Hugo Cayo[a]*, Myriam Codes[b] and Luis C. Contreras[b]

[a]*Departamento de Educación, Universidad de Antofagasta, Antofagasta, Chile;*
[b]*Departamento de Didácticas Integradas, Universidad de Huelva, Huelva, España*

*hugo.cayo@uantof.cl

Funding

The Spanish Government (RTI2018-096547-B-I00) (MECD) and the Research Centre COIDESO (University of Huelva, Spain) supported this research.

A mathematics teacher's specialised knowledge in the selection and deployment of examples for teaching sequences

This paper explores the specialised knowledge mobilised by a mathematics teacher in the selection and use of examples for teaching sequences. Taking an experimental case study approach, we analyse the examples deployed in a series of third year secondary level lessons on sequences and identify the different knowledge subdomains activated according to the Mathematics Teachers' Specialised Knowledge analytical model. We will analyse active and passive examples, pointing out the mathematical entity that is being exemplified and the aspect of this entity which is being emphasized by the example. The results identify the different subdomains and categories which are drawn on in the selection and use of examples, along with the various interconnections across knowledge subdomains which interact in the process.

Keywords: Mathematics Teacher's Specialised knowledge; exemplification; sequences

Subject classification codes: 580207

Introduction

At the end of the 80s, Shulman (2005) suggested that for the next decade an important part of the research agenda should be focused on collecting, collating and interpreting teachers' practical knowledge. As it turned out, the endeavour he initiated exceeded the following decade to the extent that the last 30 years have witnessed a vast expansion in the field of teachers' knowledge (of mathematics, in our case), as testified by the great number of studies which have been carried out (Badillo et al., 2019; Pino-Fan & Godino, 2015) and the varying models which have been developed for the purpose of deepening our understanding of mathematics teachers' knowledge (Ball et al., 2008; Carrillo et al., 2018; Pino-Fan & Godino, 2015; Rowland et al., 2005).

The importance of continuing to explore teachers' knowledge lies in the impact this research has on improving the quality of teaching (Ball et al., 2005). As the 2007 McKinsey report notes, quoting an unnamed South Korean policymaker: "the quality of an educational system cannot exceed the quality of its teachers" (Barber & Mourshed, 2007). Teachers' knowledge, in all its aspects, is a critical factor in teachers' performance and in promoting student learning (Zakaryan et al., 2018). It is what enables teachers to decide what should be taught and how, what kind of representation to choose, and how to solve the problems thrown up by any particular content (Shulman, 1986).

When Shulman (1986) establishes the existence of a specific knowledge of teachers, examples, metaphors and analogies become as relevant resources of representation. They ease to build bridges between the understanding of the teacher and the expected for the students. Therefore, these three resources can be considered as similar (Shulman, 2005).

Examples are an important element in teachers' pedagogical content knowledge and are considered one of the most useful ways of representing an idea with the aim of making it comprehensible to others. In this respect, Vinner (2011) accords exemplification a key role in the cognitive processes associated with how the mind gives shape to concepts and conjectures.

Exemplification has always played a central role in developing and teaching mathematical thinking (Bills & Watson, 2008), and are today one of the chief resources' teachers draw on in their day-to-day practice (Pascual & Contreras, 2018). Authors such as Chick and Harris (2007) and Figueiredo and Contreras (2013) assert that the majority, if not all, teachers use examples in teaching mathematical content.

Part of the importance of examples to the discipline lies in their role as a tool for mediating between students and the culture of mathematical concepts, theorems and techniques (Goldenberg & Mason, 2008). In this regard, Figueiredo and Contreras (2015) argue that definitions in themselves are insufficient for students to understand and apply a concept. Rather, they consider it necessary to illustrate the mathematical concept with examples consistent with the definition, such that the definition thus acquires its meaning principally through the examples.

As noted above, exemplification has been highly important in the development and teaching of mathematics and has become one of the most used tools by teachers. Figueiredo and Contreras (2013) underline teachers' awareness that successful learning of concepts depends to a large extent on their choice and use of examples, and it is these two aspects – the selection and implementation of appropriate examples – which can be considered the most critical and complex points in the process of working with examples (Zodik & Zaslavsky, 2007a,b). For this reason, we consider it important to explore the specialised knowledge deployed by mathematics teachers in the choice and use of examples.

Theoretical foundations

Some 35 years ago, Shulman (1986) posited the existence of a kind of knowledge which was exclusive to teachers, and which he divided into three categories: (a) content knowledge; (b) pedagogical content knowledge; and (c) curriculum knowledge. The following year, addressing the kinds of knowledge involved in teaching, he amplified the list to seven categories: content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners, knowledge of educational contexts, and knowledge of educational aims, purposes, and values, and their philosophical and historical foundations. Pedagogical content knowledge received particular interest and was defined as "that special amalgam of subject and pedagogy that is uniquely the province of teachers" (Shulman, 2005, p. 11).

Shulman's work formed the basis of various models focusing on mathematics teachers' knowledge. The development of the model Mathematical Knowledge for Teaching (MKT) drew on two of Shulman's subdomains: content knowledge and pedagogical content knowledge (Ball et al., 2008). Several of the categories proposed by Shulman were included in the Knowledge Quartet by Rowland et al. (2005), namely content knowledge, curriculum knowledge, and pedagogical content knowledge (Rowland et al., 2005). Shulman's ideas are also recirculated in some of the facets forming part of the Didactic Dimension in the model Didactic-Mathematical Knowledge by Pino-Fan and Godino (2015). Finally, two of the knowledge domains constituting the Mathematics Teachers' Specialised Knowledge (MTSK) model (see figure 1), developed by Carrillo and associates, draw on Shulman's original insights (Carrillo et al., 2018; Carrillo et al., 2017; Carrillo et al., 2013).

This latter, the MTSK model, is the analytical instrument we shall use in this study, due to the completeness of its system of categories and the ease of analytical application. For the authors, the main advantage of the model is that it enables researchers to explore mathematics teachers' knowledge through structured analysis of the components from which it is constituted (Carrillo et al., 2014; Liñán et al., 2019, 2021). The model comprises two domains, Mathematical Knowledge (MK), made up of the teacher's knowledge of mathematics as an academic discipline in an educational context, and Pedagogical Content Knowledge (PCK), comprising knowledge about the teaching and learning of mathematical content (Carrillo et al., 2018). Each of these domains is further subdivided into three subdomains, each with their corresponding categories. Together they enable the knowledge deployed by teachers to be examined with precision (Escudero-Domínguez et al., 2016).

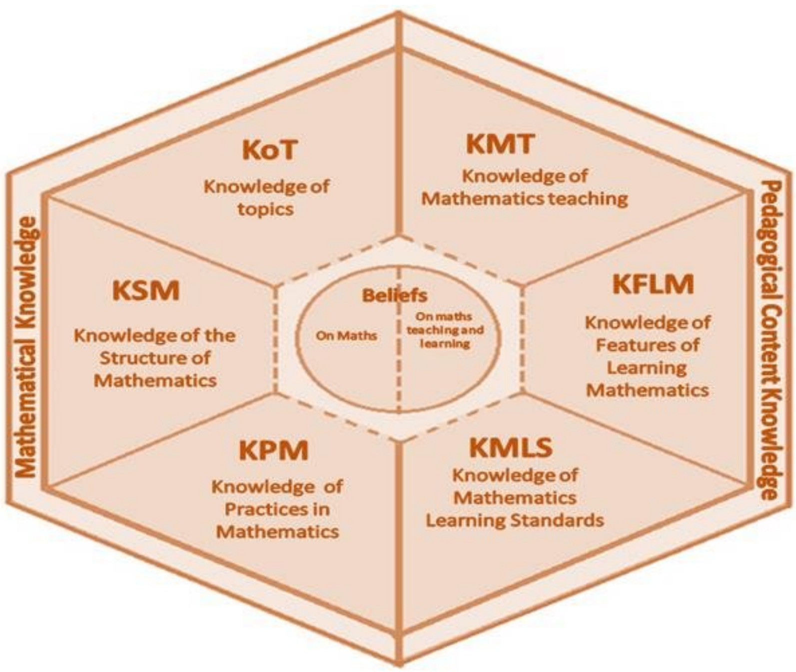


Figure 1: The MTSK model (Carrillo et al., 2018)

An inventory of the three subdomains which make up MK is given below, starting with Knowledge of Topics (KoT). (a) Following Liñán et al. (2016), KoT corresponds to mathematical knowledge per se. It includes knowledge about concepts, propositions, properties, procedures, classifications, examples (example spaces), formulae and algorithms, with their respective meanings and demonstrations. The subdomain is structured according to the following categories: procedures; definitions, properties and their foundations; registers of representation; and phenomenology and applications (Carrillo et al., 2018). (b) Knowledge of the Structure of Mathematics (KSM) is the subdomain which focuses on the interconnections between elements of mathematical knowledge. Such connections can help a teacher to increase or decrease the complexity of an item, or to draw a link to an unrelated concept, and are consequently organised into the following categories: connections based on simplification; connections based on increased complexity; auxiliary connections; and transverse connections (Carrillo et al., 2018). (c) Knowledge of Practices in Mathematics (KPM). This subdomain concerns the production and development of mathematical knowledge. It includes the knowledge teachers need to do demonstrations, justify propositions, provide adequate definitions, apply deductive and inductive reasoning appropriately, recognise the limitations of examples, and understand the role of counter-examples in the justification process (Carrillo et al., 2018).

In like fashion, the subdomains comprising PCK are as follows. (a) Knowledge of Features of Learning Mathematics (KFLM). This focuses on the kind of knowledge which is inherent to the process of learning, that is, the aspects of mathematical content which are brought to the fore when the learner engages with an item. The categories for this subdomain are: theories of learning; strengths and weaknesses; modes of interacting with mathematical content; and emotional aspects (Carrillo et al., 2018). (b) Knowledge of Mathematics Teaching (KMT). KMT covers the theoretical knowledge of mathematics teaching, at both a personal and institutional level, and is brought into play when teachers devise learning opportunities for students. The corresponding categories are: theories of teaching; teaching resources (both physical and digital); and strategies, techniques, tasks, and examples (criteria for selection and use) (Carrillo et al., 2018). (c) Knowledge of Mathematics Learning Standards (KLMS). This subdomain concerns the evaluative dimension of learning mathematics in terms of students' abilities in terms of understanding, constructing and using mathematics. It has the following categories: expected learning outcomes; expected level of conceptual or procedural development; and sequencing of topics (Carrillo et al., 2018).

This model also considers the teacher beliefs about mathematics and mathematics teaching and learning (Carrillo, et al., 2019). Beliefs are considered within the model as a subdomain that permeates both the mathematical knowledge and the didactical knowledge of the mathematical content.

It should be noted that although this compartmentalisation of knowledge provides researchers with a tidy framework for analysis, there is no intention to disregard the distributed and integrated nature of teachers' knowledge. The purpose of the division is to allow us to characterise each of the subdomains precisely and in-depth, and especially to enable connections between them to be established (Carrillo et al, 2014).

Various authors have carried out studies into the interconnections between subdomains (Delgado-Rebolledo & Zakaryan, 2020; Zakaryan et al., 2018; Zakaryan & Ribeiro, 2016). Zakaryan and Ribeiro (2016) show how the interconnections between KMT and the other subdomains enabled the researchers to identify a set of core elements central to the participating teachers' knowledge. Zakaryan et al. (2018) found that the subdomains KFLM and KMT mutually reinforced each other, although their study noted that of the two, it was KFLM which was most influential. In a case study of a secondary school teacher working on the concept of proportionality, Fuentes (2020) found connections between KMT and KFLM, and between KMT and KoT. Elsewhere, Delgado and Zakaryan (2019) observe that, when the teacher's KFLM was reinforced by KPM, they were able to provide a more detailed explanation of the topic. The researchers also found that the teacher's KMT was conditioned by their KPM to such an extent that the goal they set the students, in addition to understanding the content in question, was also to learn to think mathematically with regard to validating and constructing arguments. Finally, Carrillo at al. (2014) presented and described different interconnections between the subdomains KSM-KMLS; KoT-KMLS; KFLM-KMLS; KFLM-KMT; KoT-KPM; and MK-KMT/KFLM.

In this study, the model is used to analyse the interconnections between different subdomains in the course of a teacher selecting and deploying different kinds of examples. In the MTSK model, exemplification is explicitly accounted for in the category *strategies, techniques, tasks, and examples* within the KMT subdomain (Carrillo et al., 2018). This subdomain includes the kind of knowledge which enables teachers to select a specific resource (such as a powerful example) for the learning of a mathematical concept or procedure (Escudero-Ávila et al., 2016; Sosa et al., 2016) and the knowledge that allows the construction of an adequate sequence of examples (Sosa et al., 2017). Some authors have proposed that exemplification should constitute a category of its own, *example spaces*[1], within the KoT subdomain (Liñán et al., 2016).

Liñán et al. (2016) maintain that exemplification represents a particularly rich context for the study of mathematics teachers' knowledge, as example spaces offer the opportunity for in-depth analysis of the various subdomains across the model. This idea is shared by Sosa et al. (2016) who consider that exemplification sheds light on indicators relating to both teachers' mathematical knowledge and their pedagogical knowledge.

It is clear that examples are a fundamental tool in the teaching and learning of mathematics. Zaslavsky et al. (2006) conclude that examples are essential for working with generalisation, abstraction and analytical reasoning. Ng and Dindyal (2015) note that researchers generally define examples as individual illustrations of a larger class. A case in point is Zodik and Zaslavsky (2008), who define an example as a specific case from which more generalised instances can be reasoned.

Based on this last definition, in our work we consider that an example is a particular case from which it is possible to reason, generalize and demonstrate a particularity of a mathematical entity.

The literature offers various types of categorisations for examples. Chick and Harris (2007) review two such categorisations developed by other authors. That of Rowland and Zaslavsky (2005) divides examples into two types, (a) those used to

[1] The notion of example spaces, mainly associated with the examples that learners work with and produce (Michener, 1978; Watson & Mason, 2005; Zazkis & Leikin, 2007), corresponds to the set of examples of which an individual is aware, and the interconnections between them. These constitute a structured network available to the individual at any particular time for a specific concept or procedure (Liñán et al., 2016).

illustrate a general principle or form of inductive reasoning, and (b) those used to provide material on which to practise. The categories developed by Rissland-Michener (1978), on the other hand, offer four types of example: (a) start-up examples, which are used at the beginning of learning to draw attention to the principle at work; (b) reference examples, used to illustrate standard cases which are frequently referred to in the broader theory; (c) model examples, which foreground the typical features of a concept; and (d) counterexamples, which highlight the conditions under which a general principle is no longer applicable. Watson and Chick (2011) present a classification system in which examples are associated with different purposes: (a) for analysis, in terms of finding plausible interconnections between the elements of an example; (b) for generalisation, involving finding similarities between examples; and (c) for abstraction, that is, classifying examples according to similarities which can be identified as a concept or class with its own properties.

In a study into the kinds of examples used by teachers, Karaagac (2004) recognises two types: passive examples are those which do not require any kind of action on the part of the students, and serve to exemplify a previously presented concept or procedure; active examples, on the other hand, are those requiring some kind of response by the participants, whereby previous knowledge must be brought into play by either the teacher or students. For their part, Zodik and Zaslavsky (2007a,b) classify examples according to the kind of mathematical entity the example is intended to illustrate: a concept, a theorem, or a procedure/algorithm. Figueiredo et al. (2007) establish a system for categorising examples based on their observations of students in the process of acquiring the concept of function in mathematics:

(a) Definitions. These are examples which are given directly after a definition, or to a set of examples given beforehand with the aim of foregrounding the salient characteristics of the definition to help the students grasp it.
(b) Representations. This type of example is designed to facilitate students' initial encounters with the concept in question in the form of exercises or problems.
(c) Characteristics. Examples in this category arise after the exploratory phase and which help students surmount difficulties and clarify doubts.
(d) Internal applications. These examples link the topic under analysis to previous or to future topics.
(e) External applications. These examples are drawn from applications of the topic to real life situations or to other areas of science.

As will be seen below, we will bring these three classification systems together in order to classify the different examples employed by the teacher, analysing three aspects of the examples: whether or not they promote student participation, the mathematical entity being exemplified, and the key feature of the mathematical entity being tackled through the example.

As mentioned above, the use of examples by teachers is a very common practice in mathematics lessons (Ng & Dindyal, 2015). What has become a problem, however, is not that examples have become the main source of learning, but that at the same time they have become a major source of confusion for students (Mason, 2011), thus underlining the importance of further studies into how they used in the classroom.

There are two aspects to working with examples which have been identified as especially important: selection and use (Zodik & Zaslavsky, 2007a,b). Various authors are in agreement that these two aspects require teachers to mobilise a set of specialised knowledge. Suffian and Abdul (2010) point to pedagogical content knowledge as especially relevant in this respect, as a factor of particular influence over the choice of examples and how they are employed. Zaslavsky (2008) maintains that well-grounded knowledge, both mathematical and pedagogical, is necessary for choosing and constructing useful examples, while Zodik and Zaslavsky (2007a,b) identify three areas of knowledge governing the selection and use of examples: mathematical content knowledge; pedagogical content knowledge relating to examples; and knowledge of students' learning characteristics. Identifying these different types of knowledge at work in the teacher is a task of particular interest to researchers.

Methods

The aim of this study was to explore the nature of the interconnections between subdomains activated by the teacher when selecting and deploying examples. We try to understand these relationships, based on the observation and information provided by the teacher (Vain, 2012). In this regard the study is qualitative, an approach that various authors regard as ideal for understanding, describing and interpreting the phenomena under study (Bassey, 1999; Bisquerra, 2009; Hernández et al., 2010). We chose a case study format as the most suitable for a contextualised in-depth analysis of the interconnections involved in the process of exemplification (Duran, 2012). In like fashion, the instrumental orientation facilitated the capture of the distinct elements of knowledge involved (Bisquerra, 2009; Duran, 2012).

Our informant was a state school teacher who had originally trained as a chemical engineer. With some 35 years in the profession, he amply met Chi's (2011) criteria for being considered a teacher of expertise. In the course of his career, he had taught not only Mathematics, but also Physics, Chemistry, Technology and Geology, thus marking him as an a priori candidate for illustrating the use of tangible examples. The examples which were selected for analysis derived from a third year secondary lesson (14 – 15 year old students) which was given towards the end of the 2019/2020 academic year, in which the teacher had begun work on the concept of sequences with the intention of then moving on to arithmetic and geometric progressions.

To carry out the research, a first meeting was held with the informant teacher. In this meeting, the purpose of the research and the use that we would make of the different recordings, of the class and of the interview were explained to him. The teacher coordinated a second meeting with the management of the establishment, specifically with the director and the general inspector, with whom it was agreed that during the class only the work done by the teacher would be filmed, avoiding recording the students as much as possible, and that the recordings would not be shared with people unrelated to this investigation or disseminated.

Data was obtained through lesson observation and interview. The lessons were video-recorded and fully transcribed. Episodes involving clear use of examples were then identified for further study. Whenever possible, the complete episode was segmented into smaller analytical units to facilitate the identification of evidence and indications of the different elements of specialised knowledge brought into play while providing an example. The interview, which followed a semi-structured framework around specific instances indicative of the deployment of specialised knowledge, enabled us to delve further into the specific nature of the knowledge lying behind each choice of example, and so transform indications into firm evidence of knowledge.

As mentioned above, the categorization of the examples was carried out according to three aspects: whether or not the teacher aimed to generate the active participation of his students through using the example; the nature of the mathematical entity being exemplified; and the particular aspect of this entity being emphasised by the example. These aspects corresponded to three systems of categorisation drawn from previous studies, applied, with certain adaptations, in the manner described below.

The classification of an example as either active or passive (Karaagac, 2004) was determined solely on the basis of the interaction it was intended to generate on the part of the students. Hence, where the teacher did not aim to generate any participation, the example was classed as passive; conversely, where participation was intended, the example was classed as active. The categories presented by Zodik and Zaslavsky (2007a,b) formed the basis by which we determined the nature of the mathematical entity being exemplified, whether concept, theorem or procedure/algorithm. Finally, the classification proposed in Figueiredo et al. (2007) was applied to the examples in order to identify which aspect of the mathematical entity was being highlighted: definition, representation, characteristics, and external or internal application. We found it useful to redefine some of the categories in this latter classification system. Specifically, in the category ‘representation’, we included all examples which emphasise any kind of representation of the mathematical entity being studied (graphical, algebraic, mental and so on). Likewise, in the category ‘characteristics’, we included all examples which aimed to emphasise any characteristic of the mathematical entity, or to assist students in working with and in the mathematical entity (theorem or procedure/algorithm). The modifications to these latter two categories enabled us to better contextualise the examples.

Analysis of the data began with the categorisation of the examples observed in the lesson according to the three classification systems: the interaction on the part of the students; the mathematical entity being exemplified; and the aspect of the mathematical entity being highlighted by the example. Analysis of the transcripts of the lesson and the interview were carried out sequentially. For each episode, we identified the different subdomains of MTSK mobilised by the teacher while working through the example, and in some cases, we backed this up by supporting data drawn from the interview, a tool which also enabled us to identify the different subdomains which the teacher mobilised at the moment of making their choice of example. Finally, we sought to establish the different connections between the subdomains, in terms of both the choice and deployment of the examples.

Having a collection of examples that we observed in class, we selected those which had a greater wealth of information even if they were not part of a sequence of examples, respecting the order of appearance during the class. In each case, the purpose of the example within the lesson is given along with its classification, according to the systems described above. After this, we consider the knowledge subdomains involved in each, and the interconnections between these with respect to both the choice and the use of the example in question.

Knowledge was adjudged to be relevant to the choice of an example if it could be demonstrated to underlie the choice, that is, if it was directly confirmed in the course of the interview by the teacher himself, or subsequently identified through matching his responses to the evidence available in the lesson transcript. Knowledge relevant to the use of the example was identified as any knowledge that became evident in the teacher's performance while working through the example.

We have indexed both the examples and the interview fragments with letters E and Q, respectively.

Results

The first episode (Example 1) is taken from the teacher's introductory comments on sequences. This example can be classified as a passive example of a concept, emphasising external applications. He mentions the applications which sequences have in everyday life; in this case, the teacher did not aim to generate any participation (Karaagac, 2004).

T: There are situations in everyday life, in which the key is in the fact that they operate according to a sequence, so if you are able to find out what it is, and moreover specify it, then you can predict the whole phenomenon. In other words, you can anticipate the future because you know what is going to happen. Because you have defined the sequence, you know what is going to happen at the time which you're interested in. But you also know what happened earlier in time, because you've observed the sequence, the mechanism, the dynamics, how the phenomenon behaves over time.

(E.1.)

In order to gather evidence of the different subdomains which might have influenced the choice of this example, we asked the following question in the interview:

I. Why did you refer to everyday situations in the examples?

T. To capture their imagination and create expectation. And it's very something accessible and direct to them in their lives. It means something to them, and so in *part it was aiming to ensure that they'd pay attention to a situation which they'd then have to try and solve.*

(Q.1.)

The answer provides evidence of the different subdomains involved in the choice of example. On the one hand, he knows that in teaching mathematics, examples that appeal to everyday situations have a positive influence on his students (KMT: strategies, techniques, tasks and examples), while his understanding of how learning about the practical applications of a particular item infuses that item with greater meaning represents an instance of KFLM (KFLM: emotional aspects). At the same time, his knowledge of real-life applications based on sequences draws on his KoT (KoT: phenomenology and applications). Hence, the episode illustrates the interaction between the teacher's KMT and his KFLM at the moment he settles on this passive example – that is, his knowledge of the deeper relevance conferred on the item, in the view of the students, by practical applications (KFLM) interconnects with, and is enhanced by, the effectiveness of examples grounded in everyday situations (KMT). In addition, the interaction of these two subdomains would seem to activate the teacher's knowledge of the practical applications of sequences (KoT).

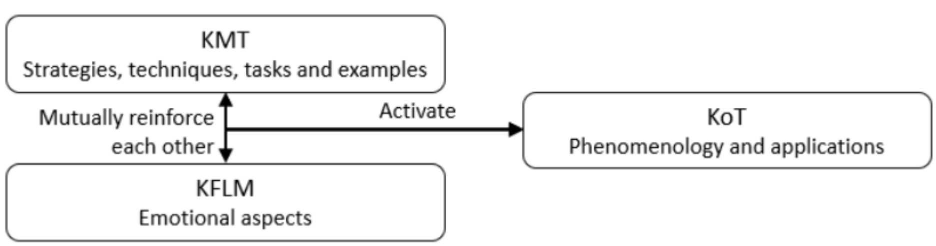


Figure 2: Interconnections between subdomains during selection of example 1.

In the course of this example, we can see that the knowledge deployed at certain moments draws on just one of the MTSK subdomains, specifically KoT. This instance corresponds to when the teacher notes that there are applications in real life which make use of sequences (KoT: phenomenology and applications).

The following episode (Example 2) shows the teacher emphasising the importance of knowing the pattern of the sequence. It corresponds to a passive example of a procedure, which emphasizes a characteristic. With this exemplification, where his students do not interact with the example, the teacher shows the importance of knowing the pattern behind a sequence. With this, the teacher intends to emphasize that this procedure allows to continue with the construction of the terms of that sequence (adapted to Figueiredo et al., 2007).

T: You have all worked on sequences before, when you were in primary school. A lot of the activities you were set were basically sequences for one simple reason. You were asked to find out what the pattern was. They gave you a bunch of data, which wasn't chosen just for nothing or any old how, but because there was some kind of pattern to it, and they said, "OK, can you tell me what comes next?" And so, you'd take a look and you'd realise what the pattern was, what the reasoning was which lay behind what you were looking at, and you could tell what came next, what came after that, and so on.

(E.2.)

Episode J.2 provides indications that, during the choice of this example, the teacher mobilises knowledge corresponding to the categories of 'connections based on simplification' (KSM) and 'sequencing of topics' (KMLS). In order to delve more deeply into these and other elements of knowledge that might have been influential in the choice of this example, we asked the teacher the following question[2]:

[2] The question itself was posed in a general sense, given that the teacher referred back to previous areas of study in several of the examples they used. The aim was to determine whether there was any specific reason for this, and if so, which MTSK subdomains were involved in the choice of example.

R: Why, when you choose the examples, do you repeatedly refer the students back to things they have done before?
T: Because knowledge is built on what you've done before. If you haven't assimilated what it's based on, if you haven't properly understood all the previous stuff, then it is difficult for you to build on that knowledge...

(Q.2.)

The teacher's answer illustrates his belief of how learning is constructed; it is a general idea, not limited to mathematics teaching, but influential in the choice of example. In this instance, he makes his choice of example drawing on previous knowledge. He recognises patterns as something the students have met in earlier years, and which are connected to sequences (KMLS: sequencing of topics), and that it can also contribute to the understanding of the concept of the general term of a sequence (KSM: connections based on simplification). Hence, we can see in the choice of this passive example a link between the teacher's KMLS and KSM – his knowledge of patterns as a topic which is connected to sequences (KMLS), and which can aid the understanding of the concept of a general term (KSM). Together, these elements of knowledge motivate the choice of an example based on previous study items, consistent with the teacher's constructivist belief of learning.

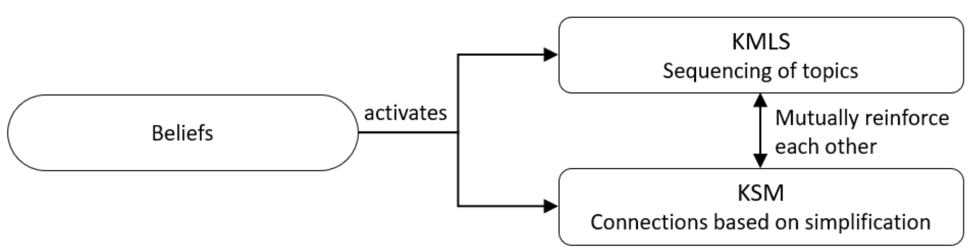


Figure 3: Interconnections between subdomains during selection of example 2.

In episode J.2 we can see that the teacher's choice of this example involves mobilising knowledge pertaining to KoT and KFLM. He is aware of the importance of perceiving the pattern underlying a sequence in order to be able to continue with the construction of terms (KoT: procedures). At the same time, he recognises the utility of the students' previous experience of studying patterns (KFLM: ways pupils interact with mathematical content). The use of this passive example thus provides evidence of an interaction between the teacher's KFLM and his KoT. The teacher knows how his students typically interact with patterns (KFLM) enhances the idea he wants to transmit – the importance of knowing the pattern underlying a sequence (KoT).

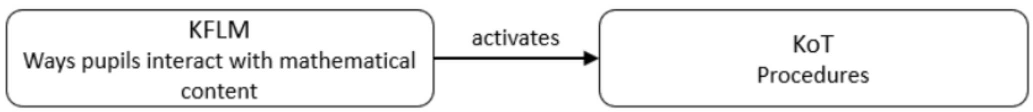
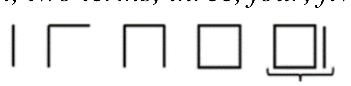


Figure 4: Interconnections between subdomains during the use of example 2.

In the following episode (Example 3), we can see how the teacher uses a non-mathematical sequence to illustrate the importance of identifying the pattern underlying the sequence before moving on to construct the terms. This example can be classified as an active example of a procedure emphasising a characteristic. The teacher asks the students to identify the characteristics of the procedure that will allow them to continue with the construction of the terms of the sequence.

T: I'm going to set you the following figure.

T: This is not numerical, you can see it's not numerical. The first ones you did were not numerical, they were situations, things, you followed a pattern, an order.
T: What pattern is this series following?
S: It has to start again, doesn't it?
T: Look at the pattern being followed here. What pattern does it follow? What should come next? This is the last term I've put up to now (indicating the fifth term). I say this because I don't want you to think that they distinct.
T: This is the same, one term, two terms, three, four, five, this is the fifth term, OK



S: You need another line.
T: OK, but tell me how.
S: The top.
T: Can you be more specific, because they have a…, can you explain in words?
T: It's vertical (pointing to the one drawn on the board), isn't it?
S: Vertical.
S: Horizontal.
T: Ah, horizontal, like this □∟ or like this □┌
S: The top.
T: OK, and the next one?
S: Going down.

(E.3.)

In order to obtain evidence of the subdomains involved in the selection of this example, we asked the questions below.

R: Why was the first sequence you did non-numerical?
T: Because it was something they had already worked on. They've been doing lots of sequences since they were small, in kindergarten and primary, and they would find out what the key was, and that it was something very easy. So I wanted to show that underlying everything there is a key that you need to find out. Whether it's easy or not is something else. So, as I say, I started from something that was meaningful for them, something they were familiar with, something they knew how to handle, something that would remind them that they had seen these things before. I wanted to make a link to something they had experience of, without getting caught up in the word 'sequences'.

(Q.3.)

We also took into consideration the question we asked with regard to Example 2, given that the teacher again uses content that the students have previously studied.

The teacher's answers enabled us to find evidence of the different subdomains mobilised in choosing this example. As seen above, the belief of knowledge construction espoused by the teacher draws on particular subdomains. He is clearly aware that the students have studied topics connected to sequences at lower levels in the curriculum (KMLS: sequencing of topics), and knows that when students work with non-numeric sequences, they have an easier time seeing patterns (KFLM: modes of interacting with mathematical content). Furthermore, the teacher knows that the rule behind the construction of a pattern serves as a support to find the general term of a sequence (KSM: auxiliary connections). There is also evidence of KMT and KFLM being brought into play. The teacher is aware of that the example he has selected is appropriate for the students in terms of finding the underlying pattern (KMT: strategies, techniques, tasks and examples) and the fact of he relays on the topics the students already know, will be meaningful to them (KFLM: emotional aspects).

In the choice of this active example, we can see that some interconnections seem to be conditioned by the teacher's belief of the construction of knowledge. The idea he stated previously that new knowledge is constructed on the foundations of existing knowledge, and his awareness of how previously studied topics are more meaningful for the students (KFLM) influence the choice of an example which is based on previous knowledge, causing the teacher to mobilise knowledge corresponding to their KMLS and their KSM during the selection of the example. He selects an example based on patterns, a topic the students have studied previously, and which is connected to sequences (KMLS). The teacher knows that the formation of patterns serves as a support for the construction of the general term of a sequence (KSM). This confluence thus provides evidence of an interconnection between the teacher's KMLS and his KSM. The example is a non-numerical sequence, as the students have interacted with this kind of sequence when they studied patterns (KFLM), and we can thus affirm an interconnection between the teacher's KFLM and the two subdomains mentioned above, KMLS and KSM. The interconnection between these three subdomains favours the selection of an example based on students' previous knowledge. It is for this reason the teacher considers it an appropriate example for the students to discover the pattern underlying the sequence (KMT), thus contributing to his KMT.

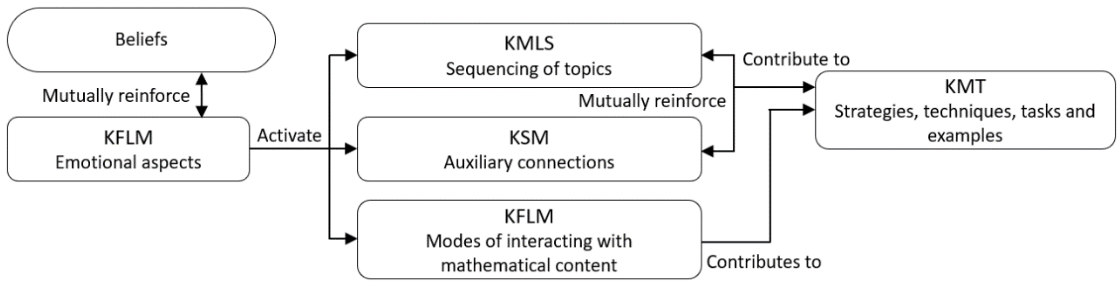


Figure 5: Interconnections between subdomains during the choice of example 3.

In episode J.3, there is evidence that the teacher mobilises knowledge from various subdomains in deploying this example. He shows he knows how to construct the representation of the chosen non-numerical sequence (KoT: procedures and registers of representation), and also that he knows the minimum number of terms necessary for the students to visualise the pattern underlying the sequence (KoT: procedures). He also identifies the obstacle generated by the fifth term in the sequence (KMT: strategies, techniques, tasks and examples), and the error which some of the students are making when they interpret this term (KFLM: strengths and weaknesses). Hence, during the deployment of this active example, there is evidence of an interconnection between the teacher's KMT and his KFLM. In other words, this confluence of knowledge enables the sequence – constructed by the teacher as a result of his KoT – to be tuned to the required degree for the students. The teacher's knowledge of the difficulty generated by the fifth term in the sequence (KMT), and of the error which some of the students were making in interpreting this term (KFLM), enables him to overcome this obstacle, hence favouring the efficient use of the sequence (constructed according to various categories of his KoT).

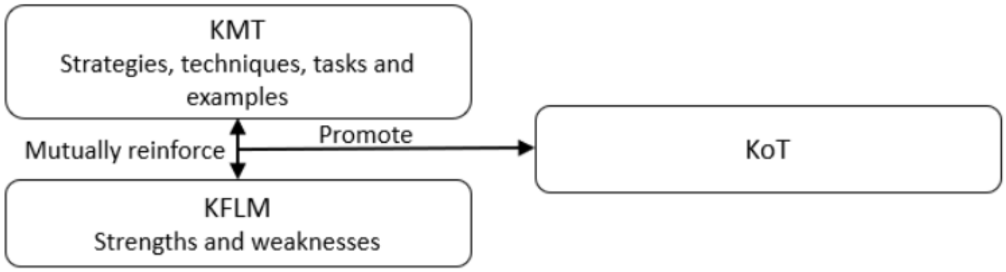


Figure 6: Interconnections between subdomains during the use of example 3.

In this episode (Example 4) the teacher presents the first numerical sequence and uses this to exemplify how the general term of a sequence can be represented mathematically. He presents the students with the sequence 2, 4, 6, 8, ..., and asks them to determine a mathematical expression, using an algebraic representation, which allows them to calculate these values. It can be classified as an active example of a concept which emphasises a representation.

2, 4, 6, 8….

T: *what's the key? What mathematical expression allows you to calculate this?*
S: *Add – add two.*
T: *But what's the mathematical expression?*
S: *Two per number minus one – multiply by two*
T: *Multiply by two to get the term you want, right? OK, that's the key. You've said it in your language, but now we are going to say it mathematically. The key to this series, or to this chest, is multiply by two the term that you want, right? What do we call the term that you want?* a_n*, no?*

$$a_n = \textcircled{2n}$$

(E. 4)

With the aim of gathering evidence of the subdomains mobilised by the teacher during the selection of this example, we asked the following questions.

R: *Why did you choose this sequence* [2, 4, 6, 8] *as the first numerical sequence to present?*
T: *Numerically, it's simple. A simple sequence to begin with. Also, it's one they know, and have known for some time. The same logic as before* [non-numerical]. *It's something that's familiar, something they've seen before and isn't unfamiliar. And*

then, if you give them a more complex sequence first, they probably won't even see that there is a sequence, they just won't see it. So, keeping things easy, mentally, for them, so they can see a relationship between one term and another. It's the simplicity of the example, really.

R: Why do you use metaphors when you introduce new concepts?

T: Because metaphors stick in the memory and are easier to remember and to compare. I think that from a pedagogical point of view they have more graphical or mental elements because when you've got a metaphor you make a mental image, and that helps you a lot to remember and to build concepts properly.

(Q. 4)

The teacher's answers enable us to visualise that, at the moment of choosing the example, he mobilises knowledge corresponding to KSM and to all the subdomains of his PCK. He knows that the students' existing knowledge of the multiplication tables can be linked to the topic of sequences (KMLS: sequencing of topics), showing a relationship between the multiplication tables, specifically between the general expression of the table of 2, 2×n, and the general term of the sequence 2,4,6,8,… (KSM: auxiliary connections). It can also be seen that he recognises the potential of the chosen example in view of its simplicity (KMT: strategies, techniques, tasks and examples), and that the students will not have any problems with this sequence when it comes to finding the general term (KFLM: strengths and weaknesses). In this instance, it can be seen that all the knowledge mobilised contributes, one way or another, to the choice of an active example which promotes the participation of the students, providing evidence of at least three interconnections. In the first place, the teacher's knowledge of multiplication as a topic appropriate for sequences (KMLS) is reinforced by his knowledge of the two-times table as a support for the construction of the general term of the sequence of even numbers (KSM). It can also be seen that this first interconnection favours the KFLM mobilised by the teacher, given that, as an example drawing on previously covered content, it strengthens the idea that the students will not have any problems in determining the general term of the sequence (KFLM). Finally, we can see an interconnection between the three subdomains mentioned above and KMT, in that awareness that the sequence draws on previous knowledge and that the students will not have any problems determining the general term, together reinforce the idea that this example is suitable for the topic being studied due to its simplicity (KMT).

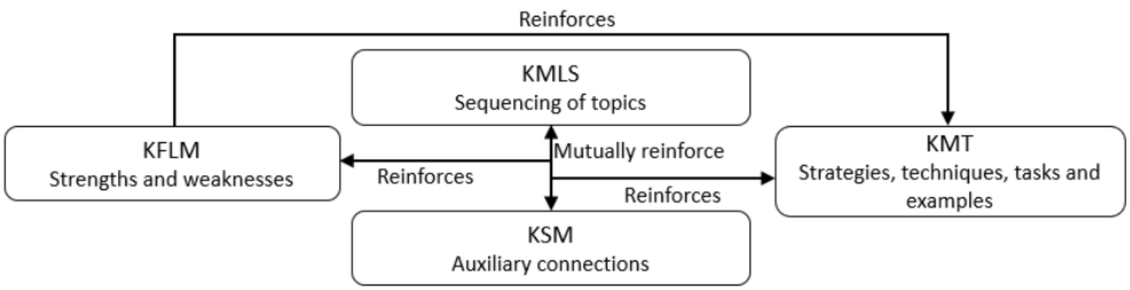


Figure 7: Interconnections between subdomains during the choice of example 4.

In terms of using the example, the episode illustrates how the teacher mobilises knowledge corresponding to KoT and KMT. He knows what the first terms of the sequence are (KoT: procedures) and know how to represent the general term algebraically (KoT: register of representations). He is also able to identify the correct answer from all those offered by the students (KoT: procedures). In the interview, we found evidence of his KMT, namely that reference to the general term as “the key” will aid comprehension of the concept of the general term (KMT: strategies, techniques, tasks and examples). In this active example, there is evidence of a clear relationship between the teacher’s KoT and his KMT: his knowledge of the algebraic expression of the general term (KoT) enables him to link this concept to that of a key through the use of a metaphor, a strategy he uses to ease the students’ comprehension (KMT).

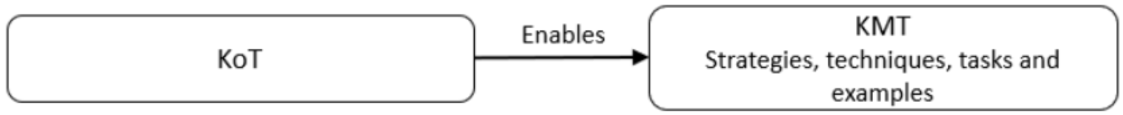


Figure 8: Interconnections between subdomains during the use of example 4.

Discussion and Conclusions

Analysing the MTSK mobilised by the teacher during the processes of selection and implementation of examples, we see that, at both stages, he mobilises and interconnects areas of knowledge pertaining to the two MTSK domains. This is consistent with the perspective of Liñán et al. (2016), who maintain that the action of exemplifying offers a rich scenario for studying MTSK.

Both stages, that of selection and that of deployment, provide an opportunity to delve into mathematics teachers' knowledge. Nevertheless, given the instances we observed, it is our belief that the most favourable point at which to study MTSK in depth is during the selection of the example, as it generates a greater number of interconnections between the different areas of knowledge mobilised by the teacher. In this regard we concur with Zakaryan and Ribeiro (2016) and Zakaryan et al. (2018), who have argued for the need for increased research into the interconnections between the different MTSK subdomains so as to enrich our understanding of teachers' knowledge.

In examples 2 and 3, it can be seen how the teacher's belief regarding the construction of knowledge determines the subdomains they mobilise during the choice of example, and the interconnections established between them. In both examples, the belief of knowledge construction displayed by the teacher favours the interconnection KMLS-KSM.

Although all examples constitute an appropriate space for exploring teacher knowledge (Liñán et al., 2016), we have noted that it is the active examples – those which the teacher uses with the aim of encouraging student participation – which seem to represent a favourable scenario for studying PCK. In the active examples observed in this study, the teacher mobilised and interconnected all three types of knowledge constituting this domain.

The active examples also provided us with evidence of how the different subdomains (KoT, KSM, KFLM and KMLS) underpin the teacher's KMT, an observation consistent with Zakaryan and Ribeiro (2016). During the selection of these examples, we observed that KMT, KFLM and KMLS underpinned the teacher's knowledge of the mathematical potential of the activity (KMT), while during the use of the example we observed the connection between his KoT and KMT.

During the selection of both active and passive examples we observed interconnections of the type KMLS-KSM. In the passive examples there was evidence of the relationship described by Carrillo et al. (2014), namely, interconnections KMLS-KSM, between the categories 'sequencing of topics' (KMLS) and 'connections based on increased complexity or simplification' (KSM). In the case of the active examples, the interconnection was between the categories of 'sequencing of topics' (KMLS) and 'auxiliary connections' (KSM).

There was evidence of the interconnection KFLM-KMT, described previously in Zakaryan and Ribeiro (2016), Carrillo et al. (2014) and Zakaryan et al. (2018), in the choice of active and passive examples. In the case of the choice of the passive example, these subdomains mutually reinforced each other, while in the active examples it was

KFLM which activated KMT, it is evident how the teacher' knowledge about the characteristics of learning mathematics contributes to his knowledge about the mathematical potentialities that the activities carried out in class can have.

The same situation could be seen in these cases as that described by Zakaryan et al. (2018). Although the reinforcement between these subdomains was mutual, KFLM had a greater impact on KMT.

Although the results of this analysis enable us to identify the interconnections mobilised by the teacher in selecting and deploying different types of examples, our study does not allow us to establish general conclusions, and it is for this reason we consider it necessary to continue exploring teacher knowledge in this kind of study. As stated above, various authors (Carrillo et al., 2014; Zakaryan et al., 2018; Zakaryan and Ribeiro, 2016) have highlighted the importance of studying the interconnections generated between the different subdomains in order to advance our knowledge of MTSK, just as others (Chick & Harris, 2007; Vinner, 2011; Figueiredo & Contreras, 2013) the importance of examples in the teaching of mathematics.

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