# AC Power and Energy Measurements based on Physical Definitions

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***Abstract*—The article presents the AC power measurements based on physical definitions as introduced by the draft German standard DIN 40110 in 1970 and its practical application as a power signal processor. The power processor is an electronic device allowing separation of the measured power signal (obtained from the multiplier circuit of voltages and currents) in two components: *P*+ and *P*- defined as the input power $P_v$ and return power $P_r$. The separation is obtained by shifting the phase angle between the current and voltage. The value of return power is negative. Such separation is natural because it is based on physical phenomena of delivered, useful energy and harmful return energy flows. Return power is considered harmful because it causes heating of cables and sources of AC power (as transformers). Presented concepts and meters can be useful for accurate billing purposes for delivered electrical energy as well as for dynamic compensation of nonlinear loads to reduce the return power. Experiments realized using the power meter WL-1 synchro (operating according to physical definitions) confirms high percentage of return power for nonlinear loads, as AC powered LED light sources where the amount of return power can reach 70% of delivered power.**

***Keywords—active power; useful power; return power; delivered power; power meter; power processor; distorted voltage; distorted current; unsymmetrical loads; nonlinear loads.***

## I. INTRODUCTION

Until recently, the electrical energy was provided from large synchronous generators, delivering almost perfectly sinusoidal voltage; power system loads were mostly linear, time-invariant and consequently, voltage distortion was almost inexistent. The concept of the active power was developed when electrical power systems were different from the situation we have now, with large proportion of nonlinear, variable, unsymmetrical loads, distributed generation and smart grids. This difference will be even greater in the future, changing power systems dramatically [8]. Therefore, it is necessary to revisit the well-established concepts of active power and power factors, as becomes obvious that active power is no longer the synonym of useful power. The energy flowing in form of voltage and current harmonics or negative-sequence components can disturb or overheat electric and electronic equipment, motors and transformers [8]. Consequently, the active power in systems with non-sinusoidal and asymmetrical voltages and currents has components that are not useful but are harmful. The proper metering of delivered useful power becomes of utmost importance when the cost of energy increases and rationalization of energy accounts in systems with non-sinusoidal and asymmetrical voltages and currents is advised by scholars [8].

The idea and physical definitions of delivered and return power has its roots in former German standard DIN40110, proposed in 1970 [10] and amended in 1975. Physically, in AC power circuit, a part of energy called $E_v$ (over one period) flows to receiver and another part called $E_r$ flows to the source, as defined in [8, 10]. The difference shown in (2), taken as an average value is equal to the active power and energy and these values are defined in current standards. The delivered power $P_d$ is equal to the sum of the powers $P_v$ and $P_r$. Both powers should be added without a sign. In the equations below $P$ is the average value of the power and powers are unsigned, because the sign of $P$ shows the energy propagation direction only, thus:

$$P_d = P_v + P_r \tag{1}$$

where: $P_v = \frac{1}{T}\int_0^T u(t)\cdot i(t)dt$ for $u(t)\cdot i(t) > 0$

and: $P_r = \frac{1}{T}\int_0^T u(t)\cdot i(t)dt$ for $u(t)\cdot i(t) < 0$

as opposed to active power $P$ (or average power), which determines the difference between the power absorbed by the device and power returned to the source and it is not always equal to the total supplied power:

$$P = P_v - P_r \tag{2}$$

where the return power $P_r$ is unsigned as in [4], the minus sign at $P_r$ showing the direction and the average value or difference as in (2) gives the active power consumed by the receiver. The concept of the triangle of powers with reactive power, which does not have any physical meaning, was introduced solely with the purpose of closing the balance of powers to the effective value of the product of the voltage and current. Attempts to compensate the reactive power shown in [9] will always be an approximation of the real conditions. In [10] the concept of power components with a negative sign is presented, which reduce the value of the first component of positive power unfolded in series. Formula (2) is equivalent to formula (7) from [8] with the reservation that authors in [8] keep the concept of negative power concept - difficult to analyze- and consequently can lead to errors.

Substituting (2) into (1), we get:

$$P_d = P + 2P_r \text{ and } P = P_d - 2P_r \tag{3}$$

where $2P_r$ is called the power of mutual exchange [2]. Power $P$ equals $P_d$ only when $P_r$ equals zero. The equation (3) shows why energy meters measuring active power $P$ (in accordance to EN 50470-1:2006) slow down and count lower number of pulses or blade rotations when a harmful return power (or power of mutual exchange $2P_r$ exists). Delivered power $P_d$ can be also defined by a simple formula [1]:

$$P_d = \frac{1}{T}\int_0^T |u(t) \cdot i(t)| dt \tag{4}$$

## II. POWER PROCESSOR

Analog power processor signal flow and operations

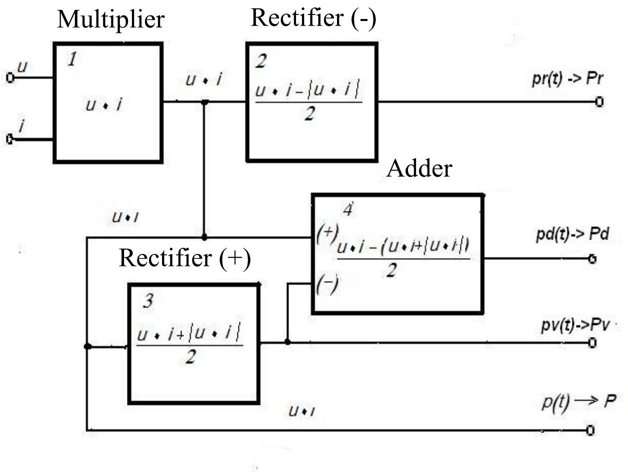

Fig. 1. Block diagram of the power processor.

are shown in Fig 1. A working model as in Fig. 2 was designed and realized. Detailed description can be found in [9]. Many other versions of power meters were built, for various purposes, another example is shown in Fig. 3.

Fig. 2. WL-1 synchronous power processor.

Fig. 3. Power meters (voltage and current measurement terminals on the back of the instrument).

## III. RESEARCH AND DISCUSSION

Power $P$, as defined by equation (2) and the definition of the integral value of the average power or as the active power that occurs as a signal on one of the outputs of the power processor can be described by

$$P = \frac{1}{T}\int_0^T |u(t) \cdot i(t)| dt = U \cdot I \cdot cos\varphi = P_v - P_r \tag{5}$$

The direction of the flow of input power $P_v$ is from the source to receiver and direction of return power $P_r$ is opposite, occurring 100 times per second in electric grid as in fig.1. The average value is called $P$ - active power as in (5). Large $P_r$ is harmful for the electric grid.

In the first experiment the proper operation of the power processor was checked, as presented in [9]. The tests were performed in the range of $\varphi = (-100°, 100°)$ as described in detail in [14]. Moreover, the experiment by the heat [14] showed, that there exists physical delivered power $P_d$, which is higher than active power $P$, lower than apparent power and thus, there exists physically the return power $P_r$ [14], defined in (1).

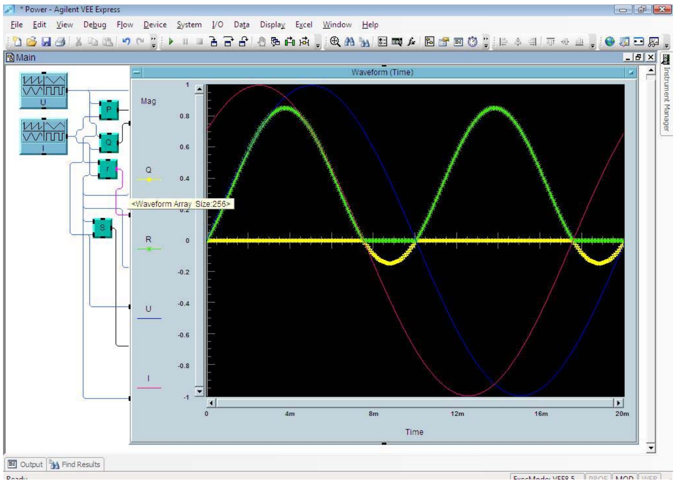

Fig. 4. Simulation of the signal power and its division into components: $P_v$ - green, $P_r$ - yellow.

The correctness of the power processor design has been confirmed in a series of tests using the arbitrary waveform generator and 4-channel digital oscilloscope. (Fig.5).

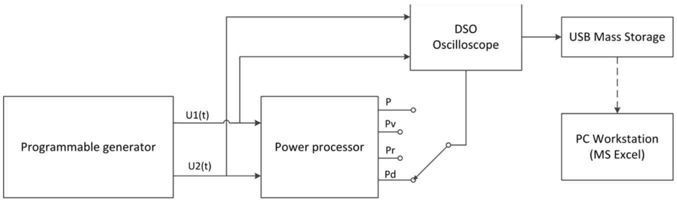

Fig. 5. Setup to nanlyse power processor accuracy and operation.

Tests were carried out with signals up to a frequency of 10 kHz and for different shapes of waveforms (sinusoidal, triangular, with the 3$^{rd}$ harmonic (positive and negative) of current as well as zero-current and peak-current triggered triac-switching). The characteristics of errors showed the need of additional trimming to obtain the accuracy better than 0.2%.

Selected results are shown below, measured at the frequency 50 Hz of the fundamental waveform.

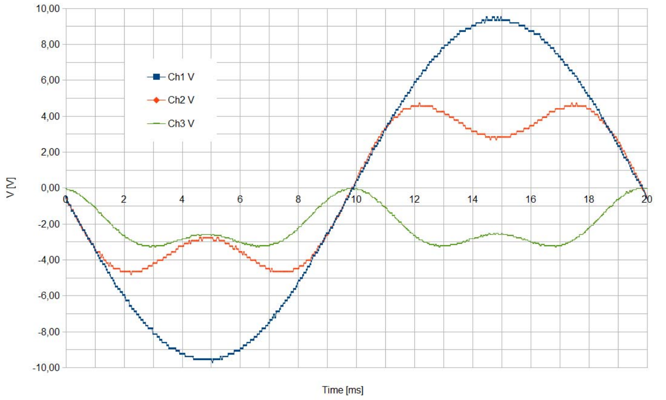

Fig. 6. Influence of 3$^{rd}$ (negative) harmonic of current on voltage waveform. The voltage is proportional to the input power $P_v$. Ch1 - voltage, Ch2 - current, Ch3 - input power $P_v$.

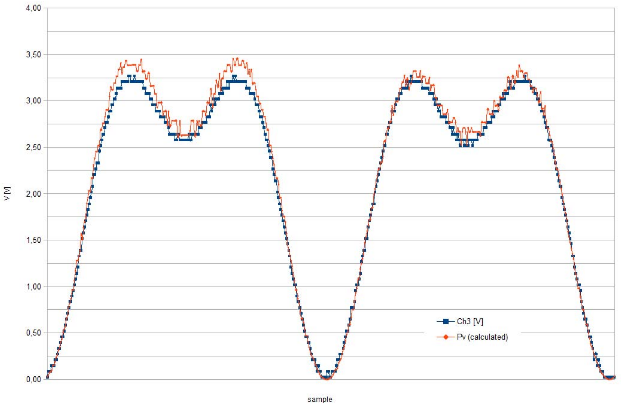

Fig. 7. Comparison of the measured voltage proportional to the input power $P_v$.measured (blue) to $P_v$. calculated (red), according to (1).

Figures 6 and 7 present the analysis of waveforms, static and dynamic errors of the system shown in Fig.5. Errors are due also to quantization error of 8-bit resolution the digital oscilloscope, as well as nonlinear errors, as shown in Fig. 8.

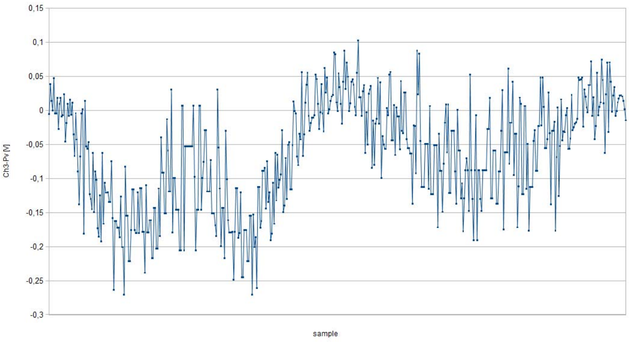

Fig. 8. Difference between measured and calculated values of input power , as in Fig 7.

Next Figures present investigations related to triac-switched waveform (in maximum current) and triangle waveforms.

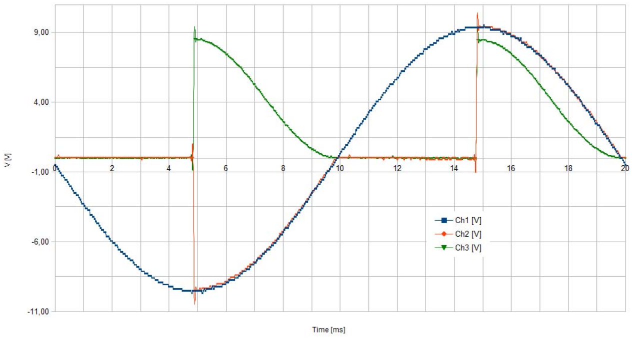

Fig. 9. Waveforms recorded for triac-switched circuit:. Ch1 (blue) voltage, Ch2 (red) current, Ch3 (green) active power $P$.

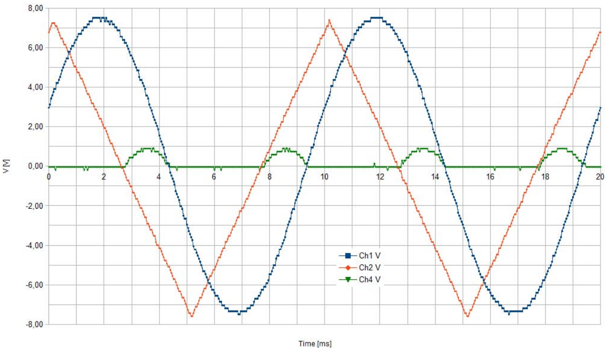

Fig. 10. Return power $P_v$. for triangle waveform of current (Ch1 (blue) – volatge, Ch2 (red) – current, Ch3 (green) – return power $P_v$.

The generator output amplitude is 9 V, thus the maximum output voltage of the multiplier equals 8.1 V. The distorted current waveform as in Fig. 6 caused decrease of voltage amplitude related to the power signal to 3.5 V. The waveform proportional to input power is presented in Fig. 3. Figure 5 shows the difference between the output signal and the corresponding values calculated from the parameters of input signals. The quantization noise due to processing in digital oscilloscope is visible. An additive error was also detected that can be compensated by the calibration of the system.

Experiments realized using the power meter WL-1 synchro (Fig. 2) confirms high percentage of return power for common nonlinear loads, as AC powered LED light sources where the amount of return power can reach 70% of delivered power.

Measurements of power processor showed that the system works properly, the errors of the power signal waveform are small and mainly result from inaccurate calibration. It was shown that the presence of harmonics in the current does not affect the formation of the return power. That one arises as a result of phase shift between voltage and current especially for the first harmonic.

Watt meters and electrical standards assume measurements of average power and energy. Experiments presented in [14] showed that the definition of active power does not describe correctly the power losses. It describes only the active power consumed by the device. Reactive power concept is useful for compensation components calculation, only. The concept of reactive power compensating balance to apparent power $S$ does not have any physical meaning.

Energy meter readings are valid only when the phase shift between voltage and current is equal to zero. This means that typical meters cannot be applied universally for various kinds of loads. There exists delivered power, as shown in the experiment in [14] and it can be measured using the power processor described in this paper.

From the above measurement it was concluded that power processor delivers correct and highly accurate results for various waveforms of voltages and currents.

## IV. PROPOSED EXTENSION OF EN50470-1:2006 STANDARD

The measurements confirmed the presence of the return power and input power. This leads to proposition of extending the standard, which would include definitions and concepts related to the physical description of the power and energy flows in the measurement node. It would allow to measure and calculate the actual real values of the power and energy. It is not competitive to the definitions using the power triangle. Proposed approach is a description of the power and energy consistent with the physics of the phenomenon and thermal effects which accompany the flow of energy in the measurement node.

Proposed extensions are as follows:

3.13a Delivered energy meter: instrument designed to measure the total energy delivered to the load by a separate integration of input and return power as functions of time, with the summation of the integrated values.

3.1.6a Delivered power $P_d$ of stationary or non-stationary, periodic or non-periodic, distorted or non-distorted waveform is the average value of input and return power throughout the integration time $T$:

$$P_v = \frac{1}{T}\int_0^T p_v(t)dt + \frac{1}{T}\int_0^T |p_r(t)|dt = \frac{1}{T}\int_0^T |p(t)|dt \quad (6)$$

where: $p_v(t)$- instantaneous input power for $u(t) \cdot i(t) > 0$

$p_r(t)$- instantaneous return power for $u(t) \cdot i(t) < 0$

NOTE 1 The delivered power is the sum of the positive instantaneous power and absolute value of the negative instantaneous power.

NOTE 2 The unit of delivered power is Watt.

3.1.7a Return power and return energy of stationary or non-stationary, periodic or non-periodic, distorted or non-distorted waveform is the average integral value of negative product of voltage and current throughout the integration time $T$, where the product is taken without a sign:

$$P_r = \frac{1}{T}\int_0^T |p_r(t)|dt \quad (7)$$

where $p_r(t) = u(t) \cdot i(t) < 0$

NOTE 3 The unit of return power is Watt.

3.1.7b Input power of stationary or non-stationary, periodic or non-periodic, distorted or non-distorted waveform is the average value of positive product of voltage and current throughout the integration time $T$:

$$P_v = \frac{1}{T}\int_0^T |p_v(t)|dt \quad (8)$$

where $p_v(t) = u(t) \cdot i(t) > 0$

NOTE 4 . The unit of input power is Watt.

3.1.7c Meter of mutual exchange energy: An instrument designed to measure the mutual exchange energy as the result of integration of the power $2P_r$ as a function of time according to the equation:

$$E_e = 2\int_{t_1}^{t_2} |p_r(t)|dt \quad (9)$$

where: $P = P_v - P_r$, $P_d = P_v + P_r$, $P_d = P + 2P_r$ and $P$ is active power.

## V. CONCLUSIONS

Power processor was designed at the Wroclaw University of Technology and its structure and electronic circuitry were developed at the Institute of Computer Systems for Automation and Measurements in order to cope with inaccurate (lowered) readings of energy meters (especially inductive but also electronic meters, where an update of the software can be done in order to obtain the values of input and return values). The return power $P_r$ and a value of the compensation $2P_r$ (mutual exchange power) are suitable for circuits with modern, nonlinear types of receivers. Power processor allows to separate the instantaneous, positive and negative values of power signal, enabling separate measurement of physical components [11, 15].

Signal distortion (especially of current) does not have a significant impact on the accuracy of the system due to the wide bandwidth (up to 1 MHz) of internal signal processing circuits. Currently, further studies of power processor with waveform shapes and frequencies of the signal are carried out. These should confirm that the simple, integral formulas with separation of power signals are sufficient to describe most anomalies in power networks.

Reactive power for large customers is commonly measured but its values are too large comparing to the actual return power, which has a physical meaning and can be observed in the measurement system by multiplying current and voltage signals [14].

An extension for the standard EN50470-1:2006 was proposed which include definitions and concepts related to the physical description of the power and energy flows in the measurement node

The performance of proposed power processor was tested and the correct operation and accuracy was confirmed. Power processor can be used together with a simple digital meter or as an extension module to control the return power, or applied for correction of readings of conventional energy meters.

## ACKNOWLEDGMENT

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## REFERENCES

[1] L. S. Czarnecki, “Powers in electrical circuits with non sinusoidal voltages and currents”, Warsaw University, 2005

[2] A. F. Krogieris, K. K. Raszczewicz, Trejmanis, J. K. Szinka, “Moszcznost pieriemiennogo toka”, Academy of science, Riga 1993 (in Russian)

[3] J. Kolanko, “Synchronous counting in power and true voltage measurement”, IEEE Transactions onTest and Measurement, no 3, 1993

[4] J. Kolanko, Patent No. 22012/2008, “System for measuring of electrical power, power processor”

[5] G. Kossobudzki, Z. Nawrocki, J. Nowak, “Measure of electric reactive power”, Metrology and Measurement Systems, Vol XI, No. 2/2005

[6] G. Kossobudzki, Z. Nawrocki, J. Nowak, “Parameters of electrical power”, Electrical Review No. 6/2007.

[7] L. Czarnecki, “Power and compensation in systems with non-sinusoidal current and voltage”, Electricity, current status and problems of its development. Conference materials INFOTECH, June 2010.

[8] L. Czarnecki, “Working, reflected and detrimental active powers”, IET Generation, Transmission & Distribution, vol. 6 iss. 3, pp. 233-239, March 2012

[9] J. Kolanko, M. Wójcik, “Power processor according to DIN 40100”, MKM 2011

[10] DIN 40110: 1976

[11] J. Kolanko, “Reactive power measurement method”, Patent No. 213060/2008

[12] J. Dudzik, J. Kolanko, P. Modzel, Z. Wierzbicki, “Method of power and eergy measurement in the power engineering systems”, Patent application No. 201/2012

[13] J. Dudzik, J. Kolanko, P. Modzel, Z. Wierzbicki, “The power processor application in energy measurement of alternating current”. Basic Problems of Metrology, 2012.

[14] J. Kolanko, P. Modzel, Z. Wierzbicki, J. Dudzik, P. Musz, “The new power processor and its application for AC power and energy measurement”, Acta IMEKO 2014, TC-4