Application of the Prony Method for Compensation of Errors in Distance Relays

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***Abstract*—In this paper we propose an application of the Prony method as a filtering technique for distance relays. First, an overview of the Prony method is presented, then the distance relay model and the impact of non-filtered frequency components such as interharmonics and subharmonics by application of typical digital filters in the operation of the distance relay is shown. We apply a solution using Prony method as a filtering technique applied to distance relay algorithms. Finally, an analysis of the proposed algorithm using simulated signals is evaluated to validate the proposed distance relay algorithm in reach error and operation time of the relay, a distorted characteristic of the Mho distance relay is shown in the impedance plane.**

Keywords-distance relay; Prony method; interharmonics; subharmonics

I. Introduction

The quality of the voltage and current signals are now a significant problem for the electric power system which affects the operation of the electric power grid, power consumers and manufacturers of electric/electronic equipment. The main problem in the quality of current and voltage signals is caused by the inclusion of equipment that its operation is based on power electronics, FACTS devices, series compensation and wind power plants in the power grid, these are the reason why certain additional requirements are necessary for a good quality of the energy in the electric power system.

The voltage and current signals are expected to be purely sinusoidal with an assigned amplitude and frequency. The modern power converters generate a wide spectral band of frequency components which compromise the quality of the energy delivered, and by consequence there's an increase in the power losses and the reliability of the electric power system is reduced [1]. In some cases, the large scale power converters systems generate in addition to the typical operational harmonics of an ideal power converter, non-characteristic frequency components as interharmonics and subharmonics will greatly deteriorate the power quality of the voltage and current signals [1-6].

The estimation of these frequency components is very important in the power system protection schemes, that is why the characterization of this frequency components allows to compensate measurement errors in the electric protection system, such as distance relay. The frequency components as interharmonics and subharmonics could not be rejected by the distance relay digital filters (Cosine or Fourier filter) [8]. The main purpose of the conventional digital filters in distance relays is to estimate the fundamental frequency phasor of the electric input signals required by the relay, but when frequency components as interharmonics or subharmonics exist in the voltage and current signals, the conventional digital filters as Cosine or Fourier will cause an error in the fundamental frequency phasors estimate, and by consequence an error in the estimate of apparent impedance, this will compromise the performance of the distance relay causing underreach or overreach (fault detection problems).

The Prony method seems to be a good alternative to get a correct apparent impedance measurement by estimating the fundamental frequency phasor of voltage and current [10-14]. This is the main focus of the analysis presented in this paper.

II. The Prony Method

Prony analysis (Prony's method) was developed by Gaspard Riche de Prony in 1795. Similar to the Fourier transform, Prony's method extracts desired information from a uniformly sampled signal and builds a series of damped complex exponentials or sinusoids. This allows for the estimation of frequency, amplitude, phase and damping components of a signal, by solving a set of linear equations for the coefficients of the recurrence equation that the signals satisfy.

The Prony's method has been studied as a power quality analysis tool [11] allowing to obtain better results in comparison with FFT, it has been also used in power system stability studies [15]. The algorithm and its practical implementation is shown in [10, 14].

III. Distance Relay Model

The distance relay operation is based on the phase comparison of two input signals, operation and polarization, to determine the trip condition.

The distance relay models have a phase comparator that responds to the phase angle displacement between input signals [7]. The input signals of phase comparator are obtained using the electric signals measured by the instrument transformers and design constants. The model of the distance relay is shown in (1).

$$S_1 = k_1\angle\alpha_1 \cdot V_r\angle 0^o + Z_{R1}\angle\theta_1 \cdot I_r\angle -\varphi_r \qquad (1)$$
$$S_2 = k_2\angle\alpha_2 \cdot V_r\angle 0^o + Z_{R2}\angle\theta_2 \cdot I_r\angle -\varphi_r$$

where S_1 and S_2 are the input signals to establish the trip signal; $\bar{k}_1$ and $\bar{k}_2$ are the constants of the design; $\bar{Z}_{R1}$ is the impedance of the protected transmission line; and $\bar{Z}_{R2}$ is the impedance multiplied by the current, resulting in a polarization voltage; $\bar{I}_r$ and $\bar{V}_r$ are the electric input signals which are estimated by the phasor estimation technique (Fourier filter or Cosine filter) to obtain the fundamental frequency phasor [8].

The phase relays are required for the detection of phase to phase faults, an important aspect in the distance relay design is that the correct values of $\bar{I}_r$ and $\bar{V}_r$ has to be selected, in Table I the electric input signals that correspond to phase distance relay units is presented:

TABLE I
ELECTRIC INPUT SIGNALS FOR DISTANCE RELAY UNIT

Unit	Voltage (V_r)	Current (I_r)
	Phase	
(AB)	V_{an} - V_{bn}	I_a - I_b
(BC)	V_{bn} - V_{cn}	I_b - I_c
(CA)	V_{cn} - V_{an}	I_c - I_a

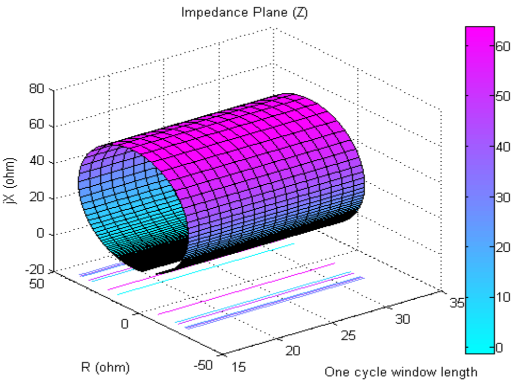

Figure 1. Mho operation characteristic (Fundamental frequency).

In Fig. 1 the Mho operation characteristic is presented in a tridimensional space of the impedance plane through time, the time is represented by the one cycle window length displacement. The estimated fundamental phasors of voltage and current are used for the relay (phase relay unit). The input signals V_r and I_r are used (see Table I) to form the operation characteristic and the phase comparator scheme.

Is necessary to incorporate two stages of filtering to eliminate the unwanted frequency components as noise, harmonics and DC component, that are considered as a source of error that could affect the selectivity of the relay (see Fig. 2).

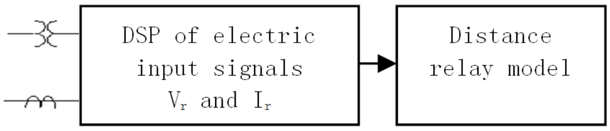

Figure 2. Distance relay structure for input signals processing.

There are two filter stages: analog and digital, intended to reduce the operation time in fault detection. Generally, the analog filter used is a Butterworth filter of 2nd or 4th order with a cut-off frequency of 360 Hz (see Fig. 3), this filter is preferred because it has flat response in passband and monotonically decreasing response in stopband [9].

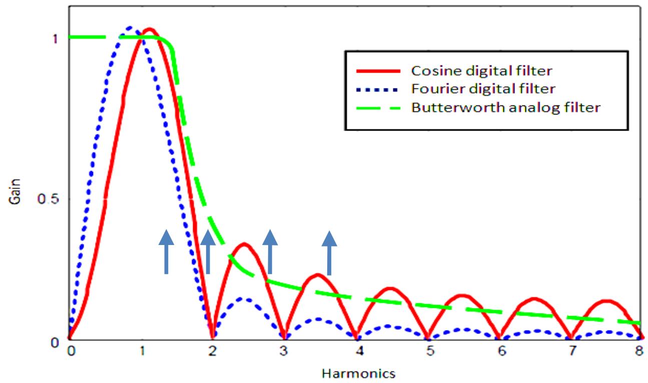

Figure 3. Analog and digital filter frequency response used in distance relays.

The distance relay algorithms used for the fundamental frequency component estimation of voltage and current phasors use the Fourier or Cosine filter (see Fig. 3). The distance relay model is based on the fundamental frequency phasor, but due to the digital filters used in distance relays errors in the fundamental frequency phasor estimate can be present due to the existence of interharmonics and/or subharmonics in the voltage and current signals during the fault period. When this frequency components are present, the digital filter generates an error in the fundamental frequency phasors estimates of voltage and current. The distance relay model needs an ezact estimate of the fundamental frequency phasors of voltage and current during the fault period, so a correct estimate of the apparent impedance could be calculated by the relay.

Due to the fact that distance relay algorithms that use Fourier and Cosine filters generate an error in the fundamental frequency phasors estimation of voltage and current signals, a characterization of the problem using the distance relay model in (1) its represented in (2) as follows:

$$S_1 = k_1\angle\alpha_1\Big[V_{h1}e^{j\theta_{h1}} + \overbrace{\sum V_i e^{j\theta_i} + \sum V_s e^{j\theta_s}}^{\text{Error in estimation}}\Big] + Z_{R1}\angle\theta_1\Big[I_{h1}e^{j\theta_{h1}} + \overbrace{\sum I_i e^{j\theta_i} + \sum I_s e^{j\theta_s}}^{\text{Error in estimation}}\Big] \qquad (2)$$
$$S_2 = k_2\angle\alpha_2\Big[V_{h1}e^{j\theta_{h1}} + \sum V_i e^{j\theta_i} + \sum V_s e^{j\theta_s}\Big] + Z_{R2}\angle\theta_2\Big[I_{h1}e^{j\theta_{h1}} + \sum I_i e^{j\theta_i} + \sum I_s e^{j\theta_s}\Big]$$

where $h1$=fundamental frequency; i=interharmonics, i>1; s=subharmonics, s<1. The graphical representation of the error in the estimate it's presented in Fig. 4, when interharmonics and subharmonics frequency components are present in the voltage and current signals. An error in the fundamental frequency phasors of voltage and current will cause an error in the operation characteristic, the operation characteristic being formed by the comparison of signals from (1), and at the same time requiring the voltage and current phasors, so from (2) the resultant operation characteristic can be plotted (see Fig. 4).

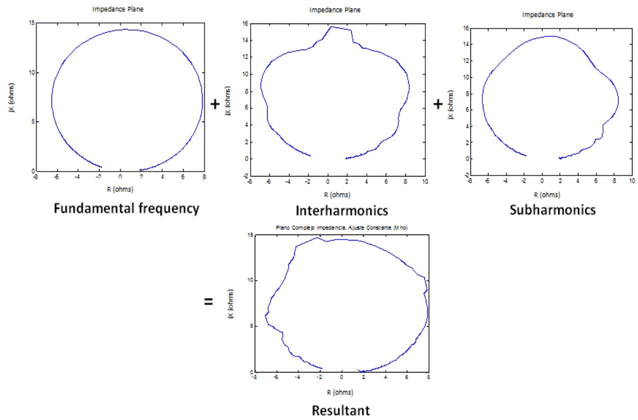

Figure. 4. Graphical representation of the error in apparent impedance estimation using digital filters.

The operation scheme for a distance relay can be described as follows:

1) Define the operational characteristic, selecting the design constants and the settings of the distance relay. This process is made off-line and is defined by the manufacturer of the relay.
2) The electric input signals measured on-line from the instrument transformers (voltage and current) are used to estimate the fundamental phasor component.
3) With the fundamental phasor component estimated, the comparison signals are formed and the trip condition is evaluated.
4) The trip condition criterion is then performed to generate a trip signal.

IV. Distance Relay Algorithm Using Prony Method as a Filtering Technique

In this section the proposed distance relay algorithm using Prony method is presented. The Prony method has been studied as a power quality analysis tool [10-14] obtaining better results in comparison with FFT, also it has been used in power system stability studies [15]. The proposed algorithm for distance relay using Prony method estimates the signal parameters (amplitude, frequency, phase angle and damping factors) using one cycle window of data, so the real fundamental frequency of the voltage and current signals can be extracted during the first fault period and that the error in the phasor estimates (see section III) could be reduced.

Consequently, if the Prony method could be implemented in distance relay algorithm it is necessary to obtain the order N of the linear prediction model (LPM). The order is obtained evaluating the mean square error (MSE) for one cycle window of data of N=1,2...Ns, where Ns is the number of samples per cycle in one window of data. The MSE for each value of N is calculated and the MSE of lower magnitude is selected for the corresponding N value, so this value of N can be assumed the optimum estimate of the model signal parameters. The proposed distance relay algorithm is shown in Fig. 5, and is intended to estimate the real signal parameters during the first period of the fault.

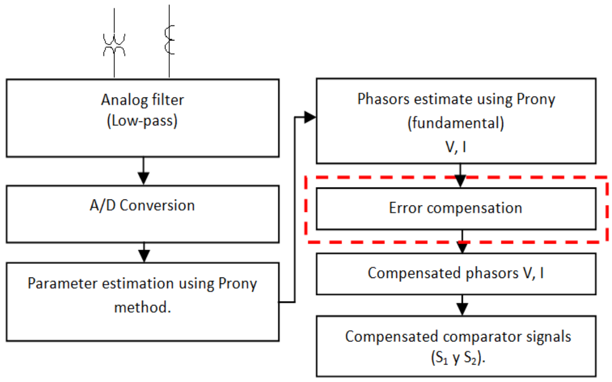

Figure. 5. Proposed distance relay algorithm using Prony method.

In Fig. 6 the Prony phasor magnitude of estimated fundamental phasors and the phase angle estimated with Prony are calculated and the compensated phasors are formed, the compensated phasors are used as the fundamental phasors in the distance relay model.

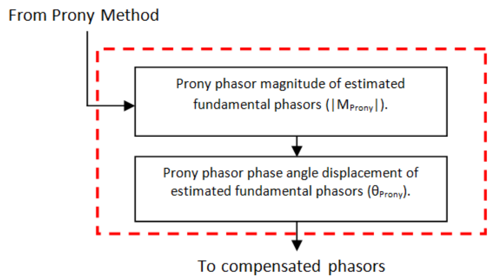

Figure 6. Proposed distance relay algorithm using Prony method.

A. Formulation

The formulation for the error compensation using the estimated phasors is presented as follows:

$$\begin{aligned} S_1 &= k_1\angle\alpha_1\left[V_{P1}e^{j\theta_{V_{P1}}}\right]+Z_{R1}\angle\theta_1\left[I_{P1}e^{j\theta_{I_{P1}}}\right] \\ S_2 &= k_2\angle\alpha_2\left[V_{P1}e^{j\theta_{V_{P1}}}\right]+Z_{R2}\angle\theta_2\left[I_{P1}e^{j\theta_{I_{P1}}}\right] \end{aligned} \quad (3)$$

where P1 is the fundamental component estimated with Prony. For the analysis of the proposed algorithm the Mho characteristic will be evaluated. The distance relay model for a Mho characteristic is:

$$\begin{aligned} S_1 &= k_1\angle\alpha_1\left[V_{P1}e^{j\theta_{V_{P1}}}\right]+Z_{R1}\angle\theta_1\left[I_{P1}e^{j\theta_{I_{P1}}}\right] \\ S_2 &= k_2\angle\alpha_2\left[V_{P1}e^{j\theta_{V_{P1}}}\right] \end{aligned} \quad (4)$$

where $Z_{R2}\angle\theta_2 = 0$ in (3) for a Mho characteristic and the distance relay model for the Mho characteristic is shown in (4).

V. ANALYSIS OF PROPOSED DISTANCE RELAY ALGORITHM

The proposed algorithm presented in section IV is validated using voltage (V_r) and current (I_r) signals with interharmonics and subharmonics during the fault period (see Fig. 7), the signals were simulated in MATLAB® using a sample frequency of 16 samples per cycle, the simulated signals emulate a fault in a transmission line, the frequencies added during fault period were 60 Hz, 48Hz and 72Hz, this frequencies (subharmonics/interharmonics) are used due to the fact that this frequencies are not filtered by the digital filters used in distance relays (Cosine and Fourier filter). It should be mentioned that here a phase distance relay unit is evaluated.

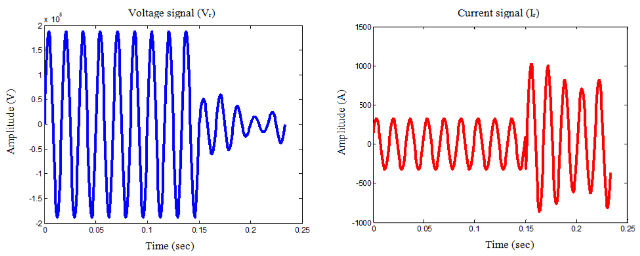

Figure. 7. Simulated signals with frequencies added during the fault period of 60 Hz, 48Hz and 72Hz. a) Voltage signal. b) Current signal.

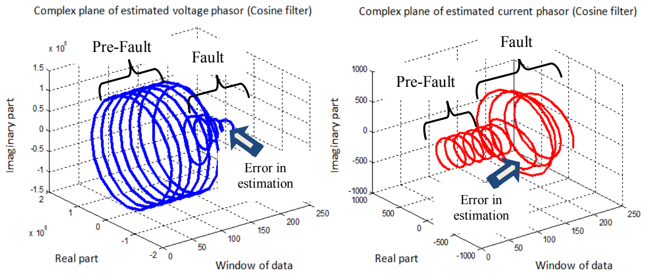

Figure. 8. Estimated fundamental frequency phasors of voltage and current using Cosine filter when interharmonics and subharmonics are present in the voltage and current signals during fault period.

The error in the estimated fundamental frequency phasors during the fault period can be observed in Fig. 8, where an alteration in the fundamental frequency phasor estimate is obtained due to the digital filters (Cosine or Fourier) used in distance relay algorithms when interharmonics or subharmonics exist in the voltage and current signals.

In Fig. 9 the Mho characteristic is presented in a tridimensional space of the impedance plane using the voltage and current signals (V_r and I_r) of Fig. 7. A reach error of the relay due to the frequency components (sub-harmonics and inter-harmonics) can be observed in Fig. 9 b), the reach error percentage of the distance relay is 15.05% and its operation time is 3.1 cycles at this fault condition due to the frequency components of the input signals in Fig. 7.

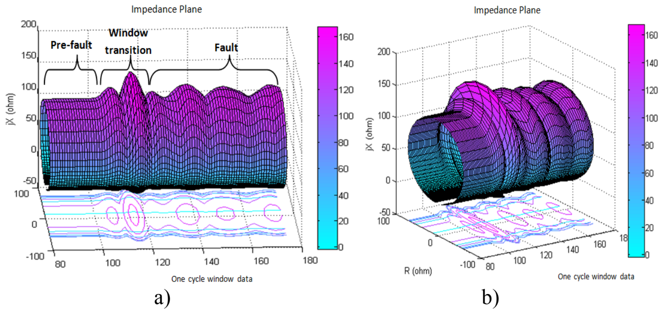

Fig. 9. Operation characteristic of a Mho distance relay when nonfiltered frequency components exists in electric input signals. a) Side view indicating transition stages. b) Fault period.

The characteristics obtained for distorted signals allows to evaluate the reach error in the distance relay. The fault impedance trajectory in the impedance plane allows to evaluate the operation time of the relay. The error measurement due to asynchronous frequency components affects the reach and the operation time of the distance relay.

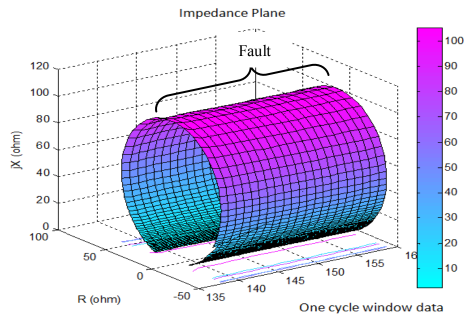

Figure. 10. Compensated Mho operation characteristic fault period (Simulated signals).

The reach error during the fault period in Fig. 9 is 15.05% and the reach error in Fig. 10 using the proposed algorithm is 0.86% only. The operation time of the proposed algorithm is 1.75 cycles and the Cosine filter 3.1 cycles, so a reduction in the operation time it's achieved. The results confirm that the use of the proposed distance relay algorithm a reach error reduction in the relay is obtained and in consequence a reduction in the operation time.

VI. SUMMARY OF RESULTS

The time of fault detection of digital distance relays are approximately 2-4 cycles, so the error in the operation time is evaluated during the first cycle during the fault period, this means that the results of the error in the operation time are significantly lower (see Table II), also the error compensation in the apparent impedance estimation is done during the fault period (see Table III), and the malfunction of the distance relay can be prevented.

TABLE II. FAULT DETECTION OPERATION TIME OF DISTANCE RELAY ALGORITHMS

Estimation method	Operation time (cycles)	% Error in operation time
Cosine filter	3.1	210
Proposed algorithm (Simulated signals)	1.75	75

.

TABLE III. REACH ERROR OF DISTANCE RELAY ALGORITHMS

Event	% Reach error (Cosine filter)	% Compensated reach error (Proposed distance relay algorithm)
Simulated signals	15.05	0.86

VII. CONCLUSIONS

During a fault condition in a transmission line where the electric input signals (voltage and current) are contaminated with frequency components as interharmonics and/or subharmonics, an error in the apparent impedance estimation due to the digital filtering (Cosine or Fourier filter) technique will compromise the performance of the distance relay.

It has been shown that the proposed distance relay algorithm causes a reach error reduction in the relay and in consequence an operation time reduction (see Table II and III).

The percentage error results shown in Table III show significant improvement- the error compensation in apparent impedance measurement during the fault period and preventing a misoperation of the distance relay. The proposed distance relay algorithm reduces consequently the reach error,. The proposed algorithm of correction of errors caused by unfiltered frequency components opens the possibility for designing new families of digital filters for power protection systems.

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