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PERFORMANCE OF PARAMETRIC SPECTRUM ESTIMATION METHODS

INTRODUCTION

The quality of voltage waveforms is nowadays an issue of the utmost importance for power utilities, electric energy consumers and also for the manufactures of electric and electronic equipment. The proliferation of nonlinear loads connected to power systems has triggered a growing concern with power quality issues. The inherent operation characteristics of these loads deteriorate the quality of the delivered energy, and increase the energy losses as well as decrease the reliability of a power system.

Methods of power quality assessment in power systems are almost exclusively based on Fourier Transform.

Parametric spectral methods, such as ESPRIT or MUSIC do not suffer from inherent limitations of resolution or dependence of estimation error on the window length (phase dependence of the estimation error) of FFT.

The author argues that the use of high-resolution spectrum estimation methods instead of Fourier-based techniques can improve the accuracy of measurement of spectral parameters of distorted waveforms encountered in power systems, in particular the estimation of the power quality indices.

PARAMETRIC METHODS

The ESPRIT and the root-Music spectrum estimation methods are based on the linear algebraic concepts of subspaces and so have been called "subspace methods"; the model of the signal in this case is a sum of sinusoids in the background of noise of a known covariance function.

MUSIC

The MUSIC method assumes the model of the signal as:

$$\mathbf{x}=\sum_{i=1}^{p}A_i\mathbf{s}_i+\eta\;;\quad A_i=|A_i|e^{j\phi_i}\quad \mathbf{s}_i=\left[1\;\; e^{j\omega_i}\;\;\cdots\;\; e^{j(N-1)\omega_i}\right]^T$$

The autocorrelation matrix of the signal is estimated from signal samples as:

$$\mathbf{R}_x=\sum_{i=1}^{p}\mathrm{E}\left\{A_iA_i^*\right\}\mathbf{s}_i\mathbf{s}_i^T+\sigma_0^2\mathbf{I}$$

N-p smallest eigenvalues of the correlation matrix (matrix dimension *N>p+1*) correspond to the noise subspace and *p* largest (all greater than the noise variance) correspond to the signal subspace.

The matrix of noise eigenvectors of the above matrix is used $\mathbf{E}_{noise}=\left[\mathbf{e}_{p+1}\;\;\mathbf{e}_{p+2}\;\;\cdots\;\;\mathbf{e}_N\right]$ to compute the projection matrix for the noise subspace: $\mathbf{P}_{noise}=\mathbf{E}_{noise}\mathbf{E}_{noise}^{*T}$

$$\mathbf{w}^{*T}\mathbf{P}_{noise}\mathbf{w}=\mathbf{w}^{*T}\mathbf{E}_{noise}\mathbf{E}_{noise}^{*T}\mathbf{w}=\sum_{i=p+1}^{N}E_i\left(e^{j\omega}\right)E_i^*\left(e^{j\omega}\right)\xrightarrow{z}\sum_{i=p+1}^{N}E_i(z)E_i^*\left(1/z^*\right)$$

The polynomial has *p* double *roots* lying on the unit circle which angular positions correspond to the frequencies of the signal components.

Powers of each component can be estimated from the eigenvalues and eigenvectors of the correlation matrix, using the relations: $\mathbf{e}_i^{*T}\mathbf{R}_x\mathbf{e}_i=\lambda_i$ $\quad\mathbf{R}_x=\sum_{i=1}^{p}P_i s_i s_i^{*T}+\sigma_0^2\mathbf{I}$

and solving for *Pi* – components' powers.

ESPRIT

The original ESPRIT algorithm is based on naturally existing shift invariance between the discrete time series, which leads to rotational invariance between the corresponding signal subspaces.

Eigenvectors **E** of the autocorrelation matrix of the signal define two subspaces (signal and noise subspaces) by using two selector matrices: $\mathbf{S}_1=\mathbf{\Gamma}_1\mathbf{E}\quad \mathbf{S}_2=\mathbf{\Gamma}_2\mathbf{E}\quad \mathbf{S}_1=\mathbf{\Phi}\mathbf{S}_2$

$$\mathbf{\Phi}=\begin{bmatrix}e^{j\omega_1} & 0 & \cdots & 0\\ 0 & e^{j\omega_2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & e^{j\omega_p}\end{bmatrix}$$

TLS (total least-squares) approach assumes that both estimated matrices can contain errors and finds the matrix as minimization of the Frobenius norm of the error matrix.

ACCURACY

MUSIC uses the *noise subspace* to estimate the signal components while ESPRIT uses the *signal subspace*.

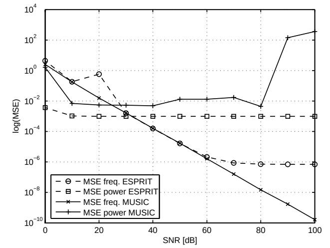
Numerous publications were dedicated to the analysis of the performance of the aforementioned methods. Unfortunately, due to many assumed simplifications, and the complexity of the problem, published results are often contradictory and sometimes misleading.

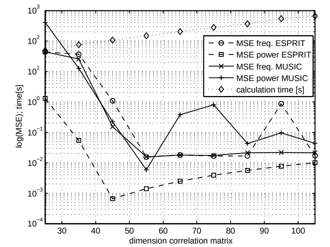
Several experiments with simulated, stochastic signals were performed, in order to compare performance aspects of both parametric methods MUSIC and ESPRIT. Testing signal is designed to belong to a class of waveforms often present in power systems. Each run of spectrum and power estimation is repeated many times (Monte Carlo approach) and the mean--square error (MSE) is computed.

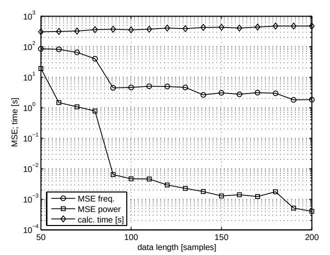
Parameters of test signals:

- one 50 Hz main harmonic with unit amplitude.
- random number of higher odd harmonic components with random amplitude (lower than 0.5) and random initial phase (from 0 to 8 higher harmonics).
- sampling frequency 5000 Hz.
- each signal generation repeated 1000 times with reinitialization of random number generator.
- SNR=40 dB.
- size of the correlation matrix = 50.
- signal length 200 samples.

RESULTS of accuracy comparison







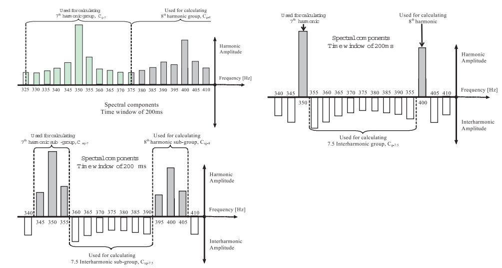
1. MUSIC performs better for SNR higher than 60 dB and lower than 20 dB. The error of power estimation is significantly lower for ESPRIT algorithm in the whole SNR range.

2. There exists an optimal size of the correlation matrix which assures the lowest possible estimation error (tradeoff between accuracy of estimation of the correlation matrix and increase of numerical errors with the size of the correlation matrix).

NEW POWER QUALITY INDICES

Several indices are in common use for the characterization of waveform distortions. However, they generally refer to periodic signals which allow an „exact" definition of harmonic components and deliver only one numerical value to characterize them.

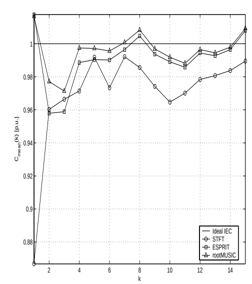
When the spectral components are time-varying in amplitude and/or in frequency (as in case of non-stationary signals), a wrong use of the term harmonic can arise and several numerical values are needed to characterize the time-varying nature of each spectral component of the signal.

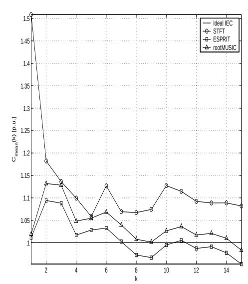


Experimental setup and results

The waveforms obtained from a power supply of a typical for dc arc furnace plant are analyzed. The IEC groups and subgroups are estimated by using DFT and the results are compared to those obtained with subspace methods: the ESPRIT and the root-MUSIC.

In order to compare the different processing techniques, a *reference technique* is adopted: "Ideal IEC", where the respective harmonic groupings are computed on the whole interval of 3s.





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