

# Numerical calculation of varistor model for sinusoidal signal

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**Abstract-** ZnO varistors are semiconductor devices with highly nonlinear current-voltage characteristic and are widely used as devices for overvoltage protection. Varistor applications range from the use of small varistors to protect electronic components to large varistors for protection of power systems. This paper presents proposed model of ZnO varistor and methodology of its mathematical analysis and simulation. The mathematical analysis of the proposed model makes it possible to simulate the current trace on a nonlinear element.

## I. INTRODUCTION

Varistors are ceramic elements whose current-voltage characteristic is highly nonlinear. The varistors are usually manufactured in the ceramic process in which pressed zinc oxide with admixtures of other metallic oxides is sintered. A matrix made up of ZnO grains enclosed by an intermediate (intergranular) layer composed of dissolved oxide admixtures forms the obtained microstructure [1].

The varistor effect is produced by phenomena, which occur at grain boundaries in the varistor's microstructure. The Schottky potential barrier is generally regarded as playing a key role in current conduction in this system [2].

Numerous, more or less complicated ZnO varistor models can be found in the literature [3-5]. The simplest model consists of a capacitance and a nonlinear resistance connected in parallel.

The simple ZnO varistor model proposed here taking under account that the additional impedance due to the finite conductance and capacitance of the intergranular phase has a significant effect on the model response. Thus that model can be presented as [6]:

- the nonlinear resistance of intergranular boundaries and the linear capacitance associated with the impoverished region, and
- the capacitance-resistance impedance associated with the intergranular phase.

## II. MATHEMATICAL ANALYSIS OF THE MODEL

Assuming that the waveform of voltage is described by function:

$$u(t) = \sqrt{2} \sin \omega t = \text{Im} \left\{ \sqrt{2} \sum_{k=1,3,5,\dots} \underline{U}_{(k\omega)} e^{jk\omega t} \right\} \quad (1)$$

and  $\underline{U}_{(k\omega)} = 0$  dla  $k > 1$  oraz  $\underline{U}_{(\omega)} = U$ ,

the waveform of current is described by:

$$i(t) = \text{Im} \left\{ \sqrt{2} \sum_{k=1,3,5,\dots} \underline{I}_{(k\omega)} e^{jk\omega t} \right\} \quad (2)$$

Mathematical analysis the equivalent circuit are based on the superposition method.

Analyzing the system for the k-th harmonic, the equations for the k-th harmonic assume form:

$$\underline{U}_{(k\omega)} = \underline{Z}_{A(k\omega)} \underline{I}_{(k\omega)} + \underline{U}_{N(k\omega)} \quad (3)$$

$$\underline{I}_{(k\omega)} = \underline{Y}_{B(k\omega)} \underline{U}_{N(k\omega)} + \underline{I}_{N(k\omega)} \quad (4)$$

The varistor model described by analytical model has been implemented in the Matlab environment. The diagram of model implementation is shown in Fig. 1. The results of simulation are shown in Figures 2-3. These trajectories are compared to trajectories of real varistor, shown in Figures 4-5. Comparing figures 2 and 3 with figures 4 and 5 it was found that there is large agreement (compatibility) between current  $i(t)$  waveform received from simulation and received from real object. This calculation could describe the influence of odd harmonic content in voltage waveform on the current waveform as described in [7].

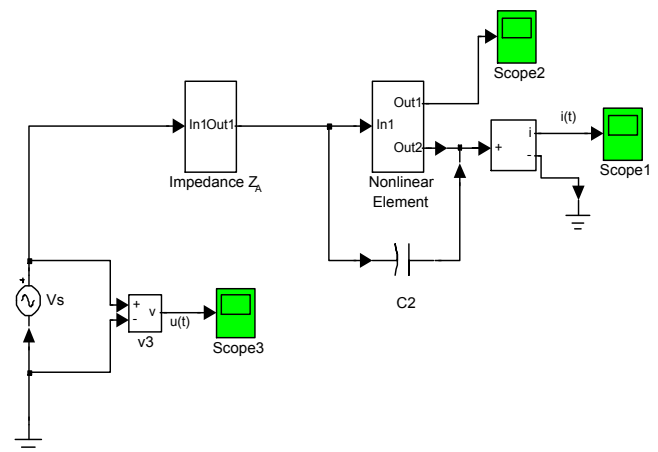


Figure 1. Analytical model implemented in the Matlab programme

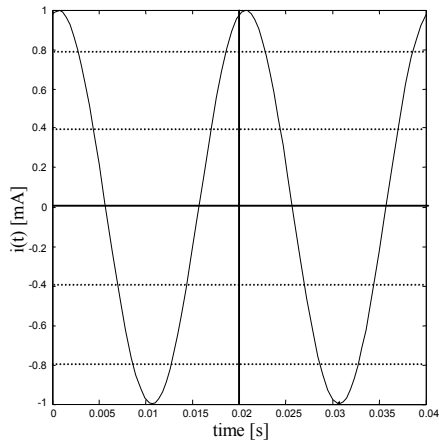


Figure 2. Calculated curve  $i(t)$  for analytical model,  $U \ll U_{ref}$ .

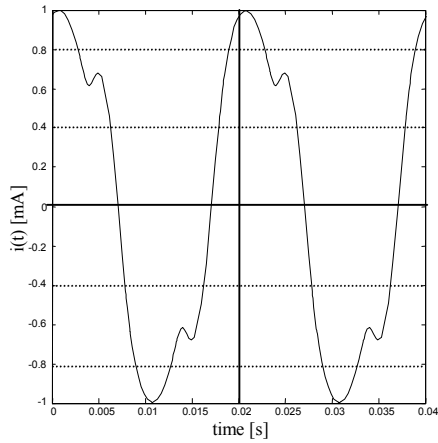


Figure 3. Calculated curve  $i(t)$  for analytical model,  $U > U_{ref}$ .

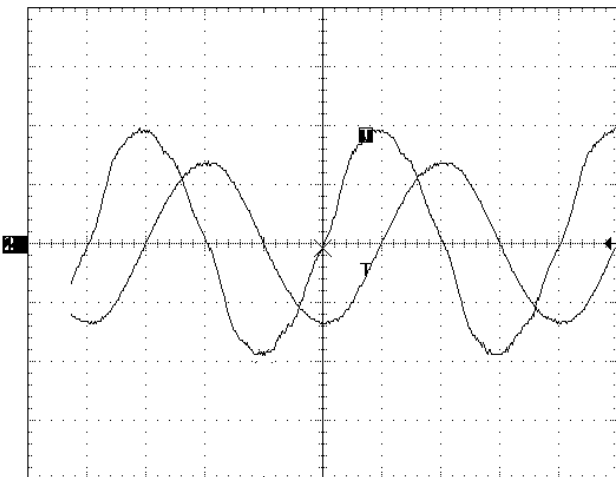


Figure 4. Trajectories of current  $i(t)$  and applied voltage  $u(t)$  of real ZnO varistor for  $U \ll U_{ref}$ .

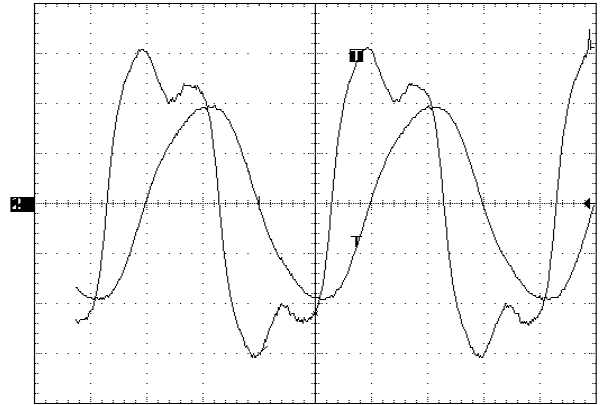


Figure 5. Trajectories of current  $i(t)$  and applied voltage  $u(t)$  of real ZnO varistor for  $U > U_{ref}$ .

### III. CONCLUSIONS

The numerical calculation of proposed analytical model are strong convergence with results from real object. It shows, that method of solution of model is proper. Moreover the solution of the analytical model based on the superposition method makes it possible to determine the current waveform on a nonlinear element.

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