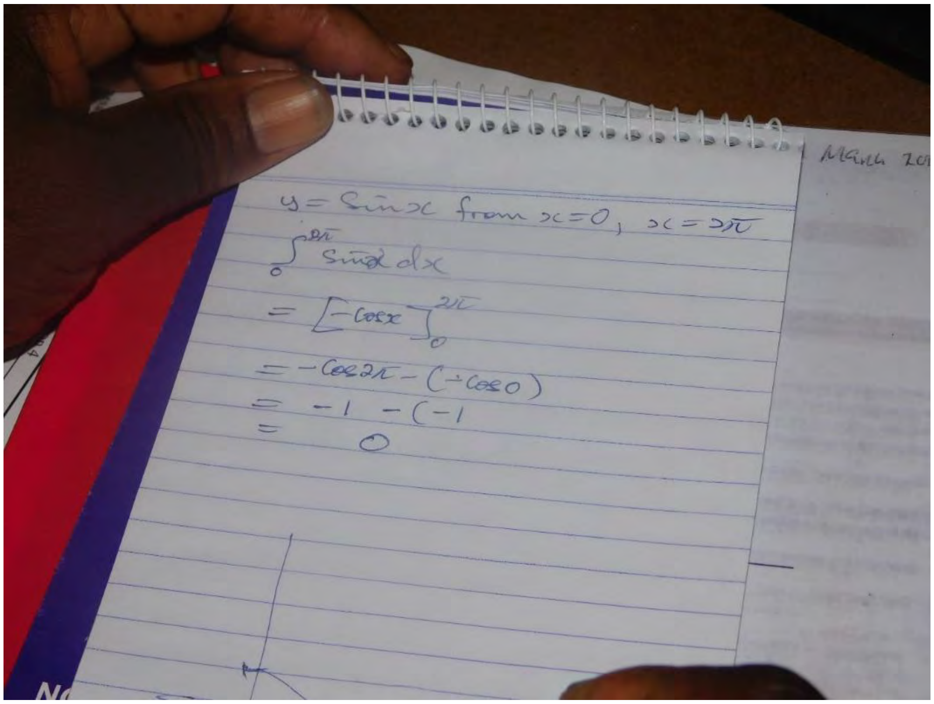

Figure 4.28: Screenshot 6.1a: Finding the area below and above the x -axis

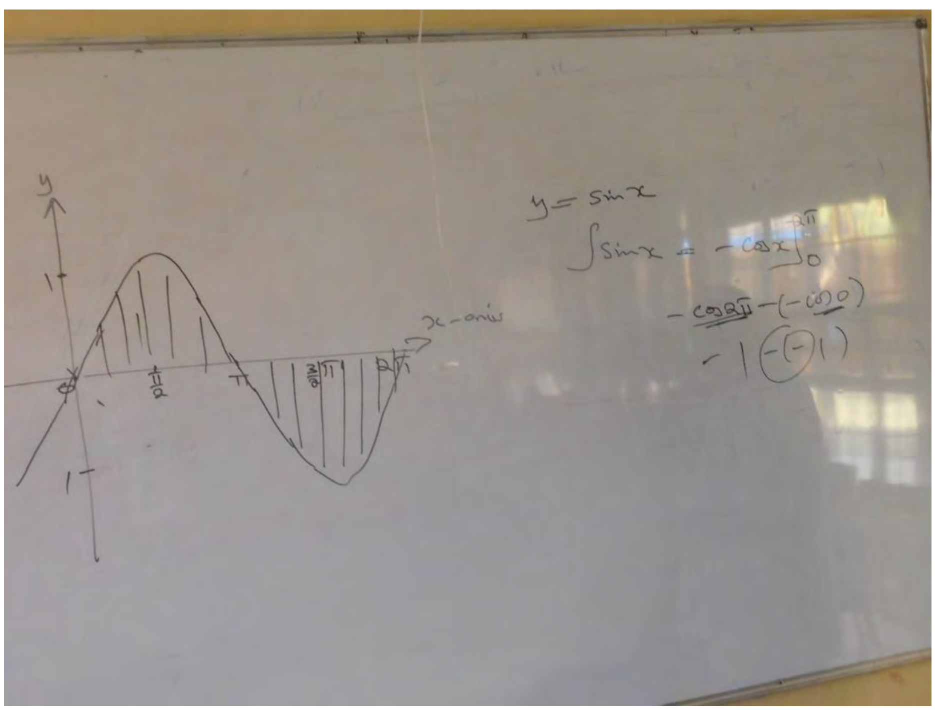

Figure 4.29: Screenshot 6.1b: Finding the area below and above the x -axis

As shown in Screenshots 6.1a and 6.1b, most of the students' methods in working out the question resulted in 0 and 1 as solutions (PCK1). The lecturer asked the students to explain the two solutions in relation to the meaning of the term 'area' (CU1). After some hesitation from the students, he reminded them that area can be defined as the space occupied by a flat shape or the amount of space taken up by a 2D shape. In view of this he asked them to give a justification for the answer 0. He went on further to explore with them that if they considered the sketches that they had made, would it be logical to have an answer 0?

The lecturer clarified that while the first solution may not necessarily be correct, it was however, a valid answer. From the visual aspect (VP 1) of the question, it was evident that it would not mathematically be correct to claim that the space covered could be zero square units, when the diagram shows that there is space covered above and below that x –axis (CK1). He emphasised the importance of visualisation in such questions, as it enhances the understanding of the question. Figure 4.27 illustrates the work that was erroneously done one of the participants during preparation with me.

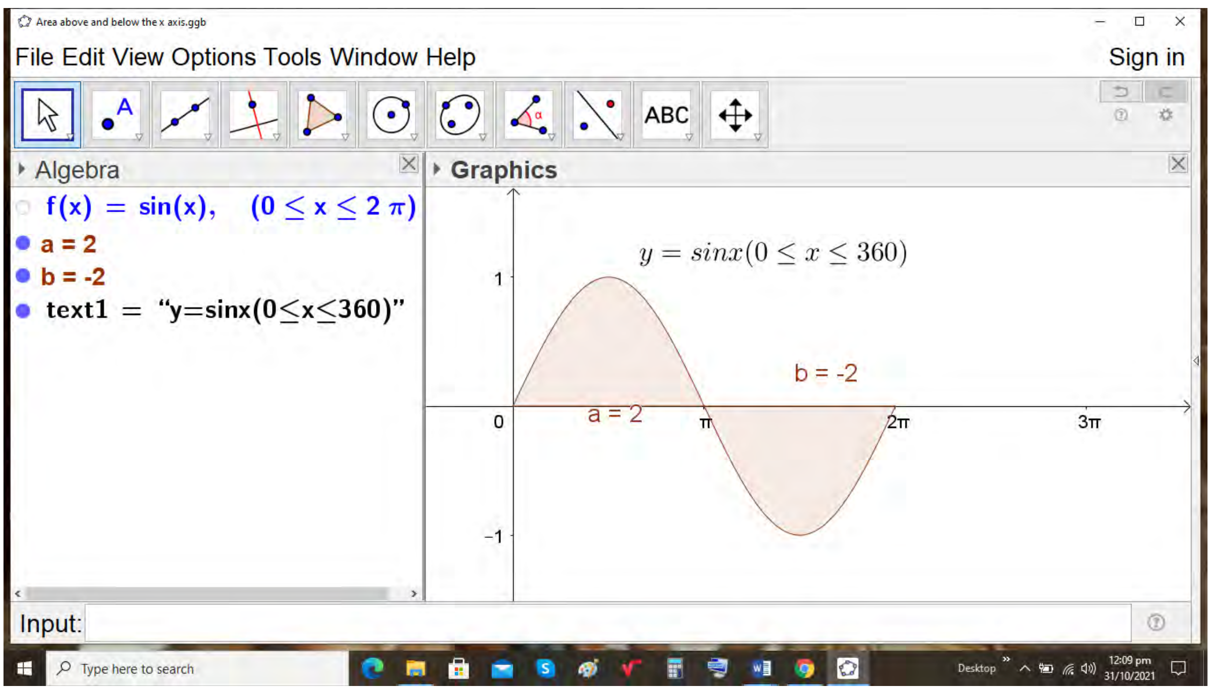

Figure 4.30: Applet 6.2: Area below and above the x –axis prepared for students

After discussing the question, the lecturer correctly worked out the solution and prepared the applet as shown in Figure 4.27, Applet 6.2 above, to obtain the solution of four square units.

4.3.6.2 Interview with Lecturer 6

Lecturer 6 explained that the *GeoGebra* applet clearly illustrated (VP1) the sine graph from the origin to π above the x-axis, and from π to 2π below the x −axis.The graphic view showed that space had been covered, but without the visual features of *GeoGebra* and going directly into computation, students were getting solutions such as -1 and 0, which were not mathematically valid considering the concept of area. He stressed that getting the solution zero indicates that no space is covered, but when it is made visual by *GeoGebra* it reveals that space is occupied. Interestingly, as the lecturers and I were preparing the applet on this cycle, some lecturers obtained -1 and 0 as solutions. When asked to justify their answers, they explained that such solutions could have been avoided had they worked out the question by using a visual aspect.

Lecturer 6 stressed:

> *Usually, as you are introducing the derivative function, we usually draw a kind of a diagram like that which we just drew in abstract* (PF1). *Learners don't even appreciate what we are doing. But if we can draw using GeoGebra, they are able to see exactly* (VP1) *how a tangent line is drawn, where it is coming from, then the secant and all those and then it will give them a picture* (VC1) *on how derivative will come out or is done. This one was quite interesting, because without visualisation you can't easily see the scope* (CU1) *at which you are finding the area because as much as we are looking at area we are interested to look at the space a particular object is covering. So the areas without GeoGebr,* (TPACK1), *you can just think of because the limits up and all have already been provided. I can quickly do the calculus part to integrate and get the solution* (PF1), *but with GeoGebra it was giving us a clear picture* (VP1). *There is this portion of the graph and then there is another potion of the graph so that consideration was quite vivid.* **(IL6L12)**

He elaborated that *GeoGebra* actually shows us (VC 1) the diagram of the shaded parts of the areas to be calculated, so it is therefore easy (EU 1) for the learners who see the actual parts that are being discussed.

Lecturer 6 stressed:

It gave us the two curves and to me it was very easy to justify the answer that was given (CU1).*It showe* (VP1), *that the curve is both above the x-axis and below, so the two, even the limits were easily noted from there. When we solved without visualising* (PF1*), we had a challenge, because of the computation process it brought some of us to zero because there was -1 and +1, but when we visualise, that is when we saw that actually there was area which was covered – area – space was occupied, so it was occupied above the x-axis and below the x-axis, hence after visualising, that is when it made sense that there is area below and above, with limits. I mean not taking the full limits but from 0 to π, then π to 2 π and it gave us now the correct answer which was 4, yes 2 + 2 is 4 so the other one without visualising*, *we didn't take the absolute value of area and we found a negative area which was not correct. It was easier* (EU 1, *to understand. It makes teaching very easy because certain concepts being taught are visualised.* **(IL6L12)**

In the FGD, Lecturer 5 further explained that:

This one was quite interesting, because without visualisation, you can't easily see the scope (CU1) *at which you are finding the area because as much as we are looking at area we are interested to look at the space a particular object is covering* (VP1). *So the areas without GeoGebra* (TPACK), *you can just think of because the limits up and down all have already been provided. I can quickly do the calculus, part to integrate and get the…, but with GeoGebra it was giving us a clear picture* (VP1). *There is this portion of the graph and then there is another potion of the graph so that consideration was quite vivid.* (**(FDGL5L304)**

Lecturer 6 summed up with the observation:

Technology simplifies matters (EU1), *that is in teaching when you want to explain a concept technology will help you to visualise* (VP1*) that concept to the learners and GeoGebra is that tool which we can use to explain these concepts in several topics, calculus and these other topics geometry, algebra, statistics. GeoGebra can easily make learners understand a concept***. (FGD L6L74)**

4.4 HORIZONTAL ANALYSIS

In this section, data was analysed across the cases to obtain deeper insights into the lecturers' experiences and perceptions of incorporating *GeoGebra* as a visualisation tool to teach calculus to pre-service teachers in TEIs. The factors that enabled or constrained the lecturers' adoption of technology in teaching and learning mathematics were also analysed. This was done by presenting a summary of the analysis by considering the emerging themes of the similarities and differences, from the perspectives of the six participants. This horizontal analysis was done with reference to my analytical framework and the three research questions of this study:

- How can *GeoGebra* be used as a visualisation tool to teach calculus to pre-service student teachers in TEIs to enhance conceptual understanding?
- What are the perceptions and experiences of lecturers in using *GeoGebra* as a visualisation tool to teach calculus in TEIs in Zambia?
- What are the enabling and constraining factors of using *GeoGebra* in the teaching and learning of mathematics?

4.4.1 Research question 1: How can *GeoGebra* be used as a visualisation tool to teach calculus to pre-service student teachers in TEIs to enhance conceptual understanding?

A variety of perspectives were expressed among the participants on this question, but a recurrent theme was a sense among the interviewees that *GeoGebra* was an extremely vital tool to both students and lecturers in the teaching and learning process of mathematics, in that the visualisation features of *GeoGebra* provided a strong link between the abstract nature or theoretical part of mathematics, to the practical aspects. The following sentiments were shared by the participants:

> *I would say it's a very important tool that every teacher and learner should use because just teaching maths on its own without providing the visual aspect doesn't add up so I would say it's a very important skill that each learner even staff should have in mathematics.* **(IL3L13)**

> *The visualisation features of GeoGebra in the teaching and learning of mathematics, provides some form of reality, whereby instead of just concentrating on abstract thinking, you bring to the learners the real situation – they can also see it. Actually, it gives them the*

idea to see how things come up, yeah, and how it is done in real life, hands on and some sort of mind zone. **(IL1L7)**

Visualisation is like using a real object to present something to the learners where they are also going to put their hands, mind on that particular activity. We say when you do, when you are involved you remember but when you just hear, it becomes difficult to recall so visualisation brings in the realities of teaching in a classroom situation. It consolidates the abstract knowledge that we want to put forward to the students. **(IL3L12)**

Visualisation on its own is very key when it comes to the teaching of mathematics, because once learners see, I think they cannot forget. Unlike a situation where you're just taught, because when they see I think they will be able to assimilate and they will be able to understand that concept easily. It helps learners to really see how the concept came about and the idea behind it because they will be able to see if you move a point from here to that point they will be able to see and follow through. **(FGD 1L21)**

I think it helps the learners to consolidate the concept as they hear whether it is the gradient or so then they now see the actual graph and all that it becomes easier for them when it comes to just the theoretical aspect so they are able to link, to relate to be convinced that really this is the result. **(Il5L15)**

The lecturers also emphasised the importance of adequate training for them to be able to effectively use *GeoGebra* as a visualisation tool to teach calculus to pre-service student teachers. They pointed out that training in the use of the software, coupled with the knowledge of the subject matter in calculus, were absolutely vital for them to effectively teach with the software. They felt that ongoing training would be of great help. One of the participants clarified:

Yes, training is very necessary. I say so because I managed to deliver a lesson using the software, the moment I was introduced or exposed to the software, I was just given the basics, but with the interest, I had to explore, I managed to build on the basics that I was given and I found it very interesting to explore, once you know the basics, you can build on the foundation to explore it. **(IL2L13)**

Another lecturer articulated that:

The information that I can share is I don't know whether it is possible to or in your personal capacity to facilitate this kind of training on a larger scale, so that lecturers can be trained from few selected colleges across the country, so that it can maybe actually be adopted as a teaching tool in colleges. I don't know whether you can facilitate, or whether you can sell the idea to the Ministry of Education since colleges of education are under Ministry of Education so that the powers that be may adopt this idea and see how they can actually implement this process, where you become one of the facilitators to improve on the results. **(IL4L13)**

This resonates with the sentiments of Koehler et al. (2017) (mentioned in the literature review in Chapter 2), that strategies that are required to better prepare future teachers for learning and teaching in the 21st Century, should take cognisance of teacher training in technology integration.

From their experience with the software, the participants felt that the incorporation of ICT in the teaching of mathematics should be one of the topics in teacher training and *GeoGebra* should be one of the components. They however, suggested that *GeoGebra* should be used to reinforce the conventional method of teaching calculus. A calculus topic should first be taught in the traditional method, and then *GeoGebra* could be used to emphasise, or used side by side, to clarify and verify the concepts, so as to enhance the learning process.

It should be used alongside the usual conventional method, so that the students can have both GeoGebra and the ordinary way of learning so that they understand better. **(FGD L2L10)**

Lecturer 2 responded that in the teaching of calculus to pre-service student teachers, *GeoGebra* should be used as a resource for innovation and for accuracy. It should also be used to facilitate the link between abstract and concrete concepts by virtue of its visualisation features.

Generally, we just teach things in abstract, but with the use of GeoGebra in the teaching process, students will be able to see what is actually happening because as we say, mathematics is a practical subject, they have to see, the practical aspect of it, in this case which is the use of GeoGebra. Yes, we have already said that GeoGebra helps both teachers and learners through visualisation and also and can also be used as a checking *tool. To verify your solution, so if teachers in learning institutions can be introduced to this app, then I think it would make their work easier when they go in the field* **(IL2L119).**

Lecturer 6 outlined that in the teaching of calculus, one of the reasons that people say mathematics is difficult, is its abstract nature. *GeoGebra* has the potential to make those mathematics concepts visual that are not easily understood by the learners. She added that it is an aid that enhances learning and can be seen graphically.

She stated:

> *I think the role of visualisation…, there is an enhancement especially in what may seem to be abstract when explaining. It brings on board, the aspect of appreciating what is being talked about. Because I remember when I was introducing differentiation, first of all, I began by displaying the ordinary, ehh, the traditional method. I drew the X o Y plane, and I drew the curve, the parabola, a curve just like that, then I also had to draw the secant line, just like that. So I explained the traditional way, but the moment, because I had already prepared it on my GeoGebra application, but the moment I finished explaining the traditional way and beamed it to the class, I could even see them now getting the actual concept of what I was saying, ooh, so this is what you meant! I think visualisation there brings in an appreciation of seeing what may seem to be abstract and the memory there is enhanced very much, they may not forget, they find it difficult to forget about the concept.* (**IL2L6)**

This view is in line with the observation by Toptaş et al. (2012) in the literature review in Chapter 2, who indicated that examples of visualisation of mathematics concepts included 2D or 3D physical manipulatives, graphs, diagrams and pictures.

Lecturer 3 responded that *GeoGebra* should be used in a manner that would make calculus concepts go beyond the theoretical perspective. He elaborated that *GeoGebra* should be used to illustrate calculus concepts such as limits, and gradient of a line at a point on the curve, so as to make students 'see' what is being talked about. He added that this would improve learning because students would actually visualise and see what is happening rather than merely learning it in an abstract fashion.

The role of visualising is actually to make it easy to follow than just listening to somebody saying this is what will happen when you do this, this is what will happen, when you do that, but if you able visualise it, it enhances your understanding. Okay, visualisation helps the learners to fully understand unlike using the traditional method, this will help learners visualise, they will get to have that picture to visualise, maybe, you are using an app, a software, let's say for instance, you are teaching them on differentiation from first principles, if you are using the ordinary method, they won't really understand the concept, but once you use that projection with GeoGebra, they will visualise and it will stick in their memory. (**IL8L3)**

Lecturer 6 reported that *GeoGebra* could be used to encourage exploration of calculus concepts by first solving questions using the traditional method, then bringing in the features of *GeoGebra.* He elaborated that:

When you look at its accuracy for instance, you might find that students feel much better if there is a problem, they solve that problem and then you have a reference point, to tell whether or not you have found the correct solution, so in an event that students are exposed to GeoGebra, they can be doing their solving in the traditional way, and by virtue of GeoGebra being able to confirm if the solution is correct and its accuracy. I think that can be a steering wheel to say, they will be using this whenever a student is dealing with calculus, it will be easier for them to refer to GeoGebra even without necessarily looking for somebody who knows the actual way of solving it. (**IL6L10)**

This conforms with what is outlined in the literature review that as students engage in problem solving, they can use *GeoGebra* to visualise their ideas, confirm or falsify their assumptions and to obtain immediate feedback.

GeoGebra should be used in a manner that lessens the focus on formulas, but rather in a manner that promotes understanding of concepts. He gave an example that lecturers normally find it a challenge to illustrate the meaning of calculus concepts such as 'delta' or 'change' when using the conventional method. The use of *GeoGebra*, he added, through its visualisation characteristics, would help illustrate the meaning of these concepts and hence enhance understanding. This is in line with the observation by Sabella and Redish (2007), who noted that a number of students enrolled in HEI calculus classes tended to acquire superficial and incomplete understanding of basic concepts of calculus, due to teaching practices that emphasise rote, algorithmic drilling and manipulative learning.

Another participant was of the view that students should be given a lot of challenging calculus tasks that require use of *GeoGebra.* This, in his view, would encourage students to use the software frequently, and since young people are often very keen on using ICTs, they would most likely discover a lot of things on *GeoGebra* on their own.

> *Ummh, to start with***,** *the learning process itself when using that software as I experienced it was quite encouraging. You could find learners on their own may even start exploring other things which they may not even been taught, the way I saw when we were trying to find the derivative itself, a learner may be encouraged to practice some more examples in the process they will learn more things.* **(IL4L7)**

The participants also brought in the aspect of access to the software. They felt that more access to the software on reliable computers for both lecturers and students would facilitate the use of *GeoGebra* as a visualisation tool to teach calculus. As noted by lecturer 5:

> *Okay, I think in order to help learners explore mathematical concepts, it would be good to give them access to the software, let us say for example, at a school in the computer lab, we install the software on those computers, so that learners may have access because some of them may have no computers where they are coming from and things like that, so we can encourage the learners, we can train them, then also encourage them to continue using the same software from the lab, and also those who are able to do it from home, that is also fine. By them doing it on their own, it would really help them because that would be like hands on.* **(IL15L11)**

This is in agreement with Ertmer et al. (2012) who identified school level barriers as lack of effective training to solve the technical problems and lack of access to resources. The participants emphasised that the visualisation characteristics of *GeoGebra* enhanced understanding. Use of *GeoGebra* through its visualisation features can be used to enhance the understanding of the application of calculus concepts in real life, as well as understanding the definition of the limit and the derivative. Lecturer 5 articulated:

So, the enhancement is through visualisation. So, they are able to see, the limit is one thing that is very difficult to understand because you don't see it. How is it coming to zero? So the movement of the straight line towards zero, that helps. The emphasis is on the enhancement of the learning process, to make the learners learn actively, especially when it comes to graphing, especially calculus requires a lot of graphs. But if they see it visually using GeoGebra, that will really help them to understand calculus at secondary school, and at tertiary, it can also actually help to enhance the teaching, even to us as lecturers, it is very helpful. (**IL5L11)**

4.4.2 Research question 2: What are the perceptions and experiences of lecturers in using *GeoGebra* as a visualisation tool to teach calculus in TEIs in Zambia?

On the issue of perceptions and experiences of lecturers in using *GeoGebra* as a visualisation tool, the aspect of using the software judiciously was one of the emerging themes. Lecturer 6 remarked:

That depends on how the lecturer or the teacher interacts with the software, because the software is eh….a very good software and so it can enhance the teaching of calculus because calculus has an algebraic abstract part which is brought into light if it is done geometrically, so when there is that combination, the abstract ideas from algebra to the geometrical functions, if there is that combination, since the app (application) caters for both, it can bring out the visual aspect of calculus. **(IL1L9)**

Contrary to the earlier suggestion by some participants, lecturer 3 argued that the software should not be merely used in a manner to extend the traditional way of teaching by using it simply to verify or prove solutions, but rather in a manner that enhanced conceptual understanding. He added that this could be achieved by making effective use of the visualisation features of *GeoGebra.* This aligns with the views of Jelatu et al. (2018) who cautioned stakeholders against using technology for its own sake. It was observed that in most learning institutions, typical uses of technology tended to simply complement conventional teaching, instead of making underlying changes to the teacher-centered instructional paradigms.

Based on their interaction with *GeoGebra* in their training as they prepared to present their lessons and during their lesson presentations, the lecturers' experience was that there was a need for more sensitisation for both students and lecturers, especially on the use of the software on topics other than calculus. They explained that the sensitisation was necessary for both, considering that the software is currently hardly used in TEIs or in secondary schools. It would accord pre-service students an opportunity to acquire the knowledge and skills of how to use the software before they graduate, and impart the skills to their learners. They added that when people became more competent in the use the of the software, the more effectively they would apply it in calculus and other mathematics topics. Lecturer 3 reflected:

> *I think what should be done is that more lecturers and students of mathematics should be sensitised on GeoGebra, that is my view, because it is very useful. Like I said in the beginning, myself, the only time that I began using it was when I was sensitised by you (referring to the researcher), now if more lecturers are sensitised, then I think that would be better for the teaching of mathematics.* **(IL3L14)**

Lecturer 4 narrated that the teaching of calculus in TEIs generally lacked the element of application and it was mostly abstract. He elaborated that such teaching did not foster the understanding of concepts but rather promoted memorisation. His experience of teaching calculus with *GeoGebra* was that visualisation helped to concretise concepts of an abstract nature.

> *Generally, the experiencee with students is that when they are dealing with calculus, when we are doing the introductory part, that part which involves limits, they are not very comfortable, even proving by First Principles. Usually they have a problem, when they are using the formula, they just multiply the power by that (coefficient and subtract 1 from the power), they have no issues. But again, when it comes to application, they have a lot of issues. Eeh, I have discovered that it is so much of theory, the way we have actually been teaching calculus, students actually rely on memorising than understanding the concepts.* **(IL3L2)**

This view was underscored by Lecturer 2, who responded:

> *Ok, my experience is that actually, students sometimes say calculus is very difficult and so on and so forth, and I have seen that in my teaching, most of my students have found calculus one of the difficult topics, where the performance in calculus hasn't been as good as other topics. Most teachers do not dwell so much on the application of calculus, and they don't even draw the curves showing that when they are differentiating, this is what is*

happening and all that, that is my experience, so many people find it difficult to understand what is this dy dx, what is this integral? **(IL15L2)**

This view aligns with the views of Little (2009) who reiterated that the use of the computer as a tool for performing the procedures of calculus and algebra could free students to explore applications. It further resonates with the views of Machromah et al. (2019), that teaching the application of derivatives and integrals with the help of *Geogebra* has the potential to impact positively on students' achievement as regards their conceptual knowledge (**FGD L1L10).**

It was also noted that *GeoGebra* is a software that is easy to use and gives instant feedback. The use of *GeoGebra* entails teaching and learning mathematics through hands-on activities and visualising what one is doing, which enhances the learning process and promotes conceptual understanding. Lecturer 1 suggested that its use should be encouraged and broadened. He emphasised:

*Yes, it must, that's what is being encouraged nowadays, instead of just using the computer to record marks (laughs), it should be used to teach, so it is something that must be done, and this is one very good software. Many times, people have told us, use ICT, but how do we use it? But here is a tool, if we have so many such tools, then, incorporation will be easier, but instead of just saying incorporate, but there is nothing to incorporate, but this is one example of something very beautiful. (***IL1L15)**

Lecturer 3 was of the view that there was a need to include other tools in the *GeoGebra* application so as to accommodate other components of calculus as well as other mathematics topics.

Aaah, yes, because some other topics, you cannot solve them using GeoGebra, so there are tools which are lacking, if it can be beefed up, I don't know if this is all, or you just picked what was appropriate for this activity. (**IL3L14)**

Lecturer 2 asserted that *GeoGebra* was the best technological tool he had so far encountered as regards the teaching and learning of mathematics in general and calculus in particular, that he could use to visualise concepts and easily but accurately draw diagrams.

He recounted:

One, it is the best tool for me I can say, because it will give the know-how for both the lecturer and the learner on how things are done and how they came up because as we are using it, we will be able to see how things are done and I can also recommend that going forward, this tool should be introduced in the teaching of mathematics especially calculus and other topics such as functions and many others. My experience after going through GeoGebra was that I was very happy to use is it because it makes my work easier. When I am teaching calculus or any other topic it will help me to easily draw diagrams, export them to word and also use them to solve certain problems and also visualise certain concepts so that learners can easily grasp the concept. My perceptions have really been, I would say the expectations have been that without the provision of GeoGebra, it has not been easy personally to bring out calculus concepts to students with the visual aspects but this time around, with the package of GeoGebra it has given me another dimension on how to address certain features of calculus and geometry on this totally. ***(*IL2L43)**

A common view among the interviewees was that *GeoGebra* enhanced better understanding of calculus concepts when compared to the conventional method of presenting calculus. In the FGD, it was disclosed that:

To me, GeoGebra enhances understanding on the part of the learners because when we are looking at the ordinary way of presenting, much of the work is in abstract, but now if you apply this, if you embrace this tool GeoGebra in teaching, certain concepts are quite vividly seen. Even when you are talking about a limit for example, you are able to see how things are done, if you are using a secant you are able to see how the gradient is coming up. So the application is quite handy and also other than that, even for the students and the teachers, it simplifies work, and it motivates them to explore calculus concepts. Most of the explanation can be done just through representation and that can be done by the students, it is easy to use – that is my experience. Moreover, calculus encompasses many topics in mathematics, so the introduction to the use of GeoGebra will really suffice because many mathematics concepts are explained using calculus. **(FGD 3L74)**

Lecturer 1 laid emphasis on the instructiveness of the software, and the aspect of giving confidence to the teacher. She claimed that as she interacted with *GeoGebra* during her lesson presentations, she discovered it was possible for students to learn a lot by themselves and even in situations where they faced challenges, each construction tool was designed in such a way that hints of how to go about finding solutions were provided. She added that the software instilled confidence in the user, as it could also be used to verify solutions.

4.4.3 Research question 3: Enabling and constraining factors of using *GeoGebra* in the teaching and learning of mathematics

4.4.3.1 Enabling factors

The findings revealed that the respondents were of the opinion that there were a number of enabling factors for using *GeoGebra* in the teaching and learning of mathematics.

In almost all cases, lecturers viewed *GeoGebra* as a motivating factor for students during problem solving. Lecturer 4 responded:

> *First, GeoGebra motivates students, they actually like things happening. ICT is actually the in thing for students, and so if you can use it as regularly as possible, I am sure everyone will be very happy, there will be more learning than when they are just listening to you explaining and may not do very well. But ICT motivates them. That part is very good for them, we have seen even when you are just writing notes on PowerPoint, just the screen and they are writing something, it motivates them than when you are just writing on the board, or you are dictating and they are writing.* **(IL11L5)**

Closely linked to the factor of motivation, were interest in and attitude to using ICTs. A variety of perspectives were expressed that interest in and a positive attitude to the use of ICTs would encourage lecturers to use technology in their teaching.

Echoing this view, another participant added that the best motivator for every teacher or lecturer is to make students understand, and this is one of the major reasons for using teaching aids. So, if we have a tool such as *GeoGebra* that is capable of helping students understand concepts more clearly, it means it is more motivating.

Lecturer 3 explained that when he was presenting his lesson using the applets, the students exhibited a lot of interest in their responses, they looked motivated and that in itself, according to him, would encourage lecturers to use *GeoGebra* as a motivating factor for the learners.

This view is acknowledged by İsleyen and Sivin-kachala (2019), who claimed that the computer is widely used as a teaching aid in mathematics in order to enhance students' self-confidence and self-motivation.

The motivation factor resonates with the constructivist approach of student-centredness observed by Mokhtar et al. (2013) and Takači et al. (2015), that student-centred approaches in general have been shown to enhance motivation in learning mathematics and to learn mathematics and realise its applicability. Additionally, TAM provides meaningful information on the link between intention and motivation to integrate technology in the teaching and learning process from a constructivist perspective. *GeoGebra* was seen as a tool that provided accurate solutions and saved time. Lecturer 5 pointed out:

> *Yes I strongly believe GeoGebra should be used because it is one software, as we are aware that the world has gone digital we are being encouraged to be resourceful, to be innovative, we can come up with other teaching aids but the accuracy that this technological software has, can, is far much better compared to others which may not be as technological in nature as it is. One, it saves time; two, it is accurate; and three, it actually gives the actual aspect, every detail that can be talked about in a particular field., it reveals, it brings out as long as you know how to use it.* **(IL5L16)**

Lecturer 6 explained that when a task is given in calculus, generally a graph or a diagram is used which requires time and is mostly 'through chalk and board', the quality of the graphs and diagrams are neither appealing nor accurate. However, *GeoGebra* is capable of drawing accurate diagrams and finding solutions in a very short time.

> *GeoGebra saves a lot of time because when you deal with calculus, then you are usually dealing with functions which usually require drawings, coming up with diagrams, and that takes a lot of time, but with GeoGebra, just with a click of the button, a function pops up, you can save a lot of time, if you use the software.* **(IIL5L10)**

This view is supported by Marrades and Gutiérrez (2000), who argued that *GeoGebra* takes care of time-consuming constructions such as graphs, with accuracy and minimal effort. They elaborated that by using tools like sliders and drag-and-drop tools, students can easily construct variations of a graphical representation that can be used to generalise and explore concepts, and allow learners and teachers more time to concentrate on mathematical processes in the lessons.

On the issue of accuracy, Lecturer 1 had a different view from the other participants, as he observed that:

> *Well for lecturers, it may be relative because the accuracy in higher mathematics is not really a factor, or it is not the focus, the focus is on deriving solutions*. **(IL1L6)**

Another enabling factor that emerged, was that *GeoGebra* was cheap. The respondent implied that it is a free, open-source software (a non-commercial software) which does not require any license fees. Additionally, it runs offline, so it does not require internet connectivity, considering the high cost of bandwidth in Zambia. Lecturer 5 substantiated his claim:

> *This software does not require internet and it is easily accessible so one will really be motivated to start applying it, use it because it is very accessible and very easy to use. I think that these factors can motivate one to use this technology.* **(IL5L49)**

The visualisation feature of *GeoGebra* was also identified as an enabling factor. Lecturer 4 observed:

> *With the visual aspect of GeoGebra, it is easy to enhance the understanding of concepts, the teaching and learning of mathematics will be real, it is like people are visualising what is happening ...it is easier for a lecturer or teacher to use GeoGebra in order for the learners to understand. because they will clearly be seeing what is happening.* **(IL14L8)**

He stressed that the use of *GeoGebra* strengthened the understanding of the concepts, can be used for proving and verification, and since it is interactive and exciting, it catches learners' attention.

Lecturer 5 echoed this view, stating:

> *Technology simplifies matters that is in teaching when you want to explain a concept, technology will help you to visualise that concept to the learners and GeoGebra is that tool which we can use to explain these concepts in several topics, calculus and these other topics, geometry, algebra, statistics, GeoGebra can easily make learners understand a concept.* **(IL5L20)**

These findings are in line with the findings of Heid and Edwards (2001), Hohenwarter and Jones (2007) and Kaput and Roschelle (2013), who reiterated that the systematic use of the visualisation features of *GeoGebra* could help students to explore, solve problems, receive prompt feedback and to engage in reasoning. Kadunz, in agreement with this view, summed up by stating that "One of the most powerful and widely recognised didactical components of dynamic mathematics software is visualisation" (1998, p. 198).

Availability of resources and the knowledge of how to use it were also cited as enabling factors.

> *I think the availability of laptops and computers and also the knowledge with some training.* **(IL1L12)**

Lecturer 4 clarified that one needs to have a computer, because without the resources, learning with this software would be very difficult. So the computer is the main resource. Farjon et al. (2019) acknowledged the factor of availability of resources and expertise by observing that countries had invested substantially in terms of money, resources, expertise and research to integrate technology into education, in a bid to make the classroom environment more conducive to enhanced learning and teaching.

Other recurrent themes among the participants on enabling factors were the availability of resources and the knowledge and skill to utilise them. It was pointed out in a FGD that:

> *Institutions need to actually supply the required equipment. I have seen that in most cases in these institutions it simply says use this and at the end those equipment are not provided, looking at the situation of COVID-19, that we had, when we closed the schools and colleges we were told to offer lessons online, but in the actual sense very few of us or let me say some of us did not have that equipment and the data to actually use in that area. I feel that institutions should come in and provide these materials. When the equipment is available, then we need to ensure* that as *lecturers, we familiarise ourselves on the use of the software.* **(FGD L72)**

Another enabling factor that was identified with regard to the use of *GeoGebra* was verification of solutions. Lecturer 3 elaborated:

> *When you use GeoGebra, you can easily check, verify, even before presenting to your students, you can verify, test, your findings, so that as you go to the learners, you are well versed with what you are going to present, that is one factor.* **(IL3L115)**

Keeping up with modern digital global trends was cited as an enabling factor. The participants indicated in the FGD that the education sector in general, and mathematics education, cannot lag behind in technological advancement, and therefore, this in itself provides impetus for lecturers to integrate technology in their teaching. Lecturer 3 in the FGD expressed that:

> *When you check at what is happening globally, all sectors, let me say all fields starting from talk time, marketing, are incorporating in their work in technology, so there is no way we who are in education and to be specific our area which is mathematics can lag behind. Even ourselves we are supposed to move so the use of those apps will definitely help us move to another level. Moreover, the coming in of COVID-19, has taught us something to say, with the help of technology learning can continue. Because as we are talking of GeoGebra, this one is self-tutoring of some sort, it can be done using the app and others will be able to get the concepts on their own.* **(FGD 3L98)**

4.4.3.2 Constraining factors

A lack of computers and expertise to use them effectively were cited as some of the major constraints of using *GeoGebra* in the teaching and learning of mathematics. Lecturer 2 pointed out that:

> *Resources, are a challenge, we do not have many computers, we only have one lab in the institution, each department, if it has to be successful, it must have its own lab, computer lab, where students can go at their own free time, probably also practice and then they can be there any time with their teacher.* **(IL2L4)**

Underscoring this view, Lecturer 6 added:

> *Lack of computers, because in schools let me not talk of colleges, in schools we don't allow phones (among students), now we know that this app can only work when you have your phone or a laptop but you will find that most schools even colleges we don't have a stocked library where we have so many computers and to learners some of the phones they have, are not smart phones, so sometimes it's very difficult and you cannot force matters they are poor, they cannot manage. Those are some of the factors that can discourage lecturers from using ICTs* **(IL6L81).**

This view is echoed by Atchoarena (2016), who acknowledged that teacher education in Zambia faces a number of challenges. The challenges cited were a lack of facilities and resources, weak capacity and qualifications of staff in TEIs, and weak utilisation of ICT due to inadequate knowledge. The Zambian (MoE, 2010, p. 36)), underscored this view, stating that qualified human resources, infrastructure and teaching and learning materials still remained the main challenges in most TEIs.

The findings further revealed that the use of *GeoGebra,* within the existing curriculum, which was examination oriented, was a challenge. This was attributed to a tendency by lecturers to focus more on completing the syllabus at the expense of engaging students in constructivist approaches to learning – a practice that requires more time. In fact, the factor of time, despite being classified as an enabling factor by some participants, was also cited as a constraining factor by other participants. This was attributed to the fact that preparing a lesson to be taught in a *GeoGebra* environment required a lot of time.

Lecturer 4 lamented:

> *Ummh…the only thing I can comment is, it's time consuming and our syllabuses are bulky also, even with just talking and writing on the board we are not completing the syllabus, but however, it can still be used as I said when a specialised lab to the department is available, the learners can go there, whenever they feel like and practice. The issue of time, because, when a software is just introduced, it's new, so, somebody needs a lot of time to practice and understand the gadgets, because this one, it will be like a teaching aid, okay, and then you need to fully understand it and that needs a lot of time. Now, if you look at the overload of time that we have, we may not have sufficient time to spend on practising so that you become perfect in the application of that), so we are looking at the time factor.* (**IL4L14)**

Inability to use the *GeoGebra* software effectively, and using it in topics other than calculus were also identified as constraining factors. In his response, Lecturer 6 commented:

> *Let me say, some apps on the GeoGebra software, they are like, if you do not know how to use them, it might be difficult, yes, they are there, but how do you use them? That is where most of the challenge is. Like, let's say for instance, you want to find the turning points,*

there is a tool on top there, which you can use, but if you are not too conversant on how to use that, you might end up getting wrong information. So, I would say, it is lack of knowledge if lecturers are not trained well, it can hinder their use of ICT in teaching. (**IL6L79)**

Lecturer 3 lamented that the challenge he had was finding the right tools and avoiding clicking on several tool icons until he found the correct one.

The participants further revealed that it was a challenge to use *GeoGebra* in calculus topics that required application, in a manner that would motivate students. Lecturer 5 elaborated:

I think ….it may not be easy for me maybe to manipulate the software especially when it comes to the questions on applications, those applications about speed, distance, velocity, volume, increment, decrease, changes. I may lack the skill to manipulate the GeoGebra, or the software may not have some of the things I need, otherwise if it was beefed up, and if it had so many functions, it would be very useful. (**IL5L107)**

He went on further to note that *GeoGebra* would not be used to explore some calculus components such as product and quotient rules, implicit differentiation and other mathematics topics, such as speed, velocity and acceleration. There was therefore a need, he added, to equip *GeoGebra* with other features that would make it more versatile. He felt that visualisation of the aforementioned concepts in a *GeoGebra* environment would greatly enhance students' understanding.

This view is in line with the observation by Little (2011), that the use of technology is not a panacea for challenges encountered in calculus teaching and learning. He observed that the software could not replace the need for learners to master certain algebraic processing skills. He further pointed out that certain aspects of teaching and learning calculus, such as differentiation techniques (the product, quotient and chain rules), which are naturally algebraic in nature, were less likely to be enhanced by interactive geometry.

The other inhibiting factors that were brought out by the participants, were resistance to change and attitude. It was argued that human beings generally tend to resist change, and that some lecturers had a negative attitude towards the integration of ICT into the teaching and learning process. Lecturer 2 reaffirmed:

> *Ummh, one**...,** maybe you know people, a human being always tends to resist change, that could be one of the factors, then maybe the other one could be lack of facilities, it could be another inhibiting factor, you know some institutions, may not have those facilities like what we discovered yesterday that the institution had no projector which has a provision for HDMI cable, the other factor could be just attitude, some lecturers may not just have positive attitude towards ICT* **(IL2L120)**

Ruggiero and Mong (2015), in affirmation to these sentiments, reiterated that teacher-level barriers included resistance to change and lack of confidence, while school level barriers included a lack of effective training to solve the technical problems and lack of access to resources. The literature also attests that teachers' attitudes towards ICTs have a strong influence on the acceptance of the usefulness of ICTs in their lessons, and has a strong bearing on whether teachers would integrate ICTs into their lessons (Teo, 2011; Huang & Liaw, 2005). On the other hand, TAM recognises attitude as one of the its major constructs, where it contends that the attitude of the user towards use of technology for teaching and learning is critical. Hew and Brush (2007) concurred with this view and elaborated that changing attitudes and beliefs about technologies was an important factor and should take precedence in teachers' ability to integrate technology into teaching.

Issues related to attitude and lack of confidence were particularly prominent in the interview data. Lecturer 1argued that:

> *I feel it is negative attitude on the part of teachers. We don't want to seriously engage into use of technology maybe because of our background, some of us feel, ah no, I will make mistake I feel I wouldn't do a good job so you just give yourself a position to say I can't do it so that attitude is what is discouraging us*. **(IL1L49)**
>
> In the FGD, Lecturer 2 concurred with Lecturer 1, recounting:*To add on, lack of confidence where you are not properly trained so you can't have confidence as some students may be better than you in ICT. So the lack of confidence maybe due to lack of training, and interaction with these gadgets*. **(FGDL2 L109)**

Lecturer 5 laid emphasis on institutional support, stating:

> *If say there isn't support from management in institutions by availing gadgets like laptops, and beamer, (projectors), if those are not provided then that becomes a discouragement to*

the lecturers. So it is the availability of the same gadgets and resources that can be used. If they are made available, then that would be fine but if they are not, if management of the institutions do not support then it would be a discouragement. **(IL5L58**)

It was pointed out that even though *GeoGebra* is a free open-source software, the cost of laptops and smartphones was on the higher side. In the FGD, Lecturer 5 affirmed:

So the other thing is that the cost attached to the gadgets that we use – for example phones, computers and so on. You would find that practically speaking even us as lecturers, it's very difficult for us to come up with some very good laptops, worse with our students who are coming from various negative economically homes. The cost, because you find that you want to present this lecture but now, are they going to afford a smartphone? So the costs attached to these will also inhibit the proper presentation because sometimes you find that yes, class let us now do hands on activity, let's use a laptop out of eighty you have five, is that going to be a success or a flop? So that's why I am emphasising on the cost of the gadget. **(IL5L66)**

4.5 CONCLUSION

This chapter articulated the findings and discussion of the data. This was done with reference to the analytical framework and the research questions of the study and by considering the common themes in the views of the participants. A brief outline of how each of four cycles was taught was presented. This was followed by the vertical and horizontal analysis. In the vertical analysis, a case by case analysis of how each lecturer interacted with the software was done, while in the horizontal analysis, the analysis was done across all cases.

CHAPTER 5: CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

This final chapter consolidates the findings of the study with reference to the research questions, the methodology and the theoretical frameworks. It further interrogates the significance of the study and its limitations, and makes recommendations for further research. The chapter concludes with personal reflections on the study. It is envisaged that the results of this study will significantly contribute to the enhancement of incorporating the use of *GeoGebra* in the teaching and learning of mathematics in general and calculus in particular, and identify areas for further research.

5.2 REVISITING THE RESEARCH GOALS AND QUESTIONS

Six lecturers – two from each of the three TEIs in Zambia – took part in this case study research. The participants developed *GeoGebra* applets based on four calculus cycles and presented lessons to student teachers in their respective TEIs. The main goal of my study was to investigate the use of *GeoGebra* as a visualisation tool, by lecturers to teach calculus in TEIs to pre–service teachers in Zambia, to enhance conceptual understanding. In order to accomplish this goal, the study aimed to answer the following three questions:

- How can *GeoGebra* be used as a visualisation tool to teach calculus to pre-service student teachers in TEIs to enhance conceptual understanding?
- What are the perceptions and experiences of lecturers in using *GeoGebra* as a visualisation tool to teach calculus in TEIs in Zambia?
- What are the enabling and constraining factors of using *GeoGebra* in the teaching and learning of mathematics?

These questions demanded a rigorous consideration and understanding of literature on mathematics education from a technological perspective. Thus, in my literature review (Chapter 2), the following concepts were key to my study: the use of information and communication technology (ICT) in education; visualisation; *GeoGebra;* calculus; mathematical proficiency; multiple reoresentations; constructivism; TPACK and TAM.

5.3 KEY RESEARCH FINDINGS

The synopsis of the findings as they relate to the research goal and research questions based on each of the four calculus cycles is presented below:

5.3.1 Summary of the Findings Related to Research Question 1

Use of *GeoGebra* as a visualisation tool to teach calculus to pre-service students in TEIs to enhance conceptual understanding

A number of visualisation processes were identified in the use of *GeoGebra* as a visualisation tool as lecturers taught the four calculus cycles. A synopsis of these activities is presented below:

5.3.1.1 Visualisation Processes

5.3.1.1.1 Link between the abstract nature of mathematics and the practical aspects

Across all the four cycles, a key finding of the research was that the visualisation features of *GeoGebra* provided a strong link between the theoretical or abstract nature of calculus concepts and their practical or concrete aspects. These links were evident in all four cycles, as illustrated in Chapter 4. The findings of the study revealed that the inherent visual aspect of *GeoGebra* made it easy to enhance the understanding of concepts, thereby making teaching and learning of mathematics real, as it enabled students to visualise what was happening. As matter of fact, in this code, visualisation processes were the highest recorded observable indicator, followed by multiple representations and use of sliders (as shown in Chapter 4). The lecturers acknowledged that the use of *GeoGebra* fostered learners' understanding because they were able to clearly see what was happening.

In the first cycle, (the slope of the tangent), the links between the abstract and concrete aspects of calculus concepts were illustrated when sliders were used to drag the points of estimate Q , S and R close to the point of tangency, P , on the secant. This helped to concretise the concept of 'one quantity approaching another', and was also illustrated when lecturers discussed the limit concept of 'h approaching zero', when the distance between the x coordinate of P and that of Q became infinitely small, so the students were able to literally 'see' h moving towards P . This dynamic feature of *GeoGebra* cannot be easily accomplished with static objects.

Additionally, the underlying concepts between a function and its derivative was illustrated by the link between their abstract and practical aspects. The participants illustrated the underlying concept of how the graph of the derivative relates to that of the original function with the help of the trace tool of *GeoGebra*, to visualise the process. In sum, from the lecturers' experience of teaching calculus with *GeoGebra,* the findings generally revealed that the aspect of visualisation helped to concretise concepts from their abstract nature.

In the second cycle (limits), the link between the abstract and concrete aspects of calculus concepts was made visual when sliders were used to drag point towards a limit. This link was further illustrated on the *GeoGebra* interface when a point was moving closer to another point as students observed the values of coordinates changing and approaching a certain limit. Hitt et al., (2017) asserted that making mathematics concepts visual is a way of transforming them into concrete from their abstract nature, and assists students to easily comprehend concepts. Tatar and Zengin (2016) reiterate that teaching calculus with dynamic software helps make the learning of abstract concepts easier.

In Cycle 3, the area above and below the x-axis, the link between visual processes of abstract and concrete were illustrated in a number of ways. Firstly, the graph (Figure 4.12, in Section 4.3.3)) of the function $y = s\,\text{in}\,x$ from x= 0 to x= 2π , illstrated one part above the x - axis while the other part was below the x- axis. This significantly helped the lecturers to explain and clarify between the area calculated as positive, from that calculated as negative. It also helped to justify why the area could not be zero as was obtained by some student-teachers, because the graph illustrated clearly that space was covered in the graph.

In Cycle 4 (the Riemann sum), the focus was on estimating the area under the curve using the lower and upper sums. The connection between the abstract and concrete aspects of calculus concepts was emphasised when the slider was used to increase or reduce the number of rectangles, by dragging the slider back and forth. As the number of rectangles changed, students could see the value of the area under the curve adjusting accordingly.

These visualisation processes help to consolidate abstract or theoretical knowledge. Haciomeroglu et al. (2009) raised similar sentiments that technology opens up possibilities for developing mathematics concepts by enabling the visualisation of the concepts to demonstrate complex abstract ideas clearly.

This is underscored by Presmeg (2014), Samuels (2010) and Gono, (2016), who alleged that visual presentations are necessary, especially in mathematics concepts that are abstract in nature and require students to consider situations which do not physically exist, as is the case with the limit process in calculus. The visualisation process provided students with enhanced understanding of mathematical concepts with an abstract and complex nature.

Mathematics is generally built on abstract concepts (Gono, 2016), and often requires students to comprehend the abstract processes and concepts. The abstract nature of mathematics makes it challenging for students to comprehend most concepts in calculus, and the findings of this research indicated that visualisation process between the theoretical and practical aspects of the same concept enabled by *GeoGebra,* played an important role during lesson presentations to overcome this challenge.

5.3.1.1.2 Multiple representations

The other key finding of the study across all the cases was that the inherent visualisation features of *GeoGebra* enabled a single concept of calculus to be presented in multiple ways. The inherent feature of *GeoGebra* is that once data has been entered in the input bar, it will be illustrated synchronously in the algebraic and the graphic windows, so the same concepts will be illustrated in multiple ways hence enhancing the conceptual understanding of concepts. The concept of the limit function was demonstrated in multiple ways: graphically, as a spreadsheet and algebraically. On the other hand, other calculus concepts such as the gradient function were illustrated symbolically, algebraically and graphically.

Božić et al. (2019) and Žilinskiene and Demirbilek (2015) acknowledge the potential of DGS and CAS that enable users to create dynamically connected multiple representations of mathematical concepts. This feature enabled the participants' to flexibly switch from algebraic illustrations to graphic ones and vice-versa to consolidate calculus concepts.

5.3.1.1.3 Use of sliders

The findings revealed that the use of sliders in *GeoGebra* is key to expressing its visualisation features. It was observed that the slider tool was used prominently in all the cycles to illustrate the visualisation aspects. It was used to drag points, increase or reduce the number of rectangles, illustrate change of sign of a slope on a curve and illustrate the limit function, among others.

5.3.2 Summary of the findings related to Research question 2

The perceptions and experiences of lecturers in using *GeoGebra* as a visualisation tool to teach calculus in TEIs in Zambia

5.3.2.1 Enhancing conceptual understanding of concepts

The findings revealed that the use of *GeoGebra* enhanced the understanding of concepts. This was attributed to the fact that in most instances, calculus was taught in an abstract manner and lacked the aspect of application, but the use of *GeoGebra* helped the practicality of calculuc to be more visible. It was revealed that the algebraic abstract aspect of calculus was understood better by understanding its connection to the corresponding geometric aspect, a feature that was aptly illustrated by the *GeoGebra* software.

As observed by Sabella and Redish, (2007), many students enrolled in HEI calculus classes tend to acquire superficial and incomplete understanding of basic concepts of calculus. They attributed the failure to develop conceptual understanding of calculus concepts to the teaching approaches that emphasise rote, algorithmic drilling and manipulative learning.

5.3.2.2 Procedural Fluency (PF1)

Related to the construct of conceptual understanding, the study revealed that most students generally acquired procedural fluency in their study of calculus in HEIs and lacked conceptual understanding, hence the failure to apply calculus in real life situations. This, as alluded to above, was attributed to conventional teaching methods that emphasise rote, algorithmic drilling and manipulative methods. As observed in the literature review (Chapter 2), many final year mathematics student teachers may possess procedural knowledge but lack the conceptual knowledge of the mathematics required to teach concepts such as calculus.

5.3.2.3 Using the software judiciously

It was revealed that while it was appreciated beyond reasonable doubt that *GeoGebra* was a very useful tool in the teaching and learning of calculus, the participants felt that it should not be used for its own sake. They pointed out that while there were some topics in calculus in which the visualisation features of *GeoGebra* would enhance understanding, there were however other topics where its use would not yield the desired results. This is underscored in the literature review where teachers were cautioned that technology should not just be used for its own sake, but should be used judiciously

5.3.2.4 Incorporating GeoGebra in the national curriculum

The findings revealed that a proposal should be made to the Ministry of Education to have *GeoGebra* incorporated in the Zambian curriculum at all levels of education. They attributed this to the fact that among the software that they had so far been exposed to, *GeoGebra* was one that they found to be user friendly and they felt could easily be successfully integrated into the teaching and learning process. It was for these reasons that they felt that if the government introduced it in schools, the chance of success was potentially high.

5.3.2.5 Visualisation characteristics

It was revealed that the visualisation features of *GeoGebra* made it conducive to a number of topics in mathematics. They added that since mathematics was generally perceived as a difficult subject, the visualisation characteristics of *GeoGebra* would not only enhance understanding of concepts, but would also provide some motivation to the students, by exposing them to approaches to teaching and learning other than the conventional methods.

5.3.3 Summary of the Findings Related to Research Question Three

5.3.3.1 Enabling Factors

5.3.3.1.1 Adequate Training

The findings revealed that adequate training, coupled with sufficient knowledge in calculus, were necessary for the effective use of *GeoGebra* as a visualisation tool. As acknowledged by Koehler et al. (2017), TPACK describes the complexities and challenges of technology integration, suggests strategies required to prepare future teachers for learning and teaching in the 21st Century, and prioritises the importance of teacher training.

5.3.3.1.2 Ease to use

Another key finding was that the software was affordable, easy to use, and being a free open-source software (a non-commercial software), it did not require any license fees. *GeoGebra* does not require internet connectivity, it runs offline, and is therefore very suited to countries like Zambia, where the cost of bandwidth is high. It was further suggested that policy makers should therefore take advantage of this and make more teachers and learners aware of the software. They elaborated that as well as being easy to use, the software was hands on, provided instant feedback, and a number of its functions could be self-taught.

5.3.3.1.3 Motivation

The study revealed that when students were exposed to the *GeoGebra* software, they exhibited a lot of enthusiasm and motivation to learn. It was perceived as a way of keeping up with modern digital global trends.

The participants indicated that the education sector in general, and mathematics education in particular, could not afford to lag behind in technological advancement, and therefore, use of *GeoGebra* as a visualisation tool to teach calculus provided some impetus for lecturers to integrate technology in their teaching. The study further revealed that interest and attitude were major factors in the participants' motivation to use *GeoGebra* in the teaching and learning process.

5.3.3.1.4 Saving time and verification of solutions

The study indicated that the use of the software saved time. It was established that students generally took less time to carry out calculations when using the software, compared to using conventional methods. As acknowledged by Heid and Edwards. (2001), *GeoGebra* took care of time-consuming procedures like drawing graphs, consequently it therefore accorded students more time to concentrate on other more cognitive aspects like reasoning and problem solving. This view is supported by Marrades and Gutiérrez (2000) who argued that *GeoGebra* takes care of time-consuming constructions such as graphs, with accuracy and minimal effort.

5.3.3.1.5 Accuracy

The findings revealed that the accuracy the software *GeoGebra* offers, provided a detailed description of mathematics concepts, compared to conventional methods. In addition to its accuracy features, it also provided prompt feedback and verification of solutions, which motivated students to explore further and be innovative. It was also revealed that students could solve questions in the traditional manner and then use *GeoGebra* to confirm whether the solution is correct and accurate.

5.3.3.1.6 Availability of resources and expertise

The study indicated that availability of resources was a major factor in the use of *GeoGebra* as a visualisation tool to teach calculus, and it was pointed out that the major resources were computers and smartphones. Countries that had made remarkable advances in integrating technology for enhanced learning and teaching, it was revealed, had invested substantially in terms money, resources, research and expertise. It was emphasised that it was only when resources were available that knowledge and skills could utilise them.

5.3.3.2 Constraining Factors

5.3.3.2.1 Lack of Resources and Expertise

The key finding on the constraints on the use of *GeoGebra* as a visualisation tool across all six cases was lack of materials and the expertise. The study revealed that institutions lacked sufficient numbers of computers, the expertise and computer laboratories. All except for one TEI (0.17 percent), the other five (83.33) had only one computer in the institution, and priority for use was given to students taking ICT programmes. Due to insufficient computers, students could not practise and familiarise themselves adequately with the software. The constraints that were identified included both internal and external barriers, which included a lack of qualified human resources, infrastructure and teaching and learning materials, still remained the main challenges in most TEIs. It was further revealed that in some institutions, even when resources were available, there was a lack of willingness to invest in ICT devices, and this was attributed to the high cost of ICT devices. It was pointed out that though some institutions possessed a reasonable amount of resources, unfortunately they were underutilised and rarely used in a manner that enhanced learners' conceptual understanding, but rather to perpetuate conventional methods of teaching.

5.3.3.2.2 Examination-oriented curriculum

The findings further revealed that optimal use of *Geogebra* in the existing examination-oriented curriculum was a challenge. The focus by lecturers was highly skewed on completing the syllabus rather than on engaging students in constructivist approaches to learning, a practice that required more time. The factor of time, despite being classified as an enabling factor, was also cited as a constraining factor by other participants, as the findings revealed that lecturers felt that preparing and teaching a lesson in a *GeoGebra* environment required a lot of time.

5.3.3.2.3 Inability to use *GeoGebra* in other calculus concepts

The study indicated that it was a challenge to use *GeoGebra* to explore other calculus topics, especially those that required application, in a manner that would motivate students. Such topics were identified as: product and quotient rules, implicit differentiation, speed, velocity and acceleration. It was felt that there was a need to equip *Geogebr*a with other features that would make it more versatile, as the visualisation of these concepts in a *GeoGebra* environment would greatly enhance students' understanding.

5.3.3.2.4 Beliefs and Attitudes

The findings revealed that some lecturers had a negative attitude towards the integration of ICT in the teaching and learning process. This was attributed to the fact that human beings generally tend to resist change and rather maintain their usual way of doing things. It was observed that most of those who resisted change, generally lacked confidence and belief that *GeoGebra* could be used as an effective teaching and learning tool. The literature attests that teachers' beliefs and attitudes towards ICTs were critical and had a strong influence on their acceptance of the usefulness of ICTs in their lessons. It also had a strong bearing on whether teachers would integrate ICTs into their lessons (Van Den Beemt & Diepstraten, 2016).

5.4 SIGNIFICANCE OF THE STUDY

This study contributes to the current debate on visualisation of mathematics concepts and the use of technological devices in the teaching and learning process by various stakeholders. By thoroughly reviewing the literature that relates to the integration of ICT tools in the teaching process, it was discovered that very little research has been done in this field in TEIs in Zambia. Therefore, the gap in knowledge that this study addresses may contribute to literature on solutions to achieving good practices in the teaching calculus. The findings may also guide and provide a basis to design appropriate instructional materials for the consolidation of the Teaching and Learning Mathematics with *GeoGebra* (TLMG) project for quality teacher professional development in the use of *GeoGebra* for mathematics teachers, with a view to incorporating *GeoGebra* in the Zambian teacher education Curriculum. The insights gained from my data analysis suggest that *GeoGebra* has the potential to positively impact on mathematics teaching and learning in Zambian education institutions. I therefore argue that my research findings on teaching and learning with *GeoGebra,* as indicated in the data analysis, inform stakeholders in mathematics education in Zambia about various factors involving incorporating *GeoGebra* in the teaching and learning process in education institutions. Effective integration of technology in the teaching and learning process requires adequate training, sufficient resources, institutional support and positive attitudes and beliefs about technology integration in education. My analysis of data and review of related literature indicates that the use of visualisation processes in the mathematics classroom has the potential to enhance conceptual understanding of mathematics and is therefore highly recommended. I therefore recommend teaching and learning practices that incorporate visualisation processes. It is hoped that teachers and researchers who read this thesis may gain insights into how *GeoGebra* can effectively be used as a visualisation tool for the teaching and learning of mathematics. While appreciating the inherent visualisation of *GeoGebra,* it is however necessary for teachers to unpack the abstract mathematical concepts and make them visible to students. The dynamic aspects of applets in this study emerge as a key factor in the incorporation of the DGS and CAS in the visualisation of calculus concepts.

I also feel that Kilpatrick's strands of mathematics proficiency, conceptual understanding and procedural fluency could be used in a complementary manner in the teaching of calculus, as they both play key roles.

5.5 LIMITATIONS OF THE STUDY

This study was designed as a case study of selected lecturers who incorporated *GeoGebra* in their lesson presentations of calculus cycles to student-teachers. The limitations of this case study included the small sample size, which only comprised six lecturers. The research was only conducted in three TEIs, and the period of data collection, six months, was relatively short.

The scope of a case study does not generally allow its findings to be generalised to whole population (Yin, 2014), and therefore, the findings of this study cannot be generalised to a bigger population. However, the small sample size enabled me to carry out in-depth interviews with the participants. Therefore, though the findings of this study cannot be generalised to a bigger population because of the small number of research participants, the observations are in tandem with previous studies on DGS, such as the study by Mavani, (2020). Notwithstanding the aspect of non-generalisability, the use of *Geogebra* applets can be replicated in a wide range of educational settings.

While I managed to assist the participants in the course of the investigations, there were however, some *Geogebr*a functions that I discovered along with the participants. Additionally, the study was conducted during the first wave of the COVID-19 pandemic, which caused a lot of disruptions in the academic calendar.

5.6 IMPLICATIONS

This research project informs various stakeholders in mathematics education. The stakeholders include teachers, lecturers, policy makers, curriculum designers and researchers. In view of the findings and discussion in Chapter 4, I suggest the following for the incorporation of technology in mathematics classrooms:

5.6.1 Implications for policy makes and curriculum developers

To enable mathematics lecturers to incorporate visualisation processes as a vital component of mathematics teaching, the mathematics education curriculum policy makers and developers need to realign the curriculum. This should include a shift from a curriculum that is largely examination oriented to that which includes visualisation processes and conceptual understanding of mathematics. This will encourage the curriculum implementers to embrace teaching approaches that embrace technology and in retrospect, motivate students to learn and explore In order to come up with an effective curriculum, it is incumbent upon curriculum developers to include various stake holders, especially those who implement the curriculum at various stages. In TEIs, the curriculum should include rigorous training in the use of technology, *Geogebra*, among them, in teaching, and not place emphasis on generic ICT education courses.

5.6.2 Implications for further research

The findings of this study brought to the fore that there is a lot of potential to use *GeoGebra* to teach mathematics effectively. The focus in this research was on teacher educators, or lecturers, in TEIs. Though the key data were generated from observations as the lecturers presented lessons to student teachers, the students were not part of the sample. It would be interesting to carry out research that would include students in the sample and get their views. Furthermore, the research could be extended to a larger sample size and more research sites, and include mathematics topics other than calculus.

Following the outbreak of COVID 19 which drastically disrupted the academic calendar, lecturers and other curriculum implementers can explore the optimal use of applets in an online platform. The online mode of instruction and the physical mode can be used to consolidate the initial initiative of the Teaching and Learning Mathematics with *GeoGebra* (TLMG) project for quality teacher professional development in the use of *GeoGebra* for mathematics teachers, with a view to incorporating *GeoGebra* in the Zambian teacher education curriculum.

5.7 PERSONAL REFLECTIONS AND CONCLUDING REMARKS

From the time l realised the interest that teachers and other stakeholders had in learning how to use *GeoGebra* at a personal level and in the teaching and learning process, I was motivated to explore the software. My interest grew further when I conducted an informal survey to get baseline data and views from teachers and lecturers. Among the responses I got in the baseline data was *'I am interested in learning how to use GeoGebra, but the software is too expensive'* and two other respondents said *'I have the software on my laptop but I don't know how to use it'*. The interest was more overwhelming, when after learning the basics which I shared with my undergraduate students we made presentations at the 2018 Zambia Mathematics Education Conference. This motivated me to conceive the Teaching and Learning Mathematics with *GeoGebra* (TLMG) project. This was done with a view of spearheading quality teacher professional development in the use of *GeoGebra* for mathematics teachers with a view of incorporating *GeoGebra* in the Zambian teacher education Curriculum. This project is still being pursued and it is my hope that with the eminent completion of my current studies, I will have more time to devote to the TLMG project.

In a nutshell, in this study, the respondents showed a lot of enthusiasm for the project and were of the view that the use of *GeoGebra* generally enhanced the teaching and learning of calculus. They acknowledged that adequate training was necessary for them to use *GeoGebra* as a visualisation tool to teach calculus to pre-service students effectively. They however insisted that the training would be more meaningful to people when coupled with sufficient knowledge of the subject matter in calculus. They were in agreement that the visualisation characteristics of *GeoGebra* enhanced conceptual understanding of calculus concepts and that use of the software was motivating to both lecturers and students. The lecturers were also of the view that integration of ICT in the teaching and learning process of mathematics should be incorporated in the teacher training curriculum of

TEIs, and that *GeoGebra* should be one of the components. They suggested an approach where *GeoGebra* could be used to reinforce the conventional method of teaching calculus, where a calculus topic is first taught in the traditional method, and then *GeoGebra* is used for clarification, and to provide a link between abstract and concrete concepts.

On the other hand, it was felt that the lack of resources and expertise were major hindrances in the use of *GeoGebra* to teach mathematics in TEIs. The participants were of the view that the existing curriculum in TEIs was more examination oriented, therefore making optimal use of *GeoGebra* is a challenge, as the focus presently is more on completing thesyllabus, thereby compelling lecturers to incline more to employing methods that do not engage students in constructivist approaches. They reiterated that teaching calculus in a *GeoGebra* environment required a lot of time, which was not feasible in the current dispensation. The participants also felt that there was a need to equip *GeoGebra* with other features that would make it more versatile.

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APPENDIX A – Ethical Clearance

RHODES UNIVERSITY
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Human Ethics subcommittee
Rhodes University Ethical Standards Committee
PO Box 94, Grahamstown, 6140, South Africa
t: +27 (0) 46 603 8055
f: +27 (0) 46 603 8822
e: ethics-committee@ru.ac.za

www.ru.ac.za/research/research/ethics
NHREC Registration no. REC-241114-045

26 May 2020

Mr. Lemmy Kangwa

Email: g13K6284@campus.ru.ac.za

Review Reference: 2020-0932-3474

Dear Mr. Kangwa

Title: The Incorporation of GeoGebra as a Visualisation tool to teach Calculus in Teacher Education Institutions: The Zambian case

Principal Investigator: Professor Marc Schafer

Collaborators: Mr. Lemmy Kangwa,

This letter confirms that the above research proposal has been reviewed and **APPROVED** by the Rhodes University Ethical Standards Committee (RUESC) – Human Ethics (HE) sub-committee.

Approval has been granted for 1 year. An annual progress report will be required in order to renew approval for an additional period. You will receive an email notifying when the annual report is due.

Please ensure that the ethical standards committee is notified should any substantive change(s) be made, for whatever reason, during the research process. This includes changes in investigators. Please also ensure that a brief report is submitted to the ethics committee on the completion of the research. The purpose of this report is to indicate whether the research was conducted successfully, if any aspects could not be completed, or if any problems arose that the ethical standards committee should be aware of. If a thesis or dissertation arising from this research is submitted to the library's electronic theses and dissertations (ETD) repository, please notify the committee of the date of submission and/or any reference or cataloging number allocated.

Sincerely,

Arthur Webb

Prof Arthur Webb

Chair: Human Ethics Sub-Committee, RUESC- HE

Request for Permission to conduct research

RHODES UNIVERSITY
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ACCESS LETTER REQUESTING PERMISSION TO CONDUCT RESEARCH

Rhodes University
P.O. Box 94
Makhanda (Grahamstown)
6140
Eastern Cape
South Africa

The Vice Chancellor
Chalimbana University
Private Bag E1
Lusaka

Date 11th May 2020

Dear Sir

REQUEST FOR PERMISSION TO CONDUCT RESEARCH

I am a registered PhD student in the Department of Mathematics Education at the Rhodes University. My supervisor is Professor Marc Shafer.

The proposed topic of my research is:

The Incorporation of GeoGebra as a Visualisation tool to teach Calculus in Teacher Education Institutions: The Zambian case

The objectives of the study are:

(a) To find out the perceptions and experiences of lecturers in using GeoGebra as a visualisation tool to teach calculus in TEIs in Zambia.

(b) To Investigate how GeoGebra can be used as a visualisation tool to teach calculus to pre – service students in TEIs to enhance conceptual understanding.

(c) To analyse the enabling and constraining factors of using GeoGebra in the teaching and learning of Mathematics.

I am hereby seeking your consent to kindly allow me conduct research in your institution with two of your mathematics lecturers and ten of your first-year students. To assist you in reaching a decision, I have attached to this letter:

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-commitee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

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Where leaders learn

(a) A copy of an ethical clearance certificate issued by the University
(b) A copy the research instruments which I intend using in my research

Should you require any further information, please do not hesitate to contact me or my supervisor. Our contact details are as follows:
Name: Lemmy Kangwa Email:kangwavitaliano@gmail.com Mobile No: +260 979 914076

Supervisor: Marc Shafer (PhD): Email: m.shafer@ru.ac.za. Telephone: +2746 603 7273

Upon completion of the study, I undertake to provide you with a feedback.

Your permission to conduct this study will be greatly appreciated.

Yours sincerely,

Signature
Name: Lemmy Kangwa

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-commitee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

Access letter requesting permission to conduct research

Gate keeper permission

REPUBLIC OF ZAMBIA

MINISTRY OF GENERAL EDUCATION

P.O. Box 50093
LUSAKA

19 May, 2020

Mr. Lemmy Kangwa (0979-914076/0966951313)
Department of Mathematics
Chalimbana University
LUSAKA.

REF: REQUEST FOR GATE KEEPER PERMISSION TO COLLECT DATA FOR RESEARCH - MR. LEMMY KANGWA

The above subject matter refers.

I am pleased to inform you that permission to collect data for Research from three (3) colleges of Education under the Ministry on the Research project titled ***"The Incorporation of GeoGebra as a Visualisation tool to teach Calculus in Teacher Education Institutions: the Zambian case"*** has been granted.

Be informed that the data will be collected from David Livingstone College of Education, Kitwe College of Education and Mufulira College of Education. During the data collection period, you will be required to exhibit high levels of professionalism, discipline and integrity, failure to which will attract termination of this permission.

Furthermore, ensure that the information collected is purely used for academic purposes.

By copy of this letter, the Provincial Education Officers for Southern and Copperbelt provinces are informed accordingly.

Kapulo Musonda

APPENDIX B

Participants' Concert Letters

Lecturer 1

RHODES UNIVERSITY

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PARTICIPANT INFORMED CONSENT

INFORMED CONSENT DECLARATION

(Participant)

Project Title: **The Incorporation of *GeoGebra* as a Visualisation tool to teach Calculus in Teacher Education Institutions: The Zambian case**

Lemmy Kangwa, a PhD student, from the Department of Mathematics Education, Rhodes University, has requested my permission to participate in the above-mentioned research project.

The nature and the purpose of the research project and of this informed consent declaration have been explained to me in a language that I understand.

I am aware that:

1. The purpose of the research project is to investigate the use of *GeoGebra* as a visualisation to teach Calculus in Teacher Education Institutions
2. The Rhodes University has given ethical clearance to this research project and I have seen/ may request to see the clearance certificate.
3. By participating in this research project,
   - I will contribute to developing innovative practices of teaching Calculus
   - Enrich my knowledge and skills of integrating of technology in the teaching of mathematics.
   - I will also contribute to professional teacher development through the consolidation of the Teaching and Learning Mathematics with *GeoGebra (TLMG)* Project in Zambia with a view of incorporating GeoGebra in the Zambian Mathematics Curriculum
4. I will participate in the project by
   - Designing GeoGebra applets based on Calculus topic: Limits, Slope of a Curve, Riemann Sum and Area between curves
   - With the aid of a computer, use *GeoGebra* applets as a visualisation tool to explore Calculus concepts while teaching students
   - Be interviewed by the researcher on my perceptions and experiences of using GeoGebra to enhance the teaching of Calculus and on my views on enabling and constraining factors of using GeoGebra to teach Calculus

5. My participation is entirely voluntary and should I at any stage wish to withdraw from participating further, I may do so without any negative consequences.

6. I will not be compensated for participating in the research, but my out-of-pocket expenses will be reimbursed.

7. There may be risks associated with my participation in the project. I am aware that

   a. The following risks are associated with my participation:
      Continuous focus on computer screen may be detrimental to sight
   b. The following steps have been taken to prevent the risks: Reducing screen resolution on computer desktop
   c. there is a 5 % chance of the risk materialising

8. The researcher intends publishing the research results in the form of a thesis. However, confidentiality and anonymity of records will be maintained and that my name and identity will not be revealed to anyone who has not been involved in the conduct of the research.

9. I will receive feedback in the form of a thesis regarding the results obtained during the study.

10. Any further questions that I might have concerning the research, or my participation will be answered by Lemmy Kangwa, email address: kangwavitaliano@gmail'com or my supervisor: Professor Marc Shafer, email address: m.schafer@ru.ac.za

11. By signing this informed consent declaration, I am not waiving any legal claims, rights or remedies.

12. A copy of this informed consent declaration will be given to me, and the original will be kept on record.

13. Request to take pictures, video and voice recording for this study

I, ...XXXXXXXXXXXXXXX........ have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wished to ask and these have been answered to my satisfaction. I fully understand what is expected of me during the research.

I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.

RHODES UNIVERSITY

Participants signature	Witness	14/05/2021 Date

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-committee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

Lecturer 2

RHODES UNIVERSITY

PARTICIPANT INFORMED CONSENT

INFORMED CONSENT DECLARATION

(Participant)

Project Title: **The Incorporation of *GeoGebra* as a Visualisation tool to teach Calculus in Teacher Education Institutions: The Zambian case**

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   - I will also contribute to professional teacher development through the consolidation of the Teaching and Learning Mathematics with *GeoGebra (TLMG)* Project in Zambia with a view of incorporating GeoGebra in the Zambian Mathematics Curriculum

4. I will participate in the project by

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   - With the aid of a computer, use *GeoGebra* applets as a visualisation tool to explore Calculus concepts while teaching students
   - Be interviewed by the researcher on my perceptions and experiences of using GeoGebra to enhance the teaching of Calculus and on my views on enabling and constraining factors of using GeoGebra to teach Calculus

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5. My participation is entirely voluntary and should I at any stage wish to withdraw from participating further, I may do so without any negative consequences.

6. I will not be compensated for participating in the research, but my out-of-pocket expenses will be reimbursed.

7. There may be risks associated with my participation in the project. I am aware that

   a. The following risks are associated with my participation: Continuous focus on computer screen may be detrimental to sight
   b. The following steps have been taken to prevent the risks: Reducing screen resolution on computer desktop
   c. there is a 5 % chance of the risk materialising

8. The researcher intends publishing the research results in the form of a thesis. However, confidentiality and anonymity of records will be maintained and that my name and identity will not be revealed to anyone who has not been involved in the conduct of the research.

9. I will receive feedback in the form of a thesis regarding the results obtained during the study.

10. Any further questions that I might have concerning the research, or my participation will be answered by Lemmy Kangwa, email address: kangwavitaliano@gmail'com or my supervisor: Professor Marc Shafer, email address: m.schafer@ru.ac.za

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12. A copy of this informed consent declaration will be given to me, and the original will be kept on record.

13. Request to take pictures, video and voice recording for this study

I, ...XXXXXXXXXXXXXX...... have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wished to ask and these have been answered to my satisfaction. I fully understand what is expected of me during the research.

I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.

RHODES UNIVERSITY

Participants signature

Witness

19/06/21

Date

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-commitee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

Lecturer 3

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PARTICIPANT INFORMED CONSENT

INFORMED CONSENT DECLARATION
(Participant)

Project Title: **The Incorporation of *GeoGebra* as a Visualisation tool to teach Calculus in Teacher Education Institutions: The Zambian case**

Lemmy Kangwa, a PhD student, from the Department of Mathematics Education, Rhodes University, has requested my permission to participate in the above-mentioned research project.

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4. I will participate in the project by

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5. My participation is entirely voluntary and should I at any stage wish to withdraw from participating further, I may do so without any negative consequences.

6. I will not be compensated for participating in the research, but my out-of-pocket expenses will be reimbursed.

7. There may be risks associated with my participation in the project. I am aware that

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13. Request to take pictures, video and voice recording for this study

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I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.

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....14 - 05 - 21......

Participants signature **Witness** **Date**

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-committee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

Lecturer 4

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PARTICIPANT INFORMED CONSENT

INFORMED CONSENT DECLARATION

(Participant)

Project Title: **The Incorporation of *GeoGebra* as a Visualisation tool to teach Calculus in Teacher Education Institutions: The Zambian case**

Lemmy Kangwa, a PhD student, from the Department of Mathematics Education, Rhodes University, has requested my permission to participate in the above-mentioned research project.

The nature and the purpose of the research project and of this informed consent declaration have been explained to me in a language that I understand.

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5. My participation is entirely voluntary and should I at any stage wish to withdraw from participating further, I may do so without any negative consequences.

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I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.

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………………	………………	1st Sept 2020
Participants signature	**Witness**	**Date**

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-commitee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

Lecturer 5

RHODES UNIVERSITY

PARTICIPANT INFORMED CONSENT

INFORMED CONSENT DECLARATION
(Participant)

Project Title: **The Incorporation of *GeoGebra* as a Visualisation tool to teach Calculus in Teacher Education Institutions: The Zambian case**

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I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.

RHODES UNIVERSITY
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Participants signature	Witness	Date

19 July 2020

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-commitee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

Lecturer 6

RHODES UNIVERSITY
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PARTICIPANT INFORMED CONSENT

INFORMED CONSENT DECLARATION

(Participant)

Project Title: **The Incorporation of *GeoGebra* as a Visualisation tool to teach Calculus in Teacher Education Institutions: The Zambian case**

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12. A copy of this informed consent declaration will be given to me, and the original will be kept on record.

13. Request to take pictures, video and voice recording for this study

I, ...XXXXXXXXXXXXX.......................... have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wished to ask and these have been answered to my satisfaction. I fully understand what is expected of me during the research.

I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.

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....	....	22nd Feb 2021
Participants signature	**Witness**	**Date**

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-commitee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

APPENDIX C

Survey for baseline data on lecturers' awareness and use of *GeoGebra*

Survey for mathematics lecturers on awareness and use of GeoGebra in Zambian Education system

I am in the process of writing my research proposal for my PhD study at Rhodes University and would like to obtain some information on the Mathematics software, GeoGebra.

I am seeking your personal viewpoints on the distribution of GeoGebra in Zambian schools

Do not write your name.

Tick in brackets ( ) or elaborate if you wish

1. How would you rate the awareness of GeoGebra in Zambian Schools at

(a) Primary School?	None ( )	Low ( )	Average ( )	Above average ( )
(b) Secondary School?	None ( )	Low ( )	Average ( )	Above average ( )
(c) Tertiary Institution?	None ( )	Low ( )	Average ( )	Above average ( )

....

....

2. How would you rate the use of GeoGebra in Zambian Schools at

(a) Primary School?	None ( )	Low ( )	Average ( )	Above average ( )
(b) Secondary School?	None ( )	Low ( )	Average ( )	Above average ( )
(c) Tertiary Institution?	None ( )	Low ( )	Average ( )	Above average ( )

....

....

3. (a) Does your institution use GeoGebra for teaching and learning Mathematics? Yes ( ) No ( )

(b) If your answer is yes, to what extent is it used ?

Low ( ) Average ( ) Above average ( )

....

....

4. Do you personally have any experience with using GeoGebra in your teaching? Please elaborate

....

....

Thank you

Lecturers' interview instrument

Interview Questions for Lecturers in Teacher Education Institutions

The Incorporation of *GeoGebra* as a visualisation tool to teach calculus in Teacher Education Institutions: The Zambian Case

The interviewees will be welcomed and encouraged to be honest about their responses and explain their views as much as possible. Where necessary, I will probe their responses

All identifying information will be deleted as soon as all data has been collected.

Personal data

Name of the institution…………………………**Gender**: Male (√) Female ()

Cycles taught……………………………………………………………………………….

Teaching experience: 0 – 5 years ( ),6-10 years ( ),11-15 ( ), 11-15 years ( ) Above 15 years (√)

1. **(**a) Have you ever used computer software before in teaching mathematics? If so, which one(s) and how did you use it/them?
   (b) What are your experiences about teaching calculus to students in Teacher Education Institutions? Explain your answer?
2. From your experience, which areas do students in TEIs find challenging in calculus? Which ones do they find easy? What do you think could be the reason for this?
3. What do you think is the role of visualisation in teaching mathematics?
4. In your presentation of calculus lessons to students, how did you use *GeoGebra* to
   (a) visualise calculus concepts?
   (b) enhance conceptual understanding of calculus concepts?
5. How did the use of *GeoGebra* facilitate your teaching of calculus?
6. How did you use *GeoGebra* applets to visualise mathematics concepts?
7. What did you find challenging in your use of *GeoGebra* applets in teaching calculus?
8. Based on the calculus lessons you presented, what do you think was advantageous and what do you think was disadvantageous about the use of *GeoGebra* in your teaching?
9. (a) How did *GeoGebra,* as a visualisation tool, enhance your teaching of each of the calculus concepts:
   (i) Limits (ii) Slope of the curve (iii) Riemann Sum (iv) Area between Curves
   (b) Which concepts in the above Calculus topics were enhanced by the use of *GeoGebra* applets in your teaching? Give reasons for your answer.
10. How did you use *GeoGebra* applets to encourage students explore calculus concepts?
11. From your experience of interacting with *GeoGebra*, how do you think *GeoGebra* can be used effectively to teach calculus to enhance conceptual understanding?
12. Do you feel *GeoGebra* has improved your understanding of the calculus concepts discussed and made your teaching easier? Explain your answer?
13. From your experience of teaching with *GeoGebra* applets, what factors do you think

(a) can encourage lecturers to use *GeoGebra* in their teaching?
(b) inhibit teachers from using *GeoGebra* in their teaching?

14. Was the training on use of *GeoGebra* prior to your interaction with students adequate for you to teach with the aid of the software. Explain
15. (a) In which ways do you think the use of *GeoGebra* made your explanations of calculus concepts clear to the students?
    (b) Explain the challenges if any, that you encountered when designing the applets and when using the applets to conduct your lessons.
16. In your view, what do you think should be done to make effective use of *GeoGebra* as a visualisation tool in the teaching of mathematics in general and calculus in particular?
17. Do you think *GeoGebra* should be used as a visualisation tool to teach mathematics in genera and calculus in particular in TEIs? Give reasons for your answer
18. Is there any information that you would like to share with me related to this interview that I have not captured in my questions?

End

Analytical Framework

Construct	Code	Definition	Observable Indicators The Lecturer:	Rubric for Indicators
Visualising Processes	VP	Using *GeoGebra* applets to visualise calculus concepts Knowledge of generating visuals appropriate to the calculus concept being discussed	VP1: Uses applets generated to visualise calculus concepts appropriately	Visualisation links one calculus concept to another and connecting mathematics to real- life situations Exhibits thorough knowledge of limit, slope of tangent, Riemann Sum and area between curves using
	USD Use of sliders and dragging	Sliding and dragging of objects on Calculus graphs and diagrams	USD 1: Uses sliders and dragging to show effect of change of parameters on calculus graphs and diagrams *GeoGebra* tools in lessons with ease	Lecturer visualises how movement of sliders and dragging of points on a *GeoGebra* applet results in different graphs E.g. Uses sliders to visualise movement of points, increase or reduction of number of rectangles using Riemann Sum Calculates the limit, slope of tangent, Riemann Sum and Area

				between Curves suing concept on a *GeoGebra* applets with ease
	Multiple Representation	Ability to represent the same concept in many forms.	(MR 1) Using the *GeoGebra* interface to show the same calculus concept in multiple ways from a constructivist perspective	Lecturer visualises the same concept symbolically, algebraically and graphically
Technological Pedagogical Content Knowledge (TPACK)	TPACK1	Knowledge required by teachers to integrate technology into their teaching	TPACK1: Uses *GeoGebra* to visualise and foster conceptual understanding of various calculus concepts	Knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face. Knowledge of how technologies can be used to build on existing knowledge and to develop new ideas
	Content Knowledge (CK 1)	Knowledge about the subject matter of the cycle	CK1: Applies calculus concepts to real situations and other contexts and Sequences concepts logically	Thorough knowledge of the content of a calculus cycle
	Pedagogical Knowledge (PK)	Knowledge about methods and processes of teaching	PK1: Uses methods that engage students actively	Addressing the worthwhile nature of mathematics with appropriate pedagogy.

		PK12: Uses methods that include assessment during lessons and good class management	Shows connection of calculus concepts in algebraic and geometric windows using virtual manipulatives
TK1 Technological Knowledge (TK)	Knowledge about various technologies	TK1: Uses *GeoGebra* tools skillfully and effectively	Introducing and illustrating technology in the context of meaningful content-based activities Uses *GeoGebra* tools skillfully during the lesson
PCK1 Pedagogical Content Knowledge	Content knowledge that deals with the teaching process	PCK 1: Uses teaching approaches appropriate to the content	Helping students discover mathematics concepts by taking advantage of the software capacities Uses *GeoGebra* visualisation characteristics to show calculus concepts in multiple representations
TCK Technological Content Knowledge	Knowledge of how technology can create new representations for specific content	TCK 1: Use of *GeoGebra* to teach calculus content.	Incorporating various representations of mathematical concepts.

		TCK 12: Shows same concept in multiple representations. TCK 12: Use GeoGebra to visualise calculus concepts.	Knowledge of how technologies afford particular representations and flexibility in navigating across them. Knowledge of the manner in which the subject matter can be changed by the application of technology.
TPK1 Technological Pedagogical Knowledge.	Knowledge of how various technologies can be used in teaching.	TPK 1: Uses GeoGebra to promote conceptual understanding of concepts.	Taking advantage of technological capabilities to enhance competence in teaching and learning. Understanding that a range of tools exist for a particular task Ability to choose a tool based on its fitness and strategies for using the tool's affordances ability to apply pedagogical strategies for use of technologies

TAM	ITUI 1 Intention to use ICT	Willingness to use technology in the teaching process.	ITUI 1: Prepares all necessary materials for *GeoGebra* lesson.	Enthusiasm in the use of *GeoGebra*
	AU 1 Actual Use	Use of technology in the teaching process.	AU 1: Presents lessons with *GeoGebra* software.	Making timely decisions on how and when to use technology appropriately in mathematics classrooms. Uses *GeoGebra* software where it is necessary.
	EU 1 Easy to use	Use of technology in an easier way	EU 1Deminstrates use of *GeoGebra* easily	EU 1Deminstrates use of *GeoGebra* prudently in an easy manner
Mathematics Proficiency	(CU1) Conceptual Understanding	Use technology to learn calculus concepts with understanding, and not superficially	(CU 1). Use *GeoGebra* to connect new calculus concepts to prior knowledge	(CU 1) Use *GeoGebra* to provide a link between abstract and concrete aspects of the same concept